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ON COMMITTEE DECISION MAKING: A GAME-THEORETICAL APPROACH. (U)
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ON COMMITTEE DECISION MAKING:
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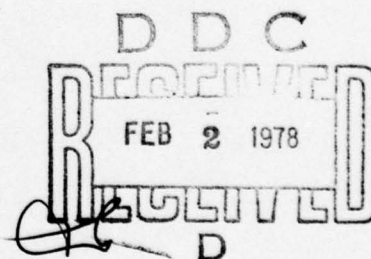
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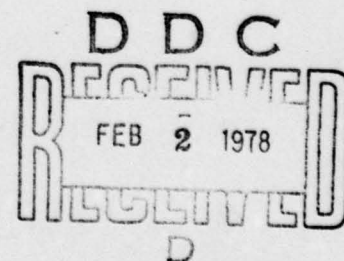
ABSTRACT

In this paper, we study the committee decision making process using game theory. The committee is modeled as a game. A new solution concept called the one-core is introduced and studied. The concept of a bargaining set, first introduced by Aumann and Maschler [4] in the context of games with side payments, is defined with appropriate modifications for committee games. These solution concepts are compared with two other well known solution concepts - the core and the Condorcet solution.

AMS(MOS) Subject Classifications: 90D12, 90D40, 90D45.

Key Words: Core, Condorcet solution, Bargaining set, One-core, Simple games.

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SIGNIFICANCE AND EXPLANATION

Committee decision making is an important aspect of most institutions and has been the subject of study in Political Science, Economics, Social choice theory and other social sciences. Yet, inspite of extensive studies, the committee decision process is far from being completely understood.

By a committee we mean any group of people who have to pick one option from a given set of alternatives. A well defined voting rule is specified by which the committee arrives at a decision. We assume that each member of the committee has a preference relation on the set of all alternatives.

A fundamental problem arising here is to characterize the option(s) that the committee can be expected to choose assuming "rational behavior" by the members. In this paper, we use game theory to analyse the decision making process. A new solution concept is defined and studied along with some well known solution concepts in game theory. Our models are primarily normative in nature, and will probably be predictive in the case that the committee members have had extensive experience in committee decision making. Our primary concern however is not so much to predict outcomes as it is to increase the understanding of the committee decision making process.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ON COMMITTEE DECISION MAKING:

A GAME-THEORETICAL APPROACH

Prakash P. Shenoy

1. Introduction

In this paper, we study the committee decision making process using game theory.

By a committee we will mean any finite group of persons who have to pick one option from a given set of outcomes. The members of the committee will be situated in one room. This assumption is important for our model. We are primarily concerned with small committees that arrive at a decision after lengthy deliberations. In this respect we differ fundamentally from the theory of elections where the decision makers (the voters) are numerous and spread out extensively.

A well defined rule is specified by which the committee will arrive at a decision. The rules are designed such that the decision of the committee will consist of a unique outcome. We do not restrict ourselves to the straight majority rule only.

The set of alternatives (outcomes) may be finite or infinite. We will assume that there are at least two alternatives. One of the alternatives will always be a 'status quo' outcome and will be denoted by a_0 . a_0 will be the decision of the committee if it cannot agree on any other outcome or if it specifically picks a_0 to be the final decision. No agenda is specified. Any member is allowed to suggest any alternative at any time for consideration by the committee.

Each member of the committee has a utility function defined on the set of all outcomes. Utility is assumed to be nontransferable and interpersonal comparison of utilities is assumed to be meaningless. For each members, his utility function induces a preference relation on the set of outcomes. We will assume that each member's preference relation is a weak order. (All undefined terms here are defined formally in Section 2.)

Each member of the committee is assumed to be a "rational player" in the sense of von Neumann and Morgenstern [17]. I.e. roughly speaking, each member is attempting to

maximize his utility. Furthermore, we assume that the members make decisions under conditions of perfect information. I.e. each member is perfectly aware of his own and everyone else's preference relation.

