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GLOBAL CONTROLLABILITY FOR A CLASS OF BILINEAR SYSTEMS. (U)
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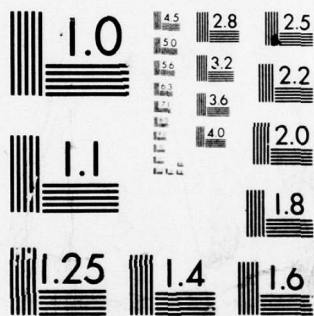
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Global Controllability for a Class of Bilinear Systems

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Abstract

Sufficient conditions are given for global controllability of the bilinear system

$$\frac{d}{dt} x(t) = \left[A(t) + \sum_{i=1}^m B_i(t) u_i(t) \right] x(t) + C(t) u(t)$$

and a related class of nonlinear systems. An example is provided to illustrate the simplicity of these conditions for certain bilinear systems.

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I. Introduction

The purpose of this note is to study the controllability property of a class of bilinear systems. Sufficient conditions are derived for the global controllability of this class of systems. These results are then generalized to several other classes of nonlinear systems.

Consider the bilinear system

$$\frac{d}{dt} x(t) = \left[A(t) + \sum_{i=1}^m B_i(t) u_i(t) \right] x(t) + C(t) u(t), \quad t \in [t_0, t_1] \quad (1)$$

where $x(t)$ is an $n \times 1$ state vector, $u(t)$ is an $m \times 1$ input vector with components u_i ; A , B_i , C are $n \times n$, $n \times n$, $n \times m$ matrix-valued functions respectively.

Given (x_0, x_1) as the initial and final state, respectively, of (1). The problem is to find a continuous input function $u(t)$, defined on $[t_0, t_1]$, which steers system (1) from x_0 to x_1 at t_1 . The usual definitions of globally completely and totally controllable are assumed [1]. The global controllability property of a class of bilinear systems using bounded controls was reported in [2] in which rather sophisticated conditions were derived to insure global controllability. Here, a completely different approach is used to derive sufficient conditions for global controllability.

Denote $C_m[t_0, t_1]$ as a Banach space of continuous R^m -valued functions on $[t_0, t_1]$ with the uniform norm $\|u(t)\| = \max_i \max_{t \in [t_0, t_1]} |u_i(t)|$, where $|u_i(t)|$ is the absolute value of $u_i(t)$, the element of $u(t)$. Define the norm of a continuous $n \times m$ matrix-valued function $F(t)$ by $\|F(t)\| = \max_i \sum_{j=1}^m \max_{t \in [t_0, t_1]} |F_{ij}(t)|$, where the F_{ij} are elements of F .

For each fixed element $v \in C_m[t_0, t_1]$, the solution to the parameterized system

$$\frac{d}{dt} x(t) = \left[A(t) + \sum_{i=1}^m B_i(t) v_i(t) \right] x(t) + C(t) u(t) \quad (2)$$

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with $x(t_0) = x_0$ is given by

$$x(t) = \Phi(t, t_0; v)x_0 + \int_{t_0}^t \Phi(t, s; v)C(s)u(s) ds \quad (3)$$

where $\Phi(t, t_0; v)$ is the state transition matrix associated with the matrix $A(t) + \sum_{i=1}^m B_i(t)v_i(t)$.

Define a controllability matrix G by

$$G(t_0, t; v) = \int_{t_0}^t \Phi(t, s; v)C(s)C'(s)\Phi'(t, s; v) ds \quad (4)$$

where prime denotes the matrix transpose operation. Obviously, $G(t_0, t; v)$ is symmetric and nonnegative-definite.

II. Main Result

Theorem 1: The bilinear system (1) is globally a) completely controllable at t_0 , or b) totally controllable, if the following conditions are satisfied:

- (1) $C(t)$ has a continuous first derivative with respect to t ,
- (2) $\Phi(t, t_0; v)$ is bounded on $[t_0, t_1] \times \mathbb{R}^m$, $A(\cdot)$ and $B_i(\cdot)$ are continuous,
- (3) there exists a positive constant ϵ such that

$$\inf_{v \in C_m[t_0, t_1]} \det G(t_0, t_1; v) \geq \epsilon, \quad (5)$$

a) for some $t_1 > t_0$, or b) for all t_0 and all $t_1 > t_0$.

