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ON THE ANALYSIS OF COMPLEX, SOFTLY DEFINED PROBLEMS.(U)
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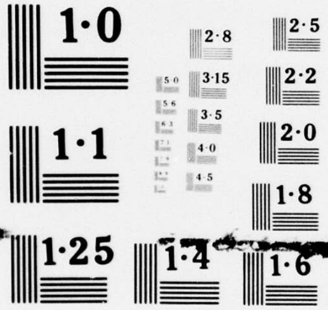

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On The Analysis of Complex, Softly Defined Problems*

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ABSTRACT

Softly defined problems are problems in which constraints and objectives are imprecise and fuzzy in the sense of the concepts of fuzzy set theory. A concept of fuzzy worth is formulated. An algorithm is presented which extracts the "most worthy" alternatives. A method is developed for obtaining all efficient solutions in the neighborhood of several most worthy alternatives. The approach is interpreted in terms of utility theory. An example of the selection of performance measures of a mobile communication system is given.

1. Introduction

There has been an increasing interest in situations where objectives and constraints are not precisely defined. Asimov [1] and Hall [13] have indicated the shortcomings of some existing approaches. The relation between system complexity and high system cost has been pointed out by Augustine [2] [3] for weapons systems. Directives of the Department of Defense have reflected this concern for complexity and cost. Cost has been emphasized as being of equal importance to performance [24].

Several approaches have been developed to deal with imprecise situations. One is based on concepts of utility theory. Several papers by Fishburn [7], [8], [9], [10] consider different aspects of the problem. Fishburn has provided an interesting summary [11] . Another approach is multiattribute decision making (see for example Raiffa [19], Huber, et al [44], Fisher [12], and Winterfeldt and Fisher [24]). A third approach which we will pursue here is based on fuzzy sets (Zadeh [25]). There are several shortcomings of many currently used approaches. These include some or all of the following: 1) difficulty in dealing with imprecision, 2) excessive effort to quantify problems, 3) algonthms which are too complex or unrealistic, 4) methods which find a unique optimal solutions rather than yield several good solutions, 5) methods which require excessive interaction with a decision maker.

The approach here is aimed at the concept definition phase in dealing with a problem. We will first develop several definitions of worth and classes of solutions. Secondly, we will develop an algorithm for extracting a subset of efficient solutions (Section 2). An example is given of a communications system (Section 3). Implementation is discussed in remarks (Section 4).

A problem will be termed softly defined if parameters of the problem vary around a value with a non-defined interval of variation. Examples of statements such as: "a Budget should be approximately $100,000." A hard, precisely defined version is that the budget cannot exceed $100,000. This is analogous to definitions in Zadeh [26] Bellman and Zadeh [4], and Lientz [16].

It is useful at this point to contrast approaches based on utility theory and fuzzy set theory. With regard to basic assumptions utility theory is based on the existence of preference among elements of a set. Fuzzy set theory is based on a gradual belonging to a set. The approaches are similar if we say that the more an object belongs to a set, the more it is preferred. However, even then utility theory assigns a single value while fuzzy set theory allows assignment of a range of values. In order to perform multiattribute optimization both require similar conditions (e.g.--transitivity and order, independence, monotonicity, convexity and/or concavity, differentiability).

2. Approach

In this section we develop the methodology and algorithm for extracting a subset of the efficient set of solutions which we will term most worthy. Worth is often defined as being approximately equal to the maximum price the customer is willing to pay for a system. More formally let a problem defined as having n attributes with the i-th attribute denoted by X_i and having levels of performance $X_{i1}, X_{i2}, \ldots, X_{ik_i}$. Let μ_w denote the fuzzy membership function (see Zadeh [27]). Worth in fuzzy set terms is a time dependent fuzzy relation between the attributes and E^1 (see Lientz [16]).