In the next section, the committee decision making process is modeled as a game[†]. Due to the special nature of the game, distinct from games with side payments, games without side payments, games in partition function form, abstract games, etc., we call such a game, a committee game. In subsequent sections, some solutions of the game are studied. In particular, a new solution concept called the one-core is introduced. Also the concept of a bargaining set first introduced by Aumann and Maschler [4] in the context of games with side payments is defined with appropriate modifications for committee games. Both these solution concepts are studied in relation to each other and in relation to two other well known solution concepts, the Condorcet solution and the core.

Like most of game theory, our models of the committee decision making process are primarily normative in nature. We feel that it is unlikely that our solution concepts will predict outcomes in real life or in laboratory conditions where the members have little or no experience in committee decision making. However, if the committee members have had extensive experience (in decision making) or if the subjects of an experiment in laboratory conditions are aware of these models, then it is quite likely (in our opinion) that these models will be predictive. This is the only true test of a normative model. Our primary concern is not so much to predict as it is to understand much better the committee decision making process.

[†]This modeling of a committee process as a game is not new. See, for example, Wilson [26].

2. The Committee Game

Let $N = \{1, 2, \dots, n\}$ denote the set of players (committee members). Let X denote the set of all outcomes (alternatives). X may consist of finite or infinite outcomes. If X is finite we will denote X by $\{a_0, a_1, \dots, a_m\}$ where $m \geq 1$. Outcome a_0 will be referred to as the status quo outcome. Nonempty subsets of N will be called coalitions. Let $2^N = \{R \subset N : R \neq \emptyset\}$ denote the set of all coalitions and 2^X denote the set of all subsets of X . The rules by which the committee members arrive at a decision is called the characteristic function $v : 2^N \rightarrow 2^X$ and v is assumed to satisfy the following conditions.

$$(2.1) \quad \forall R_1, R_2 \in 2^N, R_1 \supset R_2 \Rightarrow v(R_1) \supset v(R_2),$$

$$(2.2) \quad v(N) = X$$

$$(2.3) \quad \forall R_1, R_2 \in 2^N, R_1 \cap R_2 = \emptyset, v(R_1) \neq \emptyset, v(R_2) \neq \emptyset \Rightarrow v(R_1) \cap v(R_2) = \emptyset \text{ and } |v(R_1)| = 1.$$

$v(R)$ denotes the subset of outcomes that coalition R can realize if the decision is unanimous in R . Conditions (2.1) and (2.2) are intuitively obvious. Condition (2.3) ensures that the committee decision consists of at most one outcome.

Let $u_i : X \rightarrow E^1$ denote the real-valued utility function of player i , $i = 1, \dots, n$. Utility is assumed to be nontransferable and interpersonal comparison of utilities has no meaning. The utility function u_i of player i induces a preference relation P_i on the set of outcomes as follows. Let $a, b \in X$. We say $a P_i b$ (i.e. a is strictly preferred to b by player i) iff $u_i(a) > u_i(b)$, and not $a P_i b$ or $\sim a P_i b$ (a is not strictly preferred to b by player i) iff $u_i(a) \leq u_i(b)$. Clearly the binary relations $\{P_i\}_{i=1}^n$ on X are asymmetric, transitive and negatively transitive⁺ (i.e. a weak order).⁺⁺ $\Gamma = (N, X, v, \{P_i\}_{i \in N})$ is then called a n -person committee game. The game is said to be finite if X is a finite set and infinite otherwise. A committee game is said to be simple if

⁺ A binary relation P_i on X is said to be negatively transitive if $\forall x, y, z \in X$, $\sim x P_i y$ and $\sim y P_i z \Rightarrow \sim x P_i z$.

