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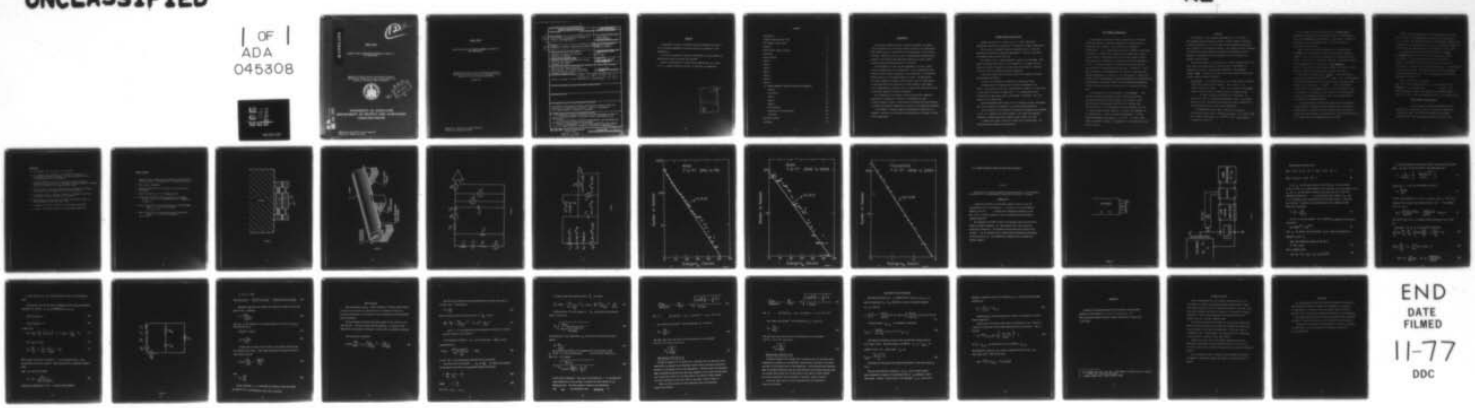

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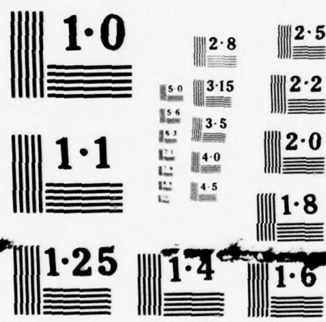

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ANNUAL REPORT

STUDY OF NOISE AND RELAXATION PHENOMENA IN SOLIDS AT LOW TEMPERATURES

UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

Supported by the Office of Naval Research
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ANNUAL REPORT

STUDY OF NOISE AND RELAXATION PHENOMENA IN SOLIDS AT LOW TEMPERATURES

August, 1977

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A demountable cryogenic installation has been developed for study of noise, relaxation phenomena, and quality factors of materials at low temperatures.

A new kind of parametric capacitor accelerometer, and new methods for mounting and isolating crystals are described.

Results of measurements at liquid helium temperatures, and a theory of an A.C. pumped parametric capacitor up converter are summarized.

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Results of measurements at liquid helium temperatures, and a theory of an A.C. pumped parametric capacitor up converter are summarized.

CONTENTS

INTRODUCTION

We have been studying the noise, relaxation phenomena, and quality factors associated with the acoustic modes of solids at low temperatures. These properties are of importance in our understanding of the structure of solids. New devices such as clocks of extraordinary stability appear possible if the observed low temperature properties of nearly perfect crystals should approach predictions of the theory.

The measurement of noise, and the development of devices, require unusual instrumentation at low temperatures. Such instrumentation must observe the noise without degradation of the properties of the materials, without addition of unacceptable amounts of noise. The mounting and isolation problems are unusually severe because there are large amounts of noise associated with operations of helium liquifiers, and because at low temperatures the internal noise is so small that large errors result if even small amounts of noise are introduced.

At the beginning of the present contract period we had completed observations on aluminum at 77° Kelvin, instrumented with lead zirconate titanate crystals. Excess noise was observed during the approach to thermal equilibrium as measured by the drift of the normal mode frequency.

It was planned to develope improved instrumentation making use of a parametric capacitor, and to extend the observations to the regime of liquid helium temperatures.

PROGRESS DURING THE PAST YEAR

Assembly and tests of a closed cycle C.T.I. Model 1400 helium refrigerator liquifier were completed. The operation is highly satisfactory.

The theory of a parametric capacitor transducer with d.c. bias was extensively developed and confirmed experimentally. This is a new kind of accelerometer which may have wide uses.

The theory of an A.C. pumped parametric capacitor was developed. This gives a real power gain prior to the pre amplifier, but requires for the pump an oscillator of unusually low noise and great stability. Reduction to practice will be attempted during the coming year.

A new type of mount and new acoustic filters with improved isolation were developed and tested.

A study of the properties of sapphire and silicon was begun.

Instrumentation was completed and tested for measurements at 4° Kelvin. Our cryostat can now cool a volume having a length of 1.7 meters and a cross section of about 1/4 square meter. It is demountable and has facilities for rapid warm up after an experiment is completed. The "turn around" time is three weeks.

Noise measurements were carried out on an aluminum cylinder instrumented with the new capacitor accelerometer, at liquid helium temperatures.

The new results on the noise and accelerometer development were presented by Prof. J.P. Richard as an invited speaker at the eighth International Conference on General Relativity, Waterloo Canda, August 7-12, 1977.

A summary will appear in the Ph.D. Thesis of Mr. William Davis. Two papers are being submitted for publication.

THE CRYOGENIC INSTALLATION

The cryogenic installation consists essentially of a CTI model 1400 helium liquifier, associated compressor and recovery tank for helium gas, and a cryostat. The cryostat has a cylindrical testing space $\sim$ 1.7 meters long and 50 cms in diameter. This space is enclosed by a double wall stainless steel cylinder, the interior of which forms a liquid helium tank with a storage capacity of 120 l. This container is surrounded by a helium gas cooled shell operating at $\sim$ 65° K. Multilayer insulation surrounds the helium tank and the gas cooled shell to reduce radiation heat leaks. A vacuum of $\sim 10^{-6}$ mhg is maintained in the vacuum chamber to keep heat conduction losses through residual gas at a low level. The testing space is sealed from the rest of the vacuum chamber so that helium pressure of a few microns can be maintained to accelerate cooling of the cylinder and help maintain thermal equilibrium between the antenna and the liquid helium storage tank.

The procedure used in the experiment to be described next has been to liquify helium gas and fill the liquid helium tank. The liquifier operation is then stopped. One compressor is then used to return the helium gas boiled off to the recovery tank. The boiloff rate is $\sim$ 15 liters/day so that an experiment can last $\sim$ 7 days without liquifier operation. If useful, the liquifier can be restarted after 7 days to continue the experiment. A 130 kg aluminum cylinder can be cooled to $\sim$ 4.2°K in $\sim$ 7 days. This makes it possible to conduct an experiment in a relatively short time and at low cost.

ALUMINUM

With aluminum, we have conducted experiments at 5 to 8 °K with a 130 kilogram cylinder resonating at 1755hz. We have studied the approach to thermal equilibrium of such a cylinder to detect possible excess noise due to relaxation of concentrated stresses. Such stresses would be introduced by otherwise very simple mechanical suspensions and possibly by sensor mounting. The experiment was also designed to test the potential of a resonant capacitor sensor.

The cylinder (Figure 1) is supported by a "ring" stage which rests on a short stack of alternate layers of felt and steel. This assembly is supported by two 150 cm. long arms which constitute a low frequency "bridge" **stage**. The arms rest at both ends on felt and steel stacks which provide additional acoustic isolation.

