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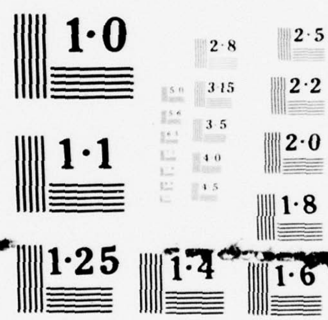
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NUSC Technical Report 5647

NUSC Technical Report 5647

Modified Image Theory Quasi-Static Range Subsurface-to-Subsurface and Subsurface-to-Air Propagation Equations

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12 October 1977

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PREFACE

This report was prepared under NUSC Project No. A-532-24, Principal Investigator - P. R. Bannister (Code 341), and Navy Subproject and Task No. B-0005, 11221; Naval Industrial Funding from David W. Taylor Naval Ship Research and Development Center, Project Director - W. J. Andahazy; Project Program Manager - W. L. Welsh (NAVSEA, SEA-03424).

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Simple engineering expressions for the quasi-static fields produced by various subsurface sources have been derived by employing finitely conducting earth image theory techniques. Both subsurface-to-subsurface and subsurface-to-air propagation cases are considered. It is shown that the resulting subsurface-to-air propagation field strength expressions are the result of simple modifications to one of the author's previously derived quasi-static range air-to-air propagation equations.

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MODIFIED IMAGE THEORY QUASI-STATIC RANGE SUBSURFACE-TO-SUBSURFACE AND SUBSURFACE-TO-AIR PROPAGATION EQUATIONS

INTRODUCTION

During the past few years, there has been considerable interest in the determination of the quasi-static field components of antennas located above or buried within the earth's surface, where the quasi-static range is defined as the range where the measurement distance is much less than a free-space wavelength. Quasi-static range results are useful for submarine radio communication and detection purposes, as well as for the buried-miner problem. They are also helpful to geophysicists engaged in determining the electrical properties of the earth.

The fields from various subsurface sources previously have been derived for many cases[1-5] when the measurement distance R $(= \sqrt{\rho^2 + Z^2})$ is much larger than the earth-skin depth, δ (i.e., the quasi-near range case), and when $R \ll \delta$ and $|z + h| \ll \delta$ (i.e., the dc case).

Some work has been done[3,6-10] on determining the fields from various subsurface sources for the quasi-static range where R is comparable to δ. However, the resulting field-strength expressions are very complicated since they involve products of modified Bessel functions of different argument. Computing the field-component expressions is a lengthy and difficult process, but some numerical results have been obtained. One method of obtaining these results has been discussed by Atzinger, Pensa, and Pigott.[11] Numerical integration techniques also have been employed.[10]

In order to provide simple engineering expressions for the general quasi-static electromagnetic fields produced by subsurface antennas, finitely conducting earth-image theory techniques will be employed. These techniques have already proved quite useful in determining the quasi-static fields of antennas located near the earth's surface — for both singlelayered and multilayered earths.[12-16]

Physically, the essence of the finitely conducting earth-image theory technique is to replace the finitely conducting earth by a perfectly conducting earth located at the (complex) depth d/2, where $d = 2/\gamma = \delta\,(1 - i)$. Analytically, this corresponds to replacing the algebraic reflection coefficient, $(u - \lambda)/(u + \lambda)$, in the exact integral equations by $\exp(-\lambda d)$, where λ is the variable of integration.

For antennas located at or above the earth's surface, the general image-theory approximation is valid throughout the quasi-static range.[12,16] However, if one or both antennas are buried, the previously derived[12] surface-to-surface and surface-to-air image-theory results (multiplied by the exponential attenuation with depth factor $\exp[\gamma\,(z + h)]$ or $\exp(\gamma h)$) generally will be valid only for $R > 3\,|z + h|$, where h and z, respectively, are the depth of the source and receiving antennas.

Therefore, we will modify further the exact subsurface-to-subsurface and subsurface-to-air integral equations and obtain fairly simple field-strength equations for the general quasi-static range — which reduce to previously derived results when $R \gg \delta$ and when $R \ll \delta$ and $|z + h| \ll \delta$.

For the purposes of this report we will consider six buried sources: the vertical electric dipole (VED); vertical magnetic dipole (VMD); horizontal electric dipole (HED); horizontal magnetic dipole (HMD); finite-length horizontal electric antenna; and long horizontal line source. All these sources are located at depth $h(h \leq 0^-)$ with respect to a cylindrical coordinate system (ρ, ϕ, z) and are assumed to carry a constant current I. The VED and HED (of infinitesimal length, ℓ) are oriented in the z and x directions, respectively, and the axes of the VMD and HMD (of infinitesimal area, A) are oriented in the z and y directions, respectively. The earth occupies the lower half-space $(z < 0)$, and the air occupies the upper half-space $(z > 0)$. Displacement currents are neglected in the ground and (for most cases) in the air. The magnetic permeability of the earth is assumed to equal μ_0, the permeability of free space. Meter-kilogram-second (MKS) units are employed and a suppressed-time factor of $\exp(i\omega t)$ is assumed.

SUBSURFACE-TO-AIR PROPAGATION

VERTICAL MAGNETIC DIPOLE (VMD)

When $h \leq 0$ and $z \geq 0$, the quasi-static range integral expression for the VMD Hertz vector is

$$\Pi_z \sim \frac{IA}{4\pi}\int_0^\infty \frac{2\lambda}{u + \lambda}\, e^{-\lambda z + uh} J_0(\lambda\rho)\, d\lambda \ . \tag{1}$$

The fields in the air are given by

$$E_\phi = i\omega\mu_0 \frac{\partial \Pi_z}{\partial \rho} \ ; \ H_\rho = \frac{\partial^2 \Pi_z}{\partial\rho\,\partial z} \ ; \ H_z = \left(-\gamma_0^2 + \frac{\partial^2}{\partial z^2}\right)\Pi_z \quad , \tag{2}$$

where

$\gamma_0 = i\omega(\mu_0\varepsilon_0)^{\frac{1}{2}} \sim 0$ (air),

$\gamma \sim (i\omega\mu_0\sigma)^{\frac{1}{2}}$ (earth),

$u = (\lambda^2 + \gamma^2)^{\frac{1}{2}}$, and

$J_0(\lambda\rho)$ is the Bessel function of the first kind, order zero, and argument $\lambda\rho$.

Utilizing the Wait and Spies image theory approximation,[14]

$$\frac{u - \lambda}{u + \lambda} \sim e^{-\lambda d} \quad , \tag{3}$$

where $d = \delta(1 - i)$, yields

$$\frac{2\lambda}{u + \lambda} \sim 1 - e^{-\lambda d} \quad . \tag{4}$$

Thus, equation (1) becomes

$$\Pi_z \sim \frac{IA}{4\pi} \int_0^\infty (1 - e^{-\lambda d}) e^{-\lambda z + uh} J_0(\lambda\rho)\, d\lambda \quad . \tag{5}$$

This expression for Π_z cannot be evaluated analytically (when $h \neq 0$) throughout the quasi-static range. However, if exp(uh) can be replaced by exp[f(λh)] times exp[f(γh)], this integral can be evaluated readily.

