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FOREWORD

This report describes the effort being conducted by the BDM Services Company Combined Arms Research and Analysis Facility (BDM/CARAF) and its subcontractors, the BDM Corporation and Vector Research, Incorporated (VRI), under Task 7-75 of U.S. Army Contract DAAG39-74-C-0018. The principal contributors to this report were Mr. David Thompson and Dr. Lee Jacobi of VRI. This report is oriented to readers familiar with the AIDM computer program and two preceding BDM/CARAF reports (R1, R2).

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TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF FIGURES

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CHAPTER I

INTRODUCTION

This working paper is the third in a series of three papers documenting methodology developed to extend the AMSAA Improved Differential Model (AIDM) as implemented at the TRADOC Systems Analysis Activity (TRASANA). This paper describes the extensions required to treat the effects of direct and indirect fire suppression. The first paper in this series (R1) discussed the representation of attack helicopters in AIDM, and the second paper in this series (R2) considered the extension to AIDM required to treat scout helicopters and information hand-off from one unit to another. The purpose of each of these papers is twofold:

(1) to document the assumptions and mathematical methodology employed in the model extensions, and

(2) to describe the procedures required to implement the extensions in the existing TRASANA AIDM computer program.

The remainder of this paper consists of six chapters. Chapter II presents an overview of the model and lists the detailed assumptions associated with the representation of suppression. Chapter III describes the representation of suppressed weapons. Chapter IV describes the determination of the probability of suppression. Chapter V discusses detections, suppressive area fire, and the changes in the methodology for the allocation of fire. Chapter VI derives the effects of suppression on the activities of attack and scout helicopters. Chapter VII discusses the changes to the computer program and data base which are required to implement these extensions. Appendix A describes a possible means of generating single-round suppression probabilities to be input to AIDM.

CHAPTER II

MODEL OVERVIEW AND DETAILED ASSUMPTIONS FOR THE REPRESENTATION OF SUPPRESSION

This chapter provides an overview of the suppression model and lists the main points and detailed assumptions associated with the models used to represent suppression in the TRASANA AIDM.

The suppression process included in the TRASANA AIDM results from behavior intended to lessen the risk of incapacitation due to received fire. The model represents the effects of reactive suppression, i.e., reactions to current (already existing) stimuli as opposed to anticipatory suppression or the result of behavior which assumes a future risk stimulus. Hence, the only suppression-causing mechanism considered is the receipt of fire. However, the suppression model currently played in AIDM, in which suppression depends on casualties, is retained (via a switch in the program code) as an alternative. This kind of suppression can be interpreted as anticipatory.

The developed suppression model includes suppression of and by every weapon group to be played with the exception of an AH (attack helicopter) group suppressing an AH group. This interaction is not currently played in AIDM. The modeled suppression is a function of the volume of fire directed at a weapon. The probability a weapon is suppressed is modeled as depending on a number of factors such as round type, suppressor and suppressee type, cover status, and the current state of suppression.

Suppression is represented via a two-state Markov chain model in which a weapon is either in the suppressed or unsuppressed state with the transition to the next state depending only on the current state and not on the past history of the weapon's suppression. The model requires the probabilities of changing states in the time interval, Δt. This probability is calculated as a function of the numbers of rounds of all types directed at the weapon during the interval.

Suppression causes changes in the acquisition and firing capabilities of both sides. The degree of vulnerability of a suppressee to acquisition and attrition and his ability to acquire and attrit the enemy while in a suppressed state depend on such factors as suppressee type, and the cover status of the group to which the suppressee belongs.

The remainder of the chapter outlines the specific assumptions together with the main points of the models used to incorporate the effects of suppression into the TRASANA AIDM. Areas discussed are: capabilities and representation of suppressed weapons; probability and duration of suppression; detections; suppressive area fire; allocation of fire; and suppression of attack helicopters.

1. Capabilities and Representation of Suppressed Weapons

   (a) The numbers of suppressed and unsuppressed weapons are treated as if they are separate groups (except for AH's) exchanging members as they become suppressed and unsuppressed.

   (b) Suppressed weapons assume new cover states, statuses, and capabilities, and differ from unsuppressed weapons of the same type (paired unsuppressed group) in ways that change their vulnerability.

(c) Suppressed groups (except AH's) can differ from their paired unsuppressed groups with respect to accuracy, lethality, rate of fire, vulnerability, and doctrines for selection of targets and round types.

(d) Suppressed weapons can have an altered ability to acquire targets and an altered line-of-sight status.

(e) Line of sight for weapons in the suppressed state depends on the weapon type and location.

(f) No mobility suppression is played.

(g) A weapon's ability to detect (acquire) and its liability to being detected (acquired) depend on the state of the weapon (suppressed or unsuppressed) and not on its history. Hence, an average probability of detection (acquisition) is applied to all members of a group.

2 Probability and Duration of Suppression

(a) Surviving weapons not already suppressed become suppressed during a time step with a probability determined from the amount of received fire.

(b) Surviving suppressed weapons become unsuppressed during a time step with a probability dependent on the weapon type and not on received fire.

(c) Hence, the duration of suppression is a geometric random variable whose mean is determined by input.

(d) If the target is receiving artillery fire during the time step, the probability of being suppressed by artillery is an input quantity, depending on target type.

7.

(e) Each round suppresses a target with a probability independent of any other fire received.

(f) Covered targets do not suffer suppressive effects.

3. Detections

(a) The suppression model distinguishes two kinds of information known to combatants about their opponents: acquisitions and detections.

(b) A detection alerts an observer to the presence of an object and its recognition as an object of military interest.

(c) An event is termed a detection only if identification of the target has not occurred.

(d) Detections are assumed lost if line of sight is lost.

(e) Detection rates are higher in handed-off grid squares.[1]

(f) Detections are themselves never handed off.

(g) Detections are forgotten with an exponentially-distributed memory time.

(h) Firers engage two types of targets, those acquired and those detected but not acquired,

(i) For N opposing weapon groups, a firer can allocate fire to up to 2N sets of targets, N possibly acquired weapon groups and N possibly detected but unacquired weapons groups.

4. Suppressive Area Fire

(a) Fire allocated to detected (unidentified and perhaps imprecisely localized) targets is termed suppressive area fire.

(b) Suppressive area fire is not allocated to acquired targets.

[1] The concepts of hand-off and grid squares are defined and discussed in the hand-off paper (R2).

(c) Targets detected and unacquired present vague stimuli that may not allow the target to be located precisely enough for the observer to lay the sights of his weapon on the target.

(d) Inability to locate precisely the target leads to a deliberate attempt by the observer to distribute his aimpoints around his mean estimate of the target's location.

(e) The accuracy of suppressive area fire is different from the accuracy of fire allocated to acquired targets due to:

(1) the error in the observer's estimate of the target's location, and

(2) the error introduced by the intentional selection of a sequence of aimpoints around this mean aimpoint.

(f) The mean aimpoint is assumed elliptical normally distributed with the mean at the point where fire would be aimed if the target were fully acquired, and with independent horizontal and vertical components.

(g) The offsets from the mean aimpoint are also assumed elliptical normally distributed with independent horizontal and vertical components.

(h) Each round fired is treated as if the mean aimpoint and the offset of the true aimpoint are sampled independently.

5. Allocation of Fire

(a) The numbers of suppressed and unsuppressed weapons belonging to "paired" groups are represented separately.[1]

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[1]Each aggregate of weapons (except attack and scout helicopters) is represented as two "paired" groups consisting of the suppressed and unsuppressed members of the aggregate.

(b) Suppressed and unsuppressed weapons belonging to "paired" groups are assumed not distinguishable once they are acquired or detected.

(c) Targets acquired or detected in "paired" groups are assumed of equal priority.

(d) Only unsuppressed groups are given on a prioritized list of targets.

(e) Suppressed and unsuppressed firers from "paired" groups are separately allocated to targets since their acquisition (detection) probabilities and attritive abilities are different.

(f) The priority list can also specify whether the entry is for suppressive area fire, or for normal fire against acquired targets. Hence, each opposing group could appear twice on the list of targets.

(g) The numbers of acquisitions (detections) in paired groups are assumed stochastically independent random variables.

(h) To distribute normal fire (suppressive area fire) between "paired" groups, assume that every acquired (detected) target in these groups is equally likely to be chosen as a target.

6. Suppression of and by Attack and Scout Helicopters

(a) Suppression by attack and scout helicopters is treated the same as that of any ground weapon.

(b) The suppression of helicopters is not modeled by exchanging AH's between suppressed and unsuppressed groups.

(c) When suppressed, an AH remasks. Hence, suppressed AH's on the average remask earlier than if not suppressed.

(d) The average time in the masked state is not directly affected by suppression.

(e) The time to suppress an exposed AH is taken to be a negative exponentially distributed random variable with a mean that depends on the volume of fire received.

(f) Suppression of AH's is independent of the suppression of all other weapons.

(g) A missile fired by an AH is lost if the AH is suppressed before the missile reaches the target.

(h) When the automatic cannon is fired, each burst is assumed fired instantaneously at the beginning of specified firing intervals. The number of bursts fired is the number fired before the AH is suppressed or remasks.

(i) AH's may either acquire or merely detect targets.

(j) AH's are assumed capable of being acquired but never merely detected.

(k) Suppressed AH's can neither acquire, fire, be acquired nor attrited.

(l) AH's search and fire for fixed length periods of time unless suppressed first.

(m) Due to target hand-off, visual (i.e., non-pinpoint) acquisition rates of and by AH's and detections by AH's differ depending on whether or not the AH (other observer) is scanning the region of the battlefield containing the other target (AH).

(n) AH's suppressed before the end of the maximum search period can hand off their acquisitions.

(o) As with other weapons, AH's do not hand off detections.

(p) Scout helicopters can behave and are treated in one of two ways: (1) either masking and unmasking like non-firing attack helicopters (type one scouts), or (2) always remaining unmasked (except possibly when suppressed) and being treated, in terms of line-of-sight processes, as ground weapons are treated (type two scouts).

CHAPTER III

THE BASIC FEATURES OF THE SUPPRESSION MODEL

This chapter presents the mathematics of the suppression model to be incorporated into AIDM. This model is intended to be an alternative to the existing AIDM suppression model. The current model is to be retained, so that users of the model can select one of the two options via a switch entered as input. The bookkeeping scheme used to determine the numbers of suppressed and unsuppressed weapons in the new model is described in Section A. Section B addresses the transition of weapons between the suppressed and unsuppressed states. The carryover of target information by suppressed and unsuppressed weapons is addressed in Section C.

A. REPRESENTATION OF SUPPRESSED WEAPONS

The differential equations of combat incorporated into the AIDM program predict the inventories of surviving weapons:

$$\frac{dn_j(t)}{dt} = -\sum_i A_{ij}(t)\; n_i(t) \qquad (1)$$

for all j, where

$n_i(t)$ = the number of survivors in group i at time t,

$A_{ij}(t)$ = the rate at which a member of group i attrits members of group j at time t (the attrition coefficient),

and where the sum is taken over all opponents of group j. The introduction of suppression processes into AIDM does not alter this basic struction, but it does introduce more coefficients into the equation, as indicated below.

The differential combat model assumes that all members of a weapon group are identical with respect to their vulnerability to attrition and acquisition by the enemy. However, a realistic portrayal of suppression can be achieved only if weapons are allowed to take on cover states and exposed areas different from their unsuppressed statuses, so as to change their vulnerability. Consequently, the attrition coefficient, $A_{ij}(t)$, should be allowed to differ for suppressed and unsuppressed targets (as well as for suppressed and unsuppressed firers). This difference in vulnerabilities can be easily reflected by bookkeeping separately the numbers of suppressed and unsurppressed weapons as if they are separate groups exchanging members as they become suppressed and unsuppressed.[1]

By treating suppressed and unsuppressed weapons to be members of separate groups, there is complete freedom in the model for suppressed weapons to take on statuses and have capabilities different from those in the unsuppressed state. In AIDM, cover status and weapon type are specified by the user for each weapon group, so that the suppressed groups can differ from their "paired" unsuppressed groups in any respect deemed realistic and significant by the user.

B. INVENTORIES OF SURVIVING WEAPONS

The model of suppression requires weapon groups to be paired in AIDM, so that the suppressed and unsuppressed portions of an aggregate that would have been a single group before the introduction of suppression are now bookkept separately. Let M(i) denote the group that is

[1] It would not be sufficient to calculate a value of A_{ij} averaged over the suppressed and unsuppressed members of a group, and then to apply this rate equally to weapons in both states, since the numbers of weapons in each state could not then be calculated.

paired with group i. That is, if group i is an unsuppressed group, M(i) is the group that contains group i's weapons when they become suppressed. Similarly, if group i is a suppressed group, group M(i) contains group i's weapons when they become unsuppressed. Then we have M(i) = j if and only if M(j) = i. This scheme does not apply to attack and scout helicopters, whose suppression is portrayed in a different fashion, as described in Chapter VI.

