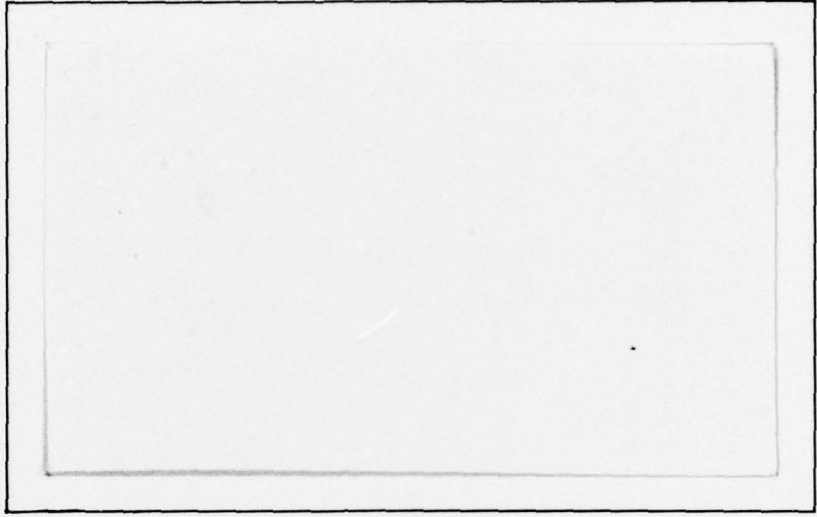
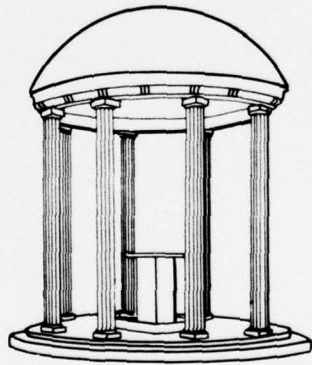


12
B.S.

UNIVERSITY OF NORTH CAROLINA
AT CHAPEL HILL



JDC NO. AD A 049523
JDC FILE COPY



DDC
RECEIVED
FEB 2 1978
F

SCHOOL OF BUSINESS ADMINISTRATION
AND
CURRICULUM IN
OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

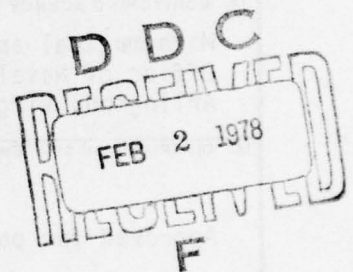
DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

**FORECASTING OPERATING CHARACTERISTICS
OF (s,S) INVENTORY SYSTEMS**

Technical Report #13

**Carl R. Schultz, Richard Ehrhardt
and Alastair MacCormick***

December 1977



Work Sponsored By

U.S. Army Research Office (DAAG29-76-G-0323)

Decision Control Models in Operations Research

and by the

Office of Naval Research under contract

with the U.S. Army Research Office

**Harvey M. Wagner
Principal Investigator
School of Business Administration
University of North Carolina at Chapel Hill**

**APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.**

***Department of Economics, University of Auckland, New Zealand and
Graduate School of Business Administration, University of North
Carolina at Chapel Hill**

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

14 TR-13

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report #13	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 FORECASTING OPERATING CHARACTERISTICS OF (s,S) INVENTORY SYSTEMS.		5. TYPE OF REPORT & PERIOD COVERED 9 Technical rept.
7. AUTHOR(s) 10 Carl R./Schultz, Richard/Ehrhardt Alastair/MacCormick		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of North Carolina at Chapel Hill Chapel Hill, North Carolina 27514		8. CONTRACT OR GRANT NUMBER(s) 15 ✓ DAAG29-76-G-0323 Now
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematics Division U.S. Army Research Office Box CM, Duke Station, Durham, N.C. 27706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS P-11639-M
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Mathematical and Information Sciences Division Office of Naval Research, Code 434 Arlington, Virginia 22217		11. REPORT DATE Dec 77 12 85 p.
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		12. NUMBER OF PAGES 47 and 29(appendices)
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		15. SECURITY CLASS. (of this report) Unclassified
18. SUPPLEMENTARY NOTES		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Inventory Control, (s,S) Policies, Forecasting		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The report We consider the management of an inventory control system operating under incomplete information about the probability distribution of demand. The system control policy is of an (s,S) form where the policy parameters, s and S, are periodically revised using a demand history of fixed length. In such a system it is often desirable to forecast values of such key system operating characteristics as: total cost per period, period-end		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

406439

inventory, period-end backlog quantity, backlog frequency, and replenishment frequency.

Several methods of forecasting system performance are presented. The criterion used to evaluate and compare these methods is the extent of prediction bias and its level of dispersion. Most of these procedures require estimation of the underlying demand distribution from a limited sample of historical data. ~~We have found for our~~ 16-item system, ~~that~~ forecasts of aggregate total cost are typically 20% below actual values. Much of this error results from underestimation of backlog quantities. This is due to the fact that the demand data is used both to set the policy parameters and to forecast system behavior.



		<input checked="" type="checkbox"/>
		<input type="checkbox"/>
		<input type="checkbox"/>
DISPATCH/AVAILABILITY CODES		
SPECIAL		
A		

- MacCormick, A. (1975), Statistical Problems in Inventory Control, ONR and ARO Technical Report, December 1974, School of Organization and Management, Yale University, 244 pp.
- Estey, A. S. and R. L. Kaufman (1975), Multi Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 1), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.
- Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.
- Kaufman, R. (1976), Computer Programs for (s,S) Policies under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.
- Kaufman, R. and J. Klinecicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.
- Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, School of Organization and Management, Yale University, 58 pp.
- Klinecicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, School of Organization and Management, Yale University, 24 pp.
- Klinecicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, School of Organization and Management, Yale University, 52 pp.
- Klinecicz, J.G. (1976), The Power Approximation: Control of Multi-item Inventory Systems with Constant Standard-deviation-to-mean Ratio for Demand, ONR and ARO Technical Report 10, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.
- Kaufman, R. L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ARO Technical Report 11, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.
- Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

FOREWORD

As part of the on-going research program in "Decision Control Models in Operations Research", the authors of this report have investigated several methods of forecasting the performance of statistically-controlled inventory systems. This work is an attempt to improve upon the bias and variability of the method of retrospective simulation which was investigated in previous studies (see Technical Report Nos. 2, 3, and 7). Although the authors report only partial success, several new avenues of research have emerged. These will be discussed in subsequent Technical Reports.

A list of other related reports dealing with this research program follows.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-principal Investigator

CONTENTS

1. Operating Characteristics of (s,S) Inventory Systems	1
1.1 The Model	3
1.2 The Inventory Replenishment Policy	4
1.3 Forecasting the Performance of Inventory Systems Under Statistical Control Policies	7
1.4 Forecasts by Retrospective Simulation	9
1.5 Experimental Design	9
2. Forecasting by Approximate Expressions for Operating Characteristics	13
2.1 Forecasts Using Analytical Approximations of Operating Characteristics	13
2.2 Forecasts Using Multiplicative Approximations of Operating Characteristics	18
2.2.1 Multiplicative Approximations for Fixed Policies	18
2.2.2 Multiplicative Approximations for Statistical Policies	21
3. Forecasting by a Future Demand Generation Technique	25
3.1 Forecasts by Future Demand Generation with a Postulated Negative Binomial Demand Distribution	26
3.1.1 Future Demand Generation Forecasting Technique Using Variable Means and Variances	32
3.2 Forecasts by Future Demand Generation with a Postulated Gamma Demand Distribution	33

4. Comparison of Forecasting Procedures	39
4.1 Properties of Single-item Forecasts	39
4.2 Properties of Multi-item Forecasts	40
4.3 Topics for Future Research	44
Bibliography	46
Appendices	

ABSTRACT

We consider the management of an inventory control system operating under incomplete information about the probability distribution of demand. The system control policy is of an (s,S) form where the policy parameters, s and S , are periodically revised using a demand history of fixed length. In such a system it is often desirable to forecast values of such key system operating characteristics as: total cost per period, period-end inventory, period-end backlog quantity, backlog frequency, and replenishment frequency.

