

AD-A044 818

HUGHES AIRCRAFT CO CULVER CITY CALIF ANTENNA DEPT
DYNAMIC IMPEDANCE MATCHING IN CONFORMAL ARRAYS. (U)
JAN 74 P C BARGELIOTES, W H KUMMER

F/G 9/5

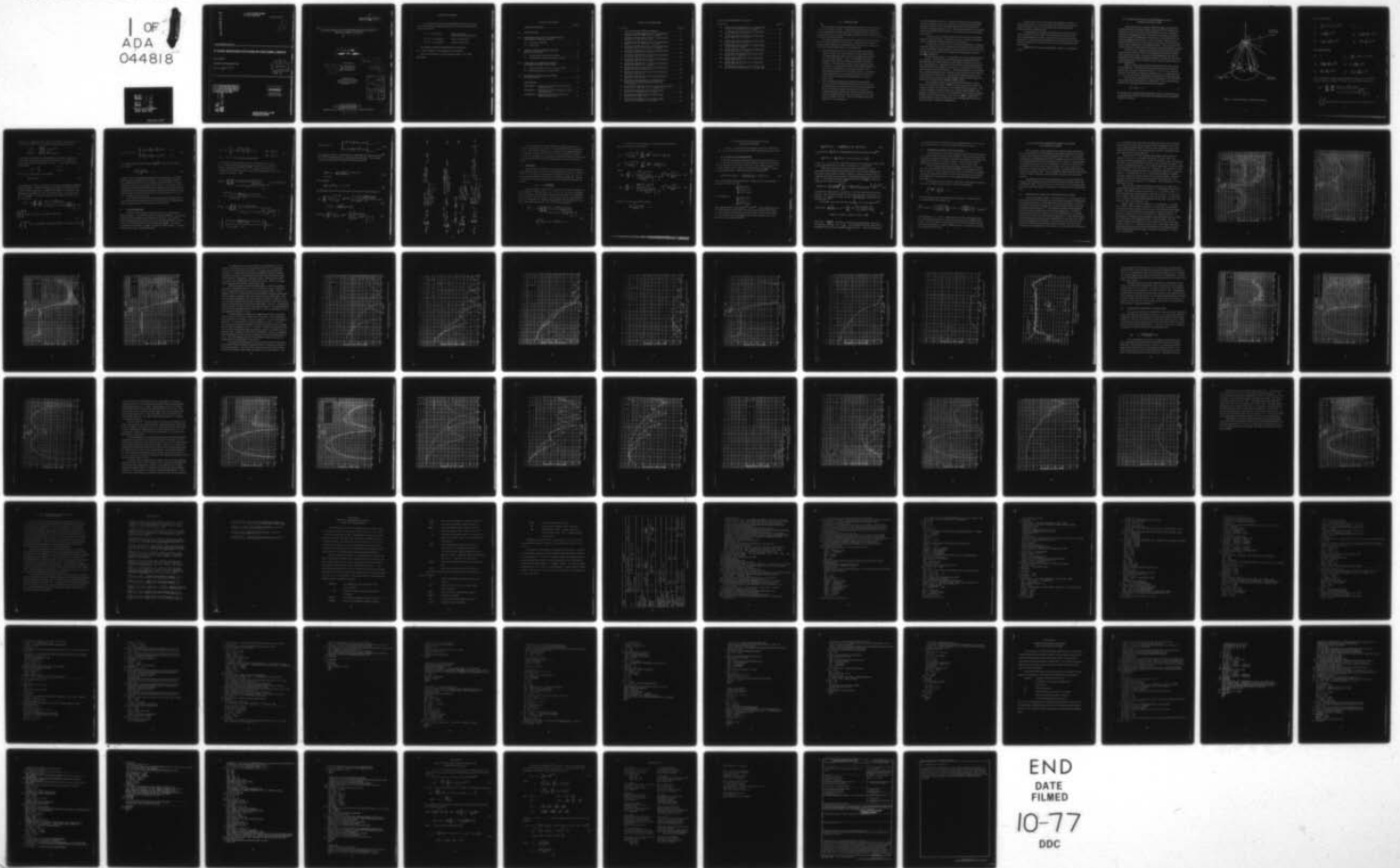
UNCLASSIFIED

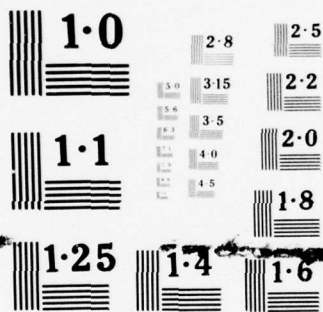
2265.30/184

N00019-73-C-0127

NL

1 OF 1
ADA
044818





NATIONAL BUREAU OF STANDARDS
 MICROCOPY RESOLUTION TEST CHART

AD A 044818

APPROVED FOR PUBLIC RELEASE
DISTRIBUTION UNLIMITED

REPORT NO. 2265 30/184
HAC REF. NO. C 1488

①

B.S.

Contract N000 19-73-C-0127

DYNAMIC IMPEDANCE MATCHING IN CONFORMAL ARRAYS

FINAL REPORT

JANUARY 1973 TO JANUARY 1974

Prepared for the AIR SYSTEMS COMMAND
Department of the NAVY

DDC
RECEIVED
SEP 26 1977
REGISTED
A

~~Distribution limited to U.S. Agencies only;
Test and Evaluation, January 1974. Other
requests for this document must be referred
to Commander, Naval Air Systems Command,
H-310B, Washington, D.C. 20360.~~

Creating a new world with electronics

HUGHES

ANTENNA DEPARTMENT
AEROSPACE GROUP
HUGHES AIRCRAFT COMPANY
CULVER CITY, CA 90230

AD NO.
DDC FILE COPY

APPROVED FOR PUBLIC RELEASE
DISTRIBUTION UNLIMITED

14
Report No/ 2265.30/184
HAC Ref - C 7488

6
DYNAMIC IMPEDANCE MATCHING IN CONFORMAL ARRAYS.

15
CONTRACT N00019-73-C-0127

15

9
FINAL REPORT

Jan 1973 to Jan 1974

Prepared by

10
Peter P. C./Bargeliotas,
Wolfgang H./Kummer
alfred T./Villeneuve

11 Jan 74

12 90p.

Prepared for
Air Systems Command
Department of the Navy
Washington, D. C.

ACCESSION for		
NTIS	Write Section	<input checked="" type="checkbox"/>
DDC	Buff Section	<input type="checkbox"/>
UNANNOUNCED		<input type="checkbox"/>
JUSTIFICATION		
BY		
DISTRIBUTION AVAILABILITY CODES		
Dist.	AVAIL.	and/or SPECIAL
A		

Antenna Department
Engineering Equipment Division
AEROSPACE GROUP
Hughes Aircraft Company - Culver City, California 90230

402 429

1477
mt

ACKNOWLEDGMENTS

The following personnel of the Hughes Radar Division contributed to the Dynamic Impedance Matching in Conformal Arrays Study during this program.

Dr. W. H. Kummer	Program Manager Radar Microwave Laboratory
Dr. P. C. Bargeliotis	Antenna Department
Dr. A. T. Villeneuve	Antenna Department

The authors gratefully acknowledge the assistance of Mrs. Yvonna Hartke in the preparation of this report.

The Technical Officer for this program is Mr. J. W. Willis (AIR-310B).

TABLE OF CONTENTS

		Page No.
	ACKNOWLEDGMENTS	ii
1.0	INTRODUCTION	1
2.0	RADIATION FIELDS OF CIRCUMFERENTIAL AND RADIAL SLOTS ON A CONE	4
2.1	Circumferential Slot	8
2.2	Radial Slot	12
3.0	COMPUTATIONAL PROCEDURES FOR SPECIAL FUNCTIONS	14
3.1	Computation of Legendre Functions	14
3.2	Computation of Spherical Bessel Functions	16
4.0	ANALYSIS OF COMPUTED ELEMENT PATTERNS FOR SLOTS ON A CONE	17
4.1	Radiation Patterns of Circumferential Slot	17
4.2	Radiation Patterns of a Radial Slot	32
5.0	RECOMMENDATIONS FOR FURTHER INVESTIGATIONS	49
	REFERENCES	50
	APPENDIX A - Computer Program for Conical Array Pattern Analysis	52
	APPENDIX B - Computer Program for Modal or Total Field Patterns	73
	APPENDIX C - Expansions for the Computation of Bessel Function $J_\nu(x)$	81

LIST OF ILLUSTRATIONS

Figure No.	Page No.
1. Conical Geometry and Slot Coordinates	5
2. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 0, 1$, and 2	19
3. θ -Polarized Modal Pattern of $\lambda/2$ Circumferential Slot for $m = 1$, $\phi = 0^\circ$, $\phi = 180^\circ$	20
4. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 3, 4$, and 5	21
5. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 6, 7$, and 12	22
6. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 0, 1$, and 2	24
7. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 3, 4$, and 5	25
8. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 6, 7$, and 12	26
9. ϕ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 6, 7$, and 12	27
10. θ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 13$	28
11. θ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 13$	29
12. ϕ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 13$	30
13. Measured and Computed ($M = 12$) θ -Polarized Pattern of Circumferential Slot on 20° Cone, $f = 8.15$ GHz	31
14. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 90^\circ$, $M = 4, 5, 6$, and $ka = 30$	33
15. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 0, 1$, and 2	34
16. ϕ -Polarized Modal Pattern of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $m = 1$	35
17. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 3, 4$, and 5	37

LIST OF ILLUSTRATIONS - Continued

Figure No.	Page No.
18. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 6, 7$, and 12	38
19. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 0, 1$, and 2	39
20. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 3, 4$, and 5	40
21. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 6, 7$ and 12	41
22. θ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 4$ and 5	42
23. θ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 6, 7$, and 12	43
24. ϕ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 13$	44
25. ϕ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 13$	45
26. θ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 13$	46
27. ϕ - Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 90^\circ$, $M = 4, 5, 6$, and $ka = 30$	48

1.0 INTRODUCTION

Several topics relating to conformal arrays have received detailed attention in the last few years. These topics include pattern synthesis, types of appropriate radiating elements, and their impedance and mutual coupling characteristics, and wide angle scanning of linear arrays located on planar and conical surfaces.

→ CONT.

Synthesis techniques for the far field patterns from current distributions on planar surfaces are well known. Since there are no proven techniques for synthesizing antenna patterns from arrays on conical surfaces, two different approaches have been pursued:

1. An equivalence principle was used to determine the distribution of sources on a cone to produce a prescribed pattern. Using this approach, planar arrays are replaced by sources on a conformal surface. Thus, it is possible to generate patterns using well known synthesis techniques for planar surfaces. The equivalent sources are continuous functions on the cone. The practical validity of the method depends on the accuracy with which the continuous functions can be approximated by physically realizable radiators. Equivalent source distributions were computed at discrete points on a conical surface to approximate the patterns of a specific planar array.
2. The computation of approximate patterns from discrete radiators judiciously positioned on the cone was carried out. An optimization was performed to obtain element weightings that yield the highest signal-to-noise ratio in the beam pointing direction neglecting the effects of mutual coupling. Uniform weighting schemes were also used for beams in the region about the cone axis. This second approach has been termed a heuristic approach.

In the Final Report on Contract N00019-72-C-0212,[1] the results from the above two approaches were reported. In the equivalence principle approach, excitations and radiation patterns were computed for two particular beam pointing directions. These patterns were then compared to the far-field pattern that would have been produced by the generating planar array.

In the use of the heuristic approach several layouts of radiating elements were considered. Parameters such as variations of sidelobe levels, grating lobes and beamwidths were investigated.

In conjunction with the use of the above two approaches, computer programs were written for the approximate computation of the fields and patterns of different configurations of elements on the surface on the cone. The program used in the heuristic approach was appended in the first Quarterly Report [2].

In the present report, the computation of an exact element pattern is described. The report begins with the general expressions for two potential functions representing the modal fields of arbitrary apertures on a conducting cone. Expressions for the electric field components are then derived from the potential functions for circumferential and radial slots with particular slot excitations. The field expressions are the basis for a computer program which computes radiation patterns from both circumferential and radial slots. The appropriate approximations for the Legendre functions and the spherical Bessel functions used for their computations are summarized in Section 3 of this report.

The computer program is a modification and extension of one described by Pridmore-Brown [3] and has been executed on the IBM 370 computer system, now operational at the Hughes Aircraft Company. The program computes the special functions in DOUBLE PRECISION, thus giving the required accuracy of the computations. The complete program together with a brief description of its use is included in Appendix A. A second program, useful in studying the exact element modal patterns at minimum cost, is included in Appendix B.

Several test cases were executed on the program in order to check its correctness. In each test case, the results compared favorably with identical cases of a circumferential slot considered by Pridmore-Brown and Stewart [4].

A comprehensive analysis of the modal series patterns is presented in Section 4 of this report for a slot 39 radians from the tip. The computed patterns are then compared with previously measured patterns [5]. Also presented are patterns of circumferential and radial slots at 30 radians from the cone tip for comparison with the previous slot location.

Suggestions for further investigations, Section 5, conclude the report.

2.0 RADIATION FIELDS OF CIRCUMFERENTIAL AND RADIAL SLOTS ON A CONE

Expressions for the radiation fields from slot-excited conical antennas were first obtained by Bailin and Silver.^[6] The fields were expressed in terms of normal-mode expansions using the orthogonal properties of the eigenfunctions. Although some corrections appeared in the literature^[7] modifying the first results, the expressions for the fields need additional examination. More recently Pridmore-Brown and Stewart^[4] presented a more rigorous calculation of the θ -polarized electric field component of circumferential slots. Their numerical calculation is based on expressions derived from integral transform methods.

In this section, complete expressions of two potential functions representing the modal fields will be presented for both the circumferential and the radial slot. The electric field components will then be obtained from the potential functions and will be available for calculation of radiation patterns of the individual slots or arrays of slots of any desired configuration.

Figure 1 shows typical circumferential and radial slots and the conical geometry. The usual spherical coordinate system centered at the cone tip is associated with the structure. The cone axis coincides with the Z-axis of the rectangular coordinates (x, y, z) associated with (r, θ, ϕ) . Primes denote coordinates of the apertures on the structure.

The two scalar potential functions in question are Π^{TE} and Π^{TM} , TE and TM to the radial direction, respectively. For an assumed time dependence of $\exp \{j\omega t\}$ both satisfy the wave equation:

$$(\nabla^2 + k^2) \Pi = 0. \quad (1)$$

The electric and magnetic field components are then conveniently given by superposition of TE and TM partial fields obtained from the potential functions as follows:

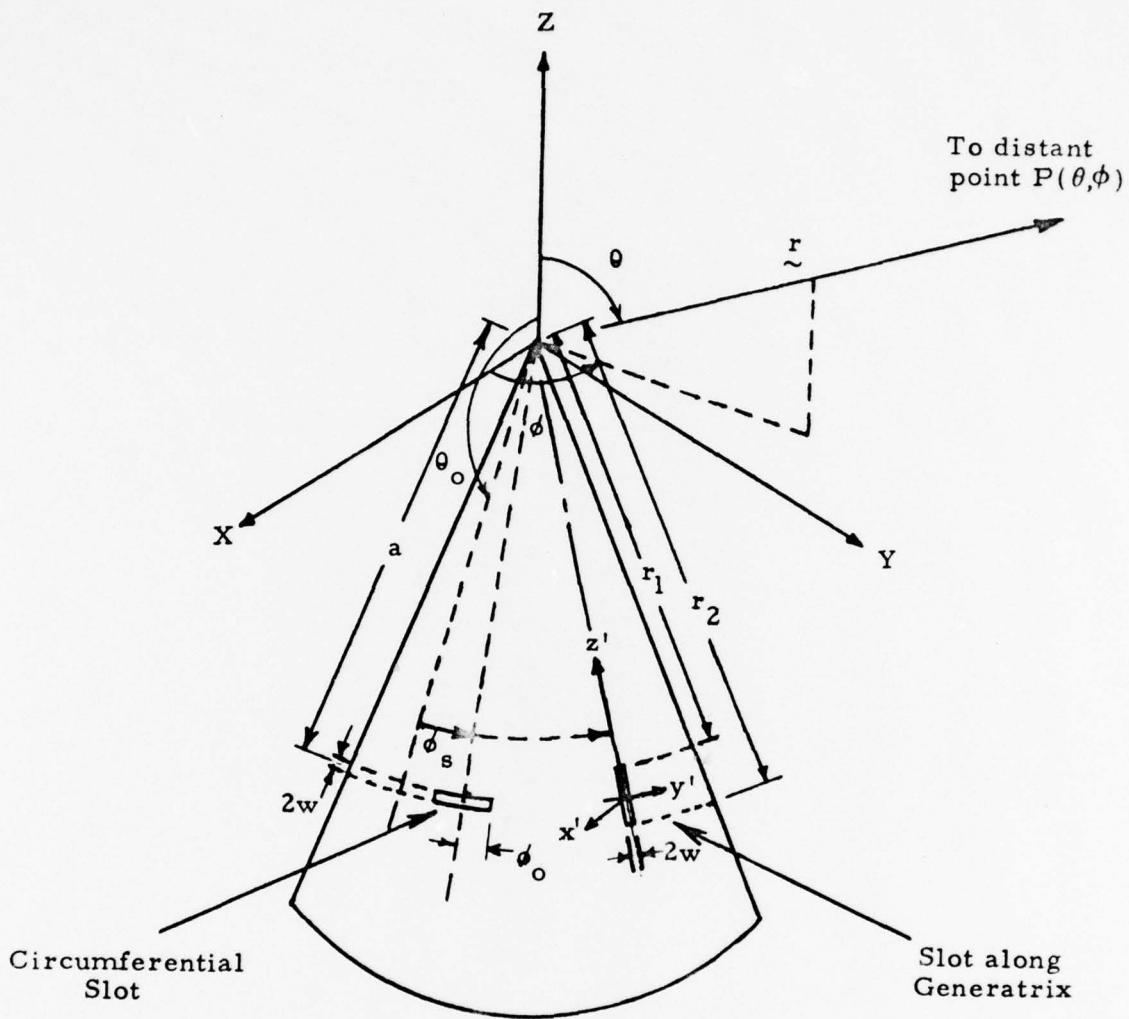


Figure 1. Conical Geometry and Slot Coordinates

TM - Partial Fields:

$$\begin{aligned}
 E_r &= \frac{\partial^2}{\partial r^2} (r \Pi^{TM}) + k^2 (r \Pi^{TM}) & H_r &= 0 \\
 E_\theta &= \frac{1}{r} \frac{\partial^2}{\partial r \partial \theta} (r \Pi^{TM}) & H_\theta &= \frac{j \omega \epsilon}{r \sin \theta} \frac{\partial}{\partial \phi} (r \Pi^{TM}) \\
 E_\phi &= \frac{1}{r \sin \theta} \frac{\partial^2}{\partial r \partial \phi} (r \Pi^{TM}) & H_\phi &= \frac{-j \omega \epsilon}{r} \frac{\partial}{\partial \theta} (r \Pi^{TM})
 \end{aligned} \tag{2}$$

TE - Partial Fields:

$$\begin{aligned}
 E_r &= 0 & H_r &= \frac{\partial^2}{\partial r^2} (r \Pi^{TE}) + k^2 (r \Pi^{TE}) \\
 E_\theta &= -\frac{j \omega \mu}{r \sin \theta} \frac{\partial}{\partial \phi} (r \Pi^{TE}) & H_\theta &= \frac{1}{r} \frac{\partial^2}{\partial r \partial \theta} (r \Pi^{TE}) \\
 E_\phi &= \frac{j \omega \mu}{r} \frac{\partial}{\partial \theta} (r \Pi^{TE}) & H_\phi &= \frac{1}{r \sin \theta} \frac{\partial^2}{\partial r \partial \phi} (r \Pi^{TE})
 \end{aligned} \tag{3}$$

For an arbitrarily shaped source excitation the expression for the Π^{TM} is given by Bailin and Silver. [7] (There is (-) sign in their expression.)

$$\begin{aligned}
 \Pi^{TM} &= \sum_{m=0}^{\infty} \sum_{i=1}^{\infty} \frac{(jk) (2\nu_i + 1) P_{\nu_i}^m (\cos \theta)}{\nu_i (\nu_i + 1) (1 + \delta_{0m}) \pi \sin \theta_0 \frac{\partial P_{\nu}^m}{\partial \nu} \Big|_{\nu = \nu_i}} \\
 &\cdot \int_{r_1}^{r_2} \int_{\phi_1}^{\phi_2} f_1(r', \phi') \cos m(\phi' - \phi) j_{\nu_i}(kr <) h_{\nu_i}(kr >) r' \sin \theta_0 dr' d\phi'
 \end{aligned} \tag{4}$$

where $r >$, $r <$ symbolizes the larger and smaller of the coordinates r , r' , respectively. $P_{\nu}^m(\cos \theta)$ is the associated Legendre function and

$$j_{\nu}(kr) = \sqrt{\frac{\pi}{2kr}} J_{\nu+1/2}(kr) \quad (5)$$

$$h_{\nu}^{(2)}(kr) = \sqrt{\frac{\pi}{2kr}} H_{\nu+1/2}^{(2)}(kr) \quad (6)$$

are the spherical Bessel and spherical Hankel functions, respectively. J_{ν} is the Bessel function of the first kind and order ν and $H_{\nu}^{(2)}$ is the Hankel function of the second kind and order ν . δ_{om} is the Kronecker delta function

$$\delta_{om} = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \quad (7)$$

while the ν_i are the roots of the equation

$$P_{\nu}^m(\cos \theta_0) = 0 \quad (8)$$

The function $f_1(r', \phi')$ in equation (4) is the electric field source excitation in the r direction. Following the general procedure described by Ballin and Silver where the boundary conditions are applied in conjunction with the Lorentz reciprocity theorem to the modal fields, an expression for Π^{TE} is obtained. The TE scalar function with arbitrary source excitation is given by,

$$\Pi^{TE} = \frac{1}{\eta} \sum_{m=0}^{\infty} \sum_{i=1}^{\infty} \frac{(2\nu_i' + 1) P_{\nu_i'}^m(\cos \theta)}{\pi (1 + \delta_{om}) \nu_i' (\nu_i' + 1) \sin \theta_0 \left. \frac{\partial^2 P_{\nu}^m}{\partial \nu \partial \theta} \right|_{\substack{\theta = \theta_0 \\ \nu = \nu_i'}}} \cdot \left\{ \int_{r_1}^{r_2} \int_{\phi_1}^{\phi_2} m f_1(r', \phi') \Gamma_1(r, r') \sin m(\phi' - \phi) dr' d\phi' \right. \\ \left. + \int_{r_1}^{r_2} \int_{\phi_1}^{\phi_2} \nu_i' (\nu_i' + 1) \sin \theta_0 f_2(r', \phi') j_{\nu_i'}(kr <) h_{\nu_i'}^{(2)}(kr >) \cos m(\phi' - \phi) dr' d\phi' \right\} \quad (9)$$

$$\text{where } \Gamma_1(r, r') = \begin{cases} j_{\nu'_i}(kr) \frac{d}{dr'} [r' h_{\nu'_i}^{(2)}(kr')] & r < r' \\ h_{\nu'_i}^{(2)}(kr) \frac{d}{dr'} [r' j_{\nu'_i}(kr')] & r > r' \end{cases} \quad (10)$$

and η is the free space wave impedance $\sqrt{\frac{\mu}{\epsilon}}$. The ν'_i are the roots of the equation

$$\frac{\partial P_{\nu}^{m_1}(\cos \theta_0)}{\partial \theta} = 0 \quad (11)$$

and $f_2(r', \phi')$ is the E_ϕ component of the source excitation. All other quantities are as defined for the TM case. It is interesting to observe that, unlike the TM mode, the TE mode is excited by both the r - and ϕ -components of the source excitation in the aperture. Conversely, the r -component of the source excitation excites both the TM and the TE modes. It should also be noted that the expression in Equation (9) differs from the expression given by Bailin and Silver [7] in the expression for $\Gamma_1(r, r')$ and a common factor of $1/\sin \theta_0$.

The modal functions and the corresponding field components from particular aperture configurations on the cone structure with appropriate field excitations are given in Sections 2.1 and 2.2 below.

2.1 Circumferential Slot

A narrow circumferential slot of width $2w \ll \lambda$ is considered as shown in Figure 1. The narrow slot has an azimuthal length $d = 2\phi_0 a \sin \theta_0$ where $2\phi_0$ is the azimuthal angle subtended by the slot, θ_0 is the cone exterior angle measured from the axis, a is the distance from the cone tip to the center of the slot and λ is the operating wavelength. The slot is assumed to be excited by a voltage V_0 across its center and has a resulting field across it in the r direction given by

$$E_r(r', \phi') = \begin{cases} V_0 \cos \left[\frac{\pi(\phi' - \phi_s)}{2\phi_0} \right] \delta(r' - a) & |\phi' - \phi_s| < \phi_0 \\ 0 & |\phi' - \phi_s| > \phi_0 \end{cases} \quad (12)$$

where $\delta(r' - a)$ is the Dirac delta function.

Substituting the expression of $E_r(r', \phi')$ in Equations (4) and (9) for $f_1(r', \phi')$ and integrating over the primed coordinates, we obtain the explicit expression for Π^{TM} and Π^{TE} . The function $f_2(r', \phi')$ is set equal to zero in the TE case, as only the E_r component of the source excitation is assumed. Thus,

$$\Pi^{TM}(\tilde{r}) = \sum_{m=0}^{\infty} \sum_{i=1}^{\infty} \frac{(jkaV_0)(2\nu_i+1)P_{\nu_i}^m(\cos\theta)}{\nu_i(\nu_i+1)(1+\delta_{om})^\pi \frac{\partial P_{\nu_i}^m(\cos\theta)}{\partial \nu}} \left. \begin{array}{l} j_{\nu_i}(kr<) h_{\nu_i}^{(2)}(kr>) \\ \theta = \theta_0 \\ \nu = \nu_i \end{array} \right\} \cdot \frac{2ka \sin \theta_0 \cos \left(\frac{m\pi}{2ka \sin \theta_0} \right) \cos m(\phi - \phi_s)}{(ka \sin \theta_0)^2 - m^2} \quad (13)$$

and

$$\Pi^{TE}(\tilde{r}) = \frac{1}{\eta} \sum_{m=0}^{\infty} \sum_{i=1}^{\infty} \frac{(2\nu_i+1)P_{\nu_i}^m(\cos\theta)}{\pi(1+\delta_{om}) \nu_i(\nu_i+1) \sin \theta_0 \frac{\partial^2 P_{\nu_i}^m(\cos\theta)}{\partial \nu \partial \theta}} \left. \begin{array}{l} \theta = \theta_0 \\ \nu = \nu_i \end{array} \right\} \cdot \left\{ \frac{-2ka \sin \theta_0 \cos \left(\frac{m\pi}{2ka \sin \theta_0} \right) m \sin m(\phi - \phi_s)}{(ka \sin \theta_0)^2 - m^2} \right\} \cdot \Gamma_1(r, a) \quad (14)$$

$$\text{where } \Gamma_1(r, a) = \begin{cases} h_{\nu_i}^{(2)}(kr) \frac{d}{dr} (r' j_{\nu_i}(kr')) \Big|_{r'=a} & r > a \\ j_{\nu_i}(kr) \frac{d}{dr} (r' h_{\nu_i}^{(2)}(kr')) \Big|_{r'=a} & r < a \end{cases} \quad (15)$$

The radiation fields are readily found by substituting the expression of Π^{TM} and Π^{TE} corresponding to $r > a$ into Equations (2) and (3). On using the asymptotic expansion for the spherical Hankel functions,

$$h_{\nu}^{(2)}(kr) = \sqrt{\frac{\pi}{2kr}} \sqrt{\frac{2j}{\pi kr}} j^{\nu+1/2} e^{-j \cdot k r} \quad (16)$$

$kr \rightarrow \infty$

and noting that

$$\frac{\partial}{\partial r} (r h_{\nu}^{(2)}(kr)) = j^{\nu} e^{-jkr} \quad (17)$$

the following expressions are obtained for the electric field components:

$$E_{\theta} = \sum_{m=0}^{\infty} \frac{V_o e^{-j(kr - \pi/4)}}{r \pi (1 + \delta_{om})} \sqrt{2\pi ka} \frac{2ka \sin \theta_o \cos\left(\frac{m\pi}{2ka \sin \theta_o}\right)}{(ka \sin \theta_o)^2 - m^2} \cos m \phi. \quad (18)$$

$$\left\{ P_{1\theta} + \frac{j m^2}{\sin \theta_o \sin \theta} P_{2\theta} \right\}$$

$$\text{where } P_{1\theta} = \sum_{i=1}^{\infty} j^{\nu_i} \frac{\nu_i}{\nu_i^2 - 1/4} J_{\nu_i}(ka) \frac{\frac{\partial}{\partial \theta} P_{\nu_i - 1/2}(\cos \theta)}{\frac{\partial P_{\nu_i - 1/2}(\cos \theta)}{\partial \nu}} \Bigg|_{\substack{\theta = \theta_o \\ \nu = \nu_i}} \quad (19)$$

$$\text{and } P_{2\theta} = \sum_{i=1}^{\infty} \sum_{j=1}^{\nu_i} \frac{\nu_i}{\nu_i^2 - 1/4} \left. \frac{P_{\nu_i - 1/2}^{-m}(\cos \theta)}{\partial \nu \partial \theta} \right|_{\nu = \nu_i} \left[\frac{(\nu_i + \frac{1}{2}) J_{\nu_i}(ka)}{ka} - J_{\nu_i + 1}(ka) \right] \quad (20)$$

$$E_{\phi} = \sum_{m=0}^{\infty} m \frac{V_0 e^{-j(kr - \pi/4)}}{r \pi} \sqrt{2\pi ka} \cdot \frac{2ka \sin \theta_0 \cos\left(\frac{m\pi}{2ka \sin \theta_0}\right)}{(ka \sin \theta_0)^2 - m^2} \sin m \phi \quad (21)$$

where

$$P_{1\phi} = \left\{ \frac{-j}{\sin \theta} P_{1\phi} + \frac{1}{\sin \theta_0} P_{2\phi} \right\} \quad (22)$$

$$P_{1\phi} = \sum_{i=1}^{\infty} \sum_{j=1}^{\nu_i} \frac{\nu_i}{\nu_i^2 - 1/4} J_{\nu_i}(ka) \left. \frac{P_{\nu_i - 1/2}^{-m}(\cos \theta)}{\partial \nu} \right|_{\nu = \nu_i} \quad \left. \begin{array}{l} \theta = \theta_0 \\ \nu = \nu_i \end{array} \right\}$$

$$\text{and } P_{2\phi} = \sum_{i=1}^{\infty} \sum_{j=1}^{\nu_i} \frac{\nu_i}{\nu_i^2 - 1/4} \left. \frac{\frac{\partial}{\partial \theta} P_{\nu_i}^{-m} - 1/2(\cos \theta)}{\partial^2 P_{\nu_i - 1/2}^{-m}(\cos \theta)} \right|_{\nu = \nu_i} \left[\frac{(\nu_i + 1/2) J_{\nu_i}(ka)}{ka} - J_{\nu_i + 1}(ka) \right] \quad (23)$$

It is pointed out that the θ -dependence is represented by the Legendre function $P_{\nu-1/2}^{-m}(\cos \theta)$ instead of $P_{\nu}^m(\cos \theta)$ as originally derived. This choice simplifies the evaluation of the Legendre function in the digital computer and is preferred. Also, except for a constant factor of $1/\pi$, the expression for E_{θ} agrees with that reported by Pridmore-Brown and Stewart^[4] derived by the Kontrovitch-Lebedev transform method.