The assumptions are similar to those employed in utility theory and are as follows:

- the $\{X_{ij}\}$ are ordered increasingly. That is, $j \leq m$ implies $\mu_w(X_{ij}) \leq \mu_w(X_{im})$ for all i. (A problem can be transformed here by ordering indices using the maximum operator).
- The attributes (denoted i=1, ...,n) are independent and separable. This has been discussed in Fishburn [6], Pollak [18], and Stork [22]. Given independence and a vector $\underline{X} = (X_i)$ there are two ways to form $\mu_w(\underline{X})$ (see for example Bellman and Zadeh [4]). We will use the product operator (i.e.-$\mu_w(\underline{X}) = \prod_{i=1}^{n} \mu_w(X_i)$) rather than a maximizing operator. The maximizing operator is less sensitive to membership function values. This assumption can be weakened in situations where dependent attributes are merged.

- The worth membership function will be assumed to be concave and a cost function c will be assumed to be convex. These assumptions have been discussed in Rowe and Bahr [20], Stork [22], and Rowe [21].

The following definitions will be employed.

<u>Definition 1</u>: Define the inclination I_w as the ratio

$$I_w(X_{ij}) = \frac{\mu_w(X_{ij}) - \mu_w(X_{ij-1})}{X_{ij} - X_{ij-1}} \tag{1}$$

Similarly define for C the inclinations I_c with c (cost function) replacing μ_w in (1). From the assumptions I_w is monotone. This can be generalized in time by letting $W_i(X_{it})$ be $f_w(X_i, T(X_i))$ where T is the time contribution factor for attribute i. Taking partial derivatives we have

$$I_w(X_{ij}) = I_{f_\omega}(X_{ij}) + I_{f_\omega}(T_{ij}) + I_T(X_{ij}) \tag{2}$$

Here f_w is a fuzzy relation in X x T and I_{f_ω} is the inclination of the shadow of the fuzzy relation f_w with respect to X.

<u>Definition 2</u>: An efficient alternative is one which has a higher worth membership function than others at the same cost.

<u>Definition 3</u>: The most worthy alternatives are a subset of the set of efficient alternatives such that every member lies on the frontier of the lowest rate of diminishing ratio of worth to cost.

With the above setting the method can be defined in stages as 1) defining the problem, 2) employing the algorithm, and 3) performing sensitivity analysis. To define the problem we determine in sequence 1) the factors-attributes to be considered, 2) the ideal performance of each attribute by a customer, 3) estimated worth of the ideal performance level (lower, average, and upper bounds).

The algorithm is defined as follows:

Step 1: Start at the highest level of performance for each attribute $(X_{1k_i}, \ldots, X_{nk_n})$.

Step 2: The ratios $R^{(1)}_{ik_i}$ given by $I_c\,(X_{ik_i})\ /\ I_w\,(X_{ik_i})$ are computed.

Step 3: The largest $R^{(1)}_{ik_i}$ is found, say $R^{(1)}_{mk_m}$. The ratio $R^{(1)}_{mk_m-1}$ is computed. (k_{m-1} replaces k_m in step 2). Return to step 2 and the super scripts of ratio R_i are incremented.

The algorithm proceeds until there are no ratios remaining ($R_{i1} = 0$ for all i). We wish to develop properties of the algorithm under the assumptions stated earlier in the section.

Theorem 1: The algorithm extracts $\sum_{i=1}^{n}(k_i-1)$ vectors (X_{ij}).

Proof: This follows easily from steps 2 and 3 of the algorithm.

Note that if all $k_i = k$, then the number of solutions is nk-n (n(k-1)).

Theorem 2: The algorithm yields the set of most worthy solutions.

Proof: We proceed by induction. At the initial stage we have

$\underline{X}^{(o)} = (X_{1k_i}, \ldots, X_{nk_n})$. Now $\mu_w\,(\underline{X}^{(o)}) = 1$

and

$$C(\underline{X}^{(o)}) = \max_{\underline{X}} C(\underline{X})$$

Therefore, the first vector $\underline{X}^{(o)}$ is efficient and with the highest worth is most worthy.