⁺⁺ The utility functions $\{u_i\}_{i=1}^n$ are introduced solely to define the preference relations. Alternatively, we could have simply assumed that $\forall i \in N$, player i has a preference relation P_i on X that is a weak order.

$$\forall R \in 2^N, v(R) = \emptyset \text{ or } v(R) = \{a_0\} \text{ or } v(R) = X.$$

If $v(R) = X$, we say R is a winning coalition and if $v(R) = \emptyset$ or $\{a_0\}$, we say R is a losing coalition. In addition, if $v(R) = \{a_0\}$, coalition R is also said to be a blocking coalition. Let W denote the set of all winning coalitions, L the set of all losing coalitions and B , the set of all blocking coalitions. Note that condition (2.3) requires the committee game to be proper, i.e. in any partition of N into coalitions, at most one coalition is winning. Player i is said to be a dictator if $\{i\} \in W$. If $\bigcap_{R \in W} R \neq \emptyset$ then $i \in \bigcap_{R \in W} R$ is said to be a veto player. A simple committee game is said to be strong if $B = \emptyset$.

See Shapley [23] for a detailed description of simple games.

The characteristic function v and the preference relation $\{P_i\}_{i \in N}$ induce a natural dominance relation on the set of outcomes, X .[†] Let $a, b \in X$ and $R \in 2^N$. We say a dominates b via R denoted by $a \text{ dom}_R b$ iff

$$(2.4) \quad a P_i b \quad \forall i \in R \quad \text{and}$$

$$(2.5) \quad a \in v(R).$$

Clearly, $\forall R \in 2^N$, dom_R is asymmetric and transitive. Let $a, b \in X$. We say a dominates b denoted by $a \text{ dom } b$ iff

$$(2.6) \quad \exists R \in 2^N \text{ such that } a \text{ dom}_R b.$$

The binary relation dom may not be transitive. However we have the following result.

Proposition 2.1. The binary relation dom is asymmetric for all committee games.

Proof: Suppose not. Then $\exists a, b \in X$ such that $a \text{ dom } b$ and $b \text{ dom } a$. I.e. $\exists R_1, R_2 \in 2^N$ such that $a \text{ dom}_{R_1} b$ and $b \text{ dom}_{R_2} a$. Now since the preference relations $\{P_i\}_{i \in N}$ are asymmetric, we have $R_1 \cap R_2 = \emptyset$. Also we have $a \in v(R_1)$, $b \in v(R_2)$ which contradicts condition (2.3) $\square$

[†] We mimic the definition of domination given by von Neumann and Morgenstern [17] in the context of games with side payments.

Before we proceed to study the solutions of these games, we introduce some notation
 $\forall a \in X$, let $D(a) = \{x \in A : a \text{ dom } x\}$ and $U(a) = X - D(a)$. Also $\forall B \in 2^X$, B nonempty,
 let $D(B) = \bigcup_{x \in B} D(x)$ and $U(B) = X - D(B)$.

3. The Condorcet Solution and the Core

Let $\Gamma = (N, X, v, \{p_i\}_{i \in N})$ be a committee game. A Condorcet solution of the game Γ is an outcome $\alpha \in X$ such that it dominates every other outcome in X . I.e. $\{\alpha\} = X - D(\alpha) = U(\alpha)$.

An obvious result is as follows.

Proposition 3.1. For all committee games, if a Condorcet solution exists, then it is unique.

The proof follows from the asymmetric property of the binary relation dom (Proposition 2.1).

The Condorcet solution was first defined by Condorcet [8] and rediscovered independently by Dodgson [10] (cf. Black [7]).

The core C of a committee game is the set of all undominated outcomes. I.e.

$$C = X - D(X) = U(X)$$

The core was first studied explicitly by Gillies [13] and Shapley.

An obvious relation between the Condorcet solution and the core is as follows.

Proposition 3.2. Let Γ be a committee game such that the Condorcet solution α exists. Then the core of the game coincides with the Condorcet solution. I.e.

$$C = \{\alpha\}.$$

The proof follows from the asymmetric property of the binary relation dom (Proposition 2.1).