Let $B_{ij}(t)$ denote the rate at which weapons in group j are being suppressed (or unsuppressed) by fire from a weapon in group i. The coefficients of the differential equations of combat then become

$$\frac{dn_j(t)}{dt} = -\sum_i \left[A_{ij}(t) + B_{ij}(t) - B_{iM(j)}(t) \right] n_i(t) \qquad (2)$$

if j is not a helicopter group, and equation (1) holds if j is a helicopter group. In equation (2), the sum is taken over all members of the side opposing group j.

The number of weapons in group j suppressed (or unsuppressed) during a small time interval, $(t, t+\Delta t)$, is approximately

$$\delta_j = \left[\sum_i B_{ij}(t)\, n_i(t) \right] \Delta t. \qquad (3)$$

If j is an unsuppressed group, the fraction of weapons in that group suppressed during Δt can be interpreted as the probability that a particular weapon in group j is suppressed during Δt:

$$Prsup_j = \frac{\delta_j}{n_j(t)} . \tag{4}$$

Similarly, if j is a suppressed group, this ratio is the probability $Prun_j$ that a particular member of the group becomes unsuppressed during Δt. Chapter 4 addresses the calculation of $Prsup_j$. The differential equations of combat can be approximated in AIDM by the following difference equation:

$$\Delta n_j(t) = -\sum_i A_{ij}(t)\ n_i(t)\Delta t - \delta_j + \delta_{M(j)} \tag{5}$$

where $\Delta n_j(t)$ is the change in the size of group j during $(t,t+\Delta t)$.

For j an unsuppressed group, $Prsup_j$ is calculated on the basis of fire received by group j, as described below and in Chapter IV. For j a suppressed group, the probability of becoming unsuppressed during Δt, $Prun_j$, is an input to the model.[1]

Since $Prun_j$ is a constant for any one group, the duration of suppression is being modeled as a geometric random variable, the mean of which is $\Delta t/Prun_j$.

C. INFORMATION CARRYOVER

The AIDM acquisition model presently bookkeeps $Q_{ij}(t)$, the probability that a given firer in group i has not acquired a particular target in group j at time t (i.e., the probability the target is not in the acquired state at that time). At time $t+\Delta t$, the value $Q_{ij}(t)$ may no longer be correct, not only because acquisitions could be gained or lost during the time step, but also because weapons may be added and subtracted

[1] $Prun_j$ might be expected to depend on the type of weapon in group j.

from the observer and observee groups as they become suppressed and unsuppressed. Thus, it is necessary to adjust $Q_{ij}(t)$ to a new value $Q'_{ij}(t)$ that takes account of the suppressions that take place during the interval $(t,t+\Delta t)$.

This updated value, $Q'_{ij}(t)$, is then available to be updated, as now calculated in AIDM, to account for acquisitions made or lost during $(t,t+\Delta t)$. This section addresses the computation of $Q'_{ij}(t)$.

Let Z_i denote the number of weapons killed in group i during $(t,t+\Delta t)$. The probability of not having acquired a particular target in group j, averaged over all observers in group i at the end of the interval $(t,t+\Delta t)$, is

$$Q'_{ij}(t) = \frac{a_{ij}\left[n_j(t) - Z_j - \delta_j\right] + b_{ij}\,\delta_{M(j)}}{n_j(t) - Z_j - \delta_j + \delta_{M(j)}}, \qquad (6)$$

where the average probability of not acquiring a target that joins group j is

$$a_{ij} = \frac{Q_{iM(j)}(t)\left[n_i(t) - \delta_i - Z_i\right] + Q_{M(i)M(j)}(t)\,\delta_{M(i)}}{n_i(t) - Z_i - \delta_i + \delta_{M(i)}}, \qquad (7)$$

and where the average probability of not acquiring a target that stays in group j is

$$b_{ij} = \frac{Q_{ij}(t)\left[n_i(t) - \delta_i - Z_i\right] + Q_{M(i)j}(t)\,\delta_{M(i)}}{n_i(t) - Z_i - \delta_i + \delta_{M(i)}} \qquad (8)$$

Figure III-1 illustrates the bookkeeping of inventories.

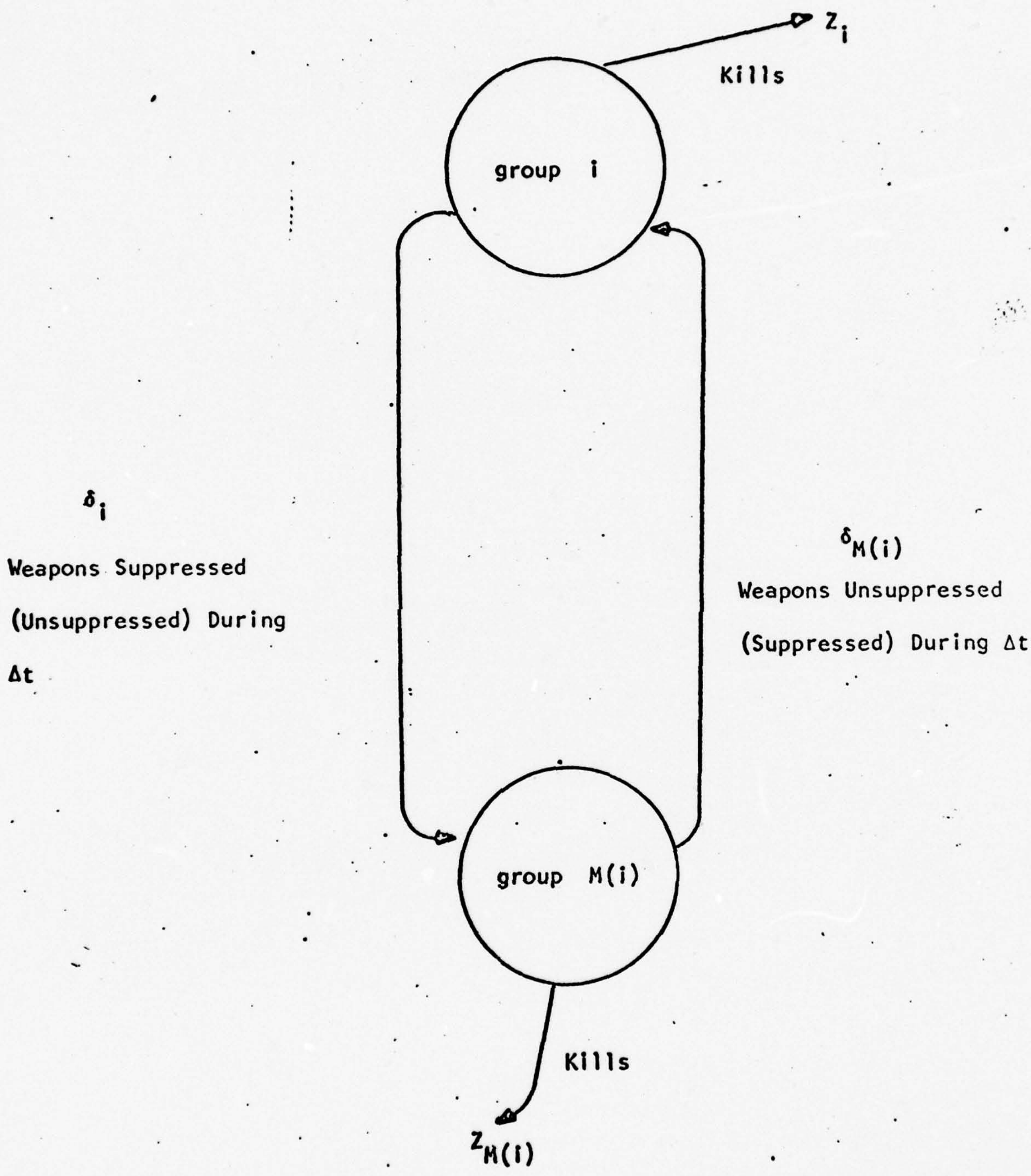

FIGURE III-1. THE BOOKKEEPING FLOW BETWEEN PAIRED SUPPRESSED AND UNSUPPRESSED GROUPS

The observing group, i, is either a suppressed or an unsuppressed group. If i is a suppressed group, then observers joining the group may lose acquisitions due to loss of line of sight to the target if a new cover status is sought in the suppressed state.[1] If this is the case, the logic in AIDM that updates acquisition probabilities will reflect this fact by setting $Q_{ij}(t+\Delta t)$ to zero. One could, however, postulate a further forgetting process, in which line-of-sight is not lost, but in which the suppressed weapon loses its ability to remember targets, just as its accuracy or rate-of-fire might be degraded. This degradation of memory is taken into account in equation (8) by replacing $Q_{M(i)j}(t)$ by $Q^*_{M(i)j}(t)$ and $Q_{M(i)M(j)}(t)$ by $Q^*_{M(i)M(j)}(t)$, defined as

$$Q^*_{M(i)K}(t) = 1 - \left[1 - Q_{M(i)K}(t)\right] \bar{P}_{M(i)} \; ; \qquad (9)$$

where $\bar{P}_x$, an input to the model, is the probability a member of unsuppressed group x does not forget an acquisition when becoming suppressed; and where $K = j$ for equation (8) and $K = M(j)$ for equation (7).

Chapter V distinguishes between two kinds of information, termed detecttions and acquisitions, that combatants are allowed to have about their opponents. With acquisitions, already incorporated in AIDM, the observer is assumed to have identified the acquired target. The new event, termed a detection, involves the alerting to the presence of an object and its recognition as an object of military interest, but without its identification

[1] In AIDM acquisitions are lost if there is no line of sight to the target.

having taken place, and possibly with information as to its location being imprecise. That is, the detector may be incapable of laying his sights exactly on the target, and fire may be placed around an estimated target location, possibly for suppressive purposes. This process of detection and engagement of imprecisely-located targets is discussed in Chapter V.

These detections will be bookkept by AIDM and must be averaged as acquisitions are averaged in equations (6), (7), and (8). Let $D_{ij}(t)$ denote the probability an observer has not detected a particular weapon in group j at time t. Then, an updated $D'_{ij}(t)$, similar to $Q'_{ij}(t)$, is

$$D'_{ij}(t) = \frac{c_{ij}\left[n_j(t) - Z_j - \delta_j\right] + d_{ij}\ \delta_{M(j)}}{n_j(t) - Z_j - \delta_j + \delta_{M(j)}}, \tag{10}$$

where

$$d_{ij} = \frac{D_{ij}(t)\left[n_i(t) - \delta_i - Z_i\right] + D_{M(i)j}(t)\ \delta_{M(i)}}{n_i(t) - \delta_i - Z_i + \delta_{M(i)}}, \tag{11}$$

and where

$$c_{ij} = \frac{D_{ij}(t)\left[n_i(t) - \delta_i - Z_i\right] + D_{M(i)M(j)}(t)\delta_{M(i)}}{n_i(t) - \delta_i - Z_i + \delta_{M(i)}}, \tag{12}$$

The forgetting of detections by suppressed weapons is taken into account as was done for acquisitions in equation (9). If the observer, i, is a suppressed group, $D_{M(i)j}(t)$ is replaced in equation (12) by $D^*_{M(i)j}(t)$:

$$D^*_{M(i)j}(t) = 1 - \left[1 - D_{M(i)j}(t)\right]\ \bar{\bar{P}}_{M(i)} \quad ; \tag{13}$$

and $D_{M(i)M(j)}(t)$ is replaced in equation (11) by $D^*_{M(i)M(j)}(t)$:

$$D^*_{M(i)M(j)}(t) = 1 - \left[1 - D_{M(i)M(j)}(t)\right] \bar{\bar{P}}_{M(i)} \quad , \qquad (14)$$

where $\bar{\bar{P}}_x$ is an input, the probability a member of group x does not forget a detection when becoming suppressed.

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CHAPTER IV

DETERMINATION OF THE PROBABILITY OF SUPPRESSION

This chapter discusses the calculation of the fraction of a group suppressed during a model time step. This fraction is equivalent to the probability a given member of the group is suppressed during the time step. Section A briefly presents the assumptions and general features of the model for this calculation. Section B describes the calculation of the suppression probability for the time step in terms of a single-round probability of suppression.

A. ASSUMPTIONS AND GENERAL FEATURES OF THE MODEL

The basic assumptions employed are listed as follows:

(1) Surviving weapons not suppressed become suppressed with a probability determined from the amount of received fire,

(2) Each round suppresses a target with a probability independent of any other received fire,

(3) There are no suppressive or attritive effects to covered targets.

Since one is seeking the fraction of a group suppressed during a model time step, one must calculate a round's average suppression probability for all members of a group (i.e., the expected proportion suppressed). A method of calculating this average single-round probability of suppression is described in Appendix A. This method could be applied externally to the model to generate the single-round suppression probabilities needed as input to the following calculations.

B. THE CALCULATION OF THE PROBABILITY OF SUPPRESSION

This section describes the calculation of $Prsup_j$, the probability of suppressing a particular weapon in unsuppressed group j during an AIDM time step, Δt.[1] The computations are presented below in a stepwise fashion, with the general organization of the calculations presented first and details presented later.

