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ELASTIC-PLASTIC DEFORMATION IN CRACKED SOLIDS AND DUCTILE FRACT--ETC(U)  
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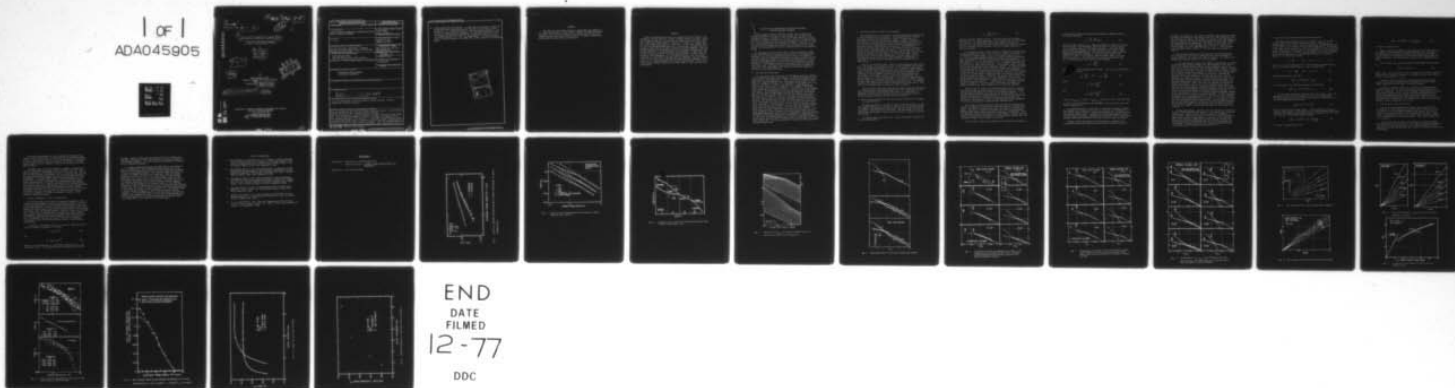
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6 ELASTIC-PLASTIC DEFORMATION IN CRACKED SOLIDS  
AND DUCTILE FRACTURE CRITERION

by

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#### ABSTRACT

Crack tip deformation and crack tip stresses have been studied. The information obtained thus far is used to analyze ductile fractures. Crack tip deformation and crack tip stresses are highly affected by plate thickness. When the necking parameter,  $(K/\sigma_y)^2/t$ , is equal to or larger than 48, strip necking takes place at a crack tip. In this case, Dugdale model of crack tip opening displacement CTOD, is applicable, and CTOD as well as thickness contraction can be used to measure fracture toughness. When the necking parameter is 18, Dugdale model of crack opening displacement, COD, is applicable, but no strip necking has been observed. In this case, the concept of crack tip stress and strain characterization by COD as well as by the far field parameters, i.e. the applied stress  $\sigma_\infty$  and the overall elongation  $\Delta$ , can be used to measure fracture toughness. For the limiting case of a very thick plate, the ASTM E24 committee has recommended a value of 0.4 for the necking parameter for valid  $K_{IC}$  measurements. The precise demarcations of various regions to use CTOD, COD,  $\sigma_\infty$  and  $\Delta$  have yet to be investigated.

ELASTIC-PLASTIC DEFORMATION IN CRACKED SOLIDS AND  
DUCTILE FRACTURE CRITERION

Linear elastic fracture mechanics has been very successful in dealing with brittle fractures of high strength and low and medium toughness materials. According to the conventional practice, a large specimen is required to measure the valid  $K_{IC}$  value of a tough material. For a very tough material, the required specimen size could be too large for a typical laboratory testing machine to handle. Therefore, a new fracture criterion consistent with the basic principles of the linear elastic fracture mechanics is needed to measure high fracture toughness values.

The development of the linear elastic fracture mechanics is based on the crack tip elastic stress and strain fields. A thorough understanding of the crack tip deformation and the crack tip stresses in the case of general yielding is necessary to develop a reliable ductile fracture theory. A program on "elastic-plastic deformation in cracked solids and ductile fracture criterion" has been conducted at Syracuse University. Considerable progress has been made during the past two and a half years. The major results obtained thus far are summarized as follows. The details have been reported in published papers and reports, see the publication list.

A. Crack Opening Displacement

The applicability and limitations of the Dugdale model are examined. Crack opening displacements are measured, and the measurements are compared with the calculated values. Figure 1 illustrates the COD measurements for a 2024-0 aluminum alloy. The tensile yield strength of the alloy is  $53.8 \text{ MN/m}^2$  (7.8 ksi). The thicknesses of Specimen 1 and 2 are 0.041 cm and 0.625 cm (0.016 in. and 0.25 in.). Both specimens are loaded to the same K-value,  $4.53 \text{ MN/m}^{3/2}$  (4.13 ksi $\sqrt{\text{in}}$ ). The dashed line represents the elastic calculation. The solid line is the Dugdale calculation. The measurements of the thin specimen agree exceedingly well with the plane stress Dugdale calculation; whereas, the measurements of the thick specimen agree well with the elastic calculation. The values of the quantity,  $(K/\sigma_Y)^2/t$ , for these two specimens are 17.5 and 1.12, where  $t$  and  $\sigma_Y$  are specimen thickness and tensile yielding strength. The quantity gives plastic zone size relative to plate thickness. This quantity is also an index of the deviation from the plane strain condition. The measurements clearly indicate that when a specimen is thin and the size of plastic zone is large, the COD measurements agree well with the Dugdale model. For this case, the condition of plane stress prevails. When the plastic zone is small relative to the plate thickness, and the condition of plane strain prevails, the COD measurements agree well with the elastic calculation. The same observation has also been made using 2024-T3 and 2024-T351 aluminum alloys. It is clear that the COD measurements are affected by specimen thickness, and it can be concluded that the Dugdale model is applicable only to the plane stress case. It is not suitable for plane strain analyses without suitable modifications.

## B. The Characteristics of Near Tip Deformation

The strains near the crack tips of a double-edge-notched specimen made of 2024-0 aluminum alloy are measured. The measurements are compared with elastic-plastic FEM calculations. Figure 2 illustrates the measurements at various load levels in general yielding. The dashed lines represent the plane stress elastic-plastic FEM calculations. Near the crack tip, the measured strains are less than those obtained from the plane stress, elastic-plastic calculations. The difference is caused by the stiffening effect due to the triaxial state of stress in the interior of the plate near the crack tip. In the interior of the plate, because that the hydrostatic tensile stress is high, and that the deviatoric stresses are low relative to those in the state of plane stress, there is less plastic deformation. In other words, close to the crack tip in the interior of a thick specimen, the material is "stiffened", and the state of deformation tends to approach that of plane strain.

Next to the "stiffened" zone, there is a characteristic plane stress zone. The slopes of the measured curves and the calculated curves agreed well with the HRR singularity,  $1/(1 + N)$ . If a sample is large enough, it is conceivable that a characteristic crack tip elastic field may exist. Further away, the stresses and strains are no longer dominated by the crack tip. They are largely controlled by the overall geometry and the loading of the sample. Figure 3 illustrates schematically the various regions in a very large specimen. One or more of these regions may not appear in a particular specimen. If a specimen is small, general yielding takes place, and the elastic region may not exist. In a bending specimen, the characteristic plane stress region may be very small. The size of the stiffened zone is affected by plate thickness.

Both the COD and the strain measurements indicate that the deformation in a cracked sample is three-dimensional in nature. Both plane strain and plane stress are idealized limiting cases. It is important to establish the conditions of validity for these two idealized cases in order to develop a reliable ductile fracture criterion.

## C. Direct Characterization of Crack Tip Stresses and Strains for Samples in General Yielding

The important result of the linear elastic fracture mechanics is that the fracture toughness of a material is invariant to the planar geometry of a sample. The derivation of the brittle fracture theory based on the principle of energy balance is well known. The same result can also be derived from the concept of direct characterization of crack tip stresses and strains.

In a small region near the tip of a crack, the elastic stresses are given by the equation:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \quad , \quad (1)$$

where  $K$  is stress intensity factor. Away from the crack tip region, Equation (1) is no longer applicable. Let us define the crack tip region by a boundary,  $r_e$ , within which the elastic stresses are given by Equation (1). The characterization of the crack tip stresses and strains by  $K$  in an elastic solid is obvious, and not of primary interest to ductile fractures.

If a specimen is made of metal, plastic deformation does take place within the plastic zone,  $r_p$ , at the crack tip. Within  $r_p$ , Equation (1) is certainly no longer valid. If  $r_p$  is much smaller than  $r_e$ , i.e.,  $r_p \ll r_e$ , the stress relaxation within  $r_p$  caused by plastic deformation will have negligible effects on the stresses along the boundary  $r_e$ . In other words, if  $r_p \ll r_e$ , the stresses on  $r_e$  are essentially those given by Equation (1). This condition of  $r_p \ll r_e$  is commonly referred to as the condition of small scale yielding. Let us examine several specimens made of the same material but of different geometry. If they are loaded to the same  $K$ -value, and if the condition of small scale yielding is satisfied by all of the specimens, the stresses on the boundaries,  $r_e$ 's, of the same size and shape of these samples, are the same. One may look at the  $r_e$ 's as free bodies of identical shape and identical size with identical boundary stresses. Therefore, the stresses and strains within the  $r_p$ 's must be the same for all of the specimens. If one of the samples fails, it is reasonable to expect that the others will fail too. In this way, one reaches the conclusion;  $K_c$  is invariant to planar geometric variation. Since the crack tip stresses and strains are highly affected by the plate thickness,  $K_c$  is thickness dependent.