Proof: The proof of the theorem is based on Schauder's fixed-point theorem.

For each fixed element $v \in C_m[t_0, t_1]$, consider the control function $u(t)$ for $t \in [t_0, t_1]$ defined by

$$u(t) = C'(t)\Phi'(t_1, t; v)G^{-1}(t_0, t_1; v)[x_1 - \Phi(t_1, t_0; v)x_0], \quad (6)$$

where $\phi(t, t_0; v)$ is defined as in (3). It should be noted that by hypothesis (3) $G^{-1}(t_0, t_1; v)$ is well-defined in the above expression. With this control, (3) can be rewritten as

$$x(t; v) = \phi(t, t_0; v)x_0 + \int_{t_0}^t \phi(t, s; v)C(s)C'(s)\phi'(t_1, s; v) \cdot \\ \cdot G^{-1}(t_0, t_1; v)ds[x_1 - \phi(t_1, t_0; v)x_0]. \quad (7)$$

It is easily seen that $x(t)$ in (7) satisfies both boundary conditions at $t=t_0$ and $t=t_1$.

Now the right side of (6) can be viewed as an operator $P(v)(t)$.

Define the nonlinear mapping $P(\cdot)$ by

$$P(v)(t) = C'(t)\phi'(t_1, t; v)G^{-1}(t_0, t_1; v)[x_1 - \phi(t_1, t_0; v)x_0].$$

Obviously, P is continuous in t by the uniform continuity of $\phi(t, t_0; v)$ in t . Therefore, P maps $C_m[t_0, t_1]$ into itself. It can also be easily verified by hypothesis (2) and the definition of $\phi(t, t_0; v)$ that P is continuous in v . Consider the subset of $C_m[t_0, t_1]$:

$$I = \{v \in C_m[t_0, t_1] : \|v\| \leq K_1, \|v(t) - v(\tilde{t})\| \leq K_2|t - \tilde{t}|, \forall t, \tilde{t} \in [t_0, t_1]\}.$$

where K_1 and K_2 are certain positive constants depending upon $A(t)$, $B_i(t)$, $C(t)$ and its derivative. It can be easily shown that the image set $P(I) \subset I$. Besides, I is closed and convex by this construction. Furthermore, each sequence $\{s_i\}_{i=1}^{\infty} \subset I$ constitutes a uniformly bounded equicontinuous family. Hence, by the Arzela-Ascoli theorem [3], I is relatively compact and, therefore, compact.

Then, Schauder's theorem [3] can be applied to conclude that P has a fixed point v^* in I , i.e., $P(v^*)(t) = v^*(t)$. Substitute this fixed point into (6) and (7). A direct differentiation of (7) with respect to t shows that $x(t; v^*)$ is a solution to the system (1) with $u(t)$ given by $v^*(t)$.

If condition 3a) holds, $v^*(t)$ drives system (1) from x_0 to x_1 on some interval $[t_0, t_1]$ for all x_0 and x_1 in R^n , and system (1) is globally completely controllable at t_0 . If condition 3b) holds, we have global total controllability. Q.E.D.

Even though the test of conditions (2) and (3) seems to be formidable, for certain bilinear systems this can be done quite simply. The next corollary characterizes one such class of bilinear systems which satisfy these conditions.

Consider the fixed bilinear system

$$\frac{d}{dt} x(t) = \left[A + \sum_{i=1}^m B_i u_i(t) \right] x(t) + Cu(t), \quad t \in [t_0, t_1]. \quad (8)$$

Corollary 1: The system (8) is globally totally controllable if

- (1) C is nonsingular,
- (2) A and B_i ($i=1, 2, \dots, n$) commute with one another,
- (3) the B_i are antisymmetric and have only zero, or purely imaginary, eigenvalues with simple elementary divisors.

