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# PARAMETRIC FREQUENCY CONVERSION

by

Daniel E. Fitch
B.S. California State University, Sacramento, 1976

THESIS

Submitted in partial satisfaction of
the requirements for the degree of

MASTER OF SCIENCE

in

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at

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## INTRODUCTION

It is generally known that the application of energy to a linear network will result in the appearance of a proportional replica of that energy at an output port of that network. Further, the application of more than one energy input will result in a weighted linear summation of those inputs. This assertion of course neglects energy level and bandwidth limitations.

If, however, one or more sources of energy are applied to a network possessing a nonlinear characteristic, a quite different situation emerges. The output will contain components of the applied energies plus sum and difference components of the original energies and their harmonics. This phenomenon is widely known as frequency conversion, modulation, or mixing.

The nonlinear operation of a network is commonly due to the presence there of one or more nonlinear impedances. This impedance may be complex, having either resistance or reactance or some combination thereof. Resistive nonlinearities may be provided by the action of diodes operating under varying levels of forward bias and three terminal devices such as junction transistors, FETs, and such which are operated over the nonlinear extremes of their transfer characteristics. Other materials and devices exist which also exhibit this behavior but are not within the scope of this paper.[1,2]

Reactive nonlinearities are found to be manifest by the junction capacitance of reverse biased semiconductor diodes and in the inductance of certain saturable reactors.[3] This paper puts forth a treatment concerned mainly with the former, although the derivations and analysis to follow applies with equal validity to nonlinear inductances.

Variable capacitance diodes, also known as voltage variable capacitors or varactor diodes, offer to the user numerous tangible benefits over more conventional components. Although much has been done to utilize the nonlinear resistance devices in the design of frequency converters, modulators, and mixers, any resistance device dissipates energy. This energy contributes thermal noise to any system in which it is used and ultimately limits the obtainable signal to noise ratio. This, in turn, limits the minimum signal which may be used for communication or data transfer.

A nonlinear reactance, however, can only store and transfer energy, contributing no thermal noise to a circuit. It is this advantage which makes possible the low noise operation experienced by a class of systems known as Parametric Amplifiers and Frequency Converters. Unlike systems which generate thermal noise in their operation, Parametric circuits may be realized in which the largest component of noise may be attributed to the thermal temperature of the circuit components used and terminations applied.

## PURPOSE

It is the purpose of this paper to analyze and discuss the operation and characteristics of the parametric frequency converter and to assess the subtleties of such circuits when applied to communication systems. A derivation put forth[4] generalizing the power relationships of signals applied to this class of circuits will be examined, from which a small signal analysis[5] may fruitfully follow. The small signal analysis will include derivations of representative gain equations for both power gain and transducer gain realizable with the various modes of operation. Further, expressions governing bandwidth and system sensitivities will be shown. Finally, an experimental endeavor will be discussed, including design considerations, results and conclusions.

## HISTORICAL PERSPECTIVE OF THE PARAMETRIC PRINCIPLE

The principles underlying the operation of systems having varying parameters are not new. As early as 1831 Michael Faraday[6] observed the "crispations" found to occur upon the surface of a liquid when the sides of the glass containing it were caused to vibrate. Faraday determined experimentally that the walls of the glass actually vibrated at twice the frequency of the liquid's surface. These crispations, or standing wave phenomena, may be produced by sliding a moistened finger around the circumference of a glass, similar to the action of a bow and violin string.

In 1859 Melde[7] reported his analysis of this principle. His experiment, in which a fine string was maintained in vigorous transverse oscillation by a massive tuning fork attached to one end, detailed the fact that the string settled into oscillation at twice the tuning fork frequency. Fundamental here is the fact that the fork was caused to oscillate in a direction parallel to the hanging string, offering no readily obvious cause for the resulting pendulum swing. Lord Rayleigh,[8,9,10] an excellent mathemetician, analyzed the works of Faraday, Melde and others while he experimented with these and similar mechanical systems.

A variable inductance system used prior to WWI for communications between Berlin and Vienna was described in 1915 by L. Kuhn.[11] This system utilized a variable inductance (a saturable reactor) to control the output of a continuous arc transmitter. Modulated in this way the signal, voice frequencies, caused sidebands to be sent along with the arc carrier to facilitate demodulation at the receiver. A year later, Zennech,[12] Alexanderson[13] and Hartley[14] contributed pioneering experimental and theoretical works. Alexanderson, in a paper delivered at an IRE meeting in New York City February 2, 1916, dubbed the system a "Magnetic Amplifier", a name still in use today, and concluded ..." The ratio of amplification is proportional to the ratio between the frequency of the radio current and that of the controlling current." This supposition was borne out by Hartley and later generalized by Manley and Rowe.[15] Alexanderson further asserted that negative resistance effects could be generated. He pointed out, in the same presentation, that "... instability and self-excited oscillations..." could exist. He concluded that they could have some useful applications for other purposes. In 1930, Peterson[16] cited that although these effects were to be avoided in telecommunication systems, the effect might have one very promising application; the negative resistance straight through amplifier.

Through the twenties and thirties there was much interest in this method of frequency conversion, or modulation, whereby harmonics and subharmonics could be generated with much more practicality than the Audion systems then in use. This interest subsided markedly, though, with the introduction of high power vacuum tube modulators and amplifiers by De Forest and others.

In 1945, L. Apker[17] of General Electric Company observed the reciprocity failure of crystal mixers. Smith,[18] of Purdue University and Torrey[19] of MIT followed with their discussion of welded contact germanium diodes made by North[31] of General Electric. Their efforts indicated that the capacitance of the diode junction varied with bias. This revelation spurred interest in the parametric concept again with hopes that noise levels comparable to or better than those available with vacuum tube devices would be obtainable. Torrey gave a stimulating discussion of frequency converters using nonlinear capacitance elements.

Waltz[21] and Pound[22] released their observations of negative conductance in intermediate frequency amplifiers in 1948. They described gain and conductance measurements made upon experimental devices. Pound supposed that converters having a better noise figure than conventional systems should be obtainable, but was unable to effect this. Later, in 1948, A van der Zeil[23] described the mixer, or frequency conversion effects of nonlinear capacitors, and pointed out the possibility of low noise operation. Further theoretical hypothesis concerned with low noise thought to be achievable with the technique was put forth by Landon.[24]

The so-called North diodes available at that time were somewhat lossy, such that the low noise operation predicted was not obtained. In 1952, Edwards[25] used diodes made by Ohl, observing the same behavior as the earlier attempts while attaining better noise temperatures. It was not until 1954 when the "Task 8"[26] diodes resulted from a U.S. Signal Corps. sponsored project at Bell Telephone Laboratories, that new techniques for low loss diodes were developed. This generated new interest and enthusiasm resulting in works by Hines,[27] Adler[28] and others,[29,30,31] while at the same time Suhl[32] proposed that a ferrite material could be used as a nonlinear inductance at microwave frequencies. He further recognized the need for idler circuits to provide for the existance of energy at frequencies other than the signal and pump frequencies and the idler's contribution to the stability of the amplifier.

Kita and Fujii,[33] working independently of Hines[27] and Adler,[28] were able at the same time to demonstrate gain and oscillations at microwave frequencies using the variable capacitance diodes.

With all of this, it was not until 1958 that the low noise predicted theoretically was actually obtained in the laboratory. In a frenzy Uenohara,[34] Engelbrecht,[35] Salzberg[36] and Heffner and Kotzebue[37] each obtained low noise operation at different frequencies ranging from 1 MHz to 6 GHz.

From this point until the present, discussions, experimental data and such have come forth with an amazing regularity and volume. As the technology advances and the low noise aspects of semiconductors improve, transistors able to operate at frequencies well into the microwave region have appeared. The presence of thermal noise, however, ultimately limits their use in this region. With recent devlopments in the area of amorphous semiconductors, it becomes a matter of speculation whether this or some equally new and interesting development will assume much of the enthusiasm now enjoyed by nonlinear elements as we now know them.

