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STUDIES IN ROTATIONAL FLOWS ESPECIALLY ASYMPTOTIC METHODS.(U)  
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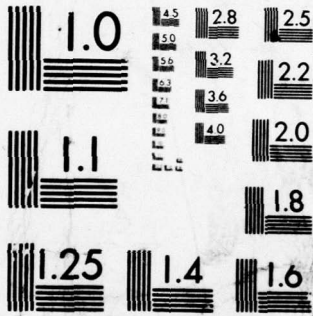
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6 STUDIES IN ROTATIONAL FLOWS ESPECIALLY ASYMPTOTIC METHODS.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Modern asymptotic methods have been used to investigate (i) combustion, in particular monopropellant decomposition in inert and reactive atmospheres, fuel-drop burning, quenching of plane deflagration waves, burning of solids and liquids, separated flames, slowly varying flames (e.g. in ducts), flame stretch, stability, quenching of multi-dimensional flames and chambered diffusion flames; (ii) entry flow and inertial effects in magnetohydrodynamic ducts under strong transverse magnetic fields; (iii) viscous sheets advancing over dry beds. Details of the results are contained in the 28 Technical Reports and 3 Theses.		

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Summary of Research Results

Achievements under the grants are contained in the 28 Technical Reports (#62-89, listed below) and 3 theses (also listed below) which were written during the period 11/1/74-9/15/77. Here we give a summary.

The dominant theme of the research since Professor J. D. Buckmaster introduced us to combustion in the summer of '74 has been activation-energy asymptotics. We have since applied the method to monopropellant decomposition in inert and reactive atmospheres, fuel-drop burning, quenching of plane deflagration waves, burning of solids and liquids, separated flames, slowly varying flames (in particular in ducts), flame stretch, stability, quenching of multi-dimensional flames and chambered diffusion flames. In every case explicit formulas are obtained for quantities of interest such as ignition and extinction conditions. The work has put a large part of combustion theory on the sound analytical footing it has not enjoyed up till now.

T.R.'s 62, 63, 65, 72, 84 and 86 are concerned with the burning of fuel droplets, a central problem in combustion involving diffusion flames. The first five reports determine, for the first time analytically, the whole steady response curve whether it is monotonic, S- or C-shaped. Unfortunately we later found that the basic questions treated in T.R.'s 62, 63 and 65 were being simultaneously considered by Law using results of Liñán on the counter-flow diffusion flame, so these reports were not publishable. However, certain important aspects were missed by Law, and these appear in T.R.'s 72 and 84 (which were published). T.R. 86 is concerned with the stability of such steady burning and shows that, on the long time scale mentioned below, the unstable branches of the S-response depend on whether the Lewis number of the fuel is less or greater than unity.

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The corresponding problem for premixed flames, namely the decompositional burning of a monopropellant droplet, is dealt with in T.R.'s 64, 66, 67 and 68. They present a complete account for all parameter values, provide practical formulas for ignition and extinction, and synthesize several isolated results which had up till then been very puzzling. In particular, T.R. 68 shows that the analysis in T.R.'s 64 and 66 is valid even onto the upper branches of S- and C-shaped response curves, not just on the presumably unstable middle and lower branches (respectively).

The series of Technical Reports 70, 76, 78 and 89 is concerned with various aspects of plane flame theory, into which activation-energy asymptotics were first introduced by Bush and Fendell: sequential flames, general perturbations and chambered diffusion flames are considered. Apart from putting the theory on a sound analytic footing, and thereby drawing together scattered results, two new questions are tackled: multiple reactions, which have hitherto hardly been mentioned (much less treated) and slow variations, which appear to be an all-pervading aspect of actual flames. A new sort of activation-energy asymptotics was introduced here, namely changes on a correspondingly long time scale.

The idea of slow temporal variations was also extended to spatial variations in Technical Report 78, which later provided the key to a rational analysis of flame stretch, a concept widely thought to determine quenching conditions. Technical Report 87 shows that stretch is not the only factor and that variations in thickness of the flame play a comparable role. In fact the flame speed is shown to be simply related to changes in volume of a flame element, rather than area, and excessive dilatation or compression can, under certain circumstances, destroy the flame.