The assumed independence of the suppressive effects of different weapons allows $Prsup_j$ to be partitioned into the effects of field artillery and direct fire weapons, as follows:

$$Prsup_j = 1 - S_{ARTY}(j)\ S_{DF}(j)\ , \qquad (15)$$

where

$S_{ARTY}(j)$ = the fraction of weapons in group j not suppressed by artillery during the time step, and

$S_{DF}(j)$ = the fraction of weapons in group j not suppressed by direct fire during the time step.

$S_{ARTY}(j)$ should be set to unity when group j is not receiving artillery fire, and should take on an input value, dependent on the type of weapon in group j, when group j is receiving artillery fire. This input could be determined from any appropriate source, such as historical data or a detailed model of artillery coverage, but should in any case reflect the suppressive effect for a time of duration Δt, the AIDM time step.

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[1] The reader should note that $Prsup_j$ is to be computed only for unsuppressed groups. The computation of the time spent in a suppressed group is treated in Chapter III.

Direct fire is portrayed in greater detail in AIDM than is fire by field artillery, and suppression caused by direct fire is consequently modeled in greater detail. The suppressive effects of fire received from more than one firer are combined as follows:

$$S_{DF}(j) = \prod_{i} S_{DF}(i,j), \qquad (16)$$

where $S_{DF}(i, j)$ is the probability that fire from group i does not suppress a particular target in group j during the time step, and where the product is taken over all groups firing at j. The total suppressive effect of fire received from group i is modeled similarly, by taking the suppressive effect of every firer to be independent:

$$S_{DF}(i,j) = \begin{cases} (1-Prsup_{ij}e_{ij})^{n_i} & \text{if } n_i \geq 1 \\ (1-Prsup_{ij}e_{ij}n_i) & \text{if } n_i < 1 \end{cases} \qquad (17)$$

where

n_i = the number of survivors in group i,

e_{ij} = the probability a member of group i is allocated to fire at group j, and

$Prsup_{ij}$ = the probability a single weapon in group i, allocated to group j, suppresses a particular target in group j during Δt.

AIDM uses e_{ij} as the fraction of group i allocated to fire at group j. Here we interpret e_{ij} as the probability a member of group i is allocated to fire on group j. Equation (17) follows by independently sampling the allocation of each member of group i. The quantities n_i and e_{ij} are computed in the existing AIDM model, but $Prsup_{ij}$ is a new quantity. By applying the assumption that the suppressive effects of different rounds are independent, we obtain:

$$Prsup_{ij} = 1-(1-P_{ij})^{\rho_{ij}(t)\Delta t}, \qquad (18)$$

where

$\rho_{ij}(t)$ = the rate-of-fire of a group -i firer when engaging group j, and

P_{ij} = the expected fraction of group j suppressed by each round fired at group j by a weapon in group i.

This latter quantity is also new to the model, and it is an input datum. Appendix A presents a specific method of calculating P_{ij} that might be used to prepare inputs to AIDM.

CHAPTER V

DETECTIONS, SUPPRESSIVE AREA FIRE, AND ALLOCATION OF FIRE

To account for the capability of weapons to suppress targets they cannot precisely acquire or localize, a kind of imprecise target acquisition, called a detection, is being introduced into AIDM. This new feature is described in Section A. Section B describes the degradation in accuracy that occurs when engaging such a detected target. Section C discusses the allocation of fire between detected and acquired targets and between suppressed and unsuppressed weapons.

A. IMPRECISE TARGET INFORMATION

The suppression model distinguishes two kinds of information, termed detections and acquisitions, that combatants can have about their opponents. With acquisitions, already incorporated in AIDM, the observer is assumed to have identified the acquired target and to use the acquisition as the basis for allocating fire to targets in order of their priority. The new event, termed a detection, involves the alerting of an observer to the presence of an object and its recognition[1] as an object of military interest. However, the event is termed a detection only if identification of the target has not taken place.

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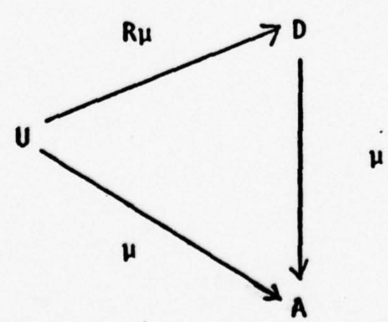
[1]There are no false detections allowed in the proposed model. Dead weapons and non-military objects are assumed to be recognized as such.

The detection process is modeled similarly to the acquisition process. However, it is possible to obtain the cumulative probability of not detecting, $D_{jk}(t)$, with many fewer computations than are required to obtain the non-acquisition probability. This simple approach postulates a simple relationship between the rates of detecting and acquiring targets. We consider an observer in group j and a weapon being observed in group k. AIDM presently computes $C_{jk}(t)$, the cumulative probability that the observer has not acquired the target. Let us postulate that the detection and acquisition processes are both exponential. (I.e., the times to acquire and detect either by pinpoint or non-pinpoint means are treated as negative exponential random variables.)

Consider that the k-group weapon can be in one of three states -- acquired (A), detected (D), and unobserved (U). It is assumed that the time to acquire the target is an exponential random variable with rate μ, and the time to detect is exponential with rate $R\mu$, where R is a constant greater than zero. It is further assumed that acquisition occurs at the same rate μ for targets in both the U and D states, so that detected targets can eventually become acquired. The flow between the states occurs in the directions and at the rates indicated below:



The acquisition rate, μ, need not be a constant for the desired result to hold, and the acquisition process is in fact allowed to be non-stationary in AIDM. However, as long as the rates are in the constant ratio R, the non-detection probability is simply related to the non-acquisition probability as

$$D_{jk}(t) = 1 - Q_{jk}(t) + Q_{jk}(t)^{R+1} \quad . \qquad (19)$$

Then, after AIDM has updated $Q_{jk}(t)$, equation (19) can be used to compute an updated value of $D_{jk}(t)$. The ratio R is required as input and could reasonably be tabulated on the weapon types of the observer and the observee.

It is interesting to note that, since the cumulative probability of non-acquisition is one if line-of-sight is lost in AIDM, the probability of non-detection is also one if line-of-sight is lost. This fact follows from equation (19) and is consistent with the assumption that detections are lost if line-of-sight is lost.

It is possible that the allocation of fire to detected, but unacquired, targets can expend a great deal of ammunition without significant attritive effect, because of the reduced accuracy of this fire (as discussed in the next section of this paper). Since weapons may conserve their ammunition for more profitable use against targets that may be acquired later, they may voluntarily limit the duration of fire missions against detected targets. This is taken into account in the model by causing detections to be forgotten, so that fire can no longer be allocated to them.[1] The time for which a detection is remembered is assumed to be a negative exponential random variable, dependent on the

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[1]Since detections by AH's and scout helicopters that maneuver like AH's are not accumulated from time step to time step in AIDM, this forgetting process is not applied to helicopters that behave in this way.

weapon type of the detector. If the mean time that a detection is remembered is $1/\beta$, then the probability of non-detection, after forgetting is taken into account,[1] is

$$D'_{jk}(t) = 1 - \left[1 - D_{jk}(t)\right] e^{-\beta \Delta t} \qquad (20)$$

B. SUPPRESSIVE AREA FIRE

This section addresses the nature of the fire that is allocated to targets that have been detected, but not acquired. This fire is termed suppressive area fire, or SAF. This is not to imply that the fire can have only suppressive effects, but only that it is allocated to unidentified and perhaps imprecisely localized targets, and, hence, might be allocated for purposes of suppression, as much as for attrition of the target. Detected, unacquired targets are assumed to present vague stimuli that do not allow the target to be located precisely enough for the observer to lay the sights of his weapon on the target.

This inability to locate the target precisely is assumed to lead in turn to a deliberate attempt by the observer to distribute his aim points around his mean estimate of the target location. Hence, there are two kinds of errors that contribute to make the accuracy of SAF different from the accuracy of fire allocated to acquired targets -- the error in the observer's estimate of the target's location and the error introduced by the intentional selection of a sequence of aim points around this mean aim point.

To model this situation mathematically the mean aim point is assumed to be elliptically normally distributed with the mean at the point where fire would be aimed if the target were fully acquired, and with independent

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[1] When implemented in AIDM, this forgetting equation should be applied before new detections for a time step are used to update $D_{jk}(t)$.

horizontal and vertical components. The offsets from the mean aim point are also assumed to be elliptically normally distributed with independent horizontal and vertical components. Each round fired is treated as if the mean aim point and the offset of the true aim point are sampled independently. This is probably not quite true in practice, since a firer probably forms an estimate of the target location and then distributes a sequence of shots around that vicinity, but the assumption of independence between rounds permits a simple mathematical model of the situation.

The following dispersions must be given as input to the model and indexed on the weapon type of the firer and the range to the target:

σ_{Mx} = standard deviation of the x-coordinate of the mean aim point,

σ_{My} = standard deviation of the y-coordinate of the mean aim point,

σ_{Ax} = standard deviation of the horizontal components of the offset of the aim point, and

σ_{Ay} = standard deviation of the vertical components of the offset of the aim point.

Then, if the horizontal and vertical standard deviations of the weapon's delivery errors are σ_x and σ_y, respectively, for fire at an acquired target, the horizontal and vertical standard deviations for SAF are $(\sigma_x^2+\sigma_{Mx}^2+\sigma_{Ax}^2)^{\frac{1}{2}}$ and $(\sigma_y^2+\sigma_{My}^2+\sigma_{Ay}^2)^{\frac{1}{2}}$, respectively. This result, and the fact that the resulting distribution is normal, follow from theorem 9B of Parzen (R4).

C. THE ALLOCATION OF FIRERS TO TARGETS

Two extensions are required to the existing methods used to allocate firers to targets in AIDM. If we assume that suppressed and unsuppressed weapons belonging to paired groups i and M(i) are not distinguishable once they are acquired or detected, then targets acquired or detected in groups i and M(i) are of equal priority, and the priority tie must be broken to allocate fire.

Consider firers in group i and assume that only unsuppressed groups are given on a prioritized list of targets. Assume also that the priority list also specifies whether the entry is for SAF or for normal fire against acquired targets.[1] We will now describe the allocation of fire to targets. Assume that j is the next group on the priority list and that E is the probability a member of the firing group, i, has already been allocated to a target of higher priority.[2] The problem is to determine the fraction of the firing group to be allocated to j and M(j) and to distribute the fire between the two groups.[3] Sections 1 and 2, below, address this problem.

1. Allocation to Acquired Targets

First consider the case when the priority list specifies fire against the acquired (not merely detected) members of groups j and M(j). The probability that a member of the firing group has not acquired at least one target in group j or M(j) is $\xi_{ij}\ \xi_{iM(j)}$, where

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[1]Thus, each opposing group could appear twice on the list of targets.

[2]The quantity E is currently computed in AIDM.

[3]The suppression of helicopters is not modeled by exchanging helicopters between suppressed and unsuppressed groups. Hence, M(j) can always be considered empty for helicopters.

$$\xi_{ik} = \begin{cases} Q_{ik}(t)^{n_k(t)} & \text{if } n_k(t) > 1 \\ n_k(t)\, Q_{ik}(t) & \text{if } n_k(t) \leq 1 \end{cases} \tag{21}$$

and where

$n_k(t)$ = the expected number of survivors in group k, and

$Q_{ik}(t)$ = the probability an observer in group i has not acquired a particular target in group k .

The fraction of group i allocated to acquired targets in groups j and M(j) can be reasonably taken to be $(1-E)\ (1-\xi_{ij}\ \xi_{iM(j)})$. To distribute fire between groups j and M(j), we assume that every acquired target in these groups is equally likely to be chosen as a target. The probability that the unsuppressed group is chosen, conditioned on an allocation to either the suppressed or unsuppressed group, is the following expectation:

$$u = E\left(\frac{A_u}{A_u+A_s} \middle|\ A_u + A_s > 0\right), \tag{22}$$

where

A_u = the number of acquisitions in group j, and

A_s = the number of acquistions in group M(j).

By definition, equation (22) is

$$u = \sum_{x+y>0} \left(\frac{x}{x+y}\right) \frac{\Pr\{A_u = x,\ A_s = y\}}{\Pr\{A_u + A_s > 0\}} \tag{23}$$

Assuming that the numbers of acquisitions in j and M(j) are stochastically independent random variables, we have

$$\Pr\{A_u = x,\ A_s = y\} = \Pr\left\{A_u = x\right\} \Pr\left\{A_s = y\right\} \tag{24}$$

To obtain the marginal distributions of A_u and A_s, we note that:

$$\Pr\left\{A_u = x\right\} = \sum_{w=x}^{\infty} \Pr\left\{A_u = x \mid N_u = w\right\} \Pr\left\{N_u = w\right\} \tag{25}$$

$$\Pr\left\{A_s = y\right\} = \sum_{w=y}^{\infty} \Pr\left\{A_s = y \mid N_s = w\right\} \Pr\left\{N_s = w\right\} \tag{26}$$

where N_u is the number of survivors in group j, and N_s is the number of survivors in group M(j).

