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# RESOURCE ALLOCATION: VALUE FUNCTIONS AND THE MARGINAL VALUE THEOREM.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper details how to use value functions to determine which method of allocating limited resources among several activities will yield the maximum amount of obtained value to the decision maker. In addition, it introduces a model from the ecological literature for use when the value of investing in new units of an activity decreases as a function of the number of units already invested in.

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# Resource Allocation: Value Functions and the Marginal Value Theorem[1]

Jay J. J. Christensen-Szalanski

In complex environments it is common to encounter problems that require decisions about how to allocate a limited resource to various competing demands such as different projects, programs, activities, or persons, all of which shall be called "activities". The problem then becomes one of adopting a fair and equitable allocation strategy that will permit each desirable activity to function adequately with its share of the resources and that will place extra resources where they will do the most good.

Before selecting an allocation strategy one must first assess the shape of the value function for the various activities. Within current technology the relative value of the activities can be obtained using either multi-attribute utility models [3] [5] or value diagrams [2]. The value of an activity corrected for the value of the resource investment then can be plotted as a function of the amount already invested in the activity. This commonly results in three types of value functions: (1) constant value functions (Figure 1a); (2) a "go-no go" step value function (Figure 1b); or a decreasing value function (Figure 1c and Figure 1d).

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Insert Figure 1 about here

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Activities that yield a constant value per unit of invested resource regardless of the amount invested and the amount of commodity already purchased have a constant value function. They are rare. Usually the value of activities decreases as the amount already invested increases. Some activities, however, may approximate a constant value function within a certain range of the amount already invested. Food to a starving person and money donated to a charity in which "every little bit helps" may be two examples of activities with constant value functions. Decisions and Designs, Inc. [2] has shown that the optimal allocation of a limited resource among several such activities can be found by calculating the value/cost ratio for each activity and then investing in them in order of the decreasing magnitudes of their value/cost ratios until all of the resource is allocated.

Some activities have value to the investor only when they are invested in specific discrete units; the value in terms of heat production is the same for one boiler as it is for 1 3/8 boilers. These activities have step value functions. The optimal allocation of a resource among several step function activities can be calculated using an integer programming technique [4].

For many activities the value of investing in new units decreases as a function of the number already invested in. A health project that consists of building a clinic and funding a team of specialists is such an activity. Its value function might resemble that in Figure 1d. An initial investment of $1,000,000 is needed to build the clinic and this must be spent before any value is obtained from the project. With additional investment the marginal value may temporarily increase because of the increased effectiveness of a group of specialists over that of a few specialists. However, as the clinic becomes sufficiently staffed, further investment will only result in slight increases in value resulting in a decreasing marginal value.

Charnov [1] has provided the optimal resource allocation strategy for those situations in which the marginal value obtained from investing in an activity is a decreasing function of amount invested. Although originally designed to model animals' allocations of time (a resource) in order to maximize their energy intake from foraging, the model can be adapted to human investors allocating money to maximize obtained value. A brief proof follows, although interested readers are recommended to see Charnov [1] for further details.

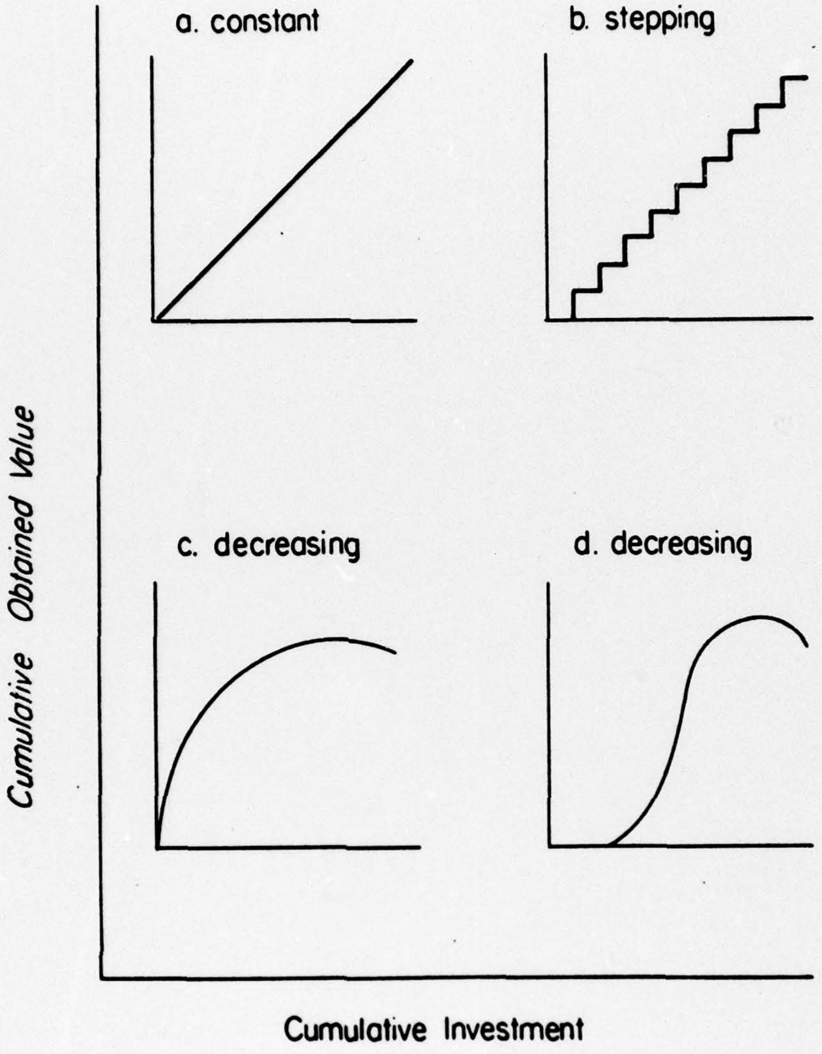

Figure 1. Possible relationships between obtained value and amount of invested resource: (a) constant, (b) stepping, (c) decreasing without initial fixed cost, and (d) decreasing with initial fixed cost.