Technical Report 88 follows up the quenching question by studying, for the first time analytically, steady two-dimensional premixed flames in the proximity of a plane boundary capable of removing heat from the combustion field. One very prominent combustion scientist has rated such a step as the first advance in this difficult and important subject in a quarter century.

T.R.'s 73, 75 and 79 treat the decompositional burning of the gases from solid pyrolysis. Again a synthesis of existing results was achieved, previous misconceptions exposed and unsuspected phenomena unearthed, in particular the concept of effective extinction (as opposed to true) and the occurrence of multiple responses. T.R. 73 was selected for the international meeting listed while we were still completely unknown in combustion circles, which gives some indication of the importance of the results.

Further recognition has come from invitations to address meetings on combustion. In particular, T.R. 71 was given at a joint meeting of the Society of Engineering Science and the Polish Academy of Sciences while T.R. 81 was written expressly for the '76 Army Mathematicians Conference. These two reports give simple introductions to activation-energy asymptotics, both the original type suggested by Williams and our later innovation.

Finally we come to T.R.'s 77, 80, 82 and 85, which form the beginning of a monograph on laminar combustion by J. D. Buckmaster and the present author. We believe that a connected account can be based on activation-energy asymptotics and indeed that such a rational approach will at last place the subject on a solid analytical foundation. Our work will figure prominently and indications are that there will be a demand for such a monograph when it is completed (hopefully next year). News of the project has led to the first

two chapters (T.R.'s 77 and 80) being published separately. We already have two offers of publication for the monograph and several other expressions of interest.

There has been a modest continuation of the extensive series on MHD duct flow in strong transverse magnetic fields written under previous ARO grants. Those results of most interest to the hydraulicist, namely estimates of the distance from entry before an MHD flow becomes fully developed, are summarized in T.R. 69. Again we emphasize that experimental work, mainly done in the U.S.S.R. but also in the U.K., has provided extraordinary confirmation of our predictions.

The series was based on the so-called inertialess approximation. We have also made a long investigation of the very difficult question of inertial effects, and the results are contained in Kapila's thesis; many of them appear in T.R. 83.

T.R. 74 is the only one on ordinary hydrodynamics. While it treats four important problems concerning the unsteady creeping motion of a thin sheet of viscous liquid as it advances over a gently sloping dry bed, it also raises a host of other questions. Unfortunately there has been no time to follow them up.

At various times the grants have supported the author (who was principal investigator), Prof. J. D. Buckmaster, Drs. A. K. Kapila and M. Matalon, Miss K. A. Ames, Messers. R. O. Ayeni, G. C. Lu, M. J. Normandia and A. K. Sen.

Kapila, Wilson and Matalon earned Ph.D.'s during the period of the grants and their theses are listed below. With the exception of parts of Kapila's and Wilson's, essentially all of the results in these theses have been published through Technical Reports.

Technical Reports

62. J. D. Buckmaster: Weak burning of a liquid fuel drop.
63. J. D. Buckmaster: Moderate burning of a liquid fuel drop.
64. G. S. S. Ludford & D. W. Yannitell: Monopropellant decomposition in a hot inert atmosphere. Part 2: maximum zone temperature. Combustion Science and Technology, 14 (1976), 133 (under the title "The decomposition of a cold monopropellant in an inert atmosphere").
65. J. D. Buckmaster: Strong burning of a liquid fuel drop.
66. G. S. S. Ludford, D. W. Yannitell & J. D. Buckmaster: Monopropellant decomposition in a cold inert atmosphere. Combustion Science and Technology, 14 (1976), 125 (under the title "The decomposition of a hot monopropellant in an inert atmosphere").
67. J. Buckmaster, A. K. Kapila & G. S. S. Ludford: Monopropellant decomposition in a reactive atmosphere. Combustion Science and Technology, 16 (1977), 254.
68. A. K. Kapila, G. S. S. Ludford & J. D. Buckmaster: Ignition and extinction of a monopropellant droplet in an inert atmosphere. Combustion and Flame, 25 (1975), 361. Presented at the SIAM National Meeting, Troy (N.Y.), 1975.
69. G. S. S. Ludford & J. S. Walker: On establishing fully developed duct flow in strong magnetic fields. Proceedings of the Bat-Sheva International Seminar on MHD-Flows and Turbulence, Beersheva 1975, p. 7. (Published by John Wiley and Israel Universities Press, 1976.) Invited address.
70. J. Buckmaster: The quenching of deflagration waves. Combustion and Flame, 26 (1976), p. 151.