The conditional distribution of A_u is

$$\Pr\left\{A_u = x \mid N_u = w\right\} = \binom{w}{x} \left(1 - Q_{ij}\right)^{x} Q_{ij}^{(w-x)} \tag{27}$$

if $w \geq x$, and zero otherwise. The conditional distribution of A_s is

$$\Pr\left\{A_s = y \mid N_s = w\right\} = \binom{w}{y} \left(1 - Q_{iM(j)}\right)^{y} Q_{iM(j)}^{(w-y)} \tag{28}$$

if $w \geq y$, and zero otherwise. Since AIDM is an expected value model, we know that the mean of N_u is n_i and the mean of N_s is $n_{M(j)}$, but we do not know the distributions of N_u and N_s. Consequently, we approximate their distributions as follows -- the probability that there are w survivors in group k is

$$\Pr\{N_k = w\} = \begin{cases} n_k - [n_k] & \text{for } w = [n_k] + 1 \\ 1 - n_k + [n_k] & \text{for } w = [n_k] \\ 0 & \text{otherwise} \end{cases} \qquad (29)$$

where

$$[n_k] = \text{the greatest integer} \leq n_k. \qquad (30)$$

Then, the approximate distribution of N_u is obtained from equations (29) and (30) with k=j, and the distribution of N_s is obtained from equations (29) and (30) with k=M(j).

From these definitions, it follows that equation (25) can contain non-zero terms for x = 0 to n_j+1 and y = 0 to $n_{M(j)+1}$. It also follows that

$$\Pr\{A_u + A_s > 0\} = 1 - \Pr\{A_u = 0\} \quad \Pr\{A_s = 0\} \qquad (31)$$

which is a quantity required in equation (23).

We now have the tie-breaking probability, u, and can use it to allocate fire between acquired targets in groups j and M(j). The fraction of the firers in group i allocated to acquired targets in group j is

$$e_{ij} = \left(1 - E\right)\left(1 - \xi_{ij}\ \xi_{iM(j)}\right) u, \qquad (32)$$

and the fraction allocated to group M(j) is

$$e_{iM(j)} = \left(1 - E\right)\left(1 - \xi_{ij}\ \xi_{iM(j)}\right)\left(1 - u\right) . \qquad (33)$$

2. <u>Allocation to Detected Targets</u>

Next, consider the case when the priority list specifies fire against the detected (and not acquired) members of groups j and M(j). The probability that a member of the firing group, i, has detected at least one unacquired target in group j or M(j) is $1 - \phi_{ij}\ \phi_{iM(j)}$, where

$$\phi_{ik} = \begin{cases} D_{ik}(t)^{n_k(t)} & \text{if } n_k(t) > 1 \\ 1 - n_k(t)\,[1 - D_{ik}(t)] & \text{if } n_k(t) \leq 1 \end{cases} \qquad (34)$$

and where $D_{ik}(t)$ is the probability an observer in group i has not detected a particular target in group k. It is assumed that suppressive area fire (SAF) is allocated only to targets that have been detected but not acquired. This assumption follows, since fire against acquired targets is assumed to be more accurate than fire against targets that are merely detected. The fraction of group i allocated to detected targets in groups j and M(j) is (1-E) $(1 - \phi_{ij}\phi_{iM(j)})$. To distribute fire between groups j and M(j), we assume that every detected target in these groups is equally likely to be chosen as a target. The probability that a target in the unsuppressed group is chosen, conditioned on an allocation to either the suppressed or unsuppressed group, is

$$s = E\left(\frac{D_u}{D_u + D_s} \mid D_u + D_s > 0\right) \qquad (35)$$

where

D_u = the number of detections in group j, and

D_s = the number of detections in group M(j)

The calculation of s exactly parallels that of u in the preceding section. Equations (23) through (24) apply, although with A_s and A_u replaced by D_s and D_u, and with $Q_{ik}(t)$ replaced with D_{ik} (t). Then, the fraction of firers in group i allocated to detected targets in group j is

$$e'_{ij} = \left(1 - E\right)\left(1 - \phi_{ij}\phi_{iM(j)}\right)s \quad , \qquad (36)$$

and the fraction allocated to detected targets in group M(j) is

$$e_{iM(j)} = \left(1 - E\right)\left(1 - \phi_{ij}\phi_{iM(j)}\right)\left(1 - s\right) . \qquad (37)$$

CHAPTER VI

THE EFFECTS OF SUPPRESSION ON THE ACTIVITIES OF ATTACK AND SCOUT HELICOPTERS

This chapter describes the mathematical models related to the suppression of and by attack and scout helicopters. Section A lists assumptions to be utilized in modeling the implications of suppression. Sections B and C give the derivation of the attrition rate equation for attack helicopters engaging ground weapons (including ADW's) by firing missiles and the automatic burst-fire cannon, respectively. Section D discusses the implications of suppression on the acquisition process and derives new acquisition probabilities for ground weapons (including ADW's) firing at helicopters. Section E treats acquisitions by helicopters.

A. ASSUMPTIONS UTILIZED IN MODELING THE SUPPRESSION OF AND BY ATTACK AND SCOUT HELICOPTERS

The following list reviews the specific assumptions used to incorporate the effects of suppression of attack and scout helicopters:

(1) Suppression by attack and scout helicopters is treated as suppression by any ground weapon,

(2) When suppressed, an AH will remask (hence, as a result of suppression, both the search period and the time spent firing either missiles or the automatic cannon are of random length),

(3) The average time in the masked state is not directly affected by suppression. Suppression does not directly cause the AH to remain masked for a longer period of time then if unsuppressed,

(4) If the helicopter is suppressed after launching a missile but before the missile reaches the target, it is assumed that the missile is ineffective and hence lost (a launch-and-forget missile, requiring no in-flight guidance, could be played by setting to zero the time required to launch and fly out the missile to the target).

(5) When the AH is firing the automatic cannon, it is assumed that each burst is fired instantaneously, and that there is some waiting time between bursts. Bursts do not have to be guided to a target so that the number of successfully fired bursts equals the number of bursts fired before suppression occurs, and

(6) AH's can neither acquire nor fire when in the suppressed state. AH's can neither be acquired nor attrited while in the suppressed state.

(7) Scout helicopters can behave and are treated in one of two ways: (1) either masking and unmasking like non-firing attack helicopters (type one scouts), or (2) always remaining unmasked except when suppressed and remaining suppressed for periods of time in the manner of ground weapons (type two scouts).[1]

B. AH MISSILE ATTRITION COEFFICIENT

The purpose of this section is to derive the attrition coefficient for an AH firing missiles and behaving according to the assumptions listed above. Since the helicopter allocation factor methodology is unchanged from that in the absence of suppression, it remains to determine the attrition rates.

The mean time to kill involves the following quantities to be determined from the interaction of AH tactics, AH performance data, and the battlefield terrain:

t_s = the time limit for the AH's search period during each exposure period. If no eligible targets (i.e., targets within range and on the AH's target priority list) are found by time t_s after unmasking, the AH remasks without launching a missile or utilizing the automatic cannon.

t_f = the time required to launch and fly out the missile to the target. It is assumed that the missile is ineffective if the AH is suppressed before

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[1]The activities of scout helicopters are discussed in more detail in the hand-off paper (R2). In general, scouts might have different parameters than AH's.

time $t_s + t_f$ (a launch-and-forget missile requiring no in-flight guidance, could be played by setting $t_f=0$). For the case of the automatic cannon, t_f is the maximum allowable time for firing a fixed number of bursts and remasking.

$\bar{T}_c$ = the expected length of an AH's masked period determined by calculations described in the handoff paper (R2).

P_E = the probability of acquiring an eligible target during T_s.

SSKP = the probability of kill for a missile that is successfully flown out to its target (i.e., suppression not occurring before time $t_s + t_f$).

The search time and the fly-out time are random variables whose maximum values t_s and t_f are constants for any given situation and whose mean values are calculated below. The duration of the covered state is a random variable, and only its mean is required for the following derivations. Its mean is determined by calculations described in the handoff paper (R2). To derive $E(T_K)$, the expected time for a missile-firing AH to kill a particular target, define the following random variables:

N_K = the number of successfully flown out missiles required to kill the target.

TBM = the time between two successive missiles that are flown out to the target without the AH being prematurely suppressed.

T_K = the time to kill the target.

If it is assumed that N_K and the sequence of TBM's are stochastically independent, then

$$E(T_K) = E(N_K) \cdot E(TBM) \qquad (38)$$

where, as in the helicopter paper (R1)

$$E(N_K) = \frac{1}{SSKP} \qquad (39)$$

and where

$$E(TBM) = \left[\frac{1}{Pr(T_E = t_s + t_f)} - 1\right]\left[E(T_E | T_E < t_s + t_f) + \overline{T}_C\right] + t_s + t_f + \overline{T}_C \qquad (40)$$

Equation (40) arises as the result of a series of Bernoulli trials where M is the number of trials (AH exposures) required until and including the first success (successfully firing a missile). Equation (40) is of the form

$$E(TBM) = [E(M)-1]\ E(T_E | \text{Unsuccessful}) + E(T_E | \text{Successful})$$

where T_E is a random variable, the duration of a helicopter's exposure time (time unmasked). E(M)-1 equals the expected number of unsuccessful trials with E(M) the reciprocal of the probability of success.

To derive the probability and the conditional expectation in equation (40), note that the time to suppress an exposed AH is taken to be a negative exponential random variable with rate λ. In addition, the probability an unsuppressed AH acquires some eligible target during the internal $(0, t_s)$ and tries to engage it during the internal $(t_s, t_s + t_f)$ is P_E. Again, let T_E denote the duration of one AH exposure period. T_E is approximated as a random variable with probability density

$$Pr\left\{t \leq T_E < t + dt\right\} = \lambda e^{-\lambda t}\,dt,\ 0 \leq t < t_s$$

$$Pr\left\{T_E = t_s\right\} = (1 - P_E)e^{-\lambda t_s}$$

$$Pr\left\{t \leq T_E < t + dt\right\} = P_E \lambda e^{-\lambda t}\,dt,\ t_s < t < t_s + t_f$$

$$Pr\left\{T_E = t_s + t_f\right\} = P_E e^{-\lambda(t_s + t_f)}$$

The conditional expectation is

$$E(T_E | T_E < t_s + t_f) = \frac{1 - (1-P_E)e^{-\lambda t_s} - P_E[\lambda(t_s + t_f) + 1]e^{-\lambda(t_s + t_f)}}{\lambda(1 - P_E e^{-\lambda(t_s + t_f)})} \quad (41)$$

Thus, with $Pr(T_E = t_s + t_f) = P_E e^{-\lambda(t_s + t_f)}$, equation (67) becomes

$$E(TBM) = t_s + t_f + \frac{\overline{T}_C + t_s(1-P_E)e^{-\lambda t_s}}{P_E e^{-\lambda(t_s + t_f)}} + \frac{(1-P_E)[1-(\lambda t_s+1)e^{-\lambda t_s}]+P_E[1-\{1+\lambda(t_s+t_f)\}e^{-\lambda(t_s+t_f)}]}{\lambda P_E e^{-\lambda(t_s+t_f)}} \quad (42)$$

Finally, from (65) the expected time to kill is

$$E(T_K) = \frac{1}{SSKP}\left\{t_s + t_f + \frac{\overline{T}_C + t_s(1-P_E)e^{-\lambda t_s}}{P_E e^{-\lambda(t_s + t_f)}} + \frac{(1-P_E)[1-(\lambda t_s+1)e^{-\lambda t_s}]+P_E[1-\{1+\lambda(t_s+t_f)\}e^{-\lambda(t_s+t_f)}]}{\lambda P_E e^{-\lambda(t_s+t_f)}}\right\} \quad (43)$$

As before, the attrition rate for an AH is the reciprocal of $E(T_K)$ as determined from equation (43). Note that as $\lambda \to 0$ in the limit of zero suppression, equation (43) becomes equation (4) of Chapter III, Section A of the helicopter paper.

RMISS, the overall rate of firing missiles (some of which may not be successfully flown out to the target) is found from Ross's theorem 3.16(R3) which states that this long-term rate is the ratio of the probability that the exposure time is greater than t_s and a target is engaged to the average length of the masked/unmasked cycle. Hence the rate of fire is

$$RMISS = \frac{Pr(T_E > t_s)}{E(X)}$$

which is

$$RMISS = \frac{P_E e^{-\lambda t_s}}{\overline{T}_c + \overline{T}_E} \quad (44)$$

determined by using eq. (51) and (52) of Section C for E(X) and $\overline{T}_E$, respectively.

The suppression rate, λ, must be computed dynamically as a function of the volume of fire directed at the helicopter group. For helicopters in group j the rate is

$$\lambda = \sum_i e_{ij} \lambda_{ij} n_i \quad (45)$$

where e_{ij} = the fraction of group i allocated to fire at group j,

λ_{ij} = the rate at which a weapon in group i suppresses weapons in group j, and

n_i = the number of surviving weapons in group i.