The fact that the value of  $K$  characterizes the crack tip stresses and strains within  $r_p$  is sufficient for concluding that  $K_c$  is constant, provided that the condition of small scale yielding is obeyed. It is not necessary to know the exact stresses and strains within  $r_p$ .  $r_e$  is linearly proportional to the specimen size. One can always satisfy the condition of small scale yielding by increasing the specimen size. Indeed, this was found to be true, and the well known size requirement for valid  $K_{Ic}$  measurements was established (ASTM STP 410).

The concept of direct characterization of crack tip stresses and strains is examined for plane stress case where general yielding takes place. The crack tip stresses and strains in small scale yielding and general yielding are calculated with FEM, and a direct correspondence between small scale yielding and general yielding is established. Thus, the crack tip stresses and strains of a small sample in general yielding can be characterized by the  $K$ -value of a corresponding specimen where the condition of small scale yielding is satisfied.

For the case of small scale yielding, the calculations are made for

a semi-circular region with boundary displacement conditions given by the elastic solution:

$$u_i = \frac{K_I}{E} \sqrt{\frac{2r}{\pi}} h_i(\theta) \quad , \quad (2)$$

where  $h_i(\theta)$  are functions of  $\theta$ . FEM calculations are made for the state of plane stress. The calculated effective stress,  $\bar{\sigma}$ , and the strain,  $\epsilon_{yy}$ , at various levels of  $K$  are shown in Figure 4. The lines within plastic zones are parallel, and their slopes are very close to those given by the HRR singularities. The same results are shown in the non-dimensionalized plots of Figure 5. The sum of the slopes of the curves of  $\bar{\epsilon}_p$  and  $\bar{\sigma}$ , in the plastic region is nearly unity, as is the sum for  $\epsilon_{yy}$  and  $\sigma_{yy}$ . But the sum for  $\epsilon_{xx}$  and  $\sigma_{xx}$  is much less than one. The plastic zone size is given by the equation:

$$r_p = \alpha \left( \frac{K_I}{\sigma_Y} \right)^2 \quad (3)$$

and  $\alpha = 0.24$  for the aluminum alloy treated.  $\epsilon_{yy}$  and  $\sigma_{yy}$  in the plastic zone can be written as

$$\epsilon_{yy} = \frac{K_\epsilon}{r^m} \quad \text{and} \quad \sigma_{yy} = \frac{K_\sigma}{r^{m'}} \quad (4)$$

where

$$K_\epsilon = \frac{\epsilon_Y}{\beta} \alpha^m \left( \frac{K_I}{\sigma_Y} \right)^{2m} \quad (5)$$

and

$$K_\sigma = \frac{\sigma_Y}{\beta'} \alpha^{m'} \left( \frac{K_I}{\sigma_Y} \right)^{2m'} \quad (6)$$

$(\epsilon_Y/\beta) = \epsilon_{yy}(r=r_p)$ ,  $(\sigma_Y/\beta') = \sigma_{yy}(r=r_p)$ , and  $\beta$  and  $\beta'$  are 1.405 and 0.983.

$K_\epsilon$  and  $K_\sigma$  can be considered as the strengths of the crack tip strain and stress singularities.

As discussed earlier, in the case of a specimen in general yielding, the material in the region very close to a crack tip is "stiffened" by the triaxial state of stress. Beyond the stiffened zone, a characteristic plane stress region exists. Therefore for the specimen geometry chosen for this study, the direct correspondence between the cases of small scale yielding and general yielding in the state of plane stress is examined.

Figure 6 shows the results of the plane stress calculation of a double-edge-notched (DEN) specimen made of batch C aluminum in general

yielding. In addition to the crack line results, the y-direction stress and strain distributions along radial lines, 45°, 60° and 90° away from the crack line are shown. The results are plotted in the same manner as those in Figure 5, and the corresponding plots for small scale yielding are also presented for comparison. The values of  $r_p$  for these plots are obtained by the linear extrapolation of effective stress,  $\bar{\sigma}$ , to yield strength,  $\sigma_Y$ , in a logarithmic plot of  $\bar{\sigma}$  vs.  $r$ . The calculated  $\bar{\sigma}$  and effective plastic strain,  $\bar{\epsilon}^P$ , are also shown and compared in Figure 7.

Plane stress calculation is also made for a single-edge-notched (SEN) specimen made of the same material loaded into the region of general yielding. The results are shown in Figure 8. The plane stress characteristic region (region II) is clearly shown.

In the characteristic plane stress region, the excellent correlation between both the general yielding cases (DEN and SEN) and the case of small scale yielding indicates that a single parameter, such as  $r_p$ , is sufficient to characterize the crack tip stresses and strains. Furthermore, the near tip stress or strain of a small sample in general yielding can be used to obtain the equivalent  $K_I$  value of a large specimen where the condition of small scale yielding is satisfied. Consider two specimens of the same thickness, one large and one small. Both are loaded to the level such that at a certain distance from crack tip,  $r$ , the y-direction stress or strain is the same for both specimens. The large specimen is wide enough such that the condition of small scale yielding is retained. Hence, the crack tip stresses and strains are characterized by the value of the stress intensity factor, which is given by the elastic solution; whereas the second specimen is so small that the state of deformation is general yielding. Since the crack tip strain and stress distributions are the same in both specimens, it is fair to say that the  $K_I$  value of the small specimen in general yielding is the same as the  $K_I$  value of the large specimen. In this connection, Equations (4,5,6) are applicable even in the case of general yielding as long as the corresponding pair of  $\epsilon_{yy}$  and  $r$ , or  $\sigma_{yy}$  and  $r$  are taken from the characteristic plane stress region. By this approach, one is able to use a small sample to evaluate a very high fracture toughness. The equivalent  $K_I$  values for the empirical data in Figure 2 are 38.5, 29.8, and 23.3 ksi $\sqrt{\text{in}}$ . The tensile yield stress of this material is 7.26 ksi.

As discussed earlier, near the crack tip, the state of stress and strain is three-dimensional in nature. It is neither plane strain nor plane stress. However, if the stiffened region is embedded in a characteristic plane stress region, if the size of the characteristic plane stress region is large relative to the size of the stiffened zone, and if the plate thicknesses are the same, the stresses and strains within the stiffened zone must be the same even though they are unknown. Therefore, the crack tip stresses and strains can be characterized by the equivalent plane stress  $K$  or  $J$ .

#### D. Near Tip Strain and Stress and Far Field Parameters

In Figure 9 the normalized crack line strain,  $\epsilon_{yy}/\epsilon_Y$ , of the DEN specimen calculated at various distances from the crack tip,  $r$ , is plotted against  $\Delta/W\epsilon_Y$ , where  $\Delta$  is the elongation of the DEN specimen over a gage length of seven inches, and  $W$  is the specimen width. The results of the plane stress calculation, which are represented by the solid lines, agree well with the limited number of measurements, i.e., the triangular points in the figure. A similar relation between the crack line stress,  $\sigma_{yy}$  at various distances,  $r$ , and the applied stress  $\sigma_\infty$  is shown in Figure 10. In the linear regions,

$$\epsilon_{yy} = \gamma \frac{\Delta}{W}, \quad \text{and} \quad \sigma_{yy} = \gamma' \sigma_\infty, \quad (7)$$

where  $\gamma$  and  $\gamma'$  are functions of  $r/W$ . These results also imply that  $K_\epsilon$  and  $K_\sigma$  are linearly proportional to  $\Delta$  and  $\sigma_\infty$  respectively

$$K_\epsilon = \Gamma \left(\frac{\Delta}{W}\right) \quad \text{and} \quad K_\sigma = \Gamma' \sigma_\infty \quad (8)$$

Combining Equations (5) and (8),

$$K_I^{2m} [(\epsilon_Y \alpha^m / \beta) \sigma_Y^{-2m}] = \Gamma \left(\frac{\Delta}{W}\right) \quad (9)$$

In a like manner, the following relation can be derived,

$$K_I^{2m'} [(\sigma_Y \alpha^{m'} / \beta') \sigma_Y^{-2m'}] = \Gamma' \sigma_\infty \quad (10)$$

If the linear relations of Equation (8) do not exist,  $\Gamma$  and  $\Gamma'$  in Equations (9) and (10) are functions of  $(\Delta/W)$  and  $\sigma_\infty$  respectively. Multiplying Equations (9) and (10) and recalling the relation  $m+m' \approx 1$ , the final approximated form becomes

$$K_I^2/E = [\beta\beta'\Gamma\Gamma'/\alpha] \frac{\sigma_\infty \Delta}{W} \quad (11)$$

$K_I^2/E$  is  $J$  in the case of plane stress. If the product of  $\epsilon_{yy} \sigma_{yy}$  is linearly proportional to the product  $\Delta\sigma_\infty$  for a given geometry and a given material, the quantity in the square bracket of Equation (11),  $\beta\beta'\Gamma\Gamma'/\alpha$ , is a constant. For the DEN specimen treated,

$$K_I^2/E = 3.37 \frac{\sigma_\infty \Delta}{W} = 16.1 \left(\frac{\sigma_\infty}{\sigma_Y}\right) \left(\frac{\Delta}{W\epsilon_Y}\right) \quad (12)$$

for batch C aluminum alloy, and

$$K_I^2/E = 3.67 \frac{\sigma_\infty \Delta}{W} = 34.5 \left(\frac{\sigma_\infty}{\sigma_Y}\right) \left(\frac{\Delta}{W \epsilon_Y}\right) \quad (13)$$

for batch B aluminum alloy.

