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# Cross Field Thermal Transport Due to Ion Acoustic Waves in Magnetized Laser Plasmas

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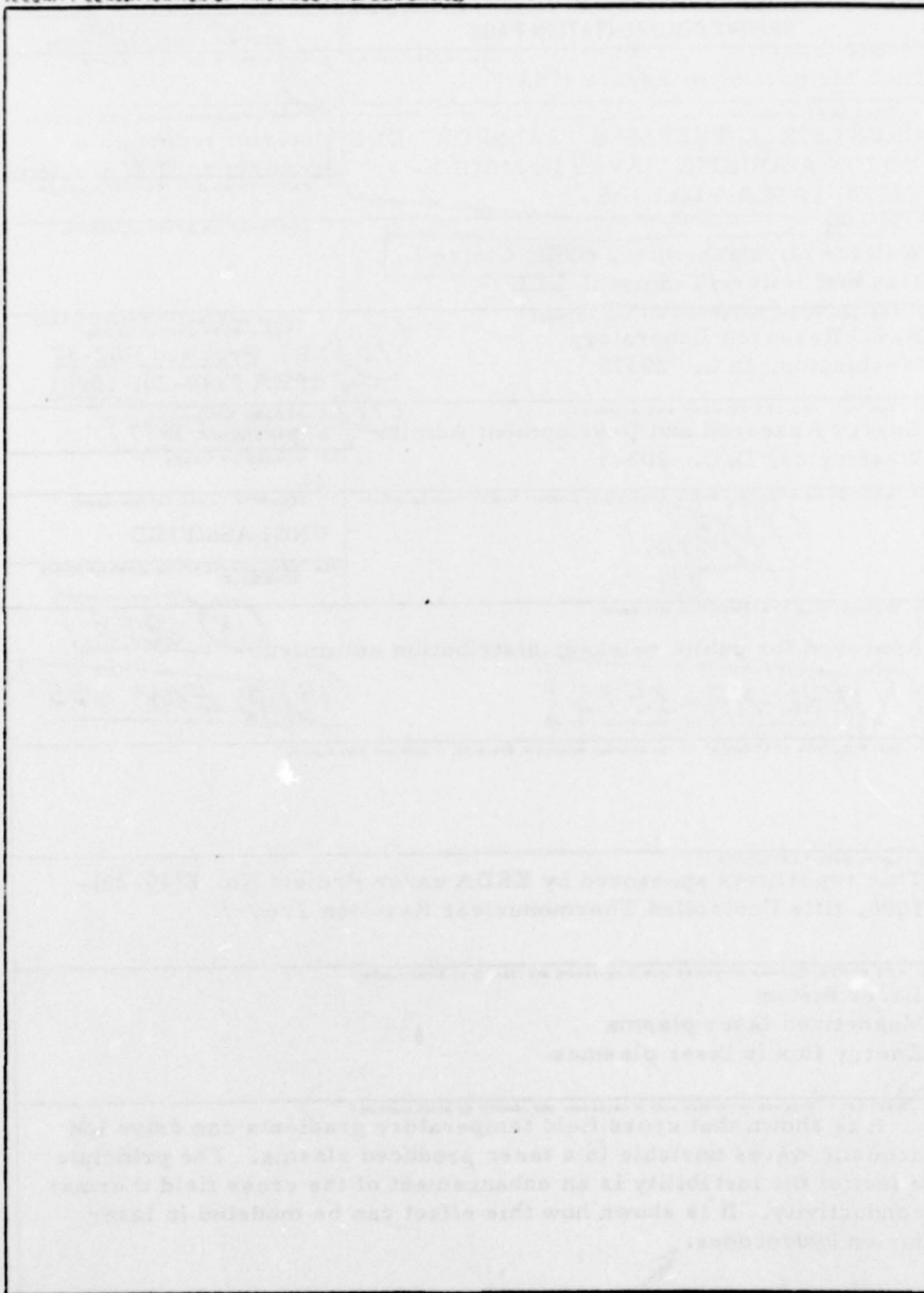
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CROSS FIELD THERMAL TRANSPORT  
DUE TO ION ACOUSTIC WAVES IN MAGNETIZED  
LASER PLASMAS

