

AD-A044 850

AEROSPACE CORP EL SEGUNDO CALIF IVAN A GETTING LABS F/G 16/2
A RATIONAL METHOD FOR DESIGN OF FEEDBACK CONTROL TO LIMIT MISSI--ETC(U)
AUG 77 D H PLATUS, J WOLKOVITCH F04701-76-C-0077
TR-0077(2240)-4 SAMSO-TR-77-193 NL

UNCLASSIFIED

| OF |
AD
A044850



12
mc

AD A 044850

A Rational Method for Design of Feedback Control to Limit Missile Dispersion

Aerophysics Laboratory
The Ivan A. Getting Laboratories
The Aerospace Corporation
El Segundo, Calif. 90245

15 August 1977

Interim Report

Prepared for
SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
Los Angeles, Calif. 90009

AD No. _____
DDC FILE COPY

DDC
RECEIVED
OCT 4 1977
A

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED

This interim report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-76-C-0077 with the Space and Missile Systems Organization, Deputy for Advanced Space Programs, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by W. R. Warren, Jr. Director, Aerophysics Laboratory. Lieutenant Batki, SAMSO/YAPT, was the project officer for Advanced Space Programs.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

The technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force Approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Dara Batki

Dara Batki, Lieutenant, USAF
Project Officer

Robert W. Lindemuth

Robert W. Lindemuth, Lt Col, USAF
Chief, Technology Plans Division

FOR THE COMMANDER

Floyd R. Stuart

Floyd R. Stuart, Colonel, USAF
Deputy for Advanced Space Programs

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SAMSO-TR- 77-193	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A RATIONAL METHOD FOR DESIGN OF FEEDBACK CONTROL TO LIMIT MISSILE DISPERSION		5. TYPE OF REPORT & PERIOD COVERED Interim report
7. AUTHOR(s) Daniel H. Platus and Julian Wolkovitch		6. PERFORMING ORG. REPORT NUMBER TR-0077(2240)-4
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, Calif. 90245		8. CONTRACT OR GRANT NUMBER(s) F04701-76-C-0077
11. CONTROLLING OFFICE NAME AND ADDRESS Space and Missile Systems Organization Air Force Systems Command Los Angeles, Calif. 90009		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 15 August 1977
		13. NUMBER OF PAGES 30
		15. SECURITY CLASS. (of this report) Unclassified
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Missile Control Missile Accuracy Missile Dispersion Missile Aerodynamics		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A method for the design of feedback control to limit cross-range dispersion of spinning missiles is described. The missile transverse velocity in the cross plane is selected as the cost function, and optimal linear feedbacks are derived that minimize this velocity in the presence of aerodynamic disturbances that cause lift nonaveraging. A closed-form solution is obtained for the open- and closed-loop missile response to simple disturbance moments. Cross-range dispersion control complements control of range error by → next page		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

angle-of-attack control of drag. The resulting two-loop system can control major sources of missile dispersion.



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

PREFACE

The authors are grateful to M. E. Brennan for her assistance with the numerical computations.

ACCESSION NO.	
NTIS	Write Section ☒
DDI	Self Section ☐
CONTRIBUTOR	☐
IDENTIFICATION	
BY	
DISTRIBUTION AVAILABILITY CODES	
DTL	AVAIL. NO. - 000000
A	

CONTENTS

PREFACE	1
I. INTRODUCTION	7
II. ANALYSIS	9
A. Control Equations	9
B. Solution for Linear Optimal Control	14
C. Response to Impulsive Yaw Moment.	17
III. NUMERICAL EVALUATION.....	21
IV. SYSTEM IMPLEMENTATION.....	29
V. CONCLUSIONS	31
REFERENCES	33
NOMENCLATURE	35

FIGURES

1.	Transverse velocity	10
2.	Open-loop angle-of-attack response to impulse	22
3.	Open-loop precession rate response to impulse	23
4.	Open-loop dispersion velocity response to impulse	24
5.	Closed-loop angle-of-attack response to impulse	25
6.	Closed-loop precession rate response to impulse	26
7.	Closed-loop dispersion velocity response to impulse	27

TABLE

1.	Net dispersion velocity in ft/sec	28
----	---	----

I. INTRODUCTION

Dispersion of missiles can be separated into two principal categories; (1) cross-range dispersion resulting from lift nonaveraging and (2) up- or down-range error associated with drag uncertainty and atmospheric variations such as winds. Winds also contribute to cross-range dispersion. Control of up- or down-range error by compensating for drag uncertainty (drag modulation) has been treated previously. It has been shown that very little energy is required to effect large changes in drag by yaw-moment control of angle of attack.^{1,2} The missile is controlled in a circular coning motion in which the drag is a strong function of the coning half-angle (angle of attack). Changes in drag caused by extraneous sources, as detected by drag deceleration, can be compensated for by appropriately increasing or decreasing the angle of attack.

Cross-range dispersion resulting from lift nonaveraging is one of the greatest contributors to missile impact error. The sources of such dispersion are quite varied and include launch errors caused by muzzle disturbance,³ small mass and configurational asymmetries,^{4,5,6,7} and asymmetric boundary-layer transition (in the case of ballistic reentry vehicles).⁸ It is well known that dispersion caused from variation in body-fixed asymmetries varies approximately inversely with roll rate,^{5,8} and that such dispersion can be controlled by maintaining the roll rate at a sufficiently large steady value (roll control). However, certain forms of disturbance moments can adversely affect the lift-vector precession rate and produce lift nonaveraging dispersion somewhat independent of the roll rate.⁸

In this report, a general method is described for the design of feedback control to limit cross-range dispersion caused by lift nonaveraging. This control would complement drag control to limit up- and down-range error; the resulting two-loop system would control major sources of missile dispersion.

II. ANALYSIS

A. CONTROL EQUATIONS

Cross-range dispersion resulting from lift nonaveraging can be described in terms of the complex missile transverse velocity $V = v + iw$ in a plane normal to the average flight path. This velocity is defined in terms of the total angle of attack θ and lift-vector precession angle ψ according to⁸ (Fig. 1).

$$V(t) = V(0) - \frac{iL_\theta}{m} \int_0^t \theta e^{i\psi} dt \quad (1)$$

where L_θ is the lift force derivative and m is the missile mass. If the average value of the transverse velocity $V(t)$ is set equal to zero prior to any disturbance in θ and ψ that causes lift nonaveraging, then the cross-range dispersion resulting from a disturbance is proportional to the magnitude of the average value of V at sufficiently large time t after the disturbance.