Figure 11 plots the product of  $\sigma_{yy}/\sigma_Y$  and  $\epsilon_{yy}/\epsilon_Y$  as a function of the product of its associated far field parameters,  $\sigma_\infty/\sigma_Y$  and  $\Delta/W\epsilon_Y$ . It is interesting to note that the deviation from linearity in Figure 11 is not as severe as in Figures 9 and 10 where stresses and strains are plotted separately. The stresses and strains tend to compensate each other.

If the product  $\epsilon_{yy} \sigma_{yy}$  is not linearly proportional to the product  $\Delta \sigma_\infty$ ,

$$K_I^2/E = (\beta \beta' \Gamma \Gamma' / \alpha) \quad (14)$$

where  $\Gamma$  and  $\Gamma'$  are functions of  $\Delta$  and  $\sigma_\infty$  respectively, and the functional relation can be derived by numerical calculations such as those shown in Figure 11 with  $(K_E K_G / r) = \epsilon_{yy} \sigma_{yy}$ .

Equations (11) and (14) relate the equivalent stress intensity factor to the overall elongation and the applied stress for a small sample loaded far into the region of general yielding.

The stress intensity factor,  $K_I$ , is plotted at various loading levels in Figure 12. The elastic solutions are also included in the plots for comparison. In the region of general yielding where the linear relationship between the near tip and far field parameters hold, the curves are approximated by the dashed lines given by Equation (11). When the linearity breaks down, the equivalent  $K_I$  values can still be obtained by the correlations of the characteristic near tip stresses and strains as given by Equations (5) and (6).

#### E. Near Tip Strain Fracture Criterion

Near tip strain measurements were made at the onset of surface crack growth in three tough and ductile materials: HY-80 steel, and two batches of a fully annealed aluminum alloy (batch B and batch C). Fatigue pre-cracked specimens were tested under tensile load. Three types of specimens were tested: WOL, SEN and DEN. All the specimens of each material were of the same thickness. Their widths ranged from 2 to 8 inches.

At the first sign of surface crack growth, the near tip strains were measured with the moire method. The results are shown in Figure 13. The cross in the second figure for batch C 2024-0 aluminum alloy was measured with a small strain gage, which agreed well with the moire strain measurements.

The strain measurements for each of these three materials fall within a narrow band in spite of the differences in specimen geometry and size. This indicates that at the first appearance of crack growth on the specimen surface, near tip strain is not affected by specimen geometry and the type of loading as long as the specimens are of the same thickness.

A careful study of the data presented in Figure 13 reveals that the three dimensional stiffening effects are shown in all the three types of specimen at the immediate vicinity of the crack tip. For the WOL and SEN specimens, the bending effect is so dominant that there is no plane stress deformed region that can be detected from the strain measurements. A longer ligament width should be considered for this observation. For the DEN type specimen, the characteristic plane stress deformed region is noticeable at approximately one thickness away from the crack tip. Based on the strains in the characteristic plane stress region of the DEN specimen, the  $K_C$  value of this particular material (batch B aluminum alloy) is estimated to be 69.1 ksi $\sqrt{in}$  according to Equation (5). The value of  $(K_C/\sigma_Y)$  is close to 50 inches. According to the size requirement of the linear elastic fracture mechanics, a ligament of 10 feet is necessary to conduct a valid fracture toughness measurement.

#### F. Fracture Toughnesses of Thin and Tough Plates

Wells has proposed to use crack tip opening displacements, CTOD, to measure fracture toughness in the case of general yielding. The crack tip opening displacements and the thickness contractions of thin steel plates were measured. For thin and ductile plates, when the crack tip plastic zone size is much larger than the plate thickness, strip necking takes place, when the necking parameter,  $(K/\sigma_Y)^2/t$ , equals to or exceeds 48. In this case, the Dugdale model is applicable to the "opening displacements" between the upper and the lower crack surfaces as well as between the upper and the lower boundaries of the strip necking zone. Furthermore, the data in Figure 14 show that the opening displacements in the strip necking zone are equal to the thickness contractions.

According to the Dugdale model,  $K$  is related to CTOD; and CTOD is equal to thickness contraction. Therefore

$$K = (E\sigma_Y\Delta t)^{1/2} \quad (15)$$

and

$$K = (E\sigma_Y t_o \epsilon_z)^{1/2} \quad (16)$$

where  $\Delta t$  is the contraction;  $t_o$ , the initial thickness; and  $\epsilon_z$ , the contraction strain. The contractions can be measured from a broken

specimen. Figure 15 shows the measured R-curves of two HY-80 steel specimens, and Figure 16 shows the variation of fracture toughness,  $K_{Ic}$ , with specimen thickness. The curves show the characteristic decrease in  $K_{Ic}$  as thickness decreases.

In summary, significant progress has been made in the studies of crack tip deformation, and the knowledge obtained thus far is very useful to develop practical methods of fracture toughness measurements for ductile and tough materials. Crack tip deformation and crack tip stresses are highly affected by plate thickness. When the necking parameter,  $(K/\sigma_Y)^2/t$ , is equal to or larger than 48, strip necking takes place at a crack tip. In this case, Dugdale's model of crack tip opening displacement, CTOD, is applicable. Therefore CTOD as well as thickness contraction can be used to measure fracture toughness. When the necking parameter is 18, Dugdale's model of crack opening displacement, COD, is applicable, but no strip necking has been observed. In this case, the concept of crack tip stress and strain characterization by COD as well as by the far field parameters, i.e., the applied stress  $\sigma_\infty$  and the overall elongation  $\Delta$ , can be used to measure fracture toughness. For the limiting case of a very thick plate, the ASTM E24 committee has recommended a value of 0.4 for the necking parameter for valid  $K_{Ic}$  measurements. The precise demarcations of various regions to use CTOD, COD,  $\sigma_\infty$  and  $\Delta$  have yet to be investigated.

#### LIST OF PUBLICATIONS

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6. H. W. Liu and Albert S. Kuo, "Fracture Toughnesses of Thin and Tough Plates," Technical Report, August 1977. Dept. of Chem. Eng. and Materials Science, to be submitted to ARO.

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Sheng-yi Kuo - Post Doctoral Fellow

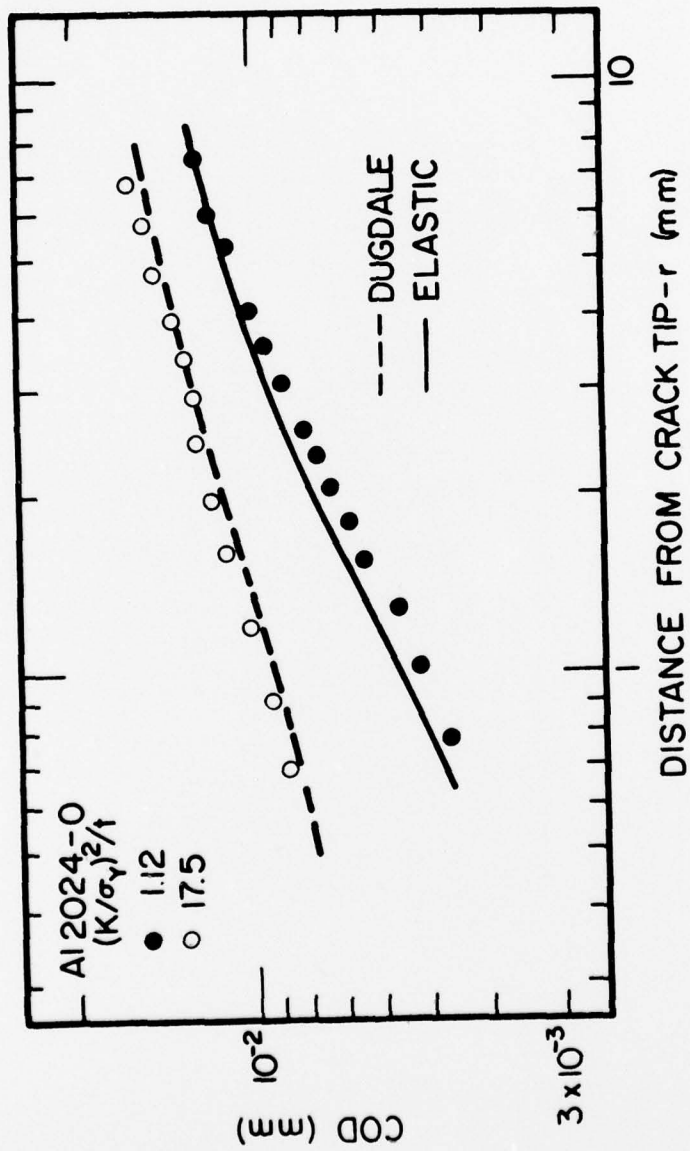


FIG. 1. EFFECT OF SPECIMEN THICKNESS ON CRACK OPENING DISPLACEMENT: Al 2024-O ALUMINUM ALLOY.