I. INTRODUCTION

It is now well established that magnetic fields can be spontaneously generated in laser produced plasmas. Recent measurements<sup>1</sup> by Faraday rotation have indicated fields larger than one megagauss in the underdense plasma. There are now many mechanisms by which magnetic fields can be produced. These include the  $\nabla n \times \nabla T$  term in the equation for  $B$ ,<sup>2,3</sup> the thermo-electric term,<sup>2-4</sup> the divergence of momentum flux of laser light,<sup>5,6</sup> currents produced by energetic electrons,<sup>7</sup> and impurity seeding<sup>8</sup> and even currents generated by turbulence in an unmagnetized plasma.<sup>9</sup>

Although the magnetic field is large, the plasmas are so dense, so hot and are flowing so rapidly that the magnetic fields observed and predicted are not nearly large enough to have any effect on the dynamics. The principle way in which the effect of the magnetic field makes itself felt is by greatly reducing the cross field electron thermal conduction. In order for the thermal conduction to be reduced, one need only have  $\omega_{ce} \tau_e \gg 1$ , where  $\omega_{ce}$  is the electron cyclotron frequency and  $\tau_e$  is the electron collision time. This condition is very easy to satisfy even for magnetic fields too small to affect the dynamics.

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Thus one expects the cross field thermal energy flux in a magnetized plasma to be much less than the flux in an unmagnetized plasma. This inhibition of energy flux would in turn lead to much steeper electron temperature gradients than what would be expected for an unmagnetized plasma. One could then ask whether the steep temperature gradients so produced could lead to some sort of instability which would enhance the electron thermal flux and reduce the temperature gradient. Experience with magnetically confined plasmas certainly indicates that this will be the case. For instance in tokamaks, electron temperature gradients drive trapped particle instabilities<sup>10</sup> which are thought to greatly enhance the electron thermal conduction. In any case it is a fact that the electron thermal conduction in tokamaks is enhanced over its classical value by many orders of magnitude.<sup>11</sup>

In this paper, we show that ion acoustic waves which propagate perpendicular to both the magnetic field and the gradients can in fact be driven unstable. Section II derives the equilibrium. Section III calculates the linear theory for ion acoustic waves in this equilibrium. Section IV calculates the electron energy flux in a given spectrum of ion acoustic fluctuations. It is shown there that the electron thermal conduction scales as  $|\epsilon\phi/T|^2$ . We also calculate how small  $|\epsilon\phi/T|^2$  must be in order that the magnetic field, rather than the turbulence in an unmagnetized plasma, is primarily responsible for inhibiting the electron thermal flux. Finally in Section V, we show how to include the effects of enhanced cross field transport in a fluid code.

## II. THE EQUILIBRIUM

The basic configuration of a one dimensional laser produced plasma is shown in Fig. 1. The laser is to the left at  $x = -\infty$ . The flow velocity is in the negative  $x$  direction, the temperature gradient is in the negative  $x$  direction and the density gradient is in the positive  $x$  direction. Also, we assume the existence of a magnetic field  $B$  in the positive  $Z$  direction (out of the plane of the paper). The magnetic field is assumed to be strong enough to affect the electron transport, i.e.,  $\omega_{ce} \tau_e \gg 1$  where  $\omega_{ce}$  is the electron cyclotron frequency and  $\tau_e$  is the electron collision time. However, it is also assumed to be sufficiently weak, that it has no effect on the dynamics, i.e.,  $n\mu^2 \gg B^2/4\pi$ ,  $\omega_{pe} \gg \omega_{ce}$  and  $\beta \gg 1$  where  $n$ ,  $M$ ,  $u$  and  $\omega_{pe}$  are respectively the number density, ion mass, flow velocity and electron plasma frequency and  $\beta = 4\pi nT/B^2$ . Also, the ions are assumed to be unmagnetized, that is  $\rho_i \gg L$  where  $\rho_i$  is the ion larmor radius and  $L$  is a macroscopic scale length. On the other hand the electrons are assumed to be strongly magnetized, or  $\rho_e \ll L$ . Therefore, in order for the electrons to flow in the negative  $x$  direction there must be an electric field in the negative  $y$  direction with magnitude  $B u/c$ . The above conditions are generally well satisfied in a laser produced plasma.

