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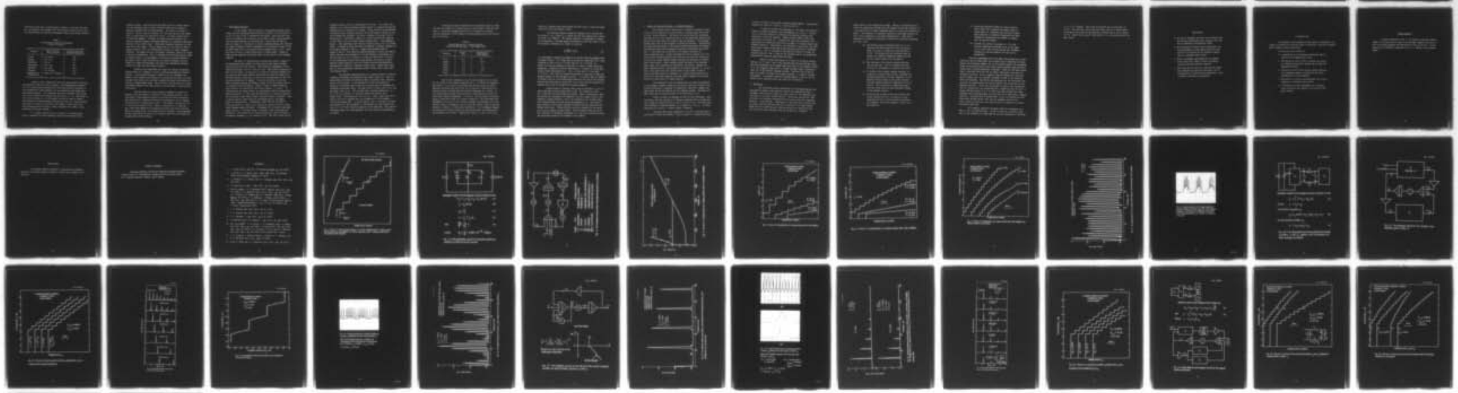

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INFLUENCE OF INTERCONNECTION AND BIASING CONFIGURATIONS ON THE OPERATION OF JOSEPHSON JUNCTIONS AS RADIATION SOURCES

Michael R. Daniel, M. Ashkin,
and M. A. Janocko

Westinghouse Electric Corporation
Research and Development Center
Pittsburgh, Pennsylvania 15235

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to the Office of Naval Research
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ABSTRACT

This final report of ONR Contract No. N00014-76-C-0609 describes some initial experimental work and then calculations on an analogue computer of properties of Josephson junctions used as radiation sources for potential application as a frequency agile source. The results of the calculations show that a junction may be used as a nearly ideal (zero impedance) dc voltage source with which to bias a second junction. The junctions must be isolated rf-wise from each other and the inductance level between them must not exceed about 10^{-8} H for operation at liquid helium temperatures. The second junction will then oscillate at a frequency determined by this bias voltage. Additionally the same junction bias source was shown to be capable of biasing a series array of junctions. The series array was shown to oscillate phase coherently at a frequency determined by the divided bias voltage across each junction.

INTRODUCTION

In work performed under the previous contract (N00014-73-C-0315) fabrication techniques were developed and the microwave properties measured of Nb_3Ge and Mo-Re superconductor Josephson junctions. Some experimental work on the coupling of Josephson junctions to microstrip was performed and the possibility of using radiation-emitting junctions as frequency-agile microwave sources was examined.

Following the completion of the above previous contract the present contract (N00014-76-C-0609) was granted for the period of March 1, 1976 to February 22, 1977 with a no-cost extension to July 30, 1977. This final report covers all work performed from March 1, 1976 to July 30, 1977 concerning the biasing and use of Josephson junctions as frequency-agile microwave sources. Additional support for this work was provided by an internal Westinghouse contract.

Appendices I and II in the previous final report to contract N00014-73-C-0315 contained a discussion of possible biasing methods of a Josephson junction and a survey of the literature. A junction when biased at voltage V will oscillate at a frequency f given by the now well known equation $V = \frac{h}{2e} f$, where h and e are respectively Plancks constant and the electron charge. Translated into practical terms 1 μV of bias causes a 0.48361 GHz oscillation. The literature survey referred to above disclosed two main biasing techniques: (a) biasing from a constant (but adjustable) current source and (b) voltage biasing by shunting the junction with a resistance, the resistance usually being much less than the junction resistance to simulate a voltage source.

An update of the survey roughly from 1974 to the present time has revealed no further or new developments in biasing junctions. This is felt to result from no published interest in Josephson junctions as narrow band frequency-agile sources. Vernet and Adde[1] and Vernet et al.[2] have added to the further development and understanding of a junction used as a self-oscillator mixer. However, they retained the constant current biasing technique used previously. Varmazis et al.[3] described a broadband tunable junction for microwave frequencies. It was presumbably frequency agile but the radiation bandwidth was large -- 1 GHz and curiously independent of frequency from 2 to 12 GHz. The reservations about using either constant current or resistive shunt biasing expressed in Appendix I of the previous report are still cogent. Thus the solution to the biasing problem proposed in Appendix II of the same report still stands as the most promising at this time of writing.

INITIAL EXPERIMENTAL WORK

In view of the outward simplicity of the idea of using a Josephson junction as a source of bias, it was felt appropriate to immediately undertake experimental work to demonstrate the idea using apparatus already assembled for work performed in the previous contract. The demonstration was to use a junction in a self biasing configuration. A niobium point contact junction was fabricated electrochemically. The point contact was chosen because it was easy to make and it readily exhibits the ac Josephson effect. A contact was mounted in an X-band waveguide parallel to the E-field at 1/4 wavelength from the shorted end, i.e., in a region of an E-field maximum. External to the waveguide, a coaxial cable was coupled (both direct and capacitance coupling were tried) to the junction through a slot in the waveguide wall. The usual dc connections and provision for mechanically adjusting the junction contact pressure were made.

The 1 GHz radiation was fed down the coaxial cable to the junction which was maintained at 4.2 K by thermally coupling it to a liquid helium bath using low-pressure helium gas. This radiation should have developed a series of steps in the dc I-V curve for the junction every 2.2 μV. Biasing the junction at the 10th step should have produced radiation from the junction at 10 GHz which could be detected from the X-band waveguide. No radiation was detected and the step structure was difficult to achieve at 1 GHz with usually only the first few steps discernable in the dc I-V curve, see Figure 1(a). However, launching 10 GHz radiation down the waveguide usually readily developed steps in the I-V curve at 22 μV intervals, see Figure 1(b). The 1 GHz radiation was unquestionably reaching the junction since an immediate reduction in the junction's critical current (typically around 3 mA) was always evident.

At this time a technical difficulty specific to a point contact junction was realized.' Vernet and Adde[4] had emphasized that sharp (small angled) points do not couple well to the transverse E-field of a waveguide. They were successful with sharp but conical shaped points where the half angle of the cone was not less than 60° fabricated from niobium rod. These were thus tried in this work but the difficulty of developing a step structure at 1 GHz remained and indeed was worse with these junctions. Similar difficulties were experienced with 3 GHz radiation -- a poorly defined step structure with few steps and small step height. From this point on work concentrated on mounting the junction external to the waveguide and trying to understand the problems in reproducibly obtaining a well defined set of steps at low frequencies (below 10 GHz).

Table 1 summarizes the junction characteristics obtained in this work and includes results of previous work done at 10 GHz. The resistance values quoted are the "ohmic" values measured for the linear region of the I-V curve well away from the origin. The number of steps was counted after adjusting the microwave power arbitrarily to suppress most of the dc critical current since under this condition the maximum number of steps was obtained. From the table the following conclusions were drawn:

(a) At a given frequency the number of steps increased as the junction resistance decreased.

(b) At a given frequency the step height increased as the junction resistance decreased.

(c) At a given frequency all step structure would disappear if the junction resistance was too high.

The effect of junction capacitance on the above conclusions remained unknown.

At this juncture it was decided to temporarily suspend the experimental work and to initiate an analysis of the Josephson junction as a circuit element. From this it was hoped to secure a better understanding of the results thus far obtained and derive guidelines for further experimental work. An analogue computer was used for the analysis.

Table 1

Nb Point Contact Junction Characteristics

Junction Resistance (Ω)	Frequency (GHz)	Number of Steps	First Step Height (mA)	Critical Current (mA)
0.05	0.96	12	0.10	3.7
0.07	1.00	> 10	0.15	3.0
0.09	10.00	∿ 20	0.30	2.5
0.14	0.96	5	0.10	2.7
0.29	10.00	> 30	0.70	3.7
0.30	0.96	5	0.05	2.7
0.50	10.00	> 30	0.08	1.6
0.53	10.00	> 10	0.10	1.5
0.53	0.96	0	--	1.5
0.60	0.95	0	--	2.1
1.10	3.00	0	--	1.9
2.00	10.00	> 30	0.02	1.2

ANALYSIS OF INTERCONNECTED JUNCTIONS

1. Preamble

The analysis of a Josephson junction as a circuit element by analogue computer is well known[5] and its advantages for the present program are many-fold. An accurate picture both graphical and on an oscilloscope of the dc and ac behavior of a junction can be readily obtained with the instant adjustment of any circuit parameters as an added feature. The simulation of a junction is accurately quantitative enabling one to derive such useful information as the frequency spectrum of the radiation and radiated power. Indeed the qualitative insight into the complex and highly nonlinear behavior of a junction was without precedence.

The starting point for the analysis was the accurate representation of a junction by an equivalent circuit -- the resistively shunted junction or RSJ model (Figure 2). The junction J, its associated shunt resistance R and capacitance C are driven by an adjustable constant dc current source i_s. The step structure in the dc I-V curve results when the junction is subject to radiation of current amplitude i_1 and frequency ω. Thus if at time t, V is the voltage across the parallel configuration we can write the total current

$$I(t) = C\frac{dV}{dt} + \frac{V(t)}{R} + i_0 \sin\phi, \qquad (1)$$

and

$$V = \frac{\hbar}{2e}\frac{d\phi}{dt} = \left(\underset{dc}{V_0} + \underset{ac}{V_1 \cos\omega t}\right) . \qquad (2)$$

The Josephson current is $i_J = i_0 \sin \phi$, where i_0 is the critical current in the absence of V_1 cost ωt and ϕ is the superconducting quantum phase difference across the junction which together with the relation between V and ϕ was originally predicted by B. D. Josephson. However, for a recent review see Waldram.[6] Equations (1) and (2) may be combined to give a differential equation to be solved by the analogue computer.

$$I(t) = \frac{\hbar}{2e} C \frac{d^2 \phi}{dt^2} + \frac{\hbar}{2e} \cdot \frac{1}{R} \frac{d\phi}{dt} + i_0 \sin \phi \,. \qquad (3)$$

The constants C and R had to be set from experiment. From Table 1 R was allowed to have values between 0.025 and 1Ω. C could only be estimated from the dimensions of a junction and was allowed values between 0.1 and 1 pF. It turned out in fact that this range of values produced only small effects upon the junction performance because $R < < \frac{1}{\omega C}$ for the frequency range of interest.

The analogue circuit is shown in Figure 3. The operational amplifiers of this circuit could not, of course, function at typical Josephson frequencies and it was necessary to simulate a junction behavior with scaled electrical parameters. The output from the amplifiers whether representing an ac or dc current or voltage was always a voltage in the range 0 to 10 volts. Table 2 lists the scaling factors used for the circuit of Figure 3. Thus in the simulation 1 GHz becomes 1 Hz which is readily available from a low-frequency oscillator. The commercial analogue computer used was a EAI Model 680.

2. The Single Junction

Figure 4 shows the results obtained for the dc I-V curve of a junction with no radiation impressed upon it. Such curves were the simplest to take and served as an initial check that the simulation was functioning correctly. The hysteretic behavior seen for the higher RC product is well known and had been readily observed for the Nb point junctions listed in Table 1. Equation (3) is soluble for this simpler situation and the type of curves shown in Figure 4 are given, for example, by Stewart[7] and McCumber.[8] The critical current value of 3 mA was selected from the previous experimental results on Nb point contacts.

Table 2

Scaling Factors Used in the Analogue Circuit of Figure 3

Parameter	Range of Values for a Junction	Scaling Factor for Analogue Simulation
Current	0 to 10 mA	10^2
Voltage	0 to 1 mV	10^3
Resistance	0 to 1Ω	10
Capacitance	0 to 1 pF	10^8
Inductance	0 to 1 nH	10^{10}
Frequency	1 to 100 GHz	10^{-9}
Magnetic Flux Quantum, Φ_0	2.068×10^{-15} Webers	10^{12}

Figures 5 and 6 respectively show the step structure resulting from applying 10 GHz and 3 GHz radiation. The experimentally observed fact of a step height decreasing with increasing junction resistance is well documented by these figures as is the decreasing step height with decreasing frequency. For a Josephson junction to operate below 10 GHz with a well defined step structure one must have $R << 1\Omega$. The effect of the critical current, i_0, on the number of steps is shown in Figure 7. This confirms the experimentally observed fact that for a given junction maximizing the value of i_0 maximizes the number of steps with the rf level being adjusted for each i_0 value to maximize this number, other parameters being held constant.

If a single junction were to be used as a frequency-agile source a parameter of prime importance would be the spectrum of the radiation output. The spectrum of the output from the analogue simulation was measured using a Federal Scientific Model UA-15A spectrum analyzer and Model 1015 spectrum averager. The results are shown in Figure 8. With in injected frequency of 3 GHz and biasing on the eighth step a strong output was expected at 24 GHz. As Figure 8 shows this is far from what happens. Evidently the highly nonlinear behavior of a junction produces frequency modulated components of substantial amplitude relative to the 24 GHz signal. Indeed many frequency components are well above the 24 GHz signal. From an energy point of view Figure 8 shows a single junction to be a very unsatisfactory source of single frequency radiation. The number and size of frequency components implies the output to be very nonsinusoidal. This is confirmed by the waveform pattern shown in Figure 9 which is a good graphic illustration of nonlinear behavior. The output from a single junction with no injected frequency was also found to be highly nonsinusoidal (see Figure 20) although the harmonic content was lower than that shown by Figure 8.

Hendricks and Lindelof[9] have shown the location of optimum biasing points on a stepped I-V curve to obtain good radiation characteristics -- narrow center frequency, absence of or low sidebands, etc. However, they confined their analysis to include only the first bias step and thus their spectra are noticeably simpler than those which pertain to biasing on higher number steps. But the authors do show that for a given step fewer sidebands occur when biasing is on the middle to upper portion of the step.