GENERAL ENERGY RELATIONS

In January of 1956, J.M. Manley and H.E. Rowe submitted the first part of a two-part manuscript treating some general properties of circuits containing nonlinear elements.[4] The substance of this work was the derivation of the now famous power relationships among energies of differing frequencies in nonlinear reactances. The following derivation parallels that presented by Manley and Rowe.

Several of the properties associated with a capacitor may be delineated by letting the voltage, v, present between its terminals be given by some function of charge, q, present.

$$v = f(q) \tag{1}$$

For the particular case of a linear capacitor, the relation is

$$v = \left(\frac{1}{C}\right) q$$

where C is the measure of capacitance. We wish to specify that f(q) is single valued, but is not restricted in any other way. The presence of hysteresis shall be considered later.

As an aid to the visualization of the derivation, but not intended to place limits upon it in any way, the capacitance may be thought to be connected to the circuit of Figure 1. Each subcircuit connected in parallel with the capacitor consists of an ideal generator of frequency $f_n$, a load impedance, and an ideal filter all series-connected. The ideal filter is further described as presenting a zero impedance at the generator frequency, $f_n$, and infinite impedance at all

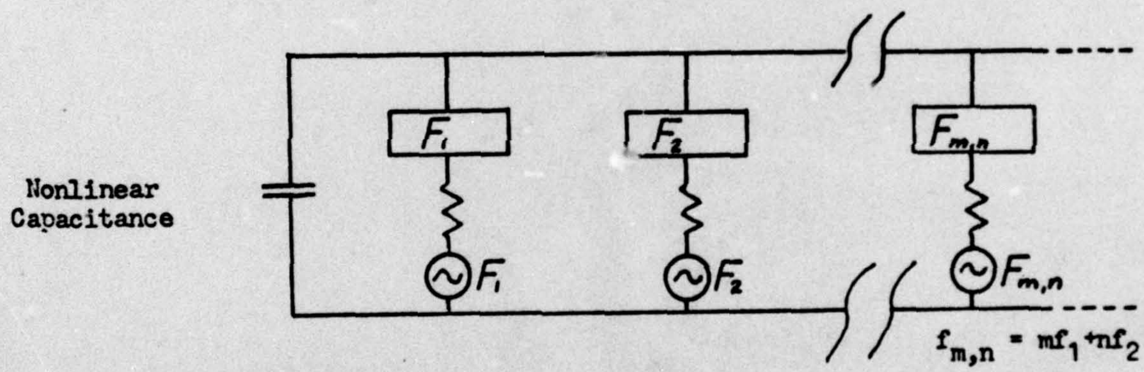


Figure 1 - Representative Parametric Circuit

Ideal Filters have zero impedance at frequency, f, infinite elsewhere.

other frequencies.

For the case of a nonlinear capacitance present in the circuit, the modulation principles discussed earlier apply. The fundamental frequencies $f_1$ and $f_2$ will be applied and present. Moreover, all of the frequencies $f_{m,n} = mf_1 + nf_2$ will be present across the capacitor, each also existing in their respective filter circuits exclusively. The fundamental frequencies are assumed to be positive and not identical. Non-essential to this discussion is the taking of $f_1$ and $f_2$ as the applied frequencies, although this will be convenient later.

The charge flowing into the nonlinear capacitor may be described by the double Fourier Series,

$$q = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} Q_{m,n}\, e^{j(mx+ny)} \tag{2}$$

where,

$$x = 2\pi f_1 t = \omega_1 t$$
$$y = 2\pi f_2 t = \omega_2 t$$

The current flowing into the capacitor is determined by taking the total derivative of q with respect to time, yielding,

$$i = \frac{dq}{dt} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} I_{m,n}\, e^{j(mx+ny)} \tag{3}$$

where,

$$I_{m,n} = j(m\omega_1 + n\omega_2)\, Q_{m,n} \tag{4}$$

Continuing, the voltage, v, is taken to be single-valued and a function of x and y such that it may be represented by a double Fourier Series in the same manner as (2),

$$v = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} V_{m,n}\, e^{j(mx+ny)} \tag{5}$$

where,

$$V_{m,n} = \frac{1}{4\pi^2}\int_0^{2\pi} dy \int_0^{2\pi} F(x,y)\, e^{-j(mx+ny)} \tag{6}$$

describe the coefficients of the series. If we next multiply both sides of (6) by $jmQ_{m,n}^*$ and take the summation over m and n from negative to positive infinity, we have;

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} jm\, Q_{m,n}^*\, V_{m,n} = \frac{1}{4\pi^2}\int_0^{2\pi} dy \int_0^{2\pi} F(x,y)\, dx \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} jm\, Q_{m,n}^*\, e^{-j(mx+ny)} \tag{7}$$

Rearranging (7) by interchanging the order of summation and integration, we may identify the double summation as the partial differential of q with respect to x, or;

$$\frac{\partial q}{\partial x} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} jm\, Q_{m,n}\, e^{j(mx+ny)} \tag{8}$$

where,

$$Q_{-m,-n} = Q_{m,n}^* \quad \text{and} \quad Q_{m,n} = Q_{-m,-n}^* \tag{9}$$

since q is taken to be real. Using this, we may then write,

$$\frac{\partial q}{\partial x} = -\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} jm\, Q_{m,n}^*\, e^{-j(mx+ny)} \tag{10}$$

which is just the negative of the double summation contained within the integral. If we now rearrange (4) by solving for $Q_{m,n}$, and substitute into (7) after taking the complex conjugate, we yield;

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{m V_{m,n} I_{m,n}^{*}}{m f_1 + n f_2} = \frac{1}{2\pi}\int_0^{2\pi} dy \int_0^{2\pi} F(x,y)\,\frac{\partial q}{\partial x}\,dx \tag{11}$$

As the partial derivative of q with respect to x times dx is actually dq with y held constant, a change of variable may be made to the variable within the integral to be,

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{m V_{m,n} I_{m,n}^{*}}{m f_1 + n f_2} \tag{12}$$

If this procedure is taken on the variable y, by multiplying (6) by $jnQ_{m,n}^{*}$ and summing from plus to minus infinity over m and n, the analysis yields the analog of (11) to become,

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{m V_{m,n} I_{m,n}^{*}}{m f_1 + n f_2} = \frac{1}{2\pi}\int_0^{2\pi} dx \int_0^{2\pi} f(q)\,dq \tag{13}$$

The limits of the second integration on f(q) are taken to be the variation of y from zero to 2 pi while holding x constant.

The above analysis has made no distinction between positive and negative frequencies, as there can be no physical distinction made. Therefore, we may combine like terms in (11) and (13) such that we may make clear the relations present between the powers, $V_{m,n}I_{m,n}^{*}$, and their respective frequencies by letting

$$S_{m,n} = W_{m,n} + j\,X_{m,n} = 2 V_{m,n} I_{m,n}^{*} \tag{14}$$

and

$$S_{m,n} = S_{-m,-n}^{*} \qquad S_{-m,-n} = S_{m,n}^{*} \tag{15}$$

Doing this, we have

$$W_{m,n} = V_{m,n} I_{m,n}^{*} + V_{m,n}^{*} I_{m,n} = W_{-m,-n} \tag{16}$$

$$X_{m,n} = j\left(V_{m,n}^{*} I_{m,n} - V_{m,n} I_{m,n}^{*}\right) = -X_{-m,-n} \tag{17}$$

A specific frequency will, in general, include both the negative and positive components $(mf_1 + nf_2)$ and $-(mf_1 + nf_2)$. If the indices m and n are restricted only such that $(mf_1 \quad nf_2)$ is greater than zero, eliminating the trivial zero-frequency condition, then $S_{m,n}$, $W_{m,n}$ and $X_{m,n}$ are, respectively, the total vector, real, and imaginary powers flowing into the nonlinear element at the given frequency.