In order to calculate whether such a plasma is stable, it is essential to consider the effect of collisions on the equilibrium. The equilibrium is governed by a flow velocity to the left, and an electron thermal conduction to the right. This thermal energy flux is equal to  $Q = -K \frac{dT}{dx}$  where  $K$  is the thermal conductivity. Classical

kinetic theory gives the result:<sup>12</sup>

$$K = 4.66 n T_e / m v_c^2 \tau_e \quad (1)$$

If the laser energy flux is  $I$  and the fractional absorption is  $\alpha$ , then conservation of energy flux (assuming for simplicity that there is no change in fluid energy flux) across the region of absorption gives

$$Q = \alpha I. \quad (2)$$

Thus thermal energy flux, and therefore a non-zero collision frequency, is inherent in the very nature of the equilibrium of a laser produced plasma. The Appendix discusses further the lack of Vlasov equilibrium for the configuration shown in Fig. 1.

As we will see, cross field thermal conduction in the  $x$  direction is always accompanied by an additional thermal flux in the  $y$  direction (along iso therms). The magnitude of this thermal flux in the  $y$  direction is independent of collisions (for  $\omega_{ce} \tau_e \ll 1$ ) at given temperature gradient. This thermal flux perpendicular to both field and gradients is well known in classical kinetic theory.<sup>13</sup> It is analogous to diamagnetic currents in an isothermal plasma, which are perpendicular to both the magnetic field and density gradient.

In the next section, it is shown that this thermal flux in the  $y$  direction gives rise to the possibility of ion acoustic instability, with wave vector in the  $y$  direction. Since this energy flux in the  $y$  direction is nearly independent of collision frequency, we will use the simplest possible collision model, a Krook collision term. The

steady state Vlasov equation becomes:

$$V_x \frac{\partial f}{\partial x} - \frac{e}{m} E_x \frac{\partial f}{\partial V_x} - \frac{e}{m} E_y \frac{\partial f}{\partial V_y} - \frac{e}{m} \frac{V}{c} \times B \cdot \frac{\partial f}{\partial V} = -\nu(f-f^0) \quad (3)$$

where  $f^0$  is a Maxwellian distribution with the local density  $n$ , flow velocity  $u$  and temperature  $T_e$ , which are  $x$  dependent. If we make the simplifying assumption that  $\nu$  is independent of  $V$ , the collision term conserves particles, momentum and energy. The quantity  $E_x$  cannot be specified independently. It is determined by the  $x$  component of the electron momentum equation,

$$0 = -ne E_x - \frac{\partial}{\partial x} n T_e, \quad (4)$$

where we have assumed  $u^2 \ll T_e/m$ . Then assuming that the first two terms on the left hand side of Eq. (3) are small, one can solve for the perturbed distribution function by standard means.<sup>14</sup> Assuming

$$f^0 = \frac{n(x)}{(2\pi T_e(x)/m)^{3/2}} \exp - \frac{m(V - u)^2}{2 T_e(x)} \quad (5)$$

and

$$f = f^0 + \delta f, \quad (6)$$

Eq. (3) can then be manipulated,<sup>14</sup> by using Eq. (4), into an equation for  $\delta f$

$$\omega_c \frac{\partial}{\partial \theta} \delta f + \nu \delta f = -(V_x - u) \frac{1}{T_e} \frac{dT_e}{dx} \left( \frac{1}{2} \frac{m(V - u)^2}{T_e} - 5/2 \right) f^0, \quad (7)$$

by making use of the transformation

$$V_x - u = V_\perp \cos \theta$$

(8)

$$V_y = V_\perp \sin \theta.$$

We have retained on the right hand side of Eq. (7) only terms which give rise to a nonzero energy flux. Solving Eq. (7) we find

$$f \equiv f_0 + \delta f = \left\{ 1 - \left[ \frac{1}{2} \frac{m(V-u)^2}{T_e} - \frac{5}{2} \right] \frac{1}{T_e} \frac{dT_e}{dx} \left( \frac{v(V_x - u) + \omega \frac{v}{c} y}{\omega_c^2 + v^2} \right) \right\} f_0 \quad (9)$$

The first term in the parentheses of Eq. (9) gives rise to an energy flux down the temperature gradient. If this energy flux is imposed as a boundary condition, then the temperature gradient is thereby specified. The second term in the parentheses of Eq. (9) gives rise to an energy flux in the positive y direction. In the limit of  $v \ll \omega_c$ , this energy flux in the y direction is independent of collisionality and it depends only on the temperature gradient.<sup>13</sup> Notice that the energy flux in the positive y direction is carried by a flux of energetic electrons moving in the positive y direction and that this flux is balanced by a return current of slower moving electrons in the negative y direction. In the next section, we will show that this return current in the negative y direction can drive unstable ion acoustic waves which propagate in the negative y direction.