2.2 Radial Slot

A narrow slot of width $2w \ll \text{circumference}$ is assumed to be positioned along a generatrix of the cone and extends from r_1 to r_2 as shown in Figure 1. It is further assumed that the length of the slot, $(r_2 - r_1)$, is such as to allow definition of the ends of the slot by constant ϕ' . The slot is excited by a voltage V_0' resulting in an electric field in the ϕ -direction given by

$$E_{\phi} = V_0' \frac{g(r') \delta(\phi')}{r' \sin \theta_0}$$

The function $g(r')$ describes the variation of the source excitation in the r' direction. Clearly, since $f_1(r', \phi') = 0$, i.e., no source excitation in the r' -direction, the TM mode of the potential function is not supported. The TE mode, on the other hand, is obtained only from the contribution of $f_2(r', \phi')$ in Equation (9). Substitution of Equation (24) into Equation (9) results in the following expression for Π^{TE} for the radial slot:

$$\begin{aligned} \Pi^{\text{TE}}(r) &= \frac{V_0'}{\eta} \sum_{m=0}^{\infty} \sum_{i=1}^{\infty} \frac{(2\nu_i' + 1) P_{\nu_i'}^m(\cos \theta) \cos m\phi}{\sin \theta_0 \pi(1 + \delta_{0m}) \frac{\partial^2 P_{\nu}^m(\cos \theta)}{\partial \nu \partial \theta}} \bigg|_{\substack{\theta = \theta_0 \\ \nu = \nu_i'}} \\ &\cdot \int_{r_1}^{r_2} \frac{g(r')}{r'} j_{\nu_i'}(kr' <) h_{\nu_i'}^{(2)}(kr >) dr' \end{aligned} \quad (25)$$

The electric field components are obtained from Equation (2) operating on Equation (25) giving:

$$E_{\theta} = \frac{V_o' e^{-j(kr - \pi/4)}}{r \sin \theta_o} \sum_{m=1}^{\infty} j \sqrt{\frac{2}{\pi}} m \sin m \phi \frac{1}{\sin \theta} P_{\theta} \quad (26)$$

$$E_{\phi} = \frac{V_o' e^{-j(kr - \pi/4)}}{r \sin \theta_o} \sum_{m=0}^{\infty} j \sqrt{\frac{2}{\pi}} \frac{\cos m \phi}{(1 + \delta_{om})} P_{\phi} \quad (27)$$

where

$$P_{\theta} = \sum_{i=1}^{\infty} \nu_i' j^{\nu_i'} \frac{P_{\nu_i' - 1/2}^{-m}(\cos \theta)}{\frac{\partial^2 P_{\nu - 1/2}^{-m}(\cos \theta)}{\partial \nu \partial \theta}} \bigg|_{\substack{\theta = \theta_o \\ \nu = \nu_i'}} \cdot \int_{r_1}^{r_2} \frac{g(r')}{r'} \frac{J_{\nu_i'}(kr')}{\sqrt{kr'}} dr' \quad (28)$$

$$P_{\phi} = \sum_{i=1}^{\infty} \nu_i' j^{\nu_i'} \frac{\frac{\partial}{\partial \theta} P_{\nu_i' - 1/2}^{-m}(\cos \theta)}{\frac{\partial^2 P_{\nu - 1/2}^{-m}(\cos \theta)}{\partial \nu \partial \theta}} \bigg|_{\substack{\theta = \theta_o \\ \nu = \nu_i'}} \cdot \int_{r_1}^{r_2} \frac{g(r')}{r'} \frac{J_{\nu_i'}(kr')}{\sqrt{kr'}} dr' \quad (29)$$

and the ν_i' are the roots of the equation (30)

$$\frac{\partial P_{\nu - 1/2}^{-m}(\cos \theta_o)}{\partial \theta} = 0$$

3.0 COMPUTATIONAL PROCEDURES FOR SPECIAL FUNCTIONS

A brief description of the procedures used in the computation of some functions appearing in the far field expressions is given below.

3.1 Computation of Legendre Functions

From the expressions of the field components given in Section 2, it is seen that the calculation of radiation patterns requires the evaluation of the function $P_\nu^m(\cos \theta)$ and its derivatives with respect to ν and θ . As stated previously, we have chosen to work with the function $P_\nu^{-m}(\cos \theta)$ rather than $P_\nu^m(\cos \theta)$. The two functions are related by^[8]

$$P_\nu^m(\cos \theta) \cos(m\pi) = \frac{\Gamma(\nu + m + 1)}{\Gamma(\nu - m + 1)} P_\nu^{-m}(\cos \theta) \quad (31)$$

where $\Gamma(k)$ is Euler's gamma function. Using the above relationship, it is easy to show that the ratio

$$\frac{\frac{\partial}{\partial \theta} P_\nu^m(\cos \theta)}{\frac{\partial}{\partial \nu} P_\nu^m(\cos \theta_0)}$$

can be replaced by

$$\frac{\frac{\partial}{\partial \theta} P_\nu^{-m}(\cos \theta)}{\frac{\partial}{\partial \nu} P_\nu^{-m}(\cos \theta_0)}$$

in the expression of the functions Π^{TM} and Π^{TE} . From equation (31) it is also evident that the roots of $P_\nu^m(\cos \theta_0)$ are also roots of $P_\nu^{-m}(\cos \theta_0)$. The same holds true for the derivatives of the two functions with respect to θ . Derivatives of the Legendre Functions with respect to θ are computed by the recurrence formula^[9]

$$\frac{d}{d\theta} P_\nu^m(\cos \theta) = m \frac{\cos \theta}{\sin \theta} P_\nu^m(\cos \theta) + P_\nu^{m+1}(\cos \theta) \quad (32)$$

The derivatives $\frac{d}{d\nu} P_\nu^m$ are computed numerically using the formula^[10]

$$\frac{d}{d\nu} f(\nu_i) = \frac{1}{12h} \left\{ f(\nu_0) - 8f(\nu_1) + 8f(\nu_2) - f(\nu_3) \right\}$$

where $\nu_i = \nu_0 + ih$, h is a selected increment in ν , and $f(\nu_i)$ are the values of the Legendre functions of order m and degree ν_i , $P_{\nu_i}^m(\cos \theta)$.

Thus far we have given expressions for computing the various derivatives of the Legendre functions in terms of the functions themselves. The computations of these functions, however, require extreme care, as valid approximations must be used for the different ranges of the variables θ and ν . For relatively small values of θ such that $\theta < \frac{9}{72+2m}$ where θ is expressed in radians, the formula^[11]

$$P_\nu^{-m}(\cos \theta) = \left(\frac{1 - \cos \theta}{1 + \cos \theta} \right)^{\frac{m}{2}} \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(\nu + n + 1)}{\Gamma(\nu - n + 1) \Gamma(m + n + 1)} \frac{\left(\frac{1}{2} - \frac{1}{2} \cos \theta\right)^n}{n!} \quad (33)$$

is suitable for computations. As before, $\Gamma(k)$ is the usual Euler's gamma function. The computation of $P_\nu^m(\cos \theta)$ is then obtained from Equation (31). However, for ν large, ($\nu > 100$), the ratio $\frac{\Gamma(\nu + m + 1)}{\Gamma(\nu - m + 1)}$ is replaced by its limiting value ν^{2m} .

For large values of θ such that $\theta \geq \frac{9}{72+2m}$ we use the expansion^[12]

$$P_\nu^{-m}(\cos \theta) = \sqrt{\frac{2}{\pi \sin \theta}} \Gamma(\nu - m + 1) \sum_{k=0}^{\infty} (-1)^k \frac{(1/2 + m)_k (1/2 - m)_k}{k! (2 \sin \theta)^k (\nu + 3/2)_k} \sin \left[(\nu + k + 1/2) \theta + \pi (1/2 k + 1/2 m + 1/4) \right] \quad (34)$$

where $(b)_k = \frac{\Gamma(b+k)}{\Gamma(b)}$. The above expansion gives satisfactory results for all values of ν and $\frac{\pi}{6} < \theta < 5\frac{\pi}{6}$. For θ outside this range, the series is asymptotic and its accuracy depends on the value of ν . This last expression

is used for numerical computations in the neighborhood of $\nu = 70 + 2m$ and the desired functions are obtained through backward recursion.

3.2 Computation of Spherical Bessel Functions

The computation of the fields involves the spherical Bessel function $j_\nu(x)$ and the spherical Hankel function $h_\nu^{(2)}(x)$. Since these functions are related to corresponding cylindrical functions by Equations (5) and (6), the cylindrical functions are computed instead. For the Hankel function, $H_\nu^{(2)}(x)$, the asymptotic expansion, Equation (16), has been used.

In the accompanying computer program, Appendix A, the Bessel functions are computed in DOUBLE PRECISION using computer system library routines. If such library routines are not available or accessible, high speed computational subroutines for these functions may be written from appropriate approximations. These approximations are given in Appendix C.

The computation of the radiation field from a slot along a generator of the conical surface requires the evaluation of the integral

$$\int_{r_1}^{r_2} \frac{g(r)}{r} \frac{J_\nu(kr)}{\sqrt{kr}} dr$$

A series solution of the indicated integral when $g(r')$ is a polynomial in r' is obtained from the formula [13]

$$\int_0^z t^\mu J_\nu(t) dt = \frac{z^\mu \Gamma(\frac{\nu + \mu + 1}{2})}{\Gamma(\frac{\nu - \mu + 1}{2})} \sum_{k=0}^{\infty} (\nu + 2k + 1) \frac{\Gamma(\frac{\nu - \mu + 1}{2} + k)}{\Gamma(\frac{\nu + \mu + 3}{2} + k)} J_{\nu + 2k + 1}(z)$$

with Real part $(\mu + \nu + 1) > 0$. Alternatively, the indicated integral may be evaluated with sufficient accuracy by known numerical integration techniques. The latter method, using Simpson's "3/8 Rule", has been chosen for the present program. Either method, however, requires the evaluation of the Bessel function $J_\nu(x)$.

4.0 ANALYSIS OF COMPUTED ELEMENT PATTERNS FOR SLOTS ON A CONE

The convergence of the modal series representing the far fields of the circumferential and radial slots was examined by looking at the pattern change in the sequential summation of higher order modes. In each case, patterns were calculated for every 10° in azimuth and every 5° in elevation for both the θ -polarization and the ϕ -polarization. However, elevation cuts were plotted only at $\phi = 0^\circ$ and $\phi = 180^\circ$ degrees. These two cuts have been plotted on an extended θ scale to give a single view of the elevation cut. Since the cone under study has a 10° half angle, the position of $\theta = 80^\circ$ was selected for azimuthal cuts. This corresponds to the broadside of the conical surface. Figure 1 shows the relative position of slots in the coordinate system.

The patterns to be shown subsequently have been normalized to the largest value computed for both polarizations for the particular modal sum or individual mode. For convenience, we shall refer to the individual mode patterns by the small letter "m" and to the sum patterns by the capital letter "M".

4.1 RADIATION PATTERNS OF CIRCUMFERENTIAL SLOTS

A slot of azimuthal length $d = \lambda/2$ located at $ka = 39$ radians ($k = 2\pi/\lambda$) from the tip of the 10° half angle cone was selected for pattern computations. This particular location was selected because it corresponds to a case of a circumferential slot at 8.15 GHz and 9 inches from the cone tip previously investigated experimentally.^[17] The computed results will be compared with the available experimental results where appropriate. The slot is assumed to be excited by a voltage across its center and has a resulting field across it in the r direction given by Equation (12). Near the tip of the cone this excitation function may be represented by a few terms of a cosine series with appropriate coefficients. A greater number of terms are required for adequate representation of the same function when the slot is located many wavelengths away from the tip. It will be seen later that there

is a correlation between the number of expansion terms in the excitation function and the number of terms required for convergence of the modal series representing the fields. However, this correlation is not easily determined beforehand. The dominant TE_{10} mode of a rectangular waveguide may be used to excite the slot.

Figures 2 and 3 show the θ -polarized pattern of the lowest two modes, $m = 0$ and $m = 1$, respectively, for $\phi = 0^\circ$ and $\phi = 180^\circ$. As expected, the modal patterns for $\phi = 180^\circ$ are identical to the $\phi = 0^\circ$ patterns, since the variation of the fields in the ϕ -direction is purely sinusoidal. The pattern corresponding to the $m = 0$ mode shows significant variation at broadside with extrema at $\theta = 0^\circ$ and $\theta = \theta_0 = 170$ degrees. The peaks at $\theta = 12^\circ$ and $\theta = 30^\circ$ of this particular mode are particularly noteworthy. In contrast to the $m = 0$ mode pattern, the $m = 1$ mode pattern of Figure 3 exhibits a relatively small and smooth variation at the broadside region. The maximum now occurs at $\theta = 0^\circ$ with a pronounced first minimum at $\theta = 15^\circ$ followed by a local maximum at $\theta = 25$ degrees. Also, the pattern is about 1 dB down at $\theta = \theta_0$.

The total or sum θ -polarized pattern of the first two modes, $M = 1$, is also shown in Figure 2 for $\phi = 0^\circ$ and for $\phi = 180^\circ$ together with the $M = 2$ pattern. The addition of the second mode, $m = 1$, to the first mode, in effect dominates the resulting pattern. The effect of the $m = 0$ mode, however, is still evident in the region near the cone axis with a -3.5 dB value at $\theta = 1$ degree. The forward-back difference varies from 1 dB at $\theta = 1^\circ$ to 9 dB at $\theta = 80^\circ$ and to 14 dB at $\theta = \theta_0$.

Higher order mode sum patterns, $M = 3$ through $M = 5$, for both $\phi = 0^\circ$ and $\phi = 180^\circ$ are shown in Figure 4. In each case, the sequential summation of a higher order mode in the sum series effectively decreases the variation at the broadside region with the average level of the $\phi = 0^\circ$ patterns at -9 dB for $M = 2$ and -4 dB for $M = 4$. Addition of the $m = 5$ mode raises the pattern level only slightly at broadside while in the vicinity of $\theta = 40^\circ$ and $\theta = 150^\circ$ changes of about -1.5 dB are noted. The next significant changes in the pattern are seen in the $M = 7$ modal sum shown in Figure 5 for $\phi = 0^\circ$. At the same time, the $\phi = 180^\circ$ pattern shows that the pattern level decreases with increased modes at the broadside region while maintaining the level of about -5.5 dB and -15.5 dB at $\theta = 1^\circ$ and $\theta = \theta_0$, respectively.

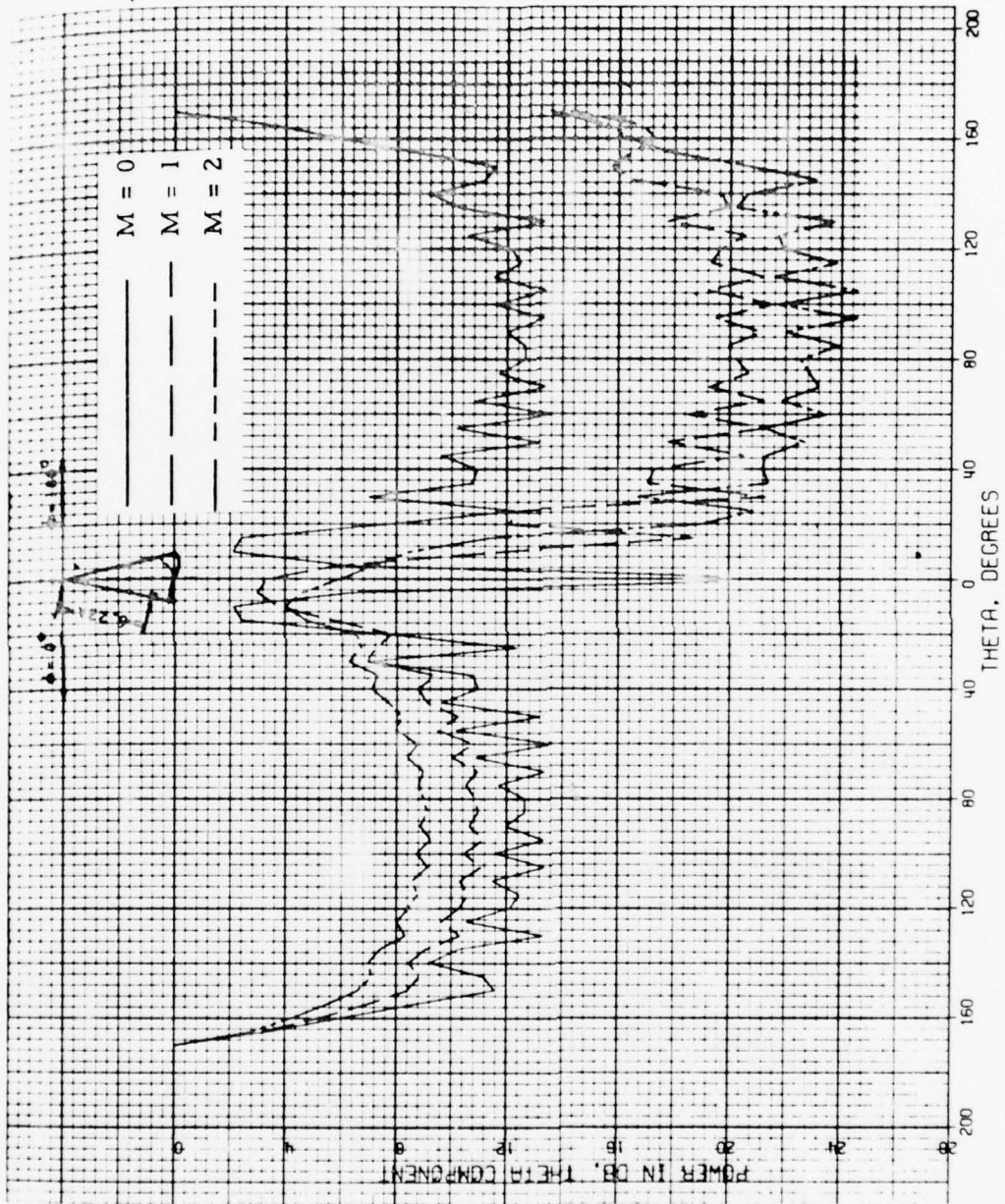


Figure 2. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot
for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 0$, 1, and 2

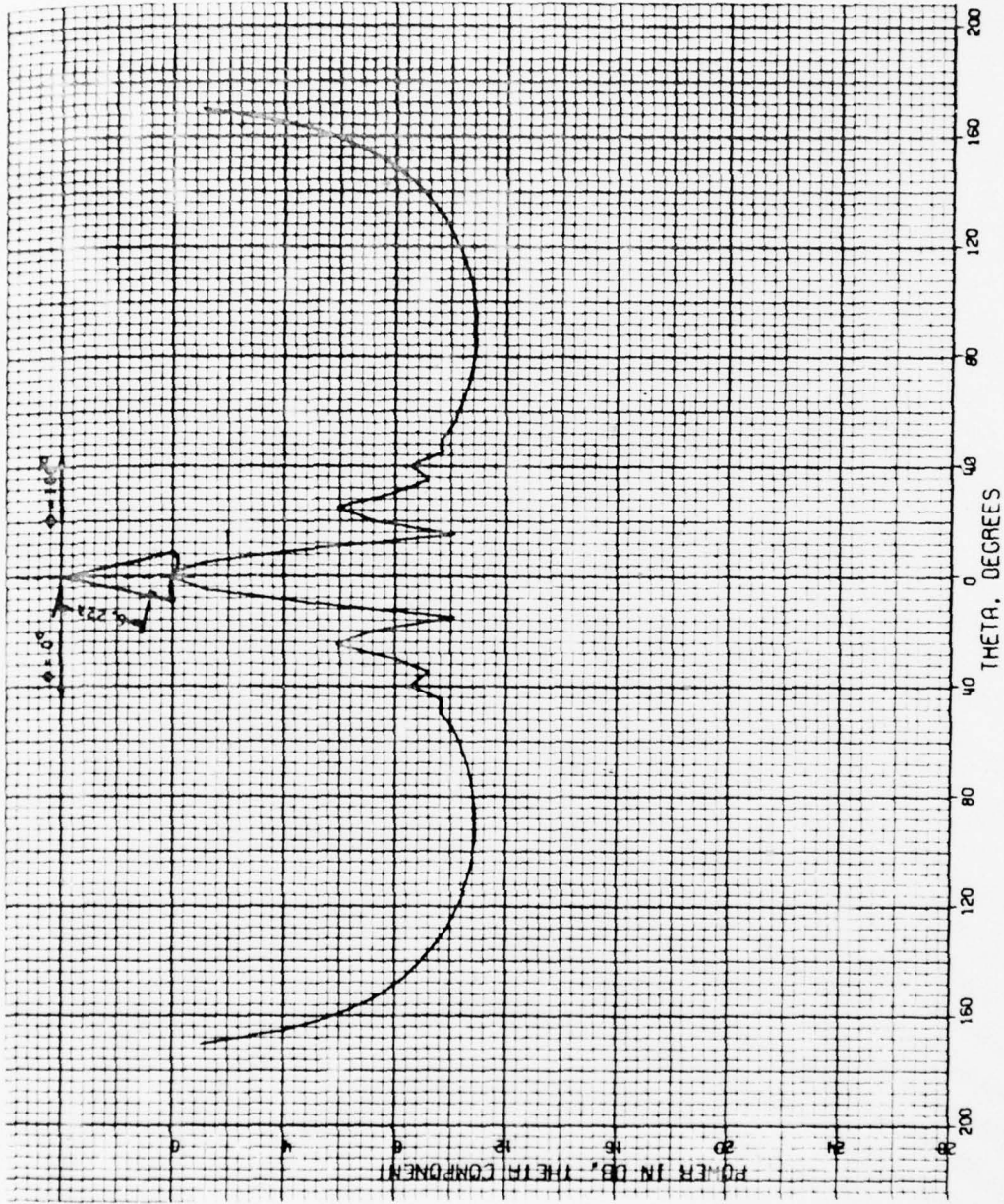


Figure 3. θ -Polarized Modal Pattern of $\lambda/2$ Circumferential Slot
for $m = 1$, $\phi = 0^\circ$, $\phi = 180^\circ$

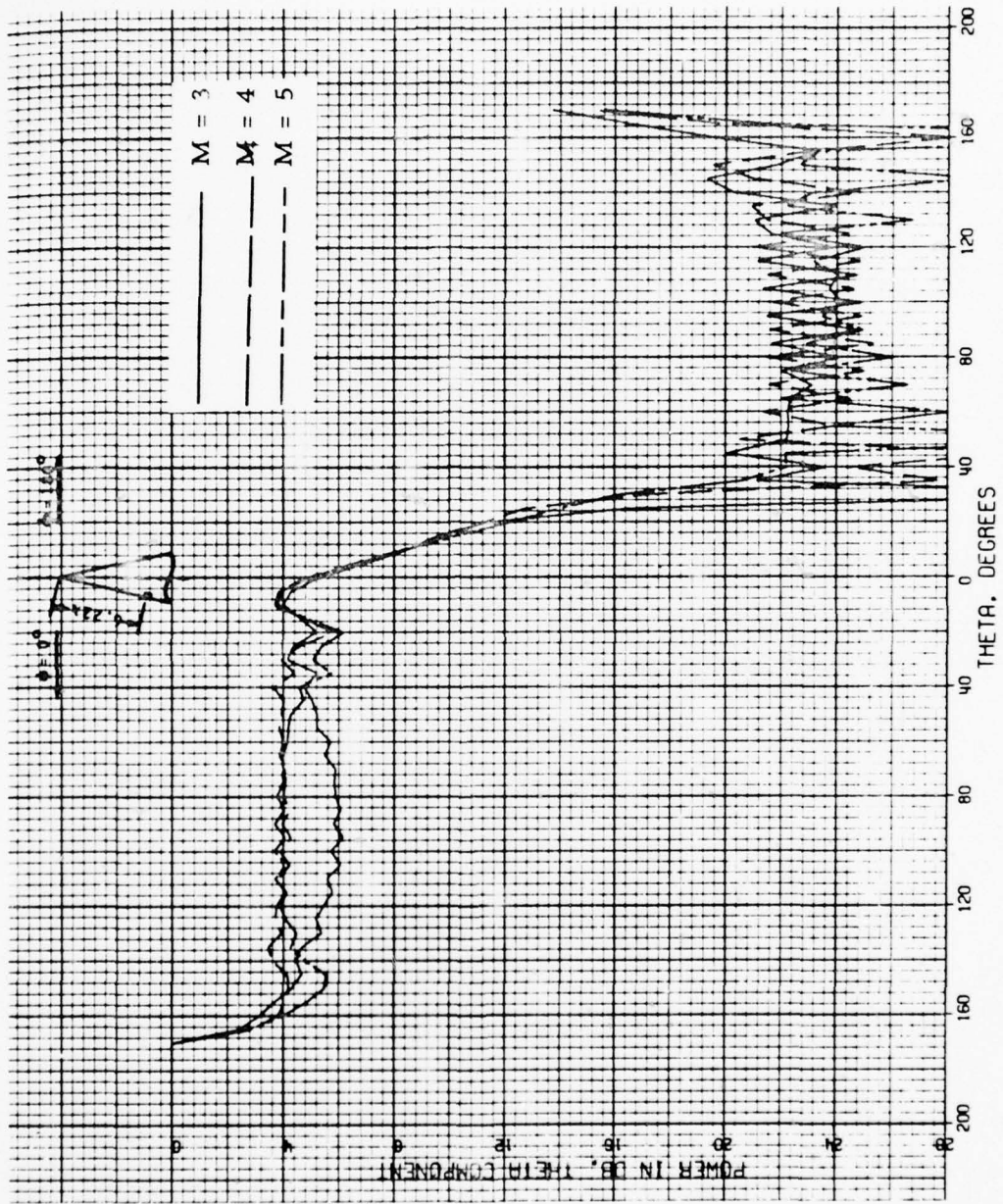


Figure 4. θ -Polarized Total Patterns of $1/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 3, 4$ and 5

