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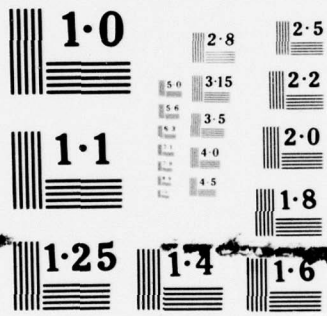
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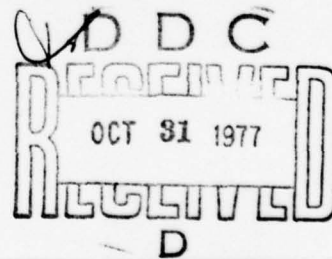
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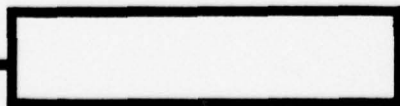
MOVEMENT OF A WING WITH A SMALL ASPECT RATIO NEAR THE INTERFACE OF FLUIDS WITH DIFFERENT DENSITIES

By

S. I. Putilin



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EDITED TRANSLATION

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MOVEMENT OF A WING WITH A SMALL ASPECT RATIO
NEAR THE INTERFACE OF FLUIDS WITH DIFFERENT
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PREPARED BY:

**TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-afb, OHIO.**

U. S. BOARD ON GEOGRAPHIC NAMES transliteration system

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
 When written as ё in Russian, transliterate as yë or ë.
 The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α	α	Nu	Ν ν	
Beta	Β β	β	Xi	Ξ ξ	
Gamma	Γ γ	γ	Omicron	Ο ο	
Delta	Δ δ	δ	Pi	Π π	
Epsilon	Ε ε	ε	Rho	Ρ ρ	ρ
Zeta	Ζ ζ	ζ	Sigma	Σ σ	ς
Eta	Η η	η	Tau	Τ τ	
Theta	Θ θ	θ	Upsilon	Υ υ	
Iota	Ι ι	ι	Phi	Φ φ	φ
Kappa	Κ κ	κ	Chi	Χ χ	
Lambda	Λ λ	λ	Psi	Ψ ψ	
Mu	Μ μ	μ	Omega	Ω ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	sin ⁻¹
arc cos	cos ⁻¹
arc tg	tan ⁻¹
arc ctg	cot ⁻¹
arc sec	sec ⁻¹
arc cosec	csc ⁻¹
arc sh	sinh ⁻¹
arc ch	cosh ⁻¹
arc th	tanh ⁻¹
arc cth	coth ⁻¹
arc sch	sech ⁻¹
arc csch	csch ⁻¹
—	
rot	curl
lg	log

GRAPHICS DISCLAIMER

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MOVEMENT OF A WING WITH A SMALL ASPECT RATIO NEAR THE INTERFACE OF
FLUIDS WITH DIFFERENT DENSITIES

S. I. Putilin (Kiev)

A. N. Panchenkov [7] studied the problem of the movement of a wing with a small aspect ratio below a free fluid surface. The integral equation for a wing with a small aspect ratio moving above the interface of fluids with different densities was obtained in a similar manner in [8]. But these studies did not determine the distribution of the load over the wing chord. In this report we will solve the problem which makes it possible to find the load distribution over the wing chord when condition $\lambda Fr^2 \rightarrow 0$, is satisfied, the Froude number being calculated for the wing span.

1. The velocity potential of an airfoil moving above a fluid interface should satisfy the following conditions on the interface (the subscript 1 refers to the upper and 2 - to the lower fluid):

$$\rho_1(\Phi_{1xx} - \mu\Phi_{1x} + v\Phi_{1z}) - \rho_2(\Phi_{2xx} - \mu\Phi_{2x} + v\Phi_{2z}) = 0;$$

$$\Phi_{1z} = \Phi_{2z},$$

where $v = \frac{g}{\omega^2}$; v is the wing speed. During inverse motion the flow is directed toward negative x .

