

AD-A047 541

COLD REGIONS RESEARCH AND ENGINEERING LAB HANOVER N H F/G 8/12
CALCULATION OF CONVECTIVE HEAT EXCHANGE DURING THE THAWING OF W--ETC(U)
NOV 77 N V UKHOVA

UNCLASSIFIED

CRREL-TL-664

NL

| OF |
AD
A047541



END
DATE
FILMED
1 -78
DDC

19 CRREL -
TL-664



Draft Translation 664

11 Nov ~~1977~~

12
12/14p.

AD A 0 4 7 5 4 1

6
CALCULATION OF
CONVECTIVE HEAT EXCHANGE DURING
THE THAWING OF WATER-SATURATED SOIL

(Uchet Konvektivnogo Teploobmena pri
Ottaiivanii Vodonasyschennogo Grunto),

10 N.V/Ukhova

AD No. _____
DDC FILE COPY

DDC
RECEIVED
DEC 14 1977
B

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

mt
037100

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

of these solutions and a comparison of the results obtained with a different statement of the problem, taking into account convective heat exchange during thawing of water-saturated soils. ←

APPROVAL for		
WSS	White Section	<input checked="" type="checkbox"/>
BSB	Blue Section	<input type="checkbox"/>
UNANNOUNCED		<input type="checkbox"/>
JUSTIFICATION		
DISTRIBUTION/AVAILABILITY CODES		
Dist.	Avail.	and/or SPECIAL
A		

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

DRAFT TRANSLATION 664

ENGLISH TITLE: CALCULATION OF CONVECTIVE HEAT EXCHANGE DURING THE THAWING OF WATER-SATURATED SOIL

FOREIGN TITLE: UCHET KONVEKTIVNOGO TEPLOOBMENA PRI OTTAIVANII VODONASYSHCHENNOGO GRUNTA

AUTHOR: N.V. Ukhova

SOURCE: Vsesoiuznoe mezhdudedomstvennoe soreshchanie po geokriologii (merzlotovedeniiu) Materialy VIII Soveshchaniia, 1966, vol. 4, p.135-143.

CRREL BIBLIOGRAPHY
ACCESSIONING NO.: 23-2217

Translated by William Grimes, Inc., Hingham, Mass., for U.S. Army Cold Regions Research and Engineering Laboratory, 1977, 11p.

NOTICE

The contents of this publication have been translated as presented in the original text. No attempt has been made to verify the accuracy of any statement contained herein. This translation is published with a minimum of copy editing and graphics preparation in order to expedite the dissemination of information. Requests for additional copies of this document should be addressed to the Defense Documentation Center, Cameron Station, Alexandria, Virginia 22314.

Calculation of Convective Heat Exchange
During the Thawing of Water-Saturated Soil

N. V. Ukhova
All-Union Correspondence
Engineering and Construction Institute

Usually, when calculating the depth of thawing of water-saturated soil as a function of time, the concepts of the theory of thermal conductivity are used. Convective heat exchange is not taken into account. However, in large-skeletal soils, and particularly in rock fill, the influence of convective heat exchange on the thawing rate may be considerable.

V. G. Gol'dtman (1958), G. V. Porkhayev, and R. M. Sarkisyan (1960) attempted to take convective heat transfer into account in studying the overall heat exchange process in thawing soil. They obtained an approximate solution to the problem of heat transfer caused by filtration of water under the influence of the head gradient, arising from the difference in water density at different temperatures. They also provided a joint solution for the equations of thermal conductivity and convection. The goal of the present paper is a development of these solutions and a comparison of the results obtained with a different statement of the problem, taking into account convective heat exchange during thawing of water-saturated soils.

Many researchers feel that the movement of water in thinly dispersed soils obeys Darcy's Law, while in coarse, sandy gravel and other materials and especially in fill made of boulders and rocks, Darcy's Law is not applicable [Aravin, Numerov (1953); Isbash (1931)]. Since this problem is of critical importance for the processes under consideration, we should like to quote the opinion on this subject expressed by Corresponding Member of the USSR Academy of Sciences M. A. Velikanov (1945); "Beginning with the analysis of the problem as a whole, we consider it particularly necessary to point out one basic defect in its statement in the literature. It is agreed that the criterion for applicability of Darcy's Law shall be the dimensions of the particles, but this is not absolutely correct. A linear law of resistance (and Darcy's Law is one of these) is valid according to the general conditions of similarity, applicable in modern hydrodynamics, at small values of the Reynold's number. Consequently, and with respect to Darcy's Law, we can speak of its applicability not in the case of finely particulate soil, but for small values of the Reynold's number." And also:

"With large heads, even in finely granular soils, the Reynold's number may turn out to be too large to allow Darcy's Law to be used; vice versa, in coarse-grained soils with very small heads (or slopes), such small velocities may be produced that Darcy's Law turns out to be quite applicable."