At the next stage let $\underline{X}^{(1)}$ be the vector extracted by the algorithm. Then $\underline{X}^{(1)}$ maximizes

$$\frac{C(\underline{X}^{(o)}) - C(\underline{X})}{1 - \mu_w(\underline{X})} \tag{3}$$

for all $\underline{X}$ such that n-1 of the indices of the components are k_i.

For any solution $\underline{X}_m$ to be most worthy and not $\underline{X}^{(1)}$ we have

$$I_c(\underline{X}_m) = C(\underline{X}^{(o)}) - C(\underline{X}_m) \geq C(\underline{X}^{(o)}) - C(\underline{X}^{(1)}) = I_c(\underline{X}^{(1)})$$

and

$$I_w(\underline{X}_m) = 1 - \mu_w(\underline{X}_m) < 1 - \mu_w(\underline{X}^{(1)}) = I_w(\underline{X}^{(1)})$$

But these combined violate (3). Hence, $\underline{X}^{(1)}$ is most worthy.

Consider now the general step. Suppose we have m most worthy solutions. We assume that $\underline{X}^{(m+1)}$,extracted by the algorithm, is not efficient. Then we proceed by proof by contradiction as in the first step above. The proof is concluded by using a similar argument to show that all most worthy solutions are reachable by the algorithm.

The method can be extended in two ways. First, if μ_w is imprecise, bounds can be set--$\check{\mu}_w$ (lower) and $\hat{\mu}_w$ (upper). We could use the algorithm with an average value $\bar{\mu}_w$ and then refine it by using $\check{\mu}_w$ and $\hat{\mu}_w$.

A second way is for a further search. The set of most worthy solutions can be shown to be a fuzzy set. The image of the most worthy solution with maximal (minimal) admissible cost to the decision maker is a fuzzy set which gives an upper (lower) bound on space for a second stage search. We can extend the algorithm to extract all efficient solutions in the neighborhood of several most worthy solutions. We will employ a dynamic programming model with c as the state variable, i as the stage, $\mu_i(d_i)$ as the worth function for decision variable d_i and f^*_{i-1} as the maximizing worth function.

The relational equation is

$$f_i(c) = \max\{(\mu_i(d_i)\ f_{i-1}{}^* \ (c - d_i)\} \qquad (4)$$

with $c_{min} < c < c_{max}$ and $\mu_{min} < f_i < \mu_{max}$

Here c_{max}, μ_{min}, and μ_{max} are obtained by the algorithm and chosen by the decision maker. It follows from dynamic programming that

Theorem 3: The dynamic programming method based on (4) extracts all efficient solutions in the neighborhood $(c_{min}, c_{max}) \times (\mu_{min}, \mu_{max})$.

Combining the above results we have a two stage method which extracts first the most worthy solutions and second the efficient solutions in a neighborhood of several most worthy solutions. This takes fewer steps than several past methods of utility theory. For example Ting [22] has given the number of steps as $kn+2^n-1$ for the utility theory method of Keeney [14]. There are several alternative search procedures to dynamic programming. Several are to adopt the method of Markowitz [16] for the portfolio selection problem and the use of interactive search of an undonominated set (Zeleny [27], Evans and Steuer [4]). The next section gives an example of the algorithm and extraction of efficient solutions.

3. Example

This section provides a simplified example in using the algorithm. The more general real example is discussed in the next section. The problem involves a mobile communications system. There are three attributes (n=3). The user (customer) is unsure of the relations between performance and cost. The statement of the problem is as follows. The budget should be about $170,000. The range should be approximately 150 nautical miles; the mean time between failures (MTBF) approximately 400 to 600 hours; mobility according to MIL-STD-"XYZ." Through user interviews the intermediate levels of performance are given in Table 1 along with costs (in thousands). Applying the algorithm we obtain the iterations (ν) shown in Table 2. In this Table S denotes the system performance and seven iterations are needed to extract all of the most worthy solutions (7= (3-1) + (4-1) + (3-1)).