The behavior of the angles θ and ψ is described approximately by the undamped equations of motion for a slowly rolling ($p = \text{const.}$), axisymmetric missile¹

$$\ddot{\theta} + (\omega_n^2 - \dot{\psi}^2) \theta = \frac{M_p}{I} + \frac{M_{\delta p}}{I} \quad (2)$$

$$\theta \ddot{\psi} + 2\dot{\theta} \dot{\psi} = \frac{M_y}{I} + \frac{M_{\delta y}}{I} \quad (3)$$

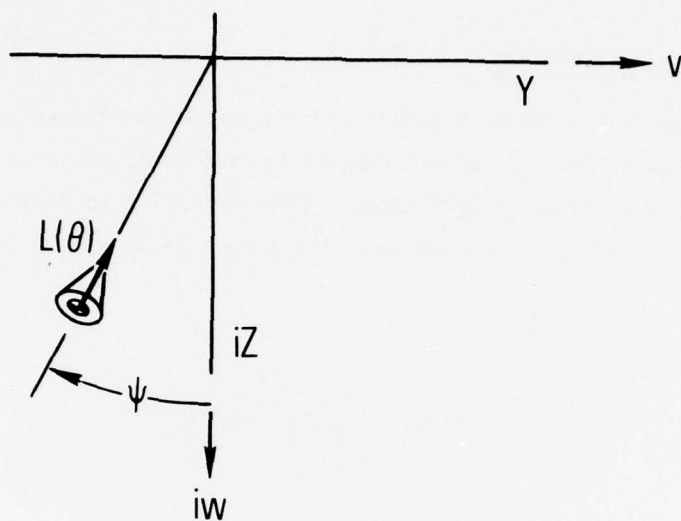


Fig. 1. Transverse velocity

where M_p and M_y are aerodynamic pitch and yaw disturbance moments in the wind-referenced axes and $M_{\delta p}$ and $M_{\delta y}$ are applied control moments.* The control problem is to define the moments $M_{\delta p}$ and $M_{\delta y}$ in terms of appropriate state variables that minimize the net transverse velocity V resulting from some disturbance. The problem is that of an optimal regulator with a cost function equal to the magnitude of the integral in Eq. (1).

It is assumed that the missile is initially untrimmed in a circular coning motion to permit a sufficient level of drag control by either increasing or decreasing the angle of attack.¹ Cross-range dispersion is then controlled by minimizing the coupling between angle-of-attack and precession angle perturbations that cause lift nonaveraging in accordance with Eq. (1). The equations of motion, Eqs. (2) and (3), can be linearized in terms of small perturbations about the quasi-steady values $\theta = \bar{\theta}$ and $\dot{\psi} = \omega_n$.¹ With $\theta = \bar{\theta} + \theta_+$ and $\dot{\psi} = \omega_n + \dot{\psi}_+$ substituted in Eqs. (2) and (3) and higher-order terms neglected (where θ_+ and $\dot{\psi}_+$ are small perturbations), the resulting linear control equations can be written

$$\ddot{\theta}_+ - 2\omega_n \bar{\theta} \dot{\psi}_+ = \frac{M_p}{I} + \frac{M_{\delta p}}{I} \quad (4)$$

$$2\omega_n \dot{\theta}_+ + \bar{\theta} \ddot{\psi}_+ = \frac{M_y}{I} + \frac{M_{\delta y}}{I} \quad (5)$$

where the control moments are assumed to be linear functions of the state variables θ_+ , $\dot{\theta}_+$, and $\dot{\psi}_+$, according to

* Aerodynamic damping is omitted, and the roll rate is assumed to be small such that $I_{xp}/2I \ll \omega_n$.

$$\frac{M_{\delta p}}{I} = -A\theta_+ - B\dot{\theta}_+ - C\dot{\psi}_+ \quad (6)$$

$$\frac{M_{\delta y}}{I} = -D\theta_+ - E\dot{\theta}_+ - F\dot{\psi}_+ \quad (7)$$

It is convenient to nondimensionalize the control equations by defining new variables**

$$\alpha \equiv \theta_+ / \bar{\theta}, \quad \lambda \equiv \dot{\psi}_+ / \omega_n, \quad \tau \equiv \omega_n t \quad (8)$$

If these variables are substituted in Eqs. (4) through (7), since $d/dt = \omega_n d/d\tau$, and the Laplace transform with respect to τ defined by

$$\mathcal{L}(\cdot) = \int_0^\infty (\cdot) e^{-s\tau} d\tau \quad (9)$$

is taken, the control equations can be written

$$\begin{bmatrix} s^2 + bs + a & c-2 \\ (2+e)s + d & s+f \end{bmatrix} \begin{bmatrix} \alpha \\ \lambda \end{bmatrix} = \begin{bmatrix} m_p \\ m_y \end{bmatrix} \quad (10)$$

where the nondimensional feedbacks $a, b, c, \dots$ are defined by

$$a \equiv \frac{A}{\omega_n^2}, \quad b \equiv \frac{B}{\omega_n}, \quad c \equiv \frac{C}{\omega_n \bar{\theta}}, \quad d \equiv \frac{D}{\omega_n^2}, \quad e \equiv \frac{E}{\omega_n}, \quad f \equiv \frac{F}{\omega_n \bar{\theta}} \quad (11)$$

** This step is not essential for the analysis, but it facilitates the numerical evaluation described later.

and the nondimensional transformed disturbance moments are

$$m_p \equiv \frac{1}{\omega_n^2 \bar{\theta} I} \mathcal{L}(M_p), \quad m_y \equiv \frac{1}{\omega_n^2 \bar{\theta} I} \mathcal{L}(M_y) \quad (12)$$

The transformed nondimensional control moments, Eqs. (6) and (7), can similarly be written

$$m_{\delta p} = -[(a + bs) \alpha(s) + c\lambda(s)] \quad (13)$$

$$m_{\delta y} = -[(d + es) \alpha(s) + f\lambda(s)] \quad (14)$$

The transverse velocity, Eq. (1), can be written in nondimensional form with the substitutions $\theta = \bar{\theta}(1 + \alpha)$ and

$$\psi = \int_0^t (\omega_n + \dot{\psi}_+) dt = \int_0^\tau (1 + \lambda) d\tau \quad (15)$$

which yields

$$V(\tau) = V(0) - i\ell\omega_n \bar{\theta} \int_0^\tau (1 + \alpha) \exp \left[i \int_0^\tau (1 + \lambda) d\tau \right] d\tau \quad (16)$$

where the relation for pitch frequency $\omega_n^2 = L_\theta x_{st}/I$ is used and a characteristic length $\ell = I/mx_{st}$ is defined. The control problem defined by Eqs. (10) and (16) is that of a linear optimal regulator. The optimal linear feedbacks a , b , c , . . . that will minimize the cost function, Eq. (16), are to be determined. It is necessary to include the additional constraint on the cost function that the control moments remain within acceptable limits to preclude the trivial case of very large feedbacks by redefining the cost function

$$I = W_1 V V^* + W_2 \int_0^\infty m_{\delta p}^2 d\tau + W_3 \int_0^\infty m_{\delta y}^2 d\tau \quad (17)$$

where V^* is the complex conjugate of V , $m_{\delta p}$ and $m_{\delta y}$ are the nondimensional control moments defined by Eqs. (13) and (14), and W_1 , W_2 , and W_3 are suitable weighting functions.