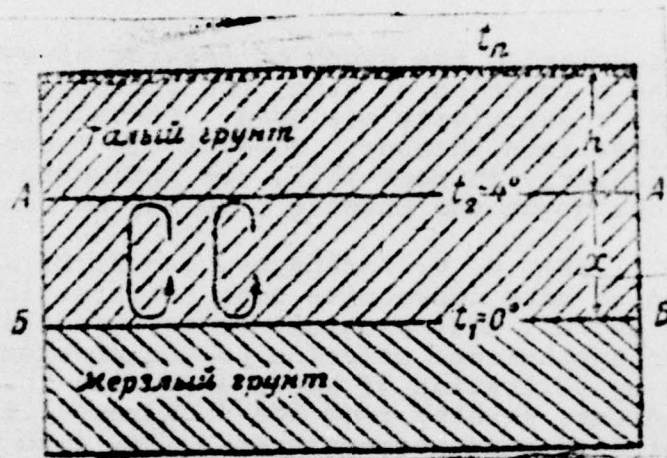


Figure 1. Calculation Diagram. (At top: thawed soil; at bottom: frozen soil.)

Since a head gradient with natural convection arises from the difference in the density of water at different temperatures and is consequently very small, we will assume that convective mass transfer is described by the process of laminar filtration.

By analogy, as pointed out by G. V. Porkhayev and R. M. Sarkisyan, let us examine the following calculation diagram (Figure 1). In the thawing layer of soil, at some depth h from the surface there is an isotherm $t_2 = 4^\circ\text{C}$. Below it, at a depth x , is the boundary between the frozen soil and the temperature $t_1 = 0^\circ\text{C}$. From surface AA to surface BB, heat transfer takes place as a result of thermal conductivity as well as convection. The water mixes along the trajectory shown in Figure 1: the movement of the water downward from AA to BB unavoidably causes an equal volume to rise from BB to AA. The soil thawing process takes place quite slowly, so that it may be assumed to be quasistationary.

Looking now at the hydrostatic pressure at levels AA and BB, we can show that the specific motive force causing mass transfer,

as a result of convection arising from the difference in the specific layers of water, will be:

$$\Delta p = \frac{x(\gamma_3 - \gamma_1)}{2} \quad (1)$$

The force p , causing the movement of the water, is actually the loss of pressure along the length of the filtration pathway. Then the head losses in this section will be expressed as follows:

$$\Delta H = \frac{\Delta p}{\gamma_{cp}} = \frac{x(\gamma_2 - \gamma_1)}{2\gamma_{cp}} \quad (2)$$

where

$$\gamma_{cp} = \frac{\gamma_1 + \gamma_2}{2}$$

The hydraulic gradient which relates to the length of the filtration path $2x$ will be:

$$I = \frac{\Delta H}{2x} = \frac{\gamma_2 - \gamma_1}{4\gamma_{cp}} \quad (3)$$

Looking at filtration in an ideal soil, where all of the "current tubes" are assumed to be cylindrical and mutually parallel, and applying Darcy's Law, we have*

$$w_1 = \frac{KI}{n} \quad (4)$$

where w_1 is the average filtration rate,
 K is the filtration coefficient of the soil, and
 n is the porosity of the soil.

By comparing (4) and (3) we have:

$$w_1 = \frac{K(\gamma_2 - \gamma_1)}{4n\gamma_{cp}} \quad (5)$$

* The usual form of Darcy's Law reads: $w_1 = KI$ (Editor's note.)

Formula (5) is valid for stable filtration when the water is flowing freely through a cross section BB. At the same time, end BB is impermeable to water, and the water will move upward as shown above. In this case, due to the movement of the water in the opposite direction, the average filtration rate will decrease relative to that determined by (5). In order to calculate the reverse movement of the water, it is necessary to introduce a reduction coefficient $\eta < 1$ into (5).