Now the customer indicated a cost of approximately $170,000. The cost of the performance levels at iteration 2 (3) is $180,300 ($168,900). A logical next step is to apply the dynamic programming method to extract all efficient solutions in this cost interval and between .595 and .665 in worth. This procedure yields an efficient solution of $(X_{1,2}, X_{2,3}, X_{3,2})$ with worth .599 and cost $172,800 (see Table 3). The most worthy solutions are shown in Figure 1.

We could now allow for different membership functions. In this example $\check{\mu}$ and $\hat{\mu}$ are defined as in Table 4. A similar approach can be used to extract efficient solutions in the larger region.

4. Remarks

The algorithm presented in Section 2 has been programmed in FORTRAN on an IBM 1130 with 16k memory. A flowchart of the program is shown in figure 2. The program utilizes three subroutines : report, order/to obtain an associative pointer in linking cost-worth ratios), and withdraw (to select the highest ratio from associative list, retain attribute pointer and performance level).

A larger problem was analyzed using the methods outlined above for a complex radar system. The system had 12 attributes (weight, range, mean time between failure (MTBF), mean time to repair (MTTR), subclutter visability, electronic countermeasures (ECCM), range resolution, range accuracy, angular accuracy, angular resolution, human factors and mobility). Each was allowed three performance levels. The method was applied in the concept phase to delineate user requirements. The method was applied successfully. However, several not totally expected events occurred. First, the user (government agency personnel) expressed the opinion that the method more clearly showed the trade-offs in performance levels. Second, the method was repeatedly used as the system was built to perform additional trade-offs as costs and performance levels became more precise.

5. Conclusions

A method has been presented for extracting the subset of efficient solutions which are most worthy in terms of a fuzzy set membership function for worth. The rate of convergence has been developed along with a method for extracting the set of efficient solutions in the neighborhood of several most worthy solutions.