Now we know that when $R \gg \delta$ and $R \gg |h|$, $\exp(uh) \sim \exp(\gamma h)$. Furthermore, if $R \ll \delta$ and $|h| \ll \delta$, $\exp(uh) \sim \exp(\lambda h)$. An approximation that is good to within 4 percent if $A > B$ is

$$\sqrt{A^2 + B^2} \sim 0.96A + 0.4B \quad . \tag{6}$$

Therefore, we will approximate exp(uh) by

$$e^{uh} \sim e^{\gamma ah} e^{\lambda bh} \quad , \tag{7}$$

with

$a = 0$ and $b = 1$ for $R \ll \delta$ and $|h| \ll \delta$,

a = 0.4 and b = 0.96 for R/δ less than approximately 1,

a = 0.96 and b = 0.4 for R/δ between approximately 1 and 10, and

a = 1.0 and b = 0 for R > |3h|.

Thus, equation (5) reduces to

$$\Pi_z \sim \frac{IAe^{\gamma ah}}{4\pi}\int_0^\infty (1 - e^{-\lambda d})e^{-\lambda(z-bh)} J_0(\lambda\rho)\,d\lambda \quad . \tag{8}$$

Since[17]

$$\int_0^\infty e^{-\lambda\beta} J_0(\lambda\rho)\,d\lambda = \frac{1}{\sqrt{\rho^2 + \beta^2}} \quad , \tag{9}$$

then

$$\Pi_z \sim \frac{IAe^{\gamma ah}}{4\pi}\left[\frac{1}{K_1} - \frac{1}{K_2}\right] \quad , \tag{10}$$

where

$$K_1^2 = \rho^2 + (z - bh)^2 \text{ and } K_2^2 = \rho^2 + (d + z - bh)^2 \quad .$$

But this expression is identical to the previously derived quasi-static range air-to-air propagation VMD Hertz potential given in a previous report,[12] equation (36), except that

1. z - h is replaced by z - bh,

2. z + h is replaced by z - bh, and

3. The equation is multiplied by $e^{\gamma ah}$.

Therefore, the subsurface-to-surface and subsurface-to-air field-component expressions produced by a buried VMD (or any other buried source) may be obtained from those given in the previous report[12] simply by

1. Replacing (z - h) by (z - bh),

2. Replacing (z + h) by (z - bh), and

3. Multiplying each equation by $e^{\gamma ah}$ (note that $h \leq 0^-$).

Thus, for the VMD subsurface-to-air propagation case (and the subsurface-to-surface case — i.e., when $z = 0^+$), equations (37) to (39) of the previous report[12] become

$$E_\phi \sim -\frac{IAi\omega\mu_0}{4\pi}(\rho)e^{\gamma ah}\left[\frac{1}{K_1^3} - \frac{1}{K_2^3}\right] , \tag{11}$$

$$H_\rho \sim \frac{IA}{4\pi} e^{\gamma ah}(3\rho)\left[\frac{(z - bh)}{K_1^5} - \frac{(d + z - bh)}{K_2^5}\right] , \tag{12}$$

and

$$H_z \sim -\frac{IAe^{\gamma ah}}{4\pi}\left\{\frac{1}{K_1^3}\left[1 - \frac{3(z - bh)^2}{K_1^2}\right] - \frac{1}{K_2^3}\left[1 - \frac{3(d + z - bh)^2}{K_2^2}\right]\right\} . \tag{13}$$

VERTICAL ELECTRIC DIPOLE (VED)

For the VED subsurface-to-air propagation case, the equations of the previous report[12] (equations (29) to (31)) must also be multiplied by $\gamma_0{}^2/\gamma^2$. Thus,

$$\Pi_z \sim \frac{I\ell e^{\gamma ah}}{2\pi\sigma}\left(\frac{1}{K_1} + 0(\gamma_0^2)\right) , \tag{14}$$

$$E_\rho \sim \frac{I\ell\rho e^{\gamma ah}}{4\pi\sigma}\left\{\frac{6(z - bh)}{K_1^5} - \frac{\gamma_0^2}{\rho^2}\left[\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right]\right\} , \tag{15}$$

$$E_z \sim -\frac{I\ell e^{\gamma ah}}{2\pi\sigma}\frac{1}{K_1^3}\left[1 - \frac{3(z - bh)^2}{K_1^2}\right] , \tag{16}$$

and

$$H_\phi \sim \frac{I\ell e^{\gamma ah}}{2\pi}\left(\frac{\gamma_0^2}{\gamma_1^2}\right)\frac{\rho}{K_1^3} . \tag{17}$$

HORIZONTAL ELECTRIC DIPOLE (HED)

The HED equations can be derived from equations (4), (7), (8), and (10) to (15) of the previous report.[12] They are

$$\Pi_x \sim \frac{I\ell e^{\gamma ah}}{4\pi i\omega\varepsilon_0}\left(\frac{1}{K_1} - \frac{1}{K_2}\right) , \tag{18}$$

$$\Pi_z \sim - \frac{I\ell \cos \phi e^{\gamma ah}}{4\pi i\omega\varepsilon_0\rho}\left\{\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right\} , \tag{19}$$

$$\vec{\nabla}\cdot\vec{\Pi} \sim - \frac{I\ell \cos \phi e^{\gamma ah}}{2\pi\sigma}\frac{\rho}{K_1^3} , \tag{20}$$

$$E_\rho \sim \frac{I\ell \cos \phi e^{\gamma ah}}{2\pi\sigma K_1^3}\left\{1 + b - \frac{3b(z - bh)^2}{K_1^2} - \gamma a(z - bh)\right\} , \tag{21}$$

$$E_\phi \sim \frac{I\ell \sin \phi e^{\gamma ah}}{2\pi\sigma K_1^3}\left\{1 + \frac{2K_1^2}{d^2}\left(1 - \frac{K_1}{K_2}\right)\right\} , \tag{22}$$

$$E_z \sim \frac{I\ell \cos \phi e^{\gamma ah}}{4\pi\sigma}\left\{\frac{6\rho(z - bh)}{K_1^5} - \frac{4}{\rho d^2}\left[\frac{(z - bh)}{K_1} - \frac{(d + z - bh)}{K_2}\right]\right\} , \tag{23}$$

$$H_\rho \sim \frac{I\ell \sin \phi e^{\gamma ah}}{4\pi}\left\{\frac{(d + z - bh)}{K_2^3} - \frac{(z - bh)}{K_1^3} + \frac{1}{\rho^2}\left[\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right]\right\} , \tag{24}$$

$$H_\phi \sim - \frac{I\ell \cos \phi e^{\gamma ah}}{4\pi\rho^2}\left\{\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right\} , \tag{25}$$

and

$$H_z \sim \frac{I\ell\rho \sin \phi e^{\gamma ah}}{4\pi}\left\{\frac{1}{K_1^3} - \frac{1}{K_2^3}\right\} . \tag{26}$$