The line width of a given frequency component in Figure 8 is believed to be that which is intrinsic to the spectrum analyzer. The line width of the radiation emitted by a junction has been discussed by many authors.[10] Essentially whether the broadening is due to thermal or shot noise it is proportional to R_D^2 -- the dynamic resistance of the I-V curve at the point of bias. Hence, if biasing is on a step then $R_D \rightarrow 0$ and there should be an indefinitely narrow line width in the absence of external noise. The single junction, therefore, should approach an ideal comb source of radiation if external noise from the current and voltage leads can be minimized.

3. The Coupled Junctions

It was evident from the results of the previous section that a single junction irradiated with microwaves to produce a step structure in its dc I-V curve is not a satisfactory oscillator. Hence the next task was to see whether such a junction can be used to bias a second junction with the second junction as the oscillator. The circuit for this is shown in Figure 10. Coupling by a series inductance between the junctions was taken as being the likely configuration in a practical circuit if for no other reason that a dc potential must be coupled from J_1 to J_2. Note here that we have J_1 current biased and J_2 voltage biased. Figure 11 shows how the equations describing Figure 10 were solved by the analogue computer.

The first I-V characteristics obtained are shown in Figure 12 and demonstrated that indeed the second junction was being dc biased (step wise) by the first junction. The biasing was independent of the coupling inductance L up to a value of 10 nH. Some further discussion of this is given later. In Figure 13 is a series of spectral responses for the output of the second junction when biased on successive steps of J_1. The main feature of note here is the ratio of the amplitudes of the first and second harmonics for example. This ratio increases as the step number increases showing the harmonic content of the oscillation from J_2 is decreasing with step number. Figure 14 shows a stepped frequency output from J_2 as a function of the dc bias current, i_s, into J_1. It confirmed that the output from J_2 was conforming to the dc biasing conditions provided by J_1. The output waveform from J_2 is shown in Figure 15 and is similar to that from a single junction as shown in Figure 9. Thus a complex spectral output would be anticipated from J_2 and indeed Figure 16 confirms this. It appeared at this time that the idea of using a junction as a source of dc bias was not going to lead to a practical frequency-agile oscillator. Fortunately the inclusion of one additional circuit -- a low pass filter -- between J_1 and J_2 overcame the difficulty. The circuit and its characteristics are shown in Figure 17. The angular frequency, ω_n, was chosen as $1/\sqrt{2}$. Thus this filter cut out frequencies above $1/\sqrt{2}\,2\pi$ or approximately 1/10 GHz. It was also used whenever dc I-V data were being recorded because it isolated the junction analogue circuits from the X-Y recorder. Figure 18 shows the improvement in spectral purity obtained with this filter and Figure 19 shows the simpler waveform also obtained. The filter isolates J_2 from the 3 GHz rf injected into J_1 and thus J_2 behaves as a single junction but biased by J_1 such that J_1 acts as a nearly ideal voltage source -- zero impedance source -- when operated on the vertical portions of any of its dc voltage steps. It should be noted here that the inductor L provides a measure of rf isolation between J_1 and J_2 -- compare the waveforms shown in Figures 9 and 15. This would be expected whenever R_1 and R_2 are significantly less than the reactance of L at the frequency of operation as is the case here ($R_1 = R_2 = 0.025\Omega$, $\omega L = 0.19\Omega$ at 3 GHz with $L = 0.01$ nH). It was interesting to ask at this stage whether the junction J_2 had a spectral output similar to that for a single junction unirradiated by microwaves but biased by a constant current source. Figure 20(b) shows that J_2 had an essentially identical output to that from a single junction [Figure 20(a)].

The nonlinear characteristics of a junction prevent it from developing a sinusoidal oscillation for any finite value of R. Thus the pulsed output of Figure 19 represents what is practical from a junction used as an oscillator. It was instructive to use the waveform of Figure 19 to calculate the rf power in a junction and its frequency distribution using Figure 20(b). The upper trace in Figure 19(b) is V_{J2} and the lower trace is i_{J2}. This average power, obtained graphically, is $\frac{1}{\tau}\int_0^\tau V_{J2}\, i_{J2}\, dt$ where τ is the pulse repetition interval from Figure 19(a). This worked out to be 25.5 nW. Table 3 shows that 72% of this or 18.4 nW is radiated into the fundamental or first harmonic -- 24 GHz from Figure 20(b). If this junction, with a resistance of 0.025Ω, were coupled to a 50Ω waveguide a 27 dB mismatch loss would result and only 3.7×10^{-11} watts would be detected in the waveguide. This level of detectable power is in good agreement with that observed experimentally by others.

Following the work of Hendricks and Lindelof[9] Figure 21 shows the spectral outputs of J_2 biased on and in the vicinity of the 8th step from J_2. Some line broadening and shifts in frequency are evident from an examination of this figure for biasing positions off the vertical part of the step.

Table 3

Energy Spectrum of a Coupled Junction
Biased on the 8th Step, -- See Figure 20(b)

Harmonic	Energy, nW	Percentage of Total Energy
First	18.4	72
Second	5.3	20.8
Third	1.3	5
Fourth	0.4	1.6
Fifth	0.1	0.4
Sixth	0.0	~ 0

Thus far the inductance L was the only parameter to be varied for its effect on the biasing conditions of J_2. R_1 and R_2 were purposely kept small ($< 0.1\Omega$) since it was known from previous work on a single junction that good step size in the dc I-V curve of a junction only resulted for small R values when the radition frequency was < 10 GHz. As also previously mentioned, the capacitance values were not critical because their reactive impedances $> > R_1$ or R_2. Now the effect of increasing R_2 on the bias across J_2 is shown in Figure 22. The step height was independent of R_2 up to 0.2Ω, and was, evidently, determined by R_1 alone. The only effect of changing R_2 was to alter the overall slope of the step structure. However, it was noticed that for $R_2 > 0.2\Omega$ inadequate damping occurred and the circuit broke into oscillation which was superposed on the Josephson oscillations. Changing the values of L and C_1 or C_2 did nothing to suppress these oscillations and thus 0.2Ω is a practical upper limit for R_2 for the present circuit.

The relation between voltage and frequency for a junction may be written $V = \Phi_o f$ where Φ_o is called the unit flux quantum and is equal to 2.068×10^{-15} Webers. For coupled junctions at temperature T linked by an inductance L the mean square fluctuation in flux $(\delta\Phi)^2$ due to L and the intrinsic fluctuation in current is given by

$$\frac{1}{2}\frac{(\delta\Phi)^2}{L} = \frac{1}{2}kT, \qquad (4)$$

k is Boltzmann's constant and Equation (4) results from the equipartition of energy theorem. If we argue that $\delta\Phi$ should not exceed $\frac{1}{2}\Phi_o$ for a step structure to exist in a dc I-V curve then at 4 K, $L \lesssim 20$ nH. In Figure 12 we saw no evidence of noise for L = 10 nH, a result which was unexpected. Equation (4) may be rewritten $Li^2 = kT$ where i^2 is the mean square current fluctuation in L. For L = 20 nH and T = 4 K we find i = 0.05 μA. This level of current fluctuation is about half of what was normally present in the analogue circuitry. It is likely that the effects of current noise in L were unwittingly removed by the use of the low pass filters which were used to record the dc I-V curves and were essential to separate the ac and dc voltages for recording purposes.

Another source of junction noise, due to resistance R_1, was already mentioned as being proportional to R_D^2. For biasing on a step $R_D \rightarrow 0$ and thus this noise should be absent. Hence, in conclusion, there are grounds for accepting 20 nH [Equation (4)] as an upper limit to the inductive coupling between J_1 and J_2 for operation at 4.2 K above which noise should destroy the step structure generated by J_1. This has not been verified by the present analogue simulation and the opportunity for further work exists. There should be no noise limitation on the values of R_1 for the reasons already stated. Shot and thermal noise due to R_2 will contribute to frequency broadening in J_2 and can be calculated using the equations on page 475 of reference 2 for example.

4. Series Fed Coupled Junctions -- Coherent Radiation

The previous section showed that it should be possible in practice to use one junction as a nearly ideal dc voltage source with which to bias a second junction. The restriction on the junction resistances that they be $\ngtr 0.1\Omega$ for frequencies up to 10 GHz immediately implies such devices will couple poorly to coaxial cable or waveguide. This problem becomes more acute as the frequency of operation decreases. Additionally, the step structure height of the biasing junction decreases with decreasing frequency making lower frequency operation increasingly more difficult. A way around this difficulty is to operate junctions in a series-connected array. Recent experimental work by Lindelof et al.[11] and by Palmer and Mercereau[12] has shown such an array will radiate or oscillate in a phase coherent manner provided the junctions are sufficiently closed spaced. Phase coherence gives the bonus of an increased power output over connected but noncoherent oscillators. However the major importance here is the possibility of using a junction, J_1, to provide a bias source with well defined steps at say 10 GHz to bias 10 series-connected junction J_2 through J_{11}. Clearly these would each oscillate at 1 GHz phase coherently and the output impedance of the array would be 10 times that of a single junction.

The simulation of such a configuration was readily implemented on the analogue computer except that the series array was reduced to two junctions because of a shortage of integrators, amplifiers, summers and logic gates for the SIN function on the computer console. Obviously the above ideas may be verified as well by two junctions as by n.

Figure 23 shows the analogue circuit for this simulation. Note, in passing, that the current i_L is actually represented by a voltage and this voltage is parallel fed to J_2 and J_3 in the circuit. Thus the situation simulates a condition whereby i_L is common to J_2 and J_3 -- a series connection of J_2 and J_3 -- which is what is required.

Initially the circuit parameters of J_2 and J_3 were made equal and Figure 24 shows that indeed J_1 could be used as a biasing source with

J_2 and J_3 sharing, in this case, the bias voltage equally. The interval between the steps for J_2 and J_3 is half that for J_1.

A question of practical importance is the effect of having different or slightly different circuit parameters of J_2 and J_3. In practice it could be difficult to fabricate two adjacent junctions with identical R values for instance. Figure 25 shows effect on the step structure of having $R_2 \neq R_3$. Although the steps are occurring at the same values of current this position in voltage is no longer regular when examined carefully. Additionally, for biasing on a given step, the junctions are no longer at the same voltage and thus oscillate at different frequencies. This is confirmed by the spectral analyses of Figure 26. Certainly when $R_3 = 2R_2 = 0.05\Omega$ the two junctions are in all probability significantly frequency modulating each other.

Figure 27 shows the outputs of J_2 and J_3 to be phase coherent when $R_2 = R_3 = 0.025\Omega$. But it was observed that the oscilloscope traces lost phase coherence and "slipped" past each other if R_2 exceeded 0.0255Ω. As R was increased from 0.025Ω to 0.0255Ω the phase angle increased to about 72° or $\frac{2\pi}{5}$ and then slippage occurred. This says that R_2 and R_3 have to be equal to within 2% for phase coherence -- a fairly stringent requirement. In practice radiation coupling between J_2 and J_3, not included in the simulation here, may relax this restriction.

5. Discussion

The simulation analysis performed on the analogue computer and discussed at length in the previous sections has demonstrated that a Josephson junction may be used as an ideal source of zero impedance bias with which to dc bias a second junction. The second junction will then oscillate at a frequency determined by this bias voltage (the Josephson relation: 0.4836 GHz/μV). Additionally it was shown that junctions may be series connected and then biased by a junction in the above manner. These series-fed junctions will then divide the bias voltage and each will oscillate phase coherently with one another at a frequency proportional to the divided bias voltage. However, the simulation provided also substantial insight into the complex and nonlinear behavior of a junction which is further compounded when two or more are connected together. The operation of connected junctions as described requires certain circuit parameters to be chosen appropriately. These may be summarized as follows if attention is confined to operation frequencies not below 1 GHz:

(a) The biasing junction resistance must be $< 0.1\Omega$ otherwise the step structure of this junction becomes very small in height for operation below 10 GHz. The second junction resistance must not exceed 0.2Ω otherwise inadequate damping results and the circuit breaks into oscillation.

(b) The inductance level between coupled junctions must be kept low, $< 10^{-8}$ H, otherwise the step structure is anticipated to dissolve into noise.

(c) A low-pass filter between coupled junctions is essential to isolate them rf-wise. Otherwise the frequency-modulated rf current components in the biasing junction couple to the second (biased) junction-producing frequency modulation of its rf current. This results in a complicated output from the second junction with substantial energy in the harmonic content.

(d) Even with the low-pass filter for isolation the second junction will only oscillate in a non-sinusoidal way for a finite junction resistance. Very roughly, 30% of the available rf power is in the harmonics.

(e) Series-fed junctions biased by a first junction need to have their junction resistances equal to within about 2%. Otherwise they oscillate without phase coherence at different frequencies and these frequencies intermodulate one another to complicate the spectral output.

(f) Provided the junction resistance is < 0.1Ω the junction capacitance estimated to be in the range 0.1 to 1 pF is not critical and has only secondary influence on junction behavior for the use described here.

Practical implementation of the above restrictions into a working device has not begun under this contract due to exhaustion of funds. However the following comments set out the problem areas to be considered. Single or multiple junctions with micron spacing between them are fairly readily made by photolithographic techniques. These are the Dayem bridge type of junctions which can also be readily fabricated to be part of a waveguide structure. Nb on a sapphire substrate wherein the junctions are an integral part of a coplanar waveguide (a CPW) is one structure worth investigating. The CPW has the advantage over the microstrip line of having its ground planes and center conductor all on one face of the sapphire substrate. It will not be possible to match a 50Ω transmission line to a junction of 0.1Ω resistance with a tapered section of a CPW. A typical CPW for 50Ω has an 0.1" or 2500μ wide space between the center conductor and the ground planes on either side of it when using a sapphire substrate. This gap could be tapered down to say 2.5μ wherein the junctions would be made and for this size the impedance would be 15Ω. Series-connected junctions could improve upon this mismatch since their resistances would add.

On a sapphire substrate the phase velocity of radiation in a CPW is 1.30 x 10^8 meters/second. If 10Ω is taken as a practical lower limit to the impedance of a CPW then for this the inductance of the guide is 7.7 x 10^{-8} H/meter. Thus 13 mm of waveguide has an inductance of 10^{-8} H. Two junctions, one to bias the other, have to be placed about 10 mm apart and no more if the inductance between them is not to exceed 10^{-8} H. Within this 10 mm of "real estate" is to be constructed the low-pass filter.

CONCLUSIONS

- The use of Josephson junctions as oscillators with any biasing configuration must recognize that they have an inherently nonsinusoidal output.
- Of the practical biasing circuits available, the one investigated in this report offers the best control of a junction for its potential application as a frequency agile source.
- This is contingent upon being able to design a filter which will isolate rf-wise the biasing junction from the oscillator junction.
- The filter must be such as to maintain the inductance between the junctions below about 10^{-8} H for operation at liquid helium temperatures. Such a filter is believed to be possible.

RECOMMENDATIONS

The program of work reported on here should be continued to implement a suitable rf filter and thence to fabricate a practical frequency agile Josephson junction oscillator.

