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Theory of a Passive, Reversible, Distributed-Coupling Transducer

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Equations are derived to describe an underwater sound transducer consisting of an acoustical transmission line coupled to a low-pass electrical transmission line. If the two transmission lines have the same phase velocity, the energy of a wave is transferred cyclically from one line to the other as a function of line length; a reversible transducer with a low Q, high efficiency, and essentially constant resistive impedance is thereby achieved. Theory and four proposed designs are described.

INTRODUCTION

The distributed-coupling transducer consists of an electrical transmission line coupled through piezoelectric elements to an acoustic transmission line. The two lines are made to have equal phase delay per unit of line length. Energy is gradually transferred from one line to the other as the wave progresses down the line.

Others have used this principle for obtaining broadband response in a loudspeaker,[1] for obtaining maximum torque with speed of response in a servo system,[2] and to produce a broad-band underwater sound projector.[3] Unlike the previous papers, this paper considers a transducer in which the transfer of electrical energy to acoustic energy is reversible and there is an optimum length of the two lines for which the energy put into one line will be transferred completely, except for line losses, into energy output from the other line.

An underwater sound transducer based on this theory will consist of an electrical transmission line coupled at equally spaced points to one or more acoustic transmission lines. The transducer is analogous to a directional coupler through which power in one waveguide is transferred to another. A directional array of these acoustic transmission lines, radiating from either end, will have a resistive radiation impedance that can be matched to the resistive image impedance of the lines by means of an area transformation. Whereas a resonant transducer builds up to a maximum power conversion

[1] R. V. L. Hartley, U. S. Patent No. 1,629,100 (May 17, 1927).
[2] J. Rabinow and M. Apstein, Electronics 27, 160 (July 1954).
[3] M. Greenspan and R. W. Wilmotte, J. Acoust. Soc. Am. 30, 528 (1958).

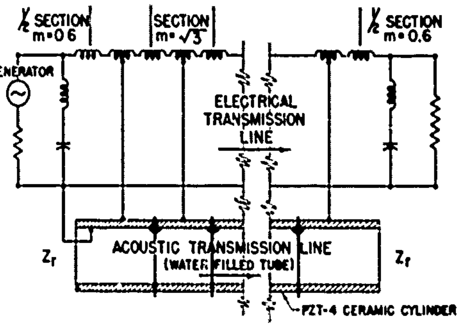

FIG. 1. Schematic diagram of end sections of distributed-coupling transducer.

over a period of time (to 92% in Q cycles), this transducer builds up to a maximum power conversion over an optimum length of coupling between the electrical and the acoustic transmission lines. This arrangement produces an electroacoustic transducer that is passive and reversible, that has broad frequency band response, $Q=1$, and a resistive electrical impedance. The new piezoelectric ceramics in the form of cylindrical shells make it possible to produce an underwater sound transducer of this type that is efficient and of reasonable length for the energy transfer.

THEORY

The transducer will consist of an acoustic transmission line which may be a solid piezoelectric line or a composite line consisting of piezoelectric elements along the axis of a liquid-filled tube. The electrical line will consist of the capacitance of the piezoelectric material and series-distributed inductances. The combination is shown schematically in Fig. 1.

Assume for the moment that the coupling between the two lines is continuous. The generalized piezoelectric equations, without subscripts, are

$$\partial\xi/\partial x=s^ET_x+dE_x,\quad \partial Q/\partial x=dT_x+\epsilon^TE_x, \tag{1}$$

where ξ is the elastic displacement, $\partial\xi/\partial x$ is the strain, T_x is the elastic stress, s^E is the elastic compliance for constant electric field, d is the piezoelectric constant relating the strain to the applied electric field, E_x is the applied electric field, and ϵ^T is the dielectric constant for constant stress. The charge Q and the current I are referred to the electric transmission line. The magnitudes of Q and I are dependent on area of cross section and length of the acoustic transmission line and on the piezoelectric coefficient used in coupling the two lines. For these reasons the charge and current are referred to a unit acoustic line length and unit cross section and corrected for acoustic line dimensions for each specific design.

