Technical Report JSR-77-21 *November 1977*

SCATTERING OF SOUND BY INTERNAL WAVES: THE ROLE OF PARTICLE VELOCITIES

By: W. H. MUNK F. ZACHARIASEN

Contract No. DAHC15-73-C-0370
ARPA Order No. 2504
Program Code No. 3K10
Date of Contract: 2 April 1973
Contract Expiration Date: 30 November 1977
Amount of Contract $3,176,255

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BOULEVARD
ARLINGTON, VIRGINIA 22209
ARPA ORDER NO. 2504

STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 · U.S.A.

Copy No. ....

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 14 SRI- JSR-77-21
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): 6 SCATTERING OF SOUND BY INTERNAL WAVES : THE ROLE OF PARTICLE VELOCITIES.
5. TYPE OF REPORT & PERIOD COVERED: 9 Technical Re[redacted]t,
6. PERFORMING ORG. REPORT NUMBER: JSR-77-21
7. AUTHOR(s): 10 W.H. Munk [redacted] F. Zachariasen
8. CONTRACT OR GRANT NUMBER(s): 15 DAHC15-73-C-0370, ARPA Order-2504
9. PERFORMING ORGANIZATION NAME AND ADDRESS: SRI, International, 1611 North Kent Street, Arlington, VA 22209
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Advanced Research Projects Agency, 1400 Wilson Boulevard, Arlington, VA 22209
12. REPORT DATE: 11 Nov[redacted]77
13. NO. OF PAGES: 36 12 38p.
14. MONITORING AGENCY NAME & ADDRESS (if diff. from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this report): Approved for public release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report)

332 500

18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ACOUSTICS
INTERNAL WAVES
SOUND TRANSMISSION

delta times c
zeta
gamma

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Internal waves scatter sound by two related perturbations: (1) those associated with vertical particle displacements $\zeta(x,y,z,t)$ in the presence of a vertical gradient of (potential) sound speed: $\delta c = \zeta \partial_z c_p$; (2) those associated with horizontal particle velocities $u(x,y,z,t)$. The combined perturbations in refractive index are $\delta c/c + u/c$. The second term, which has been generally neglected, does become important under two rather special circumstances: (a) in sound transmission along a deep downward loop, and (b) in the *difference* between transmission from a source at point A to a receiver at point B and transmission from a source at B to a receiver at A.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

DD FORM 1 JAN 73 1473 (BACK)

EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT

Internal waves scatter sound by two related perturbations: (1) those associated with vertical particle displacements $\zeta(x,y,z,t)$ in the presence of a vertical gradient of (potential) sound speed: $\delta c = \zeta \cdot \partial_z c_p$; (2) those associated with horizontal particle velocities $u(x,y,z,t)$. The combined perturbations in refractive index are $\delta c/c + u/c$. The second term, which has been generally neglected, does become important under two rather special circumstances: (a) in sound transmission along a deep downward loop, and (b) in the *difference* between transmission from a source at point A to a receiver at point B and transmission from a source at B to a receiver at A.

CONTENTS

I. PROPAGATION VELOCITY

Let $\zeta(x,y,z,t)$ designate the upward particle displacement due to internal wave motion, and

$$u,\ v,\ w = \partial_t \zeta \tag{1}$$

the three components of internal wave particle velocities. The perturbation in sound velocity arising from the vertical displacement is

$$\delta c = \partial_z c_p \cdot \zeta \tag{2}$$

where $\partial_z c_p$ is the potential sound velocity gradient. Thus

$$\delta c \pm u \tag{3}$$

is the i.w. perturbation in *propagation velocity* in the $\pm$ x-direction, ignoring the small tilt of sound rays. The refraction parameter is

$$\mu^{\pm} = \frac{\delta c}{c} \pm \frac{u}{c} \tag{4}$$

For a canonical ocean,[1] Eq. (2) can be written

$$\frac{\delta c}{c} = 24.5\ g^{-1}\ n^2(z)\zeta \tag{5}$$

where

$$n = n_o e^{z/b} \tag{6}$$

is the Brunt-Väisälä (or buoyancy) frequency. Constancy of i. w. energy flux requires

$$\text{rms } \zeta \sim n^{-\frac{1}{2}}, \qquad \text{rms } u \sim n^{\frac{1}{2}} \quad ,$$

hence

$$\delta c/c \sim n^{3/2} \quad .$$

For orientation, with a sound axis at $z = -b = -1$ km, the following numerical values obtain:[2]

	z km	$\bar{n}$ cph	rms ζ m	rms u cm/sec	rms $\delta c/c$ $\times 10^5$	rms u/c $\times 10^5$
Thermocline	0	3	7.3	4.70	49	3.1
Sound axis	-1	1.1	12.0	2.85	11	1.9
	-2	.406	19.8	1.73	2.4	1.1
bottom	-4.5	.094	41.2	0.83	0.3	0.6

Thus δc and u contributions from i.w. are comparable beneath 2 km.

II. SPECTRA

We first refer to the previous results for the effects of vertical particle displacements. From MZ (115) and MZ (118) we have $\langle \phi^2 \rangle = \langle |X|^2 \rangle$. Then from (MZ66)

$$\langle \phi\phi \rangle = \frac{2}{\pi} q^2 n_o b \int_o^R dx \sec^2\theta \sum_j j^{-1} \int_{\omega_L}^n d\omega (\omega^2 - \omega_L^2)^{-\frac{1}{2}} S(\omega,j;z) \tag{1}$$

with
$$\sum_j \int_{\omega_{in}}^n d\omega\, S(\omega,j) = \left\langle \left(\frac{\delta c}{c}\right)^2 \right\rangle \quad . \tag{2}$$

We now generalize (2.1) to

$$\langle \phi^+\phi^+ \rangle = \quad \ldots \int \quad \ldots \sum \quad \ldots \int \quad \ldots \quad S^{++}(\omega,j;z) \tag{3}$$

and similarly for $\langle \phi^-\phi^- \rangle$, $\langle \phi^+\phi^- \rangle$, where

$$\left.\sum_j \int_{\omega_{in}}^n d\omega \begin{bmatrix} S^{++}(\omega,j) \\ S^{--}(\omega,j) \\ S^{+-}(\omega,j) \end{bmatrix} \begin{matrix} = \langle \mu^+\mu^+ \rangle = \langle \frac{\delta c}{c}\frac{\delta c}{c} \rangle + \langle \frac{u}{c}\frac{u}{c} \rangle + 2\,\langle \frac{\delta c}{c}\frac{u}{c} \rangle \\ = \langle \mu^-\mu^- \rangle = \langle \qquad \rangle + \langle \quad \rangle - 2\,\langle \qquad \rangle \\ = \langle \mu^+\mu^- \rangle = \langle \qquad \rangle - \langle \quad \rangle \;. \end{matrix} \right\} \tag{4}$$