B. SOLUTION FOR LINEAR OPTIMAL CONTROL

The solution to Eq. (10) for $\alpha(s)$ and $\lambda(s)$ is of the form

$$\alpha(s) = \frac{N_1(s)}{D(s)}, \quad \lambda(s) = \frac{N_2(s)}{D(s)} \quad (18)$$

where $N_1(s)$, $N_2(s)$, and $D(s)$ are polynomials in s of order depending on the form of the disturbance moments m_p and m_y and with coefficients containing the feedbacks a , b , c , . . . to be determined. The control moment terms in Eq. (17), in the form of integral-square values, can be readily evaluated from the results of Eq. (18) in terms of Phillips integrals.^{9, 10}

For example, if the system is third order and the pitch-control moment $m_{\delta p}$, Eq. (13), has the form

$$m_{\delta p} = \frac{c_2 s^2 + c_1 s + c_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0} \quad (19)$$

the integral $I_3 = \int_0^\infty m_{\delta p}^2 d\tau$ has the value

$$I_3 = \frac{c_2^2 d_0 d_1 + (c_1^2 - 2c_0 c_2) d_0 d_3 + c_0^2 d_2 d_3}{2d_0 d_3 (d_1 d_2 - d_0 d_3)} \quad (20)$$

Evaluation of the transverse velocity term VV^* is much more difficult because of the nonlinear coupling between α and λ in Eq. (16). A reasonable approximation to Eq. (16) can be obtained, however, by writing it in the form

$$V(\tau) = V(0) - i\ell \omega_n \bar{\theta} \int_0^\tau [1 + \alpha(\tau)] e^{i\lambda_1(\tau)} e^{i\tau} d\tau \quad (21)$$

where

$$\lambda_1(\tau) = \int_0^\tau \lambda(\tau) d\tau, \quad \lambda_1(s) = \frac{\lambda(s)}{s} \quad (22)$$

and the upper limit of the integral in Eq. (21) is taken to be sufficiently large to include the net change in $V(\tau)$ resulting from perturbations $\alpha(\tau)$ and $\lambda(\tau)$. Without loss of generality, this upper limit can be taken as ∞ , and Eq. (21) becomes the Fourier transform of the function $f(\tau)$

$$V = V(0) - i\ell \omega_n \bar{\theta} \int_0^\infty f(\tau) e^{i\tau} d\tau \quad (23)$$

where

$$f(\tau) = [1 + \alpha(\tau)] e^{i\lambda_1(\tau)} \quad (24)$$

Since $\alpha(\tau)$ and $\lambda(\tau)$ are, in general, small perturbations, according to their definitions, Eq. (8), the exponential in Eq. (24) can be expanded to obtain for $f(\tau)$ the approximation

$$f(\tau) \approx 1 + \alpha(\tau) + i\lambda_1(\tau) + i\alpha(\tau)\lambda_1(\tau) - \frac{1}{2} [\lambda_1(\tau)]^2 + \dots \quad (25)$$

Equation (23) can be evaluated in terms of the Laplace transform of $f(\tau)$, Eq. (9), with $s = -i$. The constant term in Eq. (25) is the contribution to $V(t)$ from θ prior to a disturbance that causes $\alpha(\tau)$ and $\lambda(\tau)$. If $V(0)$ in Eq. (23) is defined as the average value of $V(t)$ prior to a disturbance, both the constant and $V(0)$ can be dropped, since only the net change in V from the perturbation is of interest.⁸ Equation (23) can then be written

$$\begin{aligned}\Delta V &= -i\ell\omega_n\bar{\theta}\mathcal{L}\left[\alpha(\tau) + i\lambda_1(\tau) + i\alpha(\tau)\lambda_1(\tau) - \frac{1}{2}[\lambda_1(\tau)]^2 + \dots\right]_{s=-i} \\ &= -i\ell\omega_n\bar{\theta}\left[\alpha(s) + i\frac{\lambda(s)}{s} + i\mathcal{L}[\alpha(\tau)\lambda_1(\tau)] - \frac{1}{2}\mathcal{L}[\lambda_1(\tau)]^2 + \dots\right]_{s=-i}\end{aligned}\quad (26)$$

where $\alpha(s)$, $\lambda(s)$, and $\lambda_1(s)$ are given by Eqs. (18) and (22). A first-order solution consists simply of the first two terms, which, from Eq. (18), can be written

$$\Delta V_{1st\ ord} = -i\ell\omega_n\bar{\theta}\left[\frac{N_1(-i) - N_2(-i)}{D(-i)}\right]\quad (27)$$

A second-order solution includes, in addition to Eq. (27), the terms involving the Laplace transform of a product. For the case where $\alpha(s)$ and $\lambda(s)$, given by Eqs. (18) and (22), consist of m and n first-order poles, respectively, and no others, the Laplace transforms are given by¹¹

$$\mathcal{L}[\alpha(\tau)\lambda_1(\tau)] = \sum_{k=1}^m \frac{N_1(s_k)}{D'(s_k)} \lambda_1(s - s_k)\quad (28)$$

$$\mathcal{L}[\lambda_1(\tau)]^2 = \sum_{j=1}^n \frac{N_2(s_j)}{D_1'(s_j)} \lambda_1(s - s_j)\quad (29)$$

where s_k are roots of $D(s) = 0$ and s_j are roots of $D_1(s) \equiv sD(s) = 0$. Thus, the approximate solution for V , given by Eqs. (26) through (29), is obtained directly from the s -plane solution of the control equations in the form of Eqs. (18) and (22). The optimum feedbacks are those values that minimize the cost function, Eq. (17), according to

$$\frac{\partial I}{\partial a} = 0, \frac{\partial I}{\partial b} = 0, \frac{\partial I}{\partial c} = 0, \dots \quad (30)$$