The time derivative of these equations relates the linear velocity v and the current I in the two transmission lines to the stress and electric field:

$$\begin{aligned}\partial v/\partial x&=s^E(\partial T/\partial t)+d(\partial E/\partial t),\\ \partial I/\partial x&=d(\partial T/\partial t)+\epsilon^T(\partial E/\partial t).\end{aligned} \tag{2}$$

If in addition to the piezoelectric material of density ρ, which constitutes an acoustic transmission line and an electrical capacitance, a distributed inductance L per unit of line length is added to form the electrical transmission line with a propagation constant equal to that of the acoustic transmission line, two additional equations are required for deriving the wave equation:

$$\partial T/\partial x=\rho(\partial v/\partial t),\quad \partial E/\partial x=L(\partial I/\partial t). \tag{3}$$

Loss can be represented in the two lines by acoustic and electric shunt conductance G_a and G_e and acoustic and electric series resistance R_a and R_e per unit of acoustic line length and unit cross section. Equations (2) and (3) are then written

$$\begin{aligned}\partial v/\partial x&=G_aT+s^E(\partial T/\partial t)+d(\partial E/\partial t),\\ \partial I/\partial x&=d(\partial T/\partial t)+G_eE+\epsilon^T(\partial E/\partial t),\end{aligned} \tag{4}$$

$$\begin{aligned}\partial T/\partial x&=R_av+\rho(\partial v/\partial t),\\ \partial E/\partial x&=R_eI+L(\partial I/\partial t).\end{aligned} \tag{5}$$

Since T and E are functions of $e^{j\omega t}$, Eqs. (4) and (5) can be combined as

$$\begin{aligned}\partial^2T/\partial x^2&=\gamma_1^2T-(\omega^2\rho-j\omega R_a)dE,\\ \partial^2E/\partial x^2&=-(\omega^2L-j\omega R_e)dT+\gamma_2^2E,\end{aligned} \tag{6}$$

$$\begin{aligned}\gamma_1^2&=R_aG_a-\omega^2\rho s^E+j\omega(R_as^E+\rho G_a),\\ \gamma_2^2&=R_eG_e-\omega^2L\epsilon^T+j\omega(R_e\epsilon^T+LG_e).\end{aligned} \tag{7}$$

It must be assumed that the two transmission lines are low-loss lines, and that the energy transfer through the piezoelectric constant per unit of line length is greater than the line loss. Thus, $\omega\rho>R_a$ and $\omega L>R_e$, and Eqs. (6) can be written

$$\begin{aligned}\partial^2T/\partial x^2&=\gamma_1^2T-\omega^2\rho dE,\\ \partial^2E/\partial x^2&=-\omega^2LdT+\gamma_2^2E,\end{aligned} \tag{8}$$

where

$$\gamma_1=\alpha_1+j\beta_1,\quad \gamma_2=\alpha_2+j\beta_2; \tag{9}$$

α_1 and α_2 are the attenuation constants, and β_1 and β_2 are the phase constants. For equal phase constants, $\beta_1=\beta_2=\beta_c=\omega/c$, where ω is the angular frequency and c is the speed of wave propagation,

$$c^2=1/\rho s^E=1/L\epsilon^T, \tag{10}$$

and in Eqs. (8),

$$\begin{aligned}\omega^2\rho d&=\omega^2d/c^2s^E=\beta_c^2k_{12},\\ \omega^2Ld&=\omega^2d/c^2\epsilon^T=\beta_c^2k_{21},\\ k_{12}k_{21}&=d^2s^E\epsilon^T=k^2,\end{aligned} \tag{11}$$

where k is the electromechanical coupling coefficient. Equations (8) can be rewritten as

$$\begin{aligned}\partial^2T/\partial x^2&=\gamma_1^2T+(j\beta_c)^2k_{12}E,\\ \partial^2E/\partial x^2&=(j\beta_c)^2k_{21}T+\gamma_2^2E,\end{aligned} \tag{12}$$

and the general solution of Eqs. (12) is

$$\begin{aligned}\gamma^2T&=\gamma_1^2T+(j\beta_c)^2k_{12}E,\\ \gamma^2E&=(j\beta_c)^2k_{21}T+\gamma_2^2E,\end{aligned}\tag{13}$$