It is convenient to denote the contributions to the spectrum by

$$S\left(\frac{\delta c}{c}\,\frac{\delta c}{c}\right), \qquad S\left(\frac{u}{c}\,\frac{u}{c}\right), \qquad S\left(\frac{\delta c}{c}\,\frac{u}{c}\right) \tag{5}$$

so that

$$S^{++} \equiv S(\mu^+\mu^+) = S\left(\frac{\delta c}{c}\ \frac{\delta c}{c}\right) + S\left(\frac{u}{c}\ \frac{u}{c}\right) + 2S\left(\frac{\delta c}{c}\ \frac{u}{c}\right) \quad ,$$

and similarly for S^{--} and S^{+-}, using the sign convention in (2.4). In accordance with the Garrett and Munk internal wave model we write (in the notation of MZ)

$$\begin{aligned} S\left(\frac{\delta c}{c}\ \frac{\delta c}{c}\right) &= \left\langle\frac{\delta c}{c}\ \frac{\delta c}{c}\right\rangle G_{\zeta\zeta}(\omega)\ H(j) \\ S\left(\frac{u}{c}\ \frac{u}{c}\right) &= \left\langle\frac{u}{c}\ \frac{u}{c}\right\rangle G_{uu}(\omega)\ H(j) \\ S\left(\frac{\delta c}{c}\ \frac{u}{c}\right) &= \left\langle\frac{\delta c}{c}\ \frac{u}{c}\right\rangle G_{u\zeta}(\omega)\ H(j) \end{aligned} \tag{6}$$

$$\text{with} \quad G_{\zeta\zeta} = \frac{4\omega_{in}\ (\omega^2-\omega_{in}^2)^{\frac{1}{2}}}{\pi\omega^3} \quad , \qquad G_{uu} = \frac{4\omega_{in}\ (\omega^2+\omega_{in}^2)}{3\pi\omega^3(\omega^2-\omega_{in}^2)^{\frac{1}{2}}}$$

$$G_{u\zeta} = 2\omega_{in}^2\ \omega^{-3} \quad , \qquad H(j) = (j^2+j_*^2)^{-1}\Big/\sum_1^\infty (j^2+j_*^2)^{-1} \tag{7}$$

$$\text{such that} \quad \int_{\omega_{in}}^{n} d\omega\ G(\omega) = 1 \qquad \sum_{j=1}^{\infty} H(j) = 1 \quad .$$

Further,

$$\left.\begin{aligned} \left\langle\frac{\delta c}{c}\ \frac{\delta c}{c}\right\rangle &= \left\langle\frac{\delta c}{c}\ \frac{\delta c}{c}\right\rangle_o \left(\frac{n}{n_o}\right)^3, & \left\langle\frac{\delta c}{c}\ \frac{\delta c}{c}\right\rangle_o &= \left(\frac{24.5 n_o^2}{g}\right)^2 \langle\zeta_o^2\rangle = 24.0\times10^{-8} \\ \left\langle\frac{u}{c}\ \frac{u}{c}\right\rangle &= \left\langle\frac{u}{c}\ \frac{u}{c}\right\rangle_o \frac{n}{n_o}, & \left\langle\frac{u}{c}\ \frac{u}{c}\right\rangle_o &= \frac{3n_o^2}{2c^2} \langle\zeta_o^2\rangle = 0.10\times10^{-8} \\ \left\langle\frac{\delta c}{c}\ \frac{u}{c}\right\rangle &= \left\langle\frac{\delta c}{c}\ \frac{u}{c}\right\rangle_o \left(\frac{n}{n_o}\right)^2, & \left\langle\frac{\delta c}{c}\ \frac{u}{c}\right\rangle_o &= I\ \frac{24.5\ n_o^2}{g}\sqrt{\frac{3}{2}}\,\frac{n_o}{c}\langle\zeta_o^2\rangle = 1.55\times10^{-8}\ I \end{aligned}\right\} \tag{8}$$

A. The $u\zeta$ variance for internal waves

The function I is to allow for the phase relation between u and ζ. We shall demonstrate that for free internal waves u and ζ are in fact in quadrature, hence $I=0$. But it is to be expected that for forced (growing) internal waves this is not the case, and then $0 \leq I \leq 1$. In the latter case there would be much interest in *measuring* the $u\zeta$ and $u\dot{\zeta} = uw$ covariances, as a possible indication of the vertical transfer of horizontal momentum associated with internal waves.

Constant n. The simplest demonstration is for this case; the progressive wave solution gives[3]

$$\frac{u}{\omega a} = \sin\theta \cos\phi \cos(k_x x + k_z z - \omega t) + \frac{\omega_{in}}{\omega} \sin\theta \sin\phi \sin(k_x x + k_z z - \omega t)$$

$$\zeta = a \cos\theta \sin(k_x x + k_z z - \omega t)$$

where $\quad k_x = k_H \cos\phi, \quad k_y = k_H \sin\phi, \quad k_z = k_H \tan\theta$.

But there is evidence that the waves are very nearly standing waves (equal energy flux up and down); furthermore, McComas[4] has demonstrated that if this is not the case there is strong nonlinear interaction to make it so. To obtain the standing wave solution we reverse the sign of both θ and k_z and add:

$$\frac{u}{\omega a} = -2 \sin\theta \cos\phi \sin k_z z \sin(k_x x+\omega t)$$

$$+2 \frac{\omega_{in}}{\omega} \sin\theta \sin\phi \sin k_z z \cos(k_x x-\omega t) ,$$

$$\zeta = 2a \cos\theta \cos k_z z \sin(k_x x-\omega t) .$$

For progressive waves

$$\langle u\zeta \rangle = \tfrac{1}{2} a^2 \omega_{in} \sin\theta \cos\theta \sin\phi$$

and this vanishes for a horizontally isotropic field, as assumed GM75½ , since $\langle \sin\phi \rangle = 0$. For standing waves $\langle u\zeta \rangle$ is proportional to $\langle \sin\phi \rangle$ and $\langle \cos\phi \rangle$ and so again vanishes for a horizontally isotropic field. But in addition, u and ζ are in quadrature vertically, and this again leads to cancellation.

Suppose the upward traveling waves (downward energy) have an amplitude a, and the downward traveling waves ra. Then it follows that

$$I = (1-r) \langle \sin\phi \rangle$$

which vanishes in the case of vertical and horizontal isotropy.

<u>General case</u>. One might imagine that the foregoing result is a consequence of the assumption that n is constant, or of the WKB approximation in which n varies only slowly. It is, however, quite general. Let $\zeta(\vec{k},\omega;z)$ be the Fourier component of vertical displacement corresponding to an internal wave with horizontal wavenumber $\vec{k}$ and frequency ω. Thus

$$\zeta(\vec{k},\omega;z) = \int d^2\vec{x} \int dt\, e^{-i(\vec{k}\vec{x}-\omega t)}\, \zeta(\vec{x},t)$$

and (the complex quantity) $\zeta(\vec{k},\omega;z)$ satisfies the equation

$$\left(\partial_z^2 + k^2 \frac{n^2(z)-\omega^2}{\omega^2-\omega_{in}^2}\right) \zeta(\vec{k},\omega;z) = 0 \qquad (9)$$

for any buoyancy frequency $n(z)$. The boundary conditions normally used are that ζ should vanish at the surface and on the bottom of the ocean. The quantity of interest to us is $\langle \vec{u}\zeta \rangle$. From the equations of motion, we have

$$\vec{u}(\vec{k},\omega;z) = \left(\frac{\omega}{k^2}\vec{k} - i\,\frac{\omega_{in}}{k^2}\,\hat{e}_z x \vec{k}\right) \partial_z\, \zeta(\vec{k},\omega;z)$$

where $\hat{e}_z$ is a unit vector pointed vertically upward. It is then easy to show that

$$\langle \vec{u}\zeta \rangle = \tfrac{1}{4} \left\langle \frac{\omega}{k^2}\vec{k}\, \partial_z |\zeta(\vec{k},\omega;z)|^2 \right\rangle$$

$$+ \tfrac{1}{2} \left\langle \frac{\omega_{in}}{k^2}\, \hat{e}_z x \vec{k}\, \mathrm{Im}(\zeta^*(\vec{k},\omega;z)\, \partial_z \zeta(\vec{k},\omega;z)) \right\rangle \quad .$$

Now when averaged over a vertical distance small compared to the distance over which $\zeta(\vec{k},\omega;z)$ varies appreciably, the first term is zero. And from the differential equation (9) we can readily derive the "conservation law" that

$$\mathrm{Im}(\zeta^{*}(\vec{k},\omega;z)\ \partial_z \zeta(\vec{k},\omega;z))$$

is a constant independent of depth. Since in particular it vanishes on the bottom, it vanishes everywhere. Thus $\langle \vec{u}\zeta \rangle$ is zero for any buoyancy frequency $n(z)$, without approximation, provided only that (9) holds.