Several methods of forecasting system performance are presented. The criterion used to evaluate and compare these methods is the extent of prediction bias and its level of dispersion. Most of these procedures require estimation of the underlying demand distribution from a limited sample of historical data. We have found for our 16-item system, that forecasts of aggregate total cost are typically 20% below actual values. Much of this error results from underestimation of backlog quantities. This is due to the fact that the demand data is used both to set the policy parameters and to forecast system behavior.

1. OPERATING CHARACTERISTICS OF (s,S) INVENTORY SYSTEMS

Consider an inventory manager who must design a system of replenishment rules for the stockage of possibly thousands of items. Assume that the manager knows the costs of holding inventory, making replenishment decisions, revising the stock status of an item, etc., and is able to attach a cost to each unit of demand that cannot be filled. Provided that the relevant operating characteristics for each system can be obtained, and that he has a preference ordering, the inventory manager is able to determine whether one system is better than another.

In designing the system, the manager must select a class of decision rules, and preferably pick optimal policies from the selected class. For example, the complete economic preference function of the manager may imply that if the probability distribution of demand is known, then the optimal policy is of an (s,S) form: when inventory on hand plus on order falls below s , place an order so that inventory on hand plus on order is S . However, it will often be the case that the manager faces considerable uncertainty about the underlying demand distribution. Consequently, a decision process for systems design becomes much more complex than the mere selection of values for (s,S) .

First, if the manager suspects a non-stationary demand environment, he must decide how much past demand data to actually use in updating the replenishment policy parameters and how often these revisions should be made. Second, the manager must either guess at the form of the demand distribution, statistically estimate the distribution's parameters, and then compute optimal policies for that distribution, or he can choose an

"approximately optimal" form for the replenishment policy that depends on only a few demand parameters, such as the mean and variance. Third, the manager must decide, before implementing the system, whether the design parameters are well set. To accomplish this task, the manager will need to predict the system's performance by forecasting the values of key operating characteristics. Usually, the manager will employ the same demands for forecasting as well as for policy derivations. This double use of the demand data, however, leads to biased forecasts. To eliminate this bias, the manager might consider splitting the demand sample in half; using one half of the sample to update the policy parameters, and the other half to forecast the policy's performance. However, the manager will still want to use a policy based on the entire demand sample. Thus, before this method can be investigated, we need an understanding of the relationship between the split-sample forecast and the performance of the policy based on the entire demand sample. This procedure, along with several other forecasting methods currently under investigation, is mentioned as a topic for future research in the final section of this report.

It is the purpose of this study to examine several alternatives for forecasting inventory system operating characteristics. The methods we consider fall into two major categories: methods that use approximate formulae for the operating characteristics, examined in Section 2, and methods that rely on simulation to estimate the characteristics, examined in Section 3.

1.1 The Model

Throughout this report, we deal with a single-item inventory model. It is assumed that inventory levels are reviewed periodically, and that the demand for an item is described by a stationary, discrete-time, stochastic process. The demand sequence, denoted ξ_1, ξ_2, \dots , consists of independent, identically distributed, non-negative, integer valued random variables.

As long as the amount of stock on hand is sufficient, demands are met; should a stockout occur, however, the unsatisfied demand is completely backlogged until sufficient replenishments arrive.

Items kept in inventory are assumed to be conserved, there being no losses by deterioration, obsolescence, or pilferage; disposal is not allowed. Inventory on hand at the end of a period is the inventory from the previous period plus any replenishment that arrives, less any demand. Negative stock-on-hand represents the amount backlogged. Replenishments are assumed to be delivered after a known fixed leadtime L periods after being ordered. The time sequence of events in any period is taken to be order, delivery, demand.

We assume no time discounting of costs and an unbounded horizon over which an item is demanded and stocked. Our objective is to minimize expected total cost per period.

The cost of a replenishment quantity q , denoted $c(q)$, is assumed to be of the form

$$c(q) = \begin{cases} k + cq & \text{For } q > 0 \\ 0 & \text{For } q = 0 \end{cases}$$

where k is the fixed ordering cost and c is the constant cost per item. Since costs are not discounted and unfilled demands are completely backlogged, the constant unit cost c is not a factor in choosing a minimal cost policy, and is hereafter suppressed.

Let i represent the inventory-on-hand at the end of any period. If $i > 0$, we assume that a holding cost proportional to i at unit cost h is assessed. If $i < 0$, we assume a backlog penalty cost proportional to $-i$ at unit cost p is charged.

The resulting total cost function is linear in k , p and h , and we can scale these parameters so that the value of the unit holding cost h is one. Non-trivial changes in costs arise only with changes in the ratios k/h and p/h .

1.2 The Inventory Replenishment Policy

We postulate that control over replenishment in the inventory system is exercised by an (s,S) policy: whenever inventory x on hand and on order at the start of a period drops below the value s , an order is placed for a replenishment of size $S - x$. Given our assumptions, when the demand distribution and the economic parameters are known, there is an optimal policy that has the (s,S) form [Iglehart (1963a,b), Veinott and Wagner (1965)]. When the demand distribution is not known, even though this is the only assumption relaxed, an optimal policy may no longer be of the (s,S) form. Nevertheless, in this study we use an (s,S) policy since it is in popular use in the applied situation of incomplete information.

For the purposes of this study we compute approximately optimal (s,S)

policy rules by way of the Statistical Power Approximation [Ehrhardt (1976)].

The Power Approximation is an algorithm for computing approximately optimal values for (s, S) using only the mean μ and variance σ^2 of demand. The algorithm is executed as follows. Let

$$(1) \quad D_p = (1.463)\mu^{.364}(k/h)^{.498}[(L+1)\sigma^2]^{.0691}$$

and

$$(2) \quad \begin{aligned} s_1 &= (L+1)\mu + [(L+1)\mu]^{.416}(\sigma^2/\mu)^{.603}U(z) \\ S_1 &= s_1 + D_p \end{aligned}$$

where $U(z)$ is given by

$$(3) \quad \begin{aligned} U(z) &= .182/z + 1.142 - 3.466z, \\ z &= \left\{ \frac{\mu^{.364}(k/h)^{.498}}{(1+p/h)[(L+1)\sigma^2]^{.431}} \right\}^{1/2} \end{aligned}$$

If D_p/μ is greater than 1.5, let $s = s_1$ and $S = S_1$. Otherwise, compute

$$(4) \quad S_2 = (L+1)\mu + v[(L+1)\sigma^2]^{1/2},$$

where v is the solution to

$$(5) \quad \int_{-\infty}^v (2\pi)^{-1/2} \exp(-x^2/2) dx = p/(p+h) .$$

The policy parameters are then given by

$$(6) \quad \begin{aligned} s &= \min\{s_1, S_2\} \\ S &= \min\{S_1, S_2\} . \end{aligned}$$

When demands are integer valued, s_1 , D_p , and S_2 are rounded to the nearest integer.

The Power Approximation assumes that the mean and variance of demand are known. For the more realistic situation in which only sample statistics of previous demands are available to the decision maker, the policy parameters, s and S , can be obtained by the Statistical Power Approximation, which is simply the Power Approximation with sample estimates for the mean and variance of demand substituted for μ and σ^2 in equations (1) through (6).

If the underlying demand sequence is stationary the entire demand history should be accumulated to yield progressively better performance; however, the inventory manager is not in a position to know that the conditions observed to date will continue to prevail. For this reason the policy parameters are periodically revised.