The converse, obviously, is not always true. The Condorcet solution has the strongest stability requirement of all solution concepts in game theory. However, this is attained at the cost of existence. The Condorcet solution doesn't always exist. The core represents the next level of stability requirements. These are not as strong as in the case of the Condorcet solution but strong enough to qualify as a viable solution concept. Unfortunately, the core is not always nonempty. The study of games with empty cores has been the traditional subject of study in game theory and also in the social sciences. Many different solutions

have been forwarded. We do not intend to list these here. The problem is sufficiently complex to ensure that no single solution concept will suffice for all kinds of games. In the next section, we present a new solution concept called the one-core which results from a small modification in the definition of the core. The modification is motivated by behavioral considerations.

4. The One-Core

Let Γ be a committee game. A proposal is a pair (i, x) such that $i \in N$ and $x \in X$. A proposal (i, x) represents a motion x introduced by player i . Let $P = N \times X$ denote the set of all proposals.

Define

$$(4.1) \quad \hat{C}^{(i)} = \{(i, x) \in P : x \text{ is not dominated via any coalition } R \subset N - \{i\}\}$$

$\hat{C}^{(i)}$ represents the set of proposals made by i that are undominated assuming player i 's noncooperation in any effort to dominate his proposal.

Let

$$(4.2) \quad C^{(i)} = \{(i, x) \in \hat{C}^{(i)} : \sim \exists y P_i x \forall (i, y) \in \hat{C}^{(i)}\}.$$

Intuitively, $C^{(i)}$ represents the maximal (best) proposals in the set $\hat{C}^{(i)}$ for player i . The one-core, C_1 , of the game Γ is then defined by

$$(4.3) \quad C_1 = \bigcup_{i \in N} C^{(i)}.$$

Intuitively, the one-core consists of all (maximal) proposals which are undominated assuming that the player who makes the proposal does not cooperate in any effort to dominate the proposal. For obvious reasons, assuming all proposals in $\hat{C}^{(i)}$ to be equally stable, player i picks only the maximal ones.

We now proceed to study the properties of the one-core.

Theorem 4.1. Let Γ be a committee game such that the Condorcet solution α exists. Then the one-core is given by

$$C_1 = \{(1, \alpha), (2, \alpha), \dots, (n, \alpha)\}$$

Proof: From Proposition (3.2), it follows that $(i, \alpha) \in \hat{C}^{(i)} \forall i \in N$. We claim

$C^{(i)} \supset \{(i, \alpha)\} \forall i \in N$. Suppose not. Let $(i, x) \in \hat{C}^{(i)}$ such that $x \neq \alpha$ and $x P_i \alpha$.

Since α is a Condorcet solution $\alpha \text{ dom}_R x$ for some $R \in 2^N$. If $i \in R$, then $\alpha P_i x$, a contradiction! If $i \notin R$ then $(i, x) \notin \hat{C}^{(i)}$, a contradiction again! Hence $C^{(i)} \supset \{(i, \alpha)\}$.

Using exactly the same argument as before, we can show that $(i, x) \in C^{(i)}$, $x \neq a$ leads to a contradiction. Hence $C^{(i)} = \{(i, a)\}$. $\square$

The result is a strong endorsement for the one-core when the Condorcet solution exists.

Theorem 4.2. Let Γ be a finite committee game such that the core C is nonempty. Then $C^{(i)} \neq \emptyset \forall i \in N$ and $(i, x) \in C^{(i)} \Rightarrow \sim \exists y P_i x \forall y \in C$.

Proof: $C \neq \emptyset \Rightarrow \hat{C}^{(i)} \neq \emptyset \forall i \in N$. Since X is a finite set, $P = N \times X$ is finite. Hence $C^{(i)} \neq \emptyset$. Thus $C_1 \neq \emptyset$. The second assertion follows from the definition of $C^{(i)}$. $\square$

The theorem states that each player by proposing his proposals from the one-core does as good (if not better) than any outcome in the core. The finiteness condition is merely academic and is assumed to ensure that the maximal proposal exists. We can assume a much weaker condition than finiteness of X , but we think our point is well made.