3. MEAN-SQUARE PHASE

Substitution of (2.6) into (2.1) leads to

$$\langle\phi\phi\rangle = 2\pi^{-1} \frac{n_o}{\omega_{in}} q^2 b \langle j^{-1}\rangle \int_0^R dx \sec^2\theta \left\langle\left(\frac{\delta c}{c}\right)^2\right\rangle \int_{\omega_{in}}^{n} d\omega(\omega^2-\omega_L^2)^{-\frac{1}{2}} \omega_{in} G_{\zeta\zeta}(\omega) \qquad (1)$$

plus two other terms involving $\langle(u/c)^2\rangle$ and G_{uu}, and $\langle(\delta c/c)(u/c)\rangle$ and $G_{u\zeta}$. Here $\langle j^{-1}\rangle = \Sigma\, j^{-1} H(j)$ is the i.w. weighted average of j^{-1}. The ω-integrations can be carried out explicitly.[5] Writing

$$\omega_L^2 \equiv \omega_{in}^2 + n^2 \tan^2\theta \equiv \omega_{in}^2 D^2 , \qquad y = \omega_{in}^2/\omega^2$$

we have for the three ω-integrals

$$\left.\begin{aligned}
\frac{2}{\pi} \int_{\omega_{in}/n^2}^{1/D^2} dy \sqrt{\frac{1-y}{1-Dy}} &= \frac{2}{\pi}\left(\frac{1}{D^2} + \frac{D^2-1}{2D^3} \log \frac{D+1}{D-1}\right) \\
\frac{2}{3\pi} \int dy \frac{1+y}{\sqrt{(1-y)(1-Dy)}} &= \frac{2}{3\pi}\left(-\frac{1}{D^2} + \frac{3D^2+1}{2D^3} \log \frac{D+1}{D-1}\right) \\
\int dy \sqrt{\frac{y}{1-Dy}} &= \frac{\pi}{2}\,\frac{1}{D^3}
\end{aligned}\right\} \qquad (2)$$

The x-integration in Eq. (3.1) has in general been done numerically by integration along the ray path. It can be done analytically in two special cases, as follows.

A. *The axial approximation*

Here $D=1$, $\theta=0$, $n=n_1$. We note that the second integral (involving $\langle uu \rangle$) becomes logarithmically infinite. This is treated in the Appendix. The result then is

$$\langle \phi^+ \phi^+ \rangle = \frac{2}{\pi} \frac{n_o}{\omega_{in}} \langle j^{-1} \rangle q^2 bR \left[\frac{2}{\pi} \left\langle \left(\frac{\delta c}{c}\right)_1^2 \right\rangle + \right. \tag{3}$$

$$\left. + \frac{8}{3\pi} \log \frac{0.43\ R_o}{R-R_o} \left\langle \left(\frac{u}{c}\right)_1^2 \right\rangle + 2\ \frac{\pi}{2} \left\langle \left(\frac{\delta c}{c}\ \frac{u}{c}\right)_1 \right\rangle \right]$$

where the subscript 1 denotes the sound axis value, and similarly for $\langle \phi^- \phi^- \rangle$ and $\langle \phi^+ \phi^- \rangle$. Here $R_o = 21$ km is the range of an axial loop. The logarithmic singularity is weak, and unless we are within 100 m of an axial loop, the relative contributions of the three terms are determined by the averages $\langle (\ )_1 \rangle$. From Eq. (2.8), setting $n_1/n_o = e^{-1}$, the relative magnitudes are

$$24\times10^{-8}\cdot e^{-3} = 1.2\times10^{-8}$$

$$0.10\times10^{-8}\cdot e^{-1} = .04\times10^{-8}$$

$$4\pi^{-2} I\ 0.06\times10^{-8} = .02\times10^{-8}\ I$$

The first term, involving $\delta c/c$, clearly dominates.

B. The turning point approximation

Next we discuss the apex approximation. For steep rays we note that the major contributions to the ray integrals in $\langle\phi^+\phi^+\rangle$ come from the (upper and lower) apexes of the ray. Near an apex located at $(\hat{x},\hat{z})$, the equation of the ray is

$$z(x) = \hat{z} - (x-\hat{x})^2/2\mathcal{R}$$

where $\mathcal{R}^{-1} = \gamma_A \left|1-\exp(z-z_1)/b\right|$, $\gamma_A = 1.14\times10^{-2}\ \mathrm{km}^{-1}$ (MZ85)

is the radius of curvature of the ray at the turning point. With this expression, we may approximately evaluate the ray integrals and obtain for the contribution of each apex

$$\langle\phi^+\phi^+\rangle_{apex} = 2q^2 b\mathcal{R}\,\langle j^{-1}\rangle \left|\left\langle\left(\frac{\delta c}{c}\right)^2_o\right\rangle \left(\frac{\hat{n}}{n_o}\right)^2 + \left\langle\left(\frac{u}{c}\right)^2_o\right\rangle + 2\left\langle\left(\frac{\delta c}{c}\,\frac{u}{c}\right)_o\right\rangle\left(\frac{\hat{n}}{n_o}\right)\right| \quad (4)$$

where $\hat{n}$ denotes the value of $n(z)$ at the apex: $\hat{n} = n(\hat{z})$. Thus the $(\delta c/c)^2_o$ and $(\delta c/c\cdot u/c)_o$ terms get their major contributions from upper turning points, while the $(u/c)^2_o$ has comparable contributions from both (in fact somewhat larger from the lower turning point since the radius of curvature $\mathcal{R}$ is larger there).

C. Reciprocal transmissions

Thus in any one-way experiment other than for a ray with a single deep loop) the effect of particle velocities can be ignored. If, however, we simultaneously transmit in opposite directions, then the three terms can be separately evaluated by constructing the three combinations

$$\tfrac{1}{4}\left[\langle\phi^+\phi^+\rangle + \langle\phi^-\phi^-\rangle + 2\langle\phi^+\phi^-\rangle\right]$$

$$\tfrac{1}{4}\left[\langle\phi^+\phi^+\rangle + \langle\phi^-\phi^-\rangle - 2\langle\phi^+\phi^-\rangle\right] \qquad (5)$$

$$\tfrac{1}{4}\left[\langle\phi^+\phi^+\rangle \quad \langle\phi^-\phi^-\rangle\right] \quad ,$$

respectively. The implications of this are considered in Section IV.