The maximum value for coefficient η is established from the following conditions. Half of all of the "current tubes" in the soil will be occupied by water moving from AA to BB and half by water moving from BB to AA. This will halve the actual rate of movement of the water in the soil relative to the average filtration rate. In addition, due to the friction against the walls of the tubes, the water-velocity distribution over the cross section of the tube will be in the form of a paraboloid (Figure 2). In this case, due to the formation of stagnant zones along the periphery of the tube, the average velocity of the water through the entire cross section will be halved again. Hence, the maximum value of coefficient η will be assumed to be 0.25 and (5) will have the following form

$$w = \frac{K(\gamma_2 - \gamma_1)}{16\eta \gamma_{cp}} \quad (6)$$

BEST AVAILABLE COPY

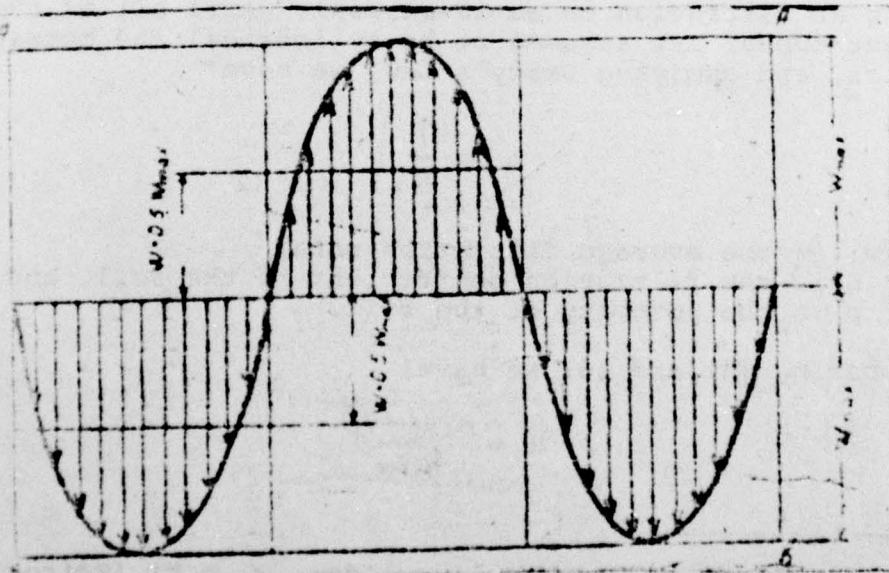


Figure 2. Diagram of Movement of Water in Current Tubes.

If we know the average filtration rate of the water, we can determine the convective heat flux:

$$q_1 = C_p \omega \gamma_{cp} (t_2 - t_1) \quad (7)$$

where C_p is the specific thermal capacity of water.

Substituting (6) in (7), we will have:

$$q_1 = 0,0625 \frac{K}{h} C_p (\gamma_2 - \gamma_1) (t_2 - t_1) \quad (8)$$

For a joint computation of heat transfer caused by convection and thermal conductivity, we can use the concept of the effective coefficient of thermal conductivity, λ_3 , assuming that all of the heat transmitted by contact (in other words, convection and thermal conductivity) is transmitted only by thermal conductivity (Kirpichev, Mikheyev, Eigensohn, 1940). Then λ_3 will be expressed as follows:

$$\lambda_3 = \frac{qx}{t_2 - t_1} \quad (9)$$

where $q = q_1 + q_2$ is the total heat flux due to convection and thermal conductivity.

The coefficient of increase in thermal conductivity due to the presence of convective flux will be:

$$\varphi = \frac{\lambda_3}{\lambda} \quad (10)$$

The heat flux, determined by the thermal conductivity, will be:

$$q_2 = \frac{\lambda(t_2 - t_1)}{x} \quad (11)$$

where λ is the coefficient of thermal conductivity of the soil in the thawed zone.

Taking (8), (9), and (11) into account, (10) may be written as follows:

BEST AVAILABLE COPY

$$\phi = 1 - 0.0625 \frac{C_p K \lambda (\gamma_2 - \gamma_1)}{\lambda} \quad (12)$$

Formula (12) makes it possible to consider jointly the heat transfers due to convection and thermal conductivity.