The following examples illustrates the advantage of the one-core over the core.

Example 4.1. Let $N = \{1, 2, 3\}$, $X = \{a_0, a_1, a_2, a_3, a_4\}$, v be given by $v(1) = v(23) = \{a_0\}$, $v(2) = v(3) = \emptyset$, $v(12) = v(13) = v(123) = X$, and the utility functions are given by

	u_1	u_2	u_3
a_0	1	3	1
a_1	3	0	2
a_2	4	1	0
a_3	2	2	3
a_4	0	4	4

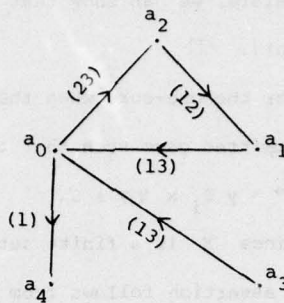


Figure 4.1 The dominance relations in Example 4.1

For this committee game, $C = \{a_3\}$ and $C_1 = \{(1, a_1), (2, a_3), (3, a_3)\}$. Note that player 1 by proposing his proposal $(1, a_1)$ can do better than the core. If after player 1 has introduced motion a_1 , player 2 introduces motion a_2 and suggests a vote between the two, player 1 will vote for motion a_1 and thus prevent motion a_1 from being defeated. Player 1 is motivated to do so because if he votes for a_2 and a_1 is defeated, it is possible that player 3 will suggest a_0 and player 1 is powerless to stop a_2 from being defeated by a_0 which put him in a worse situation than he was in a_1 . Thus player 1 benefits by anticipating more than one "step" ahead.

Example 4.2. Let $N = \{1, 2, 3, 4\}$, $X = \{a_0, a_1, a_2, a_3, a_4, a_5\}$ v be given by:

$$v(R) = \begin{cases} X & \text{if } |R| \geq 3 \\ \{a_0\} & \text{if } |R| = 2 \\ \emptyset & \text{if } |R| = 1 \end{cases}.$$

The utility functions are given as follows:

	u_1	u_2	u_3	u_4
a_0	0	0	0	0
a_1	3	1	0	4
a_2	4	3	1	0
a_3	0	4	3	1
a_4	1	0	4	3
a_5	2	2	2	2

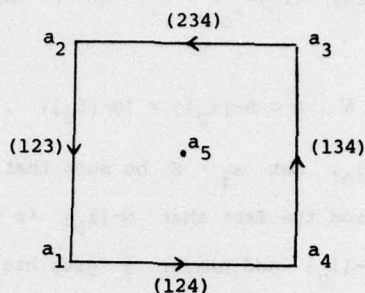


Figure 4.2 The dominance relations in Example 4.2

For this committee game, $C = \{a_5\}$ and the one-core is given by

$C_1 = \{(1, a_1), (2, a_2), (3, a_3), (4, a_4)\}$. Note that each player, by proposing his proposal in C_1 , can do better than the core

We now turn our attention to the question of existence of the one-core for finite simple committee games. The one-core always exists and is nonempty for n -person finite simple committee games when $n \leq 4$, and we exhibit a five-person finite simple committee game for which the one-core does not exist. Before we prove the above results we need a lemma.

Lemma 4.3 Let Γ be a n -person simple committee game with $n \leq 4$. Then $\exists i_0 \in N$ such that

$$\{R \in W : R \subset N - \{i_0\}\} = \{N - \{i_0\}\} \text{ or } \emptyset.$$

Proof: The proof is by simple enumeration of all proper simple games with four or less players. Shapley [23] has listed all these games. Pick i_0 to be the "strongest" player. Note that if i_0 is a veto player then

$$\{R \in W : R \subset N - \{i_0\}\} = \emptyset. \quad \square$$

Theorem 4.4 Let Γ be a n -person finite simple game with $n \leq 4$. Then $C_1 \neq \emptyset$. Also

$\exists x \in X$ such that $(i_0, x) \in C_1$ where i_0 is as given by Lemma 4.3.