### III LINEAR THEORY

The calculation of the perturbed electron distribution function, given the unperturbed distribution function in Eq. (9) can be done with standard techniques.<sup>15</sup> The result is

$$\tilde{f}_e = n \frac{eG}{T_e} \left\{ 1 + \sum_{\ell} \frac{J_{\ell}^2 \left( \frac{kV_{\perp}}{\omega_{ce}} \right) \left[ \omega + \frac{kc}{eB} \frac{dT_e}{dx} \left( \frac{1}{2} \frac{m(V_{\perp} - u_{\perp x})^2}{T_e} - \frac{5}{2} \right) \right]}{-\ell \omega_{ce} - \omega} \right\} f_e^0 \quad (10)$$

assuming that  $k$  is in the  $y$  direction the fluctuating potential is,

$$\phi(y,t) = \phi \exp i (ky - \omega t) \quad \text{c.c.} \quad (11)$$

and  $\omega_{ce} \gg v$ .

At this point, we make use of the fact that  $\omega \gg \omega_{ce}$  and  $kV_e \gg \omega_c$  where  $V_e = (T_e/m)^{1/2}$ . A great deal of analytic work and also numerical simulation has shown that in this limit the magnetic field has no effect on the microscopic dynamics of the instability. However, the  $dT/dx$  term in Eq. (10) is unchanged. (This point will be discussed more fully in the next section.)

Crudely, one can envision the situation as follows. As expressed in Eq. (10),  $\tilde{f}_e$  is a summation over a large number of resonances. It has been shown that any physical mechanism which broadens the cyclotron resonances by the cyclotron frequency<sup>15</sup> has the effect of converting the magnetized dispersion relation into an unmagnetized dispersion relation. The broadening mechanism may either be collisions, resonance broadening<sup>16</sup> or cyclotron trapping.<sup>17</sup> Furthermore, if  $k_z$  is not exactly zero but has some small value ( $k_z V_e \gtrsim \omega_{ce}$ ) it is now well established that even for linear theory in a collisionless plasma, the cyclotron resonances are washed out.<sup>18</sup>

For the case in which resonance broadening governs the resonance width, it has been shown that if  $k \lambda_{De} \sim 0.5$ , (the case we will examine

here) then the individual cyclotron resonances are washed out if

$$\frac{e\phi}{T_e} \gg \left( \frac{\omega_{ce}}{\omega_{pe}} \right)^{3/2}. \quad (12)$$

Equation (12) above shows that very small values of  $e\phi/T_e$  are sufficient to eliminate the individual cyclotron resonances. Specifically, the values of  $e\phi/T_e$  needed to wash out individual resonances are much less than that needed to wipe out the effect of the magnetic field on the equilibrium return current in the y direction. In the next section, it is shown that the magnetic field dominates the equilibrium current and transport as long as

$$2 \left( \frac{\omega_{ce}}{\omega_{pe}} \right)^{1/2} \gg \frac{e\phi}{T_e}. \quad (13)$$

Thus, there is a large range of  $\frac{e\phi}{T_e}$  for which the microscopic picture is governed by waves in an unmagnetized plasma, but for which the macroscopic currents and transport are dominated by the magnetic field. This then is similar to the case of a resistive shock in a dense plasma ( $\omega_{pe} \gg \omega_{ce}$ ) where the magnetic field governs the currents, but where the instability responsible for anomalous transport is an ion acoustic instability of an unmagnetized plasma.<sup>18-20</sup>

Then averaging over cyclotron resonances, the expression for  $\tilde{f}_e$  becomes

$$\tilde{f}_e = n f_e^0 \frac{e\phi}{T_e} \left[ 1 + \frac{\omega + \frac{kc}{eB} \frac{dT_e}{dx} \left( \frac{1}{2} \frac{m(V - u_{ix})^2}{T_e} - \frac{5}{2} \right)}{k \cdot V - \omega} \right] \quad (14)$$

Assuming that the ions are cold and unmagnetized, and  $k^2 \lambda_D^2 \ll 1$ , the

dispersion relation for waves is obtained by  $\tilde{n}_i = \tilde{n}_e$ , or

$$\frac{\omega^2}{\omega_{pi}^2} = \frac{\omega^2}{k^2 V_e^2} \left[ 1 + \int d^3V \frac{\omega + \frac{kc}{eB} \frac{dT_e}{dx} \left( \frac{1}{2} \frac{m(V_x - u_{ix})^2}{T_e} - \frac{5}{2} \right)}{k V_y - \omega} f^0 \right] \quad (15)$$