D. Worcester's reciprocal experiment[6]

This corresponds to a lower turning point at 1.5 km, and a rather small scale-depth, $B \approx 0.75$ km. Thus $\hat{n}/n_o = e^{-2}$, and the three terms in Eq. 3.4 are of a magnitude

$$\langle\phi^+\phi^+\rangle \sim \left\{0.44 + 0.10 + 0.42\ I\right\} \times 10^{-8} \quad .$$

Suppose that $I=0$, then we have

$$\langle\phi^+\ \phi^+\rangle \sim 0.54$$

$$\langle\phi^-\ \phi^-\rangle \sim 0.54$$

$$\langle\phi^+\ \phi^-\rangle \sim 0.34$$

$$\langle(\phi^+ - \phi^-)^2\rangle \sim 0.40$$

This is consistent with some preliminary results that $\langle\phi^+\phi^+\rangle$, $\langle\phi^-\phi^-\rangle$ and $\langle(\phi^+-\phi^-)^2\rangle$ have comparable magnitudes.

IV. RECIPROCAL COVARIANCES

An experiment involving reciprocal shooting will yield not only the mean-square quantities, but also the frequency content. To interpret such a spectrum we shall want to reverse the integration order, and do the x-integration first. Let

$$\langle \phi^+ \phi^+ \rangle = \int d\omega \, E^{++}(\omega)$$

etc. Then the last term of the Eq. (3.1) can be written

$$\tfrac{1}{4} \int d\omega \, [E^{++}(\omega) - E^{--}(\omega)] = E(\omega) = \frac{4}{\pi} \langle j^{-1} \rangle \, q^2 B\bar{R} \, \frac{n^2}{\omega_{in} n_o} \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right) \right\rangle_o$$

$$\cdot \frac{1}{\bar{R}} \int_0^R dx \, \sec^2\theta \, (\omega^2 - \omega_L^2)^{-\frac{1}{2}} \, G_{u\zeta}(\omega) \, \theta(n-\omega) \, \theta(\omega-\omega_L)$$

where $\theta(x)$ is a step function, $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. These θ functions restrict the part of the integral on the ray which can contribute to the spectrum for any given frequency. At any position x along the ray, the only frequencies which can interact with the ray (within the stationary phase approximation) are those satisfying

$$n(z(x)) > \omega > \sqrt{\omega_{in}^2 + n^2(z(x)) \tan^2\theta(x)} \quad .$$

Thus when the ray is steep (θ is large) frequencies near ω_{in} are excluded. But when one is near the ray apex (n large and θ small) all frequencies between ω_{in} and $\hat{n} = n(\hat{z})$ are allowed.

One may, if one wishes, pick out different parts of the ray by forming the difference between the spectrum at two nearby frequencies. If $\Delta\omega << \omega$ is a small frequency difference, then for frequencies well above ω_{in} we have

$$\Delta E(\omega) \equiv E(\omega+\Delta\omega) - E(\omega) = -\frac{8}{\pi} \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right)_o \right\rangle \left\langle \frac{1}{j} \right\rangle \frac{q^2 B \omega_{in} n^2}{n_o^4 \omega} \cdot \Delta\omega \left(1/\frac{\partial n}{\partial x} \right)_{\omega=n} \quad ;$$

thus $\Delta E(\omega)$ receives all of its contributions from that part of the ray where $n(z)x)) \approx \omega$.

V. MOMENTUM FLUX

We may wish to interpret the results of reciprocal transmission without relying so heavily on internal waves. To this end let us express the measured quantities in terms of the spectrum of momentum flux in the ocean without making any assumptions regarding the origin of this spectrum.

Let $\hat{}$ designate averages along a ray. Thus[7]

$$\hat{u}(t) = R^{-1} \int ds\ u(s,t)$$

$$\hat{\zeta}(t) = R^{-1} \int ds\ \zeta(s,t)$$

We may regard $\hat{u}$ and $\hat{\zeta}$ as the measured quantities for they are so closely related to the measured travel times; let

$$T^{\pm} = -\ (R/\bar{c})\ \hat{\mu}^{\pm} = -\ (R/\bar{c}^2)\ [\delta\hat{c} \pm \hat{u}] \quad ,$$

designate the departures from the mean travel time (averaged over several inertial periods). Then

$$\delta\hat{c} = \partial_z c_p \cdot \hat{\zeta} = -\ (\bar{c}^2/R)\ \tfrac{1}{2}(T^+ + T^-)$$

$$\hat{u} = -\ (\bar{c}^2/R)\ \tfrac{1}{2}(T^+ - T^-)$$

(Actually the vertical gradient of potential sound velocity should be properly depth weighted.)

Consider the time-lagged covariance

$$\hat{C}_{u\zeta}(\tau) = \langle \hat{u}(t)\ \hat{\zeta}(t-\tau) \rangle \quad .$$

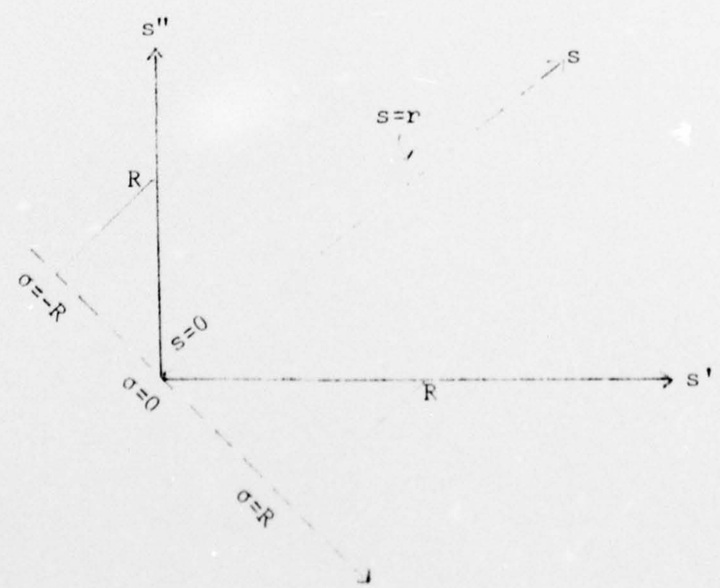
Can we manipulate $\hat{C}$ to get some measure of the momentum flux $\langle uw \rangle$? First we note that

$$\partial_\tau \hat{C}_{u\zeta}(\tau) = \langle \hat{u}(t)\ \partial_\tau \hat{\zeta}(t-\tau) \rangle$$

$$= - \langle \hat{u}(t)\ \partial_t \hat{\zeta}(t-\tau) \rangle = -\hat{C}_{uw}(\tau) \quad .$$

Now $\quad \hat{C}_{u\zeta}(\tau) = R^{-2} \int ds' \int ds''\ C_{u\zeta}(s,s';\tau)$

where $\quad C_{u\zeta}(s',s'';\tau) = \langle u(s',t)\ \zeta(s'',t-\tau) \rangle$