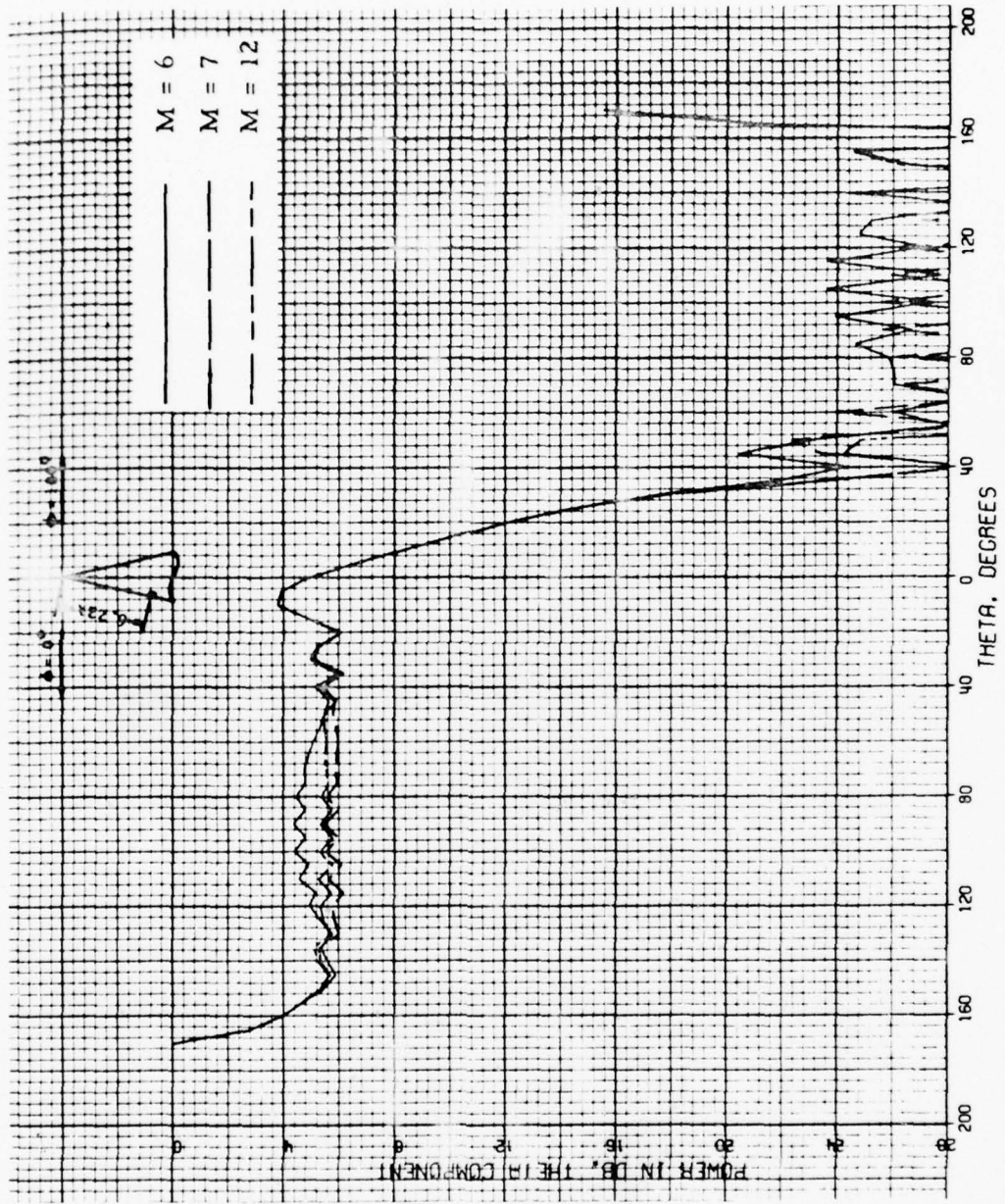


Figure 5. θ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 6, 7$, and 12

The addition of the next five modes, $m = 8$ through $m = 12$, to the $M = 7$ total pattern gives the total pattern of $M = 12$. The patterns for this case are also shown in Figure 5. Comparison of this pattern with the $M = 7$ total pattern shows that the addition of these five modes results in a decrease of the pattern level of less than .4 dB in the range of $\theta = 40^\circ$ to $\theta = 140$ degrees. Similarly, the $\phi = 180^\circ$ pattern for this case shows only a slight decrease in the same range. There is no significant change to either pattern outside this range.

Azimuthal patterns for the broadside angle $\theta = 80^\circ$ were plotted for both θ - and ϕ -polarizations. These are shown in Figures 6 through 8 for the θ -polarization and in Figure 9 for the ϕ -polarization. From Figures 6 and 7 it is seen that the θ -polarized pattern changes considerably with the addition of higher order mode almost at all azimuth angles. The $M = 7$, θ -polarized pattern, however, Figure 8, is down -5.8 dB at $\phi = 0^\circ$ and decreases almost monotonically to about -26 dB at $\phi = 100^\circ$ and remains below that level for the remaining azimuthal region. The $M = 7$, ϕ -polarized pattern is below -23 dB for all azimuth angles, Figure 9. Smoother patterns are observed in both polarizations of the $M = 12$ case as compared to corresponding pattern of the $M = 7$ case.

To study further the effects of the higher order modes to the total pattern, the $m = 13$ mode was added to $M = 12$ pattern. The resulting $M = 13$ sum pattern in elevation is shown in Figure 10. The azimuthal patterns for the $M = 13$ case have been included in Figures 11 and 12. It is seen that there are no noticeable changes in either the elevation or the azimuthal patterns with the addition of the $m = 13$ mode. It should be pointed out, however, that the normalized θ -polarized pattern of the $m = 13$ mode, has its maximum at $\theta = 90$ degrees rather than at broadside. This is a consequence of the θ -dependence of the associated Legendre functions and their derivatives. This also explains why only the broadside range of the pattern is changed with increased higher order modes.

The measured E-plane pattern of the circumferential slot fed from a half-height X-band waveguide is shown in Figure 13 together with the $M = 12$ computed pattern. The two patterns agree very well near the cone tip and for the broadside region where both patterns are quite uniform. The

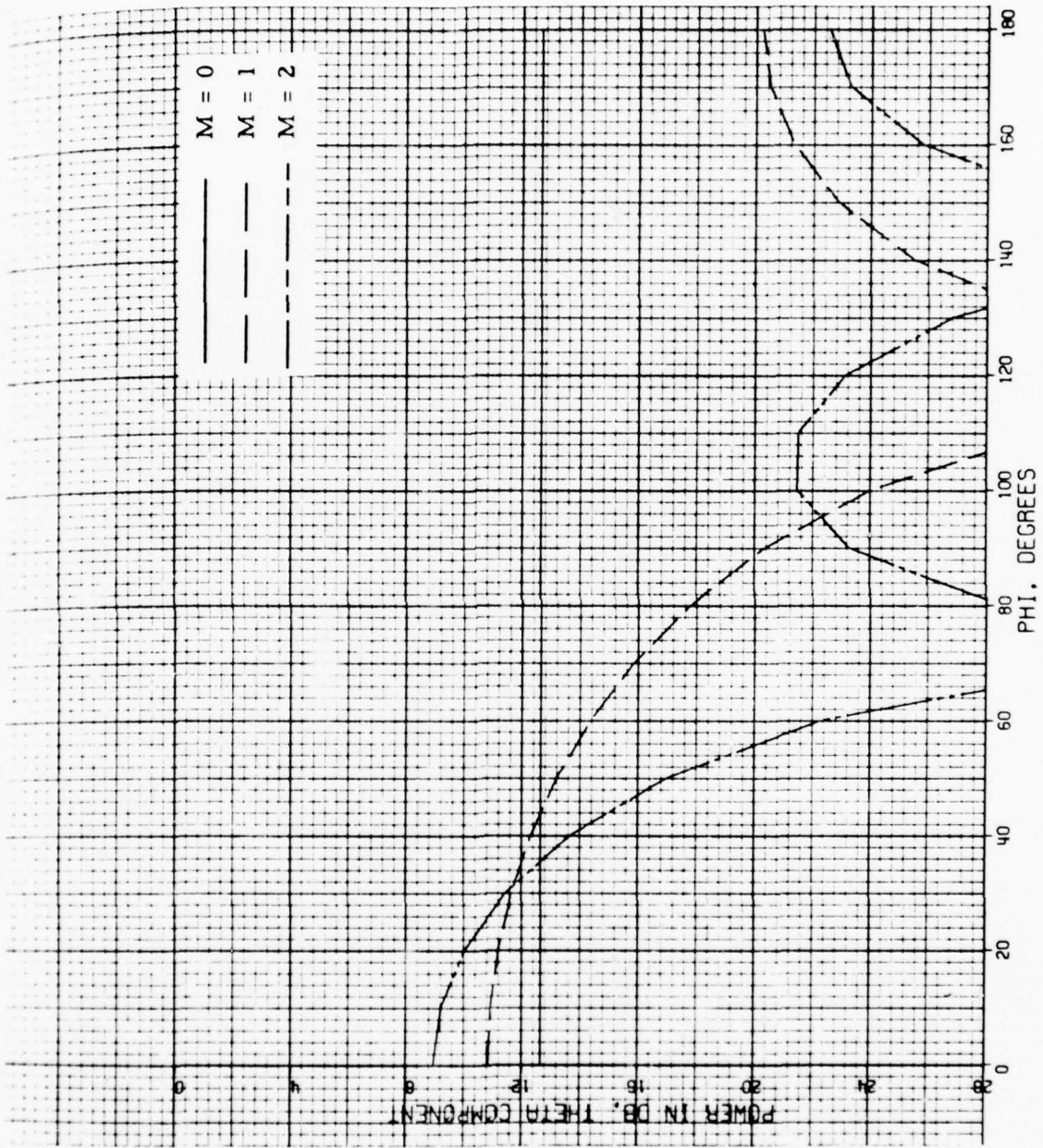


Figure 6. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 0, 1, \text{ and } 2$

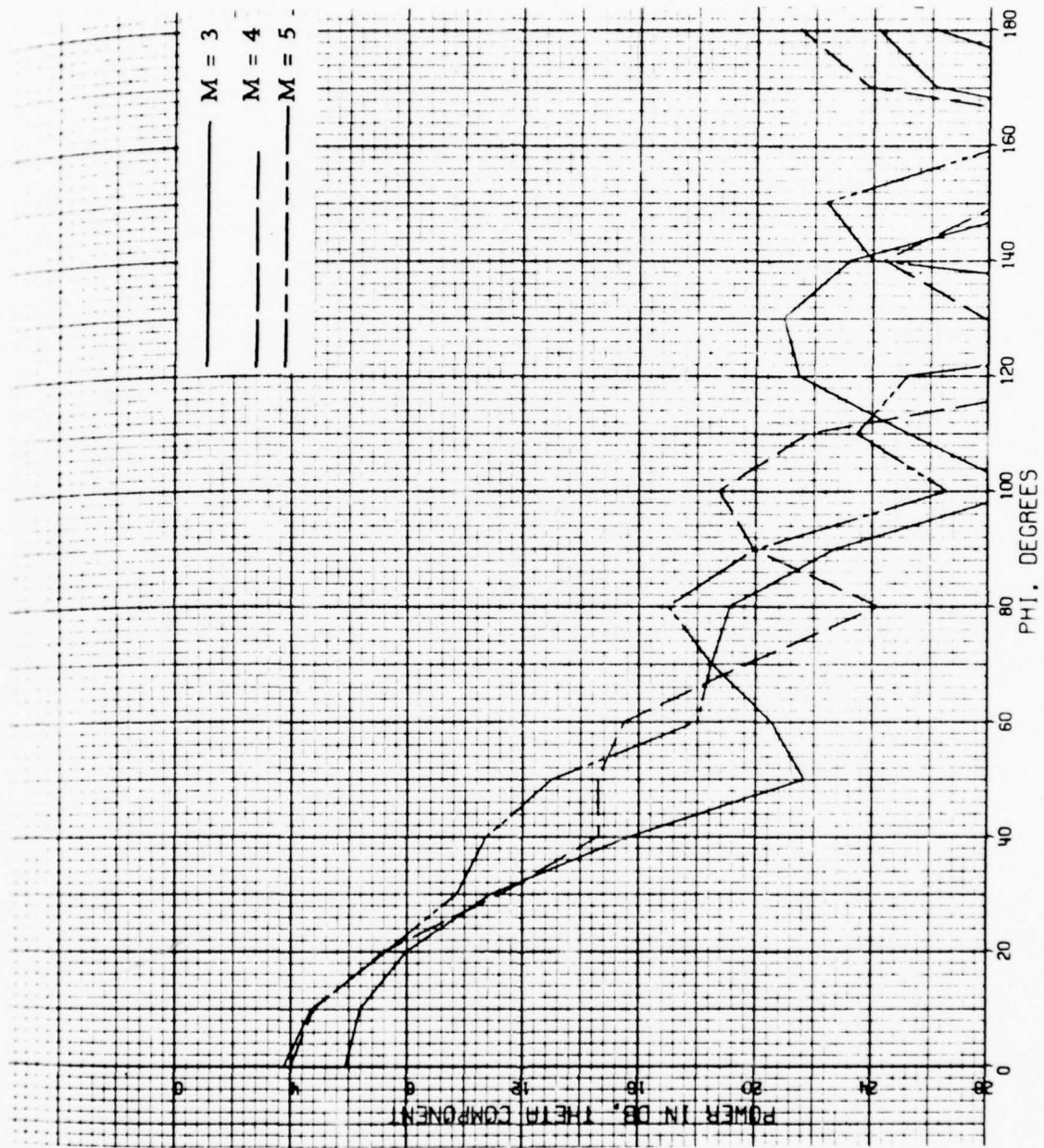


Figure 7. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 3, 4$, and 5

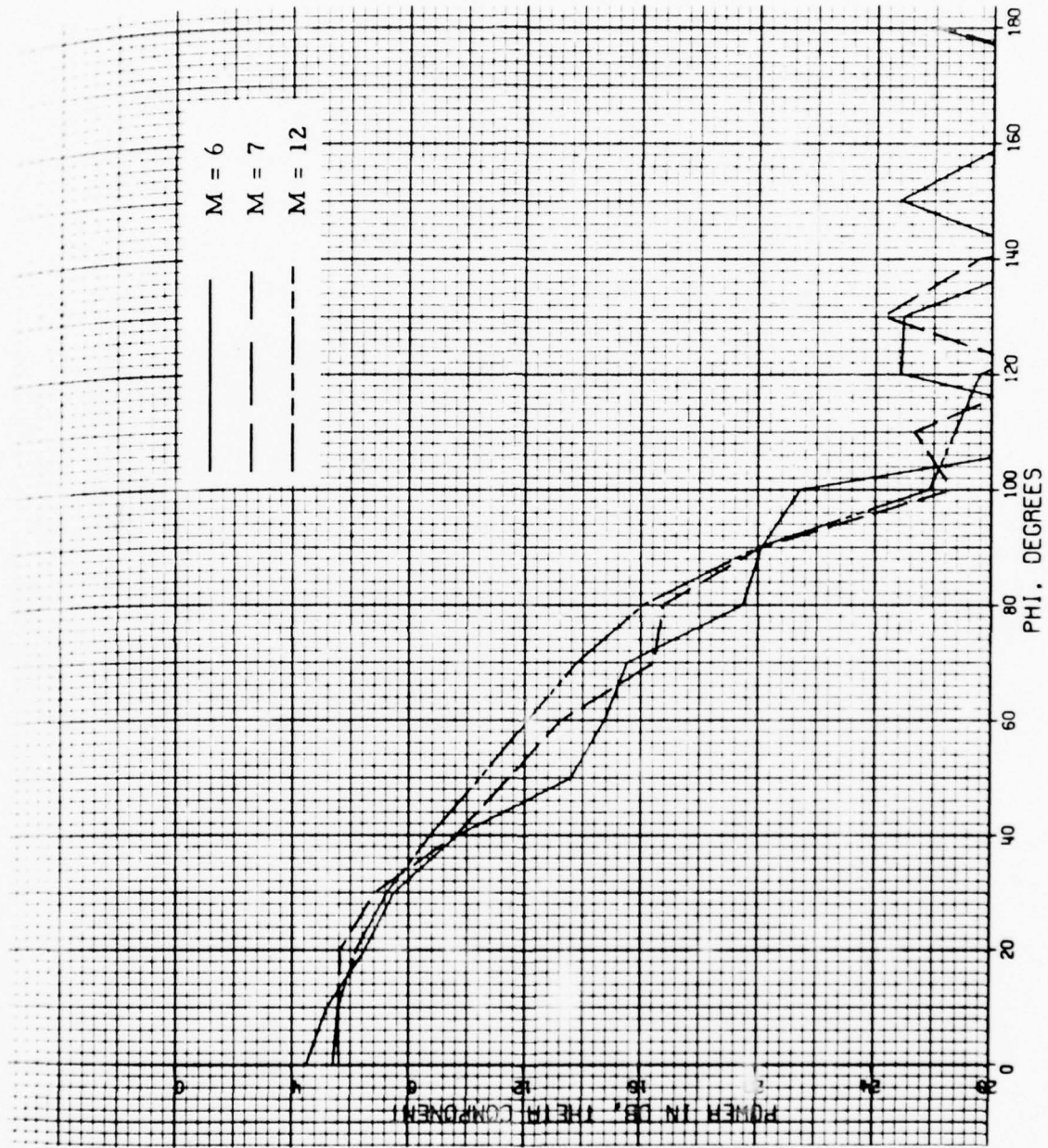


Figure 8. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 6, 7, \text{ and } 12$

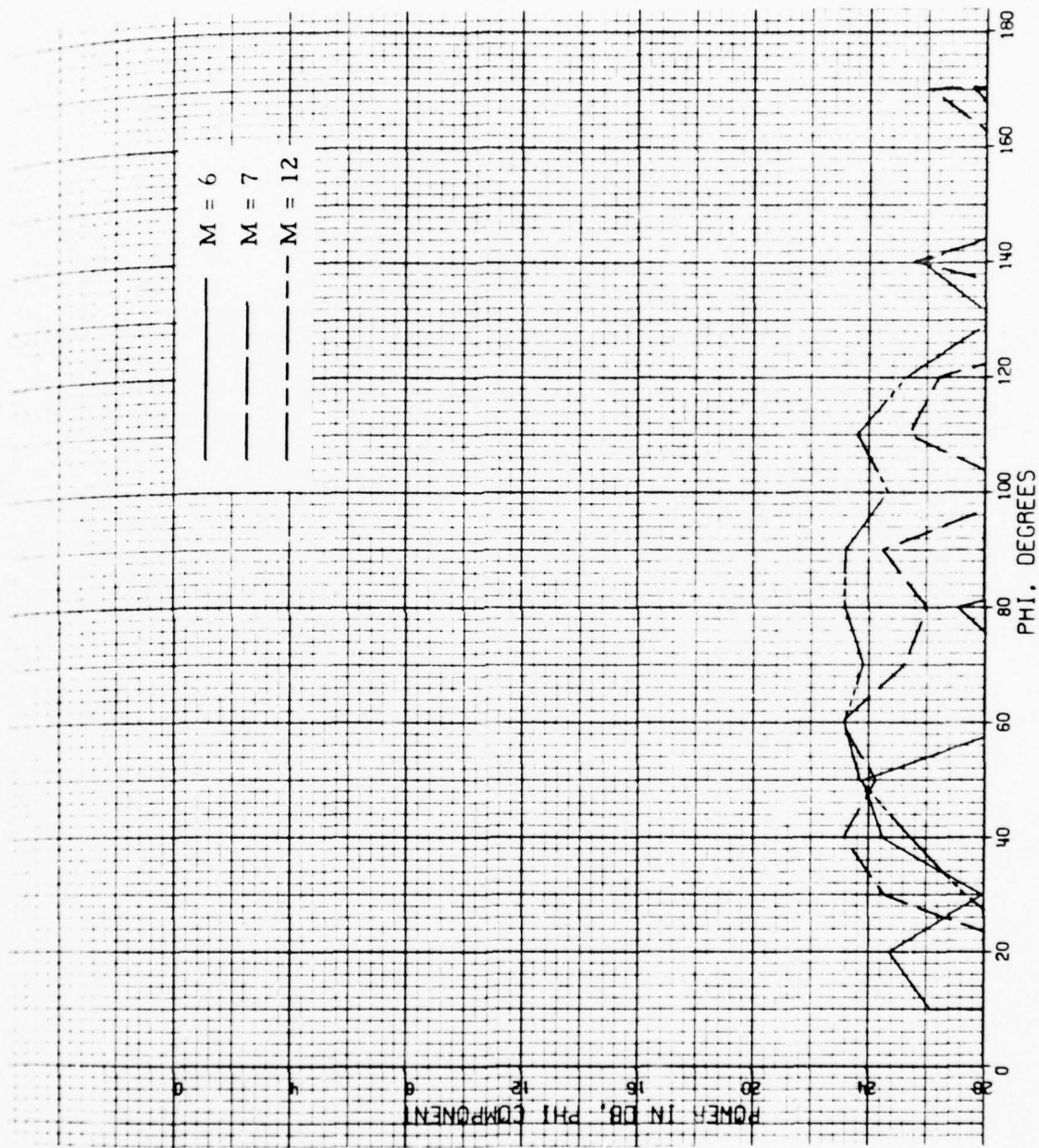


Figure 9. ϕ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\theta = 80^\circ$, $M = 6, 7, \text{ and } 12$

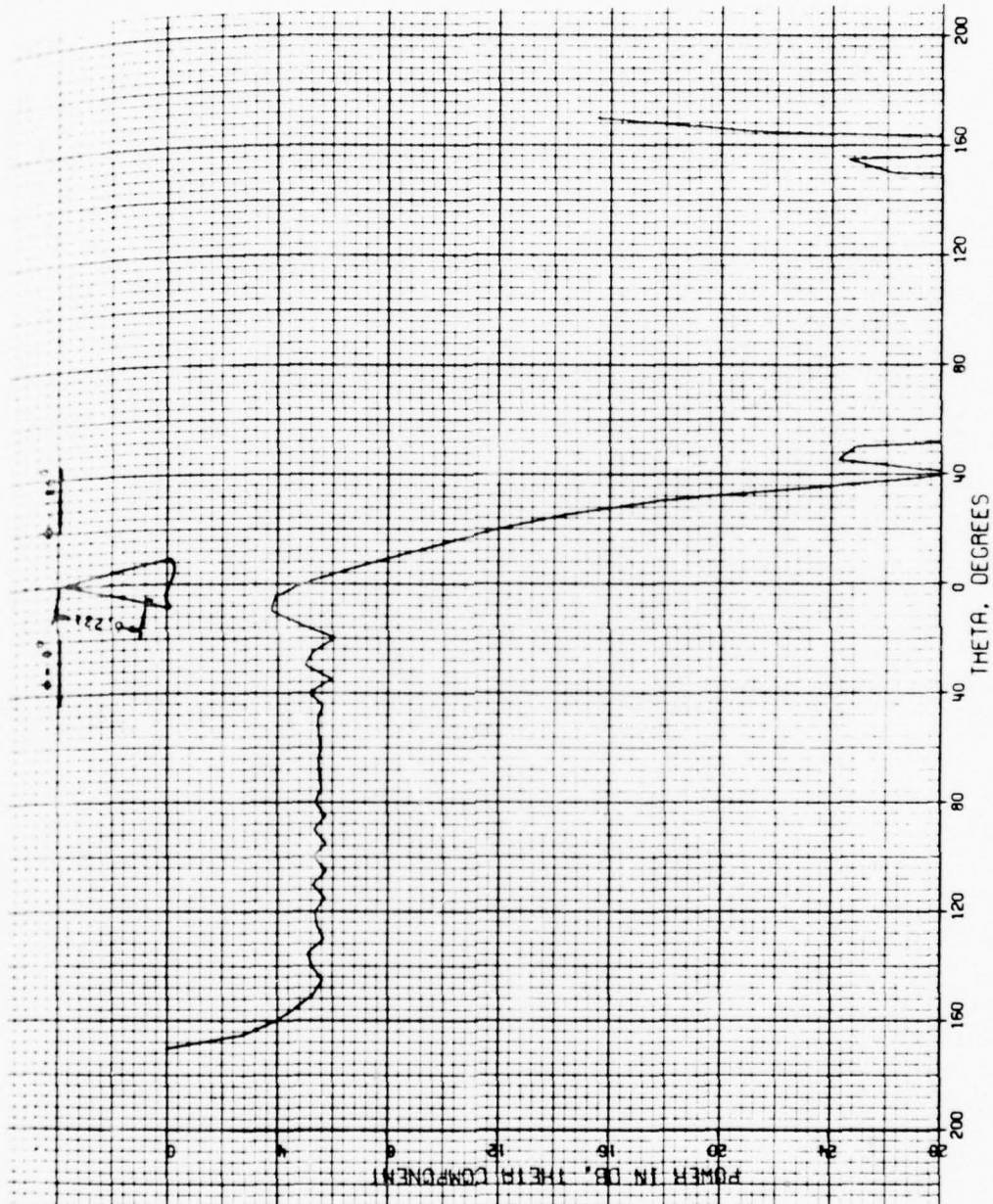


Figure 10. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot
for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 13$

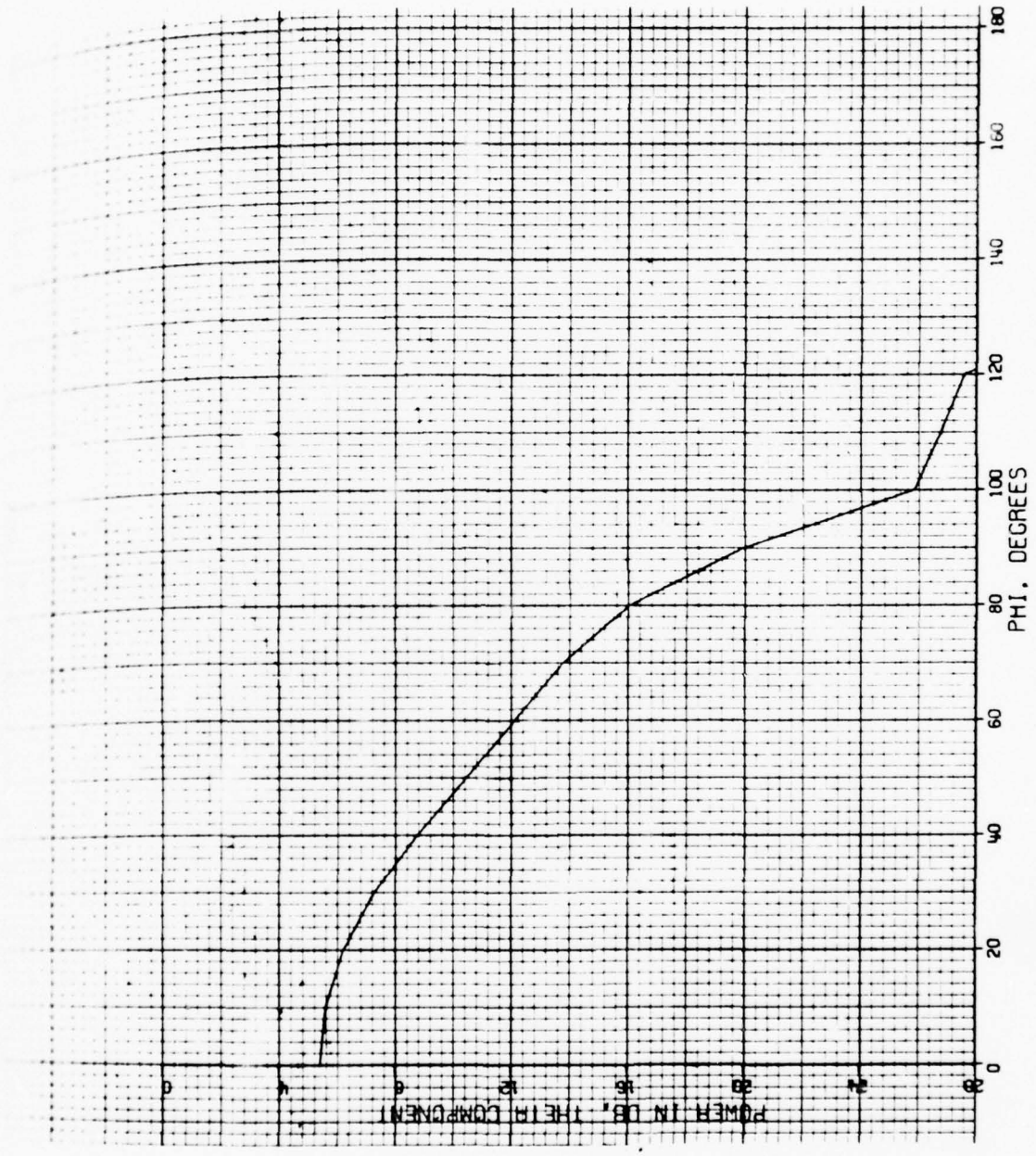


Figure 11. 9-Polarized Total Pattern of $\lambda/2$ Circumferential Slot
for $\theta = 80^\circ$, $M = 13$