The suppression rate, λ_{ij}, is obtained from the single-shot suppression probability, P_{ij}, as

$$\lambda_{ij} = -\rho_{ij} \ln(1-P_{ij}) \quad (46)$$

where ρ_{ij} is the rate at which a weapon in group i fires at group j when group j is engaged.

C. AH AUTOMATIC CANNON ATTRITION RATE

This section describes the modeling of the situation discussed in Section A in which suppression may interrupt the firing of the automatic cannon after a number of bursts fewer than the fixed number that would have been fired had no suppression occurred. In addition the allocation factor methodology is unchanged from that in the absence of suppression since only an AH remaining unsuppressed beyond t_s can be allocated to fire at targets. Hence, one need derive only the attrition rate for this case.

Let Y_i equal the number of kills achieved by the AH on the i^{th} unmasking and X_i the duration of the i^{th} cycle of masked/unmasked status. Assume the X_i form a renewal process. Define

$$Y(t) = \sum_{i=1}^{N(t)} Y_i$$

as the total number of kills in the interval (0,t),

where

$N(t)$ = the number of X_i cycles or renewals in (0,t) .

Then, even though the X_i and Y_i sequences are stochastically dependent, theorem 3.16 in Ross (R3) can be utilized to give the following result: If $E(|Y_i|)$ and $E(X_i)$ are finite, then with probability 1,

$$\frac{Y(t)}{t} \rightarrow \frac{E(Y_i)}{E(X_i)} \quad \text{as } t \rightarrow \infty \ .$$

The ratio $E(Y_i)/E(X_i)$ is interpreted as the attrition rate (α) for AH engaging a particular target group with the automatic cannon; i.e.

$$\alpha \equiv \frac{E(Y)}{E(X)} \qquad (47)$$

It now remains to find E(X) and E(Y). Again define

t_s = the limit to the search interval

T1 = time to fire a first burst (first burst starts at t_s)

TS = time to fire subsequent bursts

t_f = maximum time available for firing.

Then if suppression does not intervene, there are N subsequent bursts fired, where

$$N = \left[\frac{t_f - T1}{TS}\right], \tag{48}$$

where the [ ] stands for "greatest integer in".

It is assumed that each burst is fired instantaneously and that there is a waiting time of T1 after the first burst and TS after each subsequent burst. In reality the bursts would be distributed in some fashion over the T1 and TS intervals, but this process has been simplified by talking all bursts to be fired at the beginning of these intervals, i.e., at $t_s, t_s + T1, t_s + T1 + TS$, etc.

To obtain E(Y) note first that the number of bursts fired is a random variable since the AH could be suppressed sometime before completing the maximum sequence of N+1 bursts. Let N_B denote the number of bursts fired during a single masked/unmasked cycle (a random variable) with probability distribution (derived from the exposure time probability distribution given in Section B) as follows:

$$Pr\ \{N_B = 0\} = 1 - P_E e^{-\lambda t_s}$$

$$Pr\ \{N_B = 1\} = P_E[e^{-\lambda t_s} - e^{-\lambda(t_s+T1)}]$$

$$Pr\ \{N_B = M\} = P_E\left[e^{-\lambda[t_s+T1+(M-2)TS]} - e^{-\lambda[t_s+T1+(M-1)TS]}\right]$$

for M = 2,3,...,N.

$$Pr\ \{N_B = N + 1\} = P_E e^{-[\lambda t_s+T1+(N-1)TS]}$$

Then, E(Y) can be written as follows:

$$E(Y) = \sum_{M=1}^{N+1} Pr\left\{N_B=M\right\}[1-(1-SB1)(1-SB2)^{M-1}] \quad , \tag{49}$$

where SB1 = the probability of kill for the first burst, and

SB2 = the probability of kill for each subsequent burst.

Substituting for Pr $\{N_B=M\}$ and summing and rearranging terms, the following form for E(Y) emerges:

$$E(Y) = SB1 \cdot P_E e^{-\lambda t_s} + \frac{P_E SB2(1-SB1)\left[e^{-\lambda(t_s+T1)} - (1-SB2)^{\frac{t_f-T1}{TS}} e^{-\lambda(t_s+t_f)}\right]}{1-(1-SB2)e^{-\lambda TS}} \tag{50}$$

To obtain E(X) note that T_E, the duration of an AH exposure period, has the following probability distribution:

For $0 \le t < t_s$, $Pr\{T_E \le t\} = 1 - e^{-\lambda t}$,

$t = t_s$, $Pr\{T_E \le t_s\} = 1 - P_E e^{-\lambda t_s}$,

$t_s < t < t_s + t_f$, $Pr\{T_E \le t\} = 1 - P_E e^{-\lambda t}$,

$t = t_s + t_f$, $Pr\{T_E \le t_s + t_f\} = 1$,

where $t_f = T1 + N \cdot TS$.

E(X) is simply given by

$$E(X) = \overline{T}_C + \overline{T}_E \quad , \tag{51}$$

where $\overline{T}_E$ is the mean exposure time:

$$\overline{T}_E = \frac{1}{\lambda}[1 - (1 - P_E)e^{-\lambda t_s} - P_E e^{-\lambda(t_s + t_f)}] \tag{52}$$

Note that as $\lambda \to 0$, the expression for α obtained by inserting equations (50), (51), and (52) into equation (47) reduces to the expression for the automatic cannon attrition rates given on page IV-11 of the helicopter paper(R1).

RF, the rate of firing the automatic cannon averaged over masked and unmasked periods (including the possibility of suppression), is found by applying theorem 3.16 from Ross (R3):

$$RF = \frac{\overline{N}_B}{E(X)}, \quad (53)$$

where $\overline{N}_B$ is the average number of bursts fired during a single unmasked period:

$$\overline{N}_B = \sum_{x=1}^{N+1} x\Pr\{N_B = x\}. \quad (54)$$

$$\overline{N}_B = P_E \left[e^{-\lambda t_s} + \frac{e^{-\lambda(t_s+T1)} - e^{-\lambda(t_s+t_f)}}{1 - e^{-\lambda TS}} \right]. \quad (55)$$

Note that as $\lambda \to 0$ equation (53) becomes the equation for RF given on page IV-11 of the helicopter paper (R1).

D. ACQUISITION OF ATTACK AND SCOUT HELICOPTERS

The attrition of attack[1] helicopters by ground weapons involves three kinds of processes: (1) the acquisition of attack helicopters by ground observers, (2) the allocation of ground firers to attack helicopters as targets, and (3) the attrition of helicopters to which fire is allocated. The calculation of attrition rates against helicopters is unaffected by the introduction of suppression into AIDM, and can proceed as currently programmed. The allocation of ground firers to attack helicopters as targets is governed by the methodology described in Chapter V, and need not be discussed further in this chapter. Item 1, the acquisition of AH's, is affected by suppression and is discussed below.

The probability a ground observer has a particular AH acquired at a random point in time is given by equation (5) of the attack helicopter paper (R1):

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[1]This section also applies to type one scout helicopters if the time spent firing, t_f, is set equal to zero. The acquisition process of type two scouts is identical to that of ground weapons.

$$\pi = \frac{E(Y_i)}{E(Y_i) + E(W_i)} , \quad (56)$$

where

$E(Y_i)$ = the mean duration of one acquired period, and

$E(W_i)$ = the mean duration of one unacquired period.

For an observer in a given group k, and for a particular AH in group ℓ, $(1 - \pi)$ is the non-acquisition probability, $Q_{k\ell}(t)$. The AH paper (R1) also gives $E(Y_i)$ and $E(W_i)$ as

$$E(Y_i) = E\left(T_E \mid T_D < T_E\right) - E\left(T_D \mid T_D < T_E\right) \quad (57)$$

$$E(W_i) = \frac{\overline{T}_c}{P_D} + \left(\frac{1}{P_D} - 1\right) E\left(T_E \mid T_E < T_D\right) + E\left(T_D \mid T_D < T_E\right) \quad (58)$$

where

T_E = the duration of an AH unmasked period,

T_D = the time required for the ground observer to detect a continuously exposed AH, and

P_D = the probability the observer detects the AH during one unmasked period before it remasks.

Although the equations for π, $E(W_i)$, and $E(Y_i)$ still hold, the introduction of suppression has changed the equations for P_D, $E(T_E|T_D<T_E)$, $E(T_E|T_E<T_D)$, and $E(T_D|T_D<T_E)$ from the equations given in the attack helicopter paper (R1). Hence, equations (8), (9), (10), and (12) of that paper must be replaced

We proceed by approximating the time for a ground observer to acquire a particular exposed AH by a negative exponential random variable.[1] Let $\hat{\mu}$ be the acquisition rate when the observer is scanning the region of the battlefield containing the AH, and let μ^* be the rate when the observer is not scanning the region. Then the average rate is

$$\mu = \hat{\mu}\, H_{I\ell}(t) + \mu^*\left[1 - H_{I\ell}(t)\right] , \tag{59}$$

where I is the communications net to which the observer belongs and ℓ is the region of the battlefield (grid square) in which the AH is located.

Let $F_E(t)$ be the cumulative probability distribution function of the helicopter exposure time, T_E. Then P_D is given by

$$P_D = \Pr\left\{T_D \le T_E\right\} \tag{60}$$

$$P_D = \int_0^\infty \Pr\left\{T_D \le t\right\}\, d\, F_E(t) \tag{61}$$

$$P_D = \frac{\mu\,(1-P_E)\left(1 - e^{-(\lambda+\mu)t_s}\right)}{\lambda+\mu} + \frac{\mu P_E\left(1 - e^{-(\lambda+\mu)\,(t_s+t_f)}\right)}{\lambda+\mu} \tag{62}$$

$E\,(T_E \mid T_D < T_E)$ is given by

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[1] The addition of hand-offs into AIDM has made the acquisition times [illegible] exponential.

$$E\,(T_E \mid T_D<T_E) = \frac{1}{P_D}\int_0^{\infty} t \; Pr\,\{T_D<t\}\; dF_E\,(t) \qquad (63)$$

$$E\,(T_E \mid T_D<T_E) = \frac{-t_s\, \mu\, (1 - P_E)\; e^{-(\lambda+\mu)t_s}}{P_D\,(\mu+\lambda)}$$

$$- \frac{\lambda}{P_D(\lambda+\mu)^2} + \frac{(1 - P_E)\; e^{-(\lambda+\mu)t_s}}{P_D\,(\lambda+\mu)^2} + \frac{1}{\lambda P_D}$$

$$- \frac{(1 - P_E)e^{-\lambda t_s}}{\lambda P_D} - \frac{P_E\; e^{-\lambda(t_s+t_f)}}{\lambda P_D}$$

$$+ \frac{P_E}{P_D}\left\{\frac{\lambda}{(\lambda+\mu)^2} - \frac{(t_s+t_f)\,\mu}{(\lambda+\mu)}\right\} e^{-(\lambda+\mu)\,(t_s+t_f)} \qquad (64)$$

$E\,(T_D \mid T_D<T_E)$ is given by

$$E\,(T_D \mid T_D<T_E) = \frac{1}{P_D}\int_0^{\infty} t\; \mu e^{-\mu t}\;\; Pr\;\{T_E>t\}\;\; dt \qquad (65)$$

$E\,(T_D \mid T_D<T_E) =$

$$\frac{(1 - P_E)\,\mu^2}{\mu P_D\,(\lambda+\mu)^2}\left[1 - \{1 + (\lambda+\mu)\; t_s\}\; e^{-(\lambda+\mu)t_s}\right]$$

$$+ \frac{P_E\;\mu^2}{\mu P_D\,(\lambda+\mu)^2}\left[1 - \{1 + (\lambda+\mu)\;(t_s+t_f)\}\; e^{-(\lambda+\mu)\,(t_s+t_f)}\right]$$

$$(66)$$

Finally, $E\,(T_E | T_E < T_D)$ is given by

$$E\,(T_E | T_E < T_D) = \frac{1}{1 - P_D} \int_0^\infty t \; Pr \left\{ T_D > t \right\} d\,F_E(t) \tag{67}$$

$E\,(T_E | T_E < T_D) =$

$$\frac{1}{1 - P_D} \left[\frac{\lambda}{(\lambda+\mu)^2} + (1 - P_E) \left\{ \frac{t_s \mu \,(\lambda+\mu) - \lambda}{(\lambda+\mu)^2} \right\} e^{-(\lambda+\mu) t_s} \right.$$

$$\left. + \; P_E \left\{ \frac{(t_s + t_f)\, \mu(\lambda+\mu) - \lambda}{(\lambda+\mu)^2} \right\} e^{-(\lambda+\mu)\,(t_s + t_f)} \right] \tag{68}$$

Note that as $\lambda \to 0$ the above equations *reduce* to equations (8), (9), (10), and (12) in Section III-B of the attack helicopter paper (R1).

We assume that AH's and type one scout helicopters are acquired, but never merely detected, in the sense that acquisition and detection are defined in Chapter V. Hence, we always have $D_{ij}(t) = 0$ for all attack helicopter groups, j, and the non-detection probability, $D_{ij}(t)$, need not be calculated.