At this point let us simplify the geometry to horizontal rays. Then switching to center of mass and relative coordinates

$$s = \tfrac{1}{2}(s'-s''), \qquad \sigma = s'-s''$$

We have

$$\int_o^R ds' \int_o^R ds'' = \int_o^R d\sigma \int_{o+\sigma/2}^{R-\sigma/2} ds + \int_{-R}^{o} d\sigma \int_{o-\sigma/2}^{R+\sigma/2} ds$$

$$= \int_o^R d\sigma(R-\sigma) + \int_{-R}^{o} d\sigma(R+\sigma) \quad .$$

The second line follows if the integrand is a function only of σ, not of s. Under these conditions

$$C_{u\zeta}(\sigma,\tau) = \langle u(s,t)\ \zeta(s-\sigma,t-\tau)\rangle$$

$$\hat{C}_{u\zeta}(\tau) = R^{-2} \int_{-R}^{R} d\sigma(R-|\sigma|)\ C_{u\zeta}(\sigma,\tau)$$

$$\hat{C}_{uw}(0) = R^{-2} \int_{-R}^{R} d\sigma(R-|\sigma|)\ C_{uw}(\sigma,0)$$

$$= -2R^{-2} \int_{o}^{R} d\sigma\ \sigma\ C_{uw}(\sigma,0)$$

<u>Flux spectrum.</u> The more general question is to interpret the covariance of the "measured" quantities:

Let
$$C_{\hat{u}\hat{\zeta}}(\tau) = \langle \hat{u}(t)\ \hat{\zeta}(t-\tau) \rangle$$

$$F_{\hat{u}\hat{\zeta}}(\omega) = \int_{-\infty}^{\infty} d\tau\ e^{i\omega\tau}\ C_{\hat{u}\hat{\zeta}}(\tau) \quad .$$

Further

$$C_{u\zeta}(\sigma,\tau) = \langle u(s,t)\ \zeta(s-\sigma,\ t-\tau) \rangle$$

$$= \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\ e^{i(k\sigma-\omega\tau)}\ F_{u\zeta}(k,\omega)$$

so that

$$F_{u\zeta}(k,\omega) = \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\tau\ e^{-i(k\sigma-\omega\tau)}\ C_{u\zeta}(\sigma,\tau) \quad .$$

Hence

$$F_{\hat{u}\hat{\zeta}}(\omega) = R^{-2}\int_0^R ds' \int_0^R ds'' \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau} \int_{-\infty}^{\infty}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\, e^{i(k\sigma-\omega\tau)}\, F_{u\zeta}(k,\omega')$$

$$= R^{-2}\int_0^R ds' \int_0^R ds'' \int_{-\infty}^{\infty}\frac{dk}{2\pi}\, e^{ik\sigma}\, F_{u\zeta}(k,\omega)$$

$$= R^{-2}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\, F_{u\zeta}(k,\omega)\left[\int_{-R}^{0} d\sigma(R+\sigma)\, e^{ik\sigma} + \int_0^R d\sigma(R-\sigma)\, e^{-ik\sigma}\right]$$

$$= R^{-2}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\, F_{u\zeta}(k,\omega)\; 2\int_0^R d\sigma(R-\sigma)\cos k\sigma$$

$$= \int_{-\infty}^{\infty}\frac{dk}{2\pi}\,\frac{2}{k^2R^2}\,(1-\cos kR)\, F_{u\zeta}(k,\omega)$$

The Reynolds spectrum $F_{uw}(k,\omega) = i\omega F_{u\zeta}(k,\omega)$, and so will be related to the quadrature spectrum of the measured quantities.

REFERENCES

[1]W. Munk, "Sound Channel in an Exponentially Stratified Ocean; With Application to SOFAR," J. Acoust. Soc. Am. **55**, 220-226 (1974).

[2]W. H. Munk and F. Zachariasen, "Sound Propagation Through a Fluctuating Stratified Ocean: Theory and Observation," J. Acoust. Soc. Am. **59**, 818-838 (1976), to be referred to as MZ. Table 1 of MZ is in error with regard to u/c .

[3]For example, O. M. Phillips, *The Dynamics of the Upper Ocean* (Cambridge University Press, 1966), Eq. 5.4.6.

[4]C. H. McComas III, "Equilibrium Mechanisms Within the Oceanic Internal Wave Field," J. Phys. Oceanogr. (in press).

[5]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, 1965). Formula 2.264 is helpful. For the first integral set $x = 1-y$ and multiply numerator and denominator by $\sqrt{x}$.

[6]P. F. Worcester, "Reciprocal Transmission in a Mid-ocean Environment," J. Acoust. Soc. Am. (in press).

[7]For the time being we are sloppy about distinguishing between projections and arc lengths *along* the tilted ray.

[8]W. Munk, J. Acoust. Soc. Am. (1974) Eq. 24.

APPENDIX

The second integral in (3.2) diverges as $D \to 1$. Set

$$\Delta^2 = D^2 - 1 = (n/\omega_{in})^2 \tan^2\theta \quad .$$

Then the integral approaches

$$I = \frac{8}{3\pi} \log \frac{2}{\Delta} \quad . \tag{1}$$

We can express Δ as a function of range. Let source and receiver be on the axis at $x = \mp \frac{1}{2} R$. Then with

$$\delta z = A \cos \pi x/R$$

designating the distance of the ray above the sound axis, we find[8]

$$A = -\frac{3\pi}{4} B \frac{R-R_o}{R_o}$$

where $R_o = \frac{1}{2}\pi B\varepsilon^{-\frac{1}{2}} = 21$ km refer to the axial loop ($R < R_o$ for upward loops). It follows that

$$\theta = \frac{\delta z}{\delta x} = \frac{3\pi^2}{4} \frac{B}{R} \frac{R-R_o}{R_o} \sin \frac{\pi x}{R}$$

$$\Delta \approx \frac{n_\perp \theta}{\omega_{in}} = \frac{3\pi^2}{4} \frac{n_\perp}{\omega_{in}} \frac{B}{R} \frac{R-R_o}{R_o} \sin \frac{\pi x}{R} \quad .$$

Setting $B = 1$ km, $n_1 = 1.9\times10^{-3}$ sec^{-1}, $\omega_{in} = 7.3\times10^{-5}$ sec^{-1}, $R_o = 21$ km, we have

$$\Delta = 9.2\,\frac{R-R_o}{R_o}\sin\frac{\pi x}{R} \quad .$$

We now require

$$\frac{1}{R}\int_{-\frac{1}{2}R}^{\frac{1}{2}R} dx\ I = \frac{8}{3\pi R}\int_{-\frac{1}{2}R}^{\frac{1}{2}R} dx\left[\log\frac{2R_o}{9.2(R-R_o)} + \log 2\right]$$

$$= \frac{8}{3\pi}\log\left[\frac{4R_o}{9.2(R-R_o)}\right] \quad .$$

Thus even if we are within 0.1 km of the convergence distance, $R-R_o = 0.1$ km, the effect of currents is still relatively small. So the logarithmic singularity is of no practical interest.

As an aside, the singularity arises because $(\omega^2-\omega_L^2)^{-\frac{1}{2}} \to (\omega^2-\omega_{in}^2)^{-\frac{1}{2}}$ and $G_{uu} \sim (\omega^2-\omega_{in}^2)^{-\frac{1}{2}}$, so that the integrand of $I_{uu} \sim (\omega^2-\omega_{in}^2)^{-1}$, and I_{uu} is logarithmically infinity. This depends then on the fact that GM-spectrum for u has a $(\omega^2-\omega_{in}^2)^{-\frac{1}{2}}$ cusp (but, of course, the integral of the spectrum and hence $\langle u^2\rangle$ are finite). If the GM-spectrum had a $(\omega^2-\omega_{in}^2)^{-s}$ cusp, with $s < \frac{1}{2}$, then the logarithmic infinity for I'_{uu} does not arise. The evidence for the $-\frac{1}{2}$ exponent is very weak. Observational evidence clearly shows a cusp (see for example Carl Wunsch, 1975, *JGR* **80**, 339-343, Fig. 1) and for an integrable cusp "... we require $0 < s < 1$. We arbitrarily choose the mid-point $s = \frac{1}{2}$." (GM72, p. 249). But subsequently Cairns and Williams (Part II, Fig. 5) find the vertical displacement spectrum (which is a less critical test than a u-spectrum) in better accord with $s = 0.5$ than $s = 0.1$ and $s = 0.9$. The conclusion is that the singularity is probably too weak to be observable; otherwise, it would suggest an acceptable modification of the GM-spectrum.