$$(\gamma^2-\gamma_1^2)(\gamma^2-\gamma_2^2)=\beta_c^4k^2.\tag{14}$$

Four waves are propagated, two in each direction. The coupling per wavelength between the lines is small. It can be shown, as in a directional waveguide coupler, that only the waves in one direction are significant. In the direction of increasing x, the propagation constants γ_a and γ_b for low-loss lines are

$$\begin{aligned}\gamma_a&=\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c(1+k)^{\frac{1}{2}},\\ \gamma_b&=\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c(1-k)^{\frac{1}{2}},\end{aligned}\tag{15}$$

and the general solution, Eqs. (13), becomes

$$\begin{aligned}T&=T_a\exp(-\gamma_ax)+T_b\exp(-\gamma_bx),\\ E&=E_a\exp(-\gamma_ax)+E_b\exp(-\gamma_bx).\end{aligned}\tag{16}$$

To obtain a particular solution, the transducer can be assumed to be transmitting with power that is initially all electrical at one end or receiving with power that is initially all acoustic at one end. When the transducer is receiving sound at $x=0$ and the electrical signal is received at the other end, the boundary conditions at $x=0$ can be given as $T=1$ and $E=0$ at $x=0$. Then

$$T_b=1-T_a,\quad E_b=-E_a.\tag{17}$$

The particular solution of Eqs. (8) using these general solutions and boundary conditions is

$$\begin{aligned}T_a&=(\gamma_b^2-\gamma_1^2)/(\gamma_b^2-\gamma_a^2),\\ T_a&=\frac{\frac{1}{4}(\alpha_1+\alpha_2)^2-\alpha_1^2+\beta_c^2k+j\beta_c[\alpha_2-\alpha_1-\frac{1}{2}k(\alpha_1+\alpha_2)]}{2\beta_c^2k-j\beta_ck(\alpha_1+\alpha_2)}.\end{aligned}\tag{18}$$

For lossless lines, $\alpha_1=\alpha_2=0$, or when $\alpha_1=\alpha_2$, $T_a=\frac{1}{2}$ and $T_b=\frac{1}{2}$. Then

$$E_a=(\gamma_b^2-\gamma_1^2)/2\beta_c^2k_{12},\tag{19}$$

and from Eqs. (15),

$$2E_a=\frac{k}{k_{12}}+\frac{(\alpha_1+\alpha_2)^2}{4(1-k)\beta_c^2k_{12}}-\frac{\alpha_1^2}{\beta_c^2k_{12}}+j\frac{\alpha_2-\alpha_1}{\beta_ck_{12}}.\tag{20}$$

As long as α_1 and α_2 are negligible compared to β_c,

$$E_a=\tfrac{1}{2}(k/k_{12})=\tfrac{1}{2}(k_{21}/k_{12})^{\frac{1}{2}}=\tfrac{1}{2}(s^E/\epsilon^T)^{\frac{1}{2}}.\tag{21}$$

Substituting these constants into the general solution, Eqs. (16), produces

$$\begin{aligned}T&=\{\exp-[\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c]x\}\\ &\qquad\times[\tfrac{1}{2}\exp(j\tfrac{1}{2}\beta_ckx)+\tfrac{1}{2}\exp(-j\tfrac{1}{2}\beta_ckx)],\\ E&=-(s^E/\epsilon^T)^{\frac{1}{2}}\{\exp-[\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c]x\}\\ &\qquad\times[\tfrac{1}{2}\exp(j\tfrac{1}{2}\beta_ckx)-\tfrac{1}{2}\exp(-j\tfrac{1}{2}\beta_ckx)],\end{aligned}\tag{22}$$

or

$$\begin{aligned}T&=\{\exp-[\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c]x\}\cos\tfrac{1}{2}\beta_ckx,\\ E&=-j(s^E/\epsilon^T)^{\frac{1}{2}}\{\exp-[\tfrac{1}{2}(\alpha_1+\alpha_2)+j\beta_c]x\}\\ &\qquad\times\sin\tfrac{1}{2}\beta_ckx.\end{aligned}\tag{23}$$

If T is expressed as the pressure p_d at the acoustical input end of the transducer for the condition $x=\pi/\beta_ck$, then

$$|E|=p_d(s^E/\epsilon^T)^{\frac{1}{2}}\exp[-\tfrac{1}{2}\pi(\alpha_1+\alpha_2)/\beta_ck].\tag{24}$$