Proof: Let $X_p = \{x \in X : \sim a_0 P_k x \forall k \in R \text{ for each } R \in \mathcal{P}\}$. Since $a_0 \in X_p$, $X_p \neq \emptyset$. If Γ has veto players then let i_0 be a veto player. Let $x_{i_0} \in X_p$ such that $\sim x P_{i_0} x_{i_0} \forall x \in X_p$. Then clearly $(i_0, x_{i_0}) \in C_1$. If Γ has no veto players then let $i_0 \in N$ such that

$$\{R \subset W : R \subset N - \{i_0\}\} = \{N - \{i_0\}\}.$$

Pick any $j \in N$ such that $j \neq i_0$. Let $x_j \in X$ be such that $\sim x P_j x_j \forall x \in X$. Then $(i_0, x_j) \in \hat{C}^{(i_0)}$. This follows from the fact that $N - \{i_0\}$ is the only winning coalition not containing i_0 and since $j \in N - \{i_0\}$ and player j gets his maximum possible in x_j , x_j is undominated by any coalition not containing player i_0 . Hence $C^{(i_0)} \neq \emptyset \Rightarrow C_1 \neq \emptyset$. $\square$

The following example exhibits a five-person finite simple committee game that has no one-core.

Example 4.2. Let $N = \{1, 2, 3, 4, 5\}$, $X = \{a_0, a_1, a_2, a_3, a_4, a_5\}$, v be given by

$$v(R) = \begin{cases} X & \text{if } |R| \geq 3 \\ \emptyset & \text{if } |R| \leq 2 \end{cases},$$

and the utility functions be give by

	u_1	u_2	u_3	u_4	u_5
a_0	0	0	0	0	0
a_1	1	2	3	4	0
a_2	2	3	4	0	1
a_3	3	4	0	1	2
a_4	4	0	1	2	3
a_5	0	1	2	3	4

A simple calculation will show that C_1 does not exist.

Compared to the core, the stability requirements in the one-core was slightly relaxed. (However, considering the behavioral motivation behind the one-core, the stability requirements seems as strong as in the core.) As a consequence, the one-core has a stronger existence result than the core. As an added advantage, even when the core exists, the one-core may be favored over the core in some games by some players.

At this stage we would like to emphasize our earlier assertion that no single solution concept can be expected to be the "best" solution for all games. For some games, the stability requirement in the one-core may be "too strong". We illustrate by means of an example.

Example 4.4. (A discrete 3-person constant sum game.) Let $N = \{1, 2, 3\}$, $X = \{x = (x_1, x_2, x_3) \in E^3 : \sum_{i=1}^3 x_i = 9, x_i \geq 0, \text{ integer } \forall i \in N\}$, v be given by

$$v(R) = \begin{cases} X & \text{if } |R| \geq 2 \\ \emptyset & \text{if } |R| = 1 \end{cases},$$

and the utility functions be given by

$$\forall x \in X \quad u_i(x) = x_i, \quad i = 1, 2, 3.$$

Then, a simple calculation reveals that $C = \emptyset$ and $C_1 = \{(i, x) : x_i = 1, x \in X, i = 1, 2, 3\}$.

From the symmetry of the game, one expects an outcome $(3, 3, 3)$. This is possible if we relax further the stability requirements of the one-core. One approach is to use the idea of the bargaining set first developed by Aumann and Maschler [4] in the context of games with side payments. This is discussed in the next section.

5. The Bargaining Set of Committee Games

The concept of a bargaining set was first introduced by Aumann and Maschler [4] in the context of games with side payments. They defined several kinds of bargaining sets. These were generalized for games without side payments and studied by Peleg [18], Billera [5,6], D'Aspremont [9], and Asscher [2,3]. Since then, several other modifications of the bargaining set have been studied in different contexts by Shenoy [25], Rosenthal [21], Wilson [26] and Issac and Plott [14].