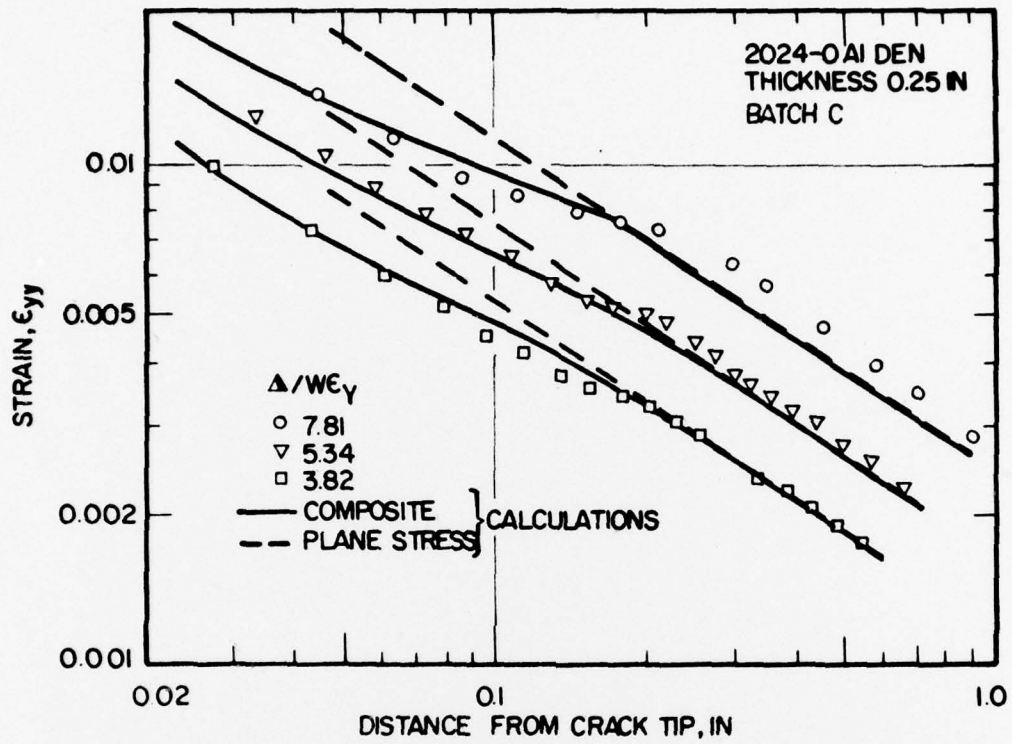


Fig. 2. Near tip strain measurements and calculations, 2024-0 aluminum alloy, batch C.

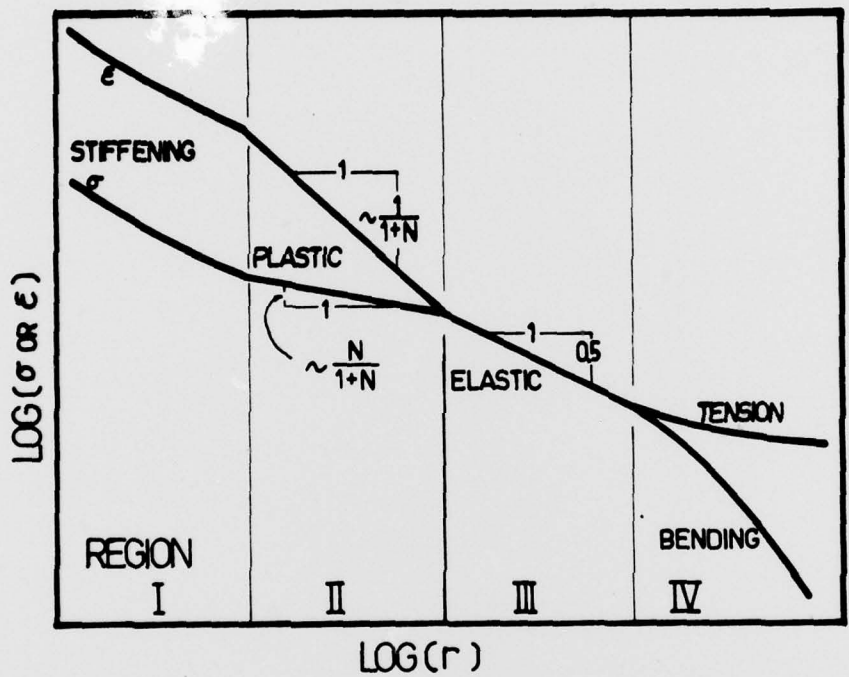


Fig. 3. Schematic plot of stress and strain distributions ahead a crack in logarithmic scale.

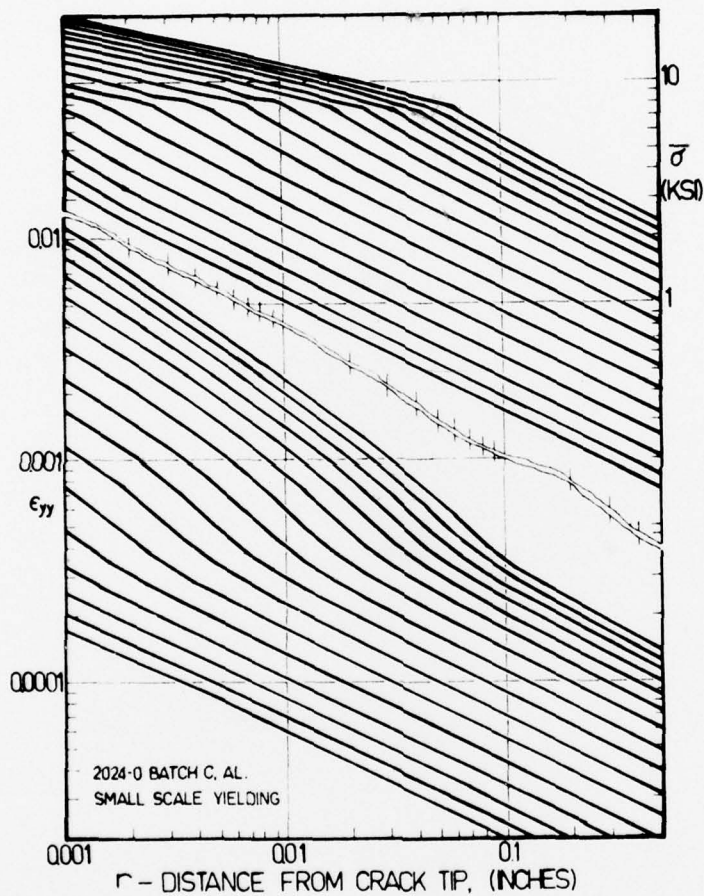


Fig. 4. Calculated  $\bar{\sigma}$  and  $\epsilon_{yy}$  for batch C aluminum along the crack line in small scale yielding case.

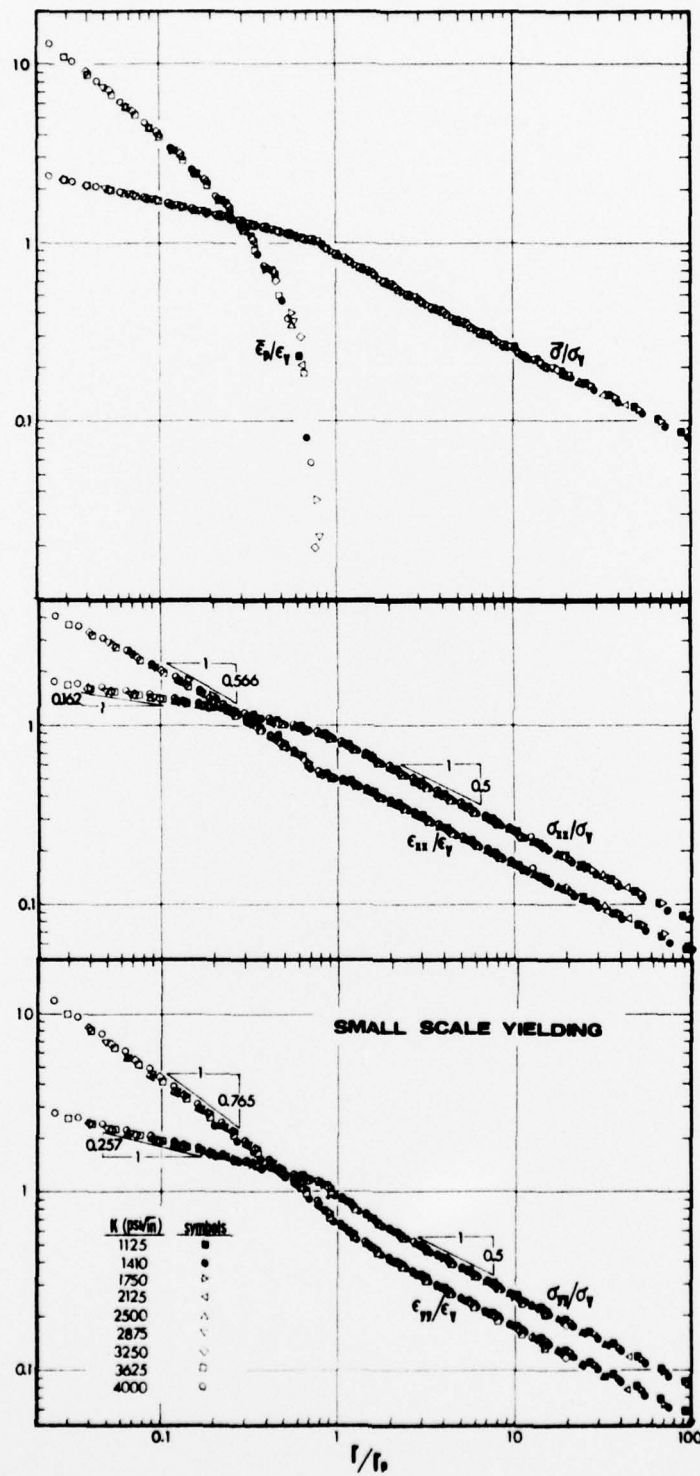


Fig. 5. Normalized plots of crack line stresses and strains.