In particular there should be:

- A computation of the frequency-attenuation characteristics of an appropriate filter.
- Fabrication of such a filter to measure its practical frequency-attenuation reponse and to check the computation results.
- Simulation of this filter and the coupled junctions on an analogue computer to ascertain that their performance is still acceptable.
- Fabrication of a device and measurement of its performance as a frequency agile source.
- Investigation of the requirements of a frequency agile oscillator with respect to the level of harmonic content.

ACKNOWLEDGEMENTS

The kind hospitality of Dr. T. H. Putman is greatfully acknowledged in allowing access to part of his analogue computer facilities as well as providing guidance and instruction in the use of such. In this connection the technical assistance of C. E. L. Johnson is also acknowledged.

PUBLICATIONS

"An Analogue Computer Simulation of Interconnected Josephson Junctions as Radiation Sources," Michael R. Daniel and M. Ashkin, to be submitted.

PATENT DISCLOSURES

Microwave Radiation from Series Connected Josephson Junctions Voltage Biased by a Complementary Josephson Junction Voltage Source, M. A. Janocko, Michael R. Daniel, and M. Ashkin.

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Curve 691976-B

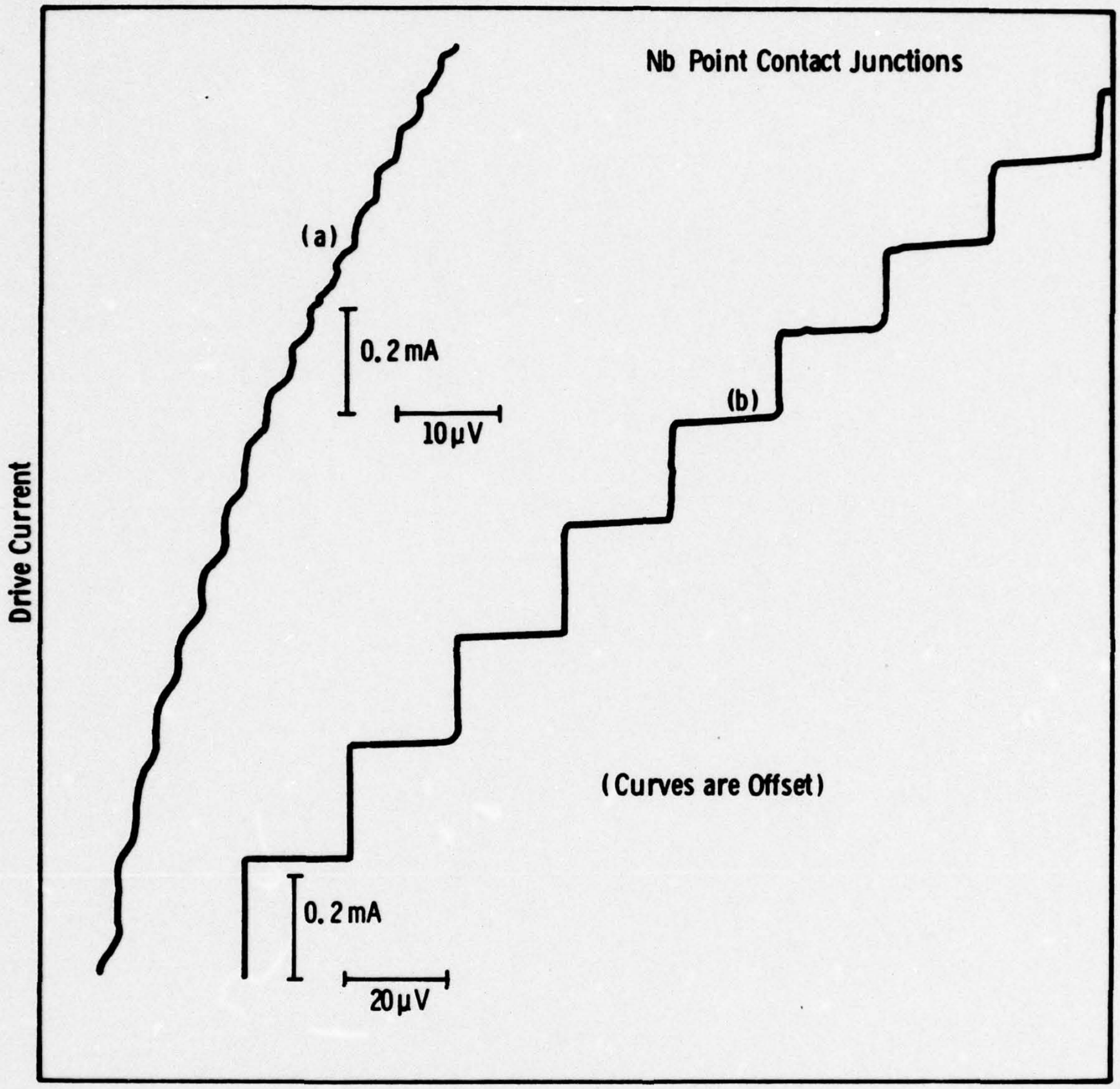
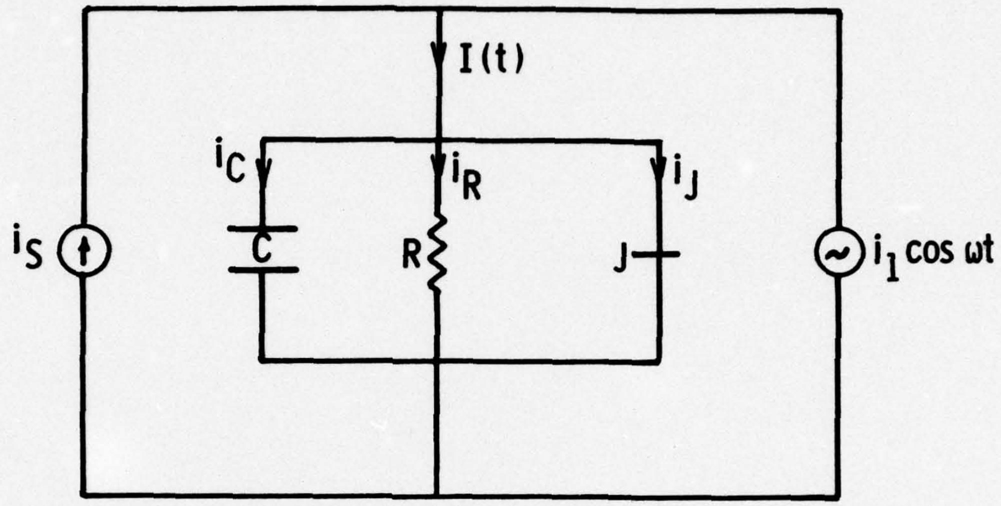


Fig. 1 — The dc I - V curves for two junctions: (a) Ohmic resistance 0.07 Ω, critical current 3 mA and receiving 1 GHz radiation (b) Ohmic resistance 0.28 Ω, critical current 3.6 mA and receiving 10 GHz radiation



Dwg. 6419A28

Equations Used in the Analogue Circuit of Figure 3 Are:

$$-i_C = i_J + i_R - i_S - i_1 \cos \omega t\,; \qquad \text{(a)}$$

$$i_J = i_0 \sin \emptyset\,; \qquad \text{(b)}$$

$$i_R = \frac{V}{R}\,; \qquad \text{(c)}$$

$$V = \frac{1}{C} \int i_C \, dt\,; \qquad \text{(d)}$$

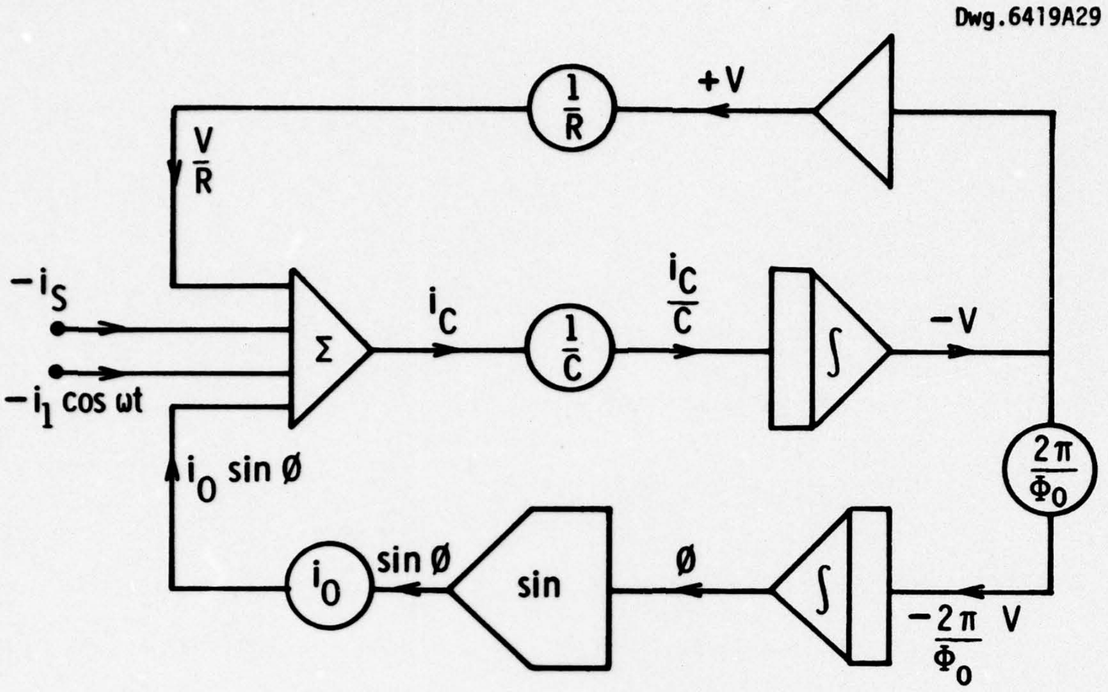
and

$$\frac{d\emptyset}{dt} = \frac{2\pi}{\Phi_0} V, \qquad \text{(e)}$$

where

$$\Phi_0 = \frac{h}{2e} = 2.068 \times 10^{-15} \text{ Webers}$$

Fig. 2— The equivalent circuit of a junction driven by constant current ac and dc sources



Symbol	Term	Analogue Operation
○	Potentiometer	Multiplies by a Constant
▷	Inverter	Multiplies by −1
Σ	Summer	Algebraically Sums and Multiplies by −1
∫	Integrater	Integrates and Multiplies by −1
sin	Sine Generator	Generates the Sine Function

Fig. 3 — The analogue circuit used to simulate the behavior of a junction

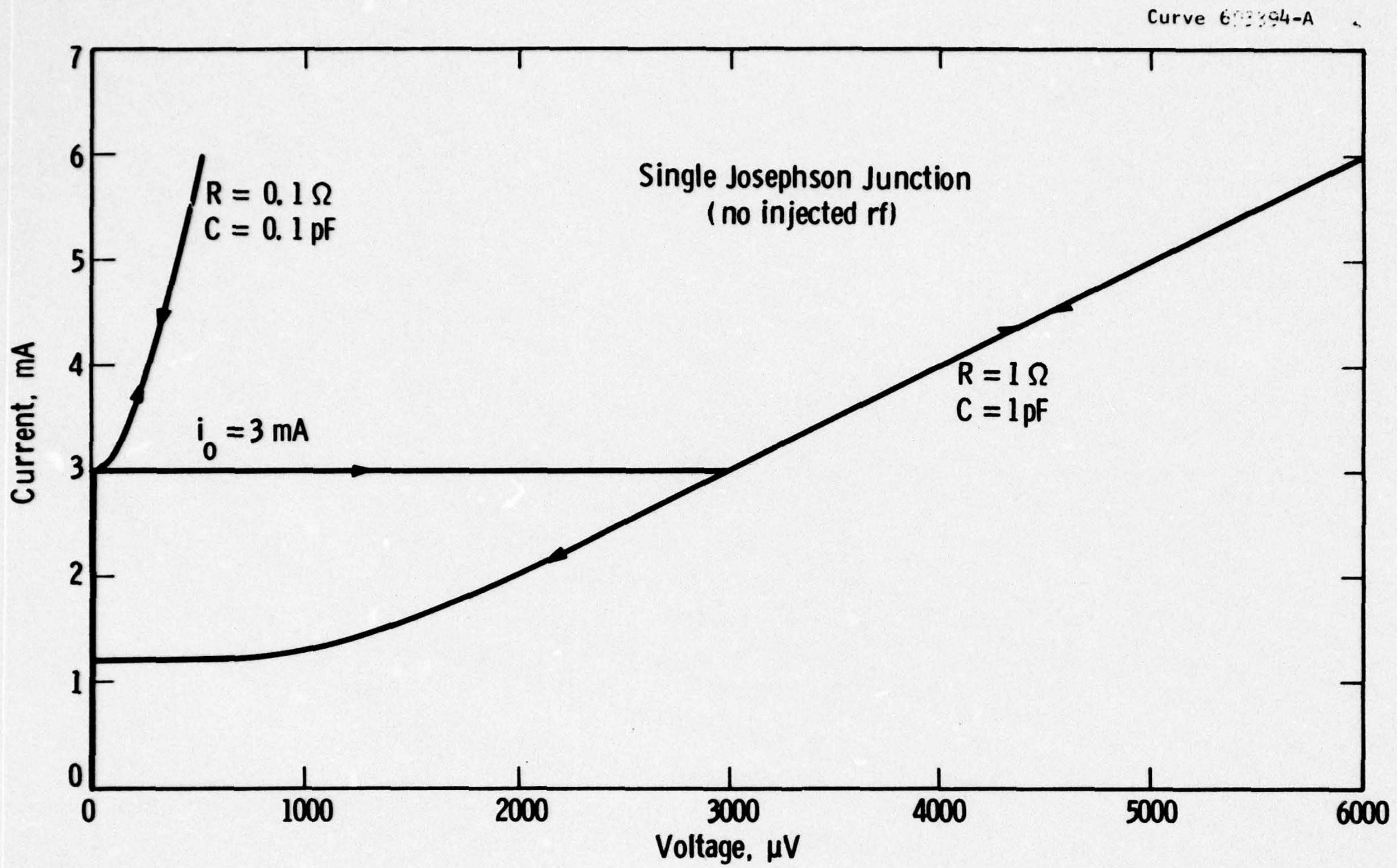


Fig. 4 — The dc I-V characteristic of a single junction with no injected rf radiation