For our study the admittedly arbitrary choice has been made to keep a history of fixed length and to give equal weight to all observations in this

history. Let T be the length of the revision interval; that is, the number of periods between revisions. Assume that a history of T periods' demand is kept for use at each revision. If t is a period at the beginning of which a revision is made, then

$$\bar{\xi} = T^{-1} \sum_{\tau=1}^T \xi_{t-\tau}$$

(7) and

$$\bar{v} = (T - 1)^{-1} \sum_{\tau=1}^T (\xi_{t-\tau} - \bar{\xi})^2$$

are, respectively, the sample estimates for the demand mean μ and variance σ^2 required by our decision rules. Thus when using the Statistical Power Approximation we periodically obtain values for (s,S) by substituting $\bar{\xi}$ and \bar{v} for μ and σ^2 in equations (1) through (6).

1.3 Forecasting the Performance of Inventory Systems Under Statistical Control Policies

In our study we are interested in predicting system performance by examining forecasts of key operating characteristics. These system operating characteristics are: period-end holding quantity, period-end backlog quantity, frequency of period-end backlogs, frequency of replenishment, and total cost.

Consider a system in which the control parameters are revised periodically. At the beginning of each revision interval the control parameters (s,S) are revised and a forecast f is made of the mean value of each operating characteristic over the revision interval. Let a be the actual realization

of the mean operating characteristic. If the process continues over an infinite horizon, statistical equilibrium will be established and the random variables f and a will be identically distributed, though correlated, from one period to the next.

We characterize the quality of the forecasting method by its bias B , defined as

$$(8) \quad B = E(a - f)$$

and by its level of dispersion D , where

$$(9) \quad D = [\text{Var}(a - f)]^{1/2} .$$

Note that our definition of bias is the negative of the definition usually given for bias, since our biases run in this direction.

In our study we estimate B and D for a given forecasting method from the output of a computer simulation. Let f_i and a_i be, respectively, the forecast and actual operating characteristics in revision interval i of the simulation experiment. Simulated data are collected for 200 consecutive revision intervals. The sample mean m and sample variance v of the quantity $(a_i - f_i)$ are constructed using an autoregressive representation of the time series $\{(a_i - f_i), i = 1, \dots, 200\}$. We construct estimates b and d of the quantities B and D , respectively, where

$$b = m,$$

and

$$d = \sqrt{v} .$$

Our objective is to obtain a forecasting procedure which produces forecasts with biases which are close to zero and whose level of dispersion is low.

1.4 Forecasts by Retrospective Simulation

Previous forecasting studies [MacCormick (1974), Estey and Kaufman (1975), and Ehrhardt (1976)] have utilized a method called "retrospective simulation". This method entails using the same sample of demands both to set the policy parameters (s,S), and to forecast the system's performance by simulating how the policy would have performed on these demands. For a complete description of this procedure see MacCormick (1974). As a result of this double demand usage, forecasts by retrospective simulation are biased towards lower total cost.

The purpose of this report is to investigate the performance of other forecasting techniques designed to decrease prediction bias and/or dispersion. The forecasting performance of the retrospective simulation method will be used as a bench-mark to gauge improvements obtained by other methods.

1.5 Experimental Design

In our statistical policy simulations we have chosen to examine a 16-item system with the parameter settings of Table 1.1 . The system is

a subset of the 72-item negative binomial system used in previous studies [Ehrhardt (1976), MacCormick (1974)]. We have chosen to examine only those items with the highest variance-to-mean ratio and shortest revision interval since these parameter settings produced the largest forecast biases in those studies.

TABLE 1.1
SYSTEM PARAMETERS

Factor	Levels	Number of Levels in Each System
Demand Distribution	Negative Binomial ($\sigma^2/\mu = 9$)	1
Mean Demand, μ	2, 8	2
Unit Holding Cost, h	1	1
Unit Backlog Penalty Cost, p	4, 99	2
Replenishment Setup Cost, k	32, 64	2
Replenishment Leadtime, L	0, 4	2
Revision Interval	13	1

In this study we discuss forecasts of system-wide operating characteristics, and of individual item characteristics. System characteristics are defined as

the sum of the individual characteristics for holding quantity, backlog quantity, and total cost, and as the average of the individual characteristics for replenishment frequency and backlog frequency.

The performance of the retrospective simulation forecasting method on our 16-item inventory system is summarized in Table 1.2 . Notice that, on the average, forecasts underestimate each average operating characteristic value. The forecasts of holding quantity and replenishment frequency incur small percentage errors; however, backlog quantity, backlog frequency, and total cost have percentage errors of 46.7%, 23.3%, and 29.9% respectively. Most of the bias in forecasting total cost is due to the extreme underestimation of backlog quantity.

Single-item data appear in Appendix A. Notice that in forecasting backlog quantity and backlog frequency the largest forecasting errors occur for those items with a high penalty cost. This is due to the fact that a high penalty cost will cause the policy parameters to be set in such a way that backlogs are rare events. Forecasting the occurrence of these rare events is difficult, especially when using retrospective simulation which uses the same demands to set the policy parameters and again to forecast the system performance. In fact for several items with high penalty costs no backlogs were ever predicted. In forecasting total cost note that, in addition to high penalty cost items, those items with small means have higher forecasting biases. The level of dispersion in the bias for forecasting total cost is higher for those items with high penalty costs and large means.

TABLE 1.2
Retrospective Simulation Forecasting Performance on a 16-Item System

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	416.8	410.0	6.8 (27.4)	1.6
BACKLOG QUANTITY	19.1	10.2	8.9 (6.3)	46.7
BACKLOG FREQUENCY	.1126	.0864	.0262 (.029)	23.3
REPLENISHMENT FREQUENCY	.1519	.1470	.0049 (.020)	3.3
TOTAL COST	806.3	565.1	241.3 (117.5)	29.9

2. FORECASTING BY APPROXIMATE EXPRESSIONS FOR OPERATING CHARACTERISTICS

Ehrhardt (1977) developed a number of expressions that give approximate values for the following operating characteristics of (s,S) inventory systems:

- H \equiv average holding cost per period
- B \equiv average backlog cost per period
- (10) P \equiv backlog protection, i.e., frequency of periods without backlogs
- R \equiv average replenishment cost per period
- T \equiv average total cost per period .

The approximations were derived by using least-squares regression to fit the parameters of functions to the operating characteristics of a large number of parameter settings. These approximations are functions of only the economic parameters, the policy parameters, and the mean and variance of demand. They are accurate to within a few percentage points of the actual values.

In this section we forecast average operating characteristic values by using these formulas with the sample mean $\bar{\xi}$, and the sample variance \bar{v} of the previous demand history substituted for the unknown mean μ and variance σ^2 of the demand distribution.

2.1 Forecasts Using Analytical Approximations of Operating Characteristics

Approximations to the exact, analytical expressions for the operating characteristics of fixed, infinite-horizon (s,S) policies were developed by Ehrhardt (1977). He approximated these exact expressions with simplified functions and then fit their parameter values to the observed characteristics of 576 items using least-squares regression. In using these approximations

for forecasting purposes we acknowledge that the approximations were derived for infinite-horizon fixed policies and assume a known mean and variance of demand. Neither situation exists in our model; however, it was hoped that the approximations were robust enough to produce accurate forecasts.