Using concept [5], by the acceleration potential method

$$\frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x-\xi)\sqrt{\lambda^2+k^2}+i\sigma} d\lambda dk,$$

where $\sigma = (x-\xi)\lambda + (y-\eta)k$, we obtain the following expression for Φ_1 :

$$\Phi_1 = -\frac{1}{4\pi} \int_s \Gamma(\xi, \eta) \left\{ \int_{-\infty}^{\infty} \frac{\partial}{\partial \zeta} \left[\frac{1}{r} + \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{N(\lambda, k)}{Q(\lambda, k)} e^{i\lambda\tau} d\lambda dk \right] d\tau \right\} ds, \quad (1)$$

where

$$N = \frac{a}{2\sqrt{\lambda^2+k^2}} [v\sqrt{\lambda^2+k^2} - \lambda^2] e^{-\sqrt{\lambda^2+k^2}(z+\zeta) + i\lambda(y-\eta)}; \quad (2)$$

$$Q = av\sqrt{\lambda^2+k^2} - \lambda^2 + i\mu\lambda; \quad (3)$$

$$a = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}.$$

The roots of equation $Q = 0$ lie in the lower half plane, and they must pass above it during integration in the final result. By transferring the integration contour to the required half plane, we obtain the asymptotic concept of the velocity potential at large negative values of $x-t$:

$$\Phi_{-\infty} = -\frac{1}{4\pi} \int_S \int \Gamma(\xi, \eta) \left\{ \int_{-\infty}^{\infty} \frac{\partial}{\partial \xi} \left(\frac{1}{r} \right) d\tau + 2 \int_0^{\infty} e^{-h(x+\xi)} \cos k(y-\eta) dk - \right. \\ \left. - \frac{2(1-a)}{\bar{v}} \int_{-\infty}^{\infty} \frac{\lambda_0^2}{\sqrt{\bar{v}^2 + 4k^2}} e^{-(x+\xi) + ih(y-\eta)} \cos \lambda_0(x-\xi) dk \right\} ds,$$

where

$$\lambda_0^2 = \frac{\bar{v}^2}{z} + \frac{\bar{v}^2}{2} \sqrt{\bar{v}^2 + 4k^2}; \\ \bar{v} = av.$$

Introducing the dimensionless values

$$\bar{\omega} = \bar{v}b; \quad h = \frac{\xi}{2b}; \quad \gamma = \frac{\Gamma}{v_0}; \quad \varphi = \frac{\Phi}{v_0 b} \quad (4)$$

and relating x and ξ to the half chord and y, η, z, ζ to the half span, after simple transformations we will obtain the integral equation

$$\begin{aligned} \Phi_x = & -\frac{1}{2\pi\lambda} \int_{-1}^{+1} \int_{-1}^{+1} \gamma_1(\xi) \gamma_2(\eta) \left\{ \frac{1}{y-\eta} - \int_0^{\infty} e^{-k\lambda} \sin k(y-\eta) dk + \right. \\ & + \bar{\omega}(1-a) \int_0^{\infty} \frac{t+1}{t} e^{-k\bar{\omega}(t+1)} \sin[\bar{\omega}\sqrt{t(t+1)}(y-\eta)] \times \\ & \left. \times \sin\left[\frac{\bar{\omega}}{\lambda}(x-\xi)\sqrt{t+1}\right] dt \right\} d\xi d\eta, \end{aligned} \quad (5)$$

where it is assumed that

$$\gamma(\xi, \eta) = \gamma_1(\xi) \gamma_2(\eta) \quad \text{and} \quad \int_{-1}^{+1} \gamma_1(\xi) d\xi = 2. \quad (6)$$

The solution is

$$\gamma_1(\xi) = \delta(\xi - 1).$$

for a flat plate with a small aspect ratio in an unbounded fluid. We can also anticipate that the majority of the load will be concentrated on the leading edge near the interface of fluids with different densities and we can assume that

$$\int_x^1 \gamma_1(\xi) \cos[\bar{\omega}(x - \xi)] d\xi = 2.$$

After this simplification, equation (5) assumes the form of the equation for a wing with a finite span and the optimum load distribution solved by A. N. Panchenkov and P. I. Zinchuk [6].

Using the results obtained in this study, we have

$$\gamma_2(y) = \lambda \sqrt{1 - y^2} (A_1 + A_2 y^2 + A_3 y^4 + \dots),$$

where A_i depend on the load and are expressed by special functions $G_{sp}(\frac{\bar{\omega}}{\lambda})$. The expressions which determine the force are given in [8].