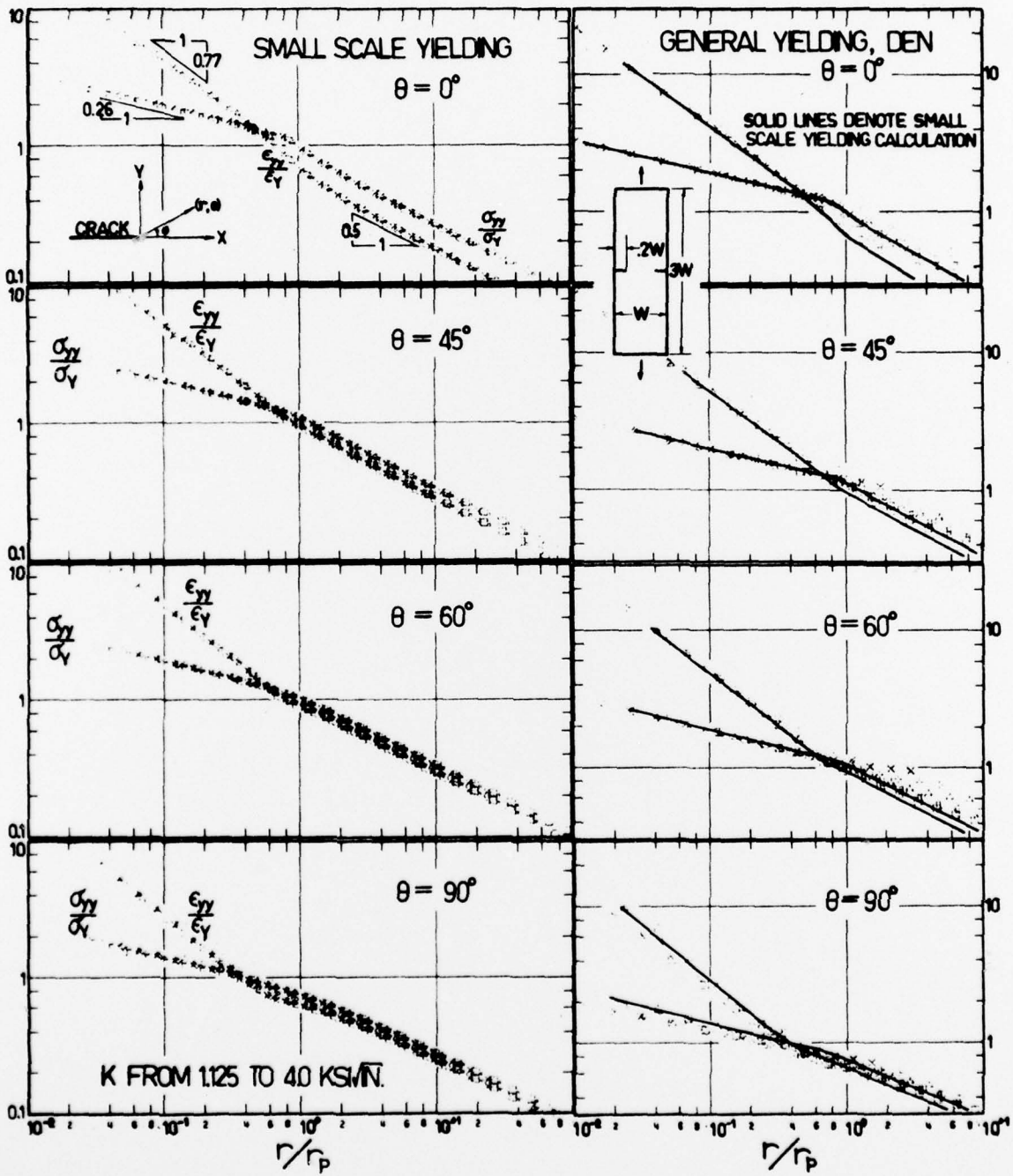


Fig. 6. Correlations of near tip stresses and strains in the loading direction between the small scale yielding and double-edge-notched specimen loaded into the region of general yielding.

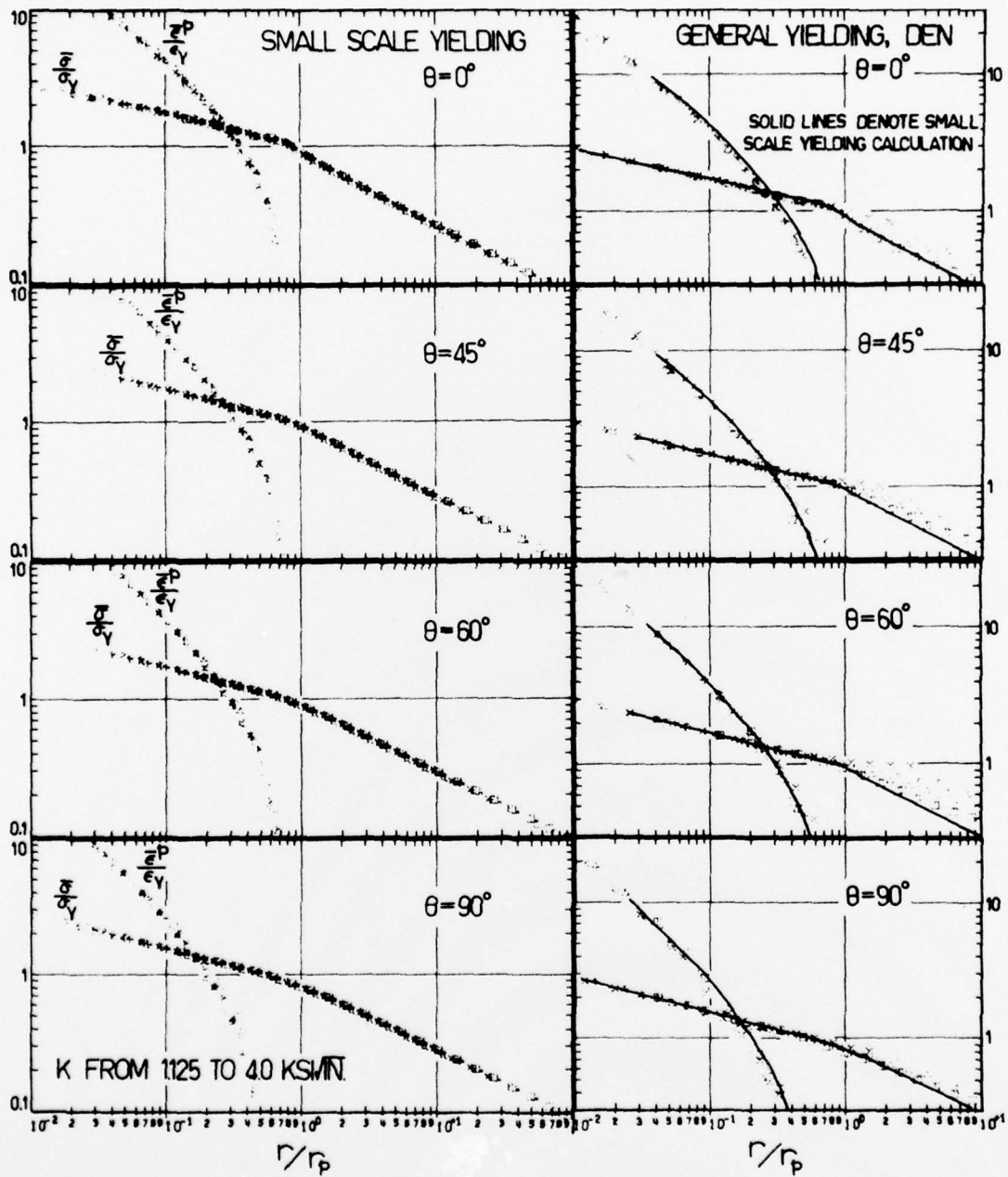


Fig. 7. Correlations of effective stress and effective plastic strain between the small scale yielding and double-edge-notched specimen loaded into the region of general yielding.

