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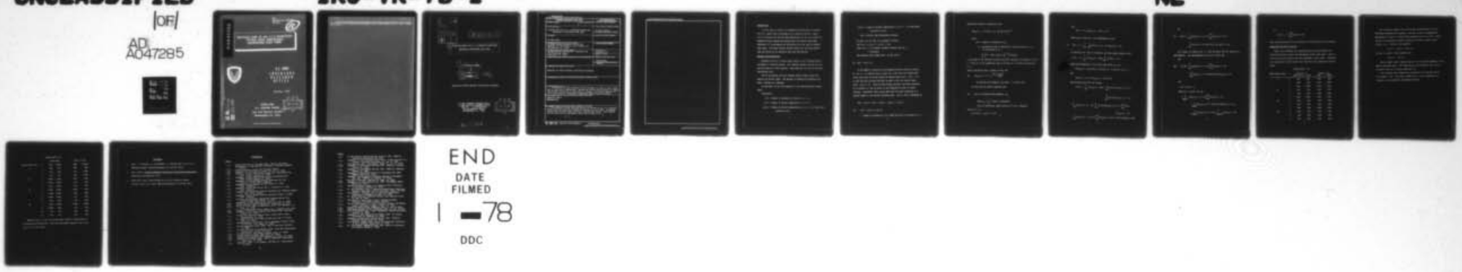

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WAITING TIME IN AN S-1,S INVENTORY SYSTEM WITH ARBITRARILY DISTRIBUTED LEAD TIMES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The customer waiting time distribution is derived for an S-1,S inventory system experiencing Poisson demands and general iid lead times. An analogy is made with the M/G/∞ queue in order to analyze the system. Some comparisons are made with an approximate result due to Higa, et al.

infinity

Introduction

In this paper we derive the probability distribution of customer wait at a supply point following an S-1,S inventory policy. Higa, et al [1] derived the waiting time distribution for an S-1,S policy for compound Poisson demands and exponentially distributed lead times. Sherbrooke [3] investigated the distribution for the case of constant lead times. This paper derives similar results for the pure Poisson case but allows for an arbitrary lead time distribution.

Waiting Time Distribution

Customers arrive at a supply point using an S-1,S inventory policy according to a Poisson process. All customer requests are for one unit, and are filled by a FIFO priority. Lead times are iid with an arbitrary distribution F(x).

We use the analogy with the infinite server Poisson queue with general iid service times. The analogy is specifically-placing of an order : arrival of a customer.

We use Ross' [2] pp 17,18 analysis of the infinite server Poisson queue.

Definitions:

$x(t)$ = number of customers in service at t, $t \geq o$

$r(t,\tau)$ = number of service completions in t to $t + \tau$

$r_1(t,\tau)$ = number of service completions in t to $t + \tau$ of those which arrived in $[0,t)$

$r_2(t,\tau)$ = number of service completions in t to t + τ of those which arrived in $(t,t+\tau)$

$F(z)$ = service time distribution function

We take t as the time of a customer's arrival.

Now $r(t,\tau) = r_1(t,\tau) + r_2(t,\tau) + \delta(\tau)$

where $\delta(\tau) = 1$ if present customer's service time is $\leq \tau$

$= 0$ otherwise

The customer waits longer than τ if and only if

(1) $x(t) - r(t,\tau) \geq S$

If the number in service at the customers arrival offset by returns in t to t + τ is greater than or equal to S, then there are either back-orders which must be filled before the current order $(x(t) - r(t,\tau) > S)$ or else the next arriving item will be used to fill the current order $(x(t) - r(t,\tau) = S)$. Since we have Poisson arrivals, the given condition of an arrival at t has no effect on the likelihood of past or future arrivals. Practically then, we may deal with the queue conditions at t without regard to the arrival occurring there. Now (1) may be expressed as