2. A general expression for the velocity potential of an airfoil moving above the interface of fluids with different densities was obtained in [6]. This expression can be rewritten as follows:

$$\begin{aligned} \varphi = & -\frac{1}{4\pi\lambda} \int_{-1}^{+1} \int_{-1}^{+1} \gamma(\xi, \eta) \left\{ \frac{z-\xi}{(y-\eta)^2+(z-\xi)^2} \times \right. \\ & \times \left[\frac{(x-\xi)\lambda^{-1}}{\sqrt{(x-\xi)^2\lambda^{-2}+(y-\eta)^2+(z-\xi)^2}} - 1 \right] - \frac{z+\xi}{(y-\eta)^2+(z+\xi)^2} \times \\ & \times \left[\frac{(x-\xi)\lambda^{-1}}{\sqrt{(x-\xi)^2\lambda^{-2}+(y-\eta)^2+(z+\xi)^2}} - 1 \right] - \frac{1-a}{2\pi i} \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{L_1}^{\frac{\pi}{2}} \frac{k \cos \theta}{k \cos^2 \theta - \bar{\omega}} \times \right. \\ & \times e^{-k(z+\xi+i\sigma)} dk d\theta - \left. \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{L_1}^{\frac{\pi}{2}} \frac{k \cos \theta}{k \cos^2 \theta - \bar{\omega}} e^{-k(z+\xi+i\sigma)} dk d\theta \right] - \\ & \left. - 2(1-a)\bar{\omega} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-\frac{\bar{\omega}(z+\xi)}{\cos^2 \theta}} \cos \left[\frac{\bar{\omega}}{\cos \theta} \frac{x-\xi}{\lambda} + \frac{\bar{\omega} \sin \theta}{\cos^2 \theta} (y-\eta) \right] \frac{d\theta}{\cos^3 \theta} \right\} d\eta d\xi. \end{aligned} \quad (7)$$

where

$$\sigma = \frac{x - \xi}{\lambda} \cos \theta + (y - \eta) \sin \theta. \quad (8)$$

The dimensionless values found with formulae (4) are introduced here. Contour L_2 passes below particular point $k = \frac{\bar{\omega}}{\cos^2 \theta}$ and contour L_1 - above it.

We will study the behavior of derivative φ_z at $\lambda \rightarrow 0$, $x < \xi$. The first two terms were studied in [2]. They can be estimated for binary integrals by successive integration by parts [9], which results in this series:

$$-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{L_1} f(k) e^{-k(z+\xi \pm i\sigma)} dk d\theta \approx -\sum_{n=0}^{\infty} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f^{(n)}(0) (z + \xi \pm i\sigma)^{-1-n} d\theta.$$

Here the plus sign refers to contour L_1 and the minus sign - to contour L_2 ;

$$f(k) = \frac{k^2 \cos \theta}{k \cos^2 \theta - \bar{\omega}}.$$

The first non-zero term is obtained at $n = 2$. It is on the order of

λ^3 :

$$A_1 = \frac{2}{\omega} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{\left\{ z + \zeta \pm i \left[\frac{x - \xi}{\lambda} \cos \theta + (y - \eta) \sin \theta \right] \right\}^3}$$

The last term can be estimated by the stationary phase method [3], [9]. For the integral

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\frac{i\omega(z+\zeta)}{\cos^2 \theta}} \cos \left[\frac{\omega(x-\xi)}{\lambda} H(\theta) \right] \frac{d\theta}{\cos^2 \theta}$$

the points where $H'(\theta) = 0$ are determining. In our case

$$H(\theta) = \frac{1}{\cos \theta} + \frac{\lambda(y-\eta) \sin \theta}{x-\xi \cos^2 \theta}$$

and the stationary points are determined by the formula

$$\sin^2 \theta = \frac{1 - 2d^2 \pm \sqrt{1 - 8d^2}}{2(1 - d^2)},$$

where

$$d = \lambda \frac{y-\eta}{x-\xi}.$$

From the four values of $\sin\theta$ it is necessary to select those whose sign is the opposite of that of d . If we consider λ to be small and disregard the terms on the order of λ^3 or higher, we will have

$$\sin^2\theta_1 = 1 - 4d^2; \quad \sin\theta_2 = -d.$$

The term of the asymptotic expansion corresponding to root θ_1 will contain the factor $\exp\left[-\frac{\bar{\omega}(z+\zeta)}{4d^2}\right]$

and can be omitted. Root θ_2 corresponds to the term

$$I \approx e^{-\bar{\omega}(z+\zeta)} \sqrt{\frac{2\pi\lambda}{\bar{\omega}|x-\xi|}} \cos\left[\frac{\bar{\omega}|x-\xi|}{\lambda} + \frac{\pi}{4}\right]. \quad (9)$$