The cylinder was instrumented with a d.c. biased resonant capacitor sensor shown schematically on Figure 2. There, plate P_1 is rigidly attached to the **end**. Plate P_2 is coupled to the **cylinder** through a high Q suspension resonating near the fundamental mode

A constant charge is maintained on the capacitance C_3 formed by P_1 and P_2 through a high impedance circuit. The optimum matching of such a system has been discussed in detail elsewhere[1,2,3]. Its performance was also tested at room temperature[2].

The preamplifier used a BF817 field effect transistor[4]. The selected transistor had a voltage noise density $e_F = 1.8\text{x}10^{-9}\ \text{v}/\sqrt{\text{hz}}$ and a current noise density $i_F = 4\text{x}10^{-15}\ \text{A}/\sqrt{\text{hz}}$ corresponding to a temperature of 0.26 °K.

Fig. 3 shows the electrical analog of the cylinder-sensor-preamplifier system and the relevant noise sources. The subscript 1 refers to the cylinder and the subscript 2, to the sensor. The mass of the sensor is 39 g. The lowest mode is at 1755.1 hz and the sensor mode is at 1799.5 hz.

Figure 4 shows the relevant parameters for the observation of the lowest mode with a quality factor Q=917,000. Proper matching required $\beta Q = 29$ and this was obtained with a relatively low bias voltage, $V_B = 63$ volts. Under these conditions, the backward influence of the preamplifier is to increase the noise temperature by 1.6 °K. The 25 MΩ resistor shown on Figure 4 was used as a noise source to calibrate the system and the following electronics.

The state of the normal mode of a solid can be specified by a vector in the complex plane $(x+iy)e^{i\omega_r t}$ where $\omega_r/2\pi$ is the frequency of a reference oscillator chosen to be very close to the normal mode monitored. The noise level $<(x^2+y^2)>$ and the fluctuations $< (x)^2 + (y)^2 > \tau^2$ were sampled every 0.1 sec. and recorded on tape. For calibration, the frequency of the reference clock was changed by 1.8 hz. and recordings were made of the noise and fluctuations with and without the calibration resistor.

Initial measurements revealed that excess noise was contributed by the liquifier, compressors and the diffusion pump. The results presented here are preliminary results obtained after the sources of excess noise were identified. By that time, no liquid helium was left in the cryostat and the bar temperature was 6.2 ± 0.2°K.

Figure 5 shows the distribution of the measurements of the noise level or energy in the cylinder mode obtained with an averaging time of $\tau = 3.5$ sec. The observed energy level corresponding to 8.0 ± 0.9 °K is reasonably well accounted for by the sum of the thermal contribution (6.2°K) and the backward influence of the preamplifier ($1.6 \pm .15$°K).

Recordings of the fluctuations were obtained at a time when the cylinder temperature was 8.1 ± 0.2°K and changing at a rate of $\sim$ 0.13°K/hr. The observed energy level of the normal mode (Figure 6) corresponded to 12.5 ± 2.6°K indicating a 2.8 ± 2.6°K excess above the thermal and preamplifier contributions. The distribution was however again normal. The averaging time used in the measurement of the fluctuations was 12 sec. The corresponding predicted average temperature fluctuation is:

$$\Delta E/K_B = T_a(\tau/\sqrt{2}\,\tau_a) = 0.45 \pm 0.01.$$

where K_B is the Boltzmann constant, T_a, the temperature of the cylinder and τ_a, the damping time (183 sec). The observed average fluctuation corresponding to the sampling shown on Figure 7 is $0.49 \pm .04$°K and corresponds closely to the predicted value.

NEARLY PERFECT SINGLE CRYSTALS

We are presently carrying out tests with a 5 kg sapphire and a 16 kg silicon crystal. New mechanical suspensions are being developed. $Q \sim 10^8$ have been obtained with aluminum suspensions for silicon and sapphire crystals at 77°K and 4.2°K.

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2. J-P. Richard, paper presented at the XXVIIth International Astronautical Congress in Anaheim, California, October 16, 1976, to be published in Acta Astronautica.

3. A detailed analysis of a d.c. biased (non resonant) capacitor sensor has also been given by H. Hirakawa, K. Narihara and M.K. Fujimoto in the J. of Phys. Soc. of Japan 41, 1093 (1976)

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FIGURE CAPTIONS

1. Capacitor sensor formed by an electrically isolated plate P_1 rigidly attached to the cylinder and a resonating plate P_2.

2. Multi stage suspension.

3. Electrical analog of the two modes of the system and the preamplifier.

4. Electrical analog near the fundamental mode.

5. Density distribution of the measurements of the energy during a 4 hr 26 min. experiment with the cylinder at 6.2 $\pm$0.2 °K.