HORIZONTAL MAGNETIC DIPOLE (HMD)

The appropriate equations for HMD subsurface-to-air propagation can be obtained from equations (48) to (56) of the previous report,[12] and are

$$\Pi_y \sim \frac{IA}{4\pi} e^{\gamma ah} \frac{2}{K_1} \;, \tag{27}$$

$$\Pi_z \sim -\frac{IA \sin \phi e^{\gamma ah}}{4\pi\rho}\left[\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right] , \tag{28}$$

$$\vec{\nabla}\cdot\vec{\Pi} \sim -\frac{IA(\rho) \sin \phi e^{\gamma ah}}{4\pi}\left[\frac{1}{K_1^3} + \frac{1}{K_2^3}\right] , \tag{29}$$

$$E_\rho \sim -\frac{i\omega\mu_0 IA \cos \phi e^{\gamma ah}}{4\pi}\left\{\frac{2(z - bh)}{K_1^3} - \frac{1}{\rho^2}\left[\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right]\right\} \tag{30}$$

$$E_\phi \sim \frac{i\omega\mu_0 IA \sin \phi e^{\gamma ah}}{4\pi}\left\{\frac{(d + z - bh)}{K_2^3} + \frac{(z - bh)}{K_1^3} + \frac{1}{\rho^2}\left[\frac{(d + z - bh)}{K_2} - \frac{(z - bh)}{K_1}\right]\right\} , \tag{31}$$

$$E_z \sim \frac{i\omega\mu_0 IA \cos \phi e^{\gamma ah}}{4\pi}\left(\frac{2\rho}{K_1^3}\right) , \tag{32}$$

$$H_\rho \sim -\frac{IA \sin \phi e^{\gamma ah}}{4\pi}\left\{\frac{1}{K_1^3}\left[1 - \frac{3\rho^2}{K_1^2}\right] + \frac{1}{K_2^3}\left[1 - \frac{3\rho^2}{K_2^2}\right]\right\} , \tag{33}$$

$$H_\phi \sim -\frac{IA \cos \phi e^{\gamma ah}}{4\pi}\left[\frac{1}{K_1^3} + \frac{1}{K_2^3}\right] , \tag{34}$$

and

$$H_z \sim \frac{IA(\rho) \sin \phi e^{\gamma ah}}{4\pi}\left[\frac{3(z - bh)}{K_1^5} + \frac{3(d + z - bh)}{K_2^5}\right] . \tag{35}$$

If $K_1 << \delta$ and $|ah| << \delta$, the modified-image theory subsurface-to-air (and subsurface-to-surface — i.e., $z = 0^+$) VED, VMD, HED, and HMD equations (equations (10) to (35) of this report) reduce to the dc equations (for example, see page 3-4 of Kraichman[2]). Furthermore, if $K_1 >> \delta$ and $R >> |h|$, the equations reduce to the quasi-near range results given by Bannister[4] (see also tables 3.13 and 3.14 of Kraichman[2]).

VMD, VED, HED, AND HMD AIR-TO-SUBSURFACE PROPAGATION

Now that simplified expressions for VMD, VED, HED, and HMD subsurface-to-air (and subsurface-to-surface) propagation have been derived, simplified expressions for VMD, VED, HED, and HMD air-to-subsurface (and surface-to-subsurface) propagation can be derived easily by utilizing the reciprocity theorem. This theorem (applicable to dipoles in the presence of any linear media) states that voltage V_2 induced in antenna 2 by current I_1 of antenna 1 is the same as voltage V_1 induced in antenna 1 by an identical current I_2 flowing in antenna 2. Further details are given in several references.[18-22]

Application of this theorem results in

$$E_z^{HM}(z,h) = i\omega\mu_0 H_\phi^{VE} \cos\phi \, \frac{A}{\ell} \, (h,z) \quad , \tag{36}$$

where (α,β) = (height or depth of transmitting dipole, height or depth of receiving dipole);

$$H_z^{HM}(z,h) = - H_\rho^{VM} \sin\phi \, \frac{A^{HM}}{A^{VM}} \, (h,z) \quad , \tag{37}$$

$$E_z^{HE}(z,h) = - E_\rho^{VE} \cos\phi \, \frac{\ell^{HE}}{\ell^{VE}} \, (h,z) \quad , \tag{38}$$

$$H_z^{HE}(z,h) = \frac{-1}{i\omega\mu_0} E_\phi^{VM} \sin\phi \, \frac{\ell}{A} \, (h,z) \quad , \tag{39}$$

$$H_\rho^{HE}(z,h) = \frac{1}{i\omega\mu_0} E_\phi^{HM} \frac{\ell}{A} \, (h,z) \quad , \tag{40}$$

$$H_\phi^{HE}(z,h) = \frac{-1}{i\omega\mu_0} E_\rho^{HM} \frac{\ell}{A} \, (h,z) \quad , \tag{41}$$

and

$$E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}(z,h) = E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}(h,z) . \tag{42}$$

BURIED FINITE-LENGTH HORIZONTAL ELECTRIC ANTENNA

The equations for subsurface-to-air and subsurface-to-surface propagation, utilizing a buried finite-length horizontal electric antenna (which is oriented in the x direction) as a source, may be obtained from equations (73) to (81) of the previous report,[12] and are

$$\Pi_x \sim \frac{Ie^{\gamma ah}}{4\pi i\omega\varepsilon_0}\left\{\ell n\left[\frac{K_{11} - (x + L/2)}{K_{12} - (x - L/2)}\right] - \ell n\left[\frac{K_{21} - (x + L/2)}{K_{22} - (x - L/2)}\right]\right\} , \tag{43}$$

$$\Pi_z \sim - \frac{Ie^{\gamma ah}}{4\pi i\omega\varepsilon_0}\left\{\ell n\left[\frac{K_{11} - (d + z - bh)}{K_{12} - (d + z - bh)}\right] - \ell n\left[\frac{K_{21} - (z - bh)}{K_{22} - (z - bh)}\right]\right\} , \tag{44}$$

and

$$\vec{\nabla}\cdot\vec{\Pi} \sim \frac{Ie^{\gamma ah}}{2\pi\sigma}\left\{\frac{1}{K_{21}} - \frac{1}{K_{22}}\right\} , \tag{45}$$

where

$$K_{11}^2 = (x + L/2)^2 + y^2 + (d + z - bh)^2,$$

$$K_{12}^2 = (x - L/2)^2 + y^2 + (d + z - bh)^2,$$

$$K_{21}^2 = (x + L/2)^2 + y^2 + (z - bh)^2, \text{ and}$$

$$K_{22}^2 = (x - L/2)^2 + y^2 + (z - bh)^2.$$

In addition

$$E_x \sim - \frac{i\omega\mu_0 Ie^{\gamma ah}}{4\pi}\left\{\ell n\left[\frac{K_{11} - (x + L/2)}{K_{12} - (x - L/2)}\right] - \ell n\left[\frac{K_{21} - (x + L/2)}{K_{22} - (x - L/2)}\right] + \frac{d^2}{2}\left[\frac{(x + L/2)}{K_{21}^3} - \frac{(x - L/2)}{K_{22}^3}\right]\right\} , \tag{46}$$