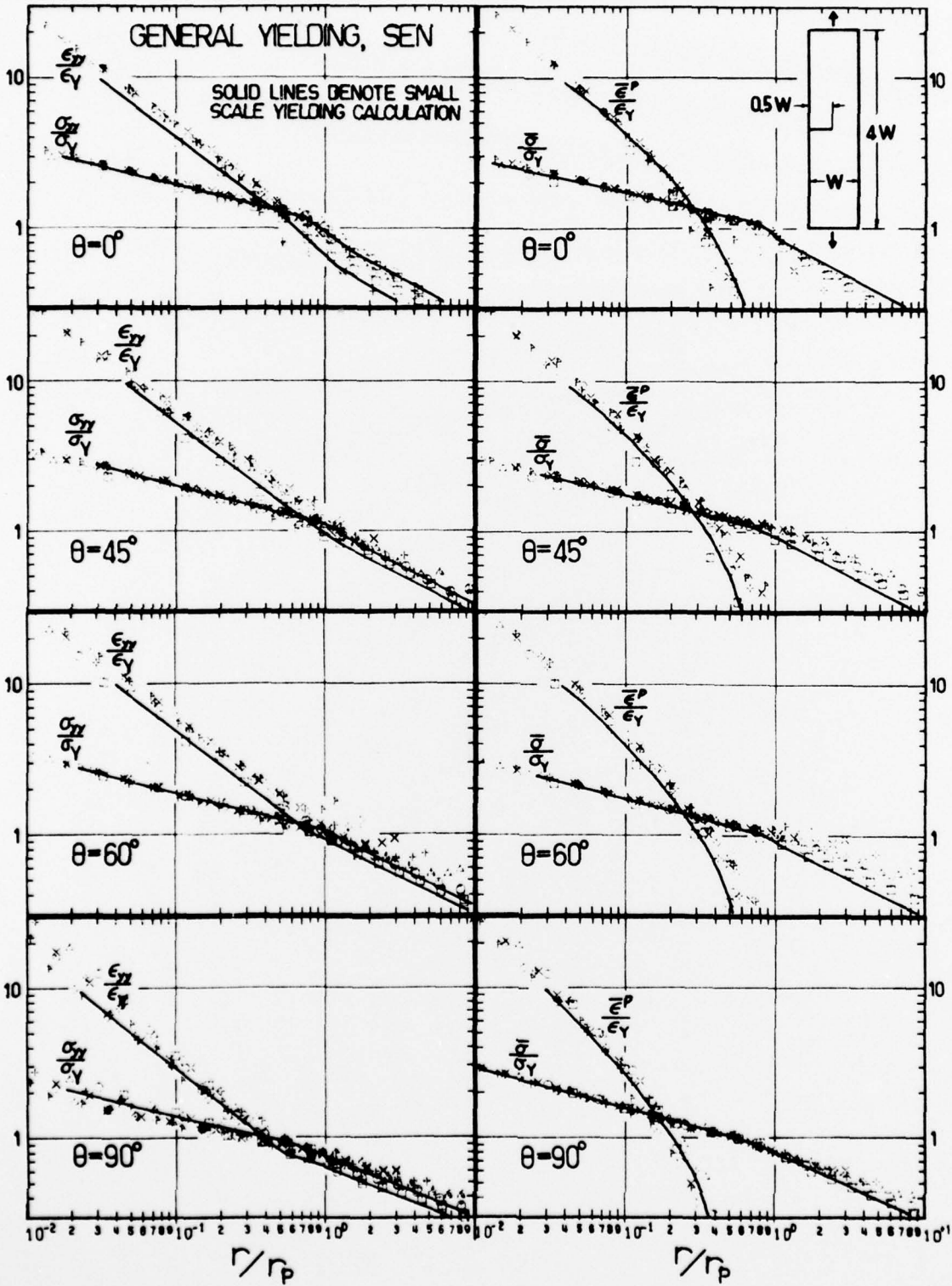


Fig. 8. Correlations of  $\sigma_{yy}$ ,  $\epsilon_{yy}$ ,  $\bar{\sigma}$  and  $\bar{\epsilon}^p$  between the small scale yielding and single-edge-notched specimen loaded into the region of general yielding.

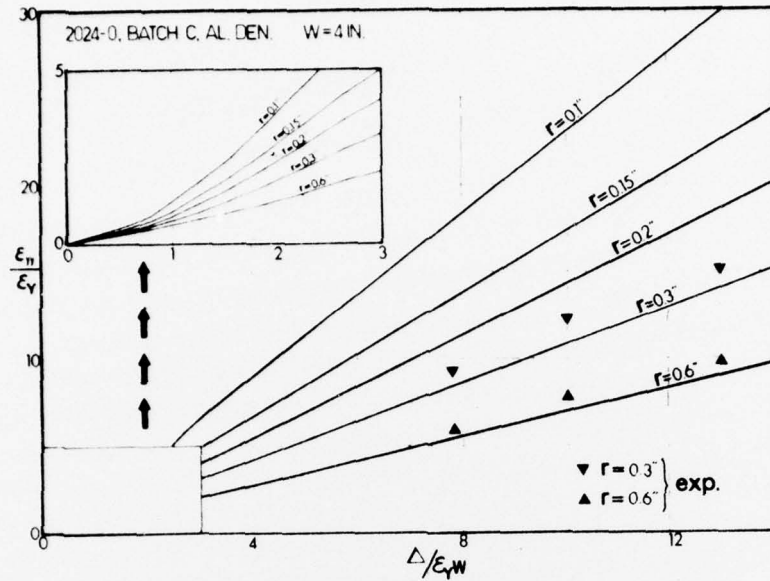


Fig. 9. The relation of near tip strain and gross elongation.

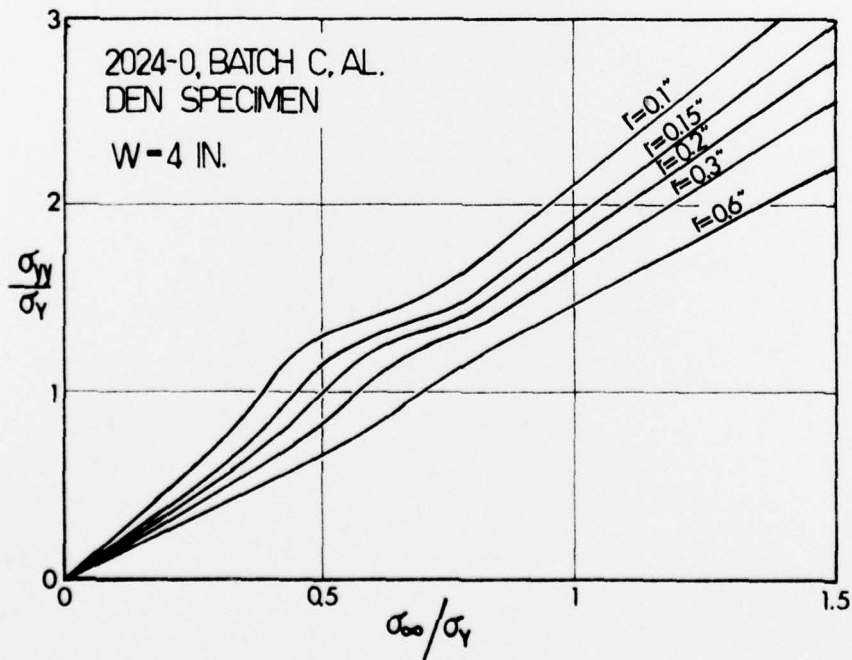


Fig. 10. The relation of near tip stress and applied stress.

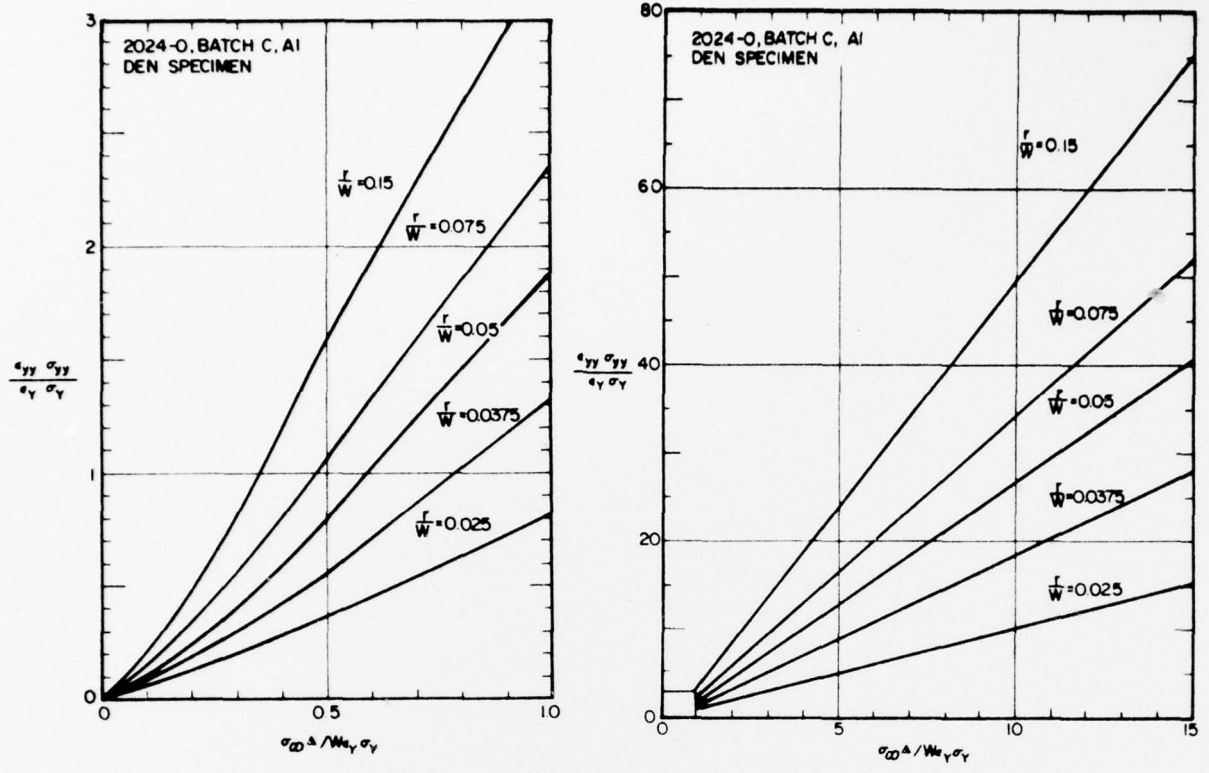


Fig. 11. Near field parameter,  $\frac{\sigma_{yy} \epsilon_{yy}}{\sigma_Y \epsilon_Y}$  versus far field parameter,  $\frac{\sigma_{\infty} \Delta}{W \sigma_Y \epsilon_Y}$ .