For no loss in transmission, Eq. (24) rearranged states that the potential energy input is equal to the potential energy output. The substitutions

$$E=IZ_{oe},\quad p_d=vZ_{oa},\tag{25}$$

where Z_{oe} and Z_{oa} are the characteristic impedances of the electrical and acoustic lines, produce

$$|I|=(s^E/\epsilon^T)^{\frac{1}{2}}(Z_{oa}/Z_{oe})v\exp[-\tfrac{1}{2}\pi(\alpha_1+\alpha_2)/\beta_ck].\tag{26}$$

If the substitutions

$$Z_{oa}=(\rho/s^E)^{\frac{1}{2}},\quad Z_{oe}=(L/\epsilon^T)^{\frac{1}{2}},\tag{27}$$

are made, then Eq. (26) becomes

$$|I|=(\rho/L)^{\frac{1}{2}}v\exp[-\tfrac{1}{2}\pi(\alpha_1+\alpha_2)/\beta_ck],\tag{28}$$

or, for no loss in transmission, the kinetic energy is conserved.

The pressure at the input to the transducer can be expressed in terms of the free-field pressure:

$$p_d/Z_{oa}=Dp_{ff}/(Z_r+Z_{oa}),\tag{29}$$

where p_{ff} is the free-field sound pressure, D is the diffraction constant defined as the ratio of the blocked diaphragm sound pressure to the free-field pressure, and Z_r is the unit area radiation impedance.

For an acoustic transmission line of cross-section area A, the acoustic power input to the line is

$$Ap_d^2/Z_{oa}=AD^2p_{ff}^2Z_{oa}/(Z_r+Z_{oa})^2\ \mathrm{w}.\tag{30}$$

In terms of open-circuit voltage E_0, the electrical power output is

$$E_0^2/4Z_{oe}.\tag{31}$$

The correction to Z_{oe} for an acoustic transmission line of area A will be deferred for the moment, because it depends on the means of connection.

If the input energy is equal to the output for lossless transmission and optimum line length, the free-field receiving voltage sensitivity is

$$M_0=\frac{E_0}{p_{ff}}=\frac{(4D^2AZ_{oe}Z_{oa})^{\frac{1}{2}}}{Z_r+Z_{oa}}\frac{\mathrm{v}}{\mathrm{newton/m^2}},\tag{32}$$

and from Eq. (23) for a line length l,

$$|M_0|=\frac{2D(AZ_{oe}Z_{oa})^{\frac{1}{2}}}{Z_r+Z_{oa}}[\exp-\tfrac{1}{2}(\alpha_1+\alpha_2)l]\sin\tfrac{1}{2}\beta_ckl.\tag{33}$$

The electrical line will consist of n sections coupled to a distributed acoustic line at n points or to an acoustic line consisting of n sections. If β_c is the phase shift per section and f_0 is the cutoff frequency for the resulting low-pass line, then

$$\tfrac{1}{2}\beta_c kl=\tfrac{1}{2}(2\pi/\lambda)k(n\lambda_0/2)=\pi kfn/2f_0, \qquad (34)$$

and Eq. (33) can be written as

$$|M_0|=\frac{D(AZ_{oe}Z_{oa})^{\frac{1}{2}}}{5(Z_r+Z_{oa})}\left[\exp-(\alpha_1+\alpha_2)\frac{n\lambda_0}{4}\right]\times\sin\frac{\pi kfn}{2f_0}\,\frac{\mathrm{v}}{\mu\mathrm{bar}}. \qquad (35)$$

The transducer is reciprocal, so the transmitting current response is

$$S_0=\tfrac{1}{2}M_0\rho_0 f(10)^2, \qquad (36)$$

$$|S_0|=\frac{10D\rho_0 f(AZ_{oe}Z_{oa})^{\frac{1}{2}}}{Z_r+Z_{oa}}\times\left[\exp-(\alpha_1+\alpha_2)\frac{n\lambda_0}{4}\right]\sin\frac{\pi kfn}{2f_0} \qquad (37)$$

μbar/amp at 1 m, where f is the operating frequency, f_0 is the cutoff frequency for one section, and ρ_0 is the density of the sound-propagating medium.