6. Density distribution of the measurements of the mode energy during a 1 hr 38 min. experiment at 8.1 $\pm$ 0.2 °K.

7. Density distribution of the measured energy fluctuations during a 1 hr 38 min. experiment at 8.1 $\pm$ 0.2 °K.

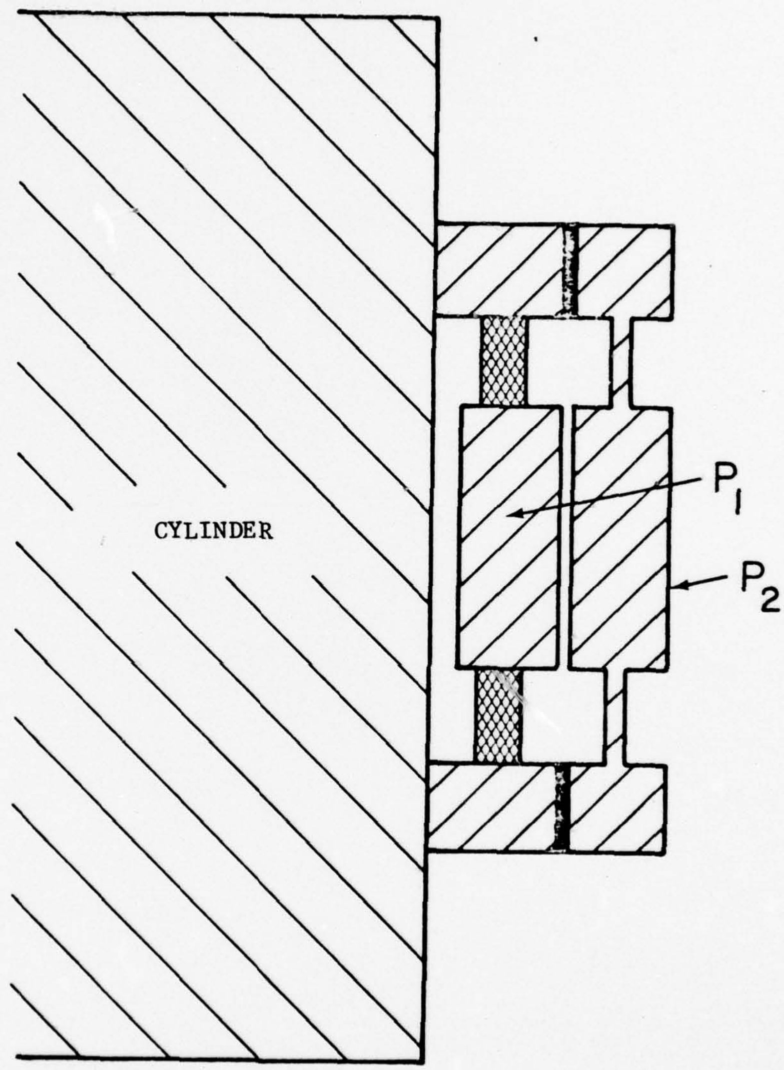


FIGURE 1

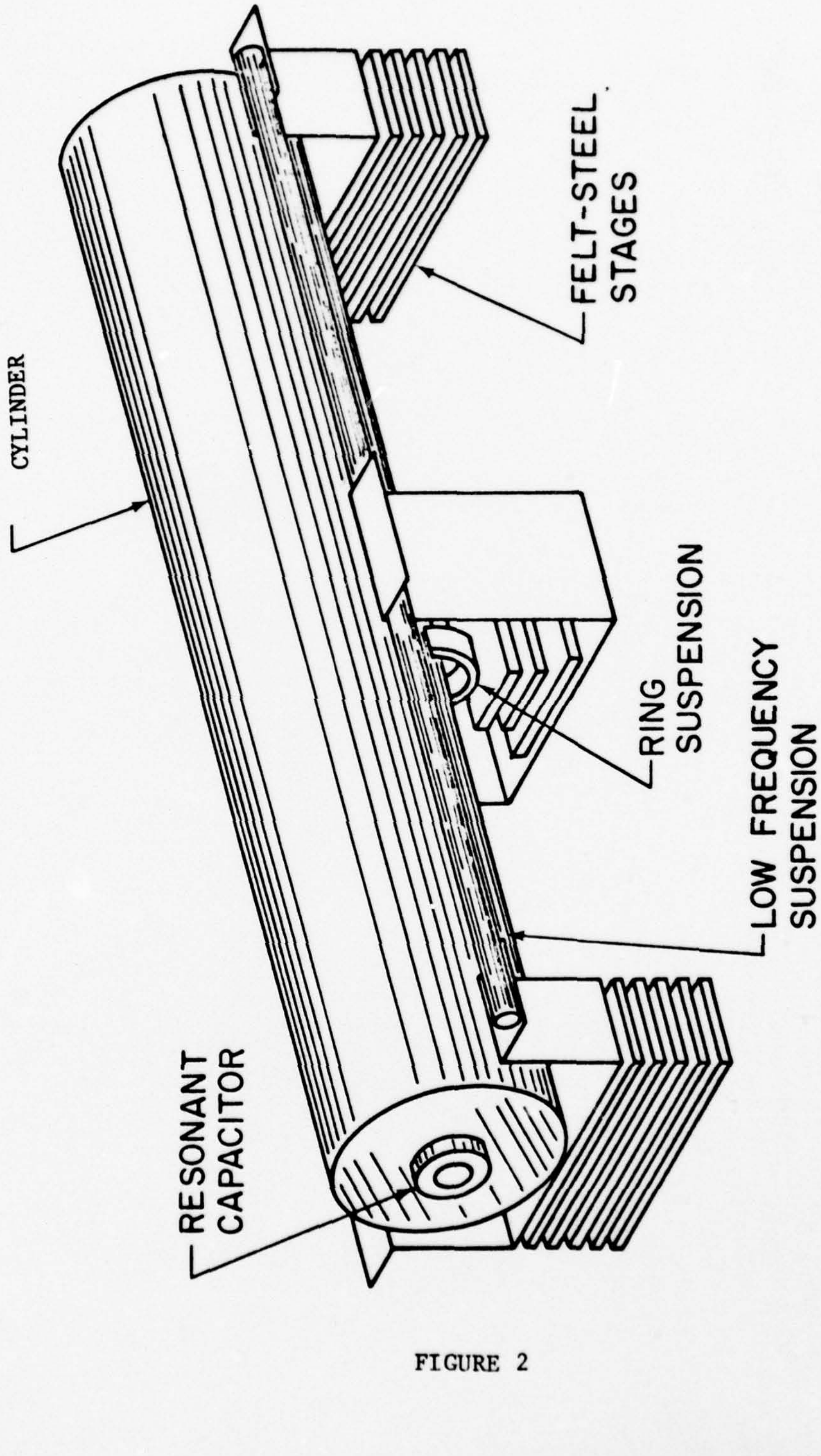

CYLINDER
FELT-STEEL STAGES
RING SUSPENSION
LOW FREQUENCY SUSPENSION
RESONANT CAPACITOR

FIGURE 2