Here we define yet another bargaining set. This definition is relevant to the context of a committee game and is presented as an extension of the one-core. The definition presented here captures some of the ideas presented in all the references mentioned above but is quite different from all of them. Along with the definition, we present a behavioral interpretation of the bargaining set.

Suppose a player $i \in N$ introduces a motion $x \in X$ in the form of a proposal (i,x) . The proposal is debated by the members. At the end of the debate there are three possible course of actions:

- (1) Player i withdraws his motion. Another proposal is made and the process continues.
- (2) The proposal (i,x) is uncontested and becomes the decision of the committee and the members go home.
- (3) Another member $j \in N$ introduces another motion $y \in X$ and the two proposals are put to vote with the members voting for one of the two motions. The motion that wins becomes the new proposal and the process continues. In case neither motion gets a decisive vote (in case of non-strong simple committee games), the motion introduced first is considered undefeated and remains as the current proposal.[†]

An objection against proposal (i,x) is a triple (j,S,y) such that

[†] We have deliberately not specified all the rules. E.g. several or all the members may wish to make a proposal. We will assume that the rules of the committee will decide which proposal is considered first. These ambiguities result in the bargaining set having several proposals. Which of these are actually realized will depend on the committee rules. At this stage we wish to be as general as possible.

$$(5.1) \quad j \in S \in 2^N, y \in X$$

$$(5.2) \quad i \notin S$$

$$(5.3) \quad y \text{ dom}_S x$$

During the process of debating the merits (or demerits) of the proposal (i,x) , player j puts forward an objection to the effect: why shouldn't I introduce a motion y against the proposal (i,x) ? Since $y \text{ dom}_S x$ (Condition (5.3)), j expects the players in coalition S to vote for y which would result in y winning against x . Condition (5.2) reflects the fact that player j cannot expect player i to cooperate with him in defeating his (player i 's) own proposal. Note that Condition (5.1) and (5.2) require i and j to be two distinct players. Player j 's objection is directed towards all players in $N-S$. A counterobjection against the objection (j,S,y) to proposal (i,x) is a triple (k,T,z) such that

$$(5.4) \quad k \in T \in 2^N, z \in X$$

$$(5.5) \quad \text{either } k = i \text{ or } x P_k y$$

$$(5.6) \quad z \text{ dom}_T y$$

$$(5.7) \quad x P_j z$$

$$(5.8) \quad \sim x P_k z$$

The counterobjection is made either by player i or by a player who stands to lose if the objection is carried out (Condition (5.5)).

The counterobjection is a reply by player k to player j to the effect: If you (player j) carry out your objection and win, then in the next round I will introduce motion z against y and since $z \text{ dom}_T y$ (Condition (5.6)), z will win against y . This will put you in a worse position than you were before (Condition (5.7)) and I will do no worse than what I started with (Condition (5.8)).

If a counterobjection does exist, there is a strong motivation for player j to withdraw his objection. On the other hand if there is no counterobjection, then player j has a justified objection and player i cannot expect to get his proposal accepted by the committee.

Proposition 5.1. Let (i,x) be a proposal, (j,S,y) be an objection and (k,T,z) be a counterobjection. Then $j \notin T$, $k \notin S$ and $T \cap S \neq \emptyset$.

Proof: $y P_j x$ and $x P_j z \Rightarrow y P_j z$. Then $z \text{ dom}_T y \Rightarrow j \notin T$. Conditions (5.2), (5.3) and (5.5) $\Rightarrow k \notin S$. $T \cap S = \emptyset$ contradicts condition (2.3) since $y \in v(S)$ and $z \in v(T)$.

A proposal is said to be M-stable if every objection has a counterobjection.

Let $\hat{M}$ denote the set of all M-stable proposals. Conceivably we could have two (or more) M-stable proposals of the type (i,a) and (i,b) . In such a case since player i is a rational player, we can trust him to introduce only those proposals in $\hat{M}$ which will maximize his utility.

A M-stable proposal (i,a) is said to be maximal if $\neg x P_i a \forall x \in X$ such that $(i,x) \in \hat{M}$.