Proof: Hypothesis (1) implies that C can be replaced by the identity matrix without loss of generality. The boundedness of $\phi(t, t_0; u)$ follows from hypothesis (2) and the second part of hypothesis (3). Finally, to test the positive-definiteness of the controllability matrix (4), we notice that the commutative property of A and B_i implies

$$G(t_0, t; u) = \int_{t_0}^t \phi_A(t, s) \prod_{i=1}^m \phi_{B_i}(t, s; u_i) \phi_{B_i}'(t, s; u_i) \phi_A'(t, s) ds$$

where $\phi_A(t, t_0)$ and $\phi_{B_i}(t, t_0; u_i)$ denote the transition matrices associated with A and $B_i u_i$ respectively. Since the B_i are antisymmetric, we have $\phi_{B_i}'(t, s; u_i) = \phi_{B_i}^{-1}(t, s; u_i)$. Hence,

$$G(t_0, t_1; u) = \int_{t_0}^{t_1} \phi_A(t_1, s) \phi_A'(t_1, s) ds = \epsilon_A > 0 \quad \text{for } t_1 > t_0,$$

where ϵ_A is some constant depending on A , t_0 and t_1 . By Theorem 1, we conclude that the system (8) is globally totally controllable. Q.E.D.

These results can be easily generalized to nonlinear systems consisting of a bilinear mode and bounded nonlinearities. This is stated in the next theorem. Consider the nonlinear system

$$\frac{d}{dt} x(t) = [A(t) + \sum_{i=1}^m B_i(t)u_i(t)]x(t) + C(t)u(t) + f(t, x, u). \quad (9)$$

Theorem 2: If conditions (1)-(3)a (or b)) in Theorem 1 hold and, furthermore, if $f(t, x, u)$, $f_x(t, x, u)$, $f_u(t, x, u)$ are continuous and bounded in $[t_0, t_1] \times \mathbb{R}^n \times \mathbb{R}^m$, then the system (9) is globally completely (or totally) controllable at t_0 .

Proof: The proof is similar to that of Theorem 1 except that the Banach space $C_{nm}[t_0, t_1]$ of continuous $\mathbb{R}^n \times \mathbb{R}^m$ matrix valued functions on $[t_0, t_1]$ with the uniform topology is considered instead of $C_m[t_0, t_1]$. The details follow similar arguments as in Wei [4].

Finally, by observing that the boundedness conditions on the system matrix $A(t, x, u)$ and its partial derivatives listed in [4] can be relaxed by introducing the boundedness of the transition matrix associated with A , we have the following theorem which extends the results of [4]. Given the nonlinear system

$$\frac{d}{dt} x(t) = A(t, x(t), u(t)) + B(t)u(t) + f(t, x(t), u(t)) \quad (10)$$

where $\Phi(t, t_0; x, u)$, $f(t, x, u)$, $f_x(t, x, u)$ and $f_u(t, x, u)$ are continuous and bounded in $[t_0, t_1] \times \mathbb{R}^n \times \mathbb{R}^m$; $A(t, x, u)$, $A_x(t, x, u)$ and $A_u(t, x, u)$ are continuous in $[t_0, t_1] \times \mathbb{R}^n \times \mathbb{R}^m$; $B(t)$ has a continuous first derivative.

Theorem 3: If there exists a positive constant δ such that

$$\inf_{(z, v) \in C_{nm}[t_0, t_1]} \det \int_{t_0}^{t_1} \Phi(t_1, s; z, v) B(s) B'(s) \Phi'(t_1, s; z, v) ds \geq \delta$$

a) for some $t_1 > t_0$, or b) for all t_0 and $t_1 > t_0$, then the system (10) is globally a) completely controllable at t_0 , or b) totally controllable.

Example: Consider the bilinear system

$$\begin{aligned} \frac{d}{dt} x_1(t) &= x_1(t) + x_2(t)u_1(t) + u_1(t) \\ \frac{d}{dt} x_2(t) &= x_2(t) - x_1(t)u_1(t) + u_2(t) \end{aligned} \quad t \in [t_0, T] \quad (11)$$

In matrix form, we have

$$A(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_1(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B_2(t) = 0, \quad C(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is easily seen that conditions (1) and (2) in Corollary 1 are satisfied. Also, B_1 is antisymmetric and has purely imaginary eigenvalues $\pm i$. Therefore, by Corollary 1, system (11) is globally totally controllable.

III. Conclusions

Sufficient conditions for global controllability of a class of bilinear and nonlinear systems have been derived. These results shed some light in applying fixed-point arguments to investigate the controllability of nonlinear systems which contain unbounded nonlinearities. An interesting problem is to link the closedness of the attainable set for commutative bilinear systems [5] with the conditions obtained here so that an explicit input function can be derived or computed through iterative schemes.

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