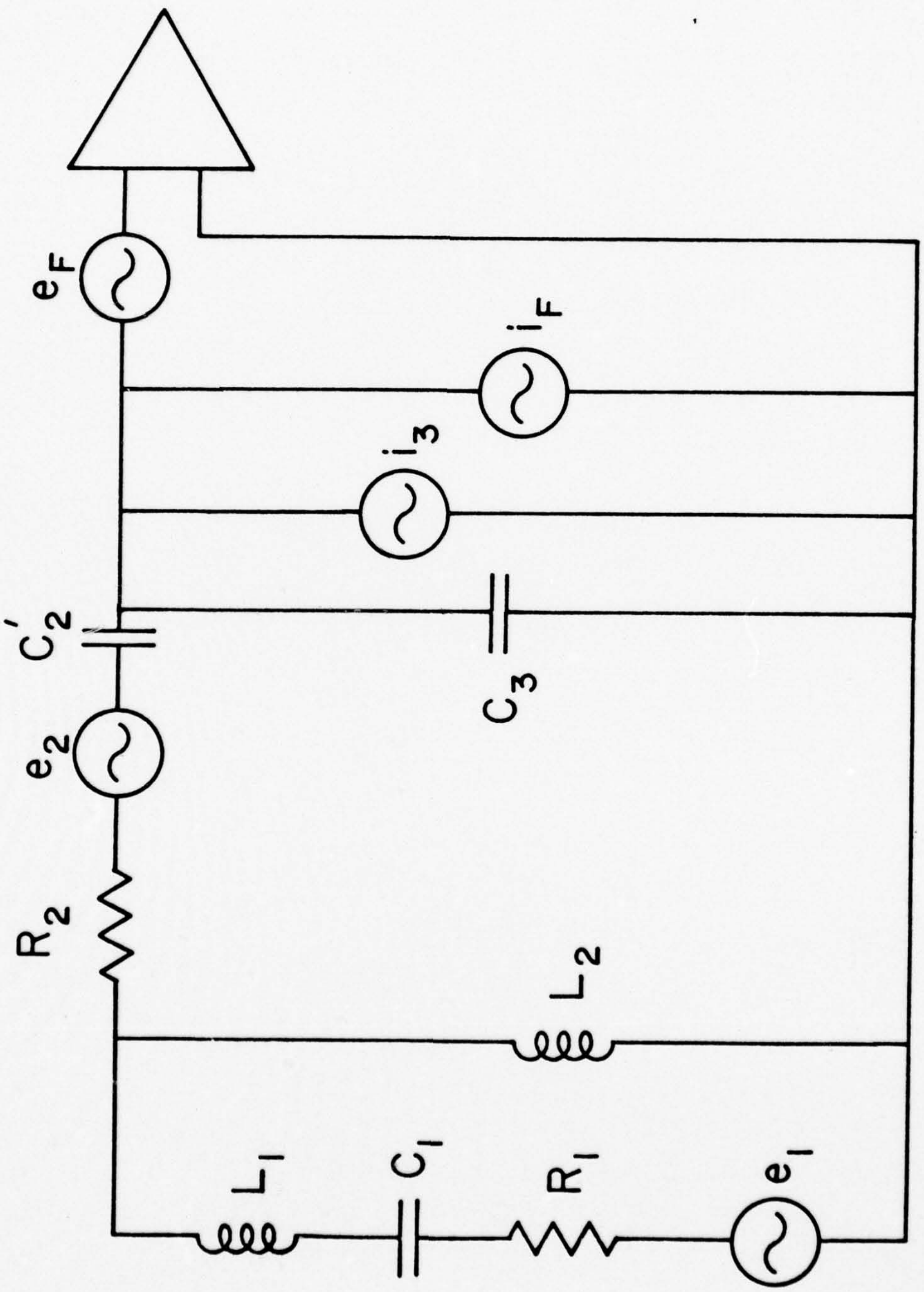


FIGURE 3

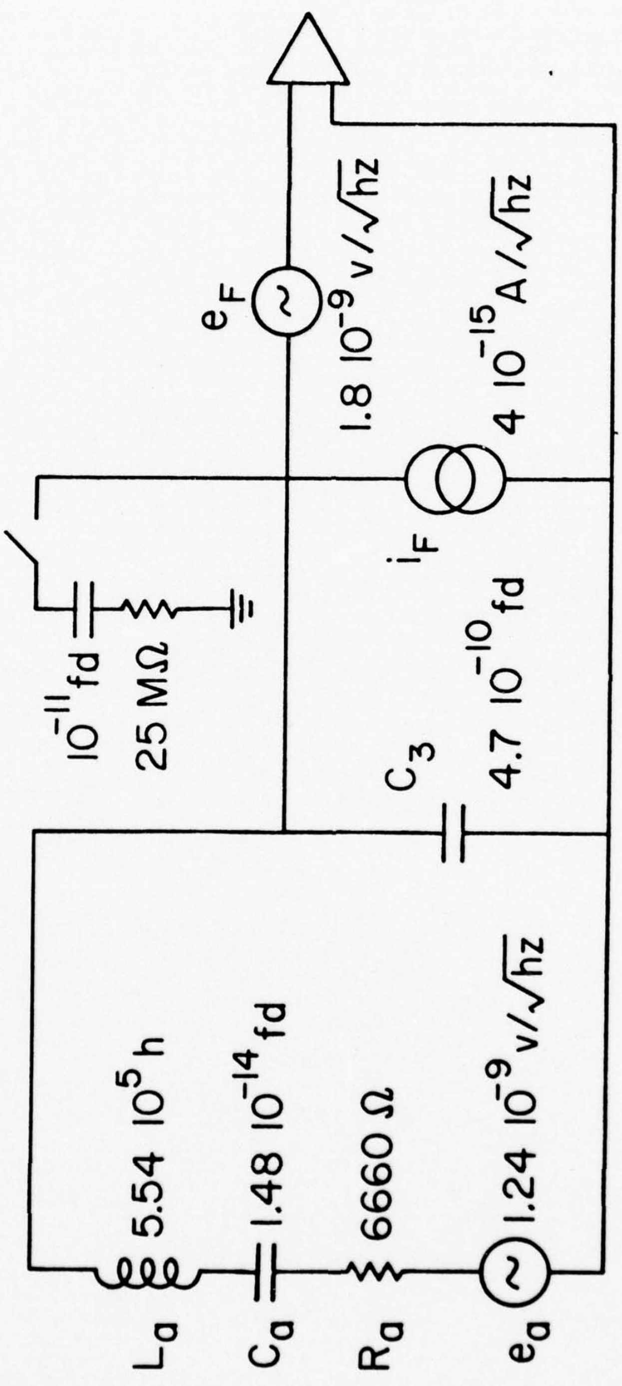
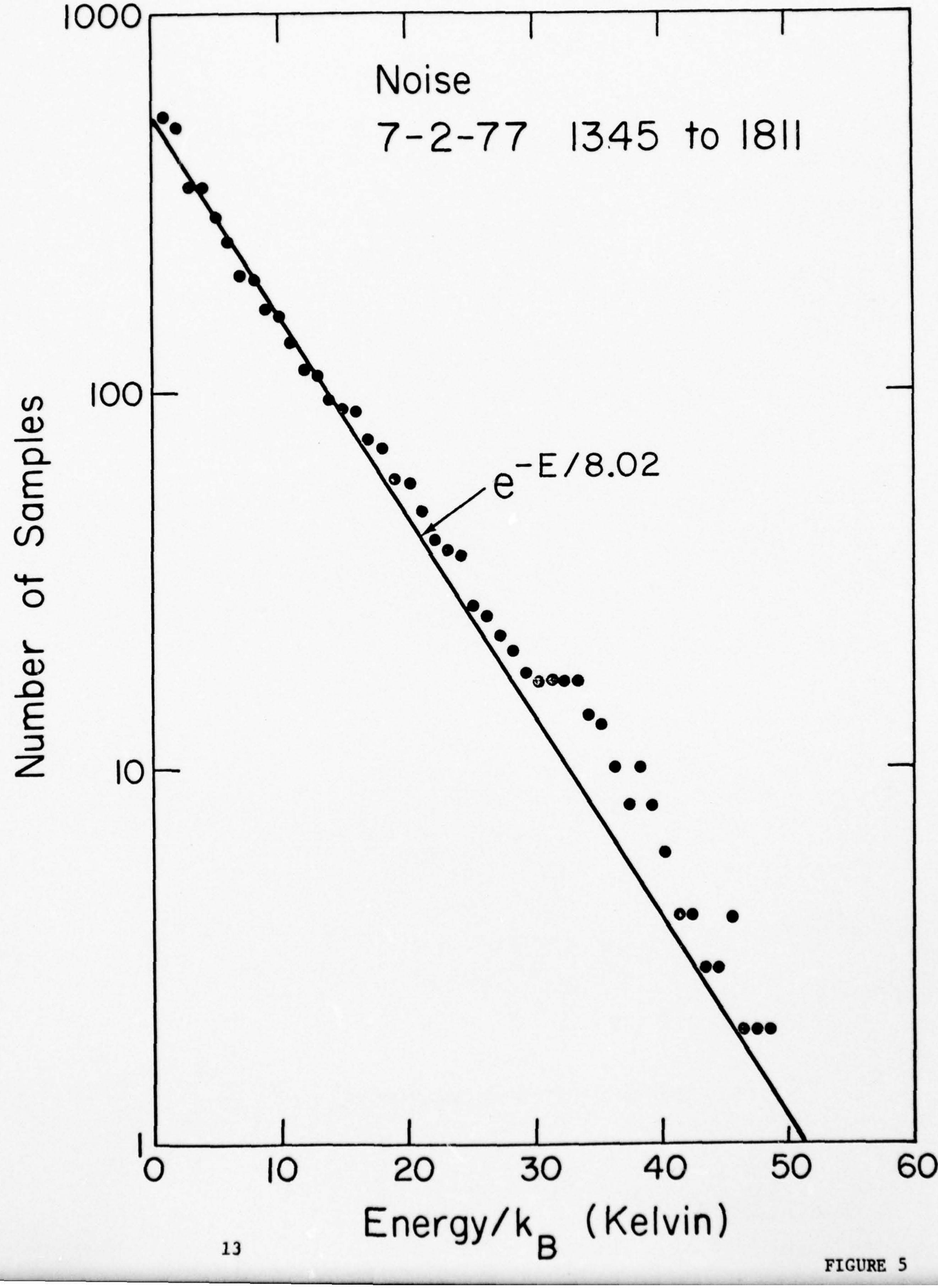

L_a
C_a
R_a
e_a
$5.54\ 10^5$ h
$1.48\ 10^{-14}$ fd
6660 Ω
$1.24\ 10^{-9}$ v/√hz
10^{-11} fd
25 MΩ
C_3
$4.7\ 10^{-10}$ fd
i_F
$4\ 10^{-15}$ A/√hz
e_F
$1.8\ 10^{-9}$ v/√hz

FIGURE 4



Noise
7-2-77 1345 to 1811
$e^{-E/8.02}$
Number of Samples
1000
100
10
1
0
10
20
30
40
50
60
Energy/k_B (Kelvin)