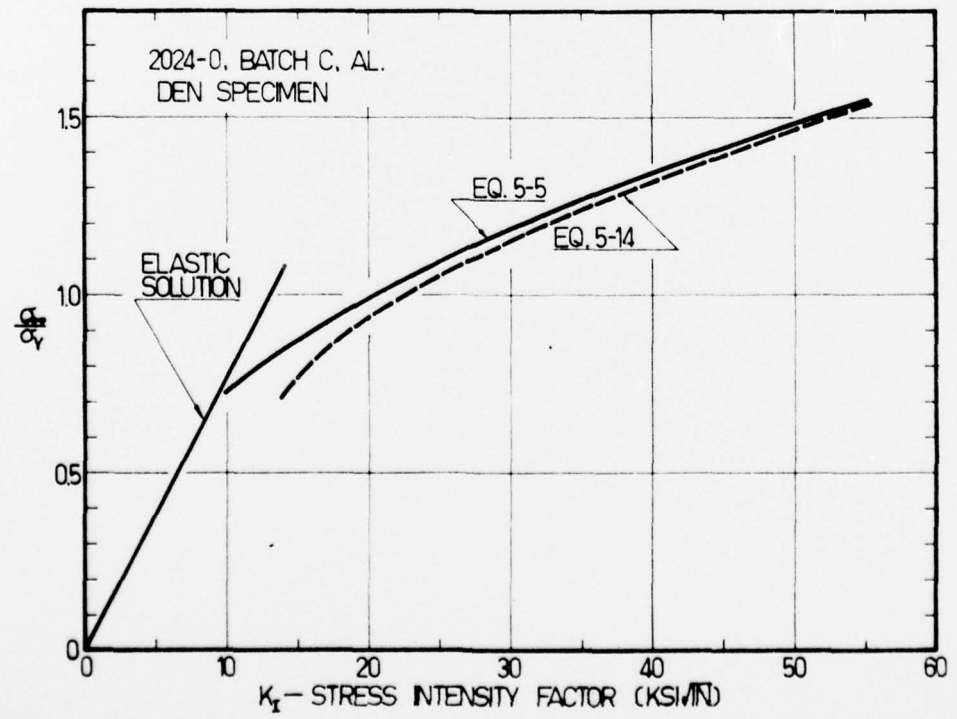


Fig. 12. Calculated stress intensity factor at various loading levels.

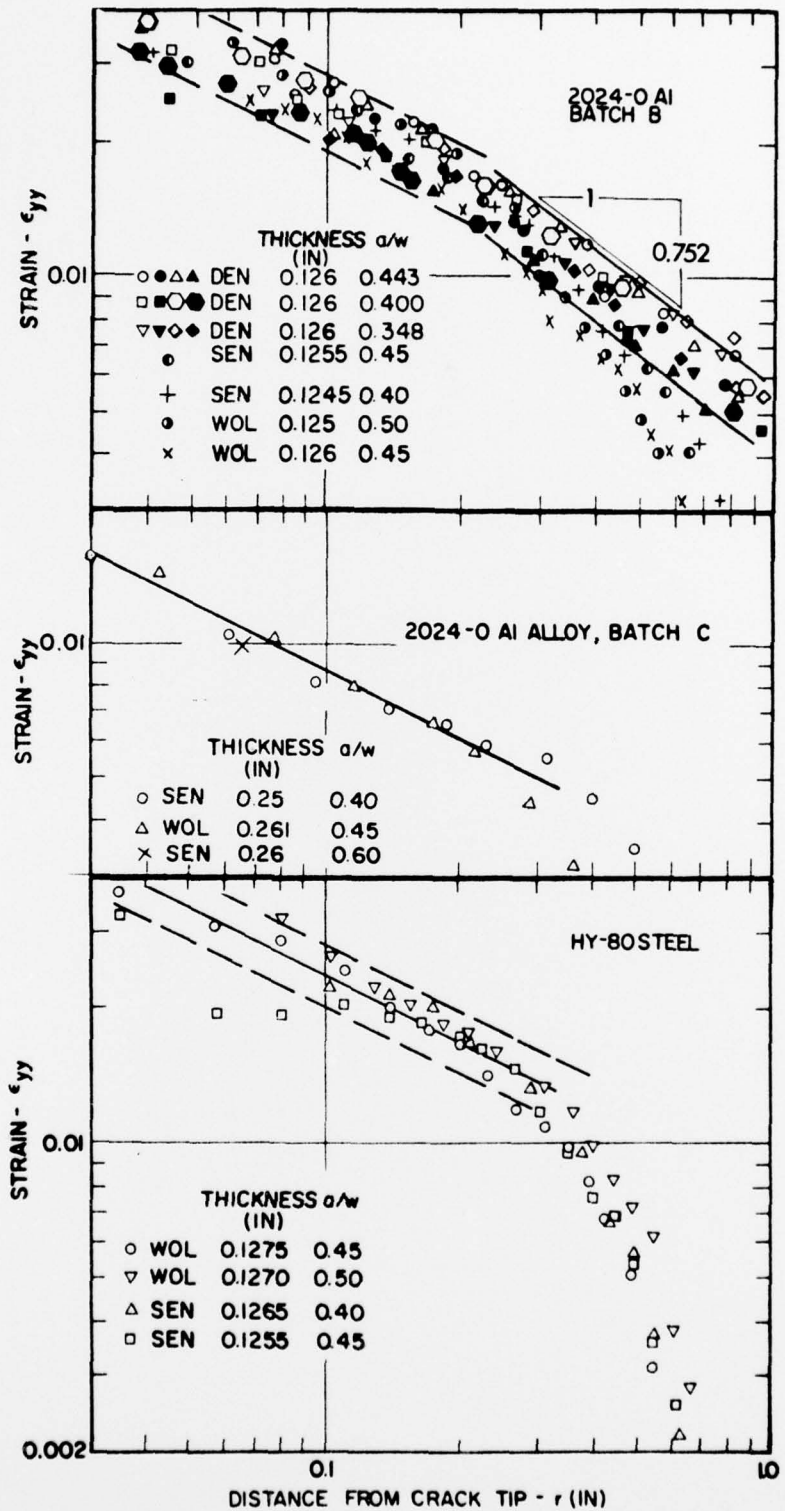


Fig. 13. Near tip strain measurements at the onset of slow crack growth on specimen surface.