The bargaining set M is the set of all maximal M-stable proposals.

Example 5.1. Consider the 3-person committee game given in Example 4.4. The bargaining set M of this game is given by

$$M = \{(1,(3,3,3)), (2,(3,3,3)), (3,(3,3,3))\}.$$

Consider the proposal $(1,(3,3,3))$. An objection by player 2 is $(2,(23), (0,4,5))$. A counterobjection by player 1 is $(1,(13), (3,0,6))$. It can easily be shown that $(1,(3,3,3))$ is a maximal M-stable proposal.

We shall now study the question of existence of the bargaining set. Comparing the bargaining set M with the Condorcet solution α , we have:

Theorem 5.2. Let Γ be a committee game such that the Condorcet solution α exists. Then $M \neq \emptyset$ and is given by $M = \{(1,\alpha), (2,\alpha), \dots, (n,\alpha)\}$.

Proof: It is clear that $\hat{M} \supset \{(1,\alpha), \dots, (n,\alpha)\}$. Consider a proposal (i,x) such that $x \neq \alpha$. Since α a condorcet solution $\alpha \text{ dom}_R x$ for some $R \in 2^N$. If $i \notin R$, then (j,R,α) is an objection to (i,x) for which there is no counterobjection. Hence $(i,x) \notin \hat{M}$. If $i \in R$, then if (i,x) is M-stable, (i,x) is not maximal since $(i,\alpha) \in \hat{M}$ and $\alpha P_i x$. $\square$

Comparing the bargaining set with the core C , we have:

Theorem 5.3. Let Γ be a finite committee game such that the core C is nonempty. Then we have

- (1) $M \neq \emptyset$
- (2) $\forall i \in N, \exists x \in X$ s.t. $(i, x) \in M$
- (3) $(i, x) \in M \Rightarrow \neg \exists y P_i x \forall y \in C$.

Proof: Clearly $\{(i, x) : i \in N \text{ and } x \in C\} \subset \hat{M}$. Since X is finite $M \neq \emptyset \iff \hat{M} \neq \emptyset$. Since $C \neq \emptyset$, the first two assertions hold. The third assertion follows from the fact that M consists of only maximal M -stable proposals in $\hat{M}$.

And finally, comparing M with the one-core C_1 we have:

Theorem 5.4. Let Γ be a finite committee game such that the one-core C_1 is nonempty.

Then we have $M \neq \emptyset$ and $(i, x) \in M \Rightarrow \neg \exists y P_i x \forall y \in X$ such that $(i, y) \in C_1$.

The proof is exactly as in Theorem 5.3.

The bargaining set is not always nonempty. For the committee game presented in Example 4.3, $M = \emptyset$. Where do we go from here? One possibility is to further relax the stability requirements of the bargaining set by modifying the definition of a counterobjection. We could drop Condition (5.7) or Condition (5.8) or both. We could also alter the definition of an objection by requiring an additional condition as follows

$$(5.9) \quad x P_i y.$$

This method is not entirely satisfactory. Each solution concept implies a definition of "rational behavior". The "rational behavior" implied by these modified bargaining sets is difficult to accept in real life.

6. Conclusion

Four different solution concepts were studied in relation to committee games. These are the Condorcet solution, the core, the one-core and the bargaining set. There is a tradeoff between stability and existence. The Condorcet solution has the strongest stability requirement but the weakest existence result. Whenever the Condorcet solution exists, all these solution concepts indicate the same solution. The bargaining set at the other extreme has the weakest stability requirements among the four solutions but the strongest existence result. However, the existence result is not strong enough to guarantee non emptiness for all games. In the Condorcet solution and the core, the players look ahead only one "step" (only objections are considered) while in the bargaining set the players anticipate two "steps" (an objection represents one step, and a counterobjection represents the second step) at a time. The one-core represents some intermediate position in this respect. Perhaps one approach is to define a solution concept that looks far ahead enough to guarantee existence and nonemptiness for all games.

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