FIGURE 5

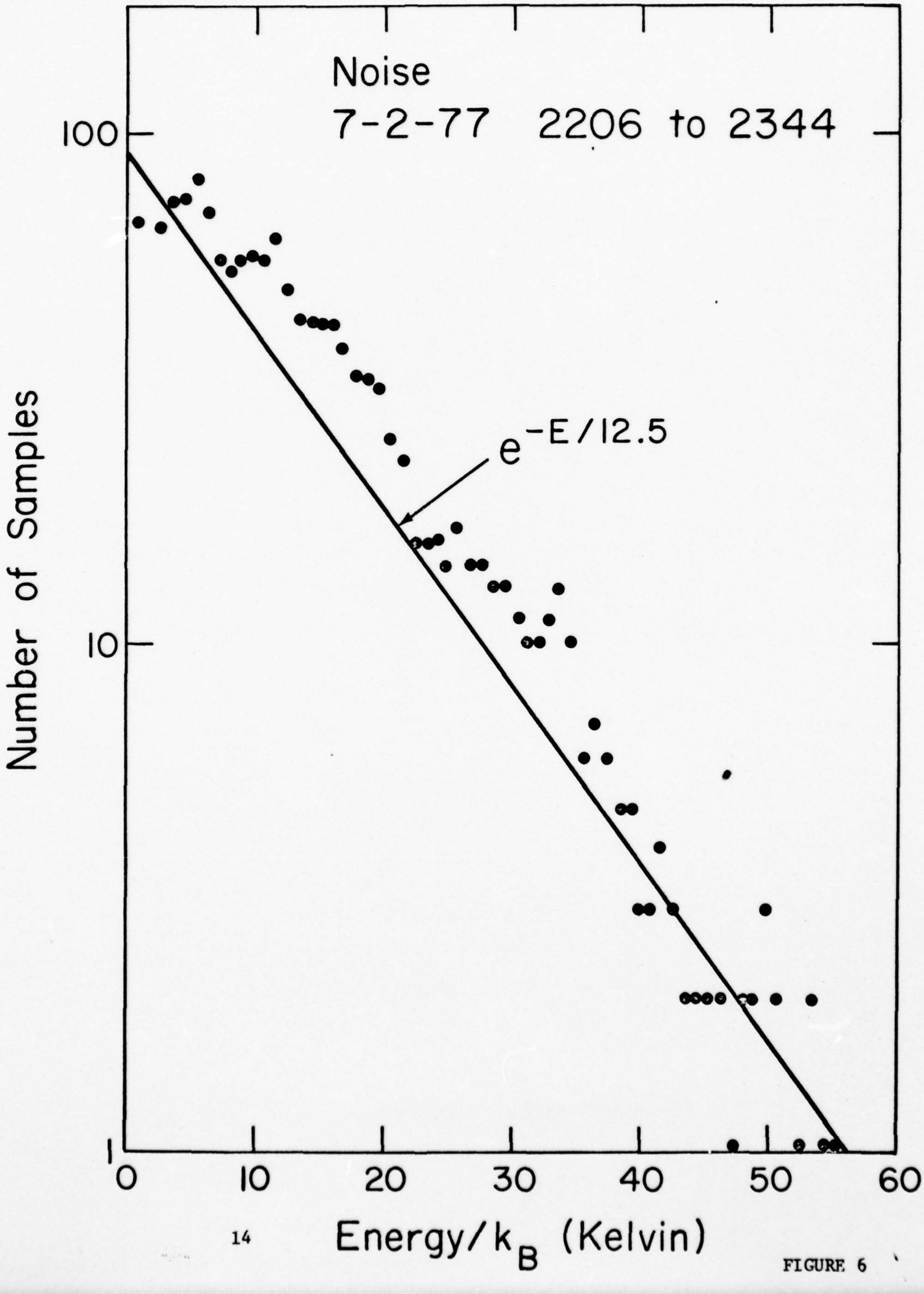

Noise
7-2-77 2206 to 2344
$e^{-E/12.5}$
100
10
1
0
10
20
30
40
50
60
Number of Samples
Energy/k_B (Kelvin)

FIGURE 6

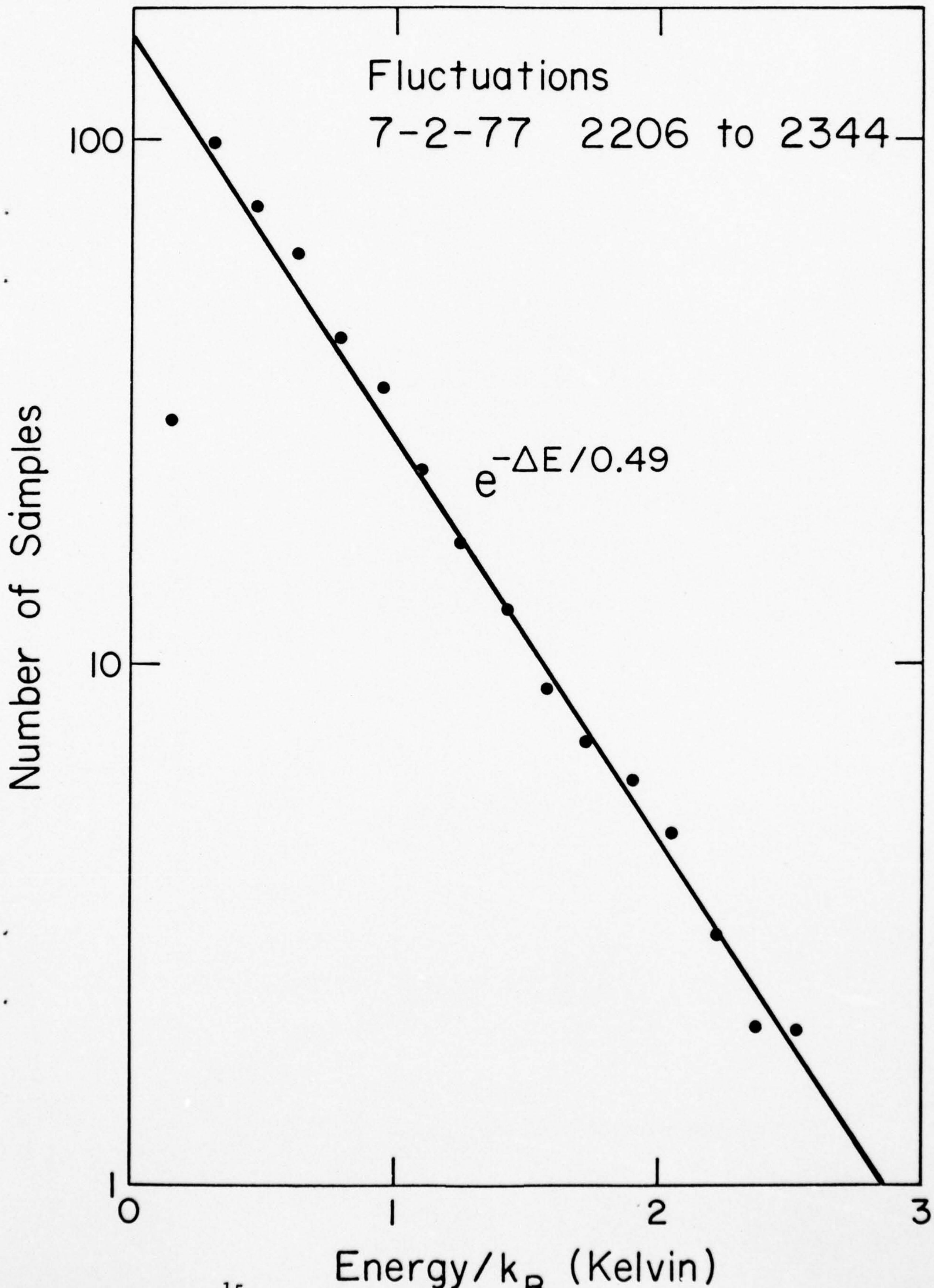

Fluctuations
7-2-77 2206 to 2344
$e^{-\Delta E/0.49}$
100
10
1
0
1
2
3
Number of Samples
Energy/k_B (Kelvin)

FIGURE 7

A.C. Pumped Parametric Amplifier Capacitive Transducer

Abstract

The equivalent circuit and design criteria for optimal noise performance are derived for an electromagnetic transducer at one end of a cylinder.