E. ACQUISITION AND DETECTION OF TARGETS BY ATTACK AND SCOUT[1] HELICOPTERS

An unsuppressed AH searches for targets during a period of length t_s, but because of suppression the duration of the search period is a random variable with maximum t_s. Since only those AH's not suppressed before time t_s after unmasking can be allocated to fire at targets, the computation of acquisition probabilities for allocation of fire is unchanged from that utilized in the absence of suppression. The computation of detection probabilities for allocation of fire is done in a parallel manner. These computations are presented in Section 1, below. For helicopters that are suppressed before reaching t_s it is assumed that they are still capable of handing off any acquisitions they may have made. Hence, for purposes of hand-off information, the probability of acquisition must be averaged over the random duration of the search period. These computations are presented in Section 2, below.

1. Acquisitions and Detections for Allocation of Fire by Helicopters

In this section we wish to obtain the following quantities:

P_E = the probability an attack helicopter that stays unsuppressed during its search period acquires or detects a target eligible to be engaged[2],

Q_{ij} = the probability an AH in group i does not acquire a particular target in group j during a search period of duration t_s, and

D_{ij} = the probability an AH in group i does not detect a particular target in group j during a search period of duration t_s.

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[1]This section also applies to type one scout helicopters if the time spent firing, t_f, is set equal to zero. The acquisition and detection processes of type two scouts are identifical to those by ground weapons.

[2]That is, a target that is within range of the helicopter's armament and is on the helicopter's priority list of targets.

The quantity P_E is used in the computations of Sections B, C, and D in this chapter. The quantities Q_{ij} and D_{ij} are used in the allocation methodology described in Chapter V.

The probability that an exposed AH in group i fails to acquire a particular target in group j during a search interval of duration t is

$$q_{ij}(t) = H'_{i\ell}(M\Delta t)\, e^{-\hat{a}_{ij}t}\, PA_{ij}^{\mathrm{NEWAMO}(j)\,\frac{t}{\Delta t}}$$

$$+ \left[1 - H'_{i\ell}\,(M\Delta t)\right] e^{-a_{ij}t}\, PA_{ij}^{\mathrm{NEWAMO}(j)\,\frac{t}{\Delta t}} , \qquad (69)$$

where

$M\Delta t$ = the time of the current time step in AIDM,

$\hat{a}_{ij}$ = the rate of making non-firing acquisitions for a single AH in group i observing a particular target in group j when the grid square occupied by group j is being scanned by group i,

PA_{ij} = the probability an AH in group i does not acquire by means of pinpoint a single round fired by a weapon in group j when the grid square occupied by group j is or is not[1] being scanned by group i,

a_{ij} = the rate of making non-firing acquisitions for a single AH in group i observing a particular target in group j when the grid square occupied by group j is not being scanned by group i,

[1] In the hand-off paper (R2) the pinpoint probability is the same when a grid square is and is not being scanned.

NEWAMO(j) = the number of rounds fired by a weapon in group j during the last time step of duration Δt, and

$H'_{l\ell}(M\Delta t)$ = the probability that an unmasked AH on communications net l is actually scanning grid square ℓ.

The probability that an exposed AH in group i fails to detect a particular target in group j during a search interval of duration t is computed similarly to $q_{ij}(t)$, but from rates of detection, rather than of acquisition. Thus, the probabilities needed for allocating AH firers to targets are:[1]

$$Q_{ij} = q_{ij}(t_s) \quad , \tag{70}$$

$$D_{ij} = 1 - q_{ij}(t_s) + q_{ij}(t_s)^{R+1}, \text{ and} \tag{71}$$

$$P_E = 1 - \prod_j g_{ij} \quad , \tag{72}$$

where the product is taken only over eligible target groups, j, and where

$$g_{ij} = \begin{cases} \left[Q_{ij} D_{ij}\right]^{n_j} & \text{if } n_j > 1 \\ 1 - n_j \,(1 - Q_{ij} D_{ij}) & \text{if } n_j \le 1 \end{cases}$$

2. Acquisitions for Hand-Offs by Helicopters

When calculating acquisition probabilities for the purpose of information hand-off, the randomness of the AH search period must be taken into account. The probability of not acquiring a particular target in group j, averaged over the duration of the search period, is the expected value of $q_{ij}(t)$:

$$E\left[q_{ij}\right] = \int_0^\infty \lambda e^{-\lambda x} q_{ij}(x)\,dx + q_{ij}(t_s)\, e^{-\lambda t_s}. \tag{73}$$

[1]Equation (71) is obtained by approximating the acquisition and detection times with negative exponential variates whose rates are assumed to be in the ratio R, as is done in Section V-A.

Let $v_{ij} = a_{ij} - \frac{NEWAMO(j)}{\Delta t} \ell n(PA_{ij})$, and

$$\hat{v}_{ij} = \hat{a}_{ij} - \frac{NEWAMO(j)}{\Delta t} \ell n(PA_{ij}).$$

Now, the expected probability of non-acquisition can be written as follows:

$$E[q_{ij}]=H'_{l\ell}(M\Delta t)E[e^{-\hat{v}_{ij}t}] + [1-H'_{l\ell}(M\Delta t)]E[e^{-v_{ij}t}], \tag{74}$$

where

$$E[e^{-\hat{v}_{ij}t}] = \frac{\frac{\lambda}{\hat{v}_{ij}+\lambda} + \left[\frac{\hat{v}_{ij}}{\hat{v}_{ij}+\lambda} - P_E\right] e^{-(\hat{v}_{ij}+\lambda)t_s}}{1-P_E e^{-\lambda t_s}}, \tag{75}$$

and where

$$E[e^{-v_{ij}t}] = \frac{\frac{\lambda}{v_{ij}+\lambda} + \left[\frac{v_{ij}}{v_{ij}+\lambda} - P_E\right] e^{-(v_{ij}+\lambda)t_s}}{1-P_E e^{-\lambda t_s}} \tag{76}$$

On page III-5 of the hand-off paper (R2) $p_{ik}(M\Delta t)$ is defined as the probability an observer in group i acquires a particular group-k target in time interval $[(M-1)\Delta t, M\Delta t]$. When the observer is an attack or type one scout helicopter, the appropriate value of this probabilitv is:

$$p_{ik}(M\Delta t) = \frac{\Delta t}{E(X)}[1-E(q_{ik})], \tag{77}$$

For $\Delta t \leqslant E(X)$, and

$$P_{ik}(M\Delta t) = 1 - E(q_{ik})^{\frac{\Delta t}{E(X)}} , \qquad (78)$$

for $\Delta t > E(X)$. These equations should be used for hand-off by attack and type one scout helicopters in place of the calculations described on page IV-2 of the hand-off paper (R2). Recall that only acquired targets are assumed to be handed off.

CHAPTER VII

MODEL IMPLEMENTATION

This chapter describes the computer program changes and data base additions required to represent suppression in the existing TRASANA AIDM. However, at the time this document is being written, programming is under way to incorporate earlier results concerning attack helicopters (R1), scout helicopters (R2), and information hand-off (R2) into the TRASANA AIDM. The final form the computer program will take is, as a result, somewhat uncertain, making it impossible to describe implementation in detail.

The first six sections in this chapter describe the changes in the computer program required to represent suppression in AIDM. Section A describes the implementation of bookkeeping procedures to keep track of the numbers of suppressed and unsuppressed surviving weapons. Section B addresses the implementation of the procedures described in Chapter IV to compute suppression probabilities. Section C describes the procedures to include detections of targets in AIDM and to account for possible reductions in accuracy of fire when detected targets are engaged. Section D describes the implementation of the information carryover process defined in Section III-C. Section E describes the new procedures to allocate fire between suppressed and unsuppressed weapons and to include allocation of fire to detected weapons. Section F addresses the program associated with the suppression of and by attack and scout helicopters. The last section, Section G, describes additions to the data base.

A. BASIC FEATURES OF THE SUPPRESSION MODEL

Three basic aspects of the suppression model are addressed in this section: the bookkeeping of the inventories of suppressed and unsuppressed survivors; the capabilities of suppressed weapons; and the use of the existing AIDM suppression model as an alternative to the one presented in this paper. These aspects are outlined below:

1. Initialize $Prsup_j$ to zero for all j before the first time step.
2. Bookkeeping weapon inventories is presently performed in AIDM in subprogram UPDATE, where the numbers of weapons in each group killed during the past time step are subtracted from the group inventories. Suppressed and unsuppressed weapons could also be bookkept in this section of code.
   (a) Before the current group inventories, n_j, are updated, determine the number of weapons leaving each group j.
      (1) If j is an unsuppressed group, use equation (4) to calculate δ_j, the number of weapons in j suppressed during the time step. In equation (4) $Prsup_j$ will have been calculated during the preceding time step.
      (2) If j is a suppressed group, replace $Prsup_j$ with $Prun_j$ in equation (4), and calculate δ_j, the number of weapons in j unsuppressed during the time step.
   (b) Modify the existing logic updating the group inventories to subtract δ_j and to add $\delta_{M(j)}$ to n_j, as done in equation (5).
   (c) If group j is a helicopter group, the logic in steps (a) and (b) does not apply. The helicopter group inventory is updated as done without the suppression model, except for type two scout helicopters (see section VI-A), which behave like ground weapons.
3. Since suppressed weapons are represented as groups separate from unsuppressed groups, there are no programming changes to be made to allow for the capabilities of suppressed weapons. Each group

in AIDM has associated with it a weapon type and line-of-sight status that are input by the user, and the performance parameters for the weapon type are specified as data.

4. The existing models of suppression in AIDM (the artillery suppression and the direct fire suppression models) are to be maintained as alternatives to the method described in this paper. Through an input switch the user will select the old or new model, and the program logic must use the logic corresponding to the selected model.

   (a) If the switch selects the old models, then the calculation of $Prsup_j$, implemented as described in Section VI-B, should be skipped and $Prsup_j$ set to zero.

   (b) If the switch selects the new model, the old models must be skipped.

      1. In subprogram UPDATE the calculation of SUPP and UNSUPP should be skipped, so as to set them to zero and unity, respectively. This can be accomplished through input by setting BETA to zero. (See lines 302-305 of UPDATE).

      2. In subprogram RATE the suppression factor, FACTOR, must be set to unity.

B. REQUIRED ADDITIONAL LOGIC TO IMPLEMENT THE CALCULATION OF THE PROBABILITY OF SUPPRESSION

1. To insert into the program the calculation of the probability of suppression, $Prsup_j$, additional logic in model UPDATE includes the following:

(a) For unsuppressed group j calculate $Prsup_j$ from eq. (15)

(b) $S_{ARTY}(j)$, the fraction of weapons in group j not suppressed by artillery during the time step, is determined from input (indexed on group j weapon type), when group j is receiving artillery fire. Set $S_{ARTY}(j) = 1$, when group j is not receiving artillery fire.

(c) For unsuppressed groups j, determine $S_{DF}(j)$, the fraction of weapons in group j not suppressed by artillery during the time step from eq. (16), with the product over all groups i firing at target j. Since this calculation involves e_{ij}, the fire allocation factor, and since e_{ij} is computed during the iteration over firer-target pairs in ATTRIT, the calculation of $S_{DF}(j)$ could most easily be done in the firer-target loop. Before entering the loop, $S_{DF}(j)$ could be set to zero for each j, and then each time group j is considered as a target, $S_{DF}(j)$ could be incremented by $S_{DF}(i,j)$:

1) Determine $S_{DF}(i,j)$ from equation (17).

2) Determine $Prsup_{ij}$ from equation (18).

3) Determine P_{ij} from input.

2. The above step applies if group j is a ground weapon or a type two scout helicopter group. If group j contains attack helicopters or type one scout helicopters, then step (1) of Section VII-F applies.

C. REQUIRED ADDITIONAL LOGIC TO IMPLEMENT THE EFFECTS OF DETECTIONS AND SUPPRESSIVE AREA FIRE

(1) To include into the program the representation of the effects of detections, additional logic in module UPDATE includes the following:

(a) Compute $D_{ij}(t)$, the cumulative probability of ground weapon and type 2 air scout detectors not detecting a target, as follows:

1) Intialize: $D_{jk}(o)=1$ before the first time step.

2) If line-of-sight does not exist, set $D_{jk}(t)=1$.

3) Compute $D_{jk}(t)$ from $Q_{jk}(t)$ using equation (19) after $Q_{jk}(t)$ has been updated. Detections by AH's are computed differently. (See Section VII-F.)

(b) Modify $D_{jk}(t)$ to D'_{jk} by eq. (20) to account for the forgetting of detections where β, the reciprocal of the mean time a detection is remembered, is determined form input and is indexed on the detector weapon type. Eq. (20) should be applied before new detections for a time step are used to update $D_{jk}(t)$.