The efficiency of the transducer is

$$\eta=(4\pi S_0^2/\rho_0 c_0 R_\theta R)10^{-2}, \qquad (38)$$

where $\rho_0 c_0$ is the wave impedance in the sound-propagating medium, R_θ is the directivity ratio, and $R=Z_{or}$ is the electrical line resistance.

For a piston source of radius a and for $a/\lambda>1$,

$$R_\theta\rightarrow 4\pi Af^2/c_0^2. \qquad (39)$$

From Eqs. (37) and (39), Eq. (38) becomes

$$\eta=\frac{D^2Z_{oa}\rho_0 c_0}{(Z_r+Z_{oa})^2}\left[\exp-(\alpha_1+\alpha_2)\frac{n\lambda_0}{4}\right]\sin^2\frac{\pi kfn}{2f_0}.$$

If the acoustic transmission line is matched to the wave impedance of the medium and the transducer is directional, the efficiency reduces to the exponential term for the frequency f_m such that $f_m=f_0/kn$. For a lossless line, $\eta\approx\sin^2 f$, and the half-power points are at frequencies f_1 and f_2 such that $\pi kf_m n/2f_0=\frac{1}{2}\pi$, $\pi kf_2 n/2f_0=\frac{3}{4}\pi$, and $\pi kf_1 n/2f_0=\frac{1}{4}\pi$.

The effective Q of a directional, resistively loaded transducer is

$$Q=f_m/(f_2-f_1)=(\tfrac{1}{2}\pi)/(\tfrac{3}{4}\pi-\tfrac{1}{4}\pi)=1. \qquad (40)$$

Thus, for a transducer of this design with low-loss electric and acoustic transmission lines, the response versus frequency characteristic will have a Q equal to 1.

TRANSDUCER DESIGNS

In this section, four designs are proposed. The fourth design was tried first, and was partially successful.

First Design

If a transducer in which the coupling between the two transmission lines would be continuous could be constructed, there would not be an upper frequency cutoff, and the transducer could be used to generate or receive extremely high-frequency sound. Such a transducer might consist of a semiconductor such as p-type silicon as one electrode of a quartz crystal. By adjustment of the voltage gradient in the plane of the electrode the propagation velocity of an electrical signal diffusing down the plane could be adjusted to the propagation velocity of the acoustic wave in the crystal.

Note added in proof. The first design proposed in this paper is a sandwich of a piezoelectric crystal and a semiconducting material. This design produces a distributed-coupling transducer in which the electric field in the semiconductor is capacitively coupled transversely to the direction of wave propagation. Hutson and White [J. Appl. Phys. **33**, 40 (1962)] have recently described a similar transducer consisting of a piezoelectric semiconductor in which the electric field is coupled in the direction of wave propagation.

Second Design

An underwater sound transducer based on the theory presented in the preceding section and having n points of coupling, Eqs. (35) and (36), can be matched to the wave impedance of water by an area transformation. A radially polarized cylindrical tube, air-filled, and capped with metal or ceramic diaphragms, can be used as the acoustic transmission line or as an element of an array of many of these acoustic transmission lines. The acoustic lines can be coupled to one electrical transmission line or to several for beam steering. For example, lead zirconate ceramic will have the characteristic impedance

$$Z_{oa}=(\rho/s^E{}_{11})^{\frac{1}{2}}=[7.5(8.15)10^{13}]^{\frac{1}{2}}=2.48(10)^7 \text{ mks units,}$$

where $\rho_0 c_0=1.5(10)^6$ mks units.

Thus an area transformation is required such that the ratio of the area A_d exposed to the sound field to the cross-section area A of the ceramic cylinder is $A_d/A=24.8/1.5=16.5$.

The outer electrode of the ceramic cylinder can be segmented and coupled to a low-pass electrical transmission line consisting of the electrical capacitance of a segment of the ceramic tube and an inductance. An m-derived low-pass line can be constructed with approximately linear phase shift versus frequency characteristics. This should be terminated by a half section with $m=0.6$ to obtain a constant characteristic impedance.