C. RESPONSE TO IMPULSIVE YAW MOMENT

The method is illustrated with a simple example that is amenable to an analytical solution. The response to an impulsive yaw moment

$$M_y(t) = M\delta(t) \quad (31)$$

is considered such that $m_y = M/\omega_n \bar{\theta}I$, from Eq. (12), is a constant and $M_p = 0$.[†] The solution to Eq. (10) for $\alpha(s)$ and $\lambda(s)$ in the form of Eq. (18) is

$$\alpha(s) = \frac{m_y(2 - c)}{D(s)} \quad (32)$$

$$\lambda(s) = \frac{m_y(s^2 + bs + a)}{D(s)} \quad (33)$$

where

$$D(s) = s^3 + (b + f)s^2 + [a + bf + (2 + e)(2 - c)]s + af + d(2 - c) \quad (34)$$

[†]Since the unit impulse function $\delta(t)$ has units of $1/t$, it must be nondimensionalized by dividing by ω_n prior to taking the Laplace transform defined in Eq. (9), which accounts for the form $M/\omega_n \bar{\theta}I$ rather than $M/\omega_n^2 \bar{\theta}I$ for m_y .

The Routh stability criterion¹⁰ requires that the feedbacks satisfy the relations

$$\begin{aligned} af + d(2 - c) &> 0 \\ a + bf + (2 + e)(2 - c) &> 0 \\ b + f &> 0 \\ (b + f) [bf + (2 + e)(2 - c)] + ab - d(2 - c) &> 0 \end{aligned} \quad (35)$$

These are satisfied with the feedbacks $a, f \neq 0$ and $b = c = d = e = 0$. Optimum values of these two feedbacks will be found that minimize dispersion in accordance with our cost function, Eq. (17). The perturbation solution, Eqs. (32) through (34), reduces to

$$\alpha(s) = \frac{2m_y}{D(s)}, \quad \lambda(s) = \frac{m_y(s^2 + a)}{D(s)}, \quad D(s) = s^3 + fs^2 + (a + 4)s + af \quad (36)$$

and the first-order solution, Eq. (27), is found to be

$$\Delta V_{1st \text{ ord}} = \frac{(M\ell/I)(3 - a) [3 + a - i(a - 1)f]}{(a - 1)^2 f^2 + (a + 3)^2} \quad (37)$$

This expression vanishes for $a = 3$, independent of f . Hence, to the first-order approximation, dispersion caused from an impulsive yaw moment can be eliminated with two simple feedbacks. In a more general case, the feedbacks would be determined to minimize both the dispersion term VV^* and the control moments in our cost function, Eq. (17), with suitable weighting of each term. Since the feedback $a = 3$ eliminates dispersion, to the first-order approximation, and a nonzero value of the feedback f is required for stability, the magnitude of f is determined to effect equal

integral-square values of the control moments in order to illustrate the method. The control moments, Eqs. (13) and (14), with $a = 3$, $b = c = d = e = 0$, and the solution, Eq. (36), can be written

$$m_{\delta p} = -\frac{2a m_y}{D(s)}, \quad m_{\delta y} = -\frac{f m_y (s^2 + a)}{D(s)} \quad (38)$$

where $D(s)$ is defined in Eq. (36). The integral-square values of these moments are given by the I_3 integral, Eq. (20), where the coefficients c_n and d_n are defined for each moment by Eqs. (36) and (38). If the numerators of Eq. (20) are equated for the two moments (since the denominators are the same), the nondimensional feedbacks a and f must satisfy the relation $f = \sqrt{a}$ for equal integral-square values of the control moments. Thus, the optimal linear first-order values of our two feedbacks for the assumed cost function are $a = 3$ and $f = \sqrt{3}$. These values should result in zero dispersion, to the first-order approximation, and require equal integral-square values of the control moments. The open-loop dispersion caused by an impulsive yaw moment M is readily obtained from Eq. (26) and the perturbation solution, Eqs. (32) through (34), with all of the feedbacks equal to zero. The first-order approximation to this dispersion is found from Eq. (37) with $a = f = 0$ to give the simple result

$$\Delta V_{\text{op lp 1st ord}} = \frac{Ml}{I} \quad (39)$$

III. NUMERICAL EVALUATION

The equations of motion, Eqs. (2) and (3), were integrated numerically to obtain both the open- and closed-loop responses to an impulsive negative yaw moment with the simple optimal feedbacks derived above. The results are compared with first- and second-order analytical approximations to the dispersion velocity. Angle of attack, precession rate, and transverse velocity histories are shown in Figs. 2 through 7 for the open- and closed-loop responses. The inputs and system parameters are as follows:

$$\begin{aligned}
 I &= 20 \text{ slug ft}^2 \\
 l &= 6.566 \text{ ft} \\
 \omega_n &= 40 \text{ rad/sec} \\
 \bar{\theta} &= 2 \text{ deg} \\
 a &= 3 \\
 f &= \sqrt{3} \\
 b &= c = d = e = 0 \\
 M &= \int M_y dt = -6.092 \text{ ft lb sec}
 \end{aligned}$$

The feedbacks used in the simulation of Figs. 5 through 7 ($a = 3$, $f = \sqrt{3}$) were determined to give zero net dispersion velocity on the basis of the first-order approximation, Eq. (37). The closed-loop velocity, computed from a numerical integration of the nonlinear equations (2) and (3), is shown in Fig. 7. The feedback control limits the dispersion to approximately 3% of the open-loop value. The second-order contributions to the net dispersion velocity, Eq. (26), can be computed from the relations, Eqs. (28) and (29), for the Laplace transform of a product, and from the control solutions, Eq. (36). The second-order terms are found to be

$$\Delta V_{\text{op lp 2nd ord}} = \frac{l}{15 \omega_n \bar{\theta}} \left(\frac{M}{I} \right)^2 = 0.0291 \text{ ft/sec} \quad (40)$$