(2) To include into the program the representation of the effects of suppressive area fire, additional logic includes the following:

In module ATTRIT, when computing the hit probabilities, check to see if the fire being allocated is suppressive area fire. If not, the existing logic applies. If it is suppressive area fire, modify the calculations as follows:

In subroutine RATE, when calculating SBP1, SBU, and SBP for the case of suppressive area fire, modify the utilized horizontal and vertical dispersions (standard deviations):.

a) $\sigma_x \leftarrow (\sigma_x^2+\sigma_{Mx}^2+\sigma_{Ax}^2)^{\frac{1}{2}}$ and

b) $\sigma_y \leftarrow (\sigma_y^2+\sigma_{My}^2+\sigma_{Ay}^2)^{\frac{1}{2}}$

where σ_x and σ_y refer to dispersions for the first round, for rounds following a miss or for rounds following a hit. σ_{Mx}, σ_{Ax}, σ_{My}, and σ_{Ay} are defined in Section V-B, are determined from input, and are indexed on firer weapon type and range.

D. REQUIRED ADDITIONAL LOGIC TO IMPLEMENT THE EFFECTS OF INFORMATION CARRYOVER

(1) To implement the representation of the effects of information carryover, additional logic in module UPDATE includes the following:

(a) Before computing new acquisitions, and before updating $Q_{ij}(t)$ (the probability that a given firer in group i has not acquired a particular target in group j at time t) to account for acquisitions made or lost during $(t,t+\Delta t)$, adjust for all i and j $Q_{ij}(t)$ to $Q'_{ij}(t)$ to account for suppressions occuring during $(t,t+\Delta t)$, as follows:

Calculate $Q'_{ij}(t)$ from eq. (6) using eqs. (7), (8), (4), an equation analogous to eq.(4) for $\delta_{M(j)}$ in terms of $Prun_j$, and equations similar to eq.(9) for replacing $Q_{M(i)j}(t)$ and $Q_{M(i)M(j)}(t)$ by $Q^*_{M(i)j}(t)$ and $Q^*_{M(i)M(j)}(t)$, respectively, to account for the forgetting of acquisitions. $Prun_j$ is defined following eq. (4), determined from input, and indexed on weapon type in group j. $\overline{P}_x$ in eq.(9) is defined following eq.(9), determined by input, and indexed on weapon type in unsuppressed group x.

(b) Using logic parallel to that for adjusting $Q_{ij}(t)$ to $Q'_{ij}(t)$, adjust $D_{ij}(t)$ to $D'_{ij}(t)$ as follows for all i and j:

Calculate $D'_{ij}(t)$ from eq. (10) using eqs. (11), (12), (4), an equation analogous to eq.(4) for $\delta_{M(j)}$ in terms of $Prun_j$, and eqs. (13) and (14) for replacing $D_{M(i)j}(t)$ and $D_{M(i)M(j)}(t)$ by $D^*_{M(i)j}(t)$ and $D^*_{M(i)M(j)}(t)$ respectively, to account for the forgetting of detections. $Prun_j$ is defined following eq.(4), determined from input, and indexed on weapon type in group j. $\overline{\overline{P}}_{M(i)}$, in eqs. (13) and (14), is defined following eq. (14), determined by input, and indexed on weapon type in unsuppressed group M(i).

E. THE ALLOCATION OF FIRERS TO TARGETS

Section V-C of this paper describes a methodology to distribute fire over suppressed and unsuppressed weapons and to allow fire at detected targets. The implementation of this allocation scheme is outlined below.

(1) The subprogram PRIRTY is called prior to the first time step's combat to compute target priorities at range zero. Provision must be made to allow each group to appear in the priority list twice -- once as a detected target and once as an acquired target -- and to tag each occurrence as fire at an acquired target or a detected target (suppressive area fire). Only the unsuppressed groups need be in the list.

(2) Target priorities are updated every time step as functions of firer-target range. The program logic must compute the priorities of each group twice, once as an acquired target and once as a detected target. The priority of a group is computed as AR + B where B is the range zero priority, R is the range, and A is the slope. Separate values of A and B, obtained from input, must be used for a group considered as an acquired or detected target. (Helicopters are never detected, and the allocation equations will never allocate to detected helicopters.)

(3) Subprogram ALLOC is called from ATTRIT to return the group of highest priority to which the current firer has not been allocated. Only the unsuppressed groups are returned to ATTRIT.

(4) If the target is an acquired group, calculate the fraction of the firing group allocated to the target and its paired group as $(1 - E)\ (1 - \xi_{ij}\xi_{iM(j)})$, using equation (21).

(5) If the target is a detected group, calculate the fraction of the firing group allocated to its target and its paired group as $(1 - E)\ (1 - \phi_{ij}\phi_{iM(j)})$, using equation (34).

(6) For an acquired target, calculate the fractions of the firer group allocated to the suppressed and unsuppressed portions of the target, $e_{iM(j)}$ and e_{ij}, using equations (32) and (33). The terms of these equations are obtained from equations (23) through (31).

(7) For a detected target calculate the fractions of the firer group allocated to the suppressed and unsuppressed portions of the target, $e'_{iM(j)}$ and e'_{ij}, using equations (36) and (37). The terms of these equations are obtained in a manner parallel to equations (23) through (31), as described in Chapter V.

(8) Loop over the suppressed and unsuppressed groups of the current target pair, allocating fire to them with the fractions calculated above, and apply the existing attrition logic to each group as a separate target.

F. REQUIRED ADDITIONAL LOGIC TO REPRESENT THE SUPPRESSION OF AND BY ATTACK AND SCOUT HELICOPTERS

This section addresses the additions and modifications of the logic of AIDM to account for suppression of and by attack and scout helicopters. Most of these changes are modifications of equations already implemented in AIDM and presented in an earlier paper that describes AH activities in the absence of suppression. The changes are described below.

(1) For each group of AH's and of type one scout helicopters the suppression rate λ must be computed. The suppression probability $Prsup_j$ need not be computed if group j is an AH or type one scout helicopter group. The computation of λ could perhaps be organized most easily in a manner similar to the computation of $Prsup_j$ for other groups. Let us call this rate λ_j for AH group j. Then λ_j could be computed from equations (45) and (46) in the same sequence that $Prsup_j$ is computed, as described in Section VII-B.

(2) To modify attrition rates for AH firers, except for zero suppression ($\lambda = 0$), required modifications of the logic in subroutine RATE includes the following:

(a) In calculating attrition rates if the AH fires a missile:

1. Replace equations (2) and (4) of the helicopter paper (R1) by eqs.(42) and (43). λ, the total suppression rate for ground weapons vs. AH's, is determined from equations (45) and (46) as described in step (1), above. Accumulate λ_j in the firer loop, save for all AH groups j, then use for AH firers in next Δt.

2. Replace the equation for RMISS, the rate of firing missiles, on page IV-10 of the helicopter paper (R1) by eq.(44).

3. Replace the equation for ALPHA, the attrition rate, on page IV-11 of the helicopter paper (R1) by the reciprocal of eq. (43).

(b) In calculating attrition rates if the AH fires a burst-fire type automatic cannon:

1. Replace the equation for RF, the rate of firing, on Page IV-11 of the helicopter paper (R1) by eq. (53) using eqs.(55), (51), and (52).

2. Replace the equation for ALPHA, the attrition rate, on Page IV-11 of the helicopter paper (R1) by eq.(47) using eqs. (50), (51) and (52).

(c) When $\lambda_j = 0$, the program should use the original equations (R1).

(3) To modify acquisitions of AH's[1] required modifications of the logic in module UPDATE and the subroutines it calls includes the following:

(a) Replace eqs. (8), (9), (10), and (12) of the helicopter paper (R1) by eqs. (52), (66) and (68), except in the case of zero suppression, $\lambda = 0$.

(b) Replace μ, the total acquisition rate of AH's, of these equations by the μ calculated from eq.(59) while utilizing eq.(4.5) of the hand-off paper (R2), with $\hat{\mu}$ and μ^* determined from inputs.

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[1]This section also applies to type one scout helicopters. Type two scout helicopters are treated identically to ground weapons, except that they are never merely detected.

(4) To modify acquisitions by and include detections by AH's,[1] required modifications of the logic in module UPDATE and the subroutines it calls includes the following:

(a) Replace the calculation of Q_{ij}, the probability an AH in group i does not acquire a particular target in group j during a search period of duration t_s, by eqs. (69), (70), and the equation for $H_{I\ell}$ $(M\Delta t)$ on pg. III-10 of the handoff paper (R2) with a_{ij}, $\hat{a}_{ij}$, and PA_{ij} defined following eq. (69) and determined from input.

(b) Using logic parallel to that for Q_{ij}, compute D_{ij}, the probability an AH in group i does not detect a particular target in group j during a search period of duration t_s, using eq. (71), the equation for $H'_{I\ell}$ $(M\Delta t)$ on pg. III-10 of the handoff paper (R2), and rates of detection analogous to those defined following eq. (69). Do not update D_{ij} each time step by multiplying by the cumulative detection probability. Rather, update D_{ij} each time step by calculating it anew via eq. (71).

(c) Replace the calculation of P_E, the probability an attack helicopter that stays unsuppressed during its search period acquires or detects a target eligible to be engaged, discussed in paragraph 14 on page IV-8 of the helicopter paper (R1) by eq. (72) using eqs. (69), (70) and (71).

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[1]This section also applies to type one scout helicopters if the time spent firing, t_f, is set equal to zero. Type two scouts are treated identically to ground weapons, except that they are never merely detected.

(d) For the purposes of information hand-off calculate $E(q_{ij})$, the probability of not acquiring a particular target in group j averaged over the duration of the search period, by utilizing eqs. (74), (75), (76), and the equation for $H'_{i\ell}$ $(M\Delta t)$ on on pg. III-10 of the hand-off paper (R2).

(e) Replace the calculation of $P_{ik}(M\Delta t)$, the probability an observer in group i acquires a particular group k target in time interval $[(M - 1)\ \Delta 5,\ M\Delta t]$, when the observer is an attack or type one scout helicopter, described on pg. IV-2 of the hand-off paper (R2), by eq. (77) using eqs. (74) and (51).

G. REQUIRED ADDITIONAL INPUT DATA

This section lists the types of required additional input data to represent the effects of suppression in the TRASANA AIDM. The data are categorized into four classes: data relating to (1) performance, (2) tactics, (3) status, and (4) options.

1. Performance Input Data

SYMBOL USED ABOVE	DEFINITION
$\overline{P}_x$	Probability a member of unsuppressed group x does not forget an acquisition upon becoming suppressed, and indexed on weapon type in unsuppressed group x.

SYMBOL USED ABOVE	DEFINITION
R	Ratio of the detection rate to the acquisition rate indexed on the types of the observing and observed weapons. The ratio must be greater than or equal to one. Values of R are not required if the observed weapon is an AH.
$\bar{P}_x$	Probability a member of unsuppressed group x does not forget a detection upon becoming suppressed, and indexed on weapon type in unsuppressed group x.
$Prun_j$	Probability a member of group j becomes unsuppressed during Δt if currently suppressed, and indexed on weapon type in group j.
$S_{ARTY}(j)$	The probability a weapon (the fraction of weapons) in group j not suppressed by artillery during the time step Δt, and indexed on group j weapon type. This input could be determined from any appropriate source such as historical data or a detailed model of artillery coverage.
P_{ij}	The expected fraction of group j suppressed by each round fired at group j by a weapon

SYMBOL USED ABOVE	DEFINITION
	in group i, indexed on the weapon types in groups i and j, round type, range, and target cover status.
$1/\beta$	The mean time group weapons and type two scout detectors remember a detection, and indexed on detector weapon type.
$\sigma_{Mx}(\sigma_{My})$	Standard deviation of the x-cordinate (y-coordinate) of the mean aim point, indexed on firer weapon type and range.
$\sigma_{Ax}(\sigma_{Ay})$	Standard deviation of the horizontal (vertical) component of the offset of the mean aim point, and indexed on firer weapon type and range.
———	Since weapon type can be specified by input as a function of group number, paired suppressed and unsuppressed groups can be assigned different weapon types to allow for different capabilities of the same weapon in the two states. No new data items need be defined, but performance data must be specified if suppressed groups contain unique weapon types.

2. Tactics Input Data

SYMBOL USED ABOVE	DEFINITION
—	Basic priorities for every weapon type considered as a detected target. (Priorities for acquired targets are in the existing data base.)

3. Status Input Data

SYMBOL USED ABOVE	DEFINITION
M (i)	Table consisting of the paired group for every suppressed and unsuppressed group. If i is unsuppressed (suppressed), then M(i) is the paired suppressed (unsuppressed) group.
—	All weapons played in AIDM require data defining their movement[1] and line-of-sight statuses. No new data items need to be defined, but all existing data items describing groups (e.g., cover, velocity), must be specified for both suppressed and unsuppressed weapons.

[1] The user should input identical coordinates and velocities for paired suppressed and unsuppressed groups.

4. Selection of Options

SYMBOL USED ABOVE	DEFINITION
____	Switch to select one of two optional suppression models: (1) the one in the existing AIDM program, or (2) the one described in this paper.