Reviewing (11) and (13), one will find that only the real powers, $W_{m,n}$, will occur in the nonlinear reactance case. In the case of a nonlinear resistance, reactive powers will occur, but are not further considered in this derivation. Combining terms of positive and negative frequency components, (16), (11) and (13) become,

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m W_{m,n}}{mf_1 + nf_2} = \frac{1}{2\pi} \int_0^{2\pi} \int_{q(0,y)}^{q(2\pi,y)} f(q)\, dq\, dy \tag{18}$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{n W_{m,n}}{mf_1 + nf_2} = \frac{1}{2\pi} \int_0^{2\pi} \int_{q(x,0)}^{q(x,2\pi)} f(q)\, dq\, dx \tag{19}$$

Restricting the nonlinear relationship between the charge and voltage to a single-valued function, we note that the variation of q is determined by taking the limits of the integrations in (18) and (19) over one period. Being sinusoidal, q returns to its initial value at the completion of each period. Consequently, these integrals will be identically zero if the nonlinear characteristic is indeed single-valued. Thus, (18) and (19) for this relationship become

$$\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\frac{mW_{m,n}}{mf_1+nf_2}=0 \tag{20}$$

$$\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\frac{nW_{m,n}}{mf_1+nf_2}=0 \tag{21}$$

Happily, these are the results sought in the derivation. They are remarkable, in that they specify two independent relations among the powers flowing into the nonlinear element at the various frequencies, independent of the shape of the applied signals or their power levels. Further, the conditions imposed upon the nonlinear element were only that its characteristic be single-valued. Hysteresis results in nonzero values for the integrations above such that power is consumed. For this case, the terms on the right do not vanish becoming zero, but represent the power lost due to the hysterisis present.

It is interesting that this derivation holds as well for the case of the linear reactance (as it should). $W_{m,n}$ for this condition is equal to zero for all m and n such that (20) and (21) are still satisfied. Further, no modulation phenomena occur.

In the nonlinear reactance without hysteresis, power may not be dissipated. Consequently, the sum of the powers flowing into the reactance must be identically zero. This may be shown by multiplying (20) by $f_1$ and (21) by $f_2$ and adding them together. This is shown in Appendix 2.

FREQUENCY CONVERTER OPERATION

Returning our attention to the circuit of Figure 1 and the equations just derived, we may now reduce some of the generalities present to a few very useful configurations and consider their respective characteristics. We have stated that, in general, when two non-identical signals of frequencies $f_1$ and $f_2$ are applied to our circuit containing a nonlinear reactance, components of the signals applied plus their harmonics and the sum and difference components and their harmonics will appear across the nonlinear element. If ideal filters are placed in the circuit as in Figure 1 for only the fundamental components and their sum of difference frequency, then the Manly-Rowe equations (20) and (21) predict certain relations among those powers allowed to flow into the nonlinear reactance. Specifically, if provision is made only for the sum frequency component and the fundamentals, the circuit may be classified as an upper sideband converter. Referring to Figure 2, energy at $f_2$ is shown to be of higher frequency than $f_1$, with their sum frequency, $f_3 = f_1 + f_2$, shown as their resultant. In the usual condition, one of the fundamental frequencies is considered to be the pump or local oscillator, while the remaining fundamental is called the signal. The resultant, known as the output, is the upper sideband of the mixing or conversion process.

Equations (20) and (21) predict that the output will have a power equal to the ratio of the pump frequency to the signal frequency, or

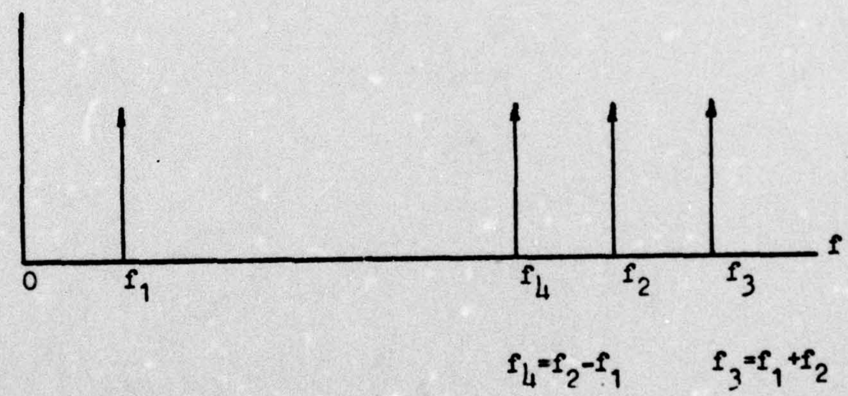


Figure 2 - Relative Input and Output Frequencies

$$\frac{W_{1,0}}{f_1} + \frac{W_{1,1}}{f_1 + f_2} = 0 \tag{22}$$

and,

$$\frac{W_{0,1}}{f_2} + \frac{W_{1,1}}{f_1 + f_2} = 0 \tag{23}$$

or,

$$\frac{W_{1,0}}{f_1} = \frac{-W_{1,1}}{f_3} \tag{24}$$

and,

$$\frac{W_{0,1}}{f_2} = \frac{-W_{1,1}}{f_3} \tag{25}$$

where,

$$f_1 = f_{1,0} \qquad f_2 = f_{0,1} \qquad f_3 = f_{11} = f_1 + f_2$$

For positive powers of $f_1$ and $f_2$, $f_3$ will be negative, i.e.; power is output from the system. Also, the power gain from (24) is

$$\frac{W_{1,1}}{W_{1,0}} = \frac{f_3}{f_1} \tag{26}$$

Consequently, the power gain is equal to the ratio of the input to output frequencies. We have removed the negative sign from (26) by adopting the general convention accepted for the definition of gain as being the ratio of power delivered to a load to the power absorbed by the input of a transducer.

If the power at frequency $f_3$ is caused to be positive, then the input is at $f_3$, down-conversion takes place and the power at frequency $f_1$ is negative. This result is classified as an upper sideband down converter. Equation (24) now predicts a conversion loss equal to the ratio of the output to the input frequency, or

$$\frac{W_{1,0}}{f_1} = \frac{-W_{1,1}}{f_3} \tag{27}$$

which becomes,

$$\frac{W_{1,0}}{W_{1,1}} = \frac{f_1}{f_3} \tag{28}$$

Again, we have used the standard definition to remove the negative sign.

It may now be noted that excursions of the signal input either up or down in frequency result in similar excursions of the output frequency. These conversion schemes are consequently referred to be noninverting.

When the pump energy is the highest in frequency that is allowed to flow in the circuit, the output signal is equal to the difference between the pump and signal. For the case where the input frequency is lower than the output frequency, the circuit is classified as a lower sideband upconverter. The reverse situation determines the lower sideband down converter. These conditions are summarized graphically in Figure 3.

Gain relations for the two lower sideband converters may be derived in like manner to the upper sideband case. In this case, equations (20) and (21) reduce to