As long as  $\frac{c}{eB} \frac{dT_e}{dx} \ll V_e$ , the integral term in the square brackets is much smaller than unity, so to lowest order the waves are just ion acoustic waves which propagate with phase speed  $V_s = (T_e/M)^{1/2}$ . Making a resonant approximation to  $(kV_y - \omega)^{-1}$ , we find that the wave frequency is given roughly by

$$\omega \approx |k| V_s \left[ 1 - i(\pi/8)^{1/2} \frac{\left( \omega - 3/2 \frac{kc}{eB} \frac{dT_e}{dx} \right)}{|k| V_e} \right] \quad (16)$$

In the configuration shown in Fig. 1,  $\frac{dT_e}{dx} < 0$ , so that if  $k < 0$ , the ion acoustic wave is unstable. This corresponds to a wave propagating in the negative  $y$  direction, or parallel to the return current in the  $y$  direction. For the case of cold ions which we have considered here, the condition for instability is given by

$$3/2 \frac{c}{eBV_e} \frac{dT_e}{dx} > (m/M)^{1/2}. \quad (17)$$

This condition is easily satisfied in a laser produced plasma. For instance if  $T_e = 5$  Kev and  $B = 1$  Megagauss, a hydrogen plasma is unstable if the temperature gradient scale length is less than about  $75 \mu$ . If the ion temperature is not zero, one can subtract from the growth rate the ion Landau damping decrement.

#### IV ANOMALOUS THERMAL CONDUCTION

In this section, we will calculate the anomalous thermal conduction arising from the ion acoustic instabilities driven by cross field temperature gradients. As discussed in Section II and III, the negative temperature gradient in the x direction gives rise to a current of low velocity particles in the negative y direction. This drives an ion acoustic instability which also propagates in the negative y direction.

One consequence of the instability is that a force is exerted on the electrons opposite to the direction of the drift, or in the positive y direction. This force acts on the electrons like an equivalent electric field in the negative y direction. This equivalent electric field crossed into the magnetic field in the positive z direction gives rise to a net electron drift in the negative x direction i.e., up the temperature gradient.

This hydrodynamic motion of the electrons of course is not real. In a one dimensional laser produced plasma characterized by  $\omega_{pe} \gg \omega_{ce}$ ,  $B \gg 1$ , and  $n\mu^2 \gg B^2/4\pi$  the magnetic field has no effect on the hydrodynamic motion. This is controlled only by the ion motion and the quasi-neutrality condition. (In a two or three dimensional plasma, the magnetic field can play a role in the electron motion as long as  $\nabla \cdot \mathbf{u}_e = \nabla \cdot \mathbf{u}_i$ .) Therefore the electric field in the y direction, which controls the x motion of the electrons must also be modified by the instability.

In the reference frame moving with speed  $cE_y/B$ , one can see how the instability gives rise to an anomalous thermal flux. In Fig. 2a,

the dotted lines are the contours of constant  $f_e^0(V)$  in the  $(V_x - u), V_y$  plane in the absence of instability. The instability exerts an average force on the electrons in the positive  $y$  direction which causes an average electron drift in the negative  $x$  direction. However this force is a decreasing function of the magnitude of the particle velocity.<sup>21</sup> Thus the lower velocity particles have a greater drift velocity in the negative  $x$  direction than do the higher velocity particles. Hence in the presence of instability, the contours of constant  $f(V)$  are the solid lines in Fig. 2a.

Transforming to the reference frame in which there is no net drift in the  $x$  direction, the contours of constant  $f(V_e)$  are as shown in Fig. 2b. Notice that the lower velocity particles have a drift to the left (up the temperature gradient), whereas the higher velocity particles have a drift to the right (down the temperature gradient). Since  $\langle V_x \rangle = 0$ , this distorted distribution function has an energy flux down the temperature gradient.

As long as  $u^2 \ll T_e/m$ , a condition easily satisfied in a laser produced plasma, the electron energy flux in Fig. 2b is just equal to the energy flux in Fig. 2a minus  $5/2 \langle nu \rangle T_e$ , where  $\langle nu \rangle$  is the instability induced particle flux. The quantity  $5/2 \langle nu \rangle T_e$  is of course just that part of the total energy flux which is convected with the fluid motion (for  $u^2 \ll T_e/m$ ).