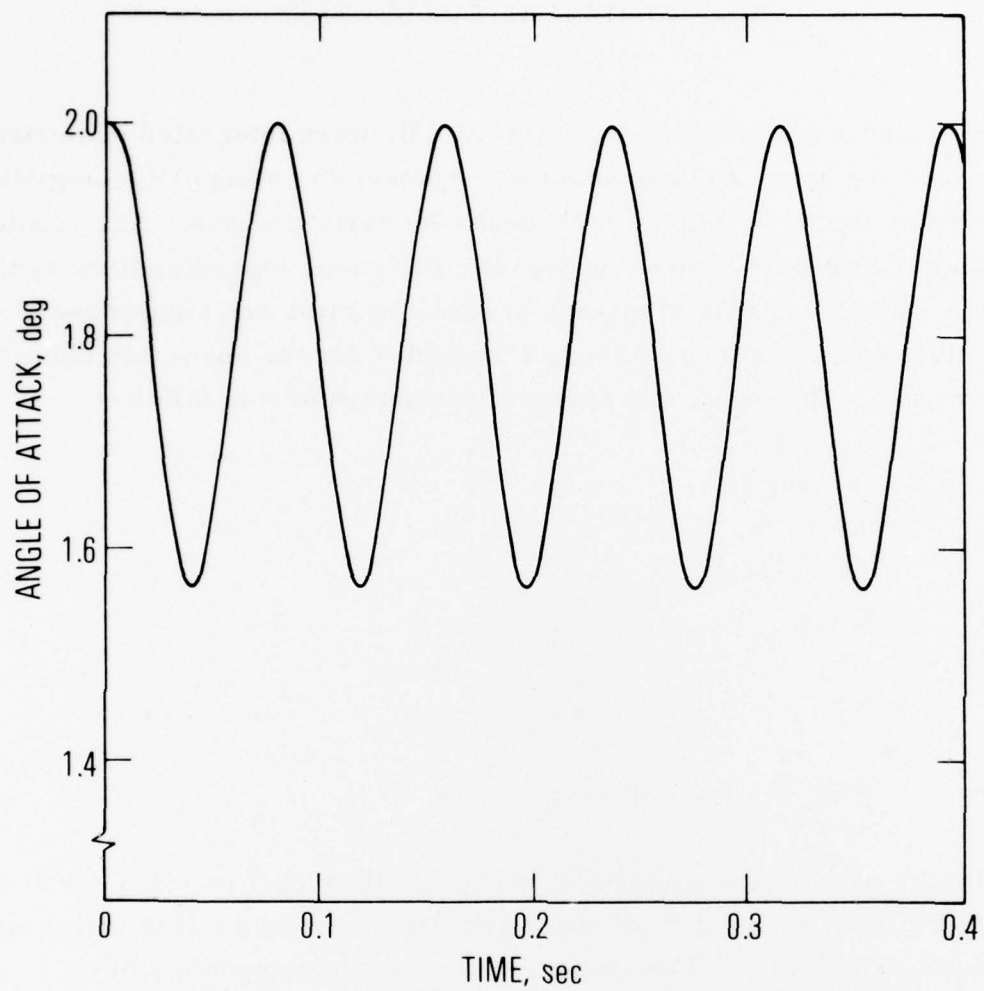


Fig. 2. Open-loop angle-of-attack response to impulse

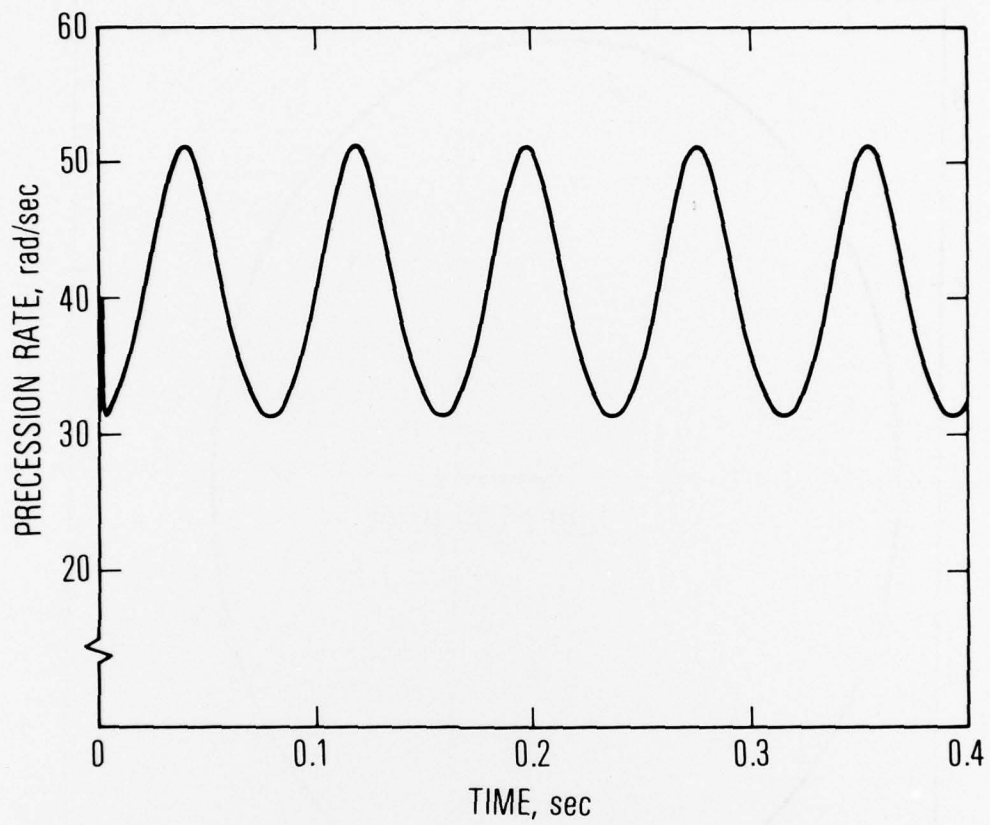


Fig. 3. Open-loop precession rate response to impulse

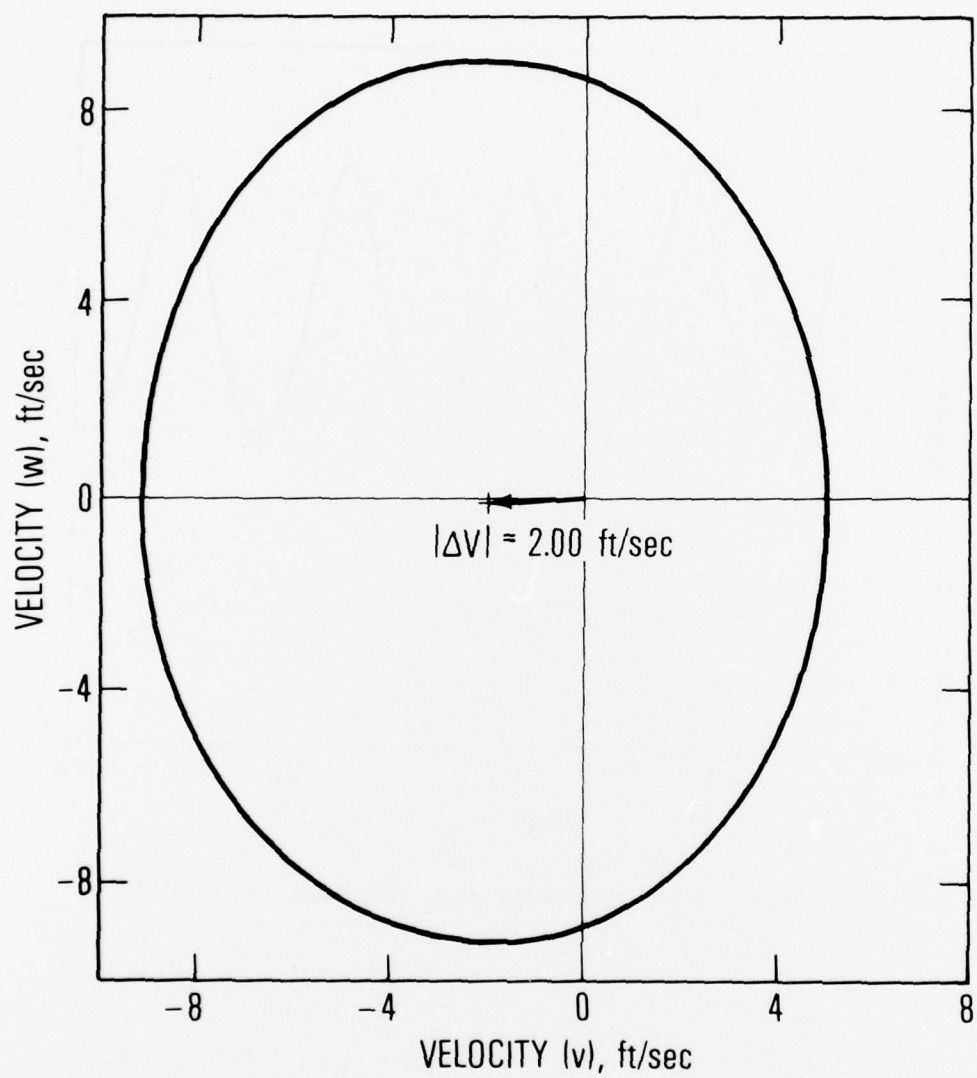


Fig. 4. Open-loop dispersion velocity response to impulse

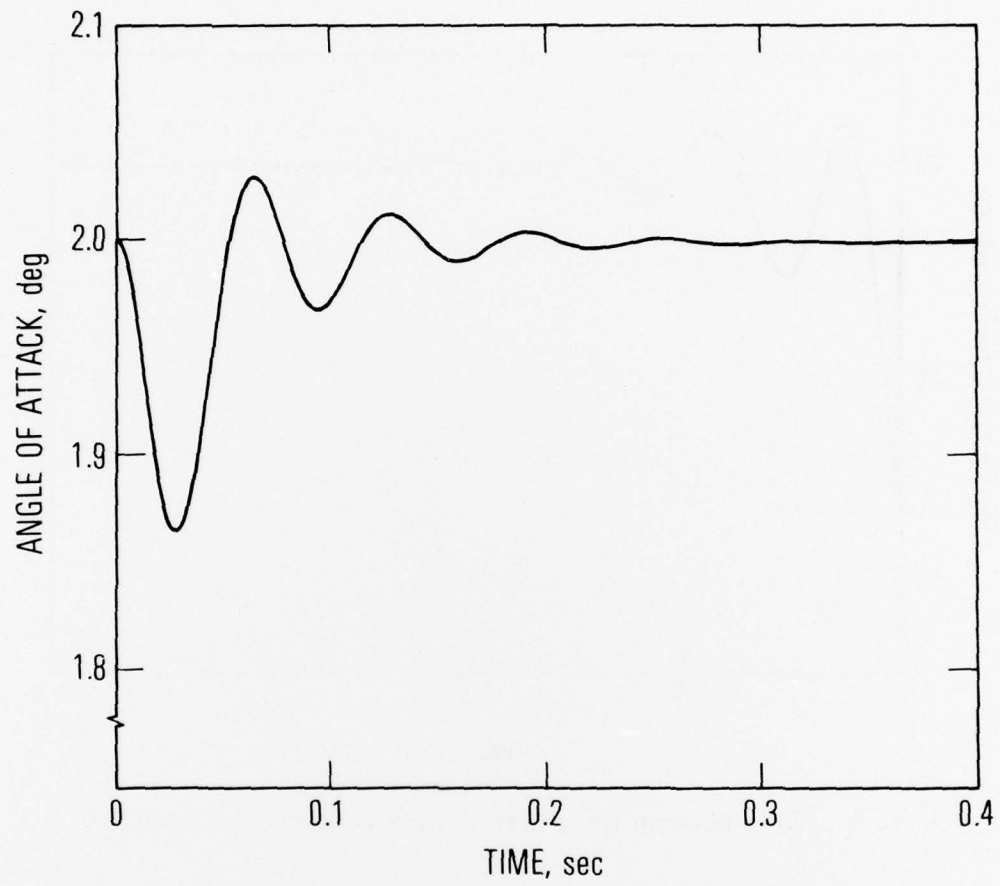


Fig. 5. Closed-loop angle-of-attack response to impulse

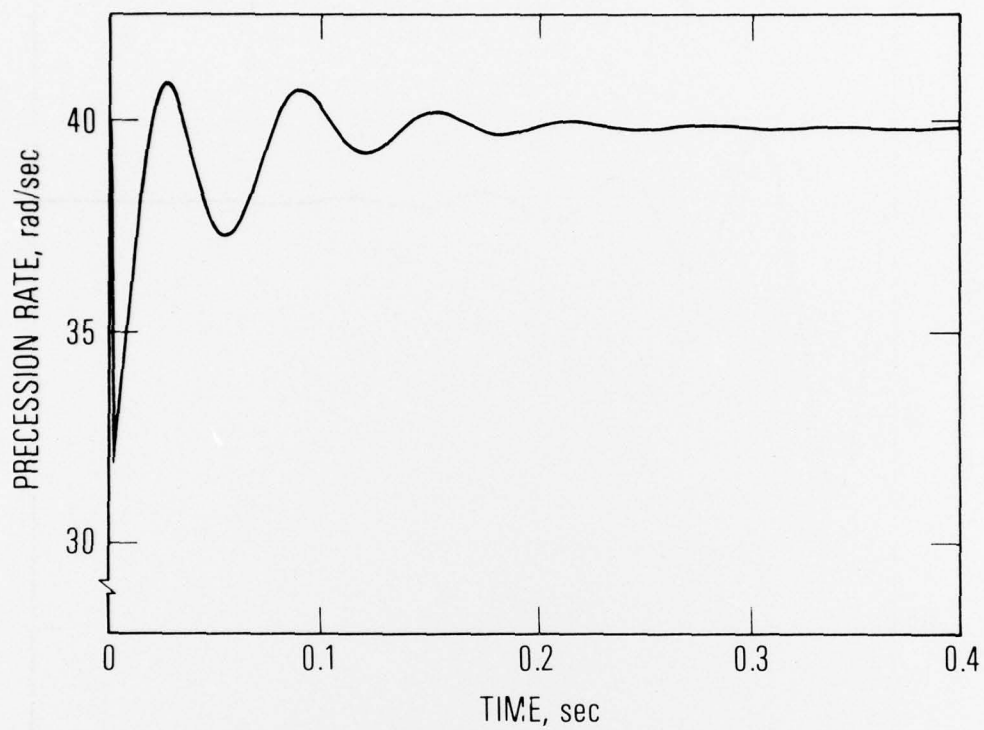


Fig. 6. Closed-loop precession rate response to impulse

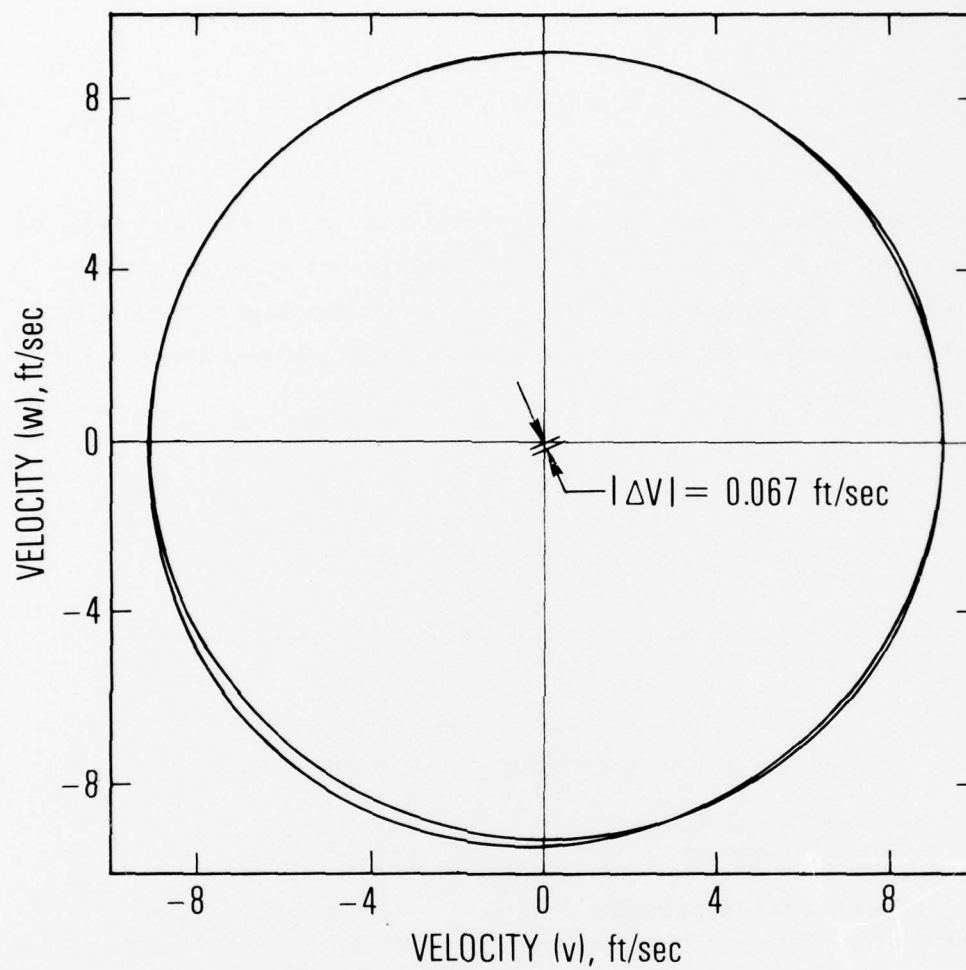


Fig. 7. Closed-loop dispersion velocity response to impulse

$$\begin{aligned}\Delta V_{\text{clsd lp 2nd ord}} &= \frac{\ell}{\omega_n \bar{\theta}} \left(\frac{M}{I}\right)^2 (0.0509 - 0.0283i) \\ &= 0.0222 - 0.0123i \text{ ft/sec} \quad (41)\end{aligned}$$

which should be added to the first-order solutions, Eqs. (39) and (37), of -2.00 ft/sec and zero, respectively. The results are summarized in Table 1, which is a comparison of the open- and closed-loop theoretical values with those computed from a numerical integration of the nonlinear equations of motion.