INTRODUCTION

A capacitor driven by an alternating electric field is among the many possible ways of instrumenting a cylinder for noise measurements. Figure 1 shows the cylinder with a capacitor transducer at one end. This is somewhat similar to the D.C. biased capacitor transducer already described.[1]

The cylinder is excited by noise or an external signal and oscillates freely at angular frequency ω_a. The capacitor has a high mechanical Q and high electrical Q. The mechanical oscillations are excited by the cylinder and the capacitor has a normal mode for mechanical oscillations in the vicinity of ω_a. For simplicity of analysis let us consider the circuit, Figure 2.

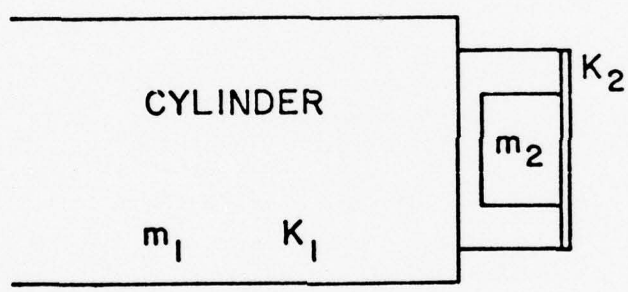
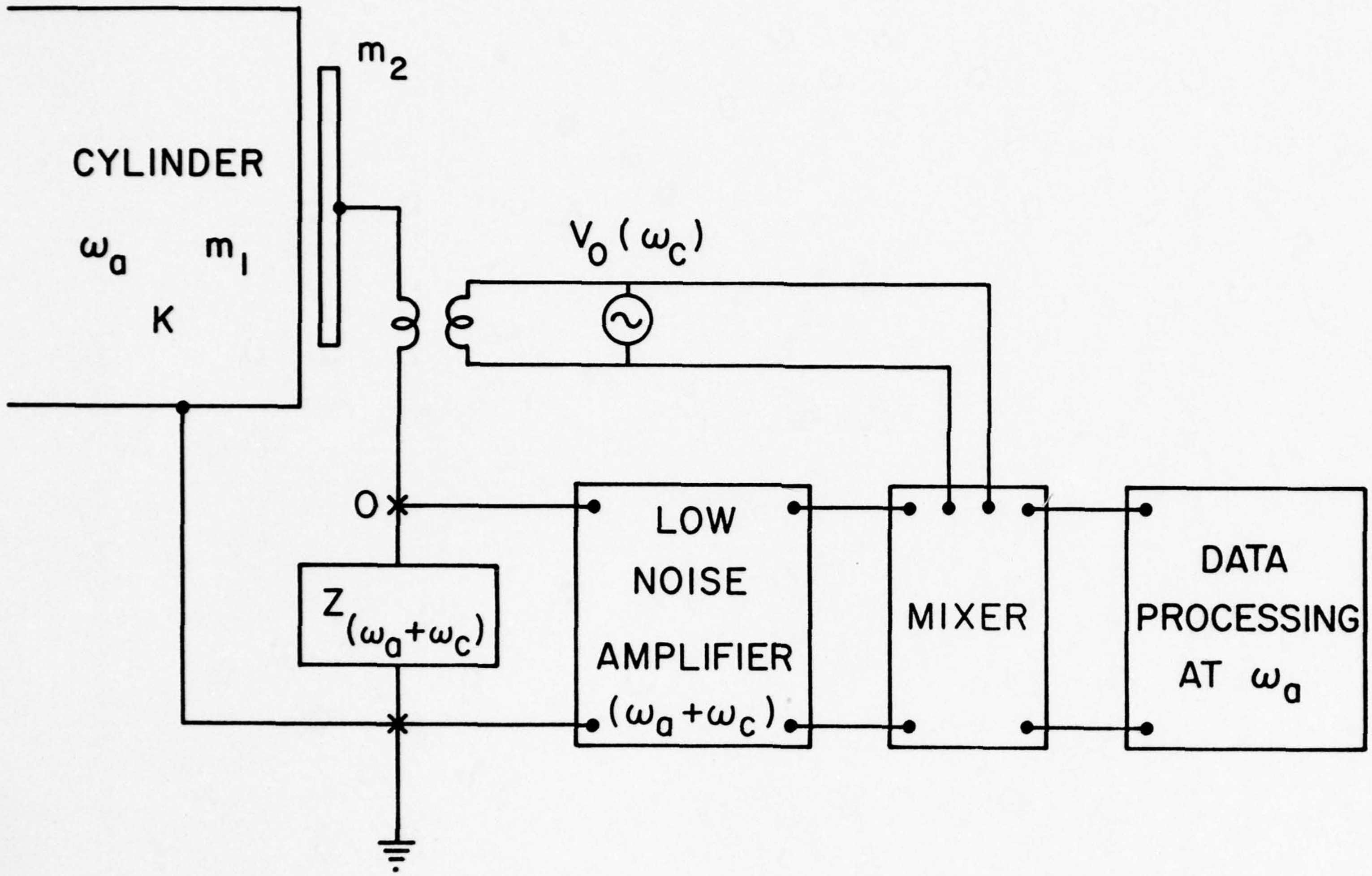


Figure 1



m2
CYLINDER
ωa m1
K
V0 (ωc)
O
Z(ωa+ωc)
LOW
NOISE
AMPLIFIER
(ωa+ωc)
MIXER
DATA
PROCESSING
AT ωa

Figure 2

The mechanical equations are

$$m_1\ddot{x}_1 = -K_1x_1 - K_2\ (x_1 - x_2) - f - \gamma_1\dot{x}_1 - \gamma_2\ (\dot{x}_1 - \dot{x}_2) + F \qquad (1)$$

$$m_2\ddot{x}_2 = -K_2(x_2-x_1) - \gamma_2(\dot{x}_2 - \dot{x}_1) + f \qquad (2)$$

In (1) m_1 is the reduced mass of the cylinder, K_1 is the stiffness of the cylinder. K_2 is the stiffness of the capacitor, m_2 is the capacitor mass. γ_1 and γ_2 are the damping constants associated with the cylinder and transducer respectively. F is an external force acting on m_1, and f is a mechanical force associated with the electric charge q and field E_c in the gap. Velocities are extremely small and the magnetic Lorentz force can be neglected.

$$f = E_c q = \frac{q^2}{C(x_2-x_1)} \qquad (3)$$

In (3) C is the gap capacity. Let us suppose the capacitor driving field is given by

$$E_c = \mathrm{Re}(E_o e^{i\omega_c t} + E_N e^{i\omega_N t}) \qquad (4)$$

with ω_c the angular driving frequency. E_N is a noise contribution at a frequency ω_N near ω_c.