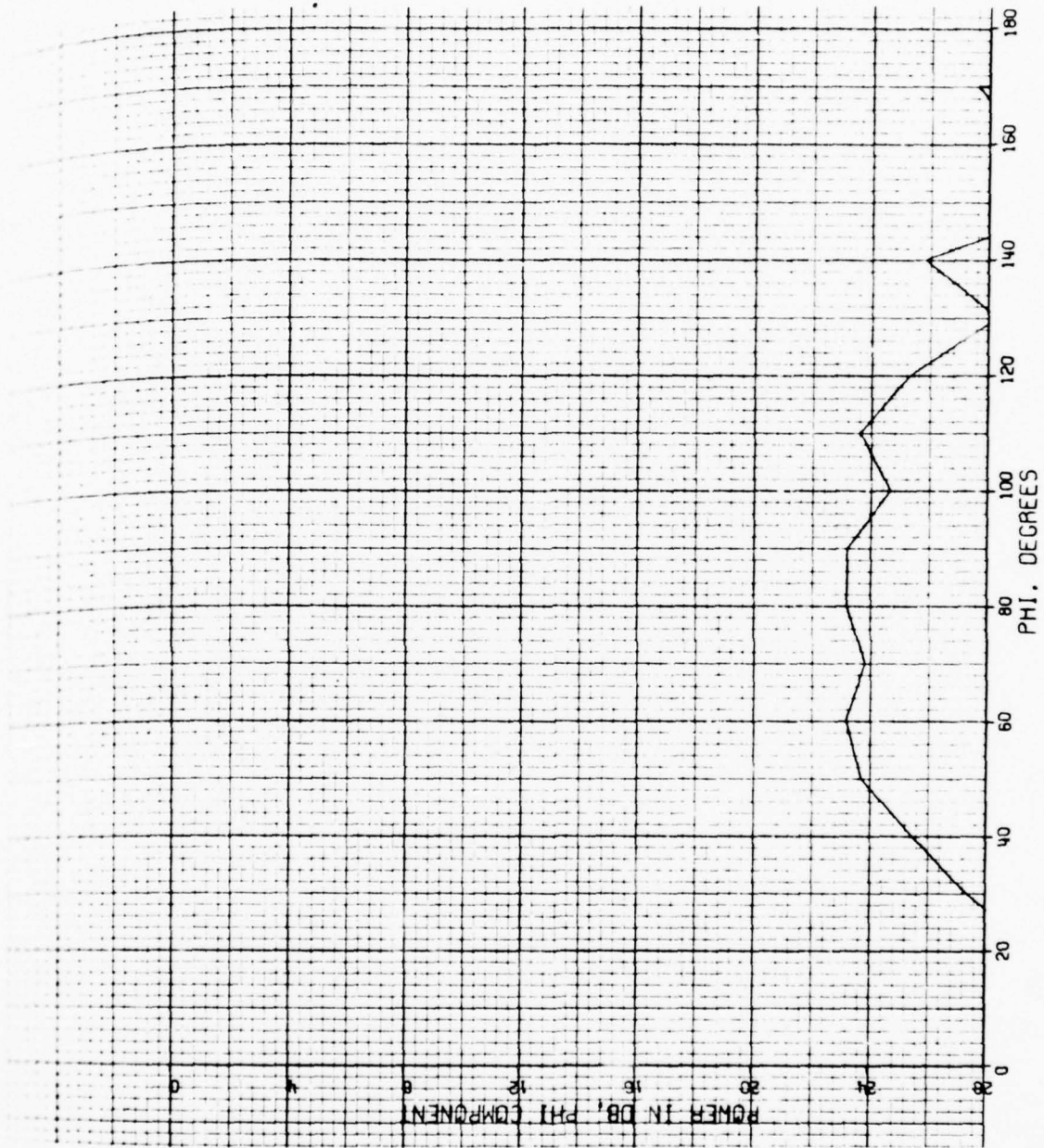


Figure 12. ϕ -Polarized Total Pattern of $\lambda/2$ Circumferential Slot
for $\theta = 80^\circ$, $M = 13$

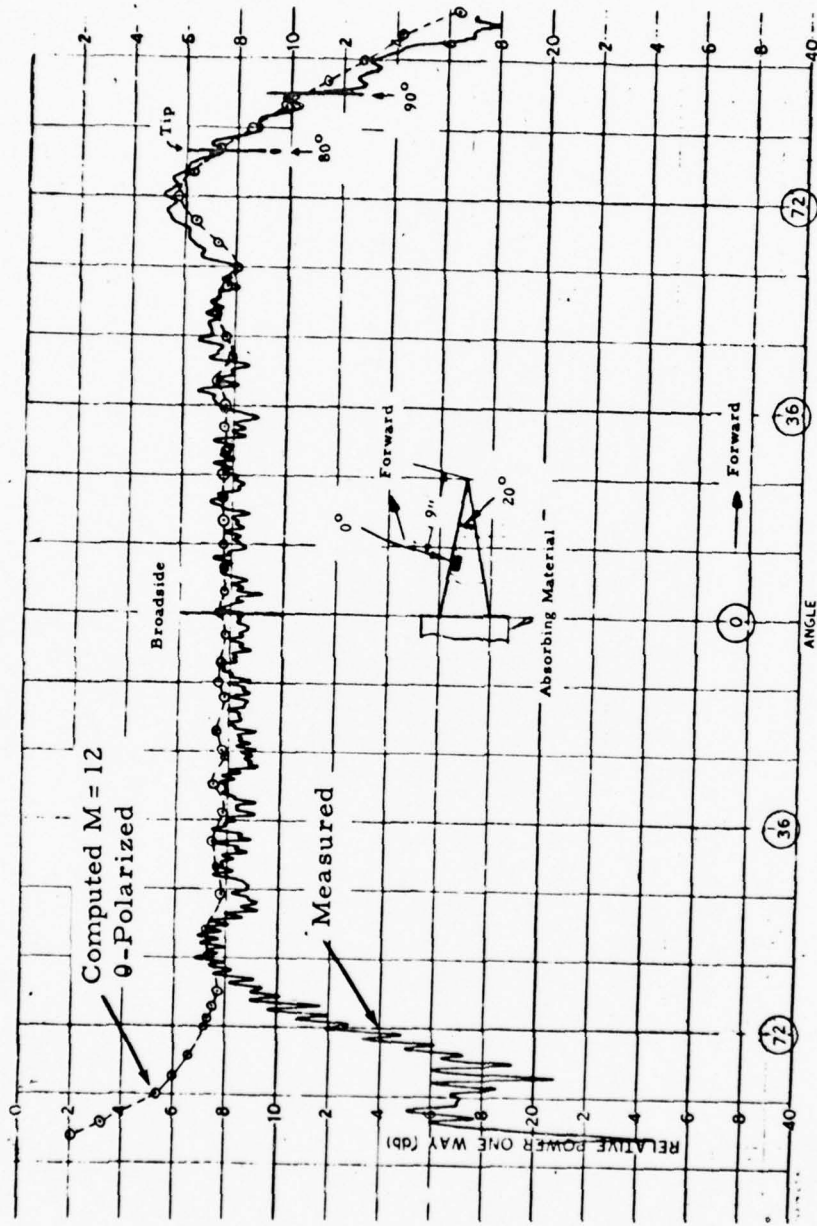


Figure 13. Measured and Computed ($M = 12$) θ -Polarized Pattern of Circumferential Slot on 20° Cone, $f = 8.15$ GHz

measured pattern drop off to the rear is a consequence of shadowing at the base of the cone by the absorbant material in which the cone was set. Thus, it is concluded that about 13 terms of the modal series are required to correctly represent the radiation from this slot configuration. Furthermore, the tip diffraction effects are generally accounted by the lower order modes of the series.

Elevation patterns were also computed for a similar slot located at $ka = 30$ radians from the tip of the cone. These three patterns corresponding to $M = 4$, $M = 5$ and $M = 6$ are shown in Figure 14. It is pointed out that the two cuts shown here are for $\phi = 0^\circ$ and $\phi = 90^\circ$. Although additional modes are required to describe the radiation characteristics of this case adequately, the figures do show the effect of tip diffraction as the slot is moved along a generatrix of the cone. The first minimum of the $\phi = 0^\circ$ patterns now occurs at $\theta = 25^\circ$ from the tip of the cone as compared to $\theta = 20^\circ$ for the $ka = 39$ position (Figure 5).

4.2 RADIATION PATTERNS OF A RADIAL SLOT

In the present section we consider the narrow slot along a generatrix of the cone which was described in Section 2.2. For the present study the length of the slot has been chosen as one-half wavelength at the operating frequency. The cone characteristics and the operating frequency, as well as the location of the center of the slot remain the same as for the circumferential slot described in Section 4.1. The slot is excited by a voltage V'_0 resulting in an electric field in the ϕ -direction given by

$$E_\phi = V'_0 \frac{\cos(kr' - ka)}{r' \sin \theta_0} \delta(\phi')$$

It is interesting to follow the convergence of the modal series of the radial slot in the same manner that was done for the circumferential slot. We first look at the $m = 0$ and $m = 1$ mode patterns of the ϕ -polarized components at $\phi = 0^\circ$ depicted in Figures 15 and 16. The effect of the cone's tip on the modal patterns is clearly seen by the behavior of the patterns near the axis of the cone. This effect continues

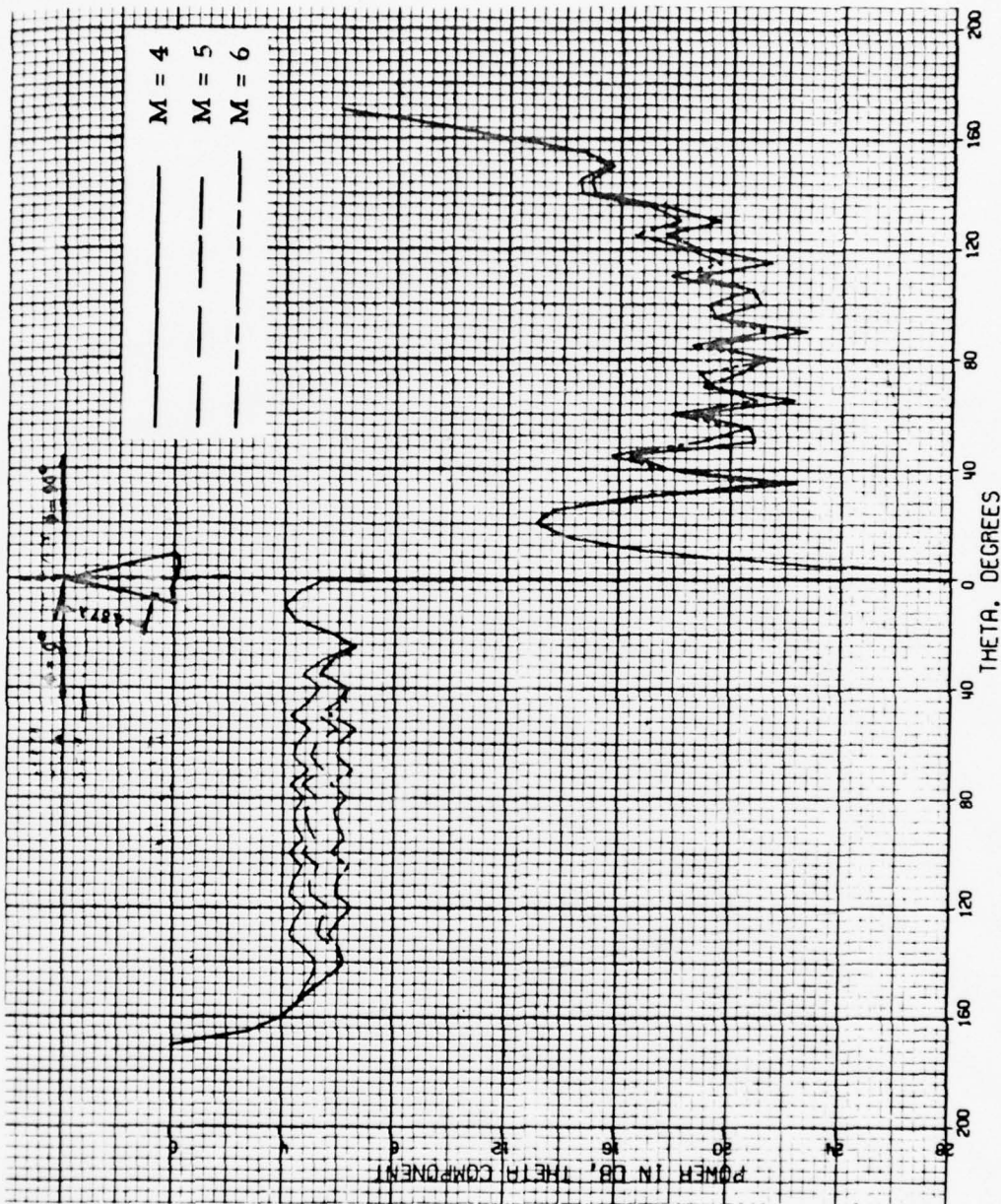


Figure 14. θ -Polarized Total Patterns of $\lambda/2$ Circumferential Slot for $\phi = 0^\circ$, $\phi = 90^\circ$, $M = 4, 5, 6$, and $ka = 30$

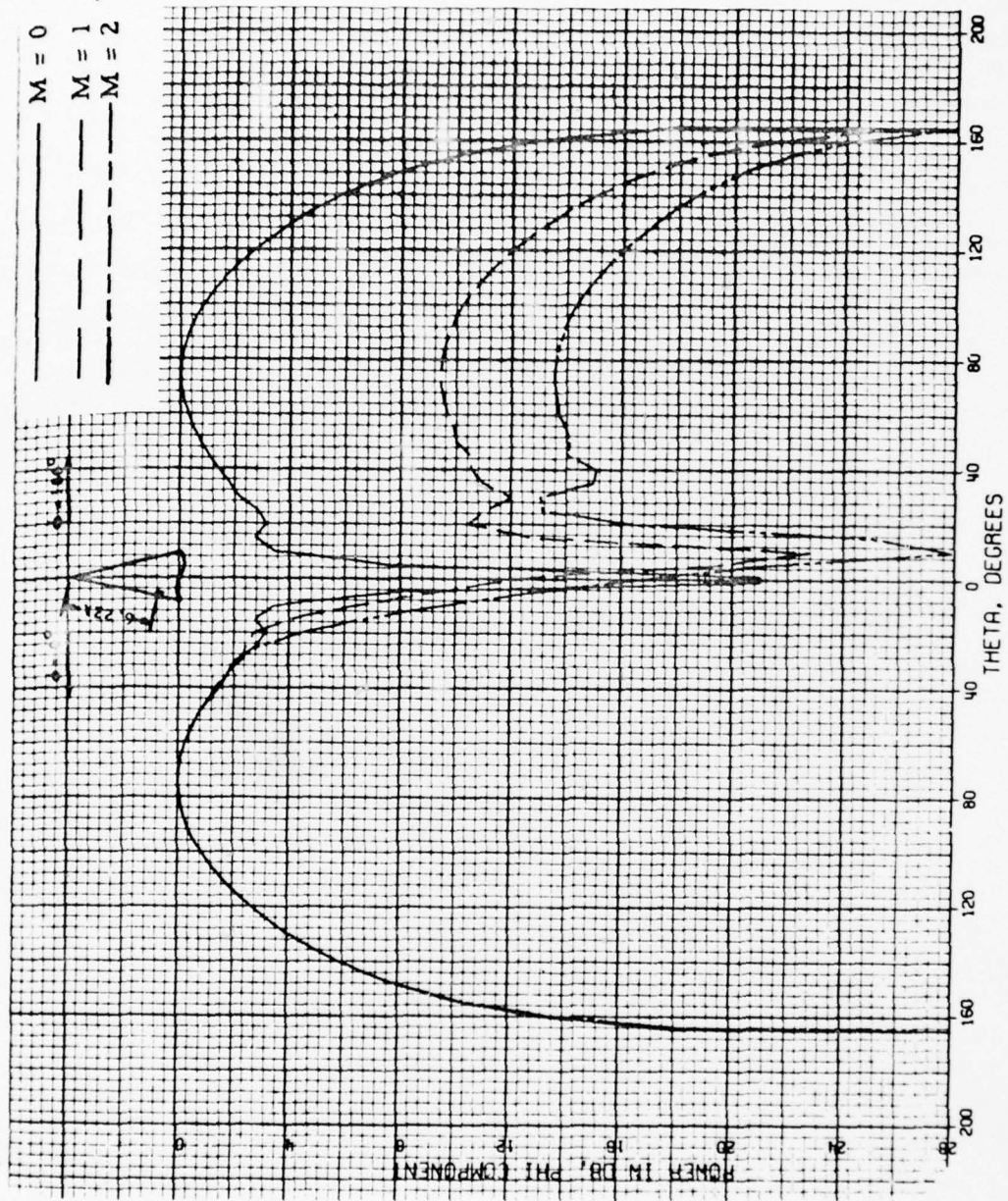


Figure 15. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 0, 1$, and 2

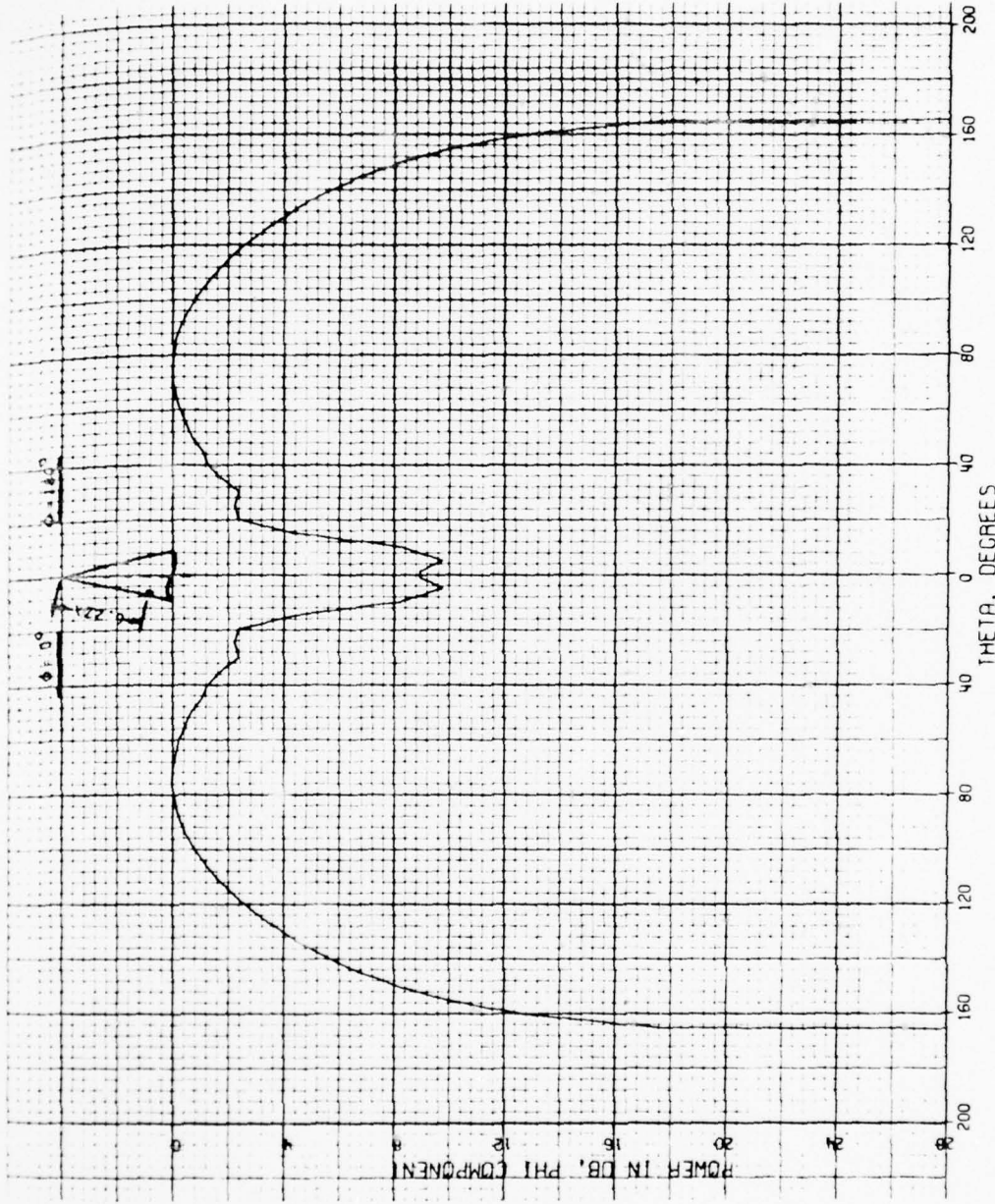


Figure 16. ϕ -Polarized Modal Pattern of $\lambda/2$ Radial Slot for $\phi = 0^\circ, \phi = 180^\circ, m = 1$

to persist also in the total patterns of $M = 1$ through $M = 5$, shown in Figures 15 and 17. It is evident from the Figures that the total patterns are dominated by the $m = 1$ mode. Both the patterns for $\phi = 0^\circ$ and $\phi = 180^\circ$ exhibit a local maximum at about 20 degrees from the axis. This maximum moves toward the broadside region with increased number of terms of the modal series. Also, the higher order total patterns are increasingly narrower. The higher order patterns of $M = 6, 7,$ and 12 are shown in Figure 18.

The $M = 12$ for $\phi = 0^\circ$ pattern shows a minimum of -18 dB at $\theta = 1^\circ$ and has its maximum at exactly broadside. The pattern for $\phi = 180^\circ$ of the same case dips to a minimum near $\theta = 15^\circ$ and then increases to about -14 dB at $\theta = 100$ degrees. Both patterns decrease rapidly near $\theta = \theta_0$. The irregularities noticed in all the previous total patterns are not present at all in the $M = 12$ patterns.

Azimuth patterns for the ϕ -polarized component and the θ -polarized component for the radial slot are shown by Figures 19 through 23. As it is seen from the Figures, all of the low order sum patterns for both polarizations exhibit considerable variation in azimuth. This can be viewed as an indication of the slow convergence of the modal series and the requirement for additional terms in the series representation. On the other hand, the $M = 12$, ϕ -polarized total pattern shows a smooth transition from 0 dB at $\phi = 0^\circ$ to -13 dB at $\phi = 140^\circ$ while maximum of the θ -polarized pattern occurs near broadside at about -17 dB.

From the above considerations, it appears that more than eight but less than fourteen terms of the modal series are required to represent the fields of this slot configuration correctly. The upper limit was further checked by looking at the $M = 13$ patterns and comparing them with the $M = 12$ patterns. These patterns are shown in Figures 24 through 26. There were no noticeable changes in either the elevation or the azimuthal patterns.