$$\frac{-W_{-1,0}}{-f_1} + \frac{-W_{-1,1}}{-f_1+f_2} = 0 \tag{29}$$

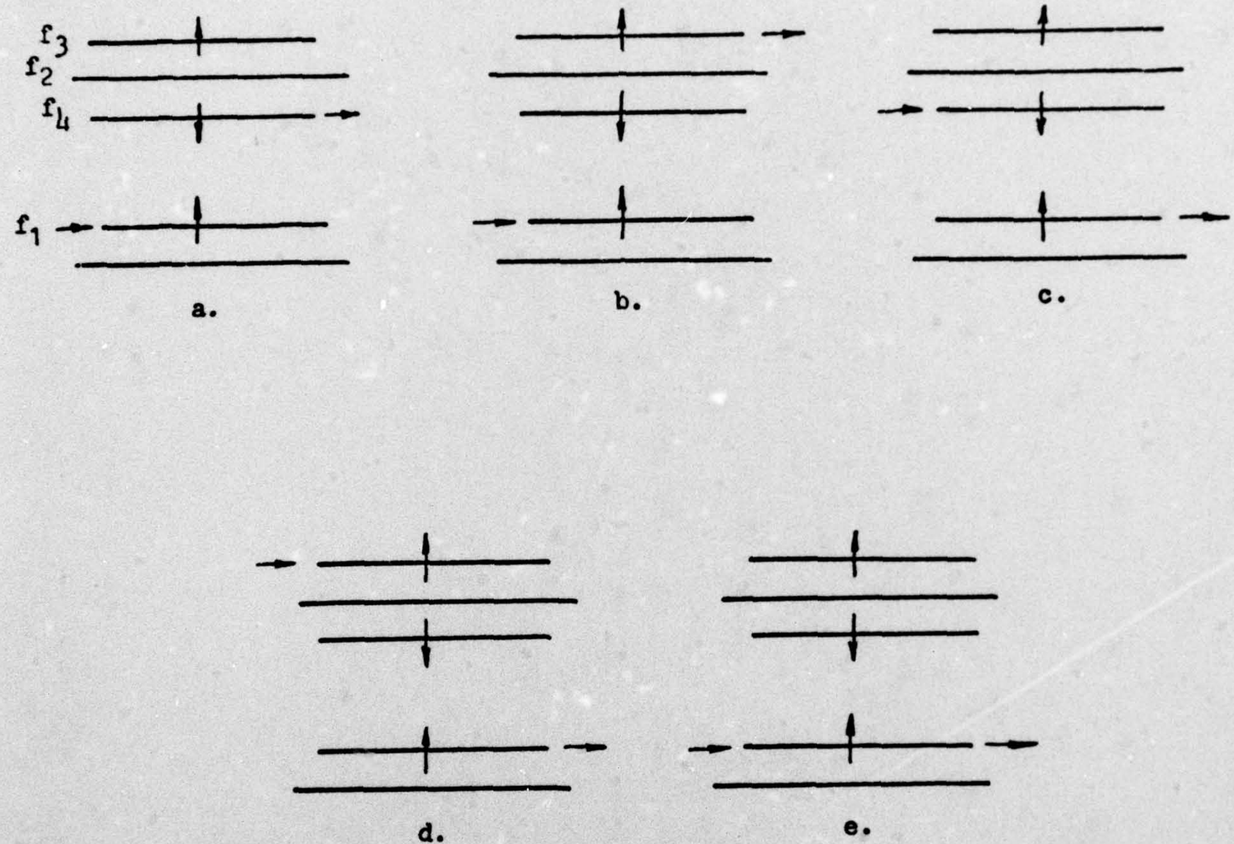


Figure 3 - Frequency Relations in various Parametric Systems.

(a) Lower sideband Up-converter
(b) Upper sideband Up-converter
(c) Lower sideband Down-converter
(d) Upper sideband Down-converter
(e) Straight-through Amplifier

From Uhlir, Proc. IRE, Vol. 46, No. 6, June, 1958, pp. 1099-1115 and,

$$\frac{W_{0,1}}{f_2} + \frac{W_{-1,1}}{-f_1 + f_2} = 0 \tag{30}$$

or,

$$\frac{W_1}{f_1} - \frac{W_4}{f_4} = 0 \tag{31}$$

and

$$\frac{W_2}{f_2} + \frac{W_4}{f_4} = 0 \tag{32}$$

where;

$$W_{-1,1} = W_4 = \text{real power at } f_2 - f_1$$

If the pump power is taken to be positive into the circuit as before and adopting the same convention on the power gain, then by equation (31) the output power is equal to the negative of the ratio of the output frequency to the input frequency. This is the same result as for the upper sideband converters excepting the negative sign which may be interpreted to be indicating the prospect of instability. This may be explained through the observation that powers may be output to passive terminations with excitation to the circuit made only by the pump. Clearly, an output may exist without a signal input. This is in contrast to the earlier case considered whereby instability could not occur with passive terminations.

Further, excursions of the input signal frequency in the upward direction results in a perturbation of the output frequency in the opposite direction. Consequently, this class of systems are known as inverting converters due to this characteristic spectrum inversion.

It should be clarified at this point that although instability is possible for the inverting converter systems, such instabilities, innomanner, preclude their use. In fact, extremely high gains may be realized when the pump power is increased toward the threshold of instability. Sensitivity to pump injection level becomes a consideration, however, and oscillations may occur due to instabilities in the pump circuit. This will be considered further in the small signal analysis.

To obtain the third class of converter, let us assume that power is allowed to flow into the nonlinear reactance at only the pump frequency and one of its harmonics. We will have then created a harmonic generator, the final class of systems to be considered here. Assuming power is input to the circuit at $f_1$, the fundamental, and removed at $f_k$, the kth harmonic of the fundamental, equation (20) yields

$$\frac{W_{1,0}}{f_1} - \frac{k W_{1,k}}{k f_1} = 0 \qquad (33)$$

or,

$$W_{1,0} = W_{1,k} \qquad (34)$$

These results indicate the ability of the parametric multiplier to provide unity power gain. This relies upon the original provision of a lossless, biunivocal nonlinear reactance in our circuit. Further, this result may be obtained only for the case whereby the terminations made to the multiplier are either of zero, infinite, or purely reactive impedances. Losses in the terminations due to the real part of the termination impedance reduce the available transducer gain.

Transducer gain may be defined[39] as the ratio of the power delivered to the load to the available power of the source. Thus the power gains determined above for the three classes of converters are the maximum gains and minimum losses that may be realized. The obtainable gains are dependant upon the power gain and a reduction due to losses in the nonlinear reactance and terminations used. The following small signal analysis considers this in detail.

## SMALL SIGNAL ANALYSIS[5]

To facilitate the determination of such parameters as transducer gain, bandwidth, and system stability for usable parametric converters, let us now derive an expression for the terminal admittances of the nonlinear reactance. To do this, we will proceed on the assumption that the pump injection level is much greater than either the signal or output levels. This assumption is justified in most useful modes of operation and may be used to successfully predict the parameters of operation for the typical system.

As before, let us assume that the charge, q, will be some function of the voltage across the nonlinear reactance, or

$$q = f(v). \tag{35}$$

Assume also the existence of a pump or energy source of frequency $f_2$ which gives rise to charge and voltage components about the capacitance consisting of the fundamental and all harmonic frequencies such that, in the absence of any applied signal,

$$q_1 = f(v_1). \tag{36}$$

If a signal is now presented to the circuit such that the signal is small compared to the pump components, then

$$\delta q = f'(v_1) \cdot \delta v$$

where,

$$f'(v) = \frac{df(v)}{dv} . \tag{37}$$

Upon observation, it may be realized that the nonlinear capacitance may be thought to be a time-varying linear capacitance, where this linear capacitance is periodic and of frequency $f_2$. If this is done, the capacitance may be expressed as a Fourier Series;

$$f'(v) = \sum_{n=-\infty}^{\infty} C_n e^{jnx} \qquad x = 2\pi f_2 t \tag{38}$$

where,

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} f'(v_1) e^{-jnx} dx \qquad C_n = C_{-n}^* \tag{39}$$

With respect to the signal input, the nonlinear capacitor may then be replaced with a linear, time varying capacitor whose capacitance is determined by the instantaneous pump amplitude and the parameters of the nonlinearity of the capacitor. More simply, the capacitance becomes a time function dependent upon the voltage presented by the pump waveform.