DISTRIBUTION LIST

ORGANIZATION	NO. OF COPIES
Dr. Henry D.I. Abarbanel National Accelerator Laboratory P.O. Box 500 Batavia, Illinois 60510	1
Mr. Bob Adams Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Dr. V.C. Anderson Scripps Institution of Oceanography University of California La Jolla, California 92037	1
Dr. Alf Andreassen OP-095T The Pentagon, Room 5D560 Washington, D.C. 20350	1
Mr. F. Andrews Catholic University 620 Michigan Avenue, NE Washington, D.C. 20017	1
Mr. H.S. Aurand, Jr. Ocean Acoustics Division Naval Ocean Systems Center San Diego, CA 92152	1
Mr. James Austin Johns Hopkins University Applied Physics Laboratory Johns Hopkins Road Laurel, Maryland 20810	1
Cdr. J. Ballou Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1
Dr. James E. Barger Bolt, Baranek & Newman, Inc. 50 Moulton Street Cambridge, MA 02138	1

ORGANIZATION	NO. OF COPIES
Mr. C. Bartberger Naval Air Development Center Warminster, Pennsylvania 18974	1
Bell Telephone Laboratories Whippany Road Whippany, New Jersey 07981	2
Dr. Joel Bengston Institute for Defense Analysis 400 Army Navy Drive Arlington, Virginia 22202	1
Dr. A. Berman Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Mr. Ange V. Bernard, Jr. Manager Anti-Submarine Warfare Systems Project Department of the Navy Washington, D.C. 20360	1
Dr. H.F. Bezdek Program Director CODE 460 NORDA Bay St. Louis, MS 39529	1
Prof. T.G. Birdsall Cooley Electronics Laboratory Cooley Bldg., North Campus University of Michigan Ann Arbor, Michigan 48105	1
Mr. George L. Boyer Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1
Mr. D.G. Browning New London Laboratory Naval Underwater Systems Center New London, Connecticut 06320	1

ORGANIZATION	NO. OF COPIES
Mr. R. Buchal Office of Naval Research Acoustic Environmental Support Detachment Arlington, Virginia 22217	1
Mr. B.M. Buck Polar Research Laboratory, Inc. 123 Santa Barbara Street Santa Barbara, California 93101	1
Dr. H. Bucker Naval Ocean Systems Center San Diego, California 92152	1
Dr. Peter J. Cable Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. D. Cacchione Office of Naval Research 495 Summer Street Boston, MA 02210	1
Dr. Curtis G. Callan, Jr. Department of Physics Princeton University Princeton, NJ 08540	1
Lt. Col. G. Canavan Advanced Research Projects Agency Strategic Technology Office 1400 Wilson Boulevard Arlington, Virginia 22209	1
Mr. G. Cann ODDR&E The Pentagon, Room 3D1048 Washington, D.C. 20301	1
Dr. Gerald Carruthers P.O. Box 1925 Main Post Office Washington, D.C. 20013	1
Dr. Peter Carruthers Lauretson Lab. for High Energy Physics California Institute of Technology Pasadena, California 91109	1

ORGANIZATION	NO. OF COPIES
Dr. Kenneth M. Case 2-11037-230 The Rockefeller University New York, New York 10021	1
Dr. Joseph W. Chamberlain 18622 Carriage Court Houston, Texas 77058	1
Mr. Robert M. Chapman Special Assistant Marine Systems Garrett Corporation 9851 Sepulveda Boulevard P.O. Box 92248 Los Angeles, California 90009	1
Dr. J.G. Clark Institute for Acoustical Research University of Miami 615 SW Second Avenue Miami, Florida 33130	1
Dr. R.H. Clarke Imperial College of Science & Technology Department of Electrical Engineering Exhibition Road London SW7 2BT ENGLAND	1
Dr. Bernard F. Cole Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Dr. W.J. Condell Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1
Mr. Randall G. Cook Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1
Mr. R. Cooper Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1

ORGANIZATION	NO. OF COPIES
Courant Institute 251 Mercer Street New York, NY 10012	2
Dr. C. Cox University of California/San Diego 9530 La Jolla Shores Drive La Jolla, California 92037	1
Capt. Henry Cox Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1
Mr. J.M. D'Albora Naval Underwater Systems Center Newport, Rhode Island 02840	1
Dr. Roger F. Dashen Institute for Advanced Study Princeton, New Jersey 08540	1
Dr. S.C. Daubin Rosential School of Marine and Atmospheric Science University of Miami Miami, Florida 33149	1
Dr. Ruth Davis ODDR&E The Pentagon, Room 3E114 Washington, D.C. 20301	2
Dr. H. DeFarrari Rosentiel School of Marine and Atmospheric Science University of Miami Miami, Florida 33149	1
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	12
Mr. John A. DeSanto Naval Research Laboratory Code 8160 Washington, D.C. 20375	1
Dr. Alvin M. Despain 1192 Grizzly Peak Boulevard Berkeley, California 94708	1
Dr. John M. Deutsch Dept. of Chemistry, Rm. 18-399 Massachusetts Institute of Technology Cambridga, Massachusetts 02139	1
Mr. Ferdinand P. Diemer Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1
Mr. F. Dinapoli Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. J. Dugan Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Prof. I. Dyer Massachusetts Institute of Technology Department of Ocean Engineering Cambridge, Massachusetts 02139	1
Prof. Freeman J. Dyson Institute for Advanced Study Princeton, New Jersey 08540	1
Mr. L. Einstein Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. A. Ellinthorpe Naval Underwater Systems Center Code TE New London, Connecticut 06320	1
Dr. David Elliott SRI, International 333 Ravenswood Avenue Menlo Park, California 94025	1

ORGANIZATION	NO. OF COPIES
Dr. J.O. Elliott Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Dr. Terry Ewart Applied Physics Laboratory University of Washington 1013 Northeast Fortieth Street Seattle, Washington 98195	1
Mr. J.I. Ewing Lamont-Doherty Geological Observatory Columbia University Palisades, New York 10964	1
Dr. A.G. Fabula Naval Ocean Systems Center San Diego, California 92132	1
Dr. F. Fisher Scripps Institution of Oceanography University of California/San Diego La Jolla, California 92037	1
Dr. R.M. Fitzgerald Naval Research Laboratory Department of the Navy Washington, D.C. 20375	1
Dr. Stanley M. Flatté 360 Moore Street Santa Cruz, CA 95060	1
Mr. E. Floyd Naval Ocean Systems Center San Diego, California 92132	1
Dr. Henry M. Foley Columbia University Department of Physics New York, NY 10027	1
Mr. H. Freese Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1