We employ the following approximation formulas from Ehrhardt (1977) to forecast average values of the Power Approximation operating characteristics in (10):

$$H \doteq h(W - .1512\mu + .1684\sigma^2/\mu + .0689)$$

$$P \doteq rG(S|\alpha, \beta) + [(1 - r)/D]\{SG(S|\alpha, \beta) - sG(s|\alpha, \beta) - \alpha\beta G(S|\alpha + 1, \beta) + \alpha\beta G(s|\alpha + 1, \beta)\}$$

$$R \doteq k\mu/(1.003D + .4942\mu + .4990\sigma^2/\mu - .5339)$$

$$T \doteq h(1.110W + .3274D + .4476\sigma^2/\mu - .2234) + p(-.001049W + .003062\sigma^2/\mu) + .3364rk$$

where

$$D = S - s$$

$$r = \mu/(1.003D + .4942\mu + .4990\sigma^2/\mu - .5339)$$

$$W = r[SG(S|\alpha, \beta) - \alpha\beta G(S|\alpha + 1, \beta)] + [(1 - r)/2D]\{x^2 G(x|\alpha, \beta) - 2\alpha\beta x G(x|\alpha + 1, \beta) + (\alpha + 1)\alpha\beta^2 G(x|\alpha + 2, \beta)\} \Big|_s^S$$

$$G(x|\alpha, \beta) = \begin{cases} [\Gamma(\alpha)\beta^\alpha]^{-1} \int_0^x y^{\alpha-1} \exp(-y/\beta) dy, & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\alpha = (L + 1)\mu^2/\sigma^2$$

$$\beta = \sigma^2/\mu$$

Due to the complexity of evaluating the function $G(x|\alpha, \beta)$ in the expressions for W and P , we used the alternatives

$$\begin{aligned} W' = & r[SN(S_0) - \alpha\beta N(S_1)] + [(1 - r)/2D] [S^2 N(S_0) \\ & - 2\alpha\beta SN(S_1) + (\alpha + 1)\alpha\beta^2 N(S_2) - s^2 N(s_0) \\ & + 2\alpha\beta s N(s_1) - (\alpha + 1)\alpha\beta^2 N(s_2)] \end{aligned}$$

and

$$P' = rN(S_0) + [(1 - r)/D]\{SN(S_0) - sN(s_0) - \alpha\beta N(S_1) + \alpha\beta N(s_1)\}$$

where $N(\cdot)$ is the unit Normal Distribution function and

$$S_i = [S - (\alpha + i)\beta]/[(\alpha + i)\beta^2]^{1/2}$$

$$s_i = [s - (\alpha + i)\beta]/[(\alpha + i)\beta^2]^{1/2}$$

Because Ehrhardt's approximations for average backlog cost per period B are inaccurate, we used the relation $B = T - H - R$ for our forecasts. We convert the above operating characteristics to ours by employing the following identities:

$$\begin{aligned} \text{average holding quantity per period} &\equiv H \\ \text{average backlog quantity per period} &\equiv B/p \\ \text{backlog frequency} &\equiv 1 - P \\ \text{replenishment frequency} &\equiv R/k \end{aligned}$$

In Table 2.1 we summarize the accuracy of this forecasting technique. All of the operating characteristics were underestimated with the exception of replenishment frequency. In comparison with the retrospective simulation results, Table 1.2, we observe that the prediction bias has been reduced substantially in backlog quantity, from 46.7% to 29.1%, and in backlog frequency, from 23.3% to 8.4%. A slight reduction in bias is also evident in forecasting total cost. However, bias in forecasting holding quantity has increased from 1.6% to 12%, and its level of dispersion has also increased from 27 to 31. The bias dispersion levels for the remaining operating characteristics are essentially the same as those in the retrospective simulation forecasts.

Single item data of the forecasts made using the analytical approximations are given in Appendix B. As mentioned above, bias in forecasting holding quantity increased. The increased bias is particularly evident for those items with low means and high lead times. As before, items with high penalty

TABLE 2.1
Analytic Approximation Formulae Forecasting
Performance on a 16-Item System

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	421.5	371.1	50.5 (31.1)	12.0
BACKLOG QUANTITY	18.9	13.4	5.5 (6.4)	29.1
BACKLOG FREQUENCY	.1120	.1025	.0095 (.022)	8.4
REPLENISHMENT FREQUENCY	.1513	.1575	-.0062 (.019)	-4.1
TOTAL COST	800.9	598.4	202.5 (117.9)	25.3

costs experienced the largest percentage errors in forecasting backlog quantity and total cost. For these same operating characteristics, prediction bias is even greater for those items with low mean values.

2.2 Forecasts Using Multiplicative Approximations of Operating Characteristics

Approximate expressions of system operating characteristics based on multiplicative functional forms were derived by Ehrhardt (1977). The parameters of the functions were fit to the observed characteristics of a large multi-item inventory system. These approximations are much easier to evaluate and interpret than the analytical approximations discussed in Section 2.1. We considered two sets of approximations; one set derived for fixed policies and the other for statistical policies.

2.2.1 Multiplicative Approximations for Fixed Policies

In this section forecasts were made by multiplicative approximations for fixed power approximation policy operating characteristics. These approximations suffer from the same limitations as the analytic approximations, in that they assume an infinite-horizon constant policy and a known mean and variance of the demand distribution. We use the following expressions from Ehrhardt (1977) for the operating characteristics in (10)

$$\begin{aligned}
 H \approx & 2.598h \exp(.1204\sigma^2/\mu)\mu^{.5076} - .004964\sigma^2/\mu \\
 & (L + 1)^{.2287} + .01472\sigma^2/\mu - .0900\mu/\sigma^2 \\
 & (p/h)^{-3.040h/p} + .01137\sigma^2/\mu - .07292\mu/\sigma^2 + .03355 \left(\frac{\sigma^2 h}{\mu p} \right) \\
 & (k/h)^{.3057} - .02667\sigma^2/\mu + .03058\mu/\sigma^2
 \end{aligned}$$

$$P \doteq 49.74(p/h)^{-7.353h/p} \exp(.01041p/h)/(1 + p/h)$$

$$R \doteq .5763h(\sigma^2/\mu)^{-.2774} \mu^{.4959} + .007906\sigma^2/\mu - .003856\mu/\sigma^2$$

$$(L + 1)^{-.05744} (k/h)^{.6046} - .06563\mu/\sigma^2$$

$$T \doteq 2.438h(\sigma^2/\mu)^{.3836} \mu^{.4964} - .005209\sigma^2/\mu$$

$$(p/h)^{-.9230h/p} + .01508\sigma^2/\mu$$

$$(L + 1)^{.1498} + .01231\sigma^2/\mu - .07050\mu/\sigma^2$$

$$(k/h)^{.3095} - .01310\sigma^2/\mu + .1073\mu/\sigma^2$$

$$B = T - H - R .$$

Table 2.2 summarizes the forecasting performance of these approximations. As with the analytical approximation method, all the operating characteristics were underestimated with the exception of replenishment frequency. In comparison with the retrospective simulation method, Table 1.2, we discover a reduction in the percentage forecasting error for backlog quantity, backlog frequency, and total cost of 6.2%, 11.6%, and 4.4% respectively. The reductions are smaller than those obtained by the analytical approximations, Table 2.1. The difference is most noticeable in forecasting backlog quantities for which the analytical approximations achieved a 17.6% reduction. Although the multiplicative approximation procedure's bias in forecasting holding quantity was less than that of the analytical approximations, 7% as compared to 12%, the level of dispersion in the bias was almost double. Levels of dispersion for the other operating characteristics are similar to those

TABLE 2.2
Fixed Policy Multiplicative Formulae Forecasting
Performance on a 16-Item System

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	421.5	392.4	29.1 (55.6)	6.9
BACKLOG QUANTITY	18.9	11.3	7.7 (5.7)	40.5
BACKLOG FREQUENCY	.1120	.0989	.0131 (.024)	11.7
REPLENISHMENT FREQUENCY	.1513	.1591	-.0078 (.019)	-5.2
TOTAL COST	800.9	596.7	204.2 (122.7)	25.5

obtained by the previous methods.

Appendix C gives the individual item data of forecasts made by the fixed-policy multiplicative approximation method. Notice that the forecasts of backlog quantity for every item were below the actual realizations and, as before, those items with large penalty costs had the highest percentage differences. For each operating characteristic the forecasts for those items with low means, long leadtimes, and high penalty costs had the largest percentage forecasting errors.

2.2.2 Multiplicative Approximations for Statistical Policies

Ehrhardt also derived multiplicative approximations for the operating characteristics of the Statistical Power Approximation. The approximations were derived only for multi-item systems having fixed variance-to-mean ratios, so using these expressions is tantamount to assuming that the variance-to-mean ratio is known with certainty. This approach avoids the shortcomings of the others in that the approximations were derived for statistical policies but they still require the substitution of our statistical estimate of mean demand in place of the actual mean.