After eliminating the terms whose order of magnitude is smaller than that of λ from ϕ_2 , we will have the equation

$$\begin{aligned} \lambda\alpha(x, y) = & -\frac{1}{2\pi} \int_{-1}^{+1} \int_x^1 \gamma_0(\xi, \eta) \left[\frac{1}{y-\eta} - \frac{y-\eta}{(y-\eta)^2 + 16h^2} \right] d\xi d\eta + \\ & + \frac{(1-a)\bar{\omega}^2}{2\pi} \int_{-1}^{+1} \int_x^1 \gamma(\xi, \eta) e^{-\eta} \sqrt{\frac{2\pi\lambda}{\bar{\omega}|x-\xi|}} \cos\left[\frac{\bar{\omega}|x-\xi|}{\lambda} + \frac{\pi}{4}\right] d\xi d\eta. \end{aligned} \quad (10)$$

Integration by ξ extends over segment $x < \xi < 1$, since $\varphi_x = 0$ at $x > \xi$ with this degree of accuracy.

We will limit ourselves to the case when the angle of attack does not change over the span. We will find function $\gamma(\xi, \eta)$ in the form of the product

$$\gamma(\xi, \eta) = \gamma_1(\xi) \gamma_2(\eta).$$

Only one of the three terms in equation (10) depends on y , which results in the equation

$$\int_{-1}^{+1} \gamma_2'(\eta) \left[\frac{1}{y-\eta} - \frac{y-\eta}{(y-\eta)^2 + 16h^2} \right] d\eta = \text{const.} \quad (11)$$

Expression (11) is in the form of the equation for a wing with a finite span and the optimum load distribution moving above a screen. The solution to this equation was obtained in [6] in the form of the series

$$\gamma_2(y) = \sqrt{1-y^2}(A_1 + A_2 y^2 + A_3 y^4 + \dots),$$

where the function of A_i is expressed by the parameter

$\tau = \sqrt{4h^2 + 1} - 2h$. Whence, setting the constant in equation (11) equal to 1, we will have

$$\int_{-1}^{+1} \gamma_2(\eta) d\eta = \pi \left(1 + \frac{1}{2} \tau^2 + \frac{1}{4} \tau^4 + \frac{7}{16} \tau^6 + \dots \right) = \pi \psi(h).$$

With consideration of this, equation (10) can be rewritten as

$$\begin{aligned} \lambda a(x) = & \int_x^1 \gamma_2(\xi) d\xi + \frac{1}{2} (1-a) \bar{\omega}^2 \psi(h) e^{-4h\bar{\omega}} \int_x^1 \gamma_2(\xi) \times \\ & \times \sqrt{\frac{2\pi\lambda}{\bar{\omega}(\xi-x)}} \cos \left[\frac{\bar{\omega}(\xi-x)}{\lambda} + \frac{\pi}{4} \right] d\xi. \end{aligned}$$

The substitution of variables $\xi = \lambda(1-\tau)$, $x = \lambda(1-t)$ reduces this equation to the form

$$\int_0^t \gamma_1(\tau) d\tau - \mu \int_0^t \gamma_1(\tau) K(t-\tau) d\tau = \alpha(t), \quad (12)$$

where

$$\frac{\gamma^*(\rho)}{\rho} - \frac{\mu}{\rho} K^*(\rho) \gamma^*(\rho) = \alpha^*(\rho). \quad (13)$$

This equation can be solved by the Laplace-Carson transformation [4]. Indicating the expression for the function by asterisks, we will have

$$\gamma^*(\rho) = \frac{\rho \alpha^*(\rho)}{1 - \mu K^*(\rho)}.$$

whence

$$p\alpha^*(p) \rightarrow \alpha'(t) + \alpha(0)\delta(t).$$

Introducing function $\Phi(t) = \int_0^t \gamma(\tau) d\tau$ and considering that

$$\mu = \frac{1}{2}(1-a)\bar{\omega}^2 \psi(h) e^{-4h\bar{\omega}};$$

$$K(t) = -\sqrt{\frac{2\pi}{\bar{\omega}t}} \cos\left(\bar{\omega}t + \frac{\pi}{4}\right).$$

we will have

$$\Phi^*(t) = \alpha(0)v(t) + \int_0^t \alpha'(\tau)v(t-\tau) d\tau, \quad (14)$$

where function $v(t)$ is determined by its representation:

$$v^*(p) = \frac{1}{1-\mu K^*(p)}. \quad (15)$$

The overall wing characteristics can be expressed directly by $\Phi(t)$; thus, the problem will be solved by finding the resolvent of $v(t)$.