Then, the alternating voltage at the gap is

$$V = (x_2 - x_1)E_c \qquad (5)$$

For a harmonic force

$$(x_2 - x_1) = (x_2 - x_1)_o + (x_2 - x_1)_1\ \mathrm{Re}\ e^{i\omega_a t} \qquad (6)$$

In (6) the subscripts zero and one refer to the zero and first order parts. The force (3) may be written in the approximate form

$$f = \frac{q^2}{C(x_2-x_1)_o}\left[1 - \frac{R_e(x_2-x_1)_1 e^{i\omega_a t}}{(x_2-x_1)_o}\right] \tag{7}$$

From Figure 2 ; let Z be the impedance at point 0.

$$q = \frac{E(x_2-x_1)}{i\omega Z} \tag{8}$$

(7) will have components at $2\omega_c$, $2\omega_c \pm \omega_a$, $2\omega_c \pm 2\omega_a$, ω_a. Only the ω_a component will interact significantly with the mode. The component of (7) at ω_a is

$$f(\omega_a) = \frac{E_o^2 (x_2-x_1)_1 \cos\omega_a t}{2C\omega_c^2 |Z|^2} + \frac{E_o E_N (x_2-x_1)_o}{C\omega_c^2 |Z|^2} \cos(\omega_a t+\phi) \tag{9}$$

The second term of (9) is a generator noise contribution due to noise E_N

Equations 1, 2, and 8 may be written in the form

$$\dot{x}_1\left[i\omega_a m_1 + \frac{K_1}{i\omega_a} + \frac{K_2}{i\omega_a} + \gamma_1\right] + (\dot{x}_1-\dot{x}_2)\left[\frac{K_2}{i\omega_a} + \gamma_2 - \frac{f_o}{i\omega_a}\right] = F - f_N \tag{10}$$

$$(\dot{x}_2-\dot{x}_1)\left[\frac{K_2}{i\omega_a} + \gamma_2 - \frac{f_o}{i\omega_a}\right] + \dot{x}_2\,(i\omega_a m_2) = 0 \tag{11}$$

$$\text{with } f_o = \frac{E^2}{2C\omega_c^2 |Z|^2} \text{ and } f_N = \frac{E_N E_o (x_2-x_1)_o}{C\omega_c^2 |Z|^2} \tag{12}$$

In (10) the term f_N is a "back reaction" of part of the electronics noise.

The equations (10) and (11) may be compared with the electrical network equations for currents i_1, i_2, and impedances Z_1, Z_2, Z_{12}

$$i_1 Z_1 + (i_1 - i_2) Z_{12} = V \tag{13}$$

$$(i_2 - i_1) Z_{12} + i_2 Z_2 = 0 \tag{14}$$

in which case

$$Z_1 = i\omega_a m_1 + \frac{1}{i\omega_a}\left[K_1 + K_2\right] + \gamma_1 \Rightarrow i\omega_a L_1 + \frac{1}{i\omega_a C_1} + R_1 \tag{15}$$

$$Z_2 = i\omega_a m_2 + i\omega_a L_2 \tag{16}$$

$$Z_{12} = \frac{K_2}{i\omega_a} + \gamma_2 - \frac{f_o}{i\omega_a} \Rightarrow \frac{1}{i\omega_a C_{12}} + R_{12} \tag{17}$$

These suggest the circuit in Figure 3. V is an external force. V_N is the generator noise back reaction. Each resistance will contribute "Johnson" noise.

From (13) and (14) we obtain

$$i_1 - i_2 = \frac{FZ_2}{Z_1 Z_2 + Z_1 Z_{12} + Z_2 Z_{12}} \tag{18}$$

Consider the denominator of (18) . We write each impedance

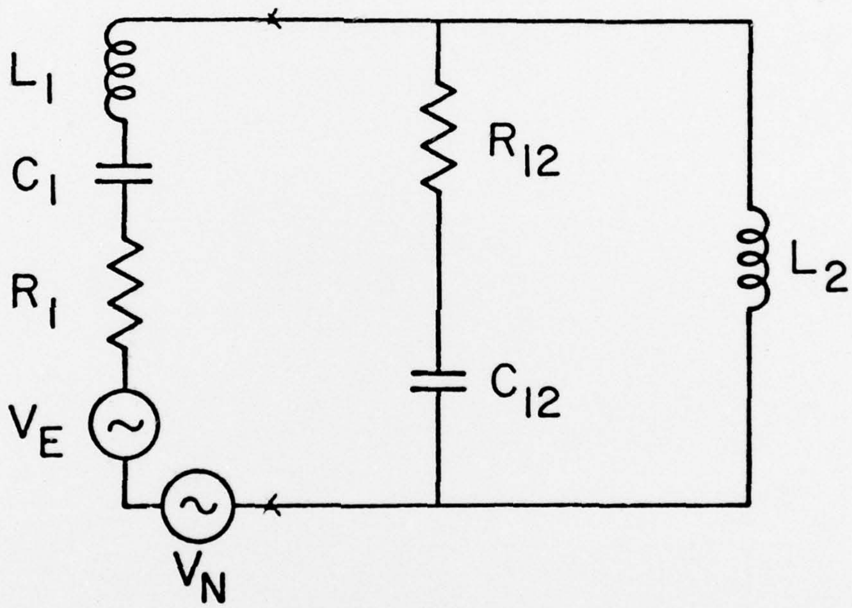


Figure 3

$Z = R + iX$. Then

$$Z_1Z_2+Z_1Z_{12}+Z_2Z_{12} = -\left[X_1X_2+X_1X_{12}+X_2X_{12}\right] + i\left[R_1X_2+R_{12}X_1+R_1X_{12}+R_{12}X_2\right] \quad (19)$$

Successful operation will require the resonance vanishing of the real part of (19). Therefore

$$X_1 = \frac{-X_2X_{12}}{(X_{12}+X_2)} \quad (20)$$

Let $(\Delta\omega)_2$ be the separation in angular frequency of the zero of $X_{12}+X_2$ from the zero of X_1

$$|X_{12}+X_2| \approx 2\, m_2\Delta\omega_2 \quad (21)$$

$$|X_1| \approx \frac{\omega X_{12}}{2\Delta\omega_2} \quad (22)$$

If then the real part of (19) vanishes, the largest imaginary part will dominate the result. Under these conditions the resonance value of (18) will be given by

$$|i_1-i_2| \rightarrow \frac{V\omega L_2}{R_{12}X_1} = \frac{2Q_2\Delta\omega_2 V}{\omega^2 L_2} \quad (23)$$

with

$$Q_2 = \frac{\omega L_2}{R_{12}} \quad (24)$$

If the frequency ω_c is increased the capacitor plate may become an element of an electromagnetic mode cavity resonator.

NOISE ANALYSIS

The resistances R_1 and R_{12} may be regarded as "Johnson Noise Sources". If their noise voltages are introduced and an integration carried out, the noise performance in the absence of noise originating in the electronics may be calculated.

We will estimate the narrow band noise output of the cylinder in the following way. Over this narrow band the terminals xx looking to the right define an equivalent resistance in series with an equivalent reactance.

For two parallel impedances

$$Z=R+jX=\frac{Z_A Z_B}{Z_A+Z_B} \simeq R_2 \left(\frac{\omega^2 L_2 C_2}{1-\omega^2 L_2 C_2}\right)^2 + \frac{i\omega L_2}{(1-\omega^2 L_2 C_2)} \tag{25}$$

The total mean squared current associated with R_1 and the real part of Z in the mode narrow band is

$$i^2 = \frac{kT}{m_1} \tag{26}$$

and the squared narrow band Voltage noise is V^2_{NB} given by

$$V^2_{NB} = \frac{kT}{m_1} \left(\frac{\omega L_2}{1 - \omega^2 L_2 C_2} \right)^2 = (\dot{x}_2 - \dot{x}_1)^2 \left(\frac{1}{\omega C_{12}} \right)^2 \tag{27}$$

In (27) T is the equivalent noise temperature including the cylinder and back reaction V_N of Figure 3.

For electronics bandwidth $\Delta\nu_e$, the wideband noise $V^2(\omega_B)$ is given approximately by

$$V^2(\omega_B) = \frac{4kT\ R_{12}\ \Delta\nu_e (\omega L_2)^2}{\left(\omega L_2 - \frac{1}{\omega C_2}\right)^2} + V_e^2 \Delta\nu_e \tag{28}$$

In (28) V_e^2 is the voltage power spectrum of the electronics.