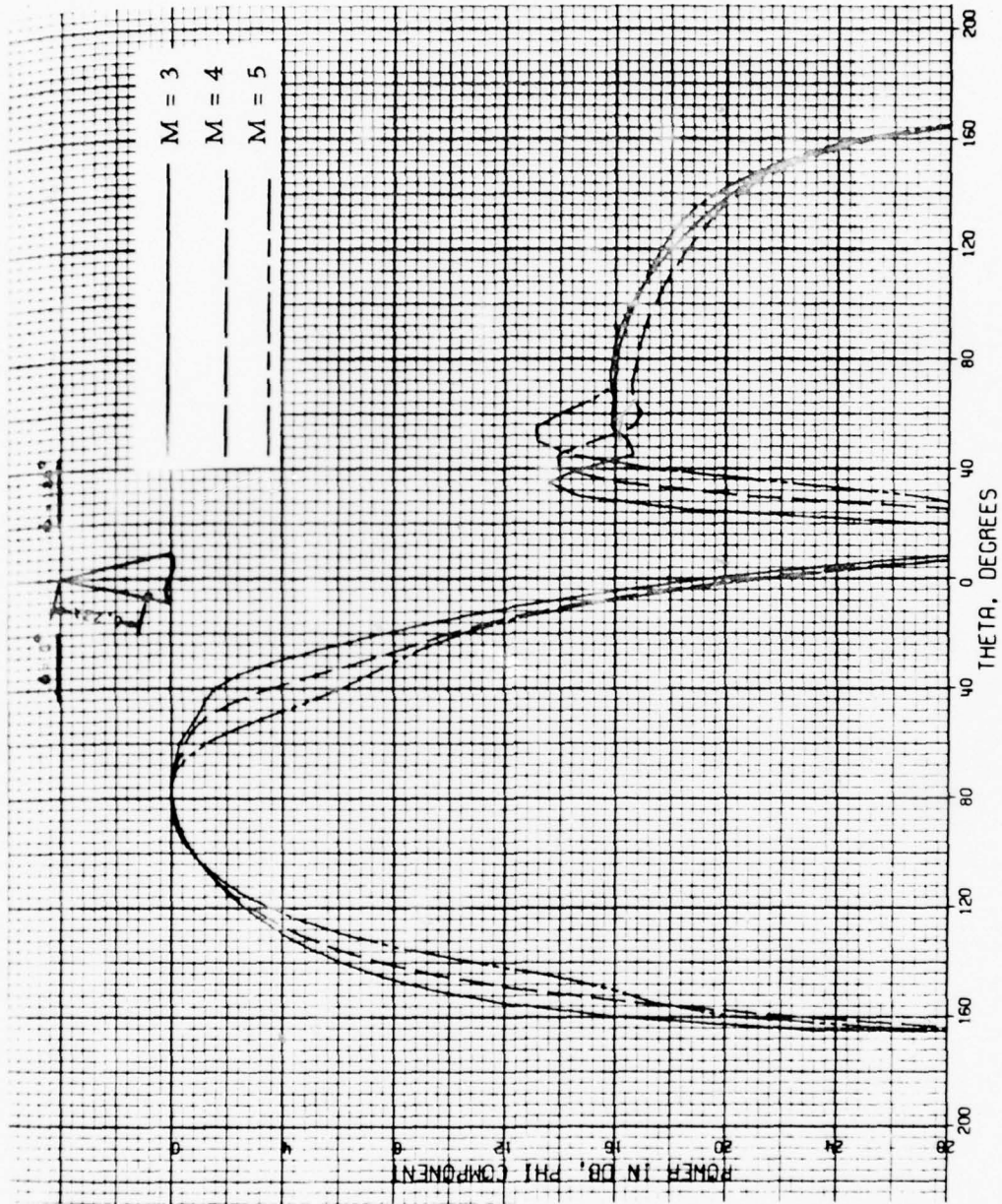


Figure 17. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 3, 4$ and 5

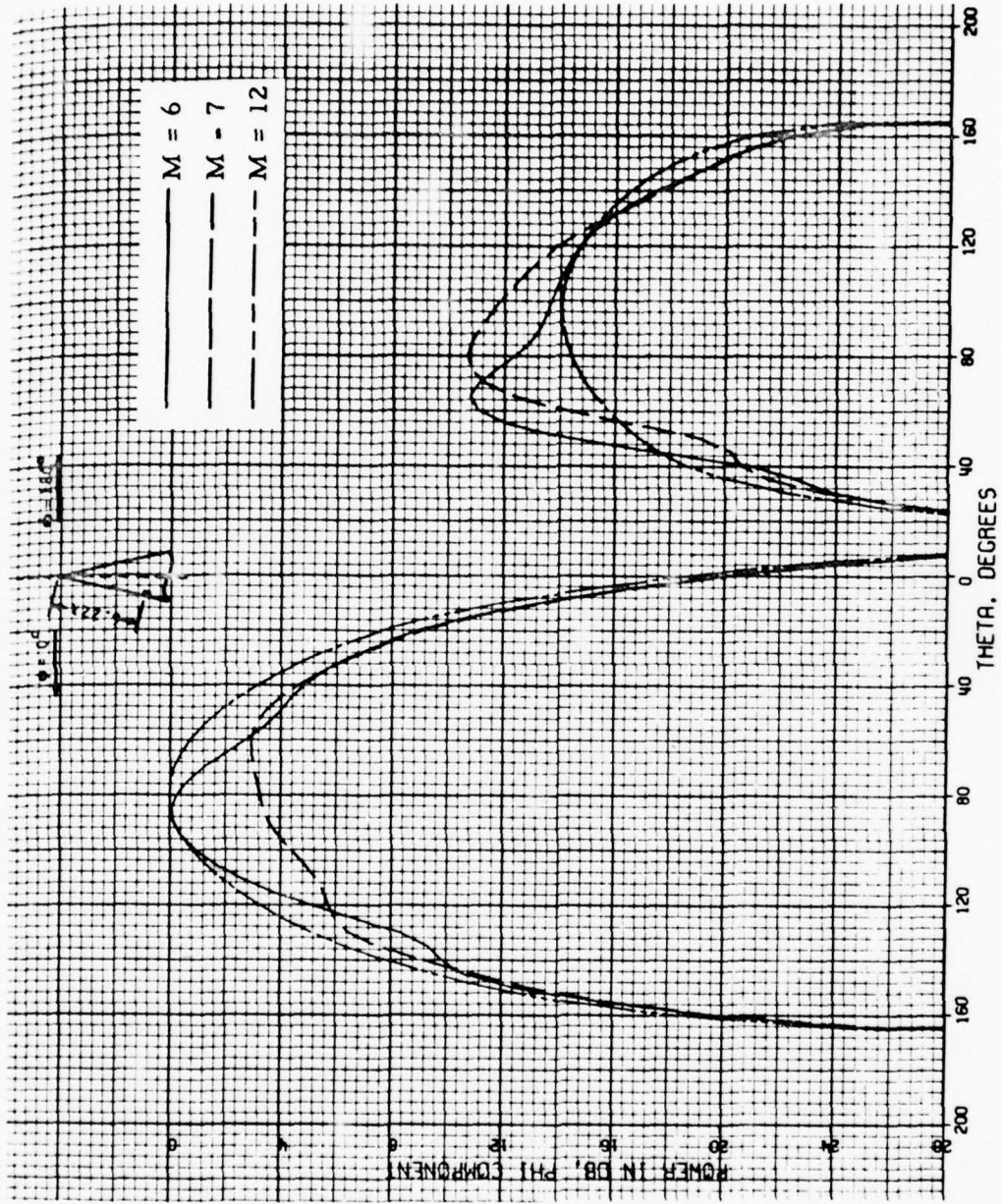


Figure 18. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 6, 7$, and 12

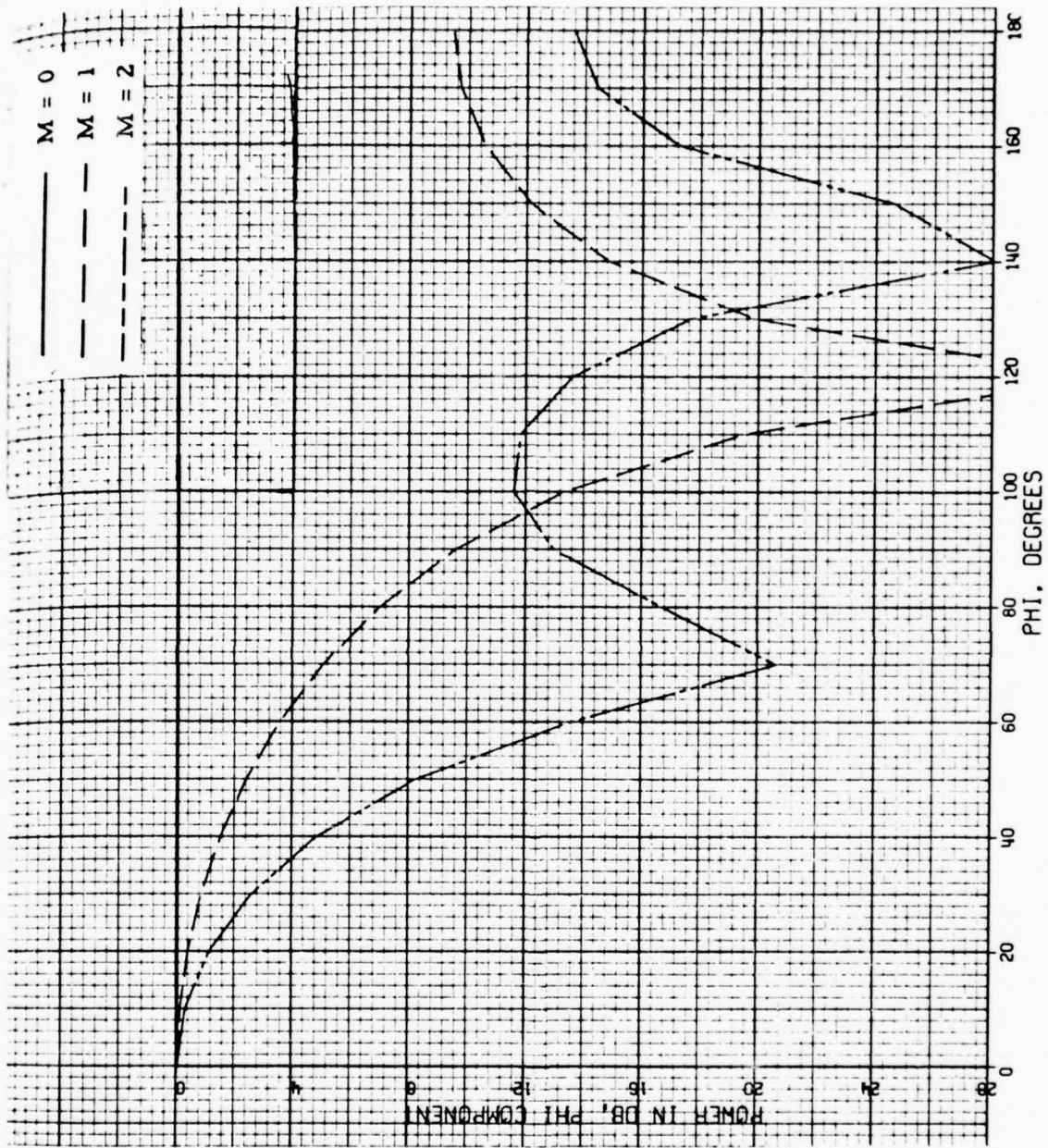


Figure 19. ϕ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 0, 1, \text{ and } 2$

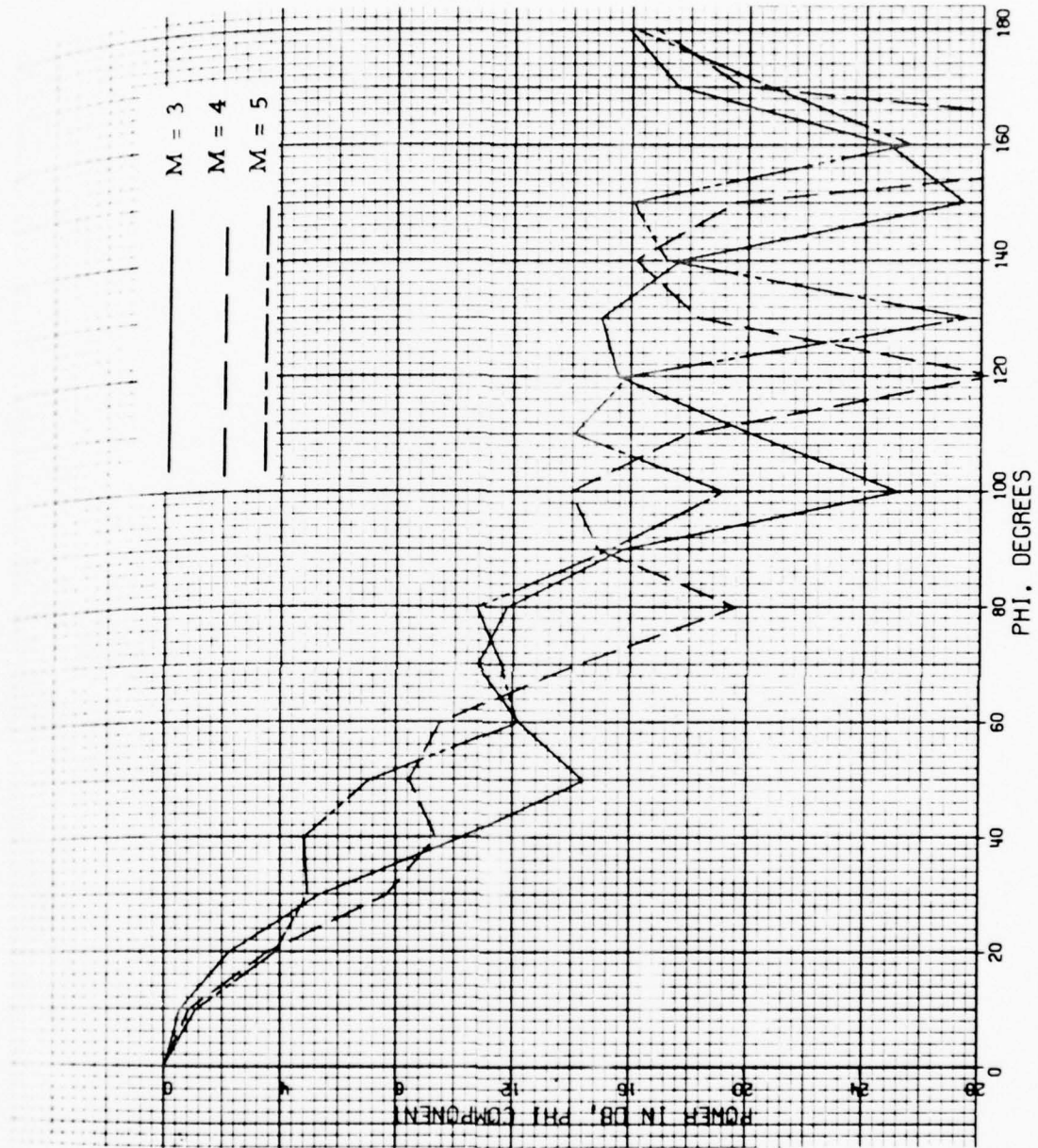


Figure 20. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 3, 4, \text{ and } 5$

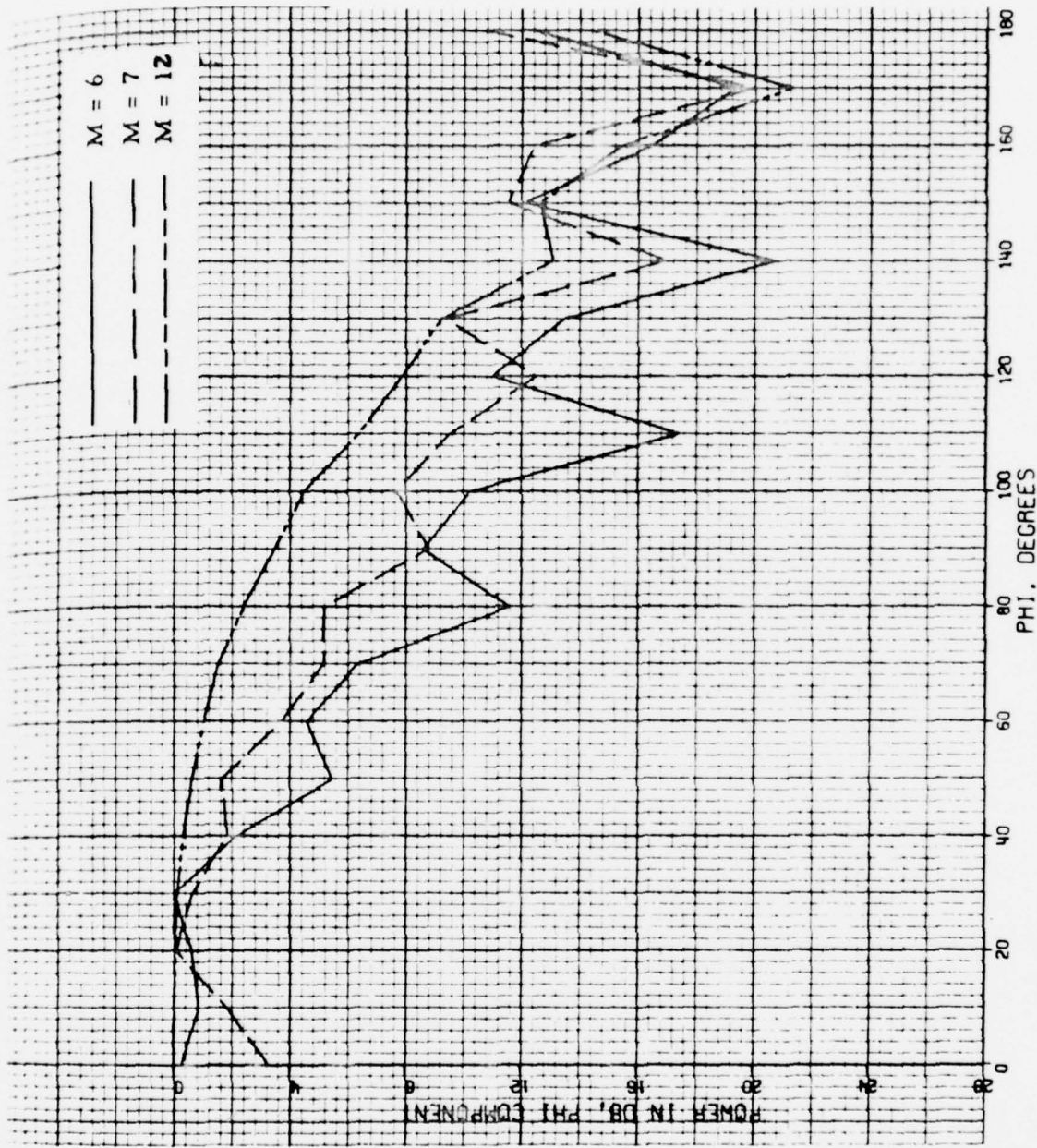


Figure 21. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 6, 7, \text{ and } 12$

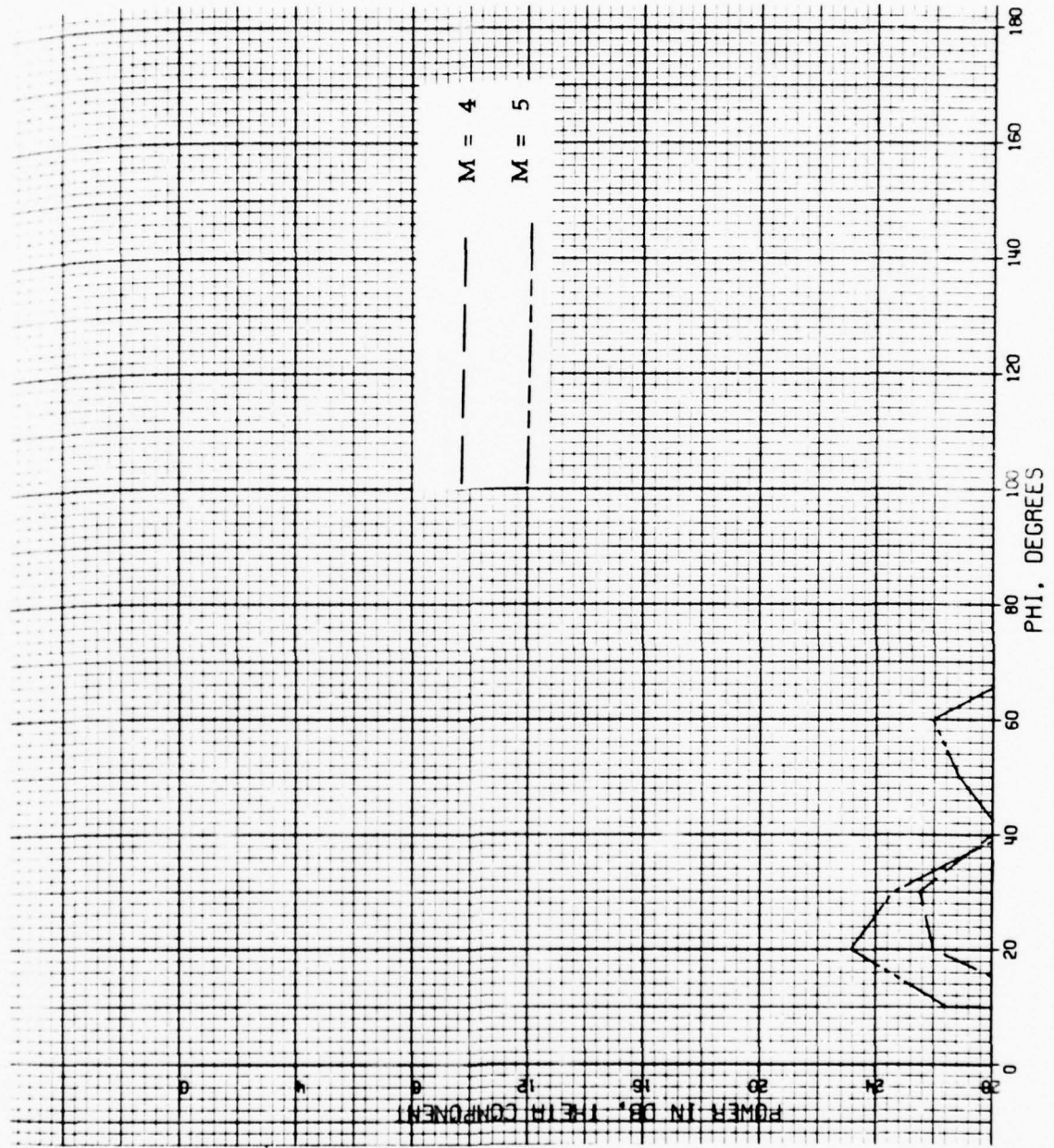


Figure 22. θ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 4$ and 5

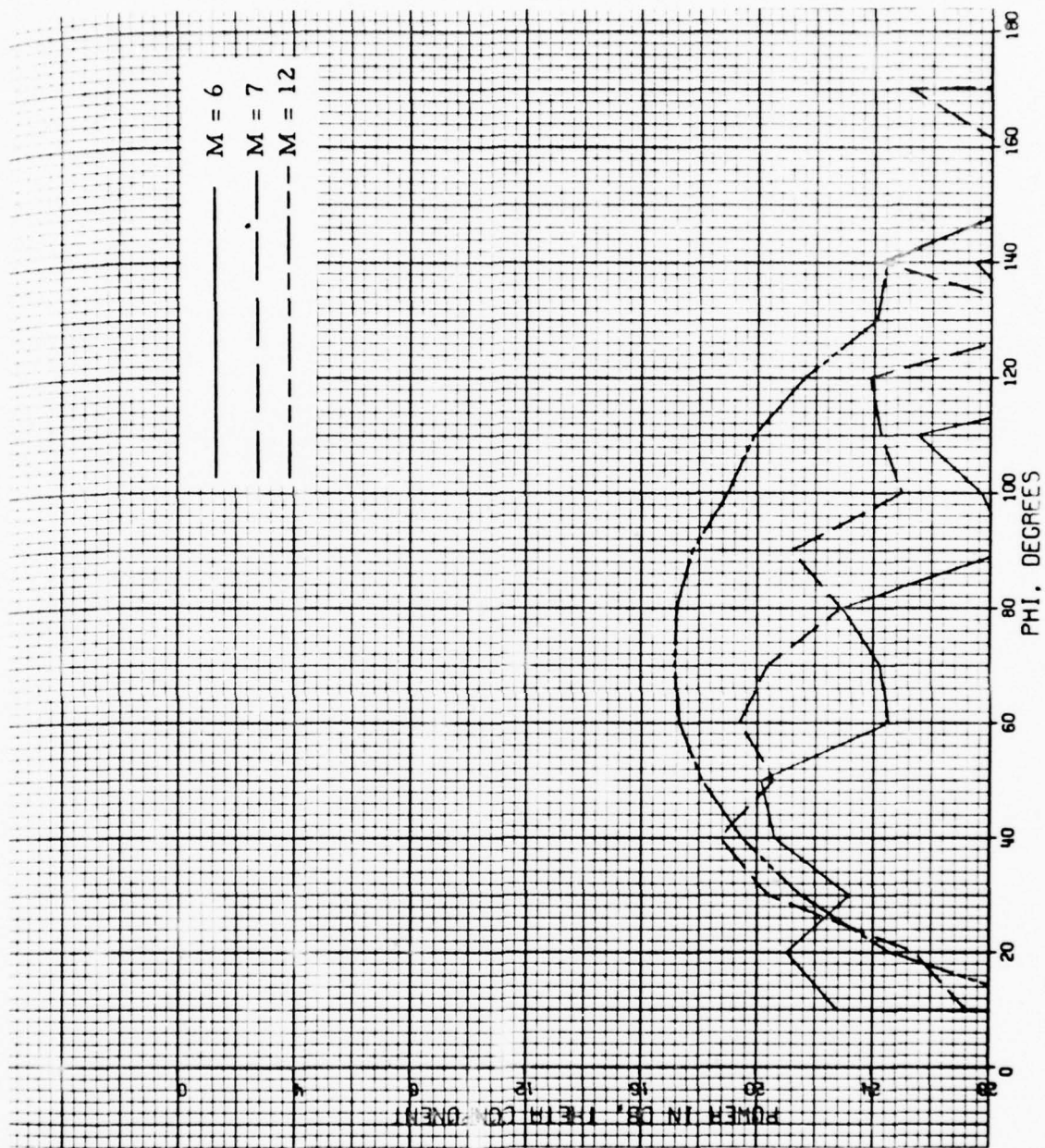


Figure 23. θ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 6, 7, \text{ and } 12$

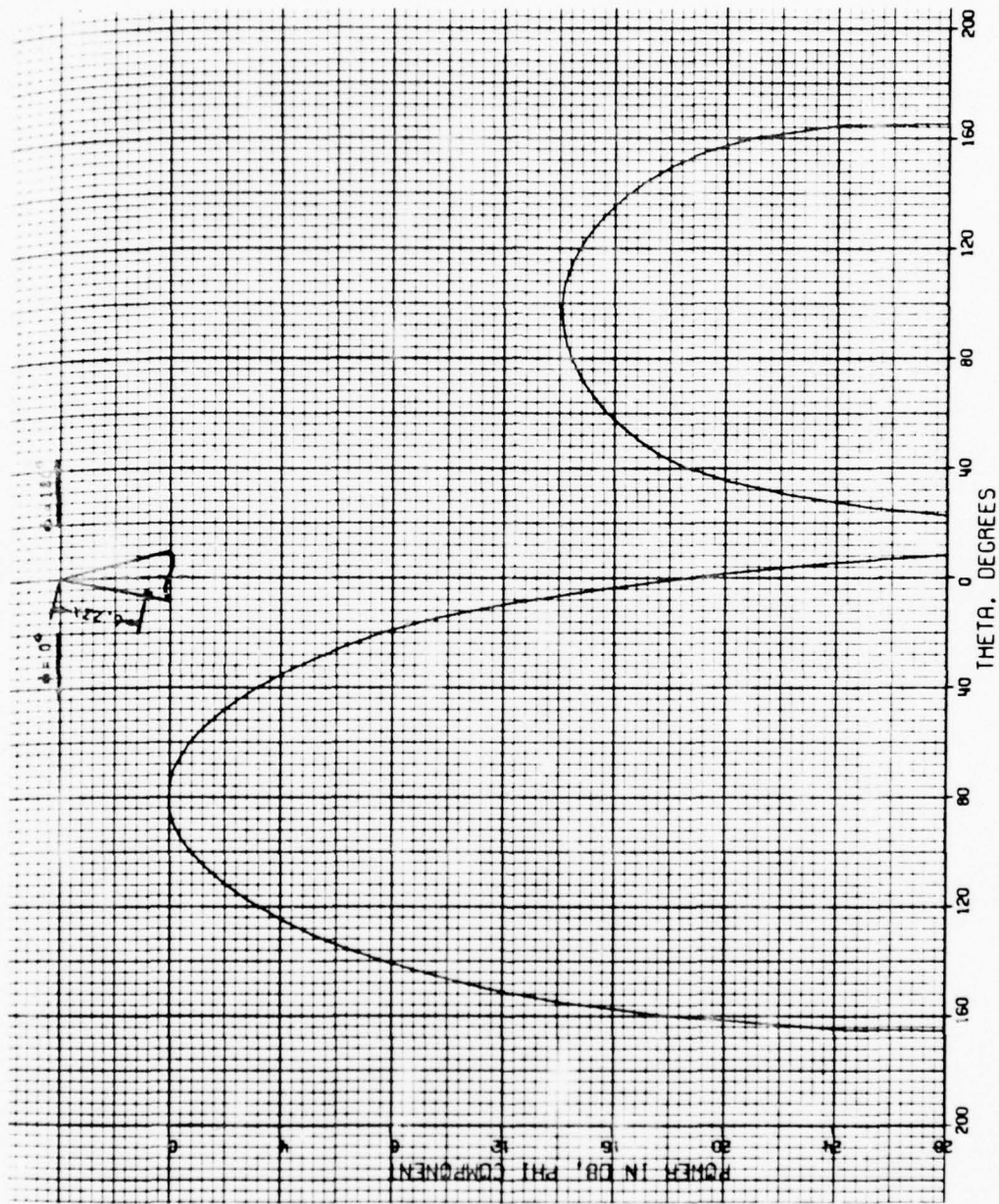


Figure 24. ϕ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 180^\circ$, $M = 13$

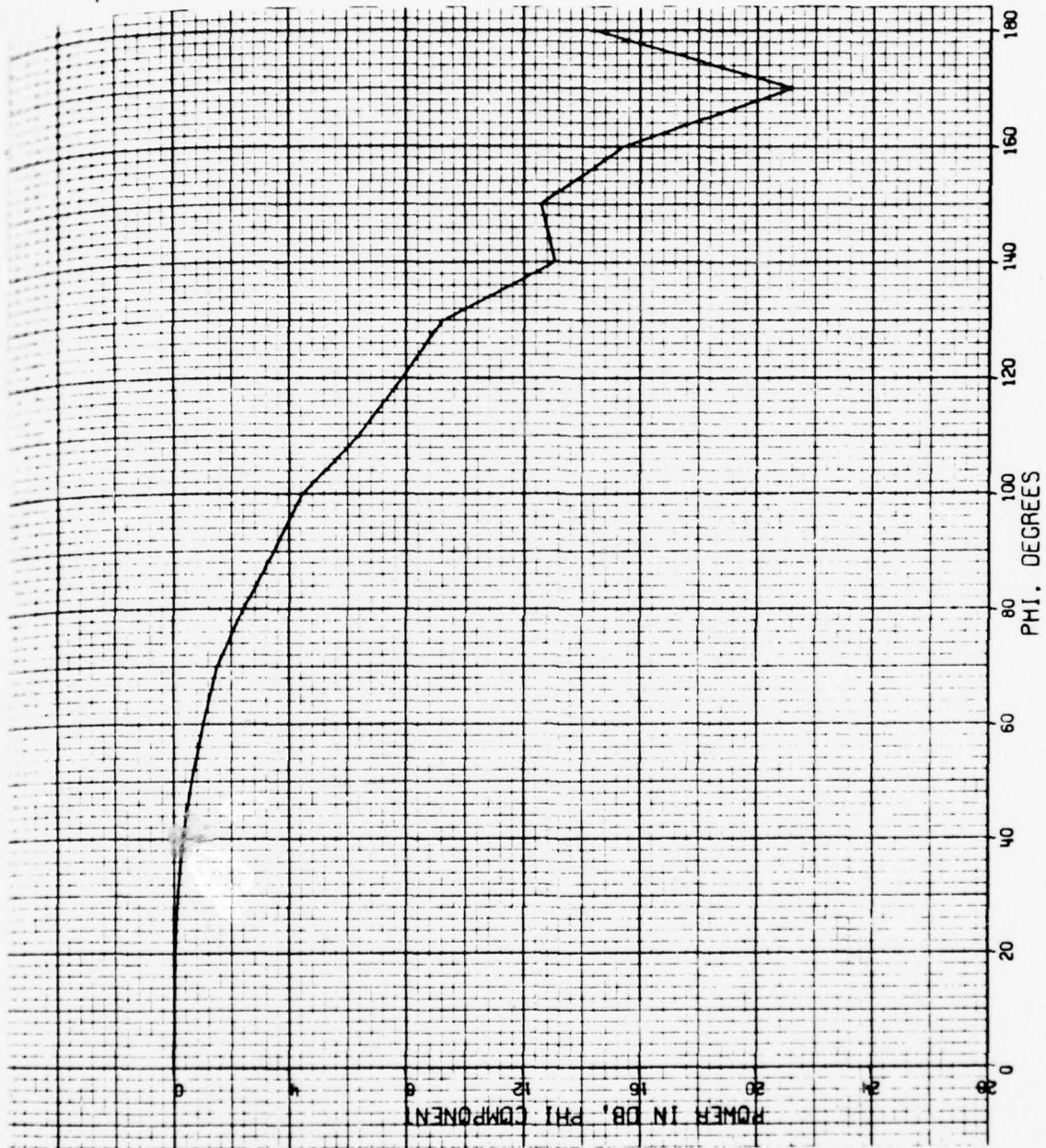


Figure 25. ϕ -Polarized Total Pattern of $\lambda/2$ Radial Slot for
 $\theta = 80^\circ$, $M = 13$