Table 1. Net dispersion velocity in ft/sec

	Open Loop	Closed Loop
First-Order Approximation	2.00	0
First- and Second-Order Approximations	1.97	0.025
Numerical Integration	2.00	0.067

IV. SYSTEM IMPLEMENTATION

Implementation of a control system to sense the required state variables and to generate wind-referenced pitch and yaw control moments has been discussed in conjunction with drag control systems.^{1,2} The principal feature of such systems, which differs from that of more conventional aerodynamic control systems, is the requirement to generate aerodynamic control moments in the wind plane, which rotates relative to body-fixed axes at a rate approximately equal to the difference between the roll rate and the natural pitch frequency when the vehicle is untrimmed. Because relatively small control moments are required to correct for aerodynamic disturbances that would otherwise cause lift nonaveraging, the power requirements are small enough that wind-referenced moments can be generated by modulation of body-fixed control surfaces or reaction jets at the windward-meridian rotation frequency. The state variables selected for this analysis consist of the angle of attack, the pitch rate, and the precession rate. Since the transverse dispersion velocity, Eq. (1), is the integral of the lateral acceleration, a more appropriate control variable than total angle of attack is lateral acceleration, which is measured directly from a resolution of body-fixed lateral accelerometers. The pitch rate $\dot{\theta}$ can similarly be obtained from a resolution of body-fixed lateral rate gyro measurements. The precession rate is the difference between the roll rate and the windward-meridian rotation frequency. With the missile untrimmed in a circular coning motion at a quasi-steady angle of attack $\bar{\theta}$, a body-fixed lateral accelerometer will measure a strong periodic signal at the windward-meridian rotation frequency. The difference between this signal frequency and the roll rate is the precession rate. Hence, all required control parameters can be obtained from conventional strapped-down sensors.