$$E_y \sim - \frac{Iye^{\gamma ah}}{2\pi\sigma}\left\{\frac{1}{K_{21}^3} - \frac{1}{K_{22}^3}\right\} , \tag{47}$$

$$E_z \sim \frac{i\omega\mu_0 I}{4\pi}\left\{\ln\left[\frac{K_{11} - (d + z - bh)}{K_{12} - (d + z - bh)}\right] - \ln\left[\frac{K_{21} - (z - bh)}{K_{22} - (z - bh)}\right] - \frac{d^2(z - bh)}{2}\left[\frac{1}{K_{21}^3} - \frac{1}{K_{22}^3}\right]\right\} e^{\gamma ah} , \quad (48)$$

$$H_x \sim \frac{Iye^{\gamma ah}}{4\pi}\left\{\frac{1}{[(x - L/2)^2 + y^2]}\left[\frac{(d + z - bh)}{K_{12}} - \frac{(z - bh)}{K_{22}}\right] - \frac{1}{[(x + L/2)^2 + y^2]}\left[\frac{(d + z - bh)}{K_{11}} - \frac{(z - bh)}{K_{21}}\right]\right\} , \quad (49)$$

$$H_y \sim \frac{Ie^{\gamma ah}}{4\pi}\left\{\frac{(d + z - bh)}{[y^2 + (d + z - bh)^2]}\left[\frac{(x + L/2)}{K_{11}} - \frac{(x - L/2)}{K_{12}}\right] - \frac{(z - bh)}{[y^2 + (z - bh)^2]}\left[\frac{(x + L/2)}{K_{21}} - \frac{(x - L/2)}{K_{22}}\right] + \frac{(x + L/2)}{[y^2 + (x + L/2)^2]}\left[\frac{(d + z - bh)}{K_{11}} - \frac{(z - bh)}{K_{21}}\right] - \frac{(x - L/2)}{[y^2 + (x - L/2)^2]}\left[\frac{(d + z - bh)}{K_{12}} - \frac{(z - bh)}{K_{22}}\right]\right\} , \quad (50)$$

and

$$H_z \sim -\frac{Iye^{\gamma ah}}{4\pi}\left\{\frac{1}{[y^2 + (d + z - bh)^2]}\left[\frac{(x + L/2)}{K_{11}} - \frac{(x - L/2)}{K_{12}}\right] - \frac{1}{[y^2 + (z - bh)^2]}\left[\frac{(x + L/2)}{K_{21}} - \frac{(x - L/2)}{K_{22}}\right]\right\} . \quad (51)$$

When the measurement distance is much greater than the source length L, equations (43) to (51) reduce to the HED equations. However, when the source length L is much greater than the measurement distance and δ, equations (43) to (51) reduce to the following long line source equations:

$$E_y \sim E_z \sim H_x \sim 0 , \quad (52)$$

$$E_x \sim -\frac{i\omega\mu_0 I e^{\gamma ah}}{2\pi}\,\ell n\sqrt{\frac{y^2 + (d + z - bh)^2}{y^2 + (z - bh)^2}} \ , \tag{53}$$

$$H_y \sim \frac{I e^{\gamma ah}}{2\pi}\left\{\frac{(d + z - bh)}{[y^2 + (d + z - bh)^2]} - \frac{(z - bh)}{[y^2 + (z - bh)^2]}\right\} \ , \tag{54}$$

and

$$H_z \sim -\frac{I y e^{\gamma ah}}{2\pi}\left\{\frac{1}{[y^2 + (d + z - bh)^2]} - \frac{1}{[y^2 + (z - bh)^2]}\right\} \ . \tag{55}$$

SUBSURFACE-TO-SUBSURFACE PROPAGATION

HORIZONTAL ELECTRIC DIPOLE (HED)

When $h \leq 0$ and $z \leq 0^-$, the quasi-static range integral expressions for the HED Hertz vectors are

$$\frac{\Pi_x}{C_1} \sim \frac{e^{-\gamma R_0}}{R_0} - \frac{e^{-\gamma R_1}}{R_1} + \int_0^\infty \left(\frac{2\lambda}{u + \lambda}\right) e^{u(z+h)} J_0(\lambda\rho)\,d\lambda \ , \tag{56}$$

$$\frac{\Pi_z}{C_1} \sim -\cos\phi \int_0^\infty \left(\frac{2\lambda}{u + \lambda}\right) e^{u(z+h)} J_1(\lambda\rho)\,d\lambda \ , \tag{57}$$

and

$$\frac{\vec{\nabla}\cdot\vec{\Pi}}{C_1} \sim -\cos\phi\left\{\frac{\rho}{R_0^3}(1 + \gamma R_0)e^{-\gamma R_0} - \frac{\rho}{R_1^3}(1 + \gamma R_1)e^{-\gamma R_1} + 2\int_0^\infty \lambda e^{u(z+h)} J_1(\lambda\rho)\,d\lambda\right\} \ , \tag{58}$$

where

$C_1 = I\ell/(4\pi\sigma)$,

$R_0^2 = \rho^2 + (z - h)^2$, and

$R_1^2 = \rho^2 + (z + h)^2$.

The fields in the earth are given by

$$\vec{E} = -\gamma^2\vec{\Pi} + \text{grad div } \vec{\Pi} \ ,$$
$$\vec{H} = \sigma \text{ curl } \vec{\Pi} \ . \tag{59}$$

These expressions (equations (56) to (58) for Π_x, Π_z, and $\vec{\nabla}\cdot\vec{\Pi}$ (and the resulting field components) can be evaluated analytically throughout the quasi-static range. In fact, they have been - see Bannister and Hart.[9] However, the resulting field-strength expressions are very complicated because they involve products of modified Bessel functions of different argument.

From equation (6), we note that

$$e^{u(z+h)} \sim e^{\gamma a(z+h)} \, e^{\lambda b(z+h)} \ , \tag{60}$$

where

a = 0 and b = 1 for $R_1/\delta << 1$,

a = 0.4 and b = 0.96 for R_1/δ less than approximately 1,

a = 0.96 and b = 0.4 for R_1/δ between approximately 1 and 10, and

a = 1 and b = 0 for $\rho > 3|z + h|$.

If we substitute equations (3) and (60) into equations (56) to (58), we can easily evaluate the HED Hertz potentials. The resulting expressions are

$$\frac{\Pi_x}{C_1} \sim \frac{e^{-\gamma R_0}}{R_0} - \frac{e^{-\gamma R_1}}{R_1} + e^{\gamma a(z+h)}\left\{\frac{1}{K_3} - \frac{1}{K_4}\right\} \ , \tag{61}$$

$$\frac{\Pi_z}{C_1} \sim -\frac{\cos\phi e^{\gamma a(z+h)}}{\rho}\left\{\frac{d - b(z + h)}{K_4} + \frac{b(z + h)}{K_3}\right\} \ , \tag{62}$$

and

$$\frac{\vec{\nabla}\cdot\vec{\Pi}}{C_1} \sim -\rho\cos\phi\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1) + \frac{2e^{\gamma a(z+h)}}{K_3^3}\right\}, \quad (63)$$

where $K_3^2 = \rho^2 + [b(z+h]^2$ and $K_4^2 = \rho^2 + [d - b(z+h)]^2$.