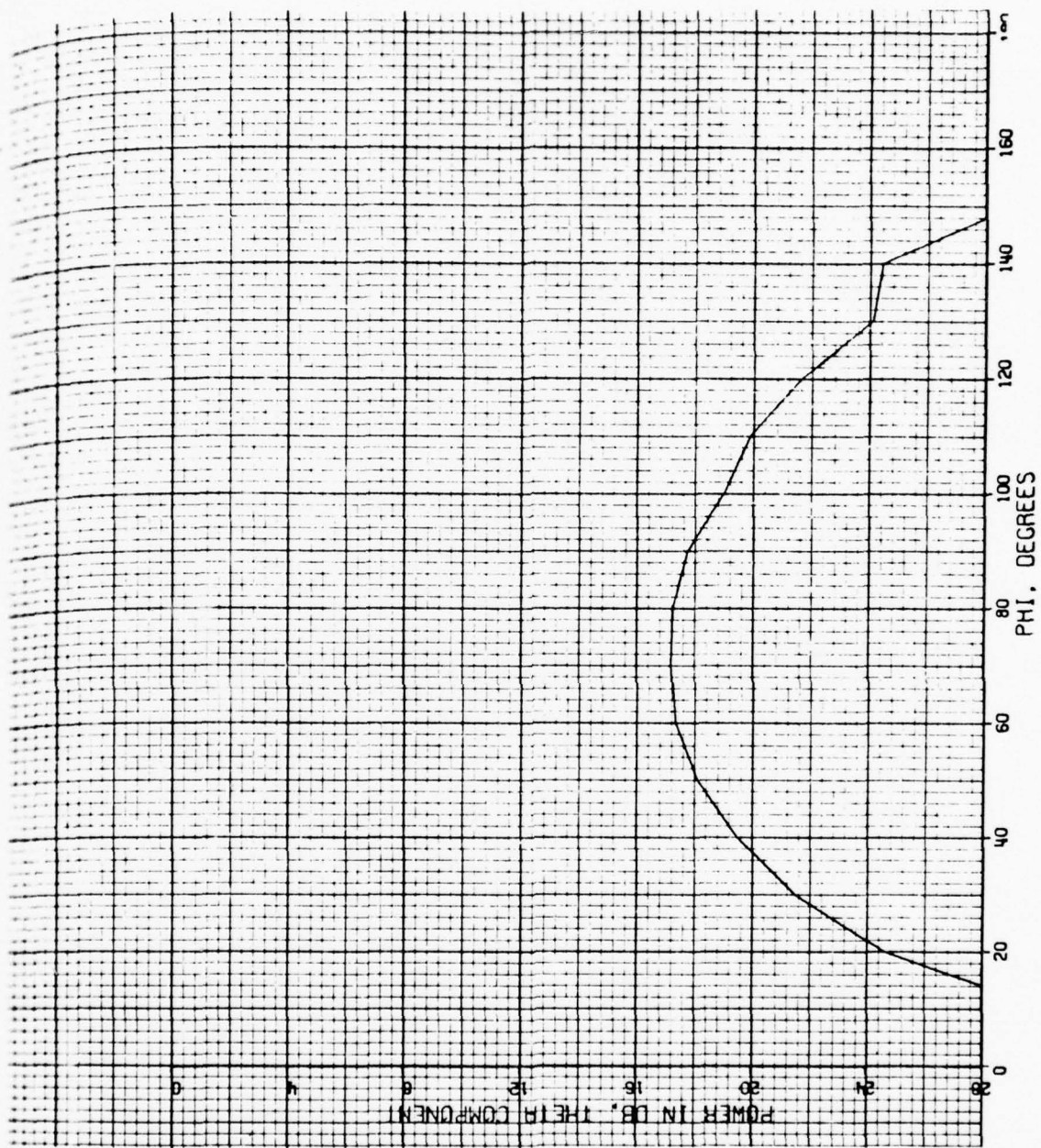


Figure 26. θ -Polarized Total Pattern of $\lambda/2$ Radial Slot for $\theta = 80^\circ$, $M = 13$

Elevation patterns of the same slot located at $ka = 30$ radians from the tip of the cone were also computed and are shown in Figure 27. As in the circumferential slot for this position, the $\phi = 0^\circ$ and the $\phi = 90^\circ$ cuts are shown. The maximum of the $M = 6$ pattern occurs at $\theta = 72^\circ$ instead of the expected broadside, $\theta = 80^\circ$. This is because of the insufficient number of modes in the truncation of the modal series. All other aspects of the pattern are the same as for the $ka = 39$ position, indicating that tip diffraction has little effect on the radiation pattern of the radial slot. A greater number of terms must be taken in the modal series, however, as the slot is moved farther away from the tip of the cone. Also, these higher order modes affect mainly the broadside region of the element pattern.

Although we do not have experimental data for the slot described in this section for comparison at this time, the above patterns do look very much like the patterns of a slot on a cylinder tipped 10 degrees from its axis. In further studies of the problem, the above results will be verified experimentally.

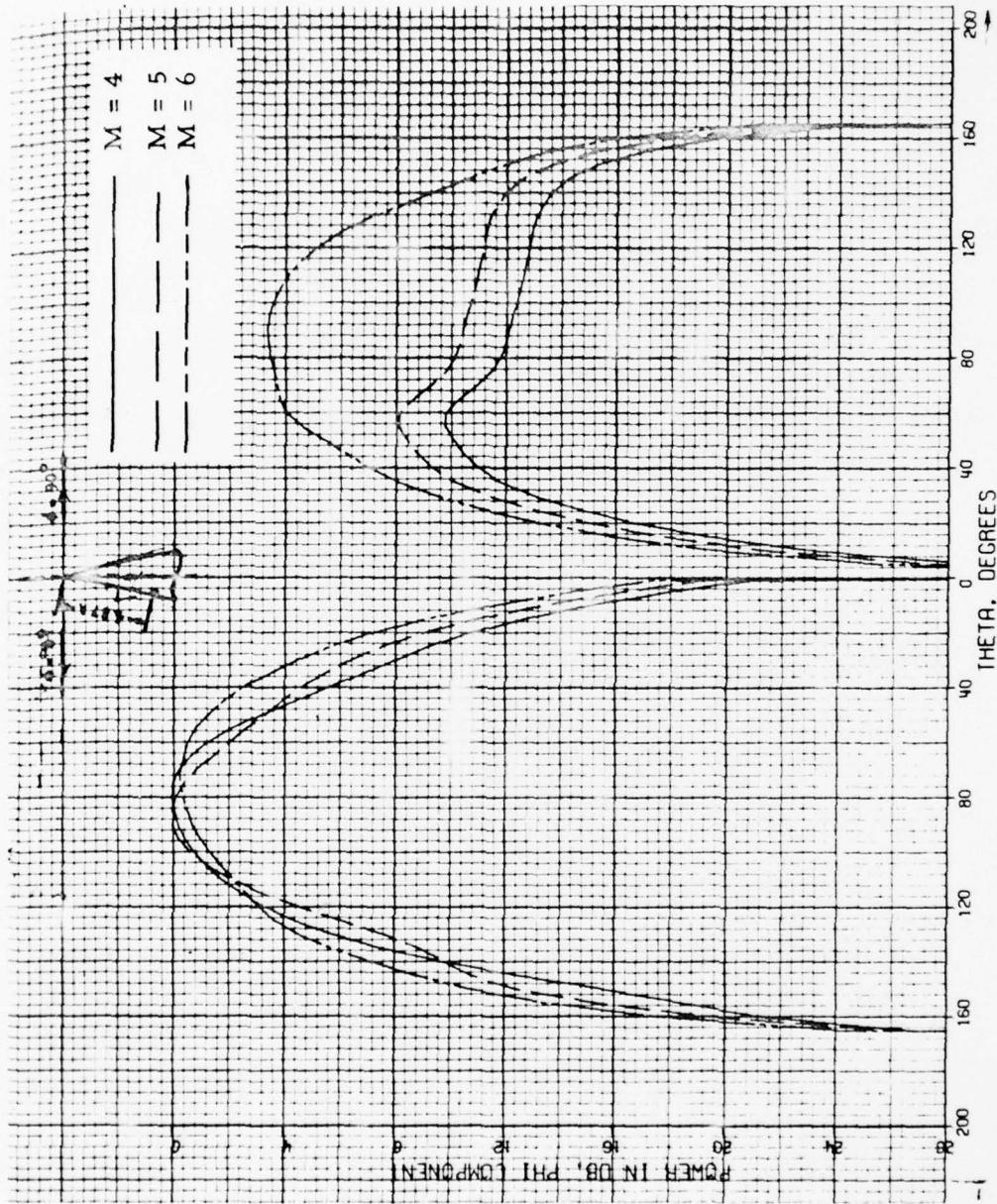


Figure 27. ϕ -Polarized Total Patterns of $\lambda/2$ Radial Slot for $\phi = 0^\circ$, $\phi = 90^\circ$, $M = 4, 5, 6$, and $ka = 30$

5.0 RECOMMENDATIONS FOR FURTHER INVESTIGATIONS

The analysis of the computed element patterns from circumferential and radial slots on a sharply tipped cone presented in Section 4 of this report is only a partial one. For more complete understanding of the radiation characteristics of such elements additional investigative work must be performed. Specifically, the developed computer program for the exact element patterns may now be used to compute patterns of slots near the tip of the cone and very far away from the tip. The results can then be analyzed so as to separate the contribution of tip diffraction from the total pattern. Such a procedure not only aids in understanding the radiation characteristics but also will result in more economical pattern calculation as discussed in the next paragraph.

The present analysis has shown that more modes are required in the truncation of the modal series for element locations far away from the tip of the cone. The computation of a higher order mode, however, requires considerable more computer time, for the index of the ν -series in the field expressions is a function of both the order of the mode and the location of the element relative to the tip of the cone. If the contribution of tip diffraction is separated from the total pattern, many repetitive computations would be eliminated and conical array problems will be solved more economically. Such an approach will allow the computation of patterns to verify the equivalence principle pattern synthesis technique.^[1]

The computed element patterns of the circumferential slot relatively close to the tip of the cone have shown good agreement with measured patterns. The experimental verification, however, needs to be accomplished for other element positions and orientations. In addition, the present analysis technique could be extended so as to take into consideration mutual coupling effects. The results could then be compared with similar results obtained from the geometrical theory of diffraction.

REFERENCES

1. Kummer, W. H., A. F. Seaton, and A. T. Villeneuve, (1973). "Conformal Antenna Arrays Study," Final Report on Contract N00019-72-C-0212, Report No. 2765, 31/421, Hughes Aircraft Company, January, 1972 to January, 1973.
2. Bargeliotas, P. C., A. F. Seaton, and A. T. Villeneuve (1973), "Dynamic Impedance Matching in Conformal Arrays", Quarterly Report, 1 Jan. to 1 April 1973, Contract N00019-73-C-0127, Hughes Aircraft Company.
3. Pridmore-Brown, D. C., 1972, "Radiation Patterns from Slot-Excited Cones," IEEE Trans. on Antennas and Propagation, Vol. AP-20, No. 6, November, 1972, pp. 815-816.
4. Pridmore-Brown, D. C. and G. Stewart, 1972, "Radiation from Slot Antennas on Cones," IEEE Trans. on Antennas and Propagation Vol. AP-20, No. 1, January, 1972, pp. 30-39.
5. Behnke, M. C., A. T. Villeneuve, and W. H. Kummer, 1972, "Advanced Conformal Array Antenna Techniques," Final Report, February 1971 to January 1972, Contract N00019-71-C-0197, Hughes Aircraft Company.
6. Bailin, L. L. and S. Silver, 1956, "Exterior Electromagnetic Boundary Value Problems for Spheres and Cones," IRE Trans. on Antennas and Propagation, Vol. AP-4, January, 1956, pp. 5-16.
7. Bailin, L. L. and S. Silver, 1957, "Exterior Electromagnetic Boundary Value Problems for Spheres and Cones," IRE Trans. on Antennas and Propagation, Vol. AP-4, January, 1956, pp. 5-16. corrections, IRE Trans., Vol. AP-5, July, 1957, p. 313.
8. Erdélyi, A., et al., Higher Transcendental Functions, Vol. 1, McGraw-Hill Book Company, Inc., New York, 1953, p. 144.
9. Erdélyi, A., et al., Higher Transcendental Functions, Vol. 1, McGraw-Hill Book Company, Inc., New York, 1953, p. 161.
10. Carnahan, B., H. A. Luther and J. O. Wilkes. Applied Numerical Methods, John Wiley and Sons, Inc., New York, 1969, p. 139.
11. Erdélyi, A., et al., Higher Transcendental Functions, Vol. 1, McGraw-Hill Book Company, Inc., N. Y., 1953, p. 147(last Eq.).
12. Erdélyi, A., et al., Higher Transcendental Functions, Vol. 1, McGraw-Hill Book Company, Inc., N. Y., 1953, p. 147, Equation 3. 5(5).

13. Abramowitz, M. and I. A. Stegun, Handbook of Mathematical Functions, U.S. Department of Commerce NBS, 1956, p. 480.
14. Jahnke, E., and F. Emde, Tables of Functions, Dover Publications, New York, 1945, p. 128.
15. Watson, G. N., Theory of Bessel Functions, Cambridge University Press, 1952, p. 199.
16. McLachlan, N. W., Bessel Functions for Engineers, Oxford Clarendon Press, 1955, pp. 87-88.

APPENDIX A
COMPUTER PROGRAM FOR CONICAL
ARRAY PATTERN ANALYSIS

A FORTRAN IV version of the program used to compute exact element patterns of circumferential and radial slots on a cone is presented at the end of this appendix. Specifically, the program uses the IBM 370 computer system presently available at the Hughes Aircraft Company facilities. The field expressions of Sections 2.2 and 2.3 are the basis for the program. The two different types of slots are considered concurrently when they are located at the same position on the conical surface or individually when located at different positions.

The input data to the program in a specified format specifies the type of slots to be considered, their location on the conical surface, and other parameters necessary for the computation of the field patterns. The program computes both modal and total radiation patterns. The modal patterns provide the necessary information for properly truncating the modal series, while the total pattern information is stored on tape for subsequent use in array analysis. The input parameters required by the READ statement are defined as follows:

- THETAO = θ_o , supplement of the half-angle of the
cone in degrees.
- KA = position of center of element from the tip
in radians.
- THETAI = initial value of spherical angle θ in degrees.
- THETAF = final value of spherical angle θ in degrees.

- DTH = increment of the spherical angle θ in degrees.
- PHI = final value of azimuthal angle ϕ in degrees.
- DPHI = increment of the azimuth angle ϕ in degrees.
 Program always evaluates pattern for $\phi = 0^\circ$.
- M = first value of the modal series. Normally $M = 0$.
 If $M \neq 0$, the lower modes are not included in the computations.
- MCR = switch specifying the type of slot to be considered. $MCR = 1$ specifies both circumferential and radial slots individually at same position. $MCR = 2$ specifies only circumferential slot. $MCR = 3$ specifies only radial slot.
- MVM = upper limit of modal series for circumferential slot.
- MVR = upper limit of modal series for radial slot.

Some of the remaining primary parameters of the program are defined as follows:

- IT = counter and identifier of different cases considered.
- MX = number of terms in the ν - series required for convergence.
- PHIN = array of values of the azimuthal angle ϕ .
- THETA = the spherical angle θ .
- ST = array of values of the spherical angle θ .
- NTH = maximum number of θ -values.

NPHI = maximum number of ϕ -values.
LX = total number of points of field evaluation.
LM = DO loop index. LM = 1 outputs patterns of
circumferential slot. LM = 2, outputs pattern
of radial slot.

Additional relationships between the program variables, printed output and the field component equations are presented for convenience in Table A-1.

In addition to printed output, the program also provides a record of all computed field component values on magnetic tape for further utilization by a separate program. The desired length in inches, the labeling information and the initial and increment values for both axes must be specified in the argument list of the CPLOT3(...) plotting routine. Azimuthal cut plots are obtained only if JS \neq 0 for $\theta = ST(JS)$ through $\theta = ST(JL)$ degrees in steps of JSTEP. The program terminates with the completion of the computations for the last data card.

TITLE	DENOTES		
	MD = 1 (TM-circ)	MD = 2 (TE-circ)	MD = 3 (Radial)
NU	N^{th} zero of $P_{\nu-1/2}^m(\cos \theta_0)$	N^{th} zero of $\frac{\partial}{\partial \theta_0} P_{\nu-1/2}^m(\cos \theta_0)$	
DPDNU (RES (J, MD))	$\frac{\partial}{\partial \nu} P_{\nu-1/2}^m(\cos \theta_0)$	$\frac{\partial^2}{\partial \nu' \partial \theta_0} P_{\nu'-1/2}^m(\cos \theta_0)$	
COEFFICIENT (BR(J, MD))	$\frac{NU}{NU * NU - .25}$	$\frac{j^{NU}}{DPDNU}$	$NU * \frac{j^{NU}}{DPDNU}$
BESSEL FUNCT (BJ (J))	$J_{\nu}(ka)$	$\frac{\partial}{\partial x} \left[\left(\frac{x}{ka} \right)^{1/2} J_{\nu'}(x) \right]_{x=ka}$	$\int_{r_1}^{r_2} \frac{g(r') J_{\nu'}(kr')}{r' \sqrt{kr'}} dr$
LEGENDRE FUNCT-P	$P_{\nu-1/2}^m(\cos \theta)$		
DR-LEGENDRE FUNCT-- PD	$\frac{\partial}{\partial \theta} P_{\nu-1/2}^m(\cos \theta)$		
THETA-PARTIAL SUM	$P_{1\theta}$	$P_{2\theta}$	P_{θ}
PHI-PARTIAL SUM	$P_{1\phi}$	$P_{2\phi}$	P_{ϕ}
ECT (IT, L) ECF (IT, L) ERT (IT, L) ERF (IT, L)	ECT = E_{θ} component ECF = E_{ϕ} component		ERT = E_{θ} component ERF = E_{ϕ} component

TABLE A-1

```

CONICAL ARRAY .
DIMENSIONING OF THE BJ,RES,AR,BR,NU ARRAYS MUST BE GREATER OR
EQUAL TO INDEX "MX", THE NUMBER OF ROOTS OF LEGENDRE FUNCTION.
COMPLEX ECT( 8,665),ECF( 8,665),ERT( 8,665),ERF( 8,665)
COMPLEX ACT(85),ACF(85),ART(85),ARF(85),AR(120,3),BR(120,3)
COMPLEX ECTM,ECFM,ERTM,ERFM
COMPLEX XEN,XNP,PSD,PS,PSDR,PSR,CF1,CF2,ZERO,OJ
DOUBLE PRECISION BJ,RES,B1,NU(99,2),NU1,NU2,XNU,XNU1,P,PD,P1,Q
DOUBLE PRECISION SNT,CST,RAD,DTR,PI02,TTD,PI
DIMENSION BJ(99),RES(99,2),PHIN(20),CMP(20),SMP(20),MS(15)
DIMENSION F(2,15),KAS(15),S(665),SF(665),SMT(20),SMF(20),ST(40)
DIMENSION ST1(40),ST2(40),ST3(40),ST4(40),SP(40),SFP(40)
EQUIVALENCE (AR,S),(BR,SF)
EQUIVALENCE (BJ,SMT),(BJ(21),SMF)
REAL KA,KAL,KAS
DATA PI,ZERO,OJ/3.14159265358979,(0.,0.),(0.,1.)/
DATA JS,JL,JSTEP/17,17,1/
MCR IS A PARAMETER FOR TYPE OF SLOT. MCR=1 TREATS CIRCUMF.
AND RADIAL CASE AT SAME POSITION, KA DISTANCE FROM TIP.
MCR= 2 TREATS CIRC. ONLY, WHILE MCR=3 TREATS RADIAL ONLY.
.....DATA JS,JL,JSTEP/.../ SELECTS THE THETA -CUT PLOTS.
109 FORMAT(1H1,49H THETA0 KA THETA1 THETA2 DTH PHI DPHI DK,
17X,14HM MCR MVM MVR)
110 FORMAT(2X,2F7.3,5F6.2,F8.5,4I4)
111 FORMAT(3E15.5)
112 FORMAT(2X,4HMCR=,I2)
113 FORMAT(2X,11HMCR,LM,KCR=,3I5)
115 FORMAT(1H1,'CONE ANGLE =',F6.1,' DEGREES, KA =',F8.3,' M =',I2)
116 FORMAT(1H0,5HMODE I1/3X,1HN,8X,2HNU,19X,11HCoefficient,20X,
15HDPDNU,12X,12HBESSEL FUNCT)
117 FORMAT(1H1, 5HMODE ,I1,5X,4HMCR=,I2)
118 FORMAT(2X,2(4H NU(,I3,1H,,I2,2H)=,F14.8))
120 FORMAT(1H0,' THETA =',F6.1,' DEGREES'//3X,1HN,4X,14HDR-LEGEN FUNCT,
111X,17H THETA-PARTIAL SUM,10X,1HN,4X,14HLEGENDRE FUNCT,
111X,17H PHI - PARTIAL SUM)
121 FORMAT(2(I4,4X,E13.5,5X,1H(,E13.5,3H , ,E13.5,1H)))
130 FORMAT(I4,F14.8,E20.8,E16.8,2E20.8,E16.8)
140 FORMAT(1H1,30X,'RADIATION PATTERN,',F4.0,' DEGREE CONE'//
12X,11HEXCITATION,25HELEM. TYPE(CIRC=1,RAD=2)=.I3,5X,
11HHE-COMP(TH=1,PH=2)=,I3/(3(4X,4HKA =,F7.2,2X,2HM=,I3,2X,
13HFC=,F7.4)))
141 FORMAT(1H+,50X,'KA =',F7.2,2X,'M =',I3,2X,'FC =',F7.4/
1(F24.2,I8,F13.4,F17.2,I8,F13.4))
143 FORMAT(//1X,10H PHI,THETA=,F5.1,16F7.1/(9X,17F7.1))
170 FORMAT(F6.1,3X,17F7.1/(9X,17F7.1))
171 FORMAT(2X,3(1X,3HAR(,I3,1H,,I2,2H)=,2E13.5)/)
172 FORMAT(2X,3HMD=,I1,2X,E13.5,5X,1H(,E13.5,3H , ,E13.5,1H),
13X,2E13.5)

```

```

173 FORMAT(2X,4E15.5,4X,2HS(,I3,2H)=,F10.3,2X,2HDB)
180 FORMAT(1H0,' THETA =',F6.1,' DEGREES'//3X,1H-,4X,14HDR-LEGEN FUNCT,
111X,17HPHI - PARTIAL SUM,10X,1HN,4X,14HLEGENDRE FUNCT,
111X,17HTHETA-PARTIAL SUM)
181 FORMAT(4HILM=,I1,5H MCP=,I1,4H M1=,I2,5H MVM=,I2,5H MVR=,I2,
14H KA=,F6.2,8H THETAC=,F6.2,6H NPHI=,I3,5H NTH=,I3,4H LX=,I4)
182 FORMAT(4H LM=,I1,5H MCR=,I1,4H M1=,I2,5H MVM=,I2,5H MVR=,I2,
14H KA=,F6.2,8H THETAC=,F6.2,6H NPHI=,I3,5H NTH=,I3,4H LX=,I4)
184 FORMAT(10F7.2)
185 FORMAT(2E15.7,F9.3,2E15.7,F9.3)
186 FORMAT(//11X,9HECT(IT,L),14X,2HDB,15X,9HECF(IT,L),14X,2HDB)
187 FORMAT(//11X,9HERI(IT,L),14X,2HDB,15X,9HERF(IT,L),14X,2HDB)
188 FORMAT(2X,'***THIS PATTERN CUT IS ZERO AND IS NOT PLOTTED.***')
189 FORMAT(1H1,30X,'RADIATION PATTERN,',F4.0,' DEGREE CONE'//
12X,11HEXCITATION,25HELEM. TYPE(CIRC=1,RAD=2)=,I3,5X,
118HE-COMP(TH=1,PH=2)=,I3/(3(4X,4HKA =,F7.2,2X,2HM=,I3)))
191 FORMAT(2X,2F10.3)
192 FORMAT(2X,2I3,5HTEST=,F8.2,3X,5HTESF=,F8.2)
    PI02 = PI/2.
    STOPI = DSQRT(2./PI)
    HSTOPI = .5*STOPI
    TPI = 2.*PI
    DTR = PI/180.
    IT = 0
    CALL PLOTI
1 READ(5,110,END=99)THETA0,KA,THETA1,THETA2,DTH,PHI,DPHI,DK,
IM,MCR,MVM,MVR
IF(THETA0.EQ.0..AND.KA.EQ.0.) STOP
IF(THETA0.EQ.0.) THETA0 = THETA2
WRITE(6,109)
WRITE(6,110)THETA0,KA,THETA1,THETA2,DTH,PHI,DPHI,DK,M,MCR,MVM,MVR
IF(DPHI.EQ.0.) DPHI=5.
M1 = M
IT = IT + 1
KZR = 0.0
NPHI=PHI/DPHI + 1.
KAS(IT) = KA
STPIKA = SQRT(TPI*KA)
STPOTP = STPIKA/TPI
STPOP = STPIKA/PI
SNT0 = DSIN(DTR*THETA0)
SNT01 = HSTOPI/SNT0
SNT02 = STOPI/SNT0
AKSO = KA*SNT0
TAKSO = 2.*AKSO
PODK = PI/DK
IM = 0
300 GO TO(310,310,330),MCR