Curve 693406-B

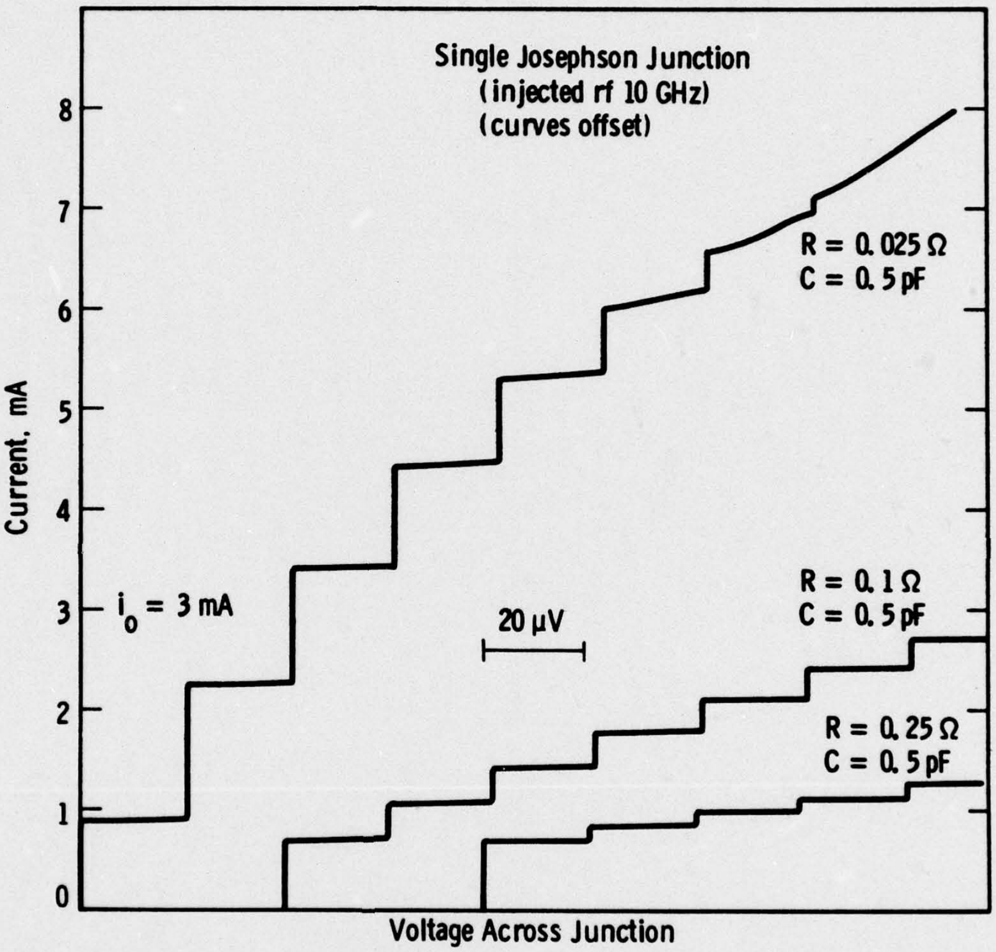


Fig. 5 — The dc I-V characteristics of a single junction with 10 GHz radiation

Curve 693393-A

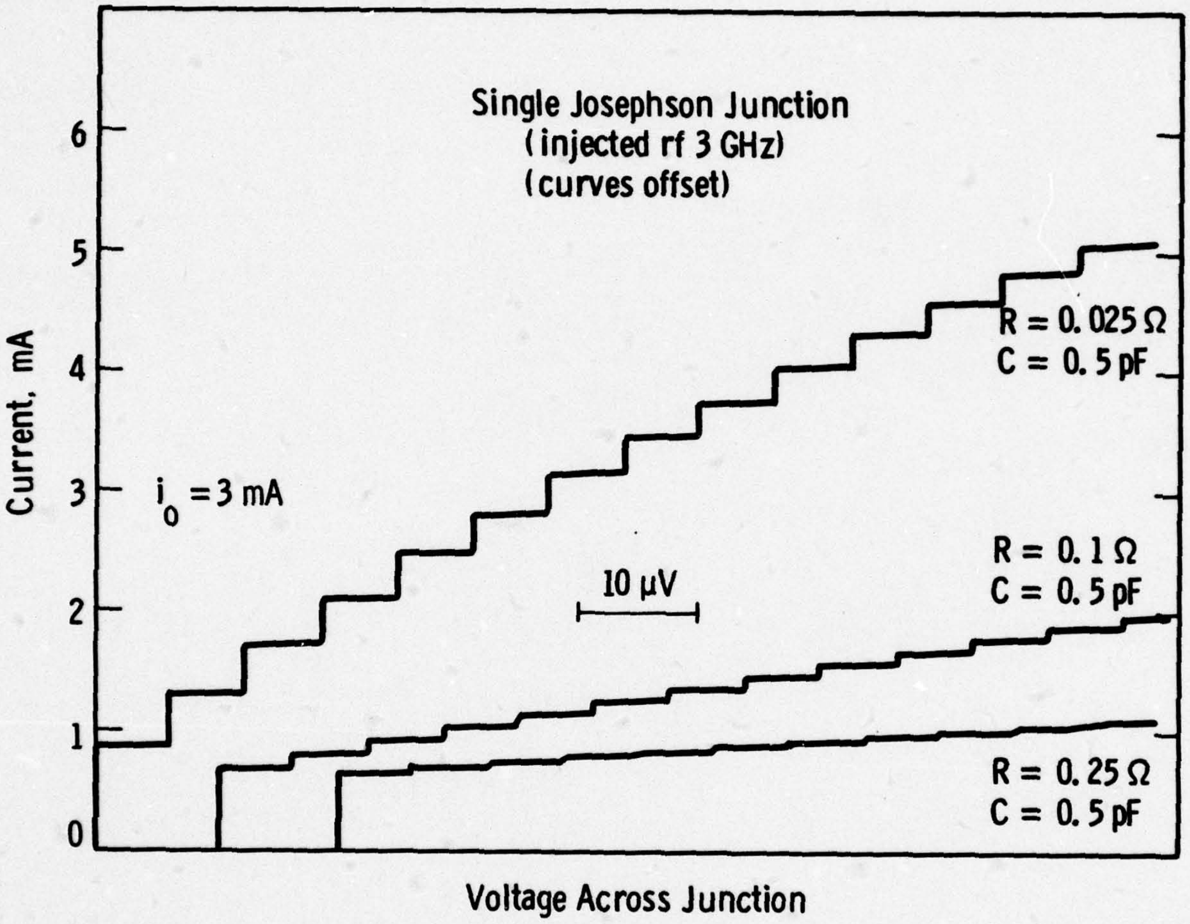


Fig. 6 — The dc I-V characteristics of a single junction with 3 GHz radiation

Curve 693398-B

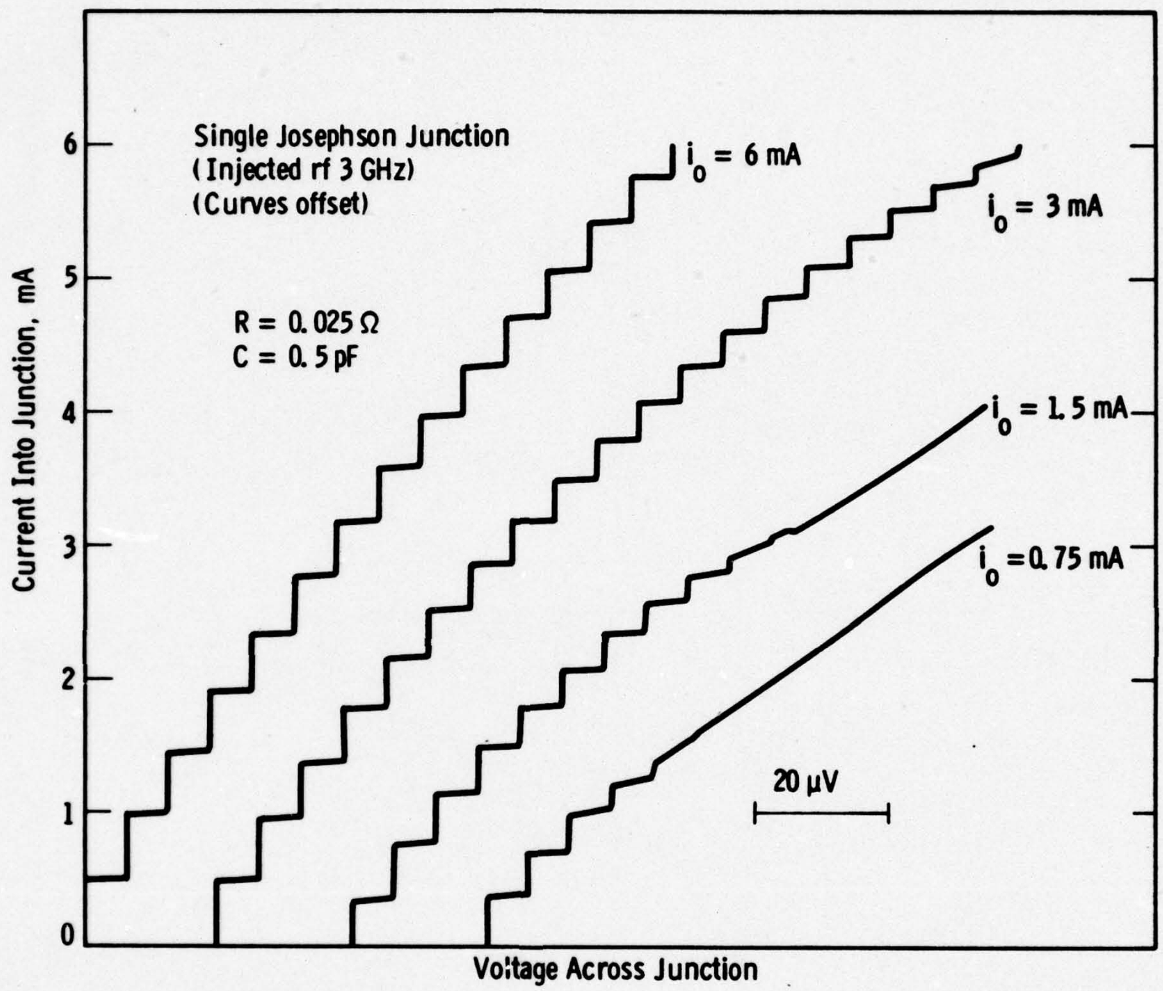


Fig. 7 — The dc I-V characteristics of a single junction with 3 GHz radiation and different critical current values

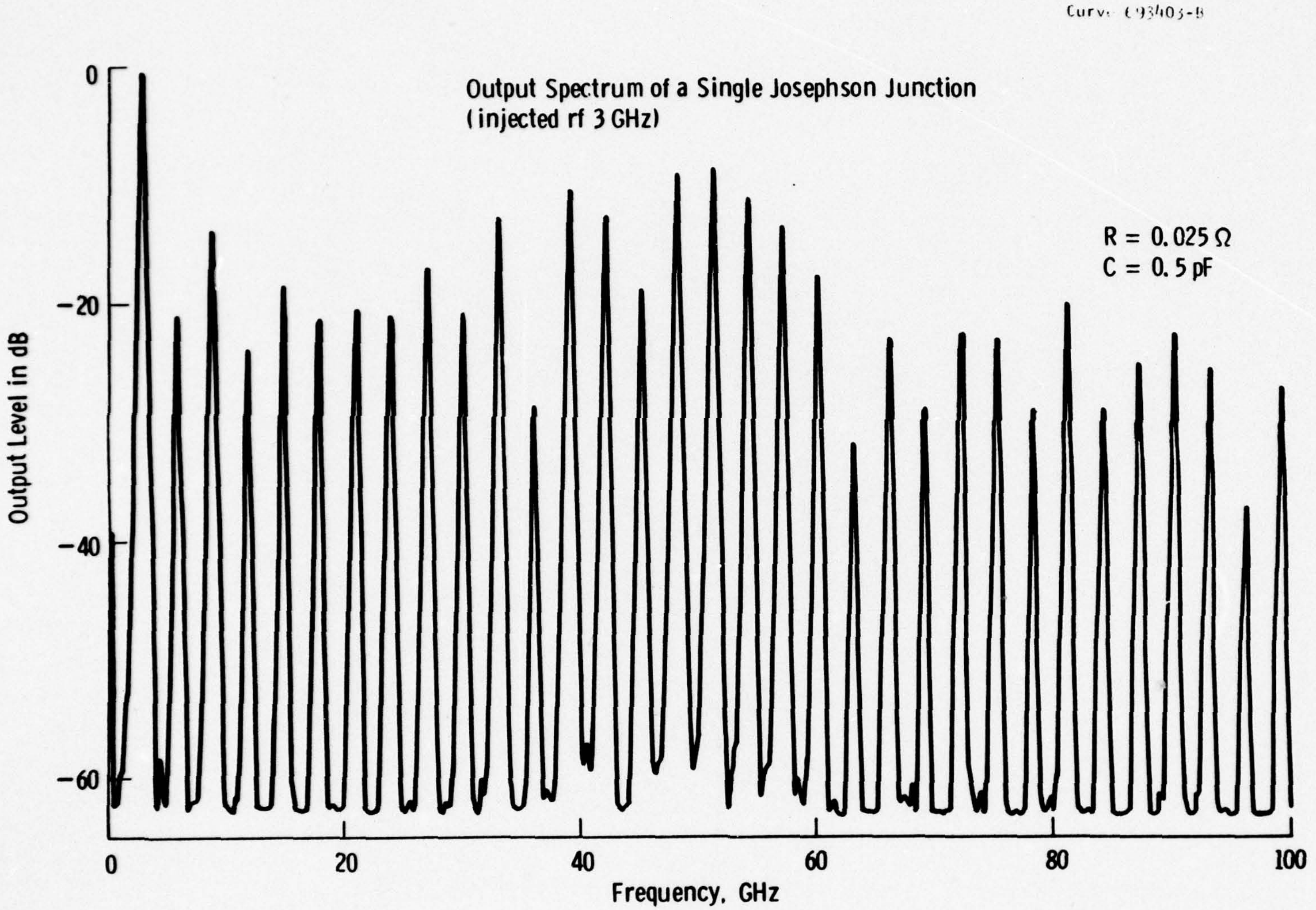


Fig. 8 – The output spectrum of a single junction biased on the 8th step with 3 GHz radiation