The field-strength expressions may be determined easily by employing equations (59) and (61) to (63). Thus,

$$E_\rho \sim \frac{I\ell\cos\phi}{4\pi\sigma}\left\{\frac{e^{-\gamma R_0}}{R_0^3}\left[\left(\frac{3\rho^2}{R_0^2} - 1\right)\left(1+\gamma R_0\right) - \gamma^2(z-h)^2\right]\right.$$

$$- \frac{e^{-\gamma R_1}}{R_1^3}\left[\left(\frac{3\rho^2}{R_1^2} - 1\right)\left(1+\gamma R_1\right) - \gamma^2(z+h)^2\right] \quad (64)$$

$$\left. + \frac{2e^{\gamma a(z+h)}}{K_3^3}\left[1 + b - \frac{3b^3(z+h)^2}{K_3^2} + \gamma ab(z+h)\right]\right\}$$

$$E_\phi \sim \frac{I\ell\sin\phi}{4\pi\sigma}\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0+\gamma^2R_0^2) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1+\gamma^2R_1^2)\right.$$

$$\left. + \frac{2e^{\gamma a(z+h)}}{K_3^3}\left[1 + \frac{2K_3^2}{d^2}\left(1 - \frac{K_3}{K_4}\right)\right]\right\}, \quad (65)$$

$$E_z \sim \frac{I\ell\cos\phi}{4\pi\sigma}\left\{\frac{\rho(z-h)}{R_0^5}(3+3\gamma R_0+\gamma^2R_0^2)e^{-\gamma R_0}\right.$$

$$\left. + \frac{\rho(z+h)}{R_1^5}(3+3\gamma R_1+\gamma^2R_1^2)e^{-\gamma R_1}\right\}, \quad (66)$$

$$H_\rho \sim \frac{I\ell\sin\phi}{4\pi}\left\{-\frac{(z-h)}{R_0^3}(1+\gamma R_0)e^{-\gamma R_0}\right.$$

$$\left. + e^{\gamma a(z+h)}\left[\frac{d-b(z+h)}{K_4\rho^2} + \frac{b(z+h)}{K_3\rho^2} + \frac{d-b(z+h)}{K_4^3}\right]\right\}, \quad (67)$$

$$H_\phi \sim -\frac{I\ell \cos\phi}{4\pi}\left\{\frac{(z-h)}{R_0^3}(1+\gamma R_0)e^{-\gamma R_0} + \frac{(z+h)}{R_1^3}(1+\gamma R_1)e^{-\gamma R_1}\right.$$
$$\left. + \frac{e^{\gamma a(z+h)}}{\rho^2}\left[\frac{d-b(z+h)}{K_4} + \frac{b(z+h)}{K_3}\right]\right\} , \tag{68}$$

and

$$H_z \sim \frac{I\ell\rho \sin\phi}{4\pi}\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1)\right.$$
$$\left. + e^{\gamma a(z+h)}\left[\frac{1}{K_3^3} - \frac{1}{K_4^3}\right]\right\} . \tag{69}$$

If either z or h = 0, then $R_0 = R_1$ and the first two terms of the E_ρ, E_ϕ, and H_z components (equations (64), (65), and (69)) cancel each other. Furthermore, if h = 0, the first two terms of the E_z and H_ϕ components (equations (66) and (68)) double, whereas if z = 0, they cancel each other.

VERTICAL MAGNETIC DIPOLE (VMD)

By following the same procedure outlined in the derivation of the HED subsurface-to-subsurface field-component expressions, the VMD subsurface-to-subsurface field-component expressions may be determined easily. They are

$$E_\phi \sim -\frac{i\omega\mu_0 IA}{4\pi}(\rho)\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1)\right.$$
$$\left. + e^{\gamma a(z+h)}\left[\frac{1}{K_3^3} - \frac{1}{K_4^3}\right]\right\} , \tag{70}$$

$$H_\rho \sim \frac{IA}{4\pi}(\rho)\left\{\frac{(z-h)}{R_0^5}(3+3\gamma R_0+\gamma^2R_0^2)e^{-\gamma R_0}\right.$$
$$\left. - \frac{3[d-b(z+h)]}{K_2^5}e^{\gamma a(z+h)}\right\} , \tag{71}$$

and

$$H_z \sim -\frac{IA}{4\pi}\left[\frac{e^{-\gamma R_0}}{R_0^3}\left\{\left[1 - \frac{3(z-h)^2}{R_0^2}\right](1+\gamma R_0) + \gamma^2\rho^2\right\}\right.$$

$$-\frac{e^{-\gamma R_1}}{R_1^3}\left\{\left[1 - \frac{3(z+h)^2}{R_1^2}\right](1+\gamma R_1) + \gamma^2\rho^2\right\} \tag{72}$$

$$\left. + e^{\gamma a(z+h)}\left\{\frac{1}{K_3^3}\left[1 - \frac{3b^2(z+h)^2}{K_3^2}\right] - \frac{1}{K_4^3}\left[1 - \frac{3[d - b(z+h)]^2}{K_4^2}\right]\right\}\right] .$$

If either z or h = 0, then $R_0 = R_1$ and the first two terms of the E_ϕ and H_z components cancel each other. Moreover, if z = h, the first term of the H_ρ component expression is equal to zero.

VERTICAL ELECTRIC DIPOLE (VED)

The VED subsurface-to-subsurface field-component expressions also may be determined by following the procedure outlined in the derivation of the HED subsurface-to-subsurface field-component expressions. Thus,

$$E_\rho \sim \frac{I\ell}{4\pi\sigma}\left\{\frac{\rho(z-h)}{R_0^5}[3 + 3\gamma R_0 + \gamma^2 R_0^2]e^{-\gamma R_0}\right.$$

$$\left. - \frac{\rho(z+h)}{R_1^5}[3 + 3\gamma R_1 + \gamma^2 R_1^2]e^{-\gamma R_1}\right\} , \tag{73}$$

$$E_z \sim -\frac{I\ell}{4\pi\sigma}\left[\frac{e^{-\gamma R_0}}{R_0^3}\left\{\left[1 - \frac{3(z-h)^2}{R_0^2}\right](1+\gamma R_0) + \gamma^2\rho^2\right\}\right.$$

$$\left. - \frac{e^{-\gamma R_1}}{R_1^3}\left\{\left[1 - \frac{3(z+h)^2}{R_1^2}\right](1+\gamma R_1) + \gamma^2\rho^2\right\}\right] , \tag{74}$$

and

$$H_\phi \sim \frac{I\ell}{4\pi}(\rho)\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1)\right\} . \tag{75}$$

If either z or h = 0^-, then $R_0 = R_1$ and $E_z \sim H_\phi \sim 0$. Furthermore, if z = 0, the E_ρ component doubles, whereas if h = 0^-, $E_\rho \sim 0$ (to the $O(\gamma_0^2)$).