3. Similarly, we can study a wing moving below the interface of two fluids. Using the results of study [4], we will obtain the following expression for the velocity potential, analogous to expression (7):

$$\begin{aligned}
 \varphi = & -\frac{1}{4\pi\lambda} \int_{-1}^{+1} \int_{-1}^{+1} \gamma(\xi, \eta) \left\{ \frac{z-\xi}{(y-\eta)^2 + (z-\xi)^2} \times \right. \\
 & \times \left[\frac{(x-\xi)\lambda^{-1}}{\sqrt{(x-\xi)^2\lambda^{-2} + (y-\eta)^2 + (z-\xi)^2}} - 1 \right] - \frac{z+\xi}{(y-\eta)^2 + (z+\xi)^2} \times \\
 & \times \left[\frac{(x-\xi)\lambda^{-1}}{\sqrt{(x-\xi)^2\lambda^{-2} + (y-\eta)^2 + (z-\xi)^2}} - 1 \right] - \\
 & - \frac{(1+a)}{2\pi i} \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \frac{k \cos \theta}{k \cos^2 \theta - \omega} e^{h(z+\xi+i\sigma)} dk d\theta - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \frac{k \cos \theta}{k \cos^2 \theta - \omega} \times \right. \\
 & \times e^{h(z-\xi-i\sigma)} dk d\theta \left. \right] - 2(1+a)\omega^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\frac{\omega(z+\xi)}{\cos^2 \theta}} \cos \left[\frac{\omega(x-\xi)}{\lambda \cos \theta} H(\theta) \right] \times \\
 & \times \frac{d\theta}{\cos^3 \theta} \left. \right\} d\xi d\eta. \quad (16)
 \end{aligned}$$

The comparison of formulae (7) and (16) shows that the final expression for this case only differs from equation (12) by the value of μ , which must be replaced by

$$\mu_1 = -\frac{1}{2} (1 + a) \bar{\omega}^2 \psi(h) e^{-4h\bar{\omega}}.$$

4. With accuracy up to the terms on the order of $\lambda^{3/2}$ asymptotic expression (13) of root $K(t)$ can be replaced by the expression

$$K(t) = \pi J_1(t),$$

where J_1 is the Bessel function. Then we will have

$$v^*(p) = \frac{1}{1 - \mu\pi \frac{p(\sqrt{p^2 + \bar{\omega}^2} - p)}{\bar{\omega}\sqrt{p^2 + \bar{\omega}^2}}}.$$

for the expression of the resolvent. Reducing this expression to the form

$$v^*(p) = \frac{P_2}{P_1} + \frac{p}{\sqrt{p^2 + \bar{\omega}^2}} \cdot \frac{P_3}{P_1},$$

where

$$P_1 = 2\pi\mu\rho^2 - \rho^2\bar{\omega}(1 + \pi^2\mu^2) + 2\pi\mu\bar{\omega}^2\rho - \bar{\omega}^2;$$

$$P_2 = \mu\pi\rho^2 - \rho^2\bar{\omega} + \pi\mu\bar{\omega}^2\rho - \bar{\omega}^2;$$

$$P_3 = \pi\mu\rho(\rho^2 + \bar{\omega}^2),$$

we will have

$$v(t) = \sum_{n=1}^3 \left\{ \frac{P_2(p_n)}{P_1'(p_n)} e^{p_n t} + \frac{P_3(p_n)}{P_1'(p_n)} \left[J_0(\bar{\omega}t) + p_n \int_0^t J_0(\bar{\omega}\tau) e^{p_n(t-\tau)} d\tau \right] \right\}.$$

where p_n are the roots of equation $P_1(p) = 0$.

For an unbounded fluid, $\mu = 0$ and $v(t) = 1$, and we will have

$$\Phi(t) = \alpha(t).$$

If the function $\alpha(t)$ is continuous, $\Phi(t)$ is also continuous and limited at $t > 0$, in particular, for the trailing edge of the wing. This provides for the satisfaction of the Zhukovskiy-Chaplygin condition [9]. It is easy to see that the equation for lack of passage is not disturbed if we place a vortex of finite intensity Γ .

on the trailing edge. If we set $\Gamma = -\Phi\left(\frac{2}{\lambda}\right)\gamma_2(\eta)$,
noncirculating flow.

we will have

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