$$x(t) - r(t,\tau) = x(t) - r_1(t,\tau) - r_2(t,\tau) - \delta(\tau) \geq S$$

But $\quad x(t) - r_1(t,\tau) \equiv x_1(t,\tau)$

= number of arrivals in o to t which are still in service at t + τ

Using Ross' method of analysis we have

$$Pr[x_1(t,\tau) = j \mid N(t) = n] = \binom{n}{j} p_1^j (1-p_1)^{n-j}$$

where

$N(t)$ = number of arrivals in $[o,t)$

p_1 = probability that an arbitrarily chosen arrival in o to t is in service at $t + \tau$

$$= \frac{1}{t} \int_o^t [1 - F(t+\tau-z)]dz = \frac{1}{t} \int_\tau^{t+\tau} (1-F(z))dz$$

z, the time of the arbitrary arrival given N(t) arrivals, is uniform in (o,t). $1 - F(t+\tau-z)$ is the probability that an arrival at z is still in service at $t + \tau$

Merely duplicating Ross' algebra we have that

$$(2) \qquad Pr[x_1(t,\tau) = j] = e^{-\lambda t p_1} \frac{(\lambda t p_1)^j}{j!}$$

ie Poisson with parameter $\lambda t p_1$ where λ = arrival rate.

We also have by similar reasoning that

(3) $r_2(t,\tau)$ is Poisson with parameter $\lambda \tau p_o$

where $p_o = \frac{1}{\tau} \int_o^\tau F(z)dz$ = probability

that an arbitrarily chosen arrival in $(t,t+\tau)$ completes service by $t+\tau$.

Let $Y(t,\tau) = r_2(t,\tau) + \delta(\tau)$

Then

$$\Pr[w > \tau] = \Pr[x_1(t,\tau) - Y(t,\tau) \geq S]$$

Since $x_1(t,\tau)$ and $Y(t,\tau)$ are independent we have

$$(4) \qquad \Pr[w > \tau] = \sum_{k=S}^{\infty} \sum_{m=k}^{\infty} \Pr[Y(t,\tau) = m - k] \Pr[x_1(t,\tau) = m]$$

On switching the order of summation, and some simple algebra we get

$$\Pr[w > \tau] = \sum_{m=o}^{\infty} \Pr[x_1(t,\tau) = S+m] \sum_{k=o}^{m} \Pr[Y(t,\tau) = k]$$

Using the definition of $\delta(\tau)$ we see that $\Pr[Y(t,\tau) = k]$

$$= \Pr[r_2(t,\tau) = k][1-F(\tau)] + \Pr[r_2(t,\tau) = k-1][F(\tau)] \text{ for } k > o$$

and

$$\Pr[Y(t,\tau) = o] = \Pr[r_2(t,\tau) = o][1-F(\tau)]$$

Substituting these into (4) we have

$$\Pr[w>\tau] = \sum_{m=o}^{\infty} \Pr[x_1(t,\tau) = S+m] \sum_{k=o}^{m} [1-F(\tau)]\Pr[r_2(t,\tau) = k] + \sum_{k=1}^{m} [F(\tau)]\Pr[r_2(t,\tau) = k - 1]$$

$$= \sum_{m=o}^{\infty} \Pr[x_1(t,\tau) = S + m] \sum_{k=o}^{m} [1-F(\tau)]\Pr[r_2(t,\tau) = k] + \sum_{k=o}^{m-1} [F(\tau)] \Pr[r_2(t,\tau) = k]$$

$$= \sum_{m=o}^{\infty} \Pr[x_1(t,\tau) = S+ m) \left[\sum_{k=o}^{m-1} \Pr[r_2(t,\tau) = k]\right] + [1-F(\tau)]\Pr[r_2(t,\tau)=m]$$

So,

$$(5) \qquad \Pr[w>\tau] = \sum_{m=1}^{\infty} \Pr[x_1(t,\tau) = S + m] \sum_{k=o}^{m-1} \Pr[r_2(t,\tau) = k] +$$

$$\sum_{m=o}^{\infty} \Pr[x(t,\tau) = S + m][1-F(\tau)] \Pr[r_2(t,\tau) = m]$$

Any attempt to compute $\Pr[w > \tau]$ from the above form (5) must use an approximation. One approximation is to use a finite sum.