```

```

C      MM IS THE NUMBER OF TERMS CONSIDERED IN THE M-SERIES, AND
C      IS DEFINED BY THE DATA VALUES OF MVM,MVR.
310 MM = MVM
    GO TO 3
330 MM = MVR
    3 RM = M
      IM = IM + 1
      MS(IM) = M
      FC = PODK*TAKSO*COS(RM*DK/TAKSO)/((PODK*AKSO)**2 - RM*RM)
      FC1 = FC*STPOTP
      FC2 = FC*STPOP
      F(IT,IM) = FC
    4 MD = 1
      THETA = THETA1
      IA = 1
C      CALCULATE AND STORE COS(MPHI(I)),SIN(MPHI(I))
      DO 2 I = 1,NPHI
        RI = I
        PHIN(I) = (RI - 1.)*DPHI
        RADMPH = RM*DTR*PHIN(I)
        SMP(I) = SIN(RADMPH)
    2 CMP(I) = COS(RADMPH)
C      MX IS NO. OF TERMS IN NU-SERIES REQ. FOR CONVERGENCE.
      MX = KA + 2.*RM + 10.
      WRITE(6,115) THETA0,KA,M
      MPI = M + 1
      DNU = 180./THETA0
      DO 11 MD = 1,2
        IF(MD.EQ.1.AND.MCR.EQ.3) GO TO 11
    5 XNU = RM + .5 - DNU
      ID = MD - 1
C      CALCULATION OF ZEROES OF LEGENDRE FUNCTIONS,NU,NUPRIME.
      DO 10 J = 1,MX
        NU1 = XNU + DNU - .04
        NU2 = NU1 + .08
        CALL ROOTS(THETA0,NU1,NU2,M,XNU,ID)
C      ID IS PARAMETER FOR MODE.
        CALL RESIDU(THETA0,XNU,M,ID,RES(J,MD))
C      EVALUATE NU-COEF IN NU-SERIES, NOTING THAT RESIDUE FOR
C      RADIAL CASE IS SAME AS FOR MODE 2.
        TTD = XNU/RES(J,MD)
        T = TTD/(XNU**2 - .25)
        TT = TTD
        XNPIM = XNU*PI02
        XNP = CMPLX(0.,XNPIM)
        XEN = CEXP(XNP)
        BR (J,MD) = T*XEN
        IF(MD.NE.2) GO TO 10

```

```

BR (J,MD+1) = TT*XEN
10 NU(J,MD) = XNU
11 CONTINUE
WRITE(6,118) ((J,MD,NU(J,MD),MD=1,2),J=1,MX)
CALCULATION OF BESSEL DEPENDENCE TIMES F(NU)/RESIDUE.
WRITE(6,112) MCR
DO 16 MD = 1,3
IF(MD.LE.2.AND.MCR.EQ.3) GO TO 16
IF(MD.EQ.3.AND.MCR.EQ.2) GO TO 16
WRITE(6,116) MD
K = MD
IF(MD.EQ.3) K = 2
EXECUTION OF ST 500 INITIALIZES AR(J,MD) TO PREVENT OVERFLOW.
IF(MD.EQ.2.AND.M.EQ.0) GO TO 500
DO 15 J = 1,MX
XNU = NU(J,K)
IF(MD.NE.3) GO TO 13
CALL BESSI(KA,DK,XNU,BJ(J))
BESSI EVALUATES INTEGRAL FOR RADIAL SLOT CASE
GO TO 14
13 CALL BESSEL(KA,XNU,BJ(J))
IF(MD.EQ.1) GO TO 14
XNUPI = XNU + 1.
CALL Bessel(KA,XNUPI,B1)
FOLLOWING EXPRESSION IS FOR D/DX(X*JNU(X)).
BJ(J) = (XNU + .5)/KA *BJ(J) - B1
14 IF(J.LT.6.OR.J.GT.MX-6)
IWRITE(6,130) J,NU(J,K),BR(J,MD),RES(J,K),BJ(J)
BJSING = BJ(J)
AR(J,MD) = BR(J,MD)*BJSING
15 CONTINUE
GO TO 16
500 CONTINUE
DO 17 J = 1,MX
17 AR(J,MD) = ZERO
16 CONTINUE
CALCULATION OF LEGENDRE FUNCTIONS AND PARTIAL SUMS.
IAT = 1. + (THETA - THETA1)/DTH
NAI = THETA1/DTH
IF(NAI.NE.1) IAT=IAT + 1
IAM = IAT/2
LOOP 18 INITIALIZES THE NU-SERIES TOTAL SUM FOR EACH NEW M.
DO 18 I = 1,IAT
ACT(I) = ZERO
ACF(I) = ZERO
ART(I) = ZERO
18 ARF(I) = ZERO
CF = RM/SNT0

```

```

      DO 200 MD = 1,2
      IF(MD.EQ.1.AND.MCR.EQ.3) GO TO 200
19  WRITE(6,117) MD,MCR
      THETA = THETA1
      DO 210 IA = 1,IAT
      GO TO (240,241), MD
240 IF(IA.EQ.1.OR.IA.EQ.IAM.OR.IA.EQ.IAT) WRITE(6,120) THETA
      GO TO 20
241 IF(IA.EQ.1.OR.IA.EQ.IAM.OR.IA.EQ.IAT) WRITE(6,180) THETA
20  RAD = DTR*THETA
      SNT = DSIN(RAD)
      CST = DCOS(RAD)
      ST(IA) = THETA
C.....THE ST1,ST2,ST3,ST4 ARRAYS ARE CREATED FOR PLOTTING ROUTINES.
      ST1(IA) = THETA
      ST2(IA) = THETA
      ST3(IA) = THETA
      ST4(IA) = THETA
      PS = ZERO
      PSD = ZERO
      PSDR = ZERO
      PSR = ZERO
      PSING = 0.
      PDSING = 0.
      P = 0.00
      P1 = 0.00
      PD = 0.00
      DO 30 J = 1,MX
      XNU = NU(J,MD)
      CALL LEGEND(THETA,XNU,MP1,PD,Q,0)
      PDSING = PD
      IF(M)25,25,22
22  CALL LFGEND(THETA,XNU,M,P1,Q,0)
      PD = RM*CST/SNT*P1 + PD
      PDSING = PD
      CALL LEGEND(THETA,XNU,M,P,Q,0)
      PSING = P
      IF(MCR.EQ.3) GO TO 26
      PS = PS + AR(J,MD)*PSING
25  PSD = PSD + AR(J,MD)*PDSING
26  IF(MD.NE.2.OR.MCR.EQ.2) GO TO 27
      PSDR = PSDR + AR(J,MD+1)*PDSING
      PSR = PSR + AR(J,MD+1)*PSING
C      PSDR AND PSR ARE TOTAL SUMS FOR THE RADIAL CASE.
27  IF(IA.EQ.1.OR.IA.EQ.IAM.OR.IA.EQ.IAT) GO TO 31
      GO TO 30
31  IF(J.LT.5.OR.J.GT.MX-8) GO TO 32
      GO TO 30

```

```

32 IF(MCR.EQ.3) GO TO 28
   WRITE(6,121) J,PD,PSD,J,P,PS
28 IF(MD.NE.2.OR.MCR.EQ.2) GO TO 30
   WRITE(6,121) J,PD,PSDR,J,P,PSR
30 CONTINUE
   EVALUATION OF FIELD COMPONENTS AT EACH THETA FOR EACH M.
   CF1AR = RM/SNT
   CF1 = CMPLX(0.,CF1AR)
   GO TO(220,230),MD
220 ACT(IA) = ACT(IA) + PSD
   ACF(IA) = ACF(IA) - PS*CF1
   GO TO 215
230 GO TO(235,237,236),MCR
235 ACT(IA) = ACT(IA) + PS*CF1*CF
   ACF(IA) = ACF(IA) + PSD*CF
236 ART(IA) = ART(IA) + PSR*CF1
   ARF(IA) = ARF(IA) + PSDR*CF1
   GO TO 215
237 ACT(IA) = ACT(IA) + PS*CF1*CF
   ACF(IA) = ACF(IA) + PSD*CF
215 IF(IA.EQ.1) THETA = FLOAT(NAI)*DTH
210 THETA = THETA + DTH
200 CONTINUE
   EVALUATION OF AMPLITUDES AND FINAL FIELD COMPONENTS.
   NTH = IAT
   MT = NTH
   LX = NPHI*NTH
   KZR IS A COUNTER FOR INITIALIZATION OF THE TOTAL FIELD ARRAYS.
   KZR = KZR + IT
   IF(KZR.GT.IT) GO TO 45
   DO 40 L = 1,LX
   FCT(IT,L) = ZERO
   ECF(IT,L) = ZERO
   ERF(IT,L) = ZERO
40 ERT(IT,L) = ZERO
45 CONTINUE
   IF(MCR.EQ.3) GO TO 59
C   THE DO 50 LOOP EVALUATES FIELD COMP. OF CIRCUM. SLOT.
C   ECTM, ECFM ARE THE FIELDS AT A POINT FOR A PARTICULAR M.
C   ECT, ECF ARE THE TOTAL FIELDS AT A POINT, CIRCUMF. CASE.
   DO 50 I = 1,NPHI
   DO 51 J = 1,NTH
   L = NTH*I - NTH + J
   IF(M.NE.0) GO TO 54
   ECTM = FCF*CMPLX(I)*ACT(J)
   ECF(IT,L) = (1.E-15,0.)
   SF(L) = -300.
   SFP(J) = SF(L)

```

```

GO TO 55
54 ECTM = FC2*CMP(I)*ACT(J)
   ECFM = FC2*SMP(I)*ACF(J)
   IF(CABS(ECFM).LT.1.E-15) ECFM = (1.E-15,0.)
   ECF(IT,L) = ECF(IT,L) + ECFM
   SF(L) = 10.*ALOG10(REAL(ECFM*CONJG(ECFM)))
   SFP(J) = SF(L)
55 IF(CABS(ECTM).LT.1.E-15) ECTM = (1.E-15,0.)
   ECT(IT,L) = ECT(IT,L) + ECTM
   S(L) = 10.*ALOG10(REAL(ECTM*CONJG(ECTM)))
   SP(J) = S(L)
51 CONTINUE
   USE "WRITE(6,170) PHIN(I),(SP(J),J=1,NTH) " AND FOR "SFP(I) HERE.
50 CONTINUE
.....
STATEMENTS THROUGH *58* ARE ONLY FOR WRITING OUT MODE PATTERNS.
WRITE(6,143)(ST(J),J=1,MT)
DO 56 I = 1,NPHI
  NIN = NTH*(I - 1)
  NIN1 = NIN + 1
  NINMT = NIN + MT
56 WRITE(6,170) PHIN(I),(S(J),J=NIN1,NINMT)
   IF(M.EQ.0) GO TO 59
   WRITE(6,143)(ST(J),J=1,MT)
   DO 58 I = 1,NPHI
     NIN = NTH*(I - 1)
     NIN1 = NIN + 1
     NINMT = NIN + MT
58 WRITE(6,170) PHIN(I),(SF(J),J=NIN1,NINMT)
.....
59 IF(MCR.EQ.2) GO TO 69
   ERTM, ERFM ARE THE FIELDS AT A POINT FOR A PARTICULAR M.
   ERT, ERF ARE THE TOTAL FIELDS AT A POINT, RADIAL CASE.
DO 60 I = 1, NPHI
DO 61 J = 1, NTH
L = NTH*I - NTH + J
IF(M.NE.0) GO TO 64
ERFM = CMP(I)*ARF(J)*SNT01
ERT(IT,L) = (1.E-15,0.)
S(L) = -300.
SP(J) = S(L)
GO TO 65
64 ERFM = CMP(I)*ARF(J)*SNT02
   ERTM = SMP(I)*ART(J)*SNT02
   IF(CABS(ERTM).LT.1.E-15) ERTM = (1.E-15,0.)
   ERT(IT,L) = ERT(IT,L) + ERTM
   S(L) = 10.*ALOG10(REAL(ERTM*CONJG(ERTM)))
   SP(J) = S(L)

```

```

65 IF(CABS(ERFM).LT.1.E-15) ERFM =(1.E-15,0.)
   ERF(IT,L) = ERF(IT,L) + ERFM
   SF(L) = 10.*ALOG10(REAL(ERFM*CONJG(ERFM)))
   SFP(J) = SF(L)
61 CONTINUE
C   USE "WRITE(6,170) PHIN(I),(SP(J),J=1,NTH) " AND FOR "SFP(I) HERE.
60 CONTINUE
C   . . . . .
C   STATEMENTS THROUGH *68* ARE ONLY FOR WRITING OUT MODE PATTERNS.
   IF(M.EQ.0) GO TO 67
   WRITE(6,143)(ST(J),J=1,MT)
   DO 66 I = 1,NPHI
     NIN = NTH*(I - 1)
     NINI = NIN + 1
     NINMT = NIN + MT
66  WRITE(6,170) PHIN(I),(S(J),J=NINI,NINMT)
67  WRITE(6,143) (ST(J),J=1,MT)
     DO 68 I = 1,NPHI
       NIN = NTH*(I - 1)
       NINI = NIN + 1
       NINMT = NIN + MT
68  WRITE(6,170) PHIN(I),(SF(J),J=NINI,NINMT)
C   . . . . .
69 IF(M.GE.MM) GO TO 350
   M = M + 1
   GO TO 3
350 IF(MCR.NE.1) GO TO 360
   MCR = 3
   ISW = 1
   IF(M.GE.MVR) GO TO 360
   M = M + 1
   GO TO 300
360 CONTINUE
C   THE "IT" CASE TOTAL PATTERNS ARE COMPUTED AND PLOTTED BELOW.
   WRITE(6,112) MCR
   THETAC = 180. - THETA0
   IF(ISW.EQ.1) MCR = 1
C..... THE WRITE(10) STATEMENTS WRITE THE COMPUTED DATA ON TAPE.
   WRITE(10) MCR
   DO 400 LM = 1,2
     MZ = MVM - M1 + 1
     IF(LM.EQ.2) MZ = MVR - M1 + 1
     WRITE(6,112) MCR
     IF(MCR.EQ.2.AND.LM.EQ.2) GO TO 400
     IF(MCR.EQ.3.AND.LM.EQ.1) GO TO 400
     KCR = 0
     DO 370 I = 1,NPHI
       TEST = -200.

```

```

      TESH = -200.
      DO 375 J = 1, NTH
      L = NTH*I - NTH + J
      GO TO(371,372),LM
371 S(L) = 10.*ALOG10(REAL(ECT(IT,L)*CONJG(FCT(IT,L))))
      SF(L) = 10.*ALOG10(REAL(ECF(IT,L)*CONJG(ECF(IT,L))))
      GO TO 374
372 S(L) = 10.*ALOG10(REAL(ERT(IT,L)*CONJG(ERT(IT,L))))
      SF(L) = 10.*ALOG10(REAL(ERF(IT,L)*CONJG(ERF(IT,L))))
374 IF(S(L).GT.TEST) TEST=S(L)
      IF(SF(L).GT.TESH) TESH=SF(L)
375 CONTINUE
      SMT(I) = TEST
      SMF(I) = TESH
      WRITE(6,192) LM,I,TEST,TESH
370 CONTINUE
      GO TO (376,377),LM
376 WRITE(6,181) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
      WRITE(10) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
      WRITE(10) (PHIN(I),I=1,NPHI)
      WRITE(10) (ST(J),J=1,NTH)
      WRITE(6,186)
      WRITE(6,185) (ECT(IT,L),S(L),ECF(IT,L),SF(L),L=1,LX)
      WRITE(10) (ECT(IT,L),S(L),ECF(IT,L),SF(L),L=1,LX)
      GO TO 378
377 WRITE(6,181) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
      WRITE(10) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
      WRITE(10) (PHIN(I),I=1,NPHI)
      WRITE(10) (ST(J),J=1,NTH)
      WRITE(6,187)
      WRITE(6,185) (ERT(IT,L),S(L),ERF(IT,L),SF(L),L=1,LX)
      WRITE(10) (ERT(IT,L),S(L),ERF(IT,L),SF(L),L=1,LX)
378 CONTINUE
C LOOP 410 FINDS LARGEST VALUE IN S(L) AND SF(L) ARRAYS.
      SMTN = -200.
      SMFN = -200.
      DO 410 I = 1,NPHI
      IF(SMT(I).GT.SMTN) SMTN=SMT(I)
410 IF(SMF(I).GT.SMFN) SMFN=SMF(I)
      SMTT = SMTN
      SMFF = SMFN
      WRITE(6,191) SMTT,SMFF
      SNORM = SMTT
      IF(SMFF.GT.SNORM) SNORM=SMFF
      WRITE(6,191) SMTT,SNORM
379 KCR = KCR + 1
      WRITE(6,113) MCR,LM,KCR
      GO TO (381,382),LM

```

```

381 WRITE(6,140) THETAC,LM,KCR,(KAS(IT),MS(I),F(IT,I),I=1,MZ)
GO TO 383
382 WRITE(6,189) THETAC,LM,KCR,(KAS(IT),MS(I),I=1,MZ)
383 WRITE(6,143)(ST(J),J=1,NTH)
CTF = -70.
DO 390 I = 1,NPHI
NIN = NTH*(I - 1)
NINI = NIN + 1
NINMT = NIN + NTH
IF(KCR.EQ.2) GO TO 386
DO 380 J = 1,NTH
L = NTH*I - NTH + J
SMT(I),SMF(I) ARE PHI-CUT NORMALIZATIONS. SMTT,SMFF ARE FOR
COMPONENT NORMALIZATION. SNORM=MAX(SMTT,SMFF) OVERALL NORMALIZAT.
S(L) = S(L) - SNORM
SF(L) = SF(L) - SNORM
SP(J) = S(L)
SFP(J) = SF(L)
380 CONTINUE
385 WRITE(6,170) PHIN(I),(S(J),J=NINI,NINMT)
THE FOLLOWING CARDS CONTROL THE NUMBER OF PLOTS DESIRED.
IF(I.GT.1) GO TO 390
IF(SP(2).LT.CTF.AND.SP(3).LT.CTF.AND.SP(6).LT.CTF) GO TO 387
CALL SYMBOL(1.,7.5,.14,36,0.,-1)
CALL LEGND1(1.3,7.5,.14,0.,1,'=',1,PHIN(I),0)
CALL CPlot3(1,ST,9.,0.,20.,14THETA, DEGREES,14,7.,-28.,4.,
128HPower IN DB, THETA COMPONENT,28,NTH,SP)
387 IF(SFP(2).LT.CTF.AND.SFP(3).LT.CTF.AND.SFP(6).LT.CTF) GO TO 390
CALL SYMBOL(1.,7.5,.14,36,0.,-1)
CALL LEGND1(1.3,7.5,.14,0.,1,'=',1,PHIN(I),0)
CALL CPlot3(1,ST,9.,0.,20.,14THETA, DEGREES,14,7.,-28.,4.,
126HPower IN DB, PHI COMPONENT,26,NTH,SFP)
GO TO 390
386 WRITE(6,170) PHIN(I),(SF(J),J=NINI,NINMT)
390 CONTINUE
IF(KCR.EQ.1) GO TO 379
THE FOLLOWING CARDS PLOT PATTERNS OF THETA VS. PHI.
. . . . .
IF(JS.EQ.0) GO TO 400
IF(JL.GT.NTH) JL=NTH
DO 590 J = JS,JL,JSTEP
DO 580 I = 1,NPHI
L = NTH*I - NTH + J
SP(I) = S(L)
SFP(I) = SF(L)
580 CONTINUE
IF(SP(2).LT.CTF.AND.SP(3).LT.CTF.AND.SP(6).LT.CTF) GO TO 587
CALL SYMBOL(1.,7.5,.14,37,0.,-1)

```

```

CALL LEGND1(1.3,7.5,.14,0.,1,'=',1,ST(J),0)
CALL CPLOT3(1,PHIN,9.,0.,20.,12HPHI, DEGREES,12,7.,-28.,4.,
128HPOWER IN DB, THETA COMPONENT,28,NPHI,SP)
587 IF(SFP(2).LT.CTF.AND.SFP(3).LT.CTF.AND.SFP(6).LT.CTF) GO TO 590
CALL SYMBOL(1.,7.5,.14,37,0.,-1)
CALL LEGND1(1.3,7.5,.14,0.,1,'=',1,ST(J),0)
CALL CPLOT3(1,PHIN,9.,0.,20.,12HPHI, DEGREES,12,7.,-28.,4.,
126HPOWER IN DB, PHI COMPONENT,26,NPHI,SFP)
590 CONTINUE
C . . . . .
400 CONTINUE
GO TO 1
99 CONTINUE
ENDFILE 10
CALL PLOT(0.,0.,999)
STOP
END

```

```

SUBROUTINE BESSEL(X,NU,JNUX)
DOUBLE PRECISION NU,JNUX,XX
REAL*8 DBSJ1
XX = X
IF(X.GT.100..OR.NU.GT.100.) GO TO 100
JNUX = DBSJ1(XX,NU)
RETURN
100 CALL RESEXT(XX,NU,JNUX)
RETURN
END

```

```

SUBROUTINE RESEXT(XX,NU,JNUXL)
COMMON/BESWRK/L1,L2,A(700)
REAL*8 XX,NU,JNUXL,A
C THE RESEXT SUBROUTINE RETURNS THE VALUE OF THE BESSEL
C FUNCTION OF ORDER P.LT.400 AND ARGUMENT X.LT.400. LARGER ORDERS
C AND ARGUMENTS ARE CONSIDERED BY EXTENDING THE DIMENSION OF A,L1.
L1 = 700
K = NU
CALL DBSJ2(XX,NU,A)
JNUXL = A(K+1)
RETURN
END

```

```

SUBROUTINE BESSI(KA,DK,NU,BINT)
C THIS IS AN INTEGRATION PROGRAM USING SIMPSON'S 3/8 RULE.
C NR IS THE NUMBER OF INCREMENTS ON RK AXIS, MULTIPLE OF 3.
DIMENSION F(4)
DOUBLE PRECISION NU,BINT,PJ
REAL KA
DATA NR/15/
R1K = KA - DK/2.
R2K = KA + DK/2.
RN = NR
HR = (R2K - R1K)/RN
A = 0.0
RK = R1K
D) 30 I = 1,NR,3
D) 20 J = 1,4,1
RKP = RK**(-1.5)
CALL BESSEL(RK,NU,PJ)
GOR = COS(RK - KA)
F(J) = RKP*PJ*GOR
IF(J.EQ.4) GO TO 30
20 RK = RK + HR
30 A = A + (3./8.)*HR*(F(1) + 3.*F(2) + 3.*F(3) + F(4))
BINT = A
RETURN
END

```

```

SUBROUTINE LEGEND(THETA,NU,M,PNU,P1,IR)
DOUBLE PRECISION P1,P2,CF,DA,A,PNU,RMGM
DOUBLE PRECISION PI,CSDA,SNDA,CSA,SNA,T,X,F,RAD,B,C,TX,Y,NU
DATA PI/3.14159265358979/
RAD = THETA*PI/180.
Y = 2.*DSIN(RAD)
DA = RAD + .5*PI
RM = M
N = (70.+RM+RM) -NU
IF(NU.GT.(69.+RM+RM)) N = -1
SNDA = DSIN(DA)
CSDA = DCOS(DA)
RA = N
NU = NU + (RN+2.)
J = 0
FM2 = RM*RM
IF(NU*PI.LT.9.) GO TO 50
10 NU = NU - 1.
A = NU*PI + (.25+.5*PM)*PI
SNA = DSIN(A)
CSA = DCOS(A)
C = 1.
P2 = P1
T = 1.
P1 = SNA
20 T = -T*((C-.5)**2 - FM2)/(C*Y*(NU+C))
SNA = SNA*CSDA + CSA*SNDA
CSA = (CSA - SNA*SNDA)/CSDA
P1 = P1 + T*SNA
C = C + 1.
IF(DABS(T).GT.1.D-9.AND.C.LT.35.) GO TO 20
R = .25 - FM2
IF(N.EQ.(-1)) GO TO 40
J = J + 1
IF(J.EQ.1) GO TO 10
C = NU + 2.
TX = 2.*DCOS(RAD)
NT = (N+1)/2
DO 30 J = 1,NT
C = C - 2.
P2 = TX*P1 - (1.+R/(C*(C+1.)))*P2
30 P1 = TX*P2 - (1.+R/(C*(C-1.)))*P1
J = 2 - MOD(N+1,2)
NU = NU - RN
IF(J.EQ.2) P1=P2
IF(IR.EQ.1) RETURN TO SUBROOTS FOR COMPLETION OF ROOTS.
40 IF(IR.EQ.1) RETURN
X = NU +RM - .5

```

```

T = NU + RM - .5
IF (M.EQ.0) X=1.
IF (M.LT.2) GO TO 45
DO 44 I = 2,M
T = T-1.
44 X = X*T
45 IF (NU.GT.110.) GO TO 46
CALL GAMRAT(NU,M,GR,1)
CF = 2./DSQRT(PI*Y)*X*GR
PNU = CF*PI
RETURN
46 T = 1./(16.*NU)
CF = 2./DSQRT(PI*Y*NU)*X*(1.-2.*T*(1.-T))
PNU = CF*PI
RETURN
50 NU = NU - RN - 2.
(NU*RAD.LE.9) NOW EXECUTING.
F = NU*NU - .25
X = DCOS(RAD)
Y = (1. - X)/2.
PI = 1.
T = 1.
C = 0.
60 C = C + 1.
T = -T*(F - C*(C-1.))/(RM+C)*Y/C
PI = PI + T
IF (DABS(T).GT.1.D-9.AND.C.LT.25.) GO TO 60
CALL GAMRAT(NU,M,GR,2)
RMGM = RM + 1.
GM1 = GAMMAR(RMGM)
IF (NU.LT.100.) CF = GR/GM1
IF (NU.GE.100.) CF = NU**(M+M)/GM1
PNU = CF*(((1.-X)/(1.+X))**(.5*RM))*((-1.)**M)*PI
RETURN
END

```

```

SUBROUTINE RESIDU(THETA,NU,M, ID,DPDNU)
SUB-RESIDUE RETURNS DPDNU,THETA0. WHEN ID=0 AND ITS
THETA DERIVATIVE WHEN ID=1.
DOUBLE PRECISION PI,F,RAD,CST,SNT,DPDNU,DPNU2D,NU,P2,PNU2
DIMENSION F(4)
DATA PI,H/3.14159265358979,.0001/
NU = NU - 2.*H
I = 1
10 CALL LEGEND(THETA,NU,M,F(I),P2,0)
IF(ID.EQ.0) GO TO 15
MPI = M+1
CALL LEGEND(THETA,NU,MPI,PNU2,P2,0)
RAD = THETA*PI/180.
CST = DCOS(RAD)
SNT = DSIN(RAD)
RM = M
F(I) =RM* CST/SNT*F(I) + PNU2
15 I = I + 1
NU = NU + H
IF(I.EQ.3) NU = NU + H
IF(I.LT.5) GO TO 10
DPDNU = (F(1)-8.*F(2)+8.*F(3)-F(4))/(12.*H)
NU = NU - 3.*H
RETURN
END

```

```

FUNCTION GAMMAR(X)
DOUBLE PRECISION X
IF(X.GT.1.) GO TO 2
Z = X
1 IF(Z.GT.0.) GO TO 3
Z = Z + 1.
GO TO 1
2 Z = X - 1.
3 T1 = Z + .5
TZG = T1 + 5.
T1 = TZG**T1
T1 = EXP(-TZG)*T1*2.50662827465
GAMZ = T1*(1.+76.18009173/(Z+1.) -86.50532033/(Z+2.)
1+24.01409922/(Z+3.) -1.231739516/(Z+4.) +.120858003E-2/
1(Z+5.) -.536382E-5/(Z+6.))
IF(X.GT.1.)GO TO 5
4 GAMZ = GAMZ/Z
IF(Z.EQ.X) GO TO 5
Z = Z - 1.
GO TO 4
5 GAMMAR = GAMZ
RETURN
END

```

```

SUBROUTINE ROOTS(THETA,NU1,NU2,M,NU,ID)
ROOTS OF PNU ARE OBTAINED WHEN ID=0, OF DPDTHETA WHEN ID=1.
DOUBLE PRECISION PI, EPS
DOUBLE PRECISION NU, NU1, NU2, NUL, P2, Q, PNU, RAD, FP, FPS, FPL, SCT, SNT
DATA PI, EPS/3.14159265358979, 1.0-7/
26 FORMAT(14H ITN,NU,FP,FPS,I5,3E15.8)
NU = NU1
ITN = 0
10 CALL LEGEND(THETA,NU,M,PNU,FP,1)
IF(ID.EQ.0) GO TO 15
MPI = M + 1
CALL LEGEND(THETA,NU,MPI,PNU,P2,1)
RAD = THETA*PI/180.
CST = DCOS(RAD)
SNT = DSIN(RAD)
RM = M
FP = RM*CST*FP + (NU+RM+.5)*SNT*P2
15 Q = NU2
ITN = ITN + 1
IF(ITN.EQ.1) GO TO 20
FPS = FP - FPL
IF(DABS(FPS).LT.EPS) FPS = DSIGN(EPS,FPS)
Q = NUL - (NU - NUL)/FPS*FPL
20 NUL = NU
FPL = FP
NU = Q
IF(DABS(NU-NUL).LT.EPS) RETURN
IF(ITN.GT.20) GO TO 30
GO TO 10
30 WRITE(6,26) ITN,NU,FP,FPS
RETURN
END

```

```

SUBROUTINE GAMRAT(NU,M,GR,IG)
SUB-GAMRAT CALCULATES AND RETURNS THE RATIO OF TWO GAMMA
FUNCTIONS.. G(NU+.5)/G(NU+1.) AND G(NU+RM+.5)/G(NU-RM+.5).
DOUBLE PRECISION NU,X,D1
10 FORMAT(3E20.10)
RM = M
IF(IG.EQ.2) GO TO 30
KNU = NU
D = NU - FLOAT(KNU)
D1 = D + 1.
X = D + .5
RAT1 = X/D1
Y = GAMMAR(X)/GAMMAR(D1)
IF(KNU.EQ.0) RAT1 = 1.
Y = Y*RAT1
IF(KNU.LT.2) GO TO 25
DO 20 I = 2,KNU
D1 = D1 + 1.
X = X + 1.
RAT = X/D1
20 Y = Y*RAT
25 GR = Y
RETURN
30 IF(M.EQ.0) GO TO 50
S = NU + RM - .5
T = S
MT = 2*M - 1
DO 40 I = 1,MT
T = T - 1.
40 S = S*T
GO TO 60
50 S = 1.
60 GR = S
RETURN
END

```

APPENDIX B
COMPUTER PROGRAM FOR MODAL
OR TOTAL FIELD PATTERNS

This program is also written for the IBM 370 system and provides complete field patterns and plots selective cuts of modal or sum patterns. The field component data is provided by one to three magnetic tapes generated by the general program of Appendix A. This auxiliary program allows for detailed study of the modal series without costly recomputations of the general program.