HORIZONTAL MAGNETIC DIPOLE (HMD)

Again, by following the procedure outlined for other radiators, the HMD subsurface-to-subsurface propagation equations are

$$E_\rho \sim \frac{i\omega\mu_0 IA \cos\phi}{4\pi}\left\{-\frac{(z-h)}{R_0^3}(1+\gamma R_0)e^{-\gamma R_0} + \frac{(z+h)}{R_1^3}(1+\gamma R_1)e^{-\gamma R_1}\right.$$
$$\left. + \frac{e^{\gamma a(z+h)}}{\rho^2}\left[\frac{d-b(z+h)}{K_4} + \frac{b(z+h)}{K_3}\right]\right\} , \qquad (76)$$

$$E_\phi \sim \frac{i\omega\mu_0 IA \sin\phi}{4\pi}\left\{\frac{(z-h)}{R_0^3}(1+\gamma R_0)e^{-\gamma R_0} + e^{\gamma a(z+h)}\left[\frac{d-b(z+h)}{K_4\rho^2}\right.\right.$$
$$\left.\left. + \frac{b(z+h)}{K_3\rho^2} + \frac{d-b(z+h)}{K_4^3}\right]\right\} , \qquad (77)$$

$$E_z \sim \frac{i\omega\mu_0 IA \cos\phi}{4\pi}(\rho)\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0) - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1)\right\} , \qquad (78)$$

$$H_\rho \sim -\frac{IA \sin\phi}{4\pi}\left[\frac{e^{-\gamma R_0}}{R_0^3}\left\{\left[1-\frac{3\rho^2}{R_0^2}\right](1+\gamma R_0) + \gamma^2(z-h)^2\right\}\right.$$
$$- \frac{e^{-\gamma R_1}}{R_1^3}\left\{\left[1-\frac{3\rho^2}{R_1^2}\right](1+\gamma R_1) + \gamma^2(z+h)^2\right\} \qquad (79)$$
$$\left. + e^{\gamma a(z+h)}\left\{\frac{1}{K_3^3}\left[1-\frac{3\rho^2}{K_3^2}\right] + \frac{1}{K_4^3}\left[1-\frac{3\rho^2}{K_4^2}\right]\right\}\right] ,$$

$$H_\phi \sim -\frac{IA \cos\phi}{4\pi}\left\{\frac{e^{-\gamma R_0}}{R_0^3}(1+\gamma R_0+\gamma^2 R_0^2)\right.$$
$$\left. - \frac{e^{-\gamma R_1}}{R_1^3}(1+\gamma R_1+\gamma^2 R_1^2) + e^{\gamma a(z+h)}\left(\frac{1}{K_3^3}+\frac{1}{K_4^3}\right)\right. , \qquad (80)$$

and

$$H_z \sim \frac{IA \sin \phi}{4\pi} (\rho) \left\{ \frac{(z - h)}{R_0^5} [3 + 3\gamma R_0 + \gamma^2 R_0^2] e^{-\gamma R_0} + \frac{3[d - b(z + h)] e^{\gamma a(z+h)}}{K_2^5} \right\} . \quad (81)$$

If either z or h = 0^-, then $R_0 = R_1$ and the first two terms of the HMD H_ρ and H_ϕ components cancel, and $E_z \sim 0$. Furthermore, if z = 0, the first two terms of the E_ρ equation double, while if h = 0 they cancel each other.

If $K_3 << \delta$ and $|a(z + h)| << \delta$, the subsurface-to-subsurface HED, HMD, VED, and VMD equations reduce to the dc equations (for example, see page 3-4 of Kraichman[2]). Furthermore, if $\rho >> \delta$ and $\rho >> |z + h|$, the equations reduce to the quasi-near range results.[1-5]

DISCUSSION

Note that we have defined loosely the variables a and b. That is, we let a = 0.96 and b = 0.4 for R_1/δ between approximately 1 and 10, and a = 0.4 and b = 0.96 for R_1/δ less than approximately 1. The specific crossover point for each field-strength component will depend not only on R_1/δ but also on $|z + h|/\delta$.

Since the resulting field-strength formulas can be calculated easily on a desk top calculator, the field strengths can be determined by using both values of a and b, thus numerically determining the crossover point. Alternatively, we could set the two expressions (involving different values of a and b) equal and solve for the crossover point. For example, consider the integral

$$N' = \frac{\partial N}{\partial Z} = \int_0^\infty e^{u(z+h)} J_0(\lambda\rho) d\lambda \quad , \quad (82)$$

where

$N = I_0 (A) K_0 (B)$,

$A = \frac{\gamma}{2} [R_1 + (z + h)]$,

$B = \frac{\gamma}{2} [R_1 - (z + h)]$, and

I_0 and K_0 are modified Bessel functions of different argument.

The (complicated) analytical solution to N' is

$$N' = \left(\frac{\gamma}{2}\right)\left\{I_0(A)K_1(B) + I_1(A)K_0(B) - \frac{(z+h)}{R_1}\left[I_0(A)K_1(B) - I_1(A)K_0(B)\right]\right\} , \qquad (83)$$

and the (simple form) modified-image theory solution is

$$N' \sim \frac{e^{\gamma a(z+h)}}{K_3} . \qquad (84)$$

The crossover point may be found by setting the two versions of (84) equal to each other and solving for ρ/δ. That is, let

$$\delta|N'| \sim \frac{e^{a_1(z+h)/\delta}}{[(\rho/\delta)^2 + b_1^2(z+h)^2/\delta^2]^{\frac{1}{2}}} = \frac{e^{a_2(z+h)/\delta}}{[(\rho/\delta)^2 + b_2^2(z+h)^2/\delta^2]^{\frac{1}{2}}} . \qquad (85)$$

Thus,

$$\rho/\delta = -\frac{(z+h)}{\delta}\left\{\frac{b_1^2 e^p - b_2^2}{1 - e^p}\right\}^{\frac{1}{2}} , \qquad (86)$$

where $a_1 = 0.4$, $b_1 = 0.96$, $a_2 = 0.96$, $b_2 = 0.4$, and $p = \alpha[(z+h)/\delta] \times (a_2 - a_1)$.

If $(z + h)/\delta = -1.0$, the crossover point is at $\rho/\delta = 0.46$.

The numerical integration result for the normalized amplitude of the function $\delta N'$ is plotted in figure 1 versus ρ/δ for the case where $(z + h)/\delta = -1$. The normalization factor (0 dB) is 0.65, the value of the function at $\rho/\delta = 0.1$. Also presented is the modified-image theory result (with a = 0.4 and b = 0.96 for $\rho/\delta < 0.46$, and a = 0.96 and b = 0.4 for $\rho/\delta > 0.46$). Here we see that the modified-image theory result is within 1 dB of the numerical-integration result over the complete range of ρ/δ (0.1 to 10.0).