We used the following approximations for the characteristics in (10). The expressions require a variance-to-mean ratio of 9 and a 13-period revision interval.

$$H \doteq 10.76h\mu^{.4499}(L + 1)^{.4187}(p/h)^{-3.691h/p}(k/h)^{.1028}$$

$$R \doteq .2676h\mu^{.5688}(L + 1)^{-.1084}(k/h)^{.6470}$$

$$P \doteq (-.0057 + .9853p/h) / (1 + p/h)$$

$$T \doteq 3.776h\mu^{.4387}(L + 1)^{.3346}(p/h)^{.2727}(k/h)^{.1843}$$

$$B = T - H - R .$$

A summary of the results of forecasting system performance using these approximations appears in Table 2.3. The results clearly demonstrate how accurate these approximations are when only an estimate of the mean demand is required. Backlog quantity, backlog frequency, and total cost are forecasted with prediction errors of 4.8, 6.0, and 3.8 percent respectively, compared with 46.6, 23.3, and 29.9 percent prediction errors of the same operating characteristics when forecasting by retrospective simulation. With the exception of holding quantity, the levels of bias dispersion for the remaining operating characteristics were slightly less than those obtained by retrospective simulation. Note also that both backlog quantity and backlog frequency were overestimated. This contrasts with the severe underestimation present when retrospective simulation or the previous approximation formulas are used.

Single-item data appear in Appendix D. The most severe biases in forecasting total cost are for those items with low means and high penalty costs. The highest levels of bias dispersion are for those items with large penalty cost parameters.

Although the forecasts in this section are accurate, it must be remembered that we have assumed a knowledge of the variance-to-mean ratio,

TABLE 2.3
 Statistical Policy Multiplicative Approximation Formulae Forecasting
 Performance on a 16-Item System

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	421.5	403.4	18.1 (45.2)	4.3
BACKLOG QUANTITY	18.9	19.8	-0.9 (5.9)	-4.8
BACKLOG FREQUENCY	.1120	.1187	-.0067 (.024)	-6.0
REPLENISHMENT FREQUENCY	.1513	.1502	.0011 (.019)	.7
TOTAL COST	800.9	770	30.3 (108.9)	3.8

3. FORECASTING BY A FUTURE DEMAND GENERATION TECHNIQUE

The bias in the forecasts produced by the retrospective simulation procedure can be attributed mainly to the double use of the demand information; once to fix the policy parameters (s, S) and then again to predict the system's performance. In this section we examine a forecasting procedure which differs from the retrospective simulation method in that, instead of subjecting the policy parameters to the previous demand history, we subject them to a sequence of forecasted demands.

The sequence of forecasted demands consists of n independent demands randomly generated from a postulated demand distribution whose parameters are estimated from the sample statistics of the previous demand history. An average value for each of the system's operating characteristics is then obtained by exposing the newly revised policy to the forecasted demand sequence. This entire procedure is repeated until we have accumulated m simulated values for each operating characteristic. The forecasted value of each operating characteristic is taken as the average of the m predictions. We shall refer to m as the number of replications. Note that although we have eliminated the explicit sequence of previous demands in our forecasting simulations, the method still requires a double use of demand information: once to set the policy parameters, and then to estimate the simulated demand parameters.

Recall that in our experiments the underlying demand distribution is negative binomial with a variance-to-mean ratio of 9. For this reason, we examined the future demand generation forecasting technique with postulated demand distributions

1) negative binomial (Section 3.1)

and

2) gamma (Section 3.2) .

The gamma was chosen because it is the continuous analogue of the negative binomial distribution. By postulating a specific form of the demand distribution, serious forecasting errors may result if, in fact, the underlying demand distribution is grossly different. In our investigation of the future demand generation technique, we did not address this issue. Nonetheless, it may be worthwhile to investigate the technique's robustness.

3.1 Forecasts by Future Demand Generation with a Postulated Negative Binomial Demand Distribution

Let $\phi(\xi)$, $\xi = 0, 1, 2, \dots$, be the probability distribution of demand per period. The mathematical form of the negative binomial distribution is

$$\phi(\xi) = \binom{r-1+\xi}{r-1} q^r p^\xi, \quad p+q=1, \quad p, q \geq 0$$

with expected value $\mu = rp/q$, variance $\sigma^2 = rp/q^2$ and variance-to-mean ratio $\sigma^2/\mu = 1/q$.

We utilize the method of moments to estimate the parameters p and r . If t is the first period in a revision interval and $\xi_{t-1}, \xi_{t-2}, \dots, \xi_{t-T}$ is the previous demand history, we calculate the solutions \hat{p}, \hat{r} of the equations

$$(11) \quad \bar{\xi} = \hat{r} \hat{p} / (1 - \hat{p}) \quad \text{and} \quad \overline{v} = \hat{r} \hat{p} / (1 - \hat{p})^2$$

where $\bar{\xi}$ and \bar{v} are the sample mean and variance, defined in (7). We solve for \hat{p} and \hat{r} to obtain

$$(12) \quad \hat{p} = 1 - \bar{\xi}/\bar{v} \quad \text{and} \quad \hat{r} = \bar{\xi}^2/(\bar{v} - \bar{\xi}) .$$

Forecasts are made by subjecting the revised policy to a sequence of n demands generated from a negative binomial distribution with parameters \hat{p} and \hat{r} . To be consistent with previous results, the value of T chosen was 13, i.e., only the 13 most recent demands were employed in computing \hat{p} and \hat{r} .

Before an assessment of the forecasting accuracy of this method can be made, the size of the generated demand sequence n , and the number of replications m must be set.

Table 3.1 was constructed to demonstrate the sensitivity of the bias to the value of n . As n increases we note a decrease in the bias of forecasts of backlog quantity, backlog frequency, and replenishment frequency, together with a reduction in the bias dispersion for each operating characteristic. In particular, the dispersion of the bias for total cost was reduced by roughly 30% when n was increased from 13 to 100. However, increasing n also results in an enormous increase in the bias of holding quantity forecasts, with a corresponding rise in the bias for total cost. This increase is due to less significant transient effects when the policy is operated over a prolonged interval. To illustrate, let y be the inventory on hand at the end of any period and suppose that upon revision of (s, S) to values, say (s', S') , y is greater than S' . Let k be the number of periods that y is above S' , and note that

TABLE 3.1

Sensitivity of Bias to the Length of the Demand Sequence
Forecasts made by Future Demand Generation Technique
with a Single Replication

OPERATING CHARACTERISTIC	LENGTH OF DEMAND SEQUENCE	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	13	416.8	416.7	0.1 (36.7)	0.023
	100		379.7	37.1 (28.6)	8.9
BACKLOG QUANTITY	13	19.1	12.5	6.6 (8.5)	34.6
	100		13.3	5.8 (6.7)	30.4
BACKLOG FREQUENCY	13	.1126	.0895	.0231 (.033)	20.5
	100		.0964	.0162 (.025)	14.4
REPLENISHMENT FREQUENCY	13	.1519	.1474	.0045 (.023)	3.0
	100		.1553	-.0034 (.019)	-2.2
TOTAL COST	13	806.3	622.7	183.6 (170.2)	22.8
	100		593.1	213.2 (122.4)	26.4

once y falls below S' it will not exceed S' in any of the following periods. As the ratio of k to the number of periods that (s', S') is in effect decreases, the effect of those k periods of high inventory levels on the average inventory level is decreased. Hence, increasing n leads to underestimation of actual holding quantities. Because of this degradation, n was set equal to the revision interval length (13 periods).

The effect of increasing the number of replications on prediction bias is shown in Table 3.2. Observe that increasing m from one to eight results in a substantial decrease in bias dispersion with only a minor change in the percentage forecasting bias for each operating characteristic. Further increasing m shows only a slight decrease in bias dispersion and, in fact, a small increase in prediction bias.

