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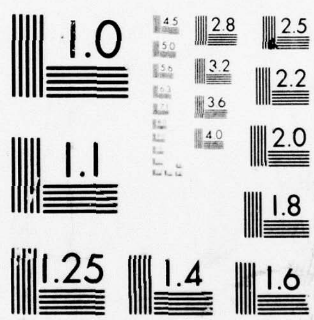

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AN UNSTABLE DYNAMICAL SYSTEM ASSOCIATED WITH MODEL REFERENCE ADAPTIVE CONTROL*

A. Feuer, B. R. Barmish, and A. S. Morse

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Department of Engineering and Applied Science

Yale University

New Haven, Ct. 06520, U.S.A.

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ABSTRACT

It is shown that a certain system of differential equations of importance to the proof of stability of the adaptive system proposed in [1], admit unbounded solutions. The implication of this result is that a much more elaborate argument than heretofore thought necessary is required to prove that the adaptive system of [1] is stable, if it is stable at all.

In studying the asymptotic behaviour of the adaptive control system proposed in [1], one encounters equations of the form

$$\dot{\eta} = - \eta + \phi(t)\delta \qquad (1)$$

$$\dot{\delta} = - \phi(t)\eta \qquad (2)$$

$$\dot{w}_1 = - w_1 + \phi^2(t)\eta \qquad (3)$$

where as in [1], η, δ, w_1 and ϕ are the augmented error, parameter error, auxiliary signal and sensitivity function respectively, of the adaptive system. These particular equations result if one assumes (for simplicity) that $D_m(p) = (p+1)D_w(p)$, $D_f(p) = p + 1$ and $N = 1$, where D_m, D_w, D_f and N are as defined in [1].

To prove that the adaptive system of [1] is stable, it is necessary to show that η, δ and w are bounded. Since the structure of the adaptive system makes it difficult to deduce very much about ϕ unless η, δ and w_1 are known apriori to be bounded, the approach in [1] and elsewhere has been to try to establish the boundedness of η, δ, w_1 without first assuming that ϕ is bounded. To get some idea of what's involved, observe that the time function

$$\alpha = \frac{1}{2} (\eta^2 + \delta^2) \qquad (4)$$

satisfies

$$\dot{\alpha} = - \eta^2 \qquad (5)$$

from which boundedness of η and δ directly follow. This and (2) imply that the output of any stable first-order linear system with input $\phi\eta$, is bounded. It is thus reasonable to expect that w_1, the output of a stable first-order linear system forced by $\eta\phi^2$, will be bounded as well. The following counterexample shows that this is not the case.

Proposition: If

$$\phi = \dot{\theta} + (\sin\theta)(\cos\theta) \tag{6}$$

where

$$\theta = e^{-t}\sin^2(e^{6t}) \tag{7}$$

then there exists an unbounded solution to (1)-(3).

Since the sensitivity function ϕ actually generated by the adaptive system of [1] is not completely arbitrary, the preceding is not quite a counterexample to the claim of stability of the adaptive system proposed in [1]. On the other hand, the example does imply that a much more elaborate argument involving the differential equations which generate ϕ is required to prove that the adaptive system is stable, if it is stable at all.

To prove the proposition, first observe from (1), (2) and (7), with $\eta(0) = \sin(\sin 1)$ and $\delta(0) = \cos(\sin 1)$, that $\eta = \zeta \sin\theta$ where $\delta = \zeta\cos\theta$ and

$$\zeta(t) = e^{-\int_0^t \sin^2\theta(\tau)d\tau} \tag{8}$$

Hence

$$\eta\phi^2 = \zeta\phi^2 \sin\theta \tag{9}$$

The definition of θ in (5) implies that

$$\sin\theta \geq \theta^2/2 \geq 0 \tag{10}$$

and that $\theta^2 \leq e^{-2t}$; from the last inequality, the trigonometric relation $\sin^2\theta \leq \theta^2$ and (8) it follows that $\zeta(t) \geq c_1 \equiv e^{-1/2}$. This (9) and (10) thus yield $\eta\phi^2 \geq c_1\phi^2 \sin\theta$. Using (4) to substitute for ϕ, we obtain

$$\eta\phi^2 \geq c_1 \sin\theta(\dot{\theta}^2 + 2\dot{\theta}(\sin\theta)(\cos\theta) + (\sin^2\theta)(\cos^2\theta)) \tag{11}$$