If we now place the restriction upon the circuit that voltages corresponding to the signal frequency and the sum and difference frequencies between the signal and pump are the only ones allowed to exist, such as through the use of appropriate ideal filters, then

$$\delta v = \left(V_1 e^{jy} + V_1^* e^{-jy}\right) + \left(V_3 e^{j(x+y)} + V_3^* e^{-j(x+y)}\right) + \left(V_4 e^{j(x-y)} + V_4^* e^{-j(x-y)}\right)$$

and,

$$\delta q = \left(Q_1 e^{jy} + Q_1^* e^{-jy}\right) + \left(Q_3 e^{j(x+y)} + Q_3^* e^{-j(x+y)}\right) + \left(Q_4 e^{j(x-y)} + Q_4^* e^{-j(x-y)}\right)$$

where, $x = 2\pi f_1 t$

and, $y = 2\pi f_2 t$

The substitution of these results into (37) will yield a matrix form for the voltage components present across the capacitor. Or,

$$\begin{bmatrix} Q_4^* \\ Q_1 \\ Q_3 \end{bmatrix} = \begin{bmatrix} C_0 & C_1 & C_{-2} \\ C_1 & C_0 & C_1 \\ C_2 & C_1 & C_0 \end{bmatrix} \begin{bmatrix} V_4^* \\ V_1 \\ V_3 \end{bmatrix} \tag{42}$$

Since current is equal to the time derivative of charge, we may make the substitution,

$$I = j2\pi f Q \quad \text{and} \quad I^* = j2\pi f Q^* \tag{43}$$

into the matrix just formed to yield;

$$\begin{bmatrix} I_4^* \\ I_1 \\ I_3 \end{bmatrix} \begin{bmatrix} -j2\pi f_4 C_0 & -j2\pi f_4 C_{-1} & -j2\pi f_4 C_{-2} \\ j2\pi f_1 C_1 & j2\pi f_1 C_0 & j2\pi f_1 C_1 \\ j2\pi f_3 C_2 & j2\pi f_3 C_1 & j2\pi f_3 C_0 \end{bmatrix} \begin{bmatrix} V_4 \\ V_1 \\ V_3 \end{bmatrix} \tag{44}$$

At this point we must note that the time origin for this analysis has been quite arbitrary. This is useful and convenient so that we may choose this origin such that $C_1 = C_{-1} =$ a positive real number. For the case considered earlier of the noninverting parametric converters, we may in like manner here limit the power flow into the nonlinear capacitor to the signal, pump and sum frequencies through the application of appropriate filters. When this is done, the matrix representation just arrived at may be reduced by setting the component of voltage for the difference frequency, $V_4{}^*$, equal to zero. When this is done, the matrix reduces to

$$\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} j2\pi f_1 C_0 & j2\pi f_1 C_1 \\ j2\pi f_3 C_1 & j2\pi f_3 C_0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} . \tag{45}$$

This is the matrix representation for the current, admittance, and voltage which exist in the circuit of the noninverting parametric converter. For the condition producing the inverting converter, $V_3$ becomes zero, yielding,

$$\begin{bmatrix} I_1 \\ I_4^* \end{bmatrix} = \begin{bmatrix} j2\pi f_1 C_0 & j2\pi f_1 C_1 \\ -j2\pi f_4 C_1 & -j2\pi f_4 C_0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4^* \end{bmatrix} . \tag{46}$$

Note that since the capacitance coefficients have been arbitrarily made real by the choice of time origin, that the admittance matrix must necessarily be purely imaginary. If we restrict ourselves to the case where the pump level is much greater than all other power levels, we may then consider the nonlinear capacitor as a two port network having just such an imaginary admittance matrix. The signal and output ports become the two ports of our network.

The powers flowing into these two ports, suppressing either the sum or difference output, are

$$W_1 = \tfrac{1}{2}\, \mathrm{Re}\, V_1 I_1^* \quad \& \quad W_2 = \tfrac{1}{2}\, \mathrm{Re}\, V_2 I_2^* . \tag{47}$$

The relation of general form characteristic of a two port having a purely imaginary admittance matrix just described is

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j\beta_{11} & j\beta_{12} \\ j\beta_{21} & j\beta_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} . \tag{48}$$

If we now express equations (47) in matrix form by taking their sum, we will have

$$\frac{W_1}{\beta_{12}} + \frac{W_2}{\beta_{21}} = \tfrac{1}{2}\, Re \begin{bmatrix} \dfrac{V_1^*}{\beta_{12}} & \dfrac{V_2}{\beta_{21}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{49}$$

where use has been made of the equality,

$$Re\; VI^* = Re\; V^*I \; . \tag{50}$$

Proceding now by substituting (48) into (49), yields

$$\frac{W_1}{\beta_{12}} + \frac{W_2}{\beta_{21}} = \tfrac{1}{2}\, Re \begin{bmatrix} \dfrac{V_1^*}{\beta_{12}} & \dfrac{V_2}{\beta_{21}} \end{bmatrix} \begin{bmatrix} j\beta_{11} & j\beta_{12} \\ j\beta_{21} & j\beta_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} . \tag{51}$$

The right side of this equation reduces to zero such that we are left with

$$\frac{W_1}{\beta_{12}} + \frac{W_2}{\beta_{21}} = 0 \tag{52}$$

which further leads to

$$G_{p12} = \frac{-W_2}{W_1} = \frac{\beta_{12}}{\beta_{21}} \tag{53}$$

and

$$G_{p21} = \frac{-W_1}{W_2} = \frac{\beta_{12}}{\beta_{21}} \; . \tag{54}$$

The end result is finally at hand. If we now use the relations determined in the analysis of the inverting and noninverting converters that produced equations (45) and (46), the power relationships that lead to the gain determinations may be found summarized below.

Case 1; Noninverting

Energy relation- $$\frac{W_1}{f_1} + \frac{W_3}{f_3} = 0 \qquad (55)$$

Modulator power gain - $$G_{p13} = \frac{W_3}{W_1} = \frac{f_3}{f_1} \qquad (56)$$

Demodulator power gain - $$G_{p31} = \frac{W_1}{W_3} = \frac{f_1}{f_3} \qquad (57)$$

Case 2; Inverting

Energy relation - $$\frac{W_1}{f_1} - \frac{W_4}{f_4} = 0 \qquad (58)$$

Modulator power gain - $$G_{p14} = \frac{-W_4}{W_1} = \frac{-f_4}{f_1} \qquad (59)$$

Demodulator power gain - $$G_{p41} = \frac{-W_1}{W_4} = \frac{-f_1}{f_4}\,. \qquad (60)$$

It is obvious that these results are in agreement with the energy relations and power gain determinations made earlier. Restating; the basic assumption made for this analysis was that the signal and output power levels be small with respect to the pump level.

## TRANSDUCER GAIN

To provide a detailed analysis concerned with the transducer gain, or that gain which we may actually obtain from the nonlinear reactance converter when connected to load and generator conductances, use may be made of the admittance matrices previously determined for the inverting and noninverting cases. The transducer gain is, as stated earlier, defined to be[39] the ratio of power delivered to the load to the available power of the source. Considering first the non-inverting case, we will assume the two port has been terminated with admittances determined by the load and current generator conductances plus terminal susceptances added for matching. This appears in Figure 4. The input admittances are given by

$$Y_{1(in)} = j\beta_{11} + \frac{\beta_{12}\beta_{21}}{g_2 + j(\beta_2 + \beta_{22})} \tag{61}$$

$$Y_{2(in)} = j\beta_{22} + \frac{\beta_{12}\beta_{21}}{g_1 + j(\beta_1 + \beta_{11})} . \tag{62}$$

These admittances are in parallel with the terminations applied, so that the total admittance connected to the current sources at the input and output become,

$$\begin{aligned} Y_{1S} &= Y_1 + Y_{1(in)} \\ &= g_1 + j(\beta_1 + \beta_{11}) + \frac{\beta_{12}\beta_{21}}{g_2 + j(\beta_2 + \beta_{22})} \end{aligned} \tag{63}$$

$$\begin{aligned} Y_{2S} &= Y_2 + Y_{2(in)} \\ &= g_2 + j(\beta_2 + \beta_{22}) + \frac{\beta_{12}\beta_{21}}{g_1 + j(\beta_1 + \beta_{11})} . \end{aligned} \tag{64}$$

Referring now to (45), we may observe that the product of $B_{12}$ and $B_{21}$ is always greater than zero. For this condition, the power gain from (53) and (54) is always positive. Further, this class of systems is always stable, as shown by the observation that

$$Re\ Y_{1(in)} > 0 \qquad Re\ Y_{2(in)} > 0 \tag{65}$$

for any two passive terminations.