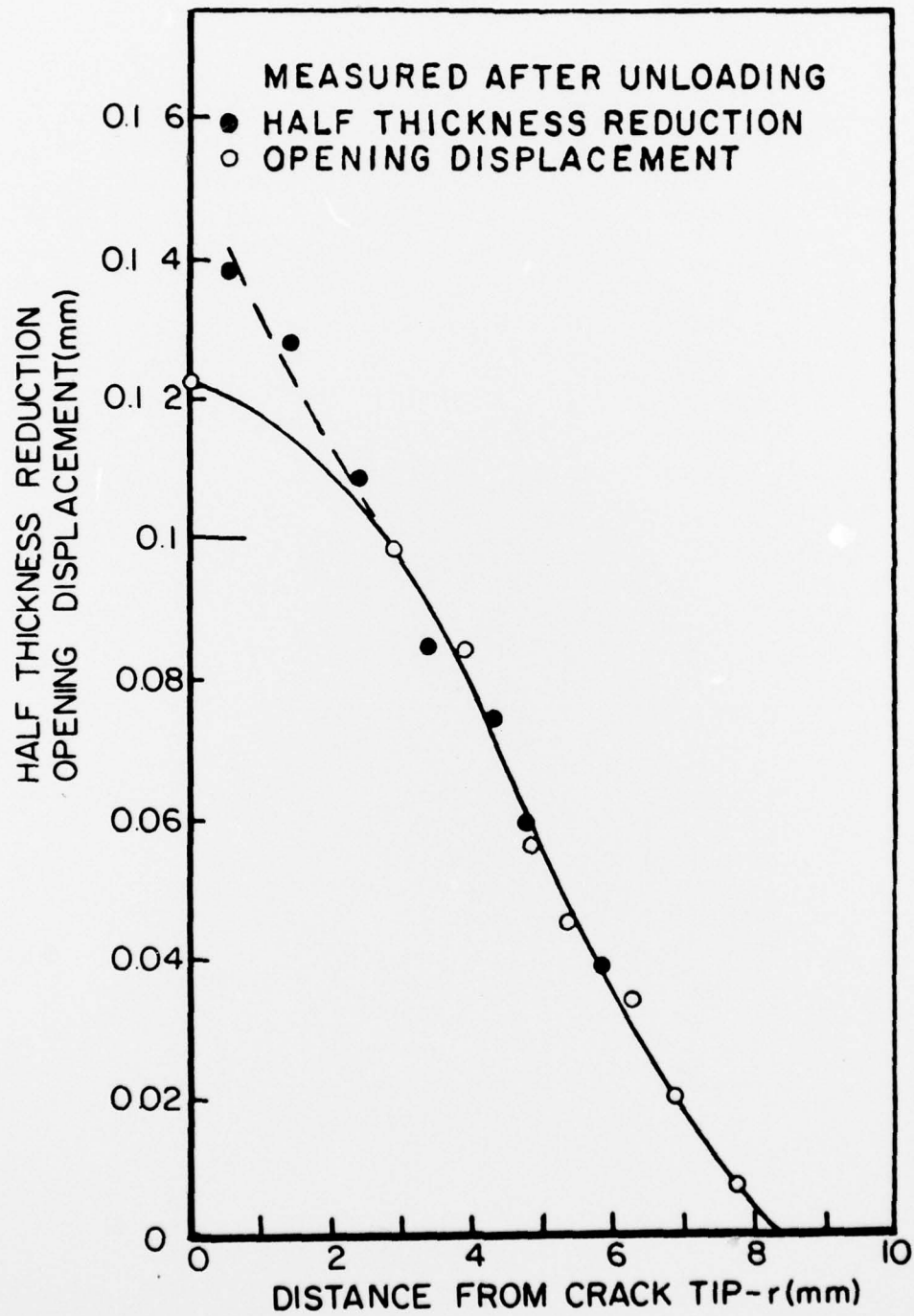


FIG. 14. HALF-THICKNESS REDUCTION AND OPENING DISPLACEMENT IN THE STRIP NECKING REGION OF A STEEL SPECIMEN:  $\sigma$ , 380 MN/m<sup>2</sup>;  $\sigma_y$ , 627 MN/m<sup>2</sup>.

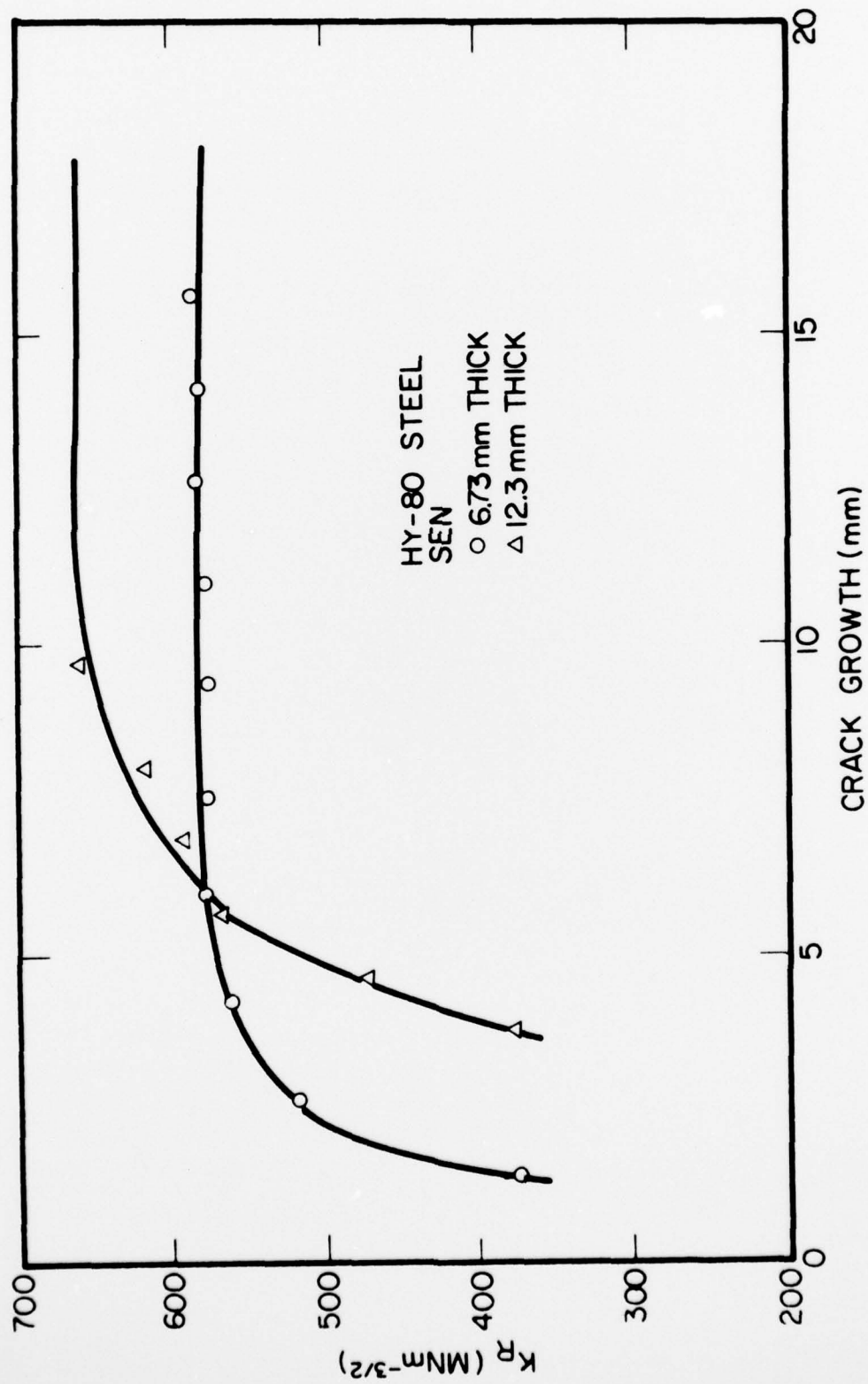


FIG. 15. R-CURVE FOR HY-80 STEEL SPECIMENS.

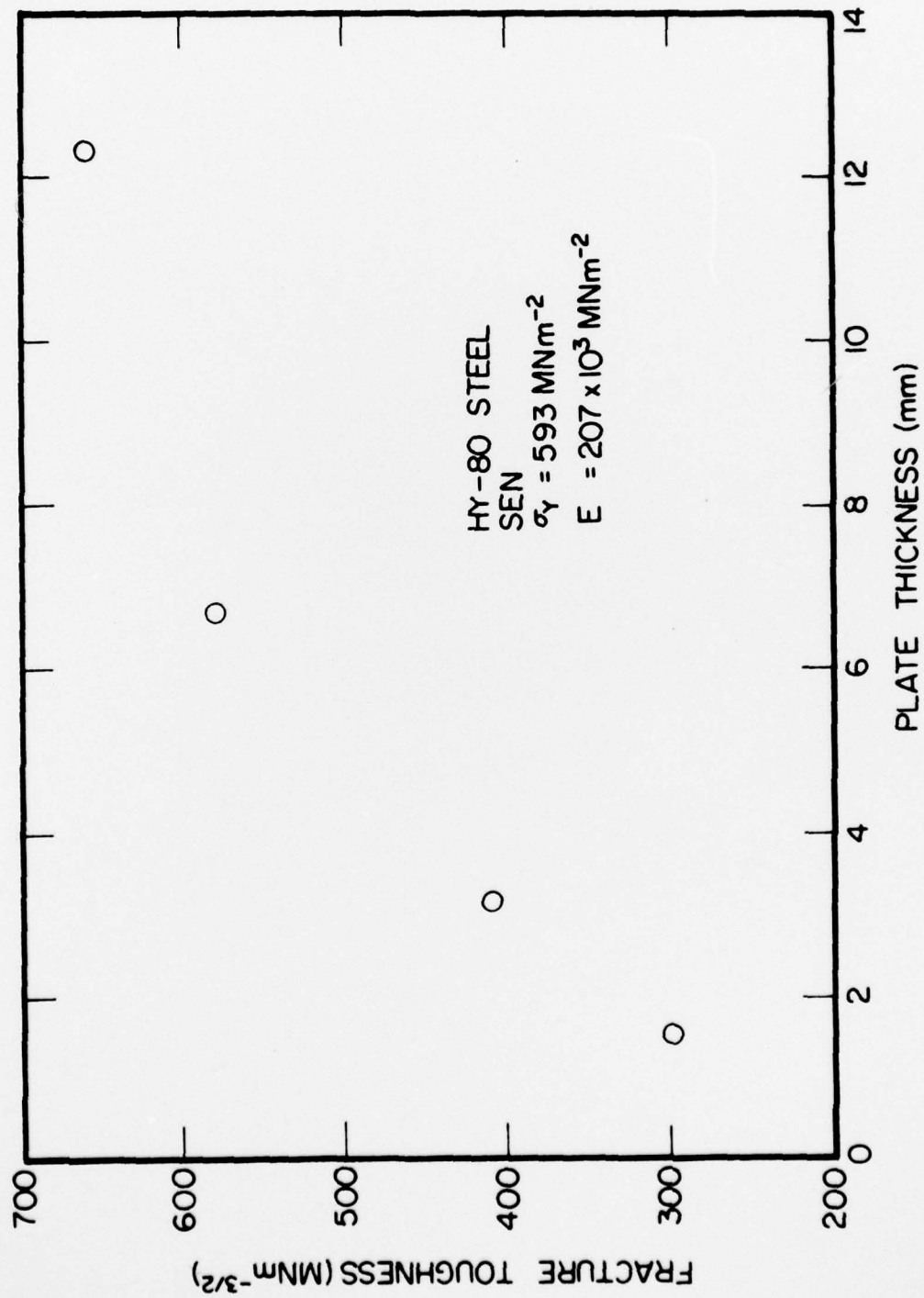


FIG. 16. FRACTURE TOUGHNESS AS A FUNCTION OF PLATE THICKNESS.