ORGANIZATION	NO. OF COPIES
Dr. Eugene G. Fubini E.G. Fubini Consultants, Ltd. 1901 N. Fort Meyer Dr., Ste. 1200 Arlington, Virginia 22209	1
Dr. Richard L. Garwin IBM, TJWatson Research Center P.O. Box 218 Yorktown Heights, NY 10598	1
Dr. Roy Gaul, Director Code 600 NORDA Bay St. Louis, MS 39529	1
Mr. A.A. Gerlach Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Dr. C. Gibson University of California/San Diego P.O. Box 119 La Jolla, California 92038	1
Dr. Marvin L. Goldberger Department of Physics Princeton University Princeton, NJ 08540	1
Dr. Ralph Goodman Technical Director NORDA, CODE 110 Bay St. Louis, MS 39529	1
Dr. D. Gordon Naval Ocean Systems Center San Diego, CA 92132	1
Mr. O.D. Grace Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Dr. Richard Gustafson Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1

ORGANIZATION	NO. OF COPIES
Mr. H. Guthart SRI, International 404B 333 Ravenswood Avenue Menlo Park, CA 94025	1
Mr. G.R. Hamilton Director, Ocean Research Office NORDA, CODE 400 Bay St. Louis, MS 39529	1
Dr. J.S. Hanna Science Applications, Inc. 8400 Westpark Drive McLean, Virginia 22101	1
Mr. R. Hardin Bell Telephone Laboratories Chester, New Jersey 07930	1
Mr. Raymond W. Hasse Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Cdr. R.K. Hastie NAVMAT-031 Washington, D.C. 20360	1
Dr. E.E. Hays Woods Hole Oceanographic Institution Woods Hole, Massachusetts 02543	1
Dr. George H. Heilmeier Director Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, VA 22209	1
Dr. John Brackett Hersey Deputy Asst. Oceanographer for Ocean Science Chief of Naval Research Naval Research Laboratory Washington, D.C. 20390	1
Mr. Robert L. Himbarger ORNICON Corporation P.O. Box 22113 San Diego, CA 92122	1
Dr. Richard Hoglund Operations Research 1400 Spring Street Silver Spring, MD 20910	1

ORGANIZATION	NO. OF COPIES
Dr. C.W. Horton, Sr. Applied Research Laboratory University of Texas P.O. Drawer 8029 Austin, Texas 78712	1
Dr. T. Horwath Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	1
Dr. Benjamin Huberman Asst. Director for National Security, International & Space Affairs Office of Science & Technical Policy New Executive Office Bldg., Rm. 3019 Washington, D.C. 20506	1
Mr. B. Hurdle Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Dr. William J. Hurley Center for Naval Analysis 1401 Wilson Boulevard Arlington, VA 22209	1
Dr. David Hyde OASN (RE&S) The Pentagon, Room 4E741 Washington, D.C. 20350	1
Dr. Francis J. Jackson Bolt, Beranek, & Newman, Inc. 1701 N. Fort Myer Drive Arlington, Virginia 22209	1
Mr. M. Jacobson Rensselaer Polytechnic Institute Department of Mathematics Troy, New York 12181	1
Mr. L. Bruce James Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1
Mr. James Janke Headquarters USAF (SAMI) Bolling AFB Washington, D.C. 20332	1

ORGANIZATION	NO. OF COPIES	ORGANIZATION	NO. OF COPIES
Mr. Finn Jensen Saclant ASW Research Centre Viale San Bartolomeo 400 I-19026 La Spezia, ITALY	1	Cdr. Alan H. Krulish Department of the Navy Office of the Chief of Naval Operations Washington, D.C. 20350	1
Mr. William John Jobst Research Scientist Palisades Geophysical Institute 615 SW Second Avenue Miami, Florida 33130	1	Dr. Richard Kurth Sperry Research Center 100 North Road Sudbury, MA 01776	1
Dr. Jonathan Katz Department of Astronomy University of California Los Angeles, CA 90024	1	Dr. F. Labianca Bell Telephone Laboratories Whippany Road Whippany, New Jersey 07881	1
Dr. A.I. Kaufman Center for Naval Analyses 1401 Wilson Boulevard Arlington, VA 22209	1	Mr. R. Lauer Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Dr. Roger N. Keeler Director of Navy Technology Department of the Navy Washington, D.C. 20360	1	Dr. Ray Leadabrand SRI, International, L1053 333 Ravenswood Avenue Menlo Park, CA 94025	1
Mr. J. Keller Courant Institute 251 Mercer Street New York, NY 10012	1	Mr. Barry Levin Naval Intelligence Supply Center 4301 Suitland Road Washington, D.C. 20390	1
Mr. Theo Kooij ARC Director Advanced Research Projects Agency ARPA Research Center, Unit 1 Moffett Field, California 94035	1	Dr. Harold W. Lewis Department of Physics University of California Santa Barbara, CA 93106	1
Dr. Norman Kroll University of California/San Diego P.O. Box 119 La Jolla, California 92038	1	Dr. Bernard Lippmann Department of Physics New York University 4 Washington Place New York, NY 10003	1
Dr. Martin Kronengold, Director Institute for Acoustical Research University of Miami 615 SW Second Avenue Miami, Florida 33130	1	Mr. Jeff Lipscomb NFOIO 4301 Suitland Road Washington, D.C. 20390	1
		Dr. Donald J. Looft Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, VA 22209	1

ORGANIZATION	NO. OF COPIES
Cdr. Terry J. McCloskey Director, CODE 200 NORDA Bay St. Louis, MS 39529	1
Dr. J. McCoy Naval Research Laboratory 4555 Overlook Avenue Washington, D.C. 20375	1
Mr. S. McDaniel Applied Research Laboratory Pennsylvania State University P.O. Box 30 State College, PA 16801	1
Mr. Mike McKisic Scientific Officer NORDA, CODE 460 Bay St. Louis, MS 39529	1
Dr. G. Maidanik Naval Ship Research and Development Center Washington, D.C. 20007	1
Dr. David E. Mann Asst. Secretary of the Navy (RE&S) The Pentagon, Room 4E732 Washington, D.C. 20350	1
Dr. S.W. Marshall NORDA, CODE 340 Bay St. Louis, MS 39529	1
Mr. R.L. Martin Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Prof. H. Medwin Naval Postgraduate School Department of Physics Monetery, CA 93940	1
Mr. R.H. Mellen Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Dr. David Middleton 127 East 91st Street New York, New York 10028	1

ORGANIZATION	NO. OF COPIES
Dr. M. Milder ARETE Associates 2120 Wilshire Boulevard Santa Monica, CA 90903	1
Dr. J. Miles University of California/San Diego P.O. Box 119 La Jolla, California 92038	1
Cdr. A.R. Miller Naval Electronic System Command PME-124 Washington, D.C. 20360	1
Mr. Robert A. Moore Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1
Dr. Paul H. Moose Naval Ocean Systems Center San Diego, CA 92132	1
Mr. John Morfit Main Post Office Box 1925 Washington, D.C. 20013	1
Dr. G.B. Morris University of California Scripps Institution of Oceanography Marine Physical Laboratory San Diego, California 92152	1
Mrs. H. Morris Naval Ocean Systems Center San Diego, CA 92132	1
Dr. William A. Moseley Supervisor, Research Physics Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1
Dr. Richard A. Muller 2831 Garber Berkeley, CA 94705	1
Dr. Walter H. Munk 9530 La Jolla Shores Drive La Jolla, CA 92037	1