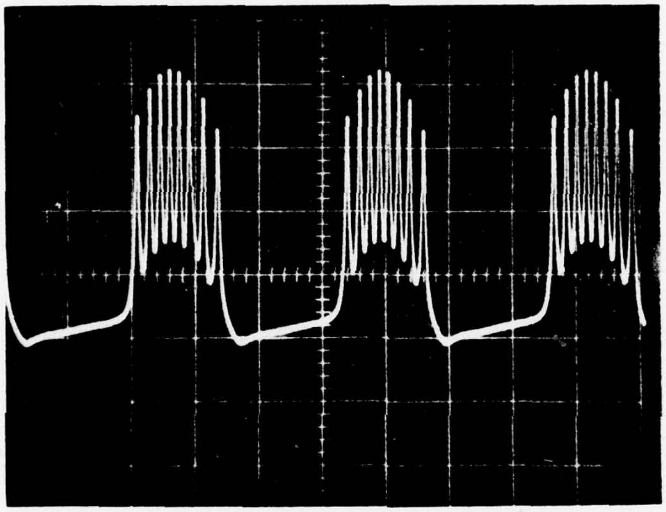
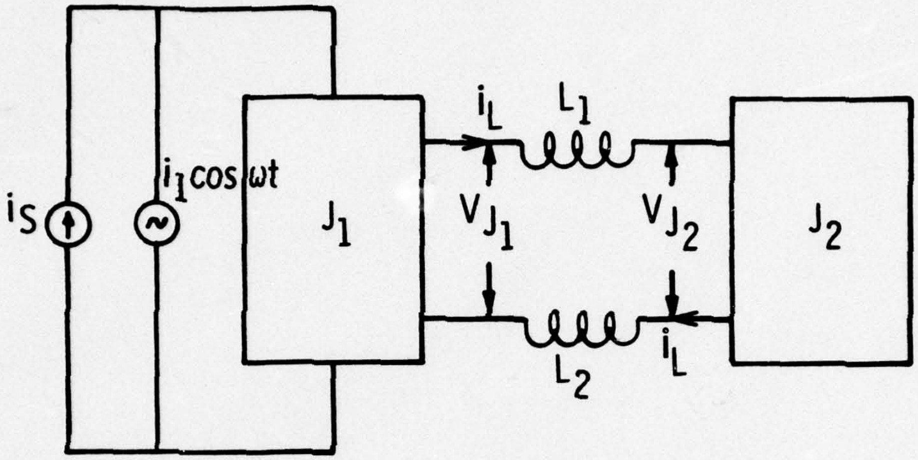


Fig. 9 — Output waveform for a single Josephson junction biased on the 8th step with 3 GHz injected frequency. Vertical axis corresponds to 50 μV/div and the horizontal axis to 0.1 nS/div R = 0.025 Ω, C = 0.5 pF

Dwg. 6419A30



Equations Used in the Analogue Circuit of Figure 11 Are:

$$i_L = \frac{1}{L} \int (V_{J_1} - V_{J_2})\, dt, \qquad \text{(a)}$$

Where $L = L_1 + L_2;$

For the First Junction J_1,

$$i_S + i_1 \cos \omega t = i_{C_1} + i_{R_1} + i_{J_1} + i_L; \qquad \text{(b)}$$

For the Second Junction J_2,

$$i_L = i_{C_2} + i_{R_2} + i_{J_2}, \qquad \text{(c)}$$

Fig. 10 — The equivalent circuit describing two coupled junctions, J_1 and J_2, together with the equations for their analogue simulation

Dwg. 6419A31

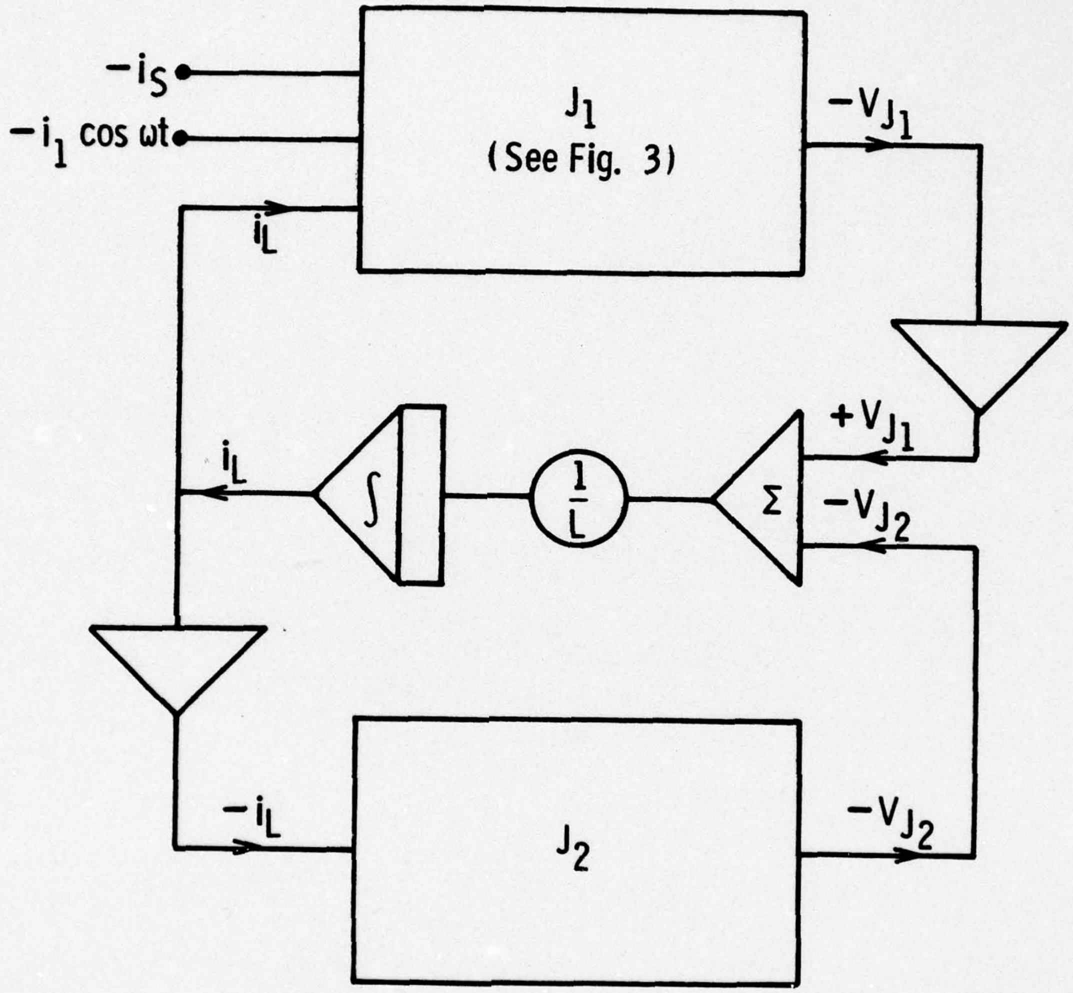


Fig. 11 — The analogue circuit for the solution of the equations given in Fig. 10

Curve 693405-B

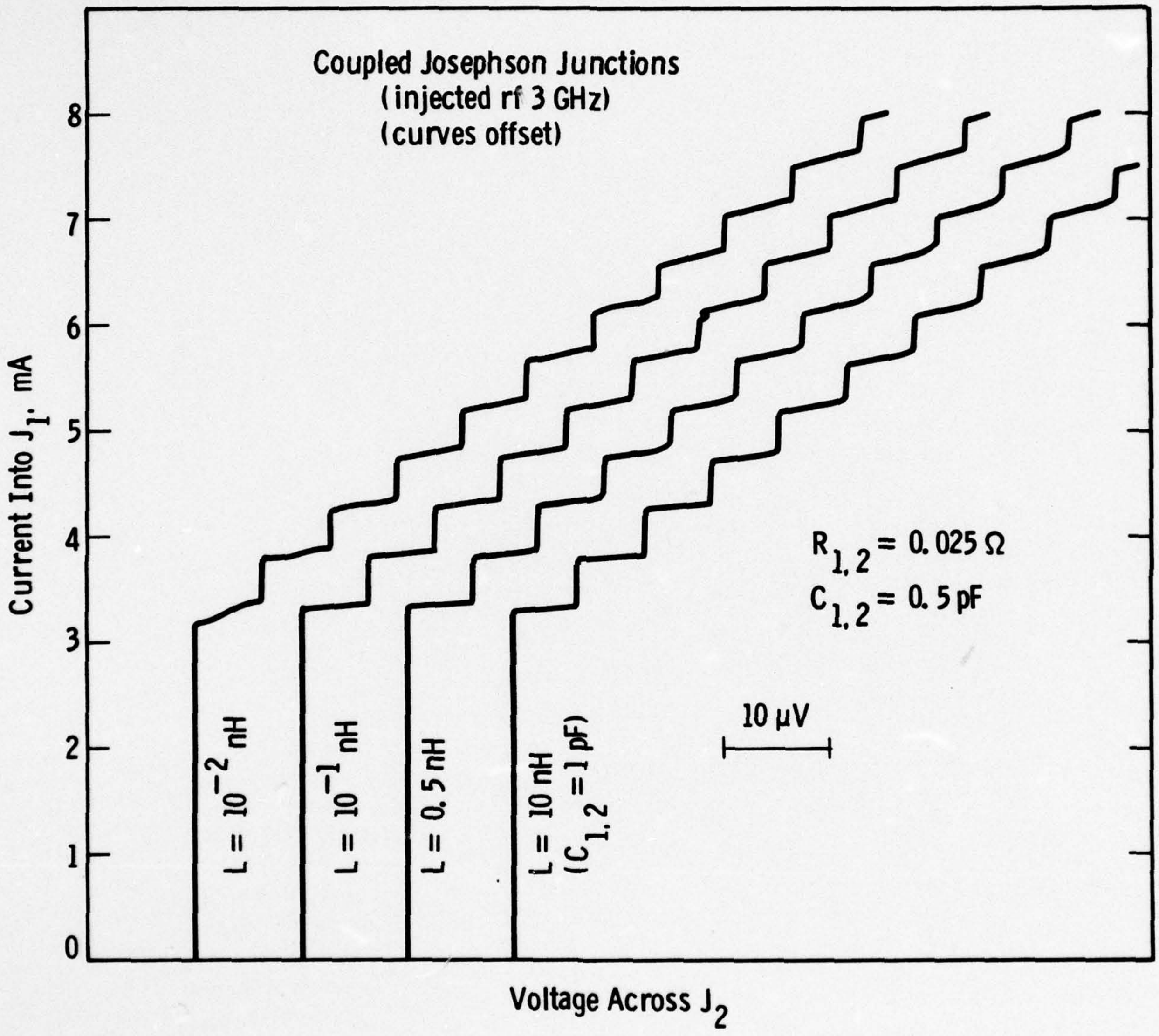


Fig. 12 — The dc I-V bias curves for junction J_2 biased from J_1 as a function of the coupling inductance

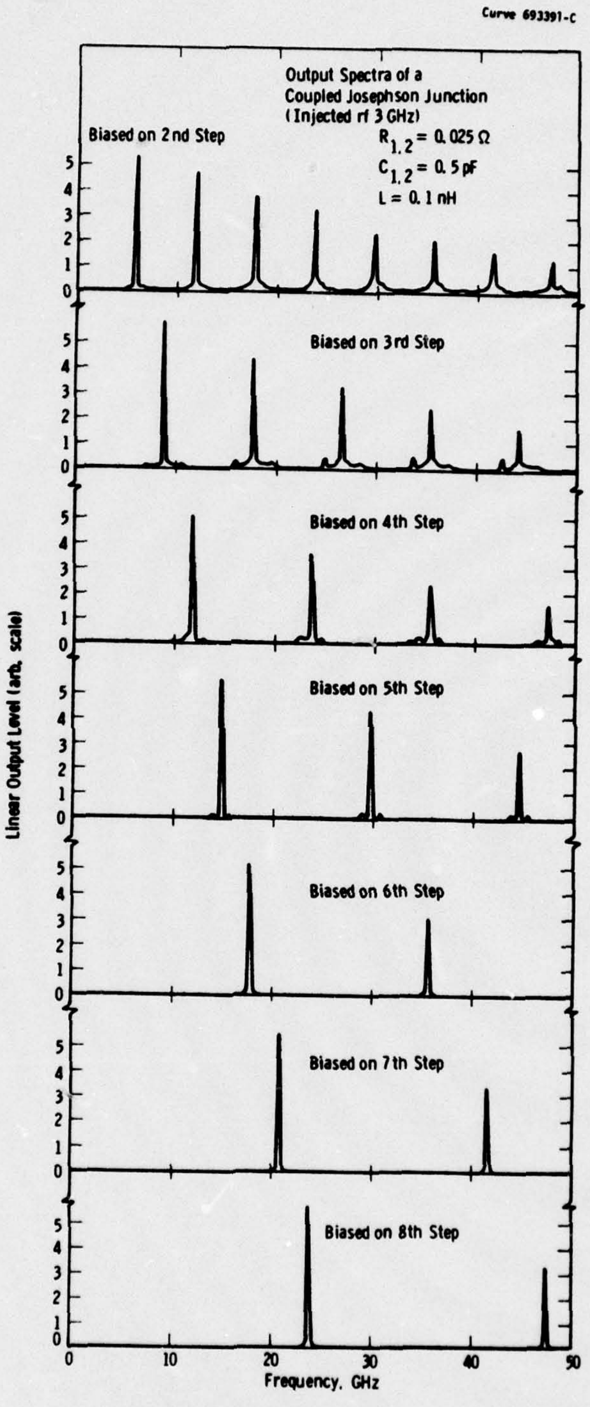


Fig. 13 – A series of spectral responses of junction J_2 when biased on successive steps of junction J_1

Curve 693404-B

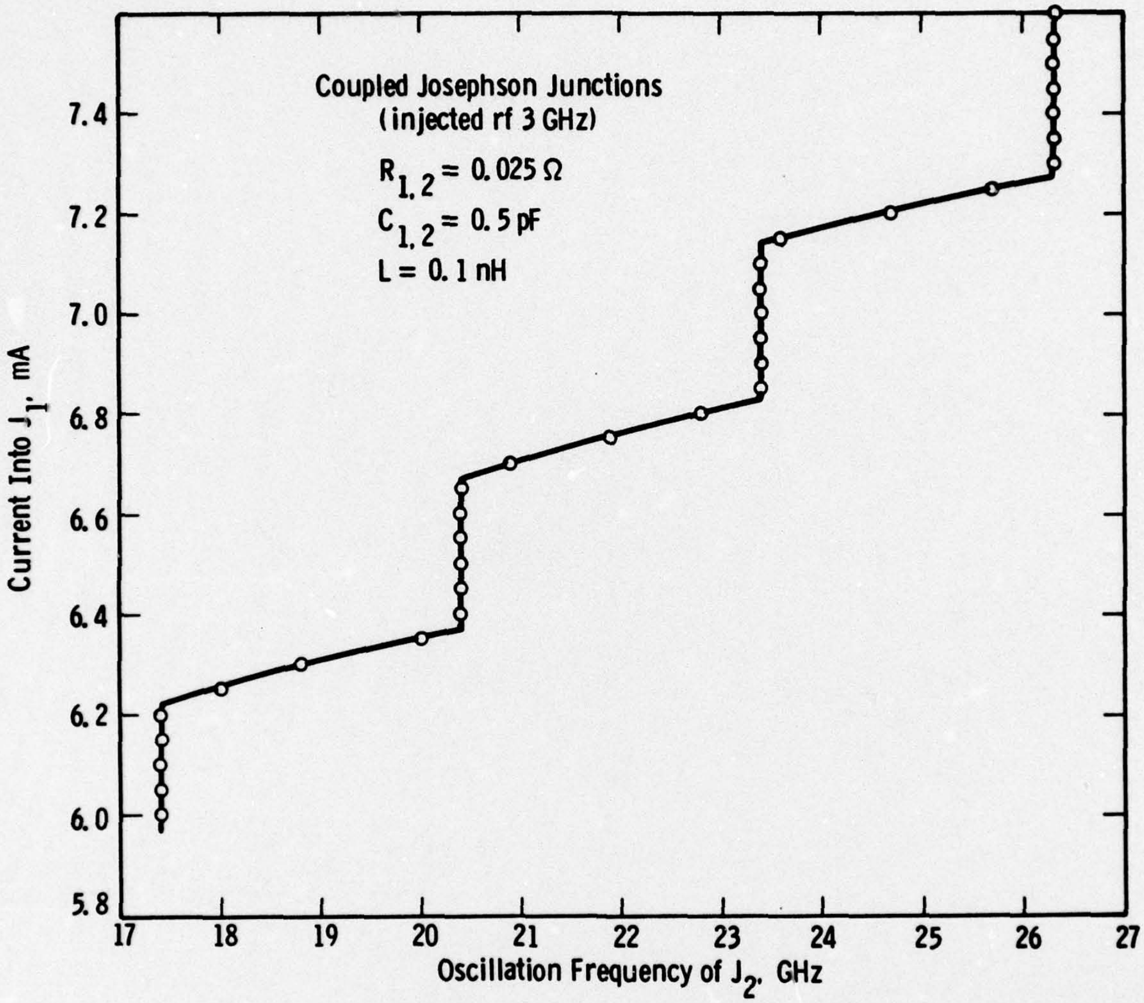


Fig. 14 – The oscillation frequency of junction J_2 as a function of the dc current into J_1