The time scale for bandwidth $\Delta\nu_e$ is $\sim \frac{\pi}{\Delta\nu_e}$ and during such periods we may expect normal mode cylinder squared current fluctuations

$$\sim \frac{kT}{m_1} \left(\frac{\pi}{\Delta\nu_e} \right) \frac{1}{\tau} \tag{29}$$

where $$\tau \simeq \frac{Q}{\omega} \tag{30}$$

Note that $m_2 \rightarrow L_2 \quad m_1 \rightarrow L_1$

To observe some short duration signal V_s^2 we require

$$V_s^2 > 4kTR_{12} \left(\frac{\omega L_2}{\omega L_2 - \frac{1}{\omega C_{12}}} \right)^2 \Delta\nu_e + V_e \Delta\nu_e + \frac{kT}{m_1} \left(\frac{\omega L_2}{1-\omega^2 L_2 C_{12}} \right)^2 \frac{\pi\omega}{Q_1 \Delta\nu_e} \quad (31)$$

Differentiating (31) with respect to $\Delta\nu_e$ and setting the derivative equal to zero gives:

$$\Delta\nu_e = \sqrt{\frac{\pi\omega_2^2 m_2 Q_2}{4m_1 Q_1 \left(1+\frac{V_e (1-\omega^2 L_2 C_{12})^2 Q_2}{4\omega^3 C_{12} kTL_2^2}\right)}} \quad (32)$$

For electronics noise temperature T_N, we have the voltage power spectral density

$$V_e = \frac{kT_N}{\omega C_{12} Q_e} \quad (33)$$

T_N includes the amplifier and uncompensated "back reaction" noise. Q_e is the Q of the electromagnetic mode at angular frequency ω_e

With (33) , (32) becomes

$$\Delta\nu_e = \sqrt{\frac{\pi\omega_2^2 m_2 Q_2}{4 m_1 Q_1 \left(1+ \frac{T_N Q_2}{T Q_e} \left[1 - \frac{\omega^2}{\omega_2^2}\right]^2\right)}} \quad (34)$$

as the optimal bandwidth. Here again the temperature T is the effective noise temperature of the cylinder, including the back reaction of V_N. The"sensitivity" for short pulses in terms of the observation time τ_{OBS} and relaxation time $\tau_{RELAXATION}$ is

$$\frac{kT\tau_{OBS}}{\tau_{RELAXATION}} = \frac{kT\omega}{2Q_1\Delta\nu_e} = kT\sqrt{\frac{4m_1\left(1+\frac{T_N}{T}\frac{Q_2}{Q_e}\left[1-\frac{\omega^2}{\omega_2^2}\right]^2\right)\omega^2}{\pi\, m_2 Q_1 Q_2 \omega_2^2}} \tag{35}$$

(35) is not valid for $\omega \simeq \omega_2$. In practice $\omega - \omega_2 \simeq 10^2$ to 10^3.

For a Maser the ultimate[2] noise temperature T_N is given by

$$T_N = \frac{\hbar\omega_c}{k\ell n_e 2} \tag{36}$$

(35) does imply that with this kind of transducer one can ultimately approach a fluctuation sensitivity

$$kT\sqrt{\frac{4m_1}{m_2 Q_1 Q_2}} \tag{37}$$

Back Reaction Noise Due to V_N

A study of equation (9), Figures 2 and 3 indicates that the generator noise contribution V_N appears in the amplifier output without significant distortion, and also in the driving force of the demodulator. Skillful design may therefore make a subtraction possible with the large reduction of this kind of noise, which has already been carried out for many years in the design of Radar systems. The noise cancellation would take place in the mixer, shown in Figure 2.

There are other sources of noise associated with the electronics, which we now discuss.

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DISCUSSION OF THE ELECTRONICS

The driving voltage V_o at ω_c (pump) and the internal motion at ω_a with the amplitude $(x_2 - x_1)_1$ contribute a current of angular frequency $\omega_a + \omega_c$ given by

$$i = \frac{d}{dt}\left[C\left(1 - \frac{(x_2-x_1)_1}{(x_2-x_1)_0}\cos\omega_a t\right) V_o \cos\omega_c t\right] = \frac{(x_2-x_1)_1}{2(x_2-x_1)_0} V_o C(\omega_a+\omega_c)\cos(\omega_a+\omega_c)t \tag{38}$$

A voltage output $V_{\omega_a+\omega_c}$ is available of magnitude

$$V_{\omega_a+\omega_c} = \frac{(x_2-x_1)_1}{(x_2-x_1)_0} V_o (\omega_a+\omega_c) C \left| Z_{\omega_a+\omega_c} \right| \tag{39}$$

(39) may be sufficiently large so that the amplifier voltage noise is not a major factor. The power output at frequency $\omega_a + \omega_c$, $P_{\omega_a+\omega_c}$ is related to the mode power output P_{ω_a} by

$$P_{\omega_a+\omega_c} = \frac{P_{\omega_a}(\omega_a+\omega_c)}{\omega_a} \tag{40}$$

The energy for this power gain originates primarily in the pump voltage V_o at ω_c.

The low noise amplifier operates at $\omega_a + \omega_c$ and its input circuit may be designed to transmit no significant noise at ω_a backwards to drive the antenna. However, current noise at the frequency $\omega_a + \omega_c$ may, by the parametric conversion process be converted to ω_a. For this noise the power relations are

$$\hat{P}_{\omega_a} = \left(\frac{\omega_a}{\omega_a + \omega_c}\right) \hat{P}_{\omega_a + \omega_c} \tag{41}$$

Nonreciprocal circuit elements may be useful in isolating the cylinder from the amplifier.

A small part of the power output (40) is contributed by the cylinder as a result of interaction of the electrostriction force and velocity. This is given by

$$P_a = C^2 V_o^2 R_{\omega_a + \omega_c} \left(\omega_a \omega_c\right) \left[\frac{(x_2 - x_1)_1}{(x_2 - x_1)_o} \right]^2 / 4 \tag{42}$$

In (42) $R_{\omega_a + \omega_c}$ is the real part of the impedance $Z_{\omega_a + \omega_c}$.

The mechanical resistance R_m' which is coupled thereby into the mode will lower the Q. From (42) we have

$$R_m' = C^2 V_o^2 \omega_c R_{\omega_a + \omega_c} / 4\omega_a (x_2 - x_1)_o^2$$

CONCLUSION

A capacitive transducer driven by an alternating field provides parametric amplification for study of the mode noise. Low noise performance appears possible through cancellation of pump noise in the mixer.

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1. J.P. Richard, Rev. Sci. Inst. 47, 4, 423 (1976); H.Hirakawa and M.K. Fujimoto J. Phys. Soc. Japan 41, 1093 (1976)
2. J. Weber, Masers, Rev. of Mod. Physics (1959)

RELAXATION EFFECTS

Earlier measurements over the frequency range 60°K-78°K of the fluctuations in acoustic mode amplitude showed large amounts of excess noise when the material was not in thermal equilibrium. The drift of the normal mode frequency was taken as a measure of the approach to thermal equilibrium. When the mean temperature was drifting less than 10^{-2} degrees C per day, the noise appeared normal.

Very different results were obtained at liquid helium temperatures. Below 17° Kelvin the drift of the normal mode frequency was smaller than we could measure with present instrumentation ~ less than a part in 3,000,000 per day. Correspondingly, the excess noise was so small that it was difficult to isolate it from other effects.

Excess noise, about 25% of the thermal equilibrium value, was observed in the aluminum cylinder warmed up from 6.6° to 8.1° Kelvin. We are not certain that this noise is associated with structural relaxation effects.

For the aluminum cylinder, relaxation effects appear much smaller, and appear to involve much longer time scales at 4° Kelvin than at 77° Kelvin.

CONCLUSION

Our new demountable cryostat and helium refrigerator-liquifier have been successfully employed to measure the performance of a new kind of accelerometer, to study the noise and acoustic quality factors of materials, and to explore new instrumentation methods at low temperatures.

The thermal noise predicted by theory was observed under equilibrium conditions. Some excess noise was observed during the warm up period.

A parametric up converter is being studied as a means of increasing the sensitivity of the accelerometer and improving the precision of our measurements.