ORGANIZATION	NO. OF COPIES
Naval Electronic Systems Command Headquarters Code PME-124 Washington, D.C. 20300	2
Naval Underwater Systems Center ATTN: Technical Library New London Laboratories New London, Connecticut 06320	1
Dr. J. Neubert Naval Ocean Systems Center San Diego, CA 92132	1
Dr. William A. Nierenberg Scripps Institution of Oceanography University of California La Jolla, California 92037	1
Mr. J.C. Nolen Institute for Defense Analyses 400 Army Navy Drive Arlington, VA 22202	1
Operations Research, Inc. 1400 Spring Street Silver Spring, MD 20910	1
Dr. David Palmer Code 8172 Naval Research Laboratory Department of the Navy Washington, D.C. 20375	1
Mr. J. Papadakis Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. M.A. Pedersen Naval Ocean Systems Center San Diego, CA 92132	1
Dr. Francis W. Perkins, Jr. Plasma Physics Laboratory Princeton University P.O. Box 451 Princeton, New Jersey 08540	1

ORGANIZATION	NO. OF COPIES
The Honorable William Perry Director, DDR&E Office of the Secretary of Defense The Pentagon, Room 3E1006 Washington, D.C. 20301	1
Dr. O.M. Phillips Hydronautics, Inc. Pindell School Road Howard County Laruel, MD 20810	1
Dr. R. Porter Woods Hole Oceanographic Institution Woods Hold, MA 02543	1
Mr. James Probus Director of Navy Laboratories The Pentagon Washington, D.C. 20350	1
Dr. Gordon Raisbeck Arthur D. Little, Inc. Cambridge, MA 02140	1
Mr. D.J. Ramsdale Acoustics Division Code 8170 Naval Research Laboratory Washington, D.C. 20375	1
Dr. Burton Richter Stanford Linear Accelerator Center P.O. Box 4349 Stanford, California 94305	1
Mr. W.T. Roderick Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. Richard R. Rojas Assoc. Director of Research for Oceanography Naval Research Laboratory Washington, D.C. 20390	1
Dr. Marshall Rosenbluth Institute for Advanced Study Princeton, New Jersey 08540	1

ORGANIZATION	NO. OF COPIES	ORGANIZATION	NO. OF COPIES
Dr. R. Ruffine ODDR&E The Pentagon Washington, D.C. 20301	1	Mr. Edward L. Smith Advanced Research Projects Agency ARPA Research Center Unit 1 Moffett Field, California 94035	1
Capt. Kenneth W. Ruggles Office of the Deputy Director for Research and Engineering (DDR&E) The Pentagon Washington, D.C. 20301	1	Dr. Gary L. Smith Johns Hopkins University Applied Physics Laboratory Johns Hopkins Road Laurel, Maryland 20810	1
Dr. R. Saenger Naval Underwater Systems Center New London Laboratory	1	Dr. Preston W. Smith Bolt, Beranek and Newman, Inc. 50 Moulton Street Cambridge, MA 02138	1
Dr. H. Schenk Naval Ocean Systems Center San Diego, CA 92132	1	Mr. H. Sonneman Office of the Assistant Secretary of the Navy The Pentagon Washington, D.C. 20360	1
Dr. M. Schulkin Naval Oceanographic Office Suitland, Maryland 20373	1	Mr. Glenn R. Spaulding Headquarters, Naval Material Command (MAT034), Room 1044 Washington, D.C. 20360	1
Prof. Peter Schultheiss Yale University New Haven, CT 16520	1	Dr. F. Spiess University of California Scripps Institution of Oceanography Marine Physical Laboratory San Diego, California 92152	1
Dr. Phil Selwyn Advanced Research Projects Agency Tactical Technology Office 1400 Wilson Boulevard Arlington, VA 22209	1	Dr. R. Spindel Woods Hole Oceanographic Institution Woods Hole, Massachusetts 02543	1
Capt. J. Shilling Strategic Systems Project Office Department of the Navy Washington, D.C. 20390	1	Mr. C.W. Spofford Science Applications, Inc. 8400 Westpark Drive McLean, VA 22101	1
Dr. Morris Schulkin 9325 Orchard Drive Potomac, MD 20854	1	Mr. D.C. Stickler Applied Research Laboratory Pennsylvania State University P.O. Box 30 State College, PA 16801	1
Mr. Carey D. Smith Commander, Naval Sea Systems Command Headquarters Department of the Navy Code 06H1 Washington, D.C. 20360	1		

ORGANIZATION	NO. OF COPIES	ORGANIZATION	NO. OF COPIES
Dr. M. Strassberg Naval Ship Research & Development Center Washington, D.C. 20007	1	Capt. Robert E. Vaughn, USN OP-095T The Pentagon, Room 5D560 Washington, D.C. 20350	1
Mr. A.O. Sykes Office of Naval Research Code 412 800 N. Quincy Street Arlington, VA 22217	1	Dr. John F. Vesecky Center for Radar Astronomy Stanford University Stanford, California 94305	1
Mr. T.E. Talpey Bell Telephone Laboratories Whippany Road Whippany, New Jersey 07981	1	Dr. William A. Von Winkle Assoc. Technical Director for Technology Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Dr. Frederick Tappert Courant Institute 251 Mercer Street New York, NY 10012	1	Dr. Kenneth Watson Lawrence Berkeley Laboratory University of California Berkeley, California 94720	1
Capt. Peter R. Tatro, USN Office of the Oceanographer of the Navy 200 Stovall Street Alexandria, Virginia	1	Mr. J. Weileman Courant Institute 251 Mercer Street New York, NY 10012	1
Dr. Alex Thompson Physical Dynamics, Inc. Berkeley, California	1	Dr. H. Weinberg Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1
Mr. Richard D. Trueblood Electronics Engineer Commander Naval Ocean Systems Center San Diego, CA 92152	1	Dr. M.S. Weinstein Underwater Systems, Inc. 8121 Georgia Avenue, Ste. 700 Silver Spring, MD 20910	1
Dr. H. Uberall Catholic University 620 Michigan Avenue, NE Washington, D.C. 20017	1	Maj. Gen. J.A. Welch, Jr. AFSA 1E 388 The Pentagon Washington, D.C. 20330	1
University of Texas Applied Research Laboratory Austin, Texas 78712	2	Capt. J.B. Wheeler Naval Electronic System Command Code 103 Washington, D.C. 20360	1
Mr. R.J. Urick TRACOR, Inc. 1601 Research Blvd. Rockville, MD 20850	1	LTC. William Whitaker Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, VA 22209	1

ORGANIZATION	NO. OF COPIES	ORGANIZATION	NO. OF COPIES
Mr. H. Wilson Science Applications, Inc. P.O. Box 351 La Jolla, CA 92037	1		
Dr. J.M. Witting Naval Research Laboratory 4555 Overlook Avenue, SW Washington, D.C. 20375	1		
Mr. Peter Worcester University of California/San Diego E-25 La Jolla, California 92093	1		
Dr. J.L. Worzel Marine Science Institute Geophysics Laboratory 700 Thestrand Galveston, Texas 77550	1		
Dr. Carl Wunsch Massachusetts Institute of Technology Room 54-1324 Cambridge, Massachusetts 02139	1		
Dr. H. Yura Aerospace Corporation P.O. Box 92956 Los Angeles, CA 90009	1		
Dr. Frederik Zachariasen 452-48 Department of Physics California Institute of Technology Pasadena, California 91109	1		