The value of the coefficient ϕ may also be determined on the basis of a completely different statement of the problem than that discussed above. Assume, as in Figure 1, that the total heat flux is moving from surface AA to surface BB (in other words, taking convection and thermal conductivity into account) with an intensity (Aerov, Umnik, 1951):

$$q = \lambda \frac{dt}{dx} - GC_p (t_2 - t_1) \quad (13)$$

where $G = w \times \gamma_{av}$ is the weight velocity of the water.

The general solution of (13), taking into account the boundary conditions shown in Figure 1, will be:

$$\frac{q}{\lambda} - \frac{GC_p}{\lambda} (t_2 - t_1) = \frac{t_2 - t_1}{x} \quad (14)$$

Then, in accordance with (9) and (10), we can write:

$$\phi = 1 - \frac{GC_p x}{\lambda} \quad (15)$$

The moving force, as before, is determined by (1). On the other hand, with a stationary regime for the movement of the flow, this force must be equal to the hydraulic resistance of the granular layer on path $2x$. With a laminar regime of movement, the hydraulic resistance of the granular layer will be expressed as in M. E. Aerov's and N. N. Umnik's paper (1951):

$$R_{10} = \frac{G}{2\sigma \gamma_{av} n^2} a / 2x \quad (16)$$

where g is the acceleration due to gravity,
 a is the surface of the grains in a unit volume of the layer, and
 f is the coefficient of friction of the granular layer.

The coefficient of friction for low flow velocities (with $Re < 20$) is:

$$f = \frac{36}{Re} = \frac{36 \cdot a \mu}{4G} \quad (17)$$

where μ is the dynamic coefficient of viscosity of the water.

Equating (1) and (16) and taking (17) into account as well, we will have:

$$G = \frac{g \gamma_{cp} n^3 (\gamma_2 - \gamma_1)}{18 a^3 \mu} \quad (18)$$

The surface of grains per unit volume of the layer may be expressed by the hydraulic radius of the "current tubes" of granular layer r [8]:

$$a = \frac{n}{R} \quad (19)$$

In turn, the hydraulic radius may be linked with the coefficient of filtration of the granular layer through the coefficient of permeability [9]:

$$R^2 = \frac{K \beta^2}{n \gamma_{cp} g} \quad (20)$$

where $\beta = 1.87$ is the parameter which takes into account the shape of the "current tube" in the soil.

Then the surface of the grains per unit volume of the layer is found from the equation:

$$a^2 = \frac{n^3 \gamma g^2}{K \beta^3 \mu} \quad (21)$$

Substituting the value a into (18) we have:

$$G = 0,194 (\gamma_2 - \gamma_1) K \quad (22)$$

From (15) and (22) we will have:

$$\varphi - 1 = 0,194 \frac{C_p K x (Y_3 - Y_1)}{\lambda} \quad (23)$$

Formula (23), like (12), makes it possible to calculate the combined heat transfer due to convection and thermal conductivity in thawing soils. The formulas obtained are approximate and based upon various assumptions. This apparently also explains the slight difference in their structures. However, the results obtained by (12) and (23) are similar.

If these formulas are converted to the form

$$\varphi - 1 = 0,0625 \frac{1}{n} A \quad (12')$$

$$\varphi - 1 = 0,194 A \quad (23')$$

where

$$A = \frac{C_p K x (Y_2 - Y_1)}{\lambda}$$

it will be easy to compare the results obtained. Table 1 shows the values for the coefficients at A according to (12') and (23') as a function of the porosity of the soil n .

TABLE 1

Formula \ Porosity	0.3	0.4	0.5	0.6
12'	0.208	0.156	0.125	0.104
23'	0.194			

It seems to us that the structure of (12') reflects real heat exchange conditions better in the soil, since in this case the coefficient of the increase in thermal conductivity due to the presence of convective currents also depends upon the coefficient of porosity of the soil.

Hence, if we know the coefficient of filtration of the soil in the thawing zone, we can calculate the value ϕ . When $\phi \leq 1.1$ to 1.2 , the influence of convection on general heat exchange will be insignificant and the rate of thawing of the soil will be determined by thermal conductivity. When $1.2 < \phi < 4$, the influence of convection becomes considerable and it must be calculated, taking into account the thawing rate of the soil.