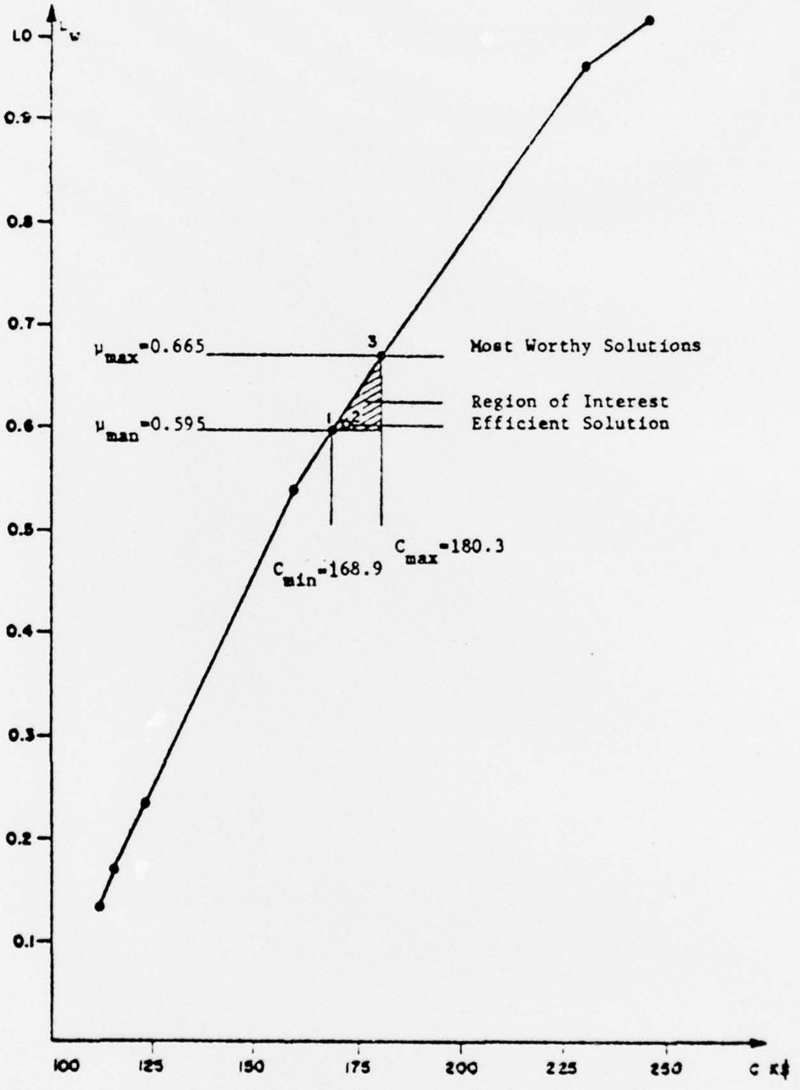

Figure 1: Illustration of results of sample problem.