As another example, consider the magnetic fields at a height of one skin depth ($z/\delta = 1.0$) produced by a HED buried at a depth of one skin depth ($h/\delta = -1.0$). The numerical-integration results for the normalized amplitude of each component (H'), where

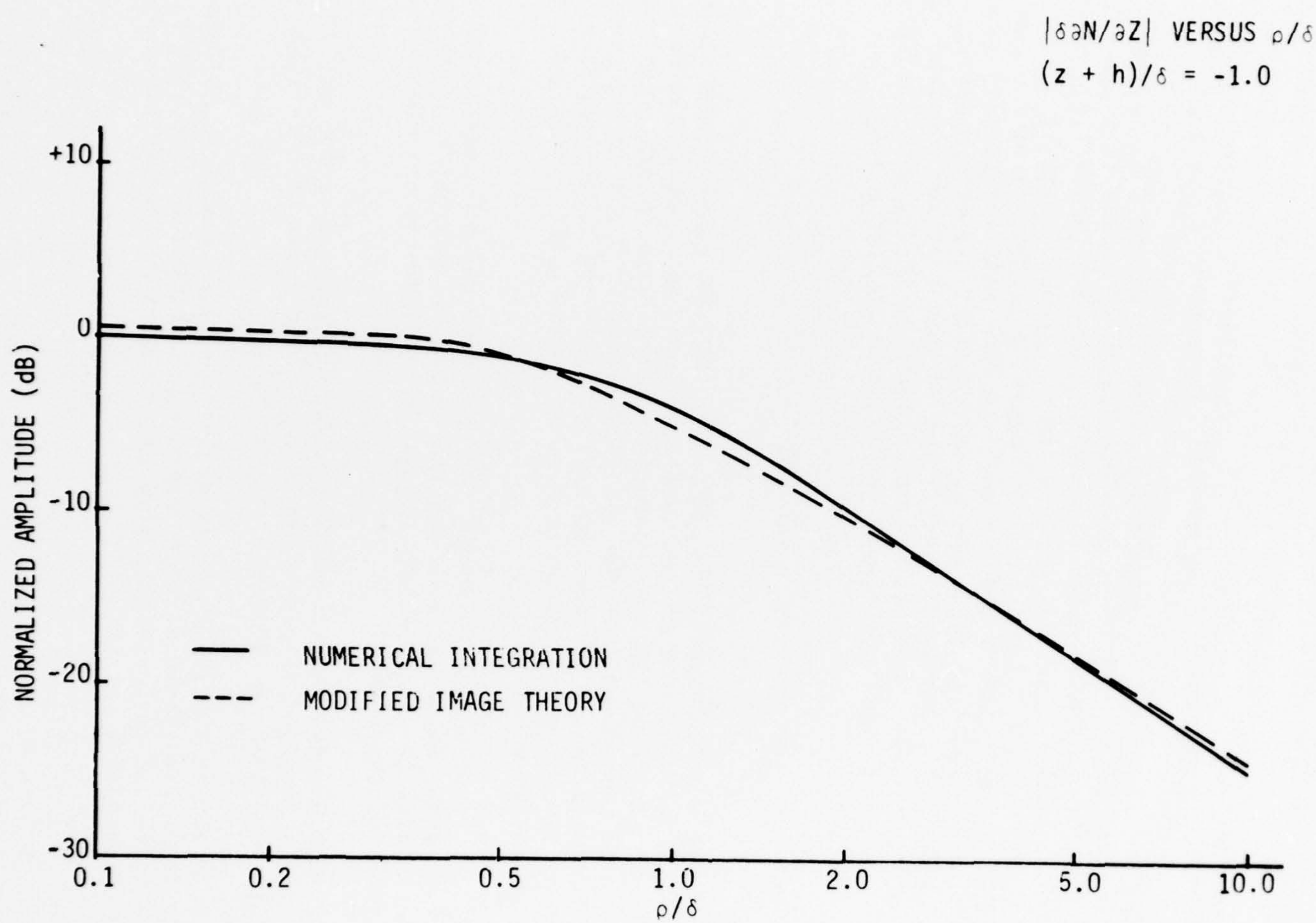

Figure 1. Comparison of Modified-Image Theory and Numerical-Integration Results for the Function N' With $(z + h)/\delta = 1.0$

$$H' = \frac{4\pi\delta^2 H}{I\ell \begin{Bmatrix} \sin\phi \\ \cos\phi \end{Bmatrix}}, \tag{87}$$

are presented in figures 2 and 3. The normalization factor (0 dB) is the numerical-integration value of each component at $\rho/\delta = 0.1$.

The modified-image theory results (from equations (24) to (26)) also are presented in figures 2 and 3. From figure 2, we see that the cross-over point for this particular example is $\rho/\delta = 1.5$ $(R/\delta = 1.8)$. That is, if $0.1 < \rho/\delta < 1.5$, then $a = 0.4$ and $b = 0.96$, whereas if $1.5 < \rho/\delta < 10$, then $a = 0.96$ and $b = 0.4$. A comparison (figure 3) of the composite modified-image theory results with the numerical-integration results shows that the modified-image theory calculations are within approximately 1 dB of the numerical-integration results over the complete range of ρ/δ (0.1 to 10). In fact, the modified-image theory even predicts the H_ρ component amplitude dip at the right place $(\rho/\delta \sim 1.5)$.

CONCLUSION

Simple expressions for HED, HMD, VED, and VMD quasi-static range subsurface-to-subsurface and subsurface-to-air propagation have been derived by employing finitely conducting earth-image theory techniques. It has also been shown that the resulting field-strength expressions for subsurface-to-air propagation are the result of simple modifications to one of the author's previously derived[12] quasi-static range air-to-air propagation equations.

These results will be useful for submarine radio-communication and detection purposes, as well as for the buried-miner problem. They also may be helpful to geophysicists engaged in determining the electrical properties of the earth.

Although displacement currents have been ignored in the analysis, they can be included simply by replacing σ by $\sigma + i\omega\varepsilon$ in the field-strength equations (as long as $|\gamma^2| >> |\gamma_0^2|$).

In a future report[23] we will present detailed numerical calculations of the simplified field-strength expressions derived in this report. They will be compared with the more exact numerical-integration results whenever possible.