We will now calculate  $\langle nu \rangle$  and  $W$ , the particle and energy flux associated with the instability. If we make transformation given in Eq. (8), the quasi-linear equation for  $f(V)$  in a time independent plasma is

$$V_{\perp} \cos \theta \frac{\partial f}{\partial x} + \omega_c \frac{\partial f}{\partial \theta} = \pi \sum_{\mathbf{k}} \frac{\partial}{\partial V} \cdot \mathbf{k} \left| \frac{e\phi(\mathbf{k})}{m} \right|^2 \delta(\mathbf{k} \cdot \mathbf{V} - \omega) \mathbf{k} \cdot \frac{\partial f}{\partial \mathbf{V}} \quad (18)$$

where  $\omega_c = eB/mc > 0$ . On the right hand side of Eq. (18), the summation over  $\mathbf{k}$  includes also a summation over  $-\mathbf{k}$ . That is for each  $\mathbf{k}$  in the summation, there is an equal term arising from  $-\mathbf{k}$ . Multiply Eq. (18) by  $V_{\perp} \sin \theta$  and integrate over velocity. The second term on the left integrates to  $-\omega_c \langle nu \rangle$ . The magnitude of the integral of the first term is roughly  $p_e/L$  times the integral of the second term. Here  $p_e$  is the electron larmor radius and  $L$  is some macroscopic scale length. Thus as long as  $p_e \ll L$ , we can neglect the first term on the left hand side of Eq. (18). Then, using Eq. (9) for  $f(V)$  on the right hand side of Eq. (18), we find

$$\langle nu \rangle = \sum_{\mathbf{k}} \left(\frac{\pi}{2}\right)^{1/2} \left| \frac{e\phi(\mathbf{k})}{T_e} \right|^2 \frac{\mathbf{k}}{|\mathbf{k}|} \frac{V_e}{\omega_c} n \left[ \frac{3}{2} \frac{kc}{eB} \frac{dT_e}{dx} - \omega \right]. \quad (19)$$

To find the energy flux  $W_1$  in the reference frame moving with speed  $c$   $E_y/B$ , multiply Eq. (18) by  $\frac{1}{2} m(V_z^2 + V_{\perp}^2)V_{\perp} \sin \theta$  and integrate over velocity. The same analysis as leading to Eq. (19) gives

$$W_1 = \sum_{\mathbf{k}} \left(\frac{\pi}{2}\right)^{1/2} \left| \frac{e\phi(\mathbf{k})}{T_e} \right|^2 \frac{\mathbf{k}}{|\mathbf{k}|} \frac{nV_e T_e}{\omega_c} \left[ \frac{1}{2} \frac{kc}{eB} \frac{dT}{dx} - \omega \right] \quad (20)$$

The thermal energy flux corrected for fluid convection is then

$$W = W_1 - \frac{5}{2} nu_1 T_e = - \sum_{\mathbf{k}} \left(\frac{\pi}{2}\right)^{1/2} \left| \frac{e\phi(\mathbf{k})}{T_e} \right|^2 \frac{\mathbf{k}}{|\mathbf{k}|} \cdot \frac{nV_e T_e}{\omega_c} \left[ \frac{13}{4} \frac{kc}{eB} \frac{dT}{dx} - \frac{3}{2} \omega \right] \quad (21)$$

Thus, as long as the plasma is unstable the energy flux is down the temperature gradient. Let us note in passing that if the spectrum is one dimensional in the y direction, the arguments leading up to Eq. (21) shows there is no instability generated heat flux in the y direction.

If one assumes

$$k \sim k_D/2, \quad (22)$$

as is characteristic of ion acoustic instabilities, Eq. (21) is an expression for the energy flux in terms of the fluctuating field strength. Clearly, it has the same form as the classical thermal energy flux. One could in fact write out an expression for the anomalous thermal conduction,

$$\kappa_{an} \approx \frac{13}{8} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \left|\frac{e\phi}{T_e}\right|^2 \frac{\omega_{pe} v^2}{\omega_{ce}^2}. \quad (23)$$