If we define as follows:

$g_i(I_i)$ = obtained value from investing in activity i corrected for the cost of the investment, and

$I_i$ = the amount of resources invested in activity i,

then the net obtained value (NV) per unit investment for funding all projects i is

$$NV = \frac{\Sigma\, g_i(I_i)}{\Sigma\, (I_i)}. \qquad (1)$$

The optimal value of $I_j$ = the amount of investment per activity of type j and is calculated by setting

$$\frac{\partial\, NV}{\partial\, I_i} = 0 \text{ for all activities simultaneously.} \qquad (2)$$

If the ratio of net obtained value per unit of invested resource = NV* when all $I_j$ are at their optimal value, then

$$NV^* = \frac{\partial\, g_j(I_j)}{\partial\, (I_j)}. \qquad (3)$$

Since we are only considering activities that have a decreasing marginal value, the $\partial\, g_i(I_i)/\partial\, (I_i)$ are always decreasing and there is a unique set of $I_i$ that satisfies equation (3). Charnov shows that this set is a maximum because its Hessian matrix is negative-definite [6].

---

Insert Table 1 about here

---

To illustrate Charnov's model imagine that a granting agency is considering the three program proposals listed in Table 1. Since the total amount of money requested by the three programs is more than the granting agency has funds for, the agency must decide how to allocate those funds in order to

## TABLE 1

### Programs for Resource Allocation Example

| Assessed Value of Completed Program | Program | Cost |
|---|---|---|
| 45 | Job Program | 4,000,000 |
| 40 | Health Program | 3,750,000 |
| 15 | Education Program | 2,000,000 |
| | Total Amount Requested | 9,750,000 |
| | Total Amount Available | 6,750,000 |

obtain the most value for its money. Equation (3) directs the granting agency to fund the projects in such a manner that the marginal value is equal for all funded projects. Figure 2 illustrates a graphical solution for the optimal allocation of money to the three projects with the hypothetical value functions of this situation.

Insert Figure 2 and Table 2 about here

Beginning with an arbitrary, steeply sloped "guide line" (Figure 2a), find points on the value curves that have the same slope as the guide line. These points are located by constructing lines parallel to the guide line and tangent to the value curves. (For S-shaped curves, two points may have the same slope as the guide line. In these cases use the point that is associated with the larger value.) In the example these points are labeled $I_E$, $I_J$, $I_H$. The sum of the investments associated with these points of tangency must equal the amount of money available for funding. If their sum is less than the available funds a new guide line is constructed with a less steep slope, consistent with the intuitive understanding that with a larger budget an investor is able to afford to invest in proportionately less valuable aspects of the activities. The slope of the guide line is then adjusted until the sum of $I_E$, $I_J$, $I_H$ equals the budget available for funding (Figure 2b and Table 2). Projects with small marginal values throughout the range of investments would not be funded unless the agency has a large budget because no points of tangency would exist on their value curves for steeply sloped guide lines.

Charnov's marginal value theorem will yield more accurate solutions to resource allocation problems for activities with decreasing marginal value functions than will other techniques that have been proposed [2] [4]. Its

## TABLE 2

### Resource Allocation Example

| Project | Preliminary Allocation | Optimal Allocation |
|---|---|---|
| Education ($I_E$) | 650,000 | 1,000,000 |
| Jobs ($I_J$) | 1,000,000 | 2,250,000 |
| Health ($I_H$) | 3,000,000 | 3,500,000 |
| Total Requested | 4,650,000 | 6,750,000 |
| Total Available | 6,750,000 | 6,750,000 |

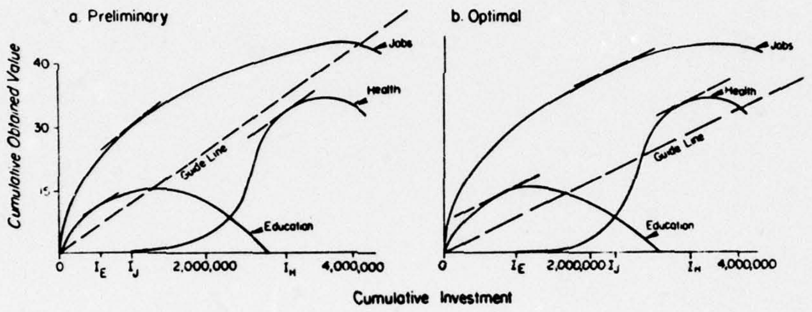

Figure 2. Graphical solution of (a) preliminary and (b) optimal resource allocation among the three hypothetical activities described in Table 1. (Actual investments are in Table 2.)

limiting degree of accuracy will depend upon the shape and quality of the assessed value curves. If one believes that the value curves for the different activities are approximated by straight lines or step functions for the range being considered, the value/cost ratio and integer programming techniques should yield a good approximate solution.

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# FOOTNOTES

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