For control of yaw-moment disturbances only, as discussed in the numerical example, the only feedbacks required are angle of attack (lateral acceleration) and precession rate. For a small or known roll rate, control could be achieved with information derived solely from lateral accelerometers.

V. CONCLUSIONS

Cross-range dispersion of spinning missiles resulting from lift nonaveraging can be controlled by application of wind-referenced pitch and yaw moments. A linear optimal control is derived that minimizes transverse dispersion velocity selected as the cost function. An approximation to the dispersion velocity permits the optimal feedbacks to be obtained directly from the s-plane solution of linearized control equations by means of classical techniques. The method also permits closed-form solutions for the open- and closed-loop system response to simple disturbances. The control loop, defined to minimize cross-range dispersion, complements control of up- and down-range error by angle-of-attack control of drag. The combined two-loop system can limit major sources of missile dispersion.

REFERENCES

1. Platus, D. H., "Angle of Attack Control of Spinning Missiles," Journal of Spacecraft and Rockets, Vol. 12, April 1975, pp. 228-234.
2. Platus, D. H., "Angle of Attack Control of Drag for Slender Re-Entry Vehicle Recovery," Journal of Spacecraft and Rockets, Vol. 13, May 1976, pp. 275-281.
3. Zaroodny, S. S., "On Jump Due to Muzzle Disturbances," BRL-703, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, June 1949.
4. Fuess, B. F., "Impact Dispersion Due to Mass and Aerodynamic Asymmetries," Journal of Spacecraft and Rockets, Vol. 4, October 1967, p. 1402.
5. Murphy, C. H. and Bradley, J. W., "Jump Due to Aerodynamic Asymmetry of a Missile with Varying Roll Rate," BRL-1077, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, May 1959.
6. Crenshaw, J. P., "Effect of Lift with Roll Rate Variation on Re-Entry Vehicle Impact," Journal of Spacecraft and Rockets, Vol. 8, May 1971, pp. 483-488.
7. Crenshaw, J. P., "Effect of Lift Variation on the Impact of a Rolling Re-Entry Vehicle," Journal of Spacecraft and Rockets, Vol. 9, April 1972, pp. 284-286.
8. Platus, D. H., "Dispersion of Spinning Missiles Due to Lift Nonaveraging," AIAA Journal, Vol. 15, July 1977.
9. James, H. M., Nichols, N. B., and Phillips, R. S., Theory of Servomechanisms, McGraw-Hill, New York, 1947.
10. Newton, G. C., Gould, L. A., and Kaiser, J. F., Analytical Design of Linear Feedback Controls, Wiley, New York, 1957.
11. Gardner, M. F., and Barnes, J. L., Transients in Linear Systems, Wiley, New York, 1942.

NOMENCLATURE

A, B, C, D, E, F	feedback gains
a, b, c, d, e, f	transformed feedback gains, Eq. (11)
$c_0, c_1, c_2, d_0, d_1, d_2$	Eq. (19)
I	pitch or yaw moment of inertia; cost function, Eq. (17)
I_x	roll moment of inertia
l	characteristic length, l/mx_{st}
L	lift force
L_g	lift force derivative
m	missile mass
m_p, m_y	transformed pitch and yaw disturbance moments, (Eq. (12))
$m_{\delta p}, m_{\delta y}$	transformed pitch and yaw control moments, Eqs. (13) and (14)
M_p, M_y	pitch and yaw disturbance moments
$M_{\delta p}, M_{\delta y}$	pitch and yaw control moments
p	roll rate
s	Laplace transform variable
t	time
v	Y-component of transverse velocity
V	transverse velocity, $v + iw$
ΔV	net change in average transverse velocity
w	Z-component of transverse velocity
w_1, w_2, w_3	weighting constants

x_{st}	static margin (distance of center of pressure aft of center of mass)
Y, Z	cross-plane coordinates
α	$\theta_+/\bar{\theta}$
$\delta(t)$	unit impulse function
θ	angle of attack (Euler angle)
$\bar{\theta}$	mean angle of attack
θ_+	angle of attack perturbation
λ	$\dot{\psi}_+/\omega_n$
λ_1	Eq. (22)
τ	nondimensional time, $\omega_n t$
ψ	precession angle (Euler angle)
$\dot{\psi}$	precession rate
$\dot{\psi}_+$	precession-rate perturbation
ω_n	undamped natural pitch frequency

THE IVAN A. GETTING LABORATORIES

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military concepts and systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the nation's rapidly developing space and missile systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

Aerophysics Laboratory: Launch and reentry aerodynamics, heat transfer, reentry physics, chemical kinetics, structural mechanics, flight dynamics, atmospheric pollution, and high-power gas lasers.

Chemistry and Physics Laboratory: Atmospheric reactions and atmospheric optics, chemical reactions in polluted atmospheres, chemical reactions of excited species in rocket plumes, chemical thermodynamics, plasma and laser-induced reactions, laser chemistry, propulsion chemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, photo-sensitive materials and sensors, high precision laser ranging, and the application of physics and chemistry to problems of law enforcement and biomedicine.

Electronics Research Laboratory: Electromagnetic theory, devices, and propagation phenomena, including plasma electromagnetics; quantum electronics, lasers, and electro-optics; communication sciences, applied electronics, semiconducting, superconducting, and crystal device physics, optical and acoustical imaging; atmospheric pollution; millimeter wave and far-infrared technology.

Materials Sciences Laboratory: Development of new materials; metal matrix composites and new forms of carbon; test and evaluation of graphite and ceramics in reentry; spacecraft materials and electronic components in nuclear weapons environment; application of fracture mechanics to stress corrosion and fatigue-induced fractures in structural metals.

Space Sciences Laboratory: Atmospheric and ionospheric physics, radiation from the atmosphere, density and composition of the atmosphere, aurorae and airglow; magnetospheric physics, cosmic rays, generation and propagation of plasma waves in the magnetosphere; solar physics, studies of solar magnetic fields; space astronomy, x-ray astronomy; the effects of nuclear explosions, magnetic storms, and solar activity on the earth's atmosphere, ionosphere, and magnetosphere; the effects of optical, electromagnetic, and particulate radiations in space on space systems.

THE AEROSPACE CORPORATION
El Segundo, California