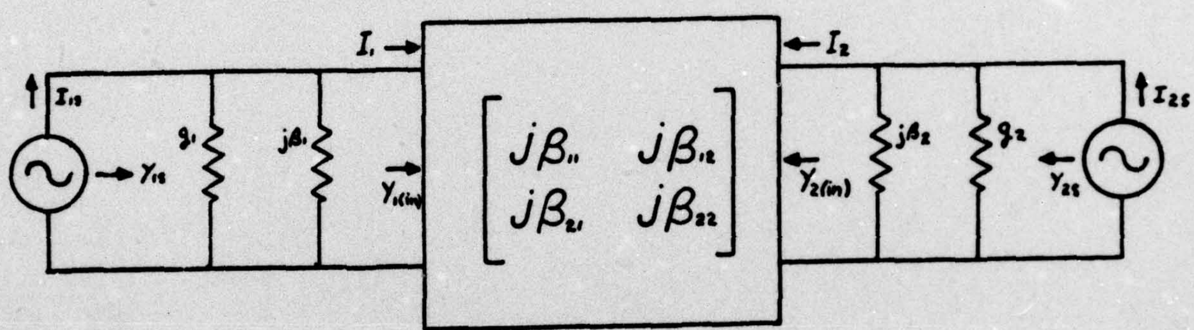


Figure 4 - Linear Two-port network driven by current sources.

The transducer gain for the noninverting modulator and demodulator may be determined using the admittances of (61) through (64) to establish a ratio of admittances multiplied by the power gains of equations (56) and (57) yielding, the transfer of power from the input signal to the output load. These gains are,

$$G_{T12} = G_{p12} \frac{4\,\mathrm{Re}\,\dfrac{Y_{1(in)}}{g_1}}{\left|\dfrac{Y_{1(s)}}{g_1}\right|^2} \tag{66}$$

and

$$G_{T21} = G_{p21} \frac{4\,\mathrm{Re}\,\dfrac{Y_{2(in)}}{g_2}}{\left|\dfrac{Y_{2(s)}}{g_2}\right|^2} \tag{67}$$

It may be implied that the transducer gain will necessarily be positive by the observation that the power gain and the real part of the input admittance are both positive. The maximum value of transducer gain will be realized when a conjugate match is made between the two ports and their respective loads. This means that the two ports may provide maximum gain when the product of the complex admittance applied to the input port times the admittance applied to the output port is equal to the product of $B_{12}$ times $B_{21}$, or

$$\left[g_1 + j\left(\beta_1 + \beta_{11}\right)\right] \cdot \left[g_2 + j\left(\beta_2 + \beta_{22}\right)\right] = \beta_{12}\beta_{21} \tag{68}$$

This requires that,

$$\angle\left[g_1 + j\left(\beta_1 + \beta_{11}\right)\right] = \angle\left[g_2 + j\left(\beta_2 + \beta_{22}\right)\right].$$

For this condition the transducer gain is equal to the power gain.

When the product of $B_{12}$ and $B_{21}$ of the admittance matrix of the two port network is negative, the inverting converter results. Correspondingly, the power gains of these circuits are also negative, implying regeneration and possible instability. If the circuit is unstable, our small signal analysis obviously does not hold. If, however, the system is stable, we may use this technique to predict the resulting gains.

For the stable case, the gains predicted by (66) and (67) again apply. Since the real part of the conductance, $Y_{in}$, and the power gain are both negative, the transducer gains of (66) and (67) must be positive. This indicates that, although the system is potentially unstable, usable gains are obtainable. In fact, the gain may be made as high as desired through the appropriate choice of $Y_s$ and g from the denominator of (66) and (67). The price paid here is in system sensitivity and narrow bandwidths resulting.

It has been determined[5] that the ratio of $Y_s$ to g just mentioned is directly dependent upon the term alpha, which is

$$\alpha = \frac{4\pi^2 f_1 f_4 C_1^2}{g_1 g_4} \tag{69}$$

The terms here are all familiar with the exception of $C_1^2$.

$C_1$ is a term in the figure of merit of the nonlinear capacitance signifying the amount of deviation that our time varying linear equivalent capacitance takes from the normal bias capacitance, $C_o$. More simply, the nonlinear capacitor, a varactor in the general case, will possess some constant capacitance dependent on the reverse bias level applied. As the pump energy is applied, the instantaneous capacitance is caused to vary about this constant. The variance from the constant is given to be $C_1$, with the constant given to be $C_0$. The figure of merit then is the ratio of this variance to the constant squared, or

$$\left(\frac{C_1}{C_0}\right)^2 \tag{70}$$

Note that for a linear capacitance, this figure of merit becomes zero, while for the ideal nonlinear capacitance, it becomes unity. The latter condition indicates that the capacitance is directly proportional to the voltage across it and possessing no constant parallel capacitance.

Returning to the gain equation, alpha is directly proportional to the term $C_1^2$ and inversely proportional to the shunt conductance of the input signal generator and the output load. Therefore, the stability of the inverting converters is dependant upon the loads to which the circuit is connected and the pump level applied. For the case of the inverting converter having resonant terminal admittances at the center, or midband, frequency, Rowe has shown[5] that for values of alpha less than unity, stability exists, while values greater than unity may indicate instability.

The sensitivity of the inverting converter, whether modulator or demodulator, increases dramatically with conditions of alpha producing greater gain. It may be shown that the sensitivity of these systems with regards to changes in load conductance or the figure of merit term, $C_1$, is equal to

$$S_g = \frac{dG_T/G_T}{dg/g} = -\frac{1+a}{1-a} \tag{71}$$

and

$$S_{C_1} = \frac{dG_T/G_T}{dC_1/C_1} = -2\,S_q \tag{72}$$

where use has been made of the definition of sensitivity given by Bode[40] to be the fractional change in transducer gain, $G_t$, to the corresponding fractional change in the parameter causing the change in gain.

A great number of relations govern the behavior of the parametric systems we have considered. It would be helpful and convenient at this juncture to summarize, briefly, the results thus far. Following is a tabular summary allowing the immediate comparison of some of the parameters of these converters.

| Power Gain | Transducer Gain | Sensitivity | Bandwidth |
|---|---|---|---|
| **Non-Inverting:** | | | |
| Up-Converter | | | |
| $G_{p13} = \frac{W_3}{W_1} = \frac{f_3}{f_1}$ | $G_{T13} = G_{p13} \frac{4\,Re\,\frac{Y_{1in}}{g_1}}{\left\lvert \frac{Y_{1s}}{g_1} \right\rvert^2}$ | $0$ | $B = \frac{C_1}{C_0}\sqrt{2 f_1 f_3}$ |
| Down-Converter | | | |
| $G_{p31} = \frac{W_1}{W_3} = \frac{f_1}{f_3}$ | $G_{T31} = G_{p31} \frac{4\,Re\,\frac{Y_{2in}}{g_2}}{\left\lvert \frac{Y_{2s}}{g_2} \right\rvert^2}$ | $0$ | $B = \frac{C_1}{C_0}\sqrt{2 f_1 f_3}$ |
| **Inverting:** | | | |
| Up-Converter | | | |
| $G_{p14} = \frac{-W_4}{W_1} = \frac{-f_4}{f_1}$ | $G_{T14} = G_{p14} \frac{4\,Re\,\frac{Y_{1in}}{g_1}}{\left\lvert \frac{Y_{1s}}{g_1} \right\rvert^2}$ | $-\frac{1+a}{1-a}$ | $B = \frac{C_1}{2C_0}\sqrt{f_1 f_4}\,(1-a)$ |
| Down-Converter | | | |
| $G_{p41} = \frac{-W_1}{W_4} = \frac{-f_1}{f_4}$ | $G_{T41} = G_{p41} \frac{4\,Re\,\frac{Y_{2in}}{g_2}}{\left\lvert \frac{Y_{2s}}{g_2} \right\rvert^2}$ | $-\frac{1+a}{1-a}$ | $B = \frac{C_1}{2C_0}\sqrt{f_1 f_4}\,(1-a)$ |

Where $a = \frac{4\pi^2 f_1 f_4 C_1^2}{g_1 g_4}$

## EXPERIMENTAL APPLICATION

The design and construction of a parametric frequency converter was begun to test the feasibility of including such a device within the framework of a system to be used for the reception of remotely sensed weather and environmental data transmitted from a geostationary satelite. The weather pictures and other data are broadcast from the satellite in the UHF region near 470 megahertz. The need for a low noise frequency converter having adequate sensitivity suggested that use could be made of the advantages offered by the use of a parametric device. It was felt that an improvement over more conventional mixers could be obtained using relatively little increase in system complexity.