We will close this section by determining an approximate maximum value for  $\frac{e\phi}{T_e}$  for which the magnetic field is important in the limitation of energy flux. An earlier study<sup>9</sup> has shown that in an unmagnetized plasma, an energy flux generates ion acoustic instability which inhibits the heat flux. There, it was found that for the unmagnetized plasma,

$$W_u \approx \frac{4}{\omega_{pe}} \frac{(2/\pi)^{\frac{1}{2}} v^2}{\left|\frac{e\phi}{T_e}\right|^2} n \frac{dT_e}{dx} \quad (24)$$

where again we have assumed  $k \sim k_D/2$ . Notice that for the unmagnetized plasma, as  $\left|\frac{e\phi}{T_e}\right|^2$  increases  $W_u$  shrinks, where  $W$  increases with  $\left|\frac{e\phi}{T_e}\right|^2$  for the magnetized plasma. If  $\left|\frac{e\phi}{T_e}\right|^2$  is so large that  $W_u$  expressed in Eq. (14) is smaller than  $W$  of Eq. (21), then one could reasonably conclude

that the magnetic field no longer plays a role in limiting the heat transport. Instead, the energy transport is inhibited only by the ion acoustic turbulence and  $W$  is given by Eq. (24). It is then a simple calculation to show that if

$$\left| \frac{e\phi}{T_e} \right|^2 > 4 \left( \frac{4}{13\pi} \right)^{\frac{1}{2}} \frac{\omega_{ce}}{\omega_{pe}} \quad (25)$$

the magnetic field does not limit the electron heat transport.

The above argument is even more plausible if one makes the reasonable assumption that the effect of the turbulence is to induce an anomalous collision frequency

$$\nu_{an} \sim \frac{1}{2} \left( \frac{\pi}{2} \right)^{\frac{1}{2}} \omega_{pe} \left| \frac{e\phi}{T_e} \right|^2. \quad (26)$$

In that case, our expression for the thermal conduction coefficient for a magnetized plasma is  $\frac{13}{4} \nu_{an} \frac{V_e^2}{\omega_{ce}^2}$ , while for a unmagnetized plasma, it is  $2 V_e^2 / \nu_{an}$ . This is then clearly analogous with the classical expression for electron thermal conduction<sup>12</sup> except for numerical factors of order unity.

#### V APPLICATION TO LASER HYDRO-CODES

As shown in the numerical example at the end of Section III, magnetized laser produced plasmas will often be unstable. Since laser fusion targets are generally designed by using hydrodynamic codes,<sup>22-25</sup> it is essential to determine how the presence of instability can be modeled in these codes. Equation (23) gives an expression for the thermal conduction in the  $x$  direction in terms of fluid parameters and also  $e\phi/T_e$ . It was also stated in Section IV that the instability has

no effect on the collisionless heat flux in the y direction. To model anomalous thermal conduction, the first thing to do is to determine if the plasma is unstable at a particular point. Of course ion Landau (and possibly collisional) damping must be figured into the growth rate. If it is stable, simply use classical thermal conduction. If it is unstable, the problem is to determine  $e\omega/T_e$ . To do this, one can make contact with a great deal of work on the nonlinear theory of ion acoustic instabilities.<sup>28-34</sup> A general consensus seems to be that some sort of trapping or resonance broadening ultimately limits the value of  $e\omega/T_e$  to somewhere around  $0.1 < e\omega/T_e < 0.2$ . Assuming such a value of  $e\omega/T_e$ , Eq. (23) then gives an approximate expression for the anomalous thermal conductivity in the unstable regions. The final thing to check is whether or not this value of  $e\omega/T_e$  is so large that the turbulence, rather than the magnetic field, dominates the thermal conduction, in other words, if the inequality, Eq. (25), is satisfied. If it is, then instead of Eq. (23), the appropriate thermal conductivity is the unmagnetized value, either classical<sup>12</sup> or anomalous.<sup>9</sup>

In summary, this paper has shown that the thermal conductivity in laser produced plasmas can be anomalously increased due to self generated ion acoustic turbulence. Also it has shown how this effect may be modeled in laser hydro-codes.