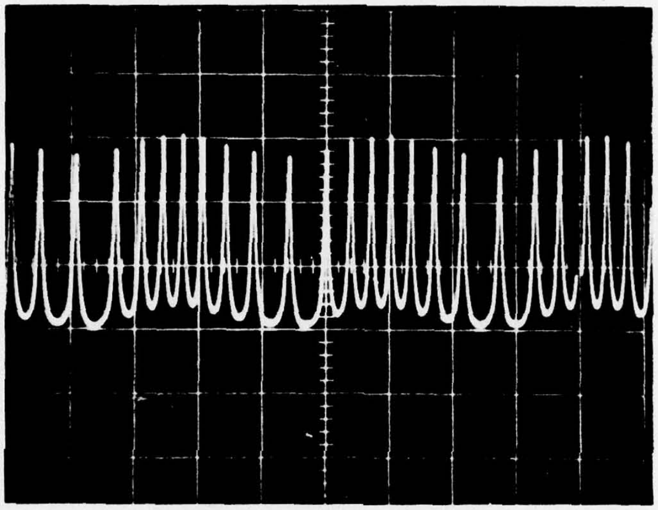


Fig. 15 — Output waveform for a coupled Josephson junction J_2 biased on the 8th step of junction J_1 with 3 GHz injected frequency. Vertical axis corresponds to 50 µV/div and the horizontal axis to 0.1 nS/div $R_{1,2} = 0.025\ \Omega$, $C_{1,2} = 0.5$ pF, $L = 0.01$ nH, $i_s = 6.9$ mA

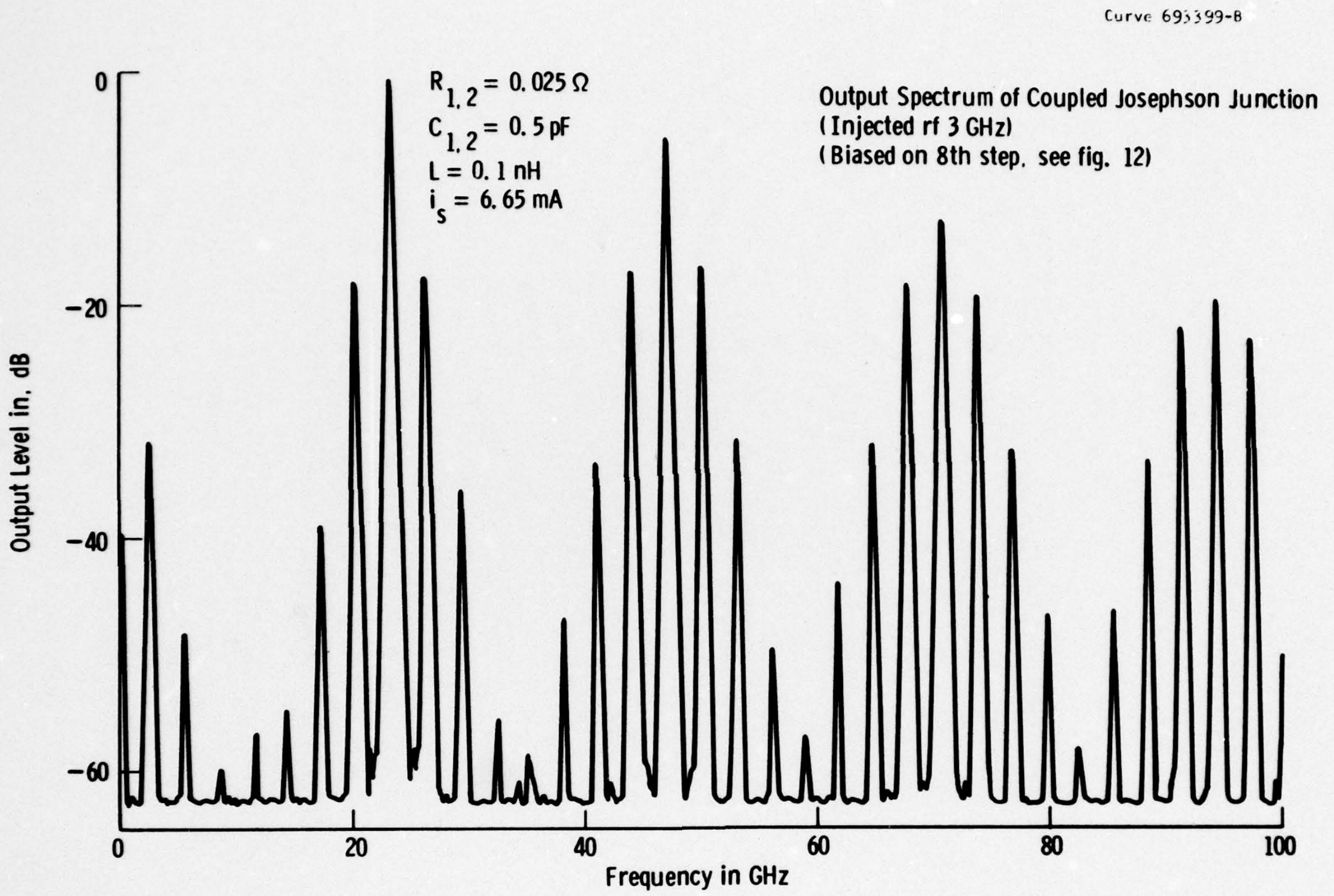
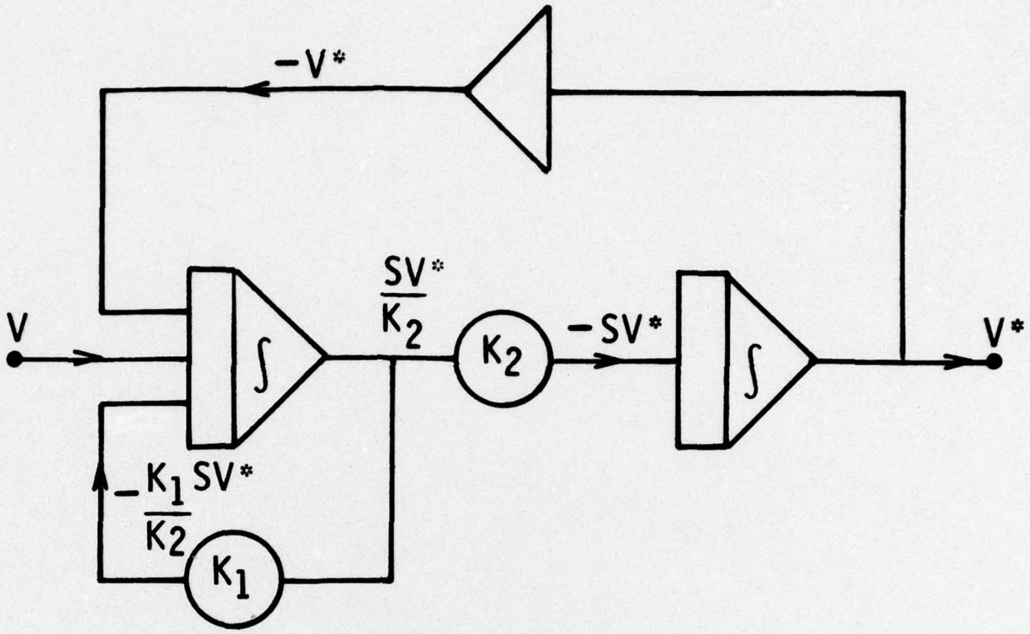


Fig. 16 – The spectral response of junction J_2 when biased on the 8th step of junction J_1

Dwg. 6419A27



Low Pass Filter

$$\frac{V^*}{V} = \left(\frac{S^2}{\omega_n K_2} + \frac{K_1 S}{\omega_n K_2} + 1\right)^{-1}$$

Where S is the Inverse of the Integration Operation

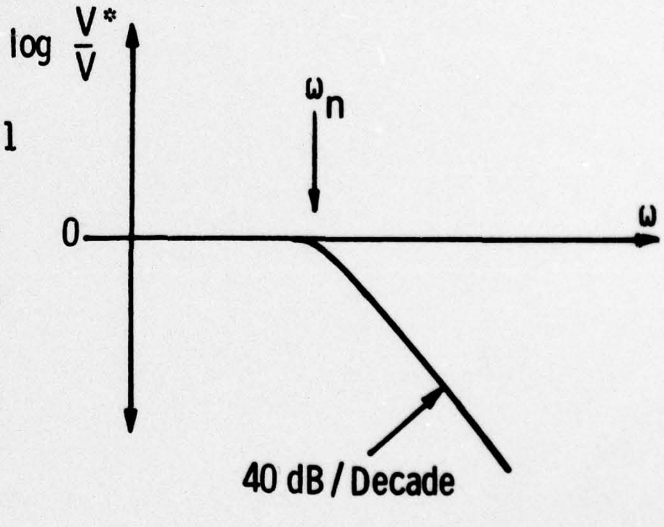


Fig. 17 — The analogue circuit for the low pass filter and its response function as used to isolate junctions J_1 and J_2

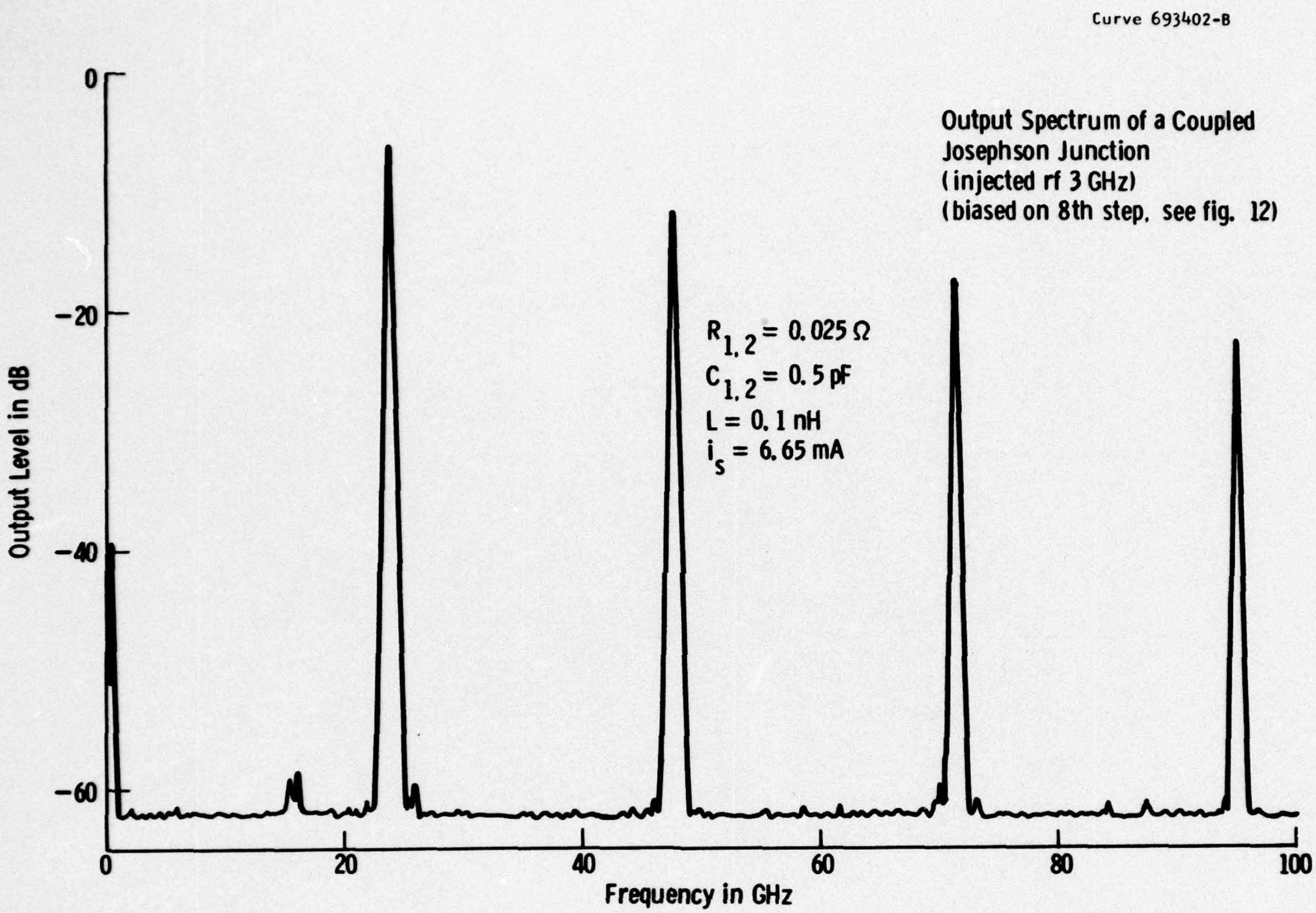
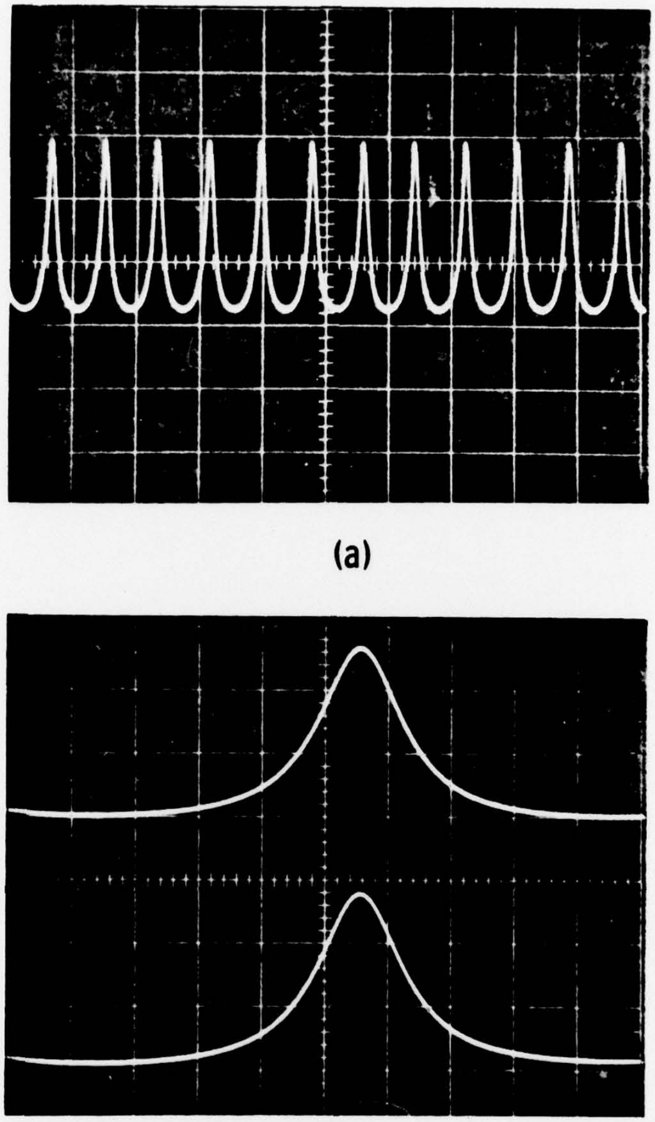