## GENERAL CONSIDERATIONS

At the time of its conception, four areas of operation were thought to need evaluation. These are sensitivity, stability, bandwidth, and complexity. The overall sensitivity of a converter depends, to a great degree, upon the loss experienced during down-conversion and the noise introduced. A theoretical loss, from the earlier derivations, equal to the ratio of frequencies involved predicted a reduction of approximately 12 dB would be realized for an ideal, lossless system. The intermediate frequency chosen was 30 MHz due to availability of IF Amplifiers for this frequency. A higher frequency was considered, but deemed impractical.

It was felt that the choice of an inverting converter could yield a system with a potential for gain instead of the predicted loss and offered the chance to test the theory regarding regeneration and the realization of negative conductance predicted by the Manley-Rowe derivations. Stability would, obviously, be a consideration here, but the prospect of gain swayed the choice made.

A bandwidth of approximately 100 KHz was deemed necessary to adequately receive the carrier and sidebands of the satellite signal, which is frequency modulated. This would allow some margin for error in tuning, but would not increase the signal to noise ratio too markedly.

The inclusion of a pump source increased the overall complexity of the system and introduced a possible source of spurious noise. A General Radio RF Oscillator of good stability and purity was available with variable power supply. This was deemed satisfactory for test purposes, after which a permanent pump under crystal control could be constructed.

## DESIGN

The signal frequency for the converter, as mentioned, is 470 MHz. The choice of intermediate frequency of 30 MHz, and a desire for an inverting class of system made imperative a pump frequency of 500 MHz. These frequencies are not harmonically related, other than their simple difference relation needed for the converter, so they were deemed appropriate for test. The frequencies chosen called for two UHF filters, which lend themselves well to distributed parameter circuits, and one high-frequency filter, which called for a lumped constant circuit.

The configuration chosen for the converter placed the varactor diode in parallel with the three filter circuits. This was done to allow each filter to be referenced directly to ground so as to make tuning and input/output of pump and signal frequencies easier. The circuit of Figure 5 describes the unit that developed. Each of the series connected filters is actually a pi-network having the varactor determine the capacitance at one end of the pi configurations. This allowed mounting of the variable capacitors directly on the ground plane to reduce hand capacitance pertubations during adjustment.

Striplines were utilized in the two UHF circuits. This method allowed higher circuit Q than lumped parameters at this frequency and greatly simplified construction. Helical resonators were considered, and could have been used, but were not used due to the simplicity offered by the stripline approach. The striplines are actually composed of narrow strips of printed circuit board material cut such that they exhibit the correct amount of inductance when placed in series with the seven picofarad tuning capacitors and the nominal capacitance of the varactor diode. These strips are affixed on the copper ground plane, also of circuit board material, such that they become small sections of transmission line, exhibiting inductance and capacitance per unit length. It was found that good immunity to stray capacitance and coupling also resulted from this technique. Although these filters could have been determined by appropriate mathematical techniques, an empirical approach using a grid dip meter was opted for after preliminary measurements of the circuit board properties gave an indication of the relative amounts of inductance and capacitance to be expected.

The intermediate frequency circuit is composed of the tunable indoctor, L3,

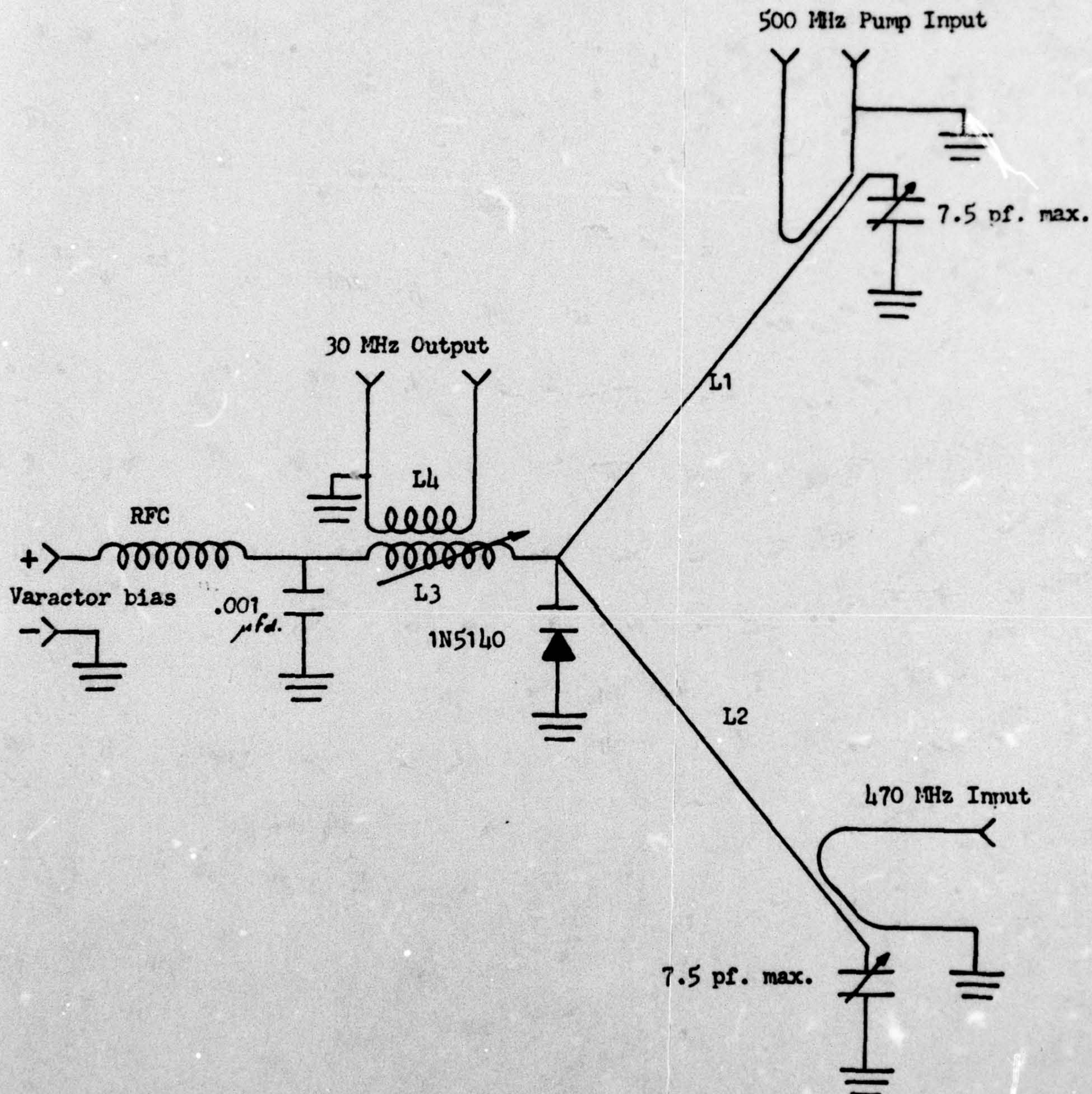


Figure 5 - Parametric Converter Circuit

and the varactor diode. The inductor is bypassed at the cold end to allow application of the reverse bias necessary for proper varactor operation.

The coupling to each of these circuits was determined empirically. No changes were necessary, however, for proper operation other than to move the coupling loop, L4, so as to be more tightly coupled to the turns of L3. This resulted in slightly more output than initially experienced.

The varactor diode chosen was a Motorola 1N5140. This device exhibits a nominal capacitance of ten picofarads at four volts of reverse bias. Maximum reverse bias is sixty volts with a capacitance ratio of 2.8.

Figure 6 details the test apparatus used. It is very nearly self-explanatory, although a few words are necessary to describe the method used to initially tune the filter circuits. The varactor diode capacitance was simulated be installing a seven picofarad capacitor in each filter, in turn, so that a Grid-dip Meter could be used to approximately set the filter's frequency before all three were paralleled at the varactor. This greatly simplified the tuning procedure, and is highly recommended. Initial attempts to tune the tree simultaneously were unsuccessful, as each variable effects the tuning of all the filters once they are connected to the varactor. The bias voltage is included in this category as well, and drastically alters circuit operation.