When $\phi > 4$ convection predominates; in other words, the thawing rate of the soil is determined not so much by thermal conductivity as by convection.

When it is necessary to calculate convection, it is important to calculate the value of the effective coefficient of thermal conductivity; the rate of thawing of the soil must be calculated using thermal conductivity formulas, substituting the values obtained into them.

For the sake of illustration to show the influence of convection upon general heat exchange, we present below calculations for three types of soils, with the characteristics shown in Table 2.

No.	Soil	Granulometric Composition					h	K cm/sec	C_p cal/g degree	λ cal/cm, sec. degree
		0.05	0.05- 1.00	1.00- 3.00	3.00- 20.0	20-80				
1	Fine sand	-	23.5	26.5	50	-	0.384	0.0091	0.404	0.00628
2	Medium sand	-	0.75	14.0	83.25	-	0.393	0.098	0.413	0.0058
3	Gravel and sand	1.5	10.5	46	42	0.35	0.406	0.408	0.0060	

The characteristics of soils 1 and 2 are taken from a paper by P. A. Bogoslovskiy (1956). Characteristic 3 of the soil is taken from the work of V. G. Gol'dtman (1958). The values of λ , C_p and n are missing from Gol'dtman's paper, and are included conditionally on the basis of corresponding characteristics of similar types of soil.

The values of $\phi - 1$ calculated by (12) were as follows:

Soil 1	$\phi - 1 = 0.0000123 x$
Soil 2	$\phi - 1 = 0.000144 x$
Soil 3	$\phi - 1 = 0.000642 x$

where x is the distance between the $+4^\circ$ and 0° isotherms in centimeters.

Convective heat transfer during the thawing of the soils in question begins to be expressed (in other words, $\phi > 1.2$) at a distance x equal respectively to the following:

Soil 1	$x = 162 \text{ m}$
Soil 2	$x = 13.9 \text{ m}$
Soil 3	$x = 3.12 \text{ m}$

Hence, thawing of the first and second types of soil will occur practically as the result of thermal conductivity alone. The thawing of the third type of soil, at a depth of more than 3 meters, requires that convective heat transfer also be taken into account.

REFERENCES

- Gol'dtman, V. G.: "Heat Exchange in Filtering Coarse-Grained Soils with Drainage and Clay Water Seepage." Transactions of VNI-1, Cryomorphology, No. 11, 1958.
- Porkhayev, G. V., Sarkisyan, R. M.: "The Influence of Convective Heat Exchange in a Water-Saturated Layer on the Thawing of the Subjacent Frozen Soil." Materials for the Basis of Calculations of Frozen Zones of the Earth's Crust. No. 5, Academy of Sciences of the USSR, 1960.
- Aravin, V. N., Numerov, S. N.: "The Theory of Movement of Fluids and Gases in Non-Deformable Porous Media." State Publishing House of Technical and Theoretical Literature, 1953.
- Izbash, S. V.: "Filtration in Coarse-Grained Material." Izvestiya NII, Gidrotehnika, No. 1, 1931.
- Velikanov, M. A.: "The Movement of Underground Waters in Coarse-Grained Soils." Izvestiya AN SSSR, Department of Technical Sciences, No. 78, 1945.
- Kirpichev, M. V., Mykheyev, M. A. and Eigensohn, A. S.: Teploperedacha (Heat Transfer), Gosenergoizdat, 1940.
- Aerov, M. E., Umnik, N. N.: "The Phenomenon of Convection in a Granular Layer." ZhTF, Vol. 21, No. 11, 1951.

- Chudnovskiy, A. F.: Teploobmen v Dipersnykh Sredakh. (Heat Exchange in Disperse Media). State Publishing House of Technical and Theoretical Literature, 1954.
- Leybenzon, L. S.: Dvyzheniye Prirodnykh Zhidkostey i Gazov v Poristoy Srede. (The Movement of Natural Fluids and Gases in a Porous Medium). OGIZ, 1947.
- Bogoslovskiy, P. A.: "The Thermal Regime of Earth Dams Under Conditions of Propagation of Permafrost Soils." Dissertation presented in support of scientific degree of Doctor of Technical Sciences, Moscow, 1956.