For an m-derived network, $\beta=2\sin^{-1}\{m/[-(1-m^2)+(\omega_0/\omega)^2]^{\frac{1}{2}}\}$. For a linear phase shift, $\beta=2Ky$, where $y=\omega/\omega_0$, and ω_0 is the cutoff angular frequency of the low-pass line.

$$\tfrac{1}{2}\beta=\sin^{-1}\{m/[(m^2-1+1/y^2)]^{\frac{1}{2}}\}=Ky,$$
$$(\sin^2 Ky)(m^2-1+1/y^2)=m^2,$$
$$m^2(\sin^2 Ky-1)=-\sin^2 Ky[(1/y^2)-1],$$
$$\tan Ky=my/(1-y^2)^{\frac{1}{2}}.$$

At cutoff, $y=1$, $\beta=\pi$, or $K=\pi/2$. At $y=\frac{1}{2}$, $\tan\pi/4=1=\frac{1}{2}m/(1-\frac{1}{2}^2)^{\frac{1}{2}}$, $m=\sqrt{3}$. Thus, for $m=\sqrt{3}$, the phase shift is nearly linear with frequency and is correct at the cutoff and $y=\frac{1}{2}$. A plot of phase shift versus frequency for $m=\sqrt{3}$ shows that the maximum error can be reduced if the cutoff for the acoustic line is 6% above the cutoff for the electrical transmission line.

Third Design

A second choice for the acoustic transmission line with n coupling points can be a liquid-filled stiff-walled metal tube as shown in Fig. 2. Short sections of ceramic cylinders with stiff end caps can be spaced along the axis of the tube as the coupling elements. Such a line, consisting of a ρ_0c_0 liquid having coupling elements of volume compliance and mass equal to that of the displaced liquid, can be matched to the wave impedance in a directional array of these liquid-filled tubes.

The change in volume of a thin-walled tube with stiff end caps is $\Delta V=(Vpd/4Yh)(5-4\nu)$, where V is the volume of the cylinder, Y is Young's modulus, h is the cylinder wall thickness, and ν is Poisson's ratio. The acoustic compliance is then $C_t=(Vd/4Yh)(5-4\nu)$ m^5/newton. This should equal the acoustic compliance of the displaced liquid, $C_t=(Vd/4Y^E_{11}h)(5-4\nu)=V/\rho_0c_0^2$ m^5/newton. Thus, $h/d=(\rho_0c_0^2/4Y^E_{11})(5-4\nu)$, $\rho_0c_0^2=2.25(10^9)$. For radially polarized lead zirconate, $Y^E_{11}=8.15(10^{10})$ newton/m^2, $\nu=0.29$, $h/d=0.0265$. This is a somewhat smaller ratio than is normally produced. It is possible, however, to add stiff end caps and make the mass equal to the mass of the displaced liquid.

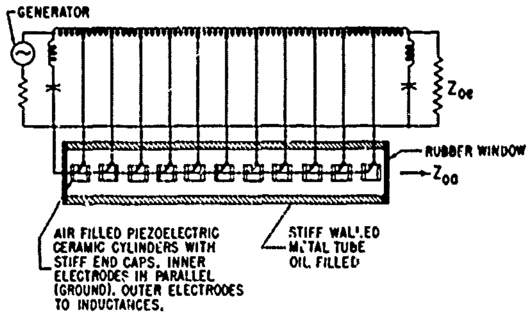

FIG. 2. Distributed-coupling transducer with air-filled ceramic cylinders coupling liquid-filled acoustic transmission line to the electric transmission line.

The lower value for Young's modulus in lead metaniobate would make the ratio $h/d=0.0636$. This ratio is normally produced, but the mass is too great to produce stiff-capped cylinders of mass equal to the displaced liquid.

Fourth Design

Because of the difficulties of manufacturing long ceramic tubes or very thin-walled short ceramic tubes, a fourth design was tried for the first model. This design is shown in Fig. 1. One-inch lengths of lead zirconate were cemented into a continuous tube using a compliant cement to decouple mechanically the elements from each other.

This tube was water filled so that the water in the tube constituted the acoustic line, and the phase delay per section of the electrical line was made equal to the phase delay in propagation of the wave in the water between centers of successive elements. The decoupling of ceramic elements was never sufficient and the single tube had a radiation impedance that was largely reactive, but the transducer did show that the theory is correct. Efficiency of 31% and Q slightly greater than 1 were achieved.

Manufacturers of piezoelectric ceramics have now shown that long cylinders can be produced. An array of seven tubes according to the first design, air-filled and capped, will soon be built.