Most of the program parameters have the same meaning as in the program of Appendix A. Other parameter definitions are as follows:

- NTAPE = number of data tapes to be read.
If NTAPE = 1 only tape identified by NC will be read.
- NC = Tape unit number.
- IT = Counter for the total number of complete files read from all the tapes. Must be \leq to first subscript of the E-arrays.

Scaling and labeling information must be provided in the arguments of the plot subroutines. Multiple graphs will be drawn with the present listing in all plots. The complete listing of the program is included in the following pages.

```

C   POWER PATTERNS AND PLOTS OF CONICAL ARRAY PROBLEM.
C   INPUT TO THIS PROGRAM IS TAKEN FROM TAPE .
COMPLEX ECT(11,665),ECF(11,665),ERT(11,665),ERF(11,665)
COMPLEX ECTI,ECFI,ERTI,ERFI
REAL KA,KA
COMMON /CFLD/ECTI(665),ECFI(665)/RFLD/ERTI(665),ERFI(665)
COMMON /PVS/ PHIN(20),ST(40),S(665),SF(665)
COMMON/KANA/KA(15),MIA(15),MVMA(15),MVRA(15)
DATA NTAPE/2/
DATA NTAPE/1/
181 FORMAT(4H1LM=,I1,5H MCR=,I1,4H M1=,I2,5H MVM=,I2,5H MVR=,I2,
14H KA=,F6.2,8H THETAC=,F6.2,6H NPHI=,I3,5H NTH=,I3,4H LX=,I4)
182 FORMAT(4H LM=,I1,5H MCR=,I1,4H M1=,I2,5H MVM=,I2,5H MVR=,I2,
14H KA=,F6.2,8H THETAC=,F6.2,6H NPHI=,I3,5H NTH=,I3,4H LX=,I4)
185 FORMAT(2E15.7,F9.3,2E15.7,F9.3)
186 FORMAT(/11X,9HECT(IT,L),14X,2HDB,15X,9HECF(IT,L),14X,2HDB)
187 FORMAT(/11X,9HERT(IT,L),14X,2HDB,15X,9HERF(IT,L),14X,2HDB)
188 FORMAT(4E15.7)
CALL PLOT1
NC = 10
IT = 0.
1 IT = IT + 1
IF(IT.EQ.10.AND.NC.EQ.11) GO TO 310
READ(NC,END=310) MCR
GO TO (301,301,302),MCR
C   IT CAN REPRESENT A MODE OR A DIFFERENT ELEMENT POSITION.
301 READ(NC) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
READ(NC) (PHIN(I),I=1,NPHI)
READ(NC) (ST(J),J=1,NTH)
READ(NC) (ECT(IT,L),S(L),ECF(IT,L),SF(L),L=1,LX)
KAA(IT) = KA
MIA(IT) = M1
MVMA(IT) = MVM
MVRA(IT) = MVR
C   USE "WRITE (6,185) (ECT(IT,L),S(L),ECF(IT,L),SF(L),L=1,LX)" HERE.
IF(MCR.EQ.1) GO TO 302
GO TO 300
302 READ(NC) LM,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX
READ(NC) (PHIN(I),I=1,NPHI)
READ(NC) (ST(J),J=1,NTH)
READ(NC) (ERT(IT,L),S(L),ERF(IT,L),SF(L),L=1,LX)
KAA(IT) = KA
MIA(IT) = M1
MVMA(IT) = MVM
MVRA(IT) = MVR
C   USE "WRITE (6,187) (ERT(IT,L),S(L),ERF(IT,L),SF(L),L=1,LX)" HERE.
300 GO TO 1
310 IT = IT - 1

```

```

IF(NTAPE.EQ.1) GO TO 312
IF(NC.EQ.12) GO TO 312
IF(NC.EQ.11) GO TO 311
NC = 11
GO TO 1
311 NC = 12
GO TO 1
312 CONTINUE
DO 340 II = 1,IT
IF(MCR.EQ.3) GO TO 317
DO 316 I = II,II
DO 315 L = 1,LX
ECTT(L) = ECTT(L) + ECT(I,L)
315 ECFF(L) = ECFF(L) + ECF(I,L)
316 CONTINUE
317 IF(MCR.EQ.2) GO TO 330
DO 321 I = II,II
DO 320 L = 1,LX
ERTT(L) = ERTT(L) + ERT(I,L)
320 ERFF(L) = ERFF(L) + ERF(I,L)
321 CONTINUE
330 CONTINUE
C THE FOLLOWING "IF" STATEMENT IS TO BE USED ONLY WHEN
C HIGHER ORDER MODE PATTERNS IN SUCCESSION ARE REQUIRED.
CALL PHICUT(II,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX)
IF(II.EQ.3.OR.II.EQ.6.OR.II.EQ.9) GO TO 340
CALL PLOT(-37.,0.,-3)
340 CONTINUE
CALL PLOT(0.,0.,999)
999 CONTINUE
STOP
END

```

```

SUBROUTINE PHICUT(IT,MCR,M1,MVM,MVR,KA,THETAC,NPHI,NTH,LX)
DIMENSION SMT(20),SMF(20),SP(40),SFP(40)
COMPLEX ECTT,ECFF,ERTT,ERFF
REAL KA,CAA
COMMON /CFLD/ECTT(665),ECFF(665)/RFLD/ERTT(665),ERFF(665)
COMMON /PVS/ PHIN(20),ST(40),S(665),SF(665)
COMMON/KAMA/CAA(15),M1A(15),MVMA(15),MVRA(15)
....EXECUTE 'DATA JS,LL,JSTEP/0,0,1/' LAST IF THETA-CUTS NOT DESIRED.
DATA JS,JL,JSTEP/ 0, 0,1/
DATA JS,JL,JSTEP/17,17,1/
113 FORMAT(2X,12HMCR,LMP,KCR=,3I5)
140 FORMAT(1H1,30X,'RADIATION PATTERN,',F4.0,'DEGREE CONE'//
12X,11HEXCITATION,25HELEM. TYPE(CIRC=1,RAD=2)=,I3,5X,
118HE-COMP(TH=1,PH=2)=,I3/(3(4X,4HKA =,F7.2,2X,3HM1=,I3,2X,
14HMVM=,I3,2X,4HMVR=,I3)))
143 FORMAT(/1X,10HPHI,THETA=,F5.1,16F7.1/(9X,17F7.1))
170 FORMAT(F6.1,3X,17F7.1/(9X,17F7.1))
181 FORMAT(4H1LM=,I1,5H MCR=,I1,4H M1=,I2,5H MVM=,I2,5H MVR=,I3,
14H KA=,F6.2,8H THETAC=,F6.2,6H NPHI=,I3,5H NTH=,I3,4H LX=,I1)
185 FORMAT(2E15.7,F9.3,2E15.7,F9.3)
191 FORMAT(2X,2F10.3)
192 FORMAT(2X,2I3,5HTEST=,F8.2,3X,5HTESEF=,F8.2)
....THE FOLLOWING TWO STATEMENTS DETERMINE CURVE IDENTIFIER
....FOR THE 'CSEG' PLOTTING ROUTINE.....
IT1 = IT - 1
KL = MOD(IT1,3)
DO 400 LMP = 1,2
IF(MCR.EQ.2.AND.LMP.EQ.2) GO TO 400
IF(MCR.EQ.3.AND.LMP.EQ.1) GO TO 400
KCR = 0
DO 370 I = 1,NPH1
TEST = -200.
TESF = -200.
DO 375 J = 1,NTH
L = NTH*I - NTH + J
GO TO(371,372),LMP
371 S(L) = 10.*ALOG10(REAL(ECTT(L)*CONJG(ECTT(L))))
SF(L) = 10.*ALOG10(REAL(ECFF(L)*CONJG(ECFF(L))))
GO TO 374
372 S(L) = 10.*ALOG10(REAL(ERTT(L)*CONJG(ERTT(L))))
SF(L) = 10.*ALOG10(REAL(ERFF(L)*CONJG(ERFF(L))))
374 IF(S(L).GT.TEST) TEST=S(L)
IF(SF(L).GT.TESF) TESH=SF(L)
375 CONTINUE
SMT(I) = TEST
SMF(I) = TESH
WRITE(6,192) LMP,1,TEST,TESF
370 CONTINUE

```

```

WRITE(6,191) (SMT(I),SMF(I),I=1,NPHI)
GO TO (376,377),LMP
376 WRITE(6,181) LMP,MCR,MIA(IT),MVMA(IT),MVRA(IT),KAA(IT),
ITHETAC,NPHI,NTH,LX
C USE "WRITE(6,185) (ECTT(L),S(L),ECFF(L),SF(L),L=1,LX)" HERE.
GO TO 378
377 WRITE(6,181) LMP,MCR,MIA(IT),MVMA(IT),MVRA(IT),KAA(IT),
ITHETAC,NPHI,NTH,LX
C USE "WRITE(6,185) (ERTT(L),S(L),ERFF(L),SF(L),L=1,LX)" HERE.
378 CONTINUE
C LOOP 410 FINDS LARGEST VALUE IN S(I) AND SF(L) ARRAYS.
SMTN = -200.
SMFN = -200.
DO 410 I = 1,NPHI
IF(SMT(I).GT.SMTN) SMTN=SMT(I)
410 IF(SMF(I).GT.SMFN) SMFN=SMF(I)
SMTT = SMTN
SMFF = SMFN
WRITE(6,191) SMTT,SMFF
SNORM = SMTT
IF(SMFF.GT.SNORM) SNORM=SMFF
WRITE(6,191) SMTT,SNORM
379 KCR = KCR + 1
WRITE(6,113) MCR,LMP,KCR
WRITE(6,140) THETAC,LMP,KCR,(KAA(I),MIA(I),MVMA(I),MVRA(I),I=1,IT)
WRITE(6,143) (ST(J),J=1,NTH)
CTF = -70.
DO 390 I = 1,NPHI
NIN = NTH*(I - 1)
NINI = NIN + 1
NINMT = NIN + NTH
IF(KCR.EQ.2) GO TO 386
C "SNORM" NORMALIZES PATTERNS WITH RESPECT TO LARGEST VALUE
C OF BOTH FIELD COMPONENTS. "SMTT" AND "SMFF" ARE FOR
C INDIVIDUAL COMPONENT NORMALIZATION.
DO 380 J = 1,NTH
L = NTH*I - NTH + J
S(L) = S(L) - SNORM
SF(L) = SF(L) - SNORM
SP(J) = S(L)
SFP(J) = SF(L)
380 CONTINUE
385 WRITE(6,170) PHIN(I),(S(J),J=NINI,NINMT)
IF(I.GT.1.AND.I.LT.NPHI) GO TO 390
IF(SP(2).LT.CTF.AND.SP(3).LT.CTF.AND.SP(6).LT.CTF) GO TO 387
387 IF(SFP(2).LT.CTF.AND.SFP(3).LT.CTF.AND.SFP(6).LT.CTF) GO TO 390
GO TO 390
388 WRITE(6,170) PHIN(I),(SF(J),J=NINI,NINMT)

```

```

390 CONTINUE
   IF(KCR.EQ.1) GO TO 379
   CALL SUB-FBCUT(KL,.....) FOR CONTINUOUS PLOT OF PHI=0 AND PHI=180.
   CALL FBCUT(KL,NTH,NPHI,LX,LMP)
   THE FOLLOWING CARDS PLOT PATTERNS OF THETA VS. PHI.
   .....
   IF(JS.EQ.0) GO TO 591
   IF(JL.GT.NTH) JL=NTH
   DO 590 J = JS,JL,JSTEP
   DO 580 I = 1,NPHI
   L = NTH*I - NTH + J
   SP(I) = S(L)
   SFP(I) = SF(L)
580 CONTINUE
   CALL PBPLOT(KL,PHIN,9.,0.,20.,12HPHI, DEGREES,12,
1SP,7.,28.,-4.,28HPOWER IN DB, THETA COMPONENT,28,NPHI,1)
   CALL PBPLOT(KL,PHIN,9.,0.,20.,12HPHI, DEGREES,12,
1SFP,7.,28.,-4.,26HPOWER IN DB, PHI COMPONENT,26,NPHI,1)
   IF(IT.EQ.3.OR.IT.EQ.6.OR.IT.EQ.9) GO TO 590
   IF(LMP.EQ.2) CALL PLOT(-24.,0.,-3)
590 CONTINUE
591 CONTINUE
   .....
   IF(IT.EQ.3.OR.IT.EQ.6.OR.IT.EQ.9) GO TO 400
   IF(LMP.EQ.2) CALL PLOT(-13.,0.,-3)
   .....
400 CONTINUE
900 RETURN
   END

```

```

SUBROUTINE PBPLOT(K,X,XN,XS,DX,XLAB,NLX,Y,YN,YS,DY,YLAB,NLY,NP, ID)
DIMENSION X(80),Y(80),XLAB(10),YLAB(10)
.....ID=1 GIVES SINGLE INCREMENT SCALE. ....
.....ID = 2 GIVES TWO-INCREMENT SCALE.....
NX = XN
NY = YN
XI = XS
YI = YS
NX1 = NX + 1
NY1 = NY + 1
IF(K.NE.0) GO TO 30
DRAW AND LABEL THE X-AXIS.
CALL PLOT(0.,0.,-3)
DO 10 I = 1,NX1
CALL PLOT(0.,-.07,2)
CALL NUMBER(-.07,-.21,.10,XI,0.,-1)
'IF STATEMENT' CHANGES INCREMENT FOR ALL POSITIVE SCALE.
GO TO (11,9), ID
9 IF(I-5) 11,11,12
11 XI = XI + DX
GO TO 13
12 XI = XI - DX
13 CALL PLOT(0.,0.,3)
IF(I.EQ.NX1) GO TO 10
CALL PLOT(1.,0.,-2)
10 CONTINUE
CALL PLOT(-XN,0.,-3)
AX = XN/2. -.14*(6./7.)*NLX/2.
CALL SYMBOL(AX,-.42,.14,XLAB,0.,NLX)
DRAW AND LABEL THE Y-AXIS.
CALL PLOT(0.,0.,-3)
DO 20 I = 1,NY1
CALL PLOT(-.07,0.,2)
CALL NUMBER(-.14,-.07,.10,YI,90.,-1)
YI = YI + DY
CALL PLOT(0.,0.,3)
IF(I.EQ.NY1) GO TO 20
CALL PLOT(0.,1.,-2)
20 CONTINUE
CALL PLOT(0.,-YN,-3)
AY = YN/2. -.14*(6./7.)*NLY/2.
CALL SYMBOL(-.35,AY,.14,YLAB,90.,NLY)
THE CURVE MAY BE DRAWN BY 'CALL LINE' IN THE X-Y ARRAYS ARE FIRST
FIRST CONVERTED INTO INCHES. 'CALL CSECG' CONVERTS AUTOMATICALLY.
.....'ST-30' SELECTS THE TWO-INCREMENT OR THE ONE-INCREMENT AXIS.
30 GO TO(31,32), ID
31 CALL CSECG(0,K,NP,X,XN,XS,DX,Y,YN,-YS,-DY)
GO TO 33

```

```

32 CALL CSEG(O,K,NP,X,XI,-XS,-DX,Y,YN,-YS,-DY)
C   SET NEW URIGIN 3 INCHES BEYOND PRESENT PLOT.
33 CALL PLOT(XN+3.,0.,-3)
   RETURN
   END

```

```

SUBROUTINE FBCUT(IT,NTH,NPHI,LX,LMP)
C   'SUB-FBCUT' PLOTS THE PHI=0 AND PHI=180 CUTS ON SAME PLOT.
   DIMENSION STT(80),SPP(80),SFPP(80)
   COMMON /FVS/PHIN(20),ST(40),S(665),SF(665)
143 FORMAT(/1X,10HPHI,THETA=,F5.1,16F7.1/(9X,17F7.1))
144 FORMAT(2X,5I5)
170 FORMAT(F6.1,3X,17F7.1/(9X,17F7.1))
   DO 600 J = 1,NTH
   JU = NTH + J
   LN = NTH + 1 - J
   LL = LX - NTH + J
   STT(J) = -ST(LN)
   STT(JU) = ST(J)
   SPP(J) = S(LN)
   SPP(JU) = S(LL)
   SFPP(J) = SF(LN)
   SFPP(JU) = SF(LL)
600 CONTINUE
C   THE ARRAYS ARE READY FOR USE BY PBPLT ROUTINES FOR PLOTTING.
   NT2 = 2*NTH
   NT1 = NTH + 1
   WRITE(6,144) LX,NTH,NPHI,NT2,NT1
   GO TO (610,611), LMP
610 CALL PBPLT(IT,STT,10.,200.,-40.,14HTheta, DEGREES,14,
1SPP,7.,28.,-4.,28HPower IN DB, THETA COMPONENT,28,NT2,2)
   WRITE(6,143) (STT(J),J=1,NTH)
   WRITE(6,170) PHIN(1),(SPP(J),J=1,NTH)
   WRITE(6,143) (STT(J),J=NT1,NT2)
   WRITE(6,170) PHIN(NPHI),(SPP(J),J=NT1,NT2)
   GO TO 612
611 CALL PBPLT(IT,STT,10.,200.,-40.,14HTheta, DEGREES,14,
1SFPP,7.,28.,-4.,26HPower IN DB, PHI COMPONENT,26,NT2,2)
   WRITE(6,143) (STT(J),J=1,NTH)
   WRITE(6,170) PHIN(1),(SFPP(J),J=1,NTH)
   WRITE(6,143) (STT(J),J=NT1,NT2)
   WRITE(6,170) PHIN(NPHI),(SFPP(J),J=NT1,NT2)
612 CONTINUE
   RETURN
   END

```

```

BLOCK DATA
COMPLEX ECTT,ECFF,ERTT,ERFF
COMMON /CFLD/ECTT(665),ECFF(665)/RFLD/ERTT(665),ERFF(665)
COMPLEX ECTT/665*(1.E-15,0.)/,ECFF/665*(1.E-15,0.)/
COMPLEX ERTT/665*(1.E-15,0.)/,ERFF/665*(1.E-15,0.)/
END

```

APPENDIX C

EXPANSIONS FOR THE COMPUTATION OF THE
BESSEL FUNCTION $J_\nu(x)$

In the event that library routines are not available three different expansions may be used for the Bessel functions $J_\nu(x)$ for different ranges of x and ν .

For $x \leq 35$ and $\nu \geq x$, the power series expansion^[14]

$$J_\nu(x) = \frac{\left(\frac{x}{2}\right)^\nu}{\nu!} \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\nu!}{(\nu+k)!} \left(\frac{x}{2}\right)^{2k} \quad (C-1)$$

is used. A more convenient form of (C-1) suitable for computation is given by

$$J_\nu(x) = \frac{\left(\frac{x}{2}\right)^\nu}{\Gamma(\nu+1)} \left[1 - S_1 + S_1 S_2 - S_1 S_2 S_3 + \dots \right] \quad (C-2)$$

$$\text{where } S_i = \frac{\left(\frac{x}{2}\right)^2}{(\nu+i)i}$$

The asymptotic expansion [15] is used to represent the Bessel function for $35 < x \leq 130$ and $\nu \leq 3\sqrt{x}$.

$$J_\nu(x) = \left(\frac{2}{\pi x}\right)^{1/2} \left[\cos\left(x - \frac{\pi}{2}\left(\nu + \frac{1}{2}\right)\right) \sum_{m=0}^{\infty} (-1)^m \frac{(\nu, 2m)}{(2x)^{2m}} \right. \\ \left. - \sin\left(x - \frac{\pi}{2}\left(\nu + \frac{1}{2}\right)\right) \sum_{m=0}^{\infty} (-1)^m \frac{(\nu, 2m+1)}{(2x)^{2m+1}} \right] \quad (C-3)$$

where (ν, m) is Hankel's symbol defined by

$$(\nu, m) = \frac{2^{-2m}}{m!} \left\{ (4\nu^2 - 1)(4\nu^2 - 3^2) \dots [4\nu^2 - (2m-1)^2] \right\} \\ = \Gamma\left(\frac{1}{2} + \nu + m\right) / [m! \Gamma\left(\frac{1}{2} + \nu - m\right)] \quad (C-4)$$

The Meissel expansion [16] valid for ν real, large and positive is used to represent the Bessel function for $x > 130$ and $\nu > 3\sqrt{x}$. This expansion is included here for convenient reference.

$$J_\nu(\nu + \epsilon) = \frac{T}{48^{1/6}} + \left(\frac{3}{4}\right)^{1/6} U \quad (C-5)$$

where

$$T = \frac{\Gamma(\frac{1}{3})}{\pi(\nu + \epsilon)^{1/3}} \sum_{r=0}^{\infty} T_r(\epsilon) \left[\frac{6}{\nu + \epsilon}\right]^r$$

$$U = \frac{\Gamma(\frac{2}{3})}{\pi(\nu + \epsilon)^{2/3}} \sum_{r=0}^{\infty} U_r(\epsilon) \left[\frac{6}{\nu + \epsilon}\right]^r$$

$$\begin{aligned} \text{and } T_0(\epsilon) &= 1 & T_1(\epsilon) &= \frac{\epsilon}{45} - \frac{\epsilon^3}{18} \\ U_0(\epsilon) &= \epsilon & U_1(\epsilon) &= -\frac{1}{420} + \frac{\epsilon^2}{36} - \frac{\epsilon^4}{36} \end{aligned}$$

$$T_2(\epsilon) = -\frac{1}{8100} + \frac{\epsilon^2}{648} - \frac{7\epsilon^4}{3240} + \frac{\epsilon^6}{1620}$$

$$U_2(\epsilon) = -\frac{13}{283500} + \frac{19\epsilon^3}{11340} - \frac{\epsilon^5}{810} + \frac{\epsilon^7}{4536}$$

Taking $x = \nu + \epsilon$ and $\epsilon = x - \nu$, the above expansion of the first three terms becomes:

$$\begin{aligned} J_\nu(x) &= C_1 x^{-1/3} \left[450 + \frac{\epsilon}{x} (60 - 150\epsilon^2) + \frac{1}{x^2} [-2 + 25\epsilon^2 - 35\epsilon^4 + 10\epsilon^5] \right] \\ &\quad - C_2 x^{-5/3} \left[450 - 31500 x \epsilon - 5250 \epsilon^2 [1 - \epsilon^2] \right. \\ &\quad \left. + \frac{\epsilon}{x} [52 - 1900 \epsilon^2 + 1400 \epsilon^4 - 2500 \epsilon^6] \right] \end{aligned} \quad (C-6)$$

$$\text{where } C_1 = \frac{\Gamma(1/3)}{450 \pi (48)^{1/6}}$$

$$C_2 = \frac{(3/4)^{1/6} \Gamma(2/3)}{31500 \pi}$$

DISTRIBUTION

Commander
Naval Air Systems Command
Department of the Navy
Washington, D. C. 20360
Attn: AIR-53321A
AIR-360
AIR-50174 (4)
AIR-310B (5)

Commander
Naval Ordnance Systems Command
Department of the Navy
Washington, D. C. 20360
Attn: ORD-034

Director
Office of Naval Research
800 North Quincy Street
Arlington, Virginia 22217
Attn: ONR-427

Raytheon Company
Missile Systems Division
Hartwell Road
Bedford, Massachusetts 01730
Attn: Mr. Walter Jeros,
MS S2-32

Commander
Department of the Air Force
Air Force Avionics Laboratory
Electronic Warfare Division
Wright-Patterson Air Force Base
Ohio 45433
Attn: Mr. Harold Weber

Commanding Officer
Naval Air Development Center
Warminster, Pennsylvania 18974
Attn: Mr. Jerry Guarini, AER-2

Director Naval Research Laboratory
Washington, D. C. 20390
Attn: Code 5360
Code 5350
Code 5256

Commanding Officer
Naval Avionics Facility
Indianapolis, Indiana 46218
Attn: Mr. Paul Brink

Commander
Naval Electronics Laboratory Center
San Diego, California 92152
Attn: Code 2330

General Dynamics
Electronics Division
P. O. Box 81127
San Diego, California 92138
Attn: Dr. G. Tricoles

Director
Electro-Sciences Laboratory
Ohio State University
1320 Kinnear Road
Columbus, Ohio 43212
Attn: Mr. Robert Fouty

Sperry-Rand Corporation
Sperry Gyroscope Division
Microwave Engineering Department
Great Neck, New York 11020
Attn: Dr. Robert J. Tims

Commander
Air Force Cambridge Research Lab.
Laurence G. Hanscom Field
Bedford, Massachusetts 01730
Attn: Mr. Philip Blacksmith

McDonnell Douglas Astronautics Co.
5301 Bolsa Avenue
Huntington Beach, California 92647
Attn: Mr. John Wright/Mail Stop 9
Department A3-830/BB10

Teledyne Ryan Company
5650 Kearny Mesa Road
San Diego, California 92119
Attn: Mr. H. Penner

DISTRIBUTION - Continued

Advanced Sensor Laboratory
U.S. Army Missile Command
Redstone Arsenal
Huntsville, Alabama 35809
Attn: Mr. W. Lindberg

Harry Diamond Laboratories
Microwave Research and
Development Branch
Connecticut Avenue and Van Ness Street, N. W.
Washington, D. C. 20438
Attn: Mr. Howard S. Jones, Jr.

Dr. Norbert N. Bojarski
16 Circle Drive
Moorestown, New Jersey 08057

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Dynamic Impedance Matching in Conformal Arrays		5. TYPE OF REPORT & PERIOD COVERED Final Report January 1973 to January 1974
		PERFORMING ORG. REPORT NUMBER 2265-307184, HAC Ref. No. C-7488
7. AUTHOR(s) Peter C. Bargelotes Wolfgang H. Kummer Alfred T. Villeneuve		8. CONTRACT OR GRANT NUMBER(s) N00019-73-C-0127
9. PERFORMING ORGANIZATION NAME AND ADDRESS Hughes Aircraft Company Culver City, California 90230		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Air Systems Command Department of the Navy Washington, D. C.		12. REPORT DATE January 1974
		13. NUMBER OF PAGES 91
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Distribution limited to U. S. Agencies only; (Test and Evaluation) (January 1974) Other requests for this document must be referred to Commander, Naval Air Systems Command, AIR-310B, Washington, D. C. 20360 APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Antenna, conformal, conical, array, element, pattern, slot, modal series, Computer Programs		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An exact element pattern from slots on a conducting cone with a full cone angle of 20 degrees was investigated during this period. The exact element pattern was computed from expressions of two potential functions representing the modal fields of arbitrary apertures on the conducting cone. Special sub- routines were written for the accurate computation of the special functions of the modal series. The various expressions used in the computer program for these functions and the complete program itself are included in the report. Continued		

20. ABSTRACT - Continued

A comprehensive analysis of the modal series patterns is presented in graphical form for circumferential and radial slots at 39 radians from the cone tip. In the case of the circumferentially oriented slot, the computed patterns agree very well with available measured patterns. Recommendations for further investigative work on the topic of conformal arrays are also included. Computer programs using FORTRAN IV, IBM 370/158 are included.