We summarize in Table 3.3 the system performance of the future demand generation forecasting method with n and m equal to 13 and 8, respectively. Notice that the forecast of holding quantity is extremely accurate, however, as with preceding methods, forecasts of backlog frequency, backlog quantity (and thus total cost), have large biases. In comparison with the retrospective simulation technique we observe a smaller prediction bias for every operating characteristic. Bias dispersion levels are approximately the same for both procedures.

Appendix E contains the single-item forecasting data. Note that on the average forecasts of backlog quantity were below the average actual realizations for every item. As before, percentage forecasting errors of backlog quantity, backlog frequency, and total cost, tend to increase with increases in the penalty cost and the leadtime, and with decreases in the mean.

Much of the bias in the forecasts produced by the future demand generation forecasting procedure is the outgrowth of a double use of the

TABLE 3.2

Sensitivity of Bias to the Number of Replications. Forecasts made using Future Demand Generation Technique with demand sequence of length $n = 13$.

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	NUMBER OF REPLICATIONS	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	416.8	1	416.7	0.1 (36.6)	.023
		8	417.1	-0.3 (29.7)	-0.65
		20	417.9	-1.0 (29.0)	-.245
BACKLOG QUANTITY	19.1	1	12.5	6.6 (8.5)	34.6
		8	12.7	6.4 (6.6)	33.7
		20	12.7	6.5 (6.5)	33.8
BACKLOG FREQUENCY	.1126	1	.0895	.0231 (.033)	20.5
		8	.0894	.0232 (.026)	20.6
		20	.0882	.0244 (.025)	21.7
REPLENISHMENT FREQUENCY	.1519	1	.1474	.0045 (.023)	3.0
		8	.1473	.0046 (.019)	3.1
		20	.1472	.0047 (.019)	3.1
TOTAL COST	806.3	1	622.7	183.6 (170.2)	22.8
		8	617.1	189.2 (121.2)	23.5
		20	613.9	192.4 (119.2)	23.9

TABLE 3.3
 Future Demand Generation Forecasting Performance
 on a 16-Item System with $n = 13$ and $m = 8$.

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	416.8	417.1	-0.3 (29.7)	-0.65
BACKLOG QUANTITY	19.1	12.7	6.4 (6.6)	33.7
BACKLOG FREQUENCY	.1126	.0894	.0232 (.026)	20.6
REPLENISHMENT FREQUENCY	.1519	.1473	.0046 (.019)	3.1
TOTAL COST	806.3	617.1	189.2 (121.2)	23.5

demand statistics; the same statistics being used to fix the policy parameters (s, S) and to estimate demand distribution parameters. This demand usage is less direct than that employed by retrospective simulation, and as a result forecast bias is reduced.

3.1.1 Future Demand Generation Forecasting Technique Using Variable Means and Variances

In Section 3.1 it was pointed out that the future demand generation technique produced biased forecasts, although less biased than those obtained with retrospective simulation. In this section we discuss a sampling procedure designed to further reduce the undesirable side effects caused by double use of demand information.

We explicitly acknowledge the uncertainty about the demand distribution parameters by estimating their probability distributions and then sampling from these distributions to obtain estimates of the parameters of the demand distribution. The estimated parameters are then used to generate a sequence of forecasted demands.

We construct an approximate probability distribution for the demand mean μ by noting that the sample mean \bar{x} of a sequence of n independent, identically distributed random variables has an asymptotic normal distribution. Hence, we sample for μ from a normal distribution with mean $\bar{\xi}$ and variance \bar{v}/n , where $\bar{\xi}$ and \bar{v} are as defined in (7). Asymptotic theory also tells us that if we assume $\bar{\xi}$ and \bar{v} are independent, then the distribution of \bar{v} will approach a chi-square distribution as n increases. Thus, we sample for the demand variance σ^2 from a chi-square distribution with $n - 1$ degrees of freedom.

For each replication s of the future demand generation technique we use the above sampling procedure to obtain estimates $\hat{\mu}_s$ and $\hat{\sigma}_s^2$ of the mean and variance of the demand distribution. Our forecasted demand sequence is then generated from a negative binomial distribution with parameters $\hat{p}_s = 1 - \hat{\mu}_s / \hat{\sigma}_s^2$ and $\hat{r}_s = \hat{\mu}_s^2 / (\bar{v}_s - \hat{\mu}_s)$. Therefore, each replication in our forecasting procedure uses demands generated from a probability distribution which is itself generated from a postulated distribution.

Table 3.4 summarizes the forecasting performance of this method. In comparison with the same procedure without sampling for demand parameters, Table 3.3, we note a reduction in forecasting bias for all operating characteristics except holding quantity which has increased slightly. Of particular importance is the fact that the percentage underestimation of backlog quantity has been reduced from 33.7% to 22.3%, and that of backlog frequency from 20.6% to 15.0%. These improvements are accompanied with only a small increase in bias dispersion.

Single-item data appear in Appendix F. Sensitivity to parameter settings is analogous to that observed without sampling for demand means and variances.

3.2 Forecasts by Future Demand Generation with a Postulated Gamma Demand Distribution

In this section we examine the forecasting performance of the future demand generation technique when the demands are generated from a Gamma distribution whose parameters are estimated using sample statistics of the previous demand history. Since the gamma is a continuous distribution, the

TABLE 3.4
 Performance of the Future Demand Generation Technique
 Using Variable Means and Variances

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS (LEVEL OF DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	416.8	423.4	-6.5 (31.1)	-1.6
BACKLOG QUANTITY	19.1	14.8	4.3 (7.3)	22.3
BACKLOG FREQUENCY	.1126	.0957	.0169 (.026)	15.0
REPLENISHMENT FREQUENCY	.1519	.1486	.0033 (.020)	2.2
TOTAL COST	806.3	639.5	166.8 (124.4)	20.7

demands generated from it constitute a sequence of nonnegative real numbers. The generated demands were not rounded-off to the nearest integer.

The gamma is a two-parameter distribution, denoted by $G(\alpha, \beta)$, where α and β are positive and real. If X is a random variable having a gamma distribution, then the probability density function of X is

$$f_X(x) = \begin{cases} x^{\alpha-1} e^{-x/\beta} / \beta^\alpha \Gamma(\alpha) & 0 \leq x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

with expectation $\mu = \alpha \beta$, variance $\sigma^2 = \alpha \beta^2$, and variance-to-mean ratio $\sigma^2/\mu = \beta$.

As was the case for the negative binomial distribution, we use the method of moments to obtain estimates of the parameters α and β . Thus, if t is the first period of a revision interval and $\xi_{t-1}, \xi_{t-2}, \dots, \xi_{t-T}$ is the previous demand history, we calculate the solutions $\hat{\alpha}, \hat{\beta}$ of the equations

$$(13) \quad \bar{\xi} = \hat{\alpha} \hat{\beta} \quad \text{and} \quad \bar{v} = \hat{\alpha} \hat{\beta}^2,$$

where $\bar{\xi}$ and \bar{v} are as defined in (7).

Hence

$$(14) \quad \hat{\alpha} = \bar{\xi}^2 / \bar{v} \quad \text{and} \quad \hat{\beta} = \bar{v} / \bar{\xi}.$$

Forecasts are made at the beginning of each revision interval by subjecting the revised control policies to a sequence of n independent

demands generated from a $G(\hat{\alpha}, \hat{\beta})$ distribution. In keeping with the practice employed in the other forecasting methods, only the 13 most recent demands were used in computing $\bar{\xi}$ and \bar{v} .