71. G. S. S. Ludford: Combustion for large activation energy. Letters in Applied and Engineering Sciences, 4 (1976), 49. Proceedings of the Symposium on Physical Fields in Material Media, Warsaw (Poland), 1975. Invited address. Also presented by invitation at the Annual Meeting of the Australian Mathematical Society, Perth (Australia), 1976.
72. J. D. Buckmaster: Combustion of a liquid fuel drop. Letters in Engineering and Applied Sciences, 3 (1975), 365.
73. J. D. Buckmaster, A. K. Kapila & G. S. S. Ludford: Linear condensate deflagration for large activation energy. Acta Astronautica, 3 (1976), 593. Proceedings of the Fifth International Colloquium on Gasdynamics of Explosions and Reactive Systems, Bourges (France), 1975.
74. J. Buckmaster: Viscous sheets advancing over dry beds. Journal of Fluid Mechanics, 81 (1977), 735. Presented at the 14th International Congress of Theoretical and Applied Mechanics, Delft (Holland), 1976.
75. A. K. Kapila & G. S. S. Ludford: Deflagration of gasifying condensates with distributed heat loss. Acta Astronautica, 4 (1977), 279.
76. A. K. Kapila & G. S. S. Ludford: Two-step sequential reactions for large activation energies. Combustion & Flame, 29 (1977), 167. Presented at the 14th International Congress of Theoretical and Applied Mechanics, Delft (Holland), 1976.
77. G. S. S. Ludford: Mathematical theory of laminar combustion. I. Governing equations. Journal de Mécanique, 16 (1977), 501 (under the title "Combustion: basic equations and peculiar asymptotics"). Invited paper.
78. J. Buckmaster: Slowly varying laminar flames. Combustion and Flame, 28 (1977), 225.

79. J. Buckmaster: Solid deflagration with varying pressure. Submitted for publication.
80. G. S. S. Ludford: Mathematical theory of laminar combustion. II. The premixed plane flame. *Journal de Mécanique*, 16 (1977), 523 (under the title "The premixed plane flame"). Invited paper.
81. J. Buckmaster & G. S. S. Ludford: Activation energy asymptotics and unsteady flames. *Transactions of the Twenty-Second Conference of Army Mathematicians, Watervliet (N.Y.) 1976*, p. 183 (ARO Report 77-1, 1977). Invited address.
82. G. S. S. Ludford: Mathematical theory of laminar combustion. III. The flame of a linear condensate.
83. A. K. Kapila & G. S. S. Ludford: MHD with inertia: flow over blunt obstacles in channels. *International Journal of Engineering Science*, 15 (1977), 465.
84. J. Buckmaster & M. Normandia: Near asphyxiated liquid fuel drop burning. *Combustion and Flame*, 29 (1977), 277.
85. G. S. S. Ludford: Mathematical theory of laminar combustion. IV. Premixed cylindrical and spherical flames.
86. J. Buckmaster: The unsteady burning of a near-adiabatic fuel drop. Submitted for publication.
87. J. Buckmaster: Some remarks on flame-stretch. Submitted for publication.
88. J. Buckmaster: The effects of a quenching surface on a premixed flame. Submitted for publication.

89. M. Matalon, G. S. S. Ludford & J. Buckmaster: Diffusion flames in a chamber. Presented at the Sixth International Colloquium on Gasdynamics of Explosions and Reactive Systems, Stockholm (Sweden), 1977. Proceedings to appear in Acta Astronautica.

Ph.D. Theses

- Ashwani Kumar Kapila: Entry flow in a channel and MHD flow in a plane channel under strong transverse magnetic fields, vi + 156 pp., June 1975.
- Sharon Slubowski Wilson: Subcharacteristic reversal, vii + 153 pp., August 1975.
- Moshe Matalon: Diffusion flames in a chamber for large activation energy, vii + 115 pp., August 1977.