Now observe that from (5), $\dot{\theta} + \theta = 6e^{5t}\sin(2\gamma)$ where

$$\gamma = e^{6t} \tag{12}$$

Hence

$$\dot{\theta}^2 \sin\theta = (6e^{5t}\sin 2\gamma)^2 \sin\theta - (2\theta\dot{\theta} + \theta^2)(\sin\theta) \tag{13}$$

If we now define

$$\left.\begin{aligned} b_1 &= ((\sin^2\theta)(\cos^2\theta) - \theta^2)\sin\theta\, c_1 \\ b_2 &= (2/3\sin^3\theta + 2(\theta\cos\theta - \sin\theta))c_1 \end{aligned}\right\} \tag{14}$$

then using (11) and (12),

$$\eta\phi^2 \geq c_1 \sin\theta(6e^{5t}\sin 2\gamma)^2 + b_1 + \dot{b}_2 \tag{15}$$

From (10), and then (7) and (12)

$$\begin{aligned} \sin\theta(6e^{5t}\sin 2\gamma)^2 &\geq 18\,\theta^2 e^{10t}\sin^2 2\gamma \\ &= 18e^{8t}(\sin^4\gamma)(\sin^2 2\gamma) \\ &= 18e^{8t}(1-\cos^2\gamma)^2(\sin^2 2\gamma) \\ &\geq 18e^{8t}(1-2\cos^2\gamma)(\sin^2 2\gamma) \\ &= -18e^{8t}(\cos 2\gamma)(\sin^2 2\gamma) \\ &= -\frac{1}{2}e^{2t}\frac{d}{dt}(\sin^3 2\gamma) \end{aligned}$$

This and (15) thus show that

$$\eta\phi^2 \geq -c_2 e^{2t}\frac{d}{dt}(\sin^3 2\gamma) + b_1 + \dot{b}_2 \tag{16}$$

where $c_2 = c_1/2 > 0$.

From the easily verified identities

$$-\int_0^t e^{3\tau} \frac{d}{d\tau} (\sin^3 2\gamma) d\tau = -e^{3t} \sin^3 2\gamma + 3 \int_0^t e^{3t} \sin^3 2\gamma d\tau$$

and

$$3e^{3t} \sin^3 2\gamma = 3/4\ b_3 + 1/4\ \dot{b}_3$$

where

$$b_3 \equiv e^{-3t}(1/3 \cos^3 2\gamma - \cos 2\gamma) \qquad (17)$$

it follows that

$$\int_0^t e^{3\tau} \frac{d}{d\tau} (\sin^3 2\gamma) d\tau = -e^{3t} \sin^3 2\gamma + \frac{1}{4} \int_0^t (3b_3 + \dot{b}_3) d\tau$$

Thus from (16)

$$\int_0^t e^{(\tau - t)} \eta(\tau) \phi^2\, d\tau \geq -\, c_2\, e^{2t} \sin^3 2\gamma + b(t) \qquad (18)$$

where

$$b(t) = -\frac{e^{-t}}{4} c_2 \int_0^t (3b_3 + b_3) d\tau + \int_0^t e^{(\tau - t)} (b_1 + \dot{b}_2) d\tau$$

Since (14) and (17) show that b_1, b_2 and b_3 are bounded, $b(t)$ is a bounded function as well.

Thus if we take $w_1(t)$ to be the zero initial condition solution to (3), then from (18), and (12)

$$w_1(t) \geq -\, c_2 e^{2t} \sin^3(2e^{6t}) + b(t).$$

Clearly $w_1(t)$ is unbounded. □

REFERENCE

[1] R. V. Monopoli, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. Auto. Control, AC-19 (5), October, 1974, pp. 474-484.

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