## RESULTS

Upon completion of the construction of the unit, the test apparatus of Figure 6 was assembled to facilitate tuning and testing. After adjustment of the filters, bias voltage, and pump level, the circuit was found to be quite remarkable. The measured loss in the down-conversion was variable, as predicted by the relations derived earlier. Early operation resulted in a measured loss of 32 dB with full pump power applied. The pump was capable of nearly 350 mW of output at 500 MHz. Further touch up of the tuning netted much better results, with instability obtainable at approximately 200 mW of pump input. Conversion loss could be varied from infinite (no output) with the pump off, to a stable gain of over 9 dB. The available gain was limited largely by instability arising out of excessive pump level variations caused by local line-voltage and power supply fluctuations.

It was found that the bandwidth attained was, indeed, a function of the gain, with a measured bandwidth of 125 KHz at a loss of 20 dB. The application of the frequency counter used injected sufficient digital harmonic energy to cause circuit instability at all higher gain levels. Estimations were made, however, that the bandwidth did not appear to decrease significantly for moderate gains, up to about 3 dB. It should be pointed out that while figures of zero to three decibels of gain are not generally considered high, the lowest loss expected from a conventional crystal diode or hot carrier mixer lies below the twelve decibel loss figure established earlier. It must be remembered that any loss contributed by a mixer must be added to the noise figure of the following stage in the determination of noise performance of a system. Therefore, improvement over the minimum loss figure is happily noted.

Incorporation of this converter into a workable system would necessitate

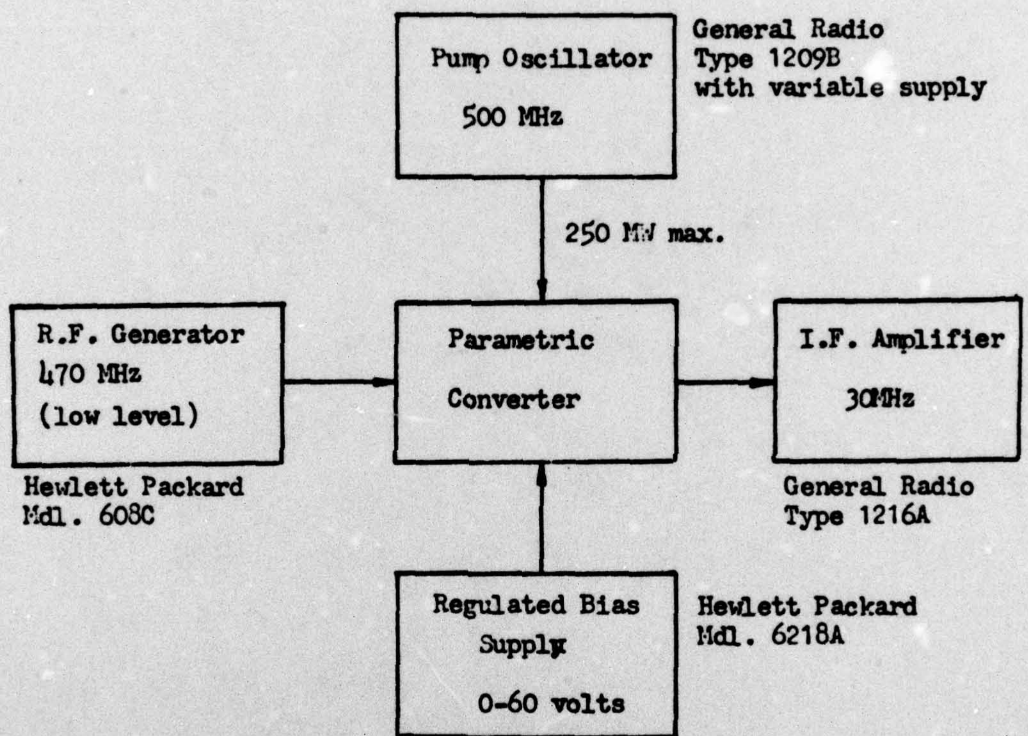


Figure 6 - Converter Test Apparatus

the construction of a suitable crystal-controlled pump circuit able to provide approximately 250 mW of output that could be regulated over a moderately variable level. The more simple requirement, a regulated bias supply, could easily be an adjunct of the pump power supply.

The construction of a low noise UHF preamplifier for use ahead of this circuit seems mandatory without a high gain antenna array. The added selectivity provided by such a preamplifier to eliminate spurious responses and noise is sufficient to warrant its construction also.

The construction of this converter verified the feasability of the inclusion of such a device in the remote sensing facility at CSUS. The completion of a helical array, now under construction, will allow an operational test to be conducted as part of a complete system at 470 MHz. Auxiliary equipment used in the initial testing should provide satisfactory service until the aforementioned pump and power supply are completed.

The negative admittances presented at the ports of the converter, as predicted by the equations derived earlier, and the method with which coupling was made in the experimental device precluded successful explicit verification of the predictions made concerning exact values of gain, stability or bandwidth obtainable. A very good agreement with the results, both predicted and measured, was realized. This leads to the conclusion that the derivations obtained may be used to good approximation to predict the behavior of such parametric systems. It is difficult, at best, to predict losses at very high and ultra high frequencies due to component quality and placement in a system. Therefore, it becomes almost imperative that breadboarding and similar techniques be employed to facilitate verification of a prospective design. The results of the small signal analysis presented appear to provide at least a very usable method of parameter determination for systems of this class.

## CONCLUSION

The general energy relationships applicable to systems having nonlinear reactance elements have been examined. Further, a small signal analysis has been detailed from which were obtained appropriate expressions concerning gain, bandwidth, and instability for the different modes of operation. Lastly, a brief description of an experimental application of this type of circuit to a communication system has been given, where the results obtained were in agreement with theory.

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## APPENDIX II

Proof that the sum of powers flowing into the nonlinear reactance = 0 for the hysteresisless case.

1) Given:

$$(20) = \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} \frac{m W_{m,n}}{m f_1 + n f_2} = 0 \qquad (21) = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} \frac{n W_{m,n}}{m f_1 + n f_2} = 0$$

2) Multiplying (20) by $f_1$ and (21) by $f_2$,

$$\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} \frac{m f_1 W_{m,n}}{m f_1 + n f_2} = 0 \qquad \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} \frac{n f_2 W_{m,n}}{m f_1 + n f_2} = 0$$

3) Separating each summation into regions of like indicies,

$$\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\cdot} \frac{m f_1 W_{m,n}}{m f_1 + n f_2} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{m f_1 W_{m,n}}{m f_1 + n f_2} = 0$$

and

$$\sum_{m=-\infty}^{\cdot}\sum_{n=0}^{\infty} \frac{n f_2 W_{m,n}}{m f_1 + n f_2} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{n f_2 W_{m,n}}{m f_1 + n f_2} = 0$$

4) Rearranging indicies and summing,

$$\sum_{m=0}^{\infty}\sum_{n=0}^{-\infty} \frac{m f_1 W_{m,n}}{m f_1 + n f_2} + \sum_{m=0}^{-\infty}\sum_{n=0}^{\infty} \frac{n f_2 W_{m,n}}{m f_1 + n f_2} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(m f_1 + n f_2) W_{m,n}}{m f_1 + n f_2} = 0$$

5) Noting that

$$(m) f_1 = (-m) f_1 \quad \& \quad (n) f_2 = (-n) f_2$$

6) Then,

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{mf_1 W_{m,n}}{mf_1+nf_2} \Rightarrow \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{mf_1 W_{m,n}}{mf_1+nf_2}$$

and

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{nf_2 W_{m,n}}{mf_1+nf_2} \Rightarrow \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{nf_2 W_{m,n}}{mf_1+nf_2}$$

7) So that,

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(mf_1+nf_2)W_{m,n}}{mf_1+nf_2} + \sum\sum\frac{(mf_1+nf_2)W_{m,n}}{mf_1+nf_2} = 0$$

or,

$$2\sum\sum W_{m,n} = 0 \,. \qquad \text{qed.}$$

Project Converter