Fig. 18 – The spectral response of junction J_2 when biased on the 8th step of junction J_1 and isolated from it by a low pass filter



(a)

(b)

Fig. 19 — Output waveforms for a coupled Josephson junction J_2 biased on the 8th step of junction J_1 with 3 GHz injected frequency and a low pass filter between J_1 and J_2

(a) $x = 0.05$ nS/div
$y = 50\ \mu V/div$

(b) $x = 0.005$ nS/div
$y_{upper} = 50\ \mu V/div$
$y_{lower} = 2$ mA/div

$R_{1,2} = 0.025\ \Omega$, $C_{1,2} = 0.5$ pF,
$L = 0.01$ nH, $i_s = 6.9$ mA

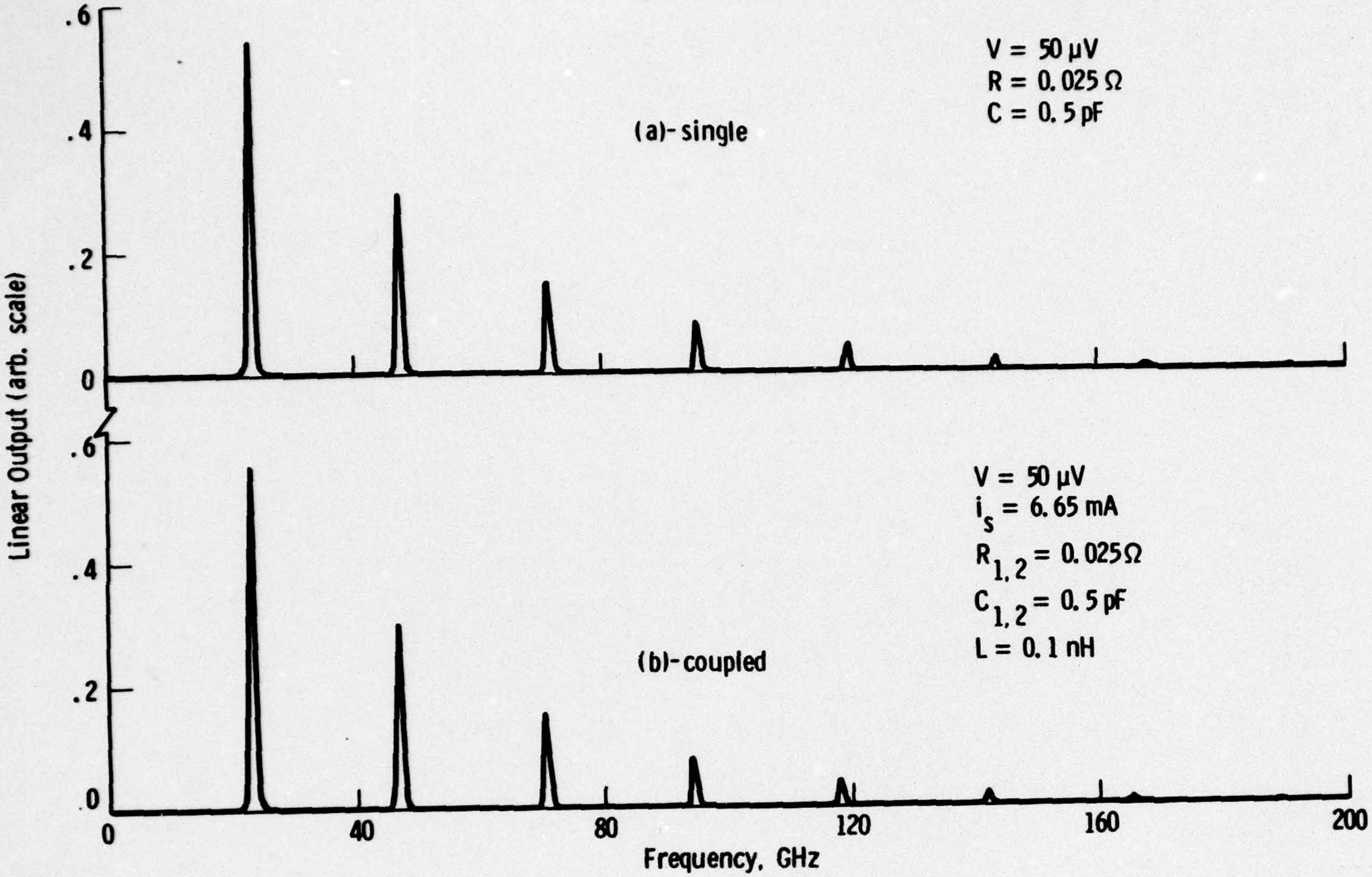


Fig. 20 – Spectral responses of two junctions: (a) single junction, current biased, unirradiated; (b) coupled junction (J_2) biased by a first junction (J_1) on the 8th step and isolated from it by a low pass filter

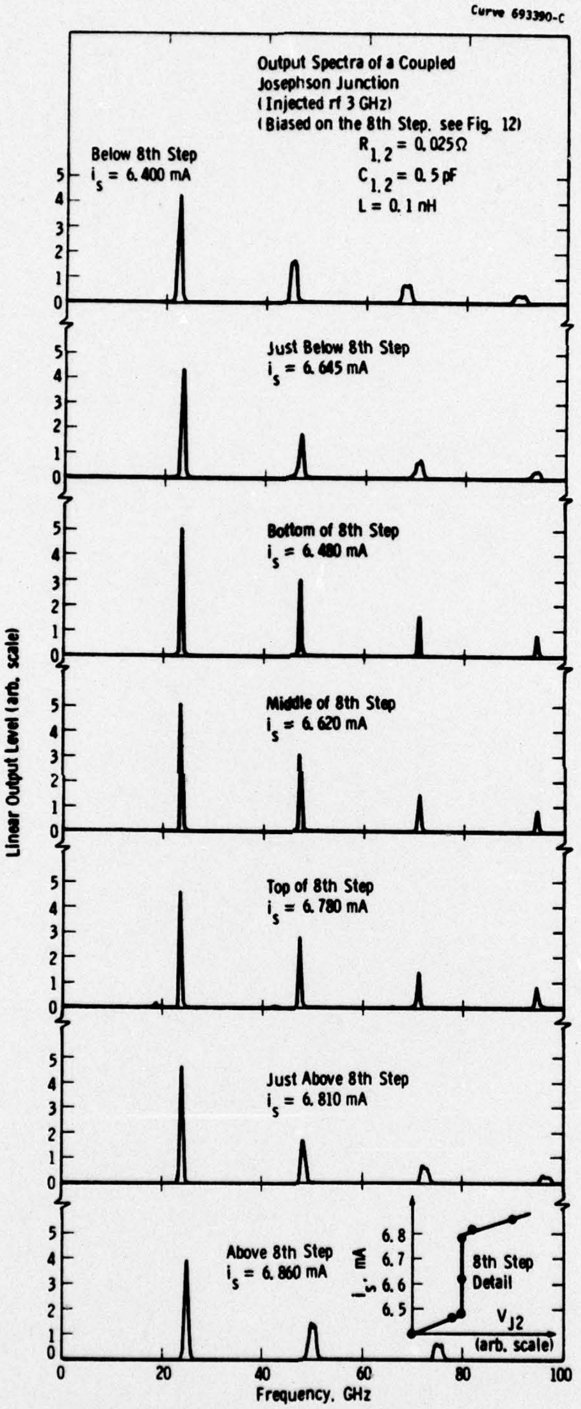


Fig. 21 – A series of spectral responses of junction J_2 when biased on and in the vicinity of the 8th step of junction J_1

Curve 693392-A

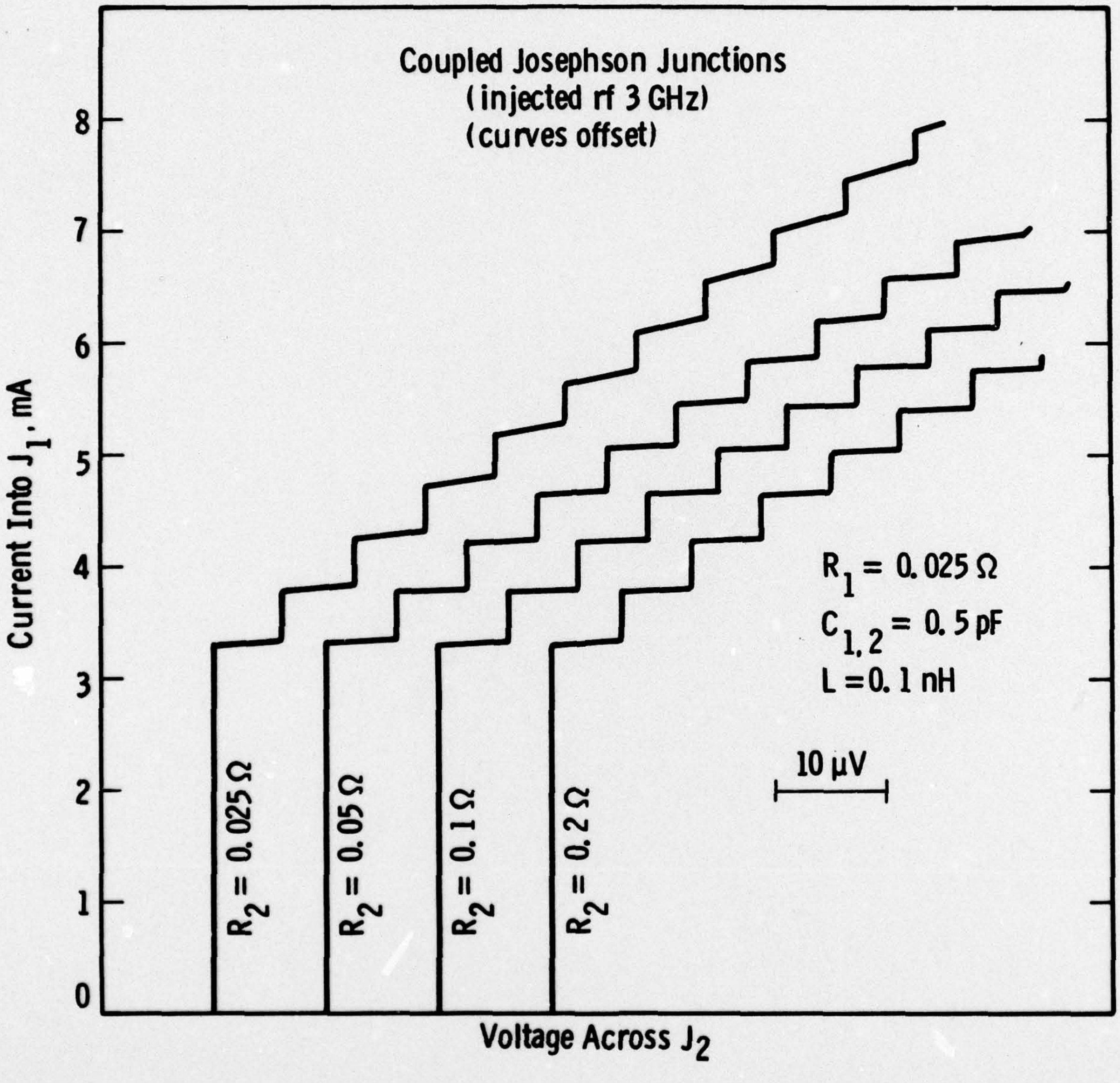
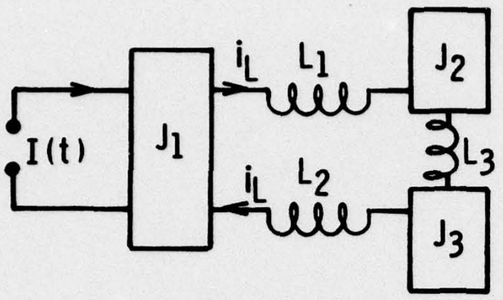


Fig. 22 — The dc I-V curves for junction J_2 biased from J_1 as a function of the resistance R_2 of J_2