A system performance summary of this forecasting method for single and multiple replications appears in Table 3.5. Notice that forecasts of backlog quantity and backlog frequency have negative biases which contrasts with the positive biases produced by the forecasting methods discussed previously. Again we see that increasing the number of replications decreases the dispersion of the bias. In particular, bias dispersion for backlog quantity and total cost has been reduced approximately 30% by increasing m from 1 to 10. A comparison of this procedure, using ten replications, with the retrospective simulation method of forecasting gives several interesting observations. First, the percentage forecasting bias for backlog quantity is 5.2% for this method as compared with the gross 46.6% bias by retrospective simulation forecasting. Second, the improved forecasts of backlog quantity lead to a smaller bias in forecasting total cost, 11% as compared with almost 30% by retrospective simulation. Third, the dispersions are greater, most notably for the total cost forecast which has a dispersion of 185 as compared to 117 for retrospective simulation.

Appendix G gives the individual-item forecasts using ten replications. Forecasting bias for total cost tends to increase as leadtime and penalty cost increase, and decrease as the mean increases. Note also that the dispersion of the bias for total cost is highly sensitive to penalty cost, and is generally greater for those items with a high setup cost.

TABLE 3.5

Future Demand Generation Forecasting Performance with Demands
Generated from an Estimated Gamma Distribution $n=13$

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	NUMBER OF REPLICATIONS	AVERAGE FORECASTED VALUE	BIAS (DISPERSION)	PERCENTAGE UNDERESTIMATES
HOLDING QUANTITY	416.8	1	400.0	16.9 (40.8)	4.0
		10	399.4	17.4 (35.0)	4.2
BACKLOG QUANTITY	19.1	1	19.7	-.6 (11.0)	-3.2
		10	20.1	-1.0 (8.5)	-5.3
BACKLOG FREQUENCY	.1126	1	.1266	-.0139(.039)	-12.4
		10	.1274	-.0148(.031)	-13.1
REPLENISHMENT FREQUENCY	.1519	1	.1558	-.0038(.024)	-2.5
		10	.1562	-.0043(.022)	-2.8
TOTAL COST	806.3	1	712.0	94.3 (238.3)	11.7
		10	716.6	89.7 (184.7)	11.1

4. COMPARISON OF FORECASTING PROCEDURES

In this section we compare the performance of the forecasting procedures discussed in the preceding sections. We also suggest topics for future research.

4.1 Properties of Single-item Forecasts

As a measure of the deviation between forecasted values f , and actual realizations a , we define the mean square error of a single-item forecast as

$$\text{MSE} \equiv E(a - f)^2 ,$$

and the root mean square error as

$$\text{RMSE} \equiv \sqrt{\text{MSE}} .$$

Using the fact that

$$\text{Var}(a - f) = E(a - f)^2 - [E(a - f)]^2 ,$$

and the definitions of bias B and dispersion D given in (8) and (9), we have

$$(15) \quad \text{MSE} = B^2 + D^2 .$$

Hence, a forecasting scheme's MSE becomes larger with increases in bias and/or dispersion.

Previous studies [Ehrhardt (1976), MacCormick (1974)] have indicated that for single-item forecasts using retrospective simulation, the dispersion

term dominates the MSE. In fact, the dispersion is so large that the forecasting method is essentially useless for single items. In our study we have observed this phenomenon. For not only the retrospective simulation method, but for the other methods investigated as well. This fact is exhibited by the single-item data in Appendices A through G.

In the next section we examine the impact of dispersion on the MSE when the single item forecasts of each operating characteristic are aggregated in a multi-item system.

4.2 Properties of Multi-item Forecasts

In the previous section we observed that for single-item forecasts the dispersion overwhelms the bias, and this was true for each of the forecasting methods discussed in this study. However, we will show that the dispersion becomes small relative to the bias when forecasts are aggregated in a multi-item system containing a sufficiently large number of items with independent demand processes.

To illustrate, consider a system consisting of n independent items. Let b_i and d_i represent, respectively, the bias and dispersion for item i . Thus, when single-item forecasts are aggregated over the multi-item system, the aggregate bias B and dispersion D are given by

$$B = \sum_{i=1}^n b_i \quad \text{and} \quad D = \left(\sum_{i=1}^n d_i^2 \right)^{1/2} .$$

The mean square error of the multi-item forecast can be expressed as

$$(16) \quad \text{MSE} \equiv B^2 + D^2 = \left(\sum_{i=1}^n b_i \right)^2 + \sum_{i=1}^n d_i^2 .$$

If we let

$$(17) \quad \bar{b} = n^{-1} \sum_{i=1}^n b_i \quad \text{and} \quad \bar{d} = n^{-1} \sum_{i=1}^n d_i^2,$$

then (16) becomes

$$\text{MSE} \equiv n \bar{b}^2 + n \bar{d}.$$

We assume that b_i and d_i are finite for all i . Therefore, we observe that for sufficiently large values of n and $\bar{b} \neq 0$, the dominant factor in the MSE of a multi-item forecast is the bias.

When comparing forecasting procedures for particular operating characteristic the above discussion suggests that the procedure which exhibits the smallest MSE in our 16-item system will not necessarily have the smallest MSE for any system size.

To illustrate, consider a hypothetical system consisting of m independent 16-item systems, each identical to ours. Suppose that in our 16-item system forecasting procedure A has bias and dispersion quantities \bar{b}_A and \bar{d}_A as defined in (17). Let forecasting procedure B be characterized by bias and dispersion quantities \bar{b}_B and \bar{d}_B , defined in the same way. Further suppose that $\bar{b}_A > \bar{b}_B$ and $\bar{d}_A < \bar{d}_B$. Then for any m we have

$$\text{MSE}_A - \text{MSE}_B = m^2 (\bar{b}_A^2 - \bar{b}_B^2) + m (\bar{d}_A - \bar{d}_B).$$

Let $\alpha = (\bar{b}_A^2 - \bar{b}_B^2)$ and $\beta = (\bar{d}_A^2 - \bar{d}_B^2)$. Then

$$\text{MSE}_A - \text{MSE}_B = \alpha m^2 + \beta m$$

is quadratic in m with roots at 0 and at $-\beta/\alpha$. Thus, for every m less than $m^* = -\beta/\alpha$ method A will have lower MSE than method B, and method B will have lower MSE than method A for every m greater than m^* . We can interpret m^* as the forecasting system size for which the better method changes.

Tables 4.1 and 4.2 provide a summary of the performance of the forecasting procedures discussed in this study. The entries in Table 4.1 are the bias, dispersion and root mean square error of each operating characteristic forecast, and Table 4.2 has these same entries expressed as the percentage of the actual value. Observe that for holding quantity the retrospective simulation method has the smallest RMSE, but the future demand generation technique with postulated negative binomial demand distribution has the smallest bias: Therefore, for sufficiently large systems the latter method will dominate. For backlog frequency the analytic approximation method has the smallest bias and lowest dispersion, and is, therefore, superior for all system sizes. For backlog quantity and replenishment frequency the analytic approximation method also has the lowest RMSE; however, the smallest biases for these operating characteristics are produced by the future demand generation method with postulated negative binomial demand distribution. We, therefore, can conclude that in larger systems this method will dominate the analytic approximation technique. Finally, for total cost the future demand generation technique with postulated gamma demand distribution has the smallest RMSE, but it has a

TABLE 4.1
 Comparison of Forecasting Methods' Performance on a 16-item Inventory System

FORECASTING METHOD	HOLDING QUANTITY	BACKLOG QUANTITY	BACKLOG FREQUENCY	REPLENISHMENT FREQUENCY	TOTAL COST
RETROSPECTIVE SIMULATION	6.8 (27.4) [28.2]	8.9 (6.3) [10.9]	.0262 (.029) [.0391]	.0049 (.020) [.0206]	241.3 (117.5) [268.4]
ANALYTIC APPROXIMATION FORMULAE	50.5 (31.1) [59.3]	5.5 (6.4) [8.4]	.0095 (.022) [.0239]	-.0062 (.019) [.0200]	202.5 (117.9) [234.3]
FIXED POLICY MULTIPLICATIVE APPROXIMATIONS	29.1 (55.6) [62.8]	7.7 (5.7) [9.6]	.0131 (.024) [.0274]	-.0078 (.019) [.0205]	204.2 (122.7) [238.2]
FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)	-6.5 (31.1) [31.8]	4.3 (7.3) [8.5]	.0169 (.026) [.0310]	.0033 (.020) [.0203]	166.8 (124.4) [208.1]
FUTURE DEMAND GENERATION (GAMMA)	17.4 (35.0) [39.1]	-1.0 (8.5) [8.6]	-.0148 (.031) [.0344]	-.0043 (.022) [.0224]	89.7 (184.7) [205.3]