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## APPENDIX

In order to calculate whether a magnetized plasma is stable, one's first instinct is generally to find a Vlasov equilibrium which describes the unperturbed situation. This is not possible to do in the plasma pictured in Fig. 1. The problem is that the pressure gradient in the x direction is balanced by an electric field in the x direction  $E_x$ , and the electron flow in the x direction is generated by an electric field in the y direction  $E_y = \frac{u}{c} B$ . Thus neither  $P_y = mV_y - eAy/c$  nor  $P_x = mV_x - eAx/c$ , the canonical momenta in either the y or x direction, are constants of the motion. This is quite different from the case of a magnetically confined plasma, where there is no flow velocity in the x direction so that  $P_y$  is a constant of the motion. This allows distributions with x variations in density and temperature to be constructed by exploiting the constancy of  $P_y$ .

In the laser produced plasma shown in Fig. 1 one can see in another ray that there is no relevant Vlasov equilibrium. To do so, notice that a Vlasov equilibrium implies no dissipation. Then, if the distribution function is nearly Maxwellian, ideal fluid equations should apply. That is, as the fluid flows into lower density regions, it should cool adiabatically, as it expands or  $T \sim n^{2/3}$ . The density and temperature gradients then could not possibly be in opposite directions as shown in Fig. 1.

The reason the density and temperature gradients are opposite for the laser produced plasma is that the fluid is not ideal; its equilibrium properties are dominated by electron thermal conductivity.

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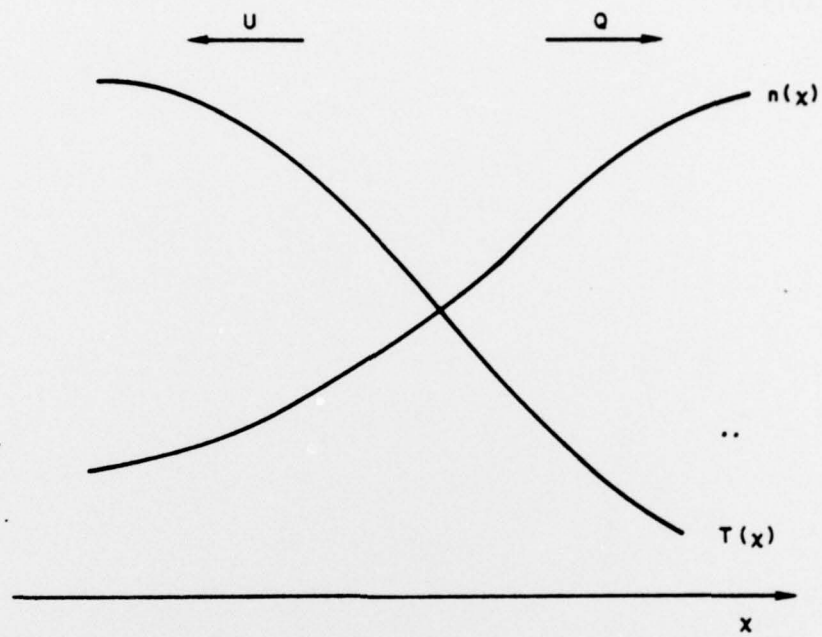


Fig. 1 - The spatial dependence of density and temperature in a laser produced plasma

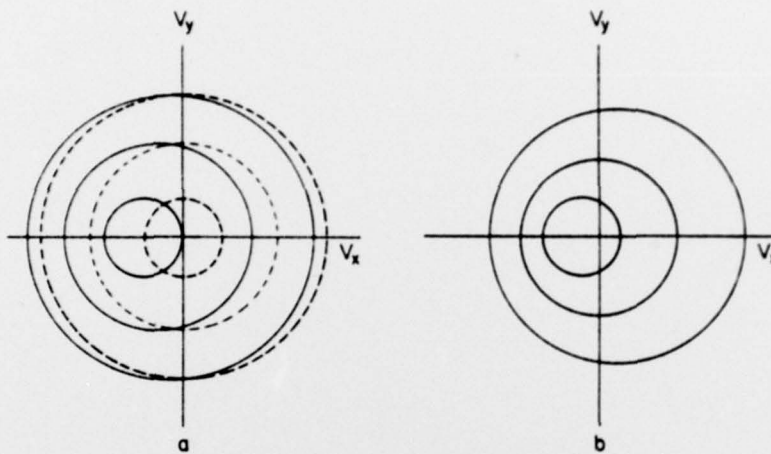


Fig. 2 - (a) The lines of constant  $f(V_x V_y)$  in the frame moving with speed  $cE_y/B$ . Dotted lines are for the absence of instability, solid lines are for the presence of instability. (b) Lines of constant  $f(V_x V_y)$  in the frame with  $u = 0$  with the presence of instability.