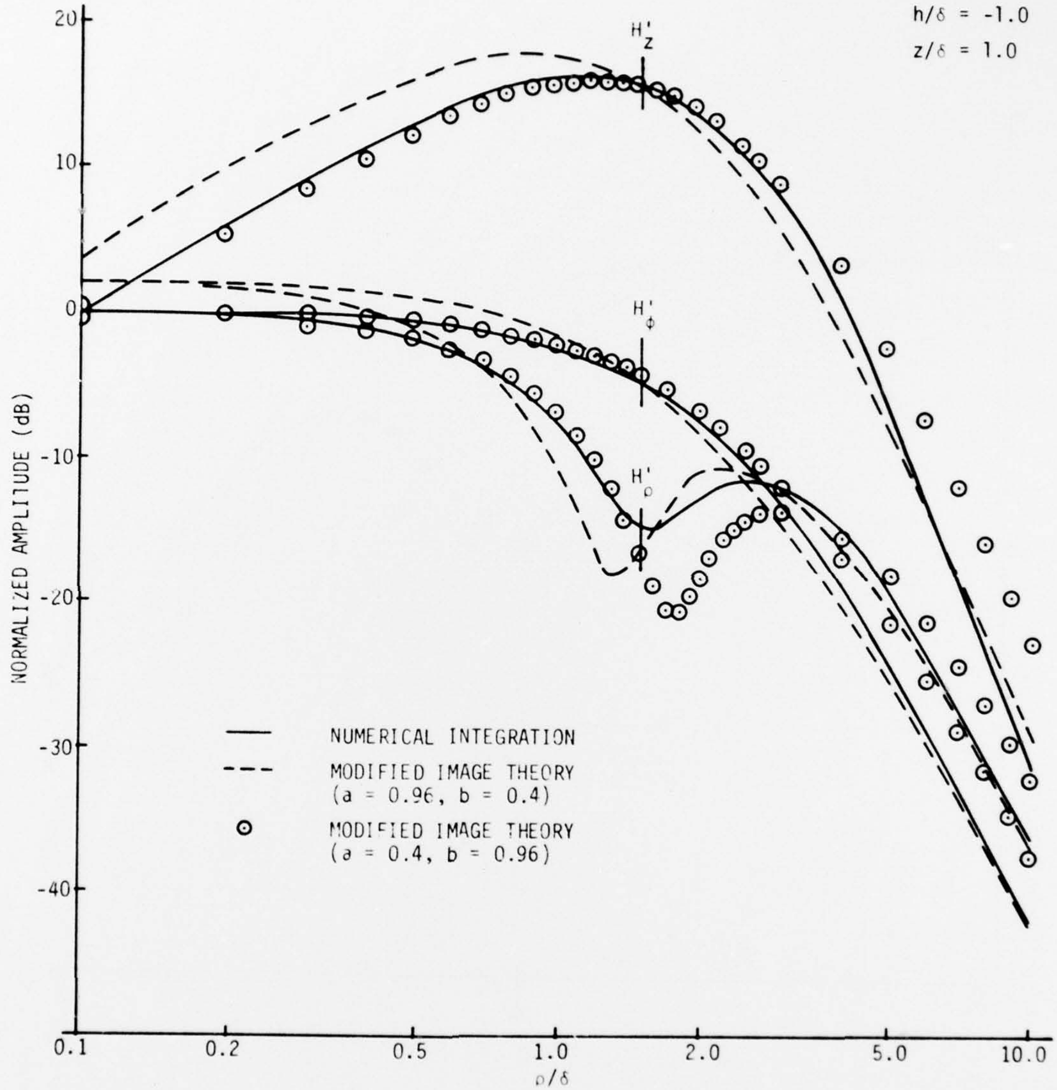

Figure 2. Comparison of Modified-Image Theory and Numerical-Integration Results for the Magnetic Fields in Air Produced by a Buried HED ($h/\delta = -1.0$, $z/\delta = 1.0$)

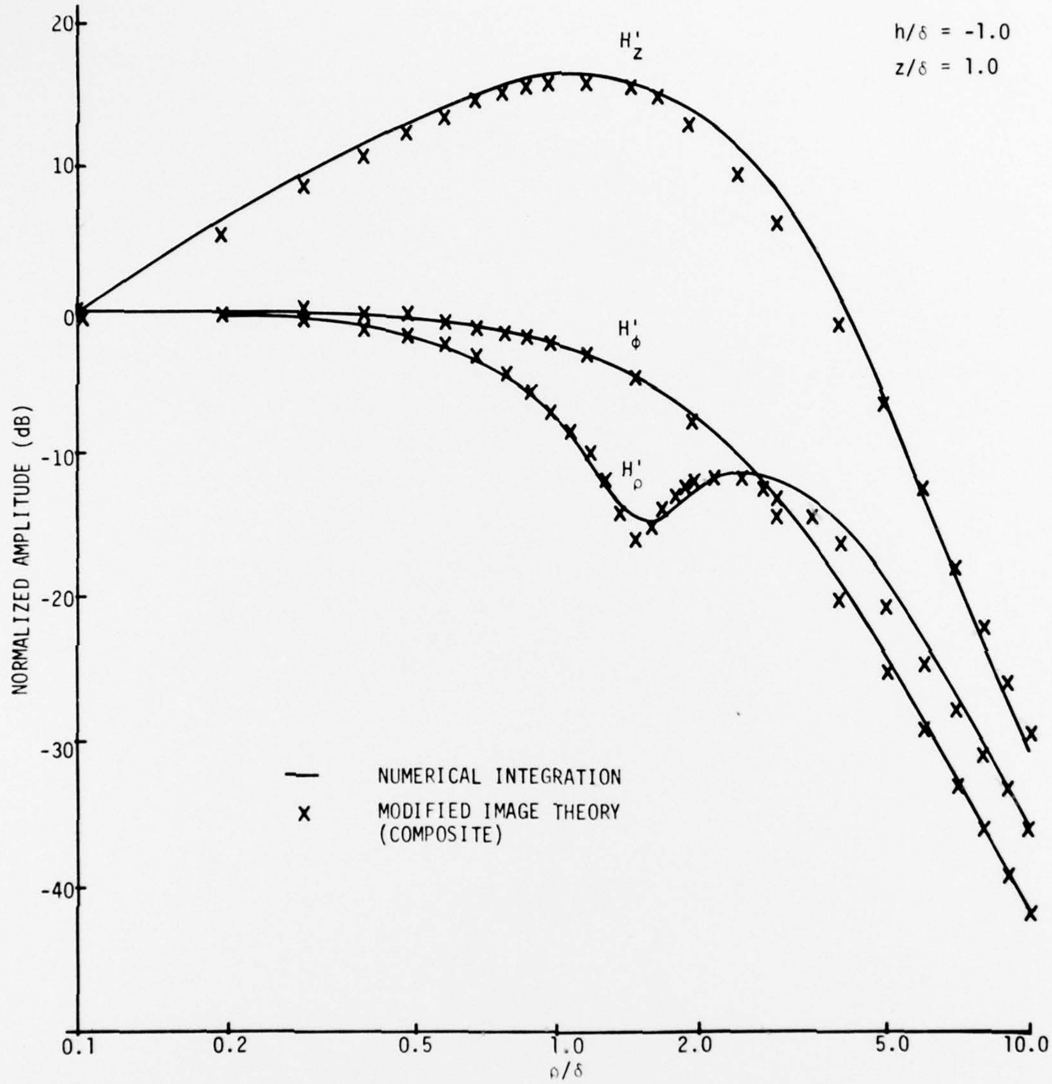

Figure 3. Comparison of Composite Modified-Image Theory and Numerical-Integration Results for the Magnetic Fields in Air Produced by a Buried HED ($h/\delta = -1.0$, $z/\delta = 1.0$)

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