APPENDIX A

A MODEL FOR THE SINGLE SHOT SUPPRESSION PROBABILITY

Chapter IV models the computation of the fraction of a group suppressed in an AIDM time increment from the single shot suppression probability, P_{ij}. This appendix presents a model from which P_{ij} might be generated as input to AIDM. Section 1 addresses the averaging of suppressive effects over all members of the target group. Section 2 addresses the averaging of hit and near miss probabilities over rounds with different accuracies. Section 3 computes the hit probabilities, and Section 4 computes near-miss probabilities.

1. The Single-Round Suppression Probability

All groups with more than one member can be thought of as spread over an area. The coordinate of the centroid of the group is bookkept in AIDM, but AIDM has no geometry of the spacing of weapons within groups. To determine the fraction of a group that is suppressed due to the effects of a single round, this section introduces a simple model of intra-group geometry and averages the suppression probability of a single round over the members of the group.

Consider a weapon which has been acquired or detected, and at which fire is directed. Other members are taken to be located[1] around the target, each at a distance which is a function of weapon type and the number of survivors in the group. This distance is furnished via an input table indexed on weapon type and number of survivors. The target weapon is taken to be at the center of a circle, and the n_j-1 other group members

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[1]The geometry of the target group is described in a plane perpendicular to the line of sight between firer and target.

(secondary suppressees) are assumed oriented around the target at the specified distance and separated by an angle of $360°/(n_j-1)$. (If $n_j \leq 1$, only the target weapon is considered.) Then, in terms of a coordinate system centered at the target weapon (primary suppressee), the coordinates of the other weapons (secondary suppressees) are taken to be (x_i, y_i) for i=1 to n_j-1, where

$$x_i = r \cos \theta_i, \tag{79}$$

$$y_i = r \sin \theta_i, \tag{80}$$

$$\theta_i = \frac{360°(i-1)}{n_j-1} \tag{81}$$

and where r is the tabulated radius of the circle. Let $P_S(x, y)$ denote the probability that a round aimed at the origin suppresses a target centered at coordinates (x, y). Then, the average fraction of the group suppressed by a single round is P_{ij}, where

$$P_{ij} = \frac{P_S(0,0) + \sum_{i=1}^{n_j-1} P_S(x_i, y_i)}{n_j} \tag{82}$$

for $n_j \geq 2$, and where

$$P_{ij} = P_S(0, 0) \tag{83}$$

for $n_j \leq 1$. If n_j is non-integer and ≥ 1, then these equations are used to calculate P_{ij} for the integer values nearest to n_j and the results are interpolated to n_j.[1]

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[1] In practice, a simple linear interpolation technique will suffice.

Suppression is assumed to be caused by non-lethal hits and near misses. For a weapon that is offset at coordinates (x, y) from an aim point at the origin, the probability of being suppressed by a single round is:

$$P_S(x, y) = P_{NLH}(x, y)\, P_{S|NLH} + P_{NM}(x, y)\, P_{S|NM}, \tag{84}$$

where

$P_{NLH}(x, y)$ = the probability of a non-lethal hit,

$P_{NM}(x, y)$ = the probability of a near miss (an impact in a certain area around the target),

$P_{S|NLH}$ = the probability of suppression given a non-lethal hit (an input dependent on round and target type), and

$P_{S|NM}$ = the probability of suppression given a near miss (an input dependent on round and target type).

The probability of a non-lethal hit is:

$$P_{NLH}(x, y) = P_H(x, y) \left[1-P_{K|H}\right], \tag{85}$$

where

$P_{K|H}$ = the probability of a kill given a hit, and

$P_H(x, y)$ = the probability of hitting a target at offset (x, y) from the aim point.

The probability of a kill given a hit, $P_{K|H}$, is a datum presently in AIDM. The probabilities of a hit and a near miss are the integrals of the distribution of impact points over the target and the target's surrounding near-miss areas, respectively. These integrals are presented in the remaining sections of this chapter.

2. The Probabilities of Hit and Near-Miss

The suppression probability derived in Section 1 of this chapter is a function of P_H (x, y) and P_{NM}(x, y), the single-round probabilities of a hit and a near-miss, respectively, for a target (either primary or secondary) centered at an offset (x, y) from the mean aim point. The problem would simply be one of integrating the elliptical normal delivery distribution over the target area or near-miss area if it were not for the fact that not all of a firer's rounds directed at a target are necessarily sampled from the same probability distribution of impact points. Specifically, first rounds, rounds following misses, and rounds following hits are allowed to have different variances in AIDM. Consequently, the hit and near-miss probabilities must be obtained for an average mix of first rounds, rounds following hits, and rounds following misses.

We wish to ascertain the proportions of rounds that are first rounds, rounds following hits, and rounds following misses and to use these proportions to calculate average values of the hit and near miss probabilities. However, since a firing sequence can be interrupted when a target remasks or is killed from any one of a number of firers, these proportions are governed by a very complex stochastic process. Consequently, the proportions cannot be determined exactly. Instead, the corresponding proportions will be determined for a simpler process in which a single firer and single target are isolated. If we consider a passive target that is engaged until it is killed (ignoring the possibility of a suppression occurring first), then we can utilize quantities now present in AIDM to determine the fractions of first rounds and rounds following hits and misses of the target. If $\overline{N}$ is the expected number of rounds to kill the primary target,[1] then the expected fraction of rounds that are first in the firing sequence is

[1] $\overline{N}$ is the variable ENCALC in AIDM.

$$f_1 = \frac{1}{\bar{N}} . \qquad (86)$$

Let $P_{K|H}$ be the probability of a kill given a hit.[1] Since the expected number of hits to kill the target is $1/P_{K|H}$, and since one of these hits terminates the sequence, the expected fraction of rounds following hits is:

$$f_H = \frac{1}{\bar{N}} \left(\frac{1}{P_{K|H}} - 1\right). \qquad (87)$$

Since the enumerated situations exhaust the possibilities, the fractions sum to unity, and the fraction of rounds that follows misses is:

$$f_M = 1 - \frac{1}{\bar{N}\, P_{K|H}} \qquad (88)$$

Using these fractions, the unconditional probabilities of hit and of near miss are:

$$P_H (x, y) = f_1 P_H (x, y, a_1, b_1) + f_H P_H (x, y, a_H, b_H) + f_M P_H (x, y, a_M, b_M) \qquad (89)$$

$$P_{NM}(x, y) = f_1 P_{NM} (x, y, a_1, b_1) + f_H P_{NM} (x, y, a_H, b_H) + f_M P_{NM} (x, y, a_M, b_M) \qquad (90)$$

where

$P_H (x, y, a, b)$ = the probability of hitting a target located at an offset (x, y) with a round having horizontal and vertical standard deviations a and b, respectively, and

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[1] $P_{K|H}$ is the variable SBPK in AIDM.

P_{NM} (x, y, a, b) = the probability of a near-miss for a target located at an offset of (x, y) with a round having horizontal and vertical standard deviations a and b, respectively.

The standard deviations are defined as follows:

a_1 = horizontal dispersion for the first round,

b_1 = vertical dispersion for the first round,

a_H = horizontal dispersion for rounds following a hit,

b_H = vertical dispersion for rounds following a hit,

a_M = horizontal dispersion for rounds following a miss, and

b_M = vertical disperson for rounds following a miss.

The functions P_H (·) and P_{NM} (·) are displayed in the following section.

3. The Conditional Hit Probabilities

The hit probabilities P_H (0, 0, ·, ·) are calculated presently in AIDM as part of the computation of the attrition rate.[1] However, the hit probabilities with non-zero offsets (i.e., for secondary targets) are new and are derived in this section. Section a presents the computations for hull defilade targets, and Section b presents the computations for fully exposed targets. These computations are applicable both to primary and secondary suppressees (that is, both zero and non-zero biases).

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[1] In AIDM the variable names are SBP1, SBU, and SBP for probabilities of hit on a first round, a round following a hit, and a round following a miss, respectively.

a. Hit Probabilities for Hull Defilade Targets

The hull defilade target is represented as a rectangle of width X_T and height Y_T, the center of which is offset at coordinate (x, y) from the aim point at the origin. The hit probability is

$$P_H(x, y, a, b) = I_{XT} I_{YT}, \tag{91}$$

where

$$I_{XT} = \frac{1}{a\sqrt{2\pi}} \int_{x-X_T/2}^{x+X_T/2} \exp\left[-\frac{Z^2}{2a^2}\right] dZ, \tag{92}$$

which becomes, after change of variables:

$$I_{XT} = 1/2 \ \text{erf}\left(\frac{2x + X_T}{2\sqrt{2}\,a}\right) - 1/2 \ \text{erf}\left(\frac{2x - X_T}{2\sqrt{2}\,a}\right). \tag{93}$$

Similarly:

$$I_{YT} = 1/2 \ \text{erf}\left(\frac{2y + Y_T}{2\sqrt{2}\,b}\right) - 1/2 \ \text{erf}\left(\frac{2y - Y_T}{2\sqrt{2}\,b}\right), \tag{94}$$

where erf is the error function,

$$\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-z^2)\, dz .$$

In what follows, we use the fact that erf(-x) = - erf(x)

b. Hit Probabilities for Fully Exposed Targets

Fully exposed targets are represented as a rectangular turret of dimensions (X_T, Y_T) and a rectangular hull of dimensions (X_H, Y_H). The bottom of the turret is contiguous with the top of the hull and the center of these sides coincide. The center of the bottom of the turret and of the top of the hull are assumed to be located at the offset (x, y), with the aim point at the origin. The hit probability in this case is:

$$P_H(x, y, a, b) = I'_{XT} I'_{YT} + I'_{XH} I'_{YH} \quad , \tag{95}$$

where

$$I'_{XT} = 1/2 \text{ erf} \left(\frac{2x + X_T}{2\sqrt{2}\, a} \right) - 1/2 \text{ erf} \left(\frac{2x - X_T}{2\sqrt{2}\, a} \right), \tag{96}$$

$$I'_{YT} = 1/2 \text{ erf} \left(\frac{y + Y_T}{\sqrt{2}\, b} \right) - 1/2 \text{ erf} \left(\frac{y}{\sqrt{2}\, b} \right), \tag{97}$$

$$I'_{XH} = 1/2 \text{ erf} \left(\frac{2x + X_H}{2\sqrt{2}\, a} \right) - 1/2 \text{ erf} \left(\frac{2x - X_H}{2\sqrt{2}\, a} \right), \tag{98}$$

and

$$I'_{YH} = 1/2 \text{ erf} \left(\frac{y}{\sqrt{2}\, b} \right) - 1/2 \text{ erf} \left(\frac{y - Y_H}{\sqrt{2}\, b} \right) . \tag{99}$$

4. <u>The Conditional Near Miss Probabilities</u>

This section presents the near-miss probability, $P_{NM}(x,y,a,b)$. Section a gives the computations for hull defilade targets, and Section b presents the computations for fully exposed targets. The computations apply to both primary and secondary suppressees (i.e., for x and y zero or non-zero).

a. <u>Near Miss Probabilities for Hull Defilade Targets</u>

The target is assumed to be surrounded by an area of dimensions X_S, Y_S within which rounds that do not hit the target can have suppressive effects. The suppressive area is assumed to be centered at the offset (x,y) as is the turret which is of dimensions X_T, Y_T. The probability of a near-miss is the probability a round lands in the suppressive area but not in the turret area. Hence:

$$P_{NM}(x,y,a,b) = P_{AREA} - P_{TURRET} \, , \tag{100}$$

where the probability of hitting the suppressive area is:

$$P_{AREA} = I_{XA} \, I_{YA} \, , \tag{101}$$

and where

$$I_{XA} = \frac{1}{2} \operatorname{erf}\left(\frac{2x + X_S}{2\sqrt{2}\, a}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{2x - X_S}{2\sqrt{2}\, a}\right) , \tag{102}$$

and

$$I_{YA} = \frac{1}{2} \operatorname{erf}\left(\frac{2y + Y_S}{2\sqrt{2}\, b}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{2y - Y_S}{2\sqrt{2}\, b}\right) . \tag{103}$$

The probability of hitting the turret has been calculated already in Section 3-a of this chapter.

Hence,

$$P_{TURRET} = I_{XT} \, I_{YT} \, , \tag{104}$$

which is the result from equation (91).

b. Near Miss Probabilities for Fully Exposed Targets

The center of the suppressive area is taken to be located at the offset (x , Y), as is the midpoint of the bottom of the turret. Fire is aimed at the origin. The probability of a near miss is the probability a round lands in the rectangular suppressive area but not in the fully exposed target area. It should be noted that the target is not necessarily centered equally distant from the top and bottom of the suppressive area, but rather that the midpoint of the line joining the hull and turret is assumed located at the center of the suppressive area. Hence, the near-miss probability is:

$$P_{NM}(x,y,a,b) = P_{AREA} - P_{TARGET} , \qquad (105)$$

where P_{AREA}, the probability of hitting the suppressive area, is given in Section 4-a of this chapter, and P_{TARGET}, the probability of hitting a fully exposed target, is computed in Section 3-b of this chapter with equation (95).

APPENDIX B

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