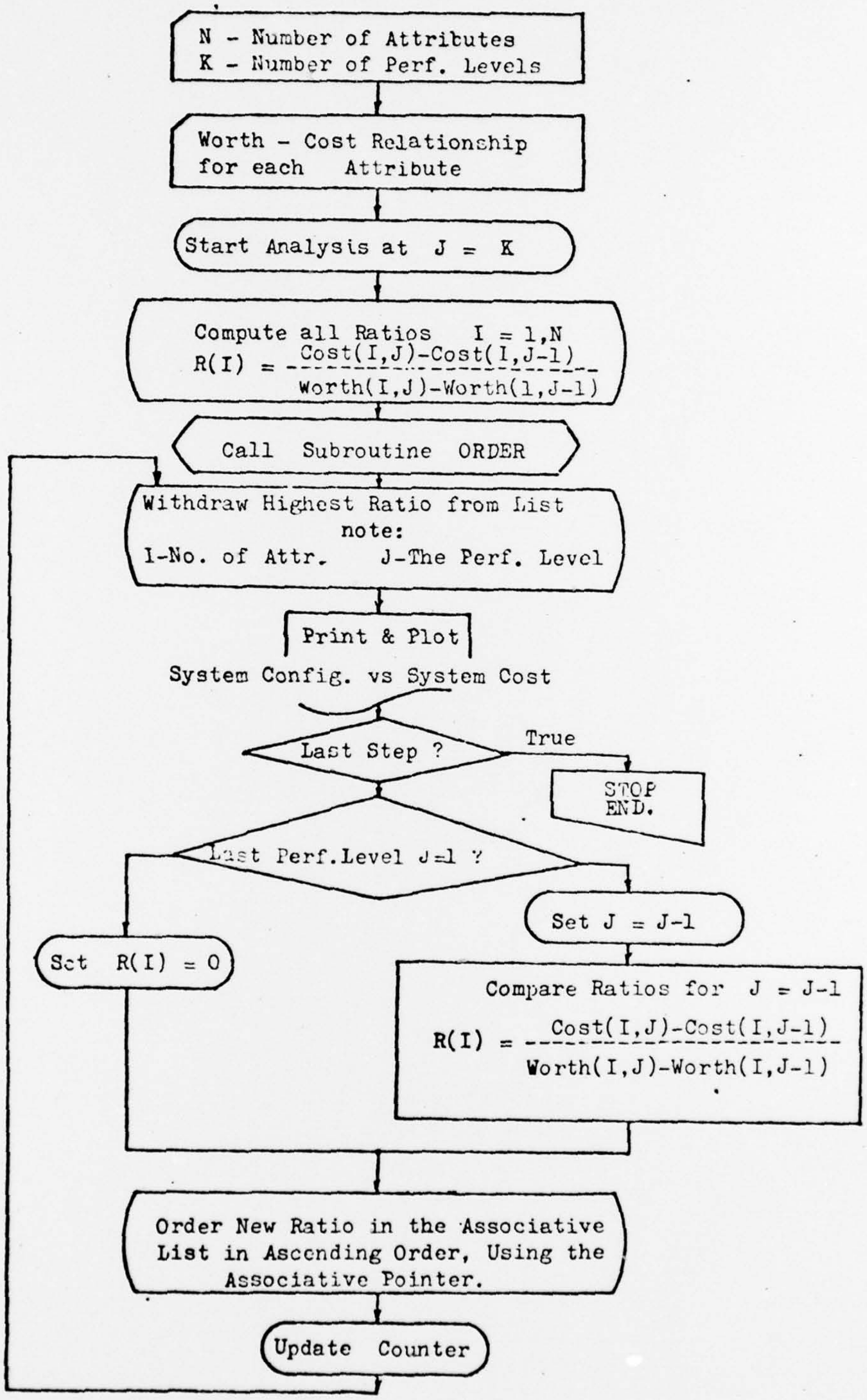

Figure 2: Flowchart of Algorithm

TABLE 1.

WORTH MEMBERSHIP FUNCTIONS AND CORRESPONDING SYSTEM COSTS FOR MOBILE COMMUNICATION SYSTEM

i	1			2			3		
j	Range			MTBF			Mobility		
	N.M.	$\mu(1,j)$	$C(1,j)$	Hours	$\mu(2,j)$	$C(2,j)$	MIL-STD	$\mu(3,j)$	$C(3,j)$
1	100	0.3	87.5	200	0.70	7.5	Class 1	0.65	17.5
2	150	0.7	122.5	400	0.85	11.4	Class 2	0.90	24.5
3	200	1.0	175.5	600	0.95	22.8	Class 3	1.00	35.0
4	---	---	---	800	1.00	38.0	---	---	---

TABLE 2.

THE SET OF MOST WORTHY ALTERNATIVES AS OBTAINED IN EACH ITERATION

v	$R_1^{(v)}$	$R_2^{(v)}$	$R_3^{(v)}$	$R^{(v)}$	S			$\mu^{(v)}$	$C^{(v)}$
					$X_{1,j}$	$X_{2,j}$	$X_{3,j}$		
0	--	--	--	--	3	4	3	1.0	248.5
1	176.6	305	105	$R_2^{(1)}$	3	(3)	3	0.95	232.3
2	176.6	114	105	$R_1^{(2)}$	(2)	3	3	0.665	180.3
3	87.5	114	105	$R_2^{(3)}$	2	(2)	3	0.595	168.9
4	87.5	26	105	$R_3^{(4)}$	2	2	(2)	0.535	158.4
5	87.5	26	28	$R_1^{(5)}$	(1)	2	2	0.229	123.4
6	0	26	28	$R_3^{(6)}$	1	2	(1)	0.166	116.4
7	0	26	0	$R_2^{(7)}$	1	(1)	1	0.136	112.5

TABLE 3.

EFFICIENT AND MOST WORTHY SOLUTIONS IN THE ENLARGED REGION OF INTEREST

Most Worthy Alternatives

V	$X_{1,j}$	$X_{2,j}$	$X_{3,j}$	μ	C
0	3	4	3	1.000	248.5
1	3	3	3	0.950	232.3
2	2	3	3	0.665	180.3
3	2	2	3	0.595	168.9
4	2	2	2	0.535	158.4
5	1	2	2	0.229	123.4
6	1	2	1	0.166	116.4
7	1	1	1	0.136	112.5

Efficient Alternatives in Region of Interest

$X_{1,j}$	$X_{2,j}$	$X_{3,j}$	μ	C
3	3	2	0.855	222.8
3	2	3	0.850	221.9
3	2	2	0.765	211.4
2	4	3	0.700	195.5
2	3	2	0.599	172.8

TABLE 4.

RANGES OF WORTH MEMBERSHIP FOR THE IMPRECISE WORTH ASSESSMENT CASE

	j \ i	1 Range	2 MTBF	3 Mobility
$\hat{\mu}(X_{i,j})$	1	0.45	0.80	0.70
	2	0.85	0.90	0.95
	3	1.00	0.99	1.00
	4	-	1.00	-

	j \ i	1 Range	2 MTBF	3 Mobility
$\check{\mu}(X_{i,j})$	1	0.15	0.60	0.60
	2	0.55	0.80	0.86
	3	1.00	0.90	1.00
	4	-	1.00	-

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