Dwg. 6419A32



Equations Used in the Analogue Circuit Below Are:

$$V_{J_1} = V_{J_2} + V_{J_3} + (L_1 + L_2 + L_3)\frac{di_L}{dt} \qquad \text{(a)}$$

and $$i_L = \frac{1}{L}\int (V_{J_1} - V_{J_2} - V_{J_3})\,dt, \qquad \text{(b)}$$

where $L = L_1 + L_2 + L_3$

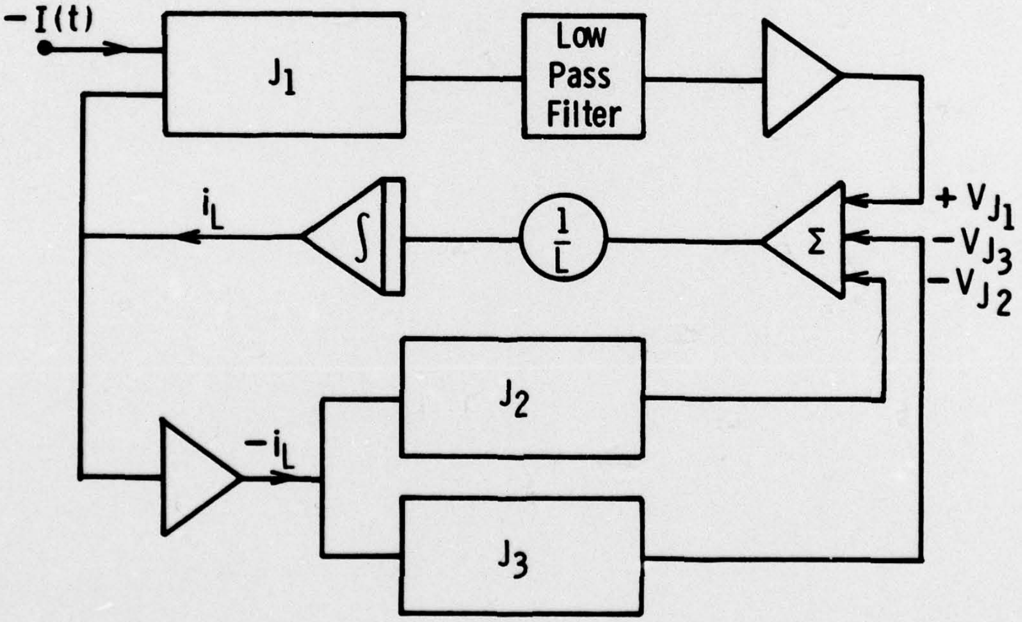


Fig. 23— Block diagram and analogue circuit for the coupled series fed junctions

Curve 693397-B

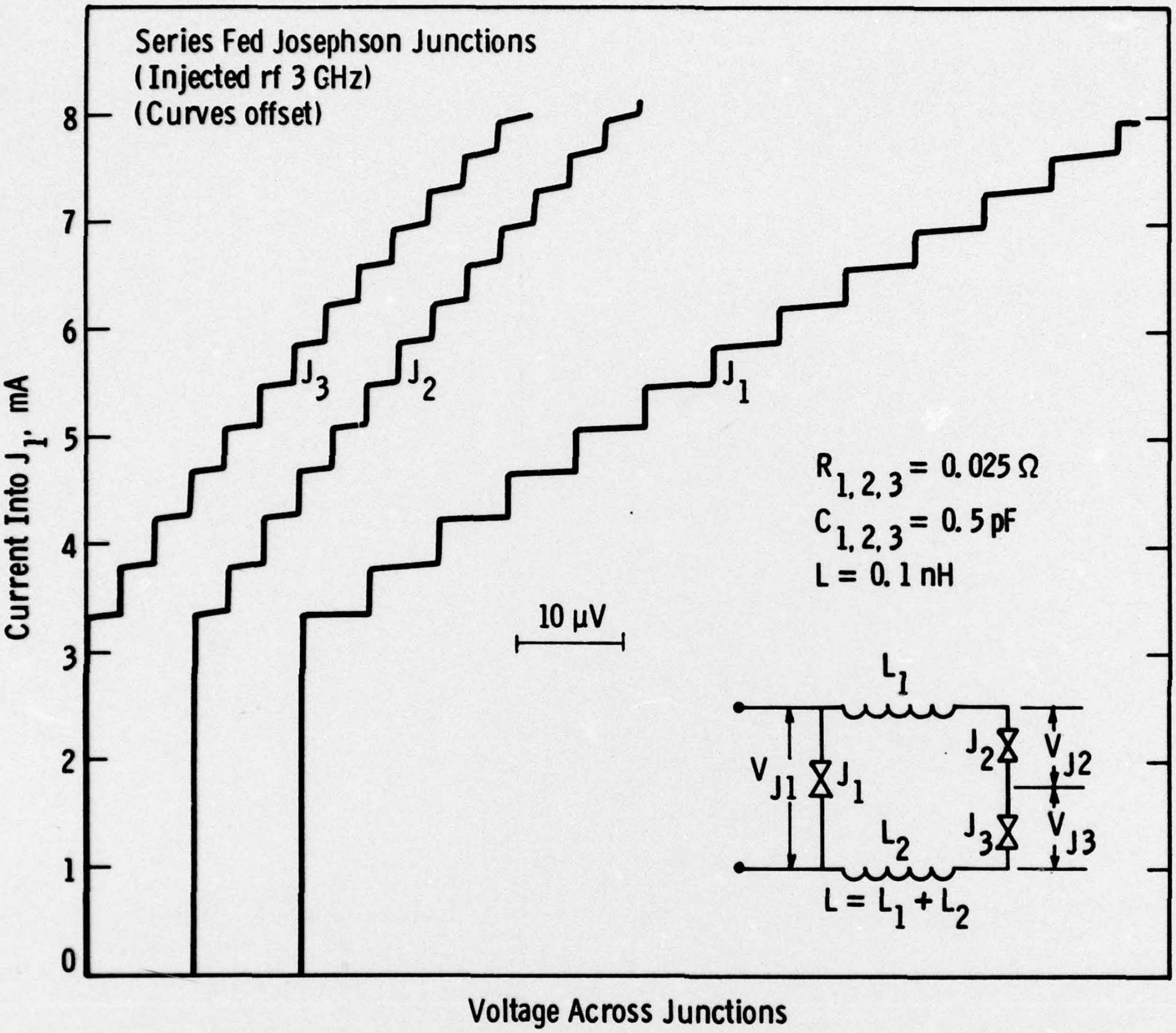


Fig. 24 — The dc I-V curves for two series fed junctions J_2 and J_3 biased by a complementary junction J_1

Curve 693396-B

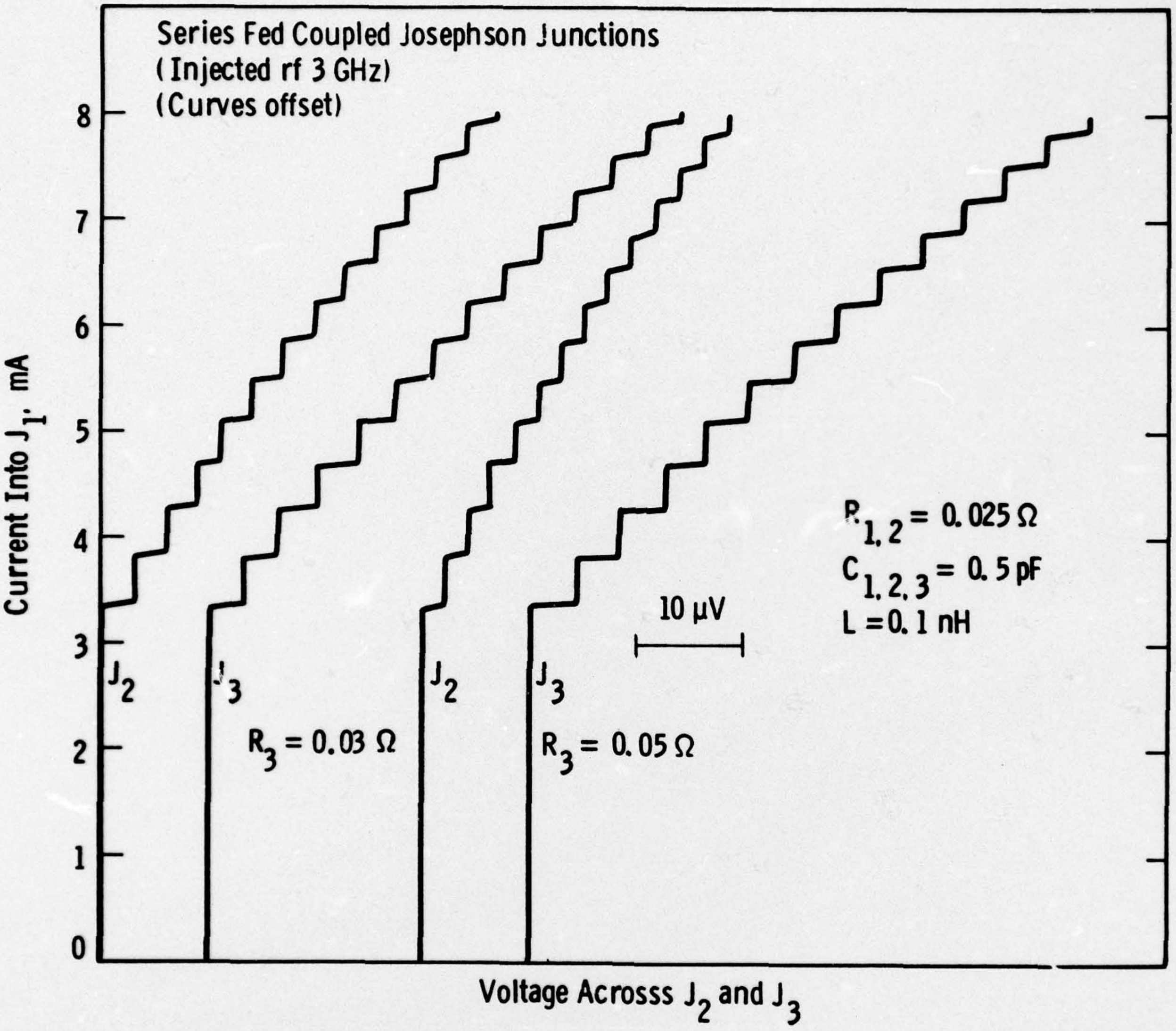


Fig. 25 – The dc I-V curves for the series fed junctions with the junction resistance as a parameter

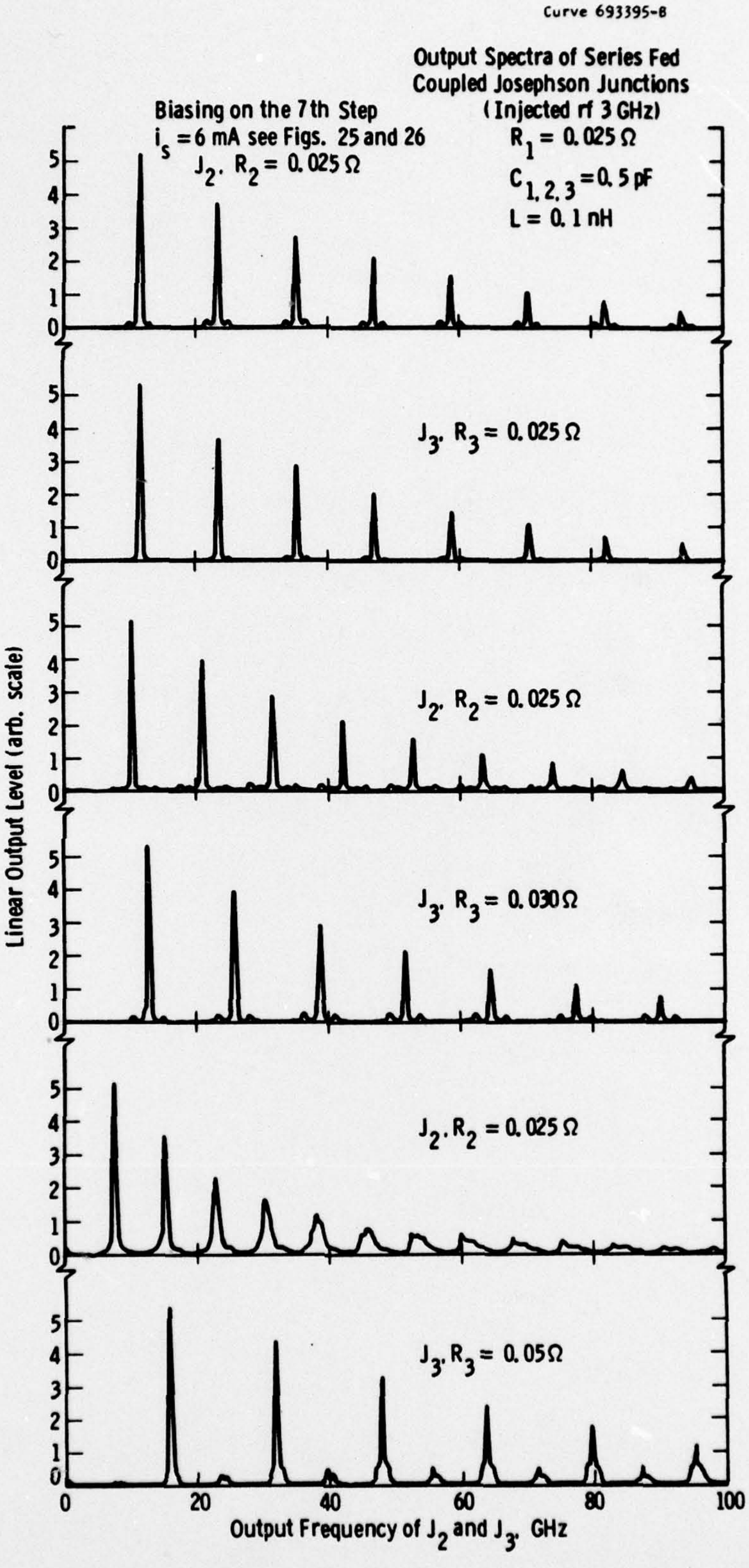


Fig. 26 — A series of spectral responses for the series fed junctions with the junction resistance as a parameter

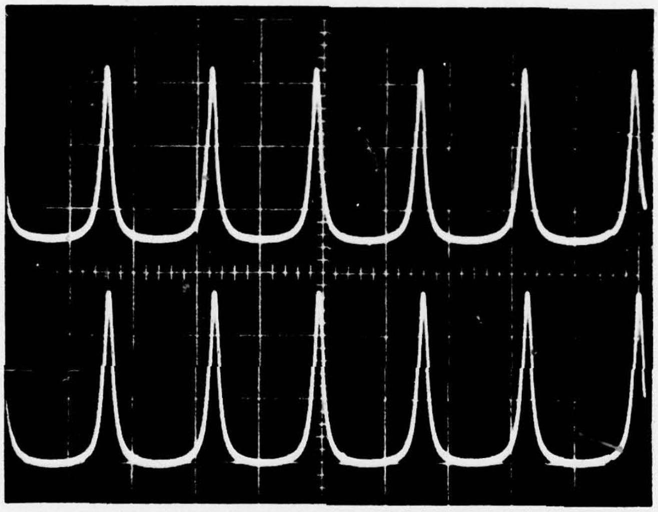


Fig. 27 — Output waveforms as a function of time for two series fed junctions J_2 and J_3 when biased by a complementary junction J_1. Coherent phase response is evident