Let

$$(6) \qquad G(\tau,N) = \sum_{m=1}^{N} \Pr(x_1(t,\tau) = S + m] \sum_{k=o}^{m-1} \Pr[r_2(t,\tau) = k]$$

$$+ \sum_{m=o}^{N} \Pr[x_1(t,\tau) = S + m][1-F(\tau)]\Pr[r_2(t,\tau) = m]$$

and

$$G(\tau) = \Pr[w > \tau]$$

Then $G(\tau) - G(\tau,N) = E(\tau,N)$

$$= \sum_{m=N+1}^{\infty} \Pr[x_1(t,\tau) = S + m] \sum_{k=o}^{m-1} \Pr[r_2(t,\tau) = k]$$

$$+ \sum_{m=N+1}^{\infty} \Pr(x_1(t,\tau) = S + m][1-F(\tau)]\Pr[r_2(t,\tau) = m]$$

$$\leq \sum_{m=N+1}^{\infty} \Pr[x_1(t,\tau) = S + m] = \sum_{m=s+N+1}^{\infty} \Pr[x_1(t,\tau) = m]$$

Or

$$E(\tau,N) \leq 1 - \sum_{m=o}^{S+N} Pr[x_1(t,\tau) = m]$$

Thus, we may choose N with the assurance of achieving any error tolerance.

Comparison With Higa's Results

Higa, et al developed the customer waiting time distribution for stuttering Poisson arrivals and exponential service lead times. Some results for the pure Poisson case were published in their paper. Following is a comparison of their results with this work with exponential service times.

Arrival Rate = .01

Mean Service Time	S	Prob[w = o] Higa, et al	Prob[w = o] Kruse	Prob[w ≤ 10] Higa	Prob[w ≤ 10] Kruse
5	1	.951	.951	.999	.999
	2	.999	.999	1.000	1.000
	3	1.000	1.000	1.000	1.000
10	1	.905	.905	.986	.987
	2	.995	.995	1.000	1.000
	3	1.000	1.000	1.000	1.000
20	1	.819	.819	.928	.929
	2	.982	.982	.996	.996
	3	.999	.999	1.000	1.000
40	1	.670	.670	.783	.785
	2	.938	.938	.968	.969
	3	.992	.992	.997	.997

The differences shown in the above table, although small, are real differences between the two models. They are not due to computational approximations, since our results have been made accurate to 10^{-4}. Higa's results are approximate because he ignores the effect of what we have called $r_2(t,\tau)$. Where we have required

$$x_1(t,\tau) - r_2(t,\tau) - \delta(t) \geq S$$

for wait to exceed τ, Higa requires only

$$x_1(t,\tau) - \quad (t) \geq S$$

The two models seem to agree because of the particular parameter values used in the above table. For these values, the probability that $r_2(t,\tau)$ is greater than zero is always less than .05.

The following table compares the two models for an arrival rate of 1.0 instead of .01. This tends to make $r_2(t,\tau)$ a more significant contributor to the probability distribution of wait.

ARRIVAL RATE = 1.0

Mean Service Time	S	Prob [w=o] Higa	Prob [w=o] Kruse	Prob [w ≤ 10] Higa	Prob [w ≤ 10] Kruse
5	1	.006	.006	.806	.998
	2	.040	.040	.952	1.000
	3	.125	.125	.991	1.000
10	1	0.000	0.000	.084	.663
	2	0.000	0.000	.226	.785
	3	0.003	0.003	.421	.876
20	1	0.000	0.000	0.000	.005
	5	0.000	0.000	.012	.085
	10	0.005	0.005	.271	.501
	15	.105	.105	.790	.898
40	1	0.000	0.000	0.000	0.000
	25	0.004	.004	.123	.178
	30	.043	.043	.410	.491
	40	.479	.479	.933	.952

Ignoring $r_2(t,\tau)$, then, may in some cases result in large errors in the probability distribution. Note that both models agree for G(o) since $r_2(t,\tau) = o$ in this case.

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