Entries are: Bias (Dispersion)
[RMSE]

TABLE 4.2

Comparison of Forecasting Methods' Performance on a 16-item System

FORECASTING METHOD	HOLDING QUANTITY	BACKLOG QUANTITY	BACKLOG FREQUENCY	REPLENISHMENT FREQUENCY	TOTAL COST
RETROSPECTIVE SIMULATION	1.6 (6.6) [6.8]	46.6 (33.0) [57.1]	23.3 (25.8) [34.7]	3.3 (13.2) [13.6]	29.9 (14.6) [33.3]
ANALYTIC APPROXIMATION FORMULAE	12.0 (7.4) [14.1]	29.1 (33.9) [44.4]	8.4 (19.6) [21.3]	-4.1 (12.6) [13.2]	25.3 (14.7) [29.3]
FIXED POLICY MULTIPLICATIVE APPROXIMATIONS	6.9 (13.2) [14.9]	40.5 (30.2) [50.8]	11.7 (21.5) [24.5]	-5.2 (12.6) [13.5]	25.5 (15.3) [29.7]
FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)	-1.6 (7.5) [7.6]	22.3 (38.2) [44.5]	15.0 (23.1) [27.5]	2.2 (13.2) [13.5]	20.7 (15.4) [25.8]
FUTURE DEMAND GENERATION (GAMMA)	4.2 (8.4) [9.4]	-5.3 (44.5) [45.0]	-13.1 (27.5) [30.6]	-2.8 (14.5) [14.7]	11.1 (22.9) [25.5]

Entries are: Bias (Dispersion)
[RMSE]
expressed as percentage of actual

higher level of dispersion than any of the other methods. As a result, we would expect one of the other methods to dominate for systems having fewer items.

In conclusion, we observe that no single method is uniformly superior for each operating characteristics. The two most promising procedures appear to be the analytic approximation method and the future demand generation techniques.

One additional issue that must be considered in selecting a forecasting procedure, is its simplicity of implementation. In this respect the analytic approximations are clearly superior to the future demand generation techniques. The analytic approximation method requires only the computation of a formula for each operating characteristic. Although these formulas are rather complicated, their evaluation requires considerably less effort than the future demand generation techniques which require the generation of many random variates and the explicit simulation of system behavior.

4.3 Topics for Future Research

In this section we suggest several topics for future research in forecasting operating characteristic values.

Recall that in Section 2.2.2 we observed that the statistical policy multiplicative approximations perform extremely well, but are not applicable in actual practice since they were derived for a specific variance-to-mean ratio. Consequently, it may be promising to investigate statistical-policy operating characteristic formulas that include the variance-to-mean ratio as a variable if and when these formulas are derived.

Another possible avenue for future research is the investigation of alternative demand estimation methods in the future demand generation

technique. In our research we have used the method of moments. One may be able to obtain better estimates of demand parameters by using other methods. For example, in the case of the negative binomial distribution we might consider the maximum likelihood method.

Further research is also needed in determining the robustness of the future demand generation technique to misspecification of the form of the underlying demand distribution.

A possible improvement in the forecasting performance of the retrospective simulation method may lie in the derivation of an analytical expression for the bias in its forecasts. This expression could then be used as a correction factor for the forecasts. Research along this line is presently underway.

Another possible extension of this study could consider hybrid forecasting procedures, which would use several methods of forecasting simultaneously. In principle, one could then use the best method for each operating characteristic and also choose the best combination of holding cost, backlog cost, and replenishment cost forecasts to obtain a total cost estimate.

Finally we describe a forecasting approach that was mentioned earlier in this report. The method reduces the double-use of information by splitting the sample of demand observations in half. One half of the sample is used to derive a policy rule while the other half is used to forecast the policy's performance. However, the decision-maker will want to use a policy based on the entire sample of demands. Therefore, we need to understand the relationship between the split-sample forecast and the performance of the policy based on the entire sample. This method is presently under investigation.

APPENDICES

<u>Appendix</u>	<u>Description</u>	<u>Pages</u>
A	Single item forecasts using retrospective simulation	A-1, A-2, A-3
B	Single item forecasts using analytical approximations	B-1, B-2, B-3
C	Single item forecasts using fixed policy multiplicative approximations	C-1, C-2, C-3
D	Single item forecasts using statistical policy multiplicative approximations	D-1, D-2, D-3
E	Single item forecasts using future demand generation (negative binomial)	E-1, E-2, E-3
F	Single item forecasts using future demand generation with variable means and variances	F-1, F-2, F-3
G	Single item forecasts using future demand generation (gamma)	G-1, G-2, G-3

BIBLIOGRAPHY

- Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report #7, School of Organization and Management, Yale University.
- Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ARO Technical Report #12, Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill.
- Estey, A. and R. Kaufman (1975), Multi-item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9), ONR and ARO Technical Report #3, School of Organization and Management, Yale University.
- Iglehart, D. (1963a), "Optimality fo (s,S) Policies in the Infinite Horizon Dynamic Inventory Problem," Management Science, 9, pp. 259-267.
- Iglehart, D. (1963b), "Dynamic Programming and Stationary Analysis of Inventory Probelms," Multistage Inventory Models and Techniques, edited by H. Scarf, D. Gilford and M. Shelly, Stanford University Press.
- MacCormick, A. (1974), Statistical Problems in Inventory Control, ONR and ARO Technical Report #2, School of Organization and Management, Yale University.
- Veinott, A. and H. Wagner (1965), "Computing Optimal (s,S) Inventory Policies," Management Science, 11, pp. 525-552.

APPENDIX A

<u>Table</u>	<u>Description</u>	<u>Page</u>
A1	Single item forecasts of holding and backlog quantity using retrospective simulation	A-1
A2	Single item forecasts of backlog and replenishment frequency using retrospective simulation	A-2
A3	Single item forecasts of total cost using retrospective simulation	A-3

TABLE A1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING RETROSPECTIVE SIMULATION

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	5.6002	5.9907	-0.3905	2.2639	6.572	6.572
2	0	4	64	6.4698	6.6864	-0.2166	3.2001	3.349	3.349
2	0	99	32	17.3214	17.3214	-0.19647	3.0996	1.147	1.147
2	0	99	64	17.2720	17.9968	-0.72483	3.9219	4.197	4.197
2	4	4	32	10.2225	9.9711	0.25140	5.0215	2.459	2.459
2	4	4	64	12.1502	10.5591	1.59107	6.4146	13.095	13.095
2	4	99	32	38.7644	38.7345	0.02991	8.1739	0.077	0.077
2	4	99	64	34.9989	35.9469	-0.94799	8.7170	2.709	2.709
8	0	4	32	11.2149	10.7234	0.49158	2.4595	4.383	4.383
8	0	4	64	12.6777	12.9210	-0.04332	3.6236	0.336	0.336
8	0	99	32	31.5995	31.9705	-0.46089	3.6828	1.463	1.463
8	0	99	64	33.2635	33.8453	-0.58183	3.9835	1.749	1.749
8	4	4	32	23.7531	19.9626	3.79076	8.6187	15.959	15.959
8	4	4	64	22.1750	19.8691	2.30595	8.7471	10.399	10.399
8	4	99	32	70.7571	69.6061	1.15128	12.3736	1.627	1.627
8	4	99	64	68.6904	67.8507	0.84005	12.2216	1.223	1.223
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				416.8840	409.9551	6.88955	27.4116	1.653	1.653

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	0.7915	0.6815	0.10993	0.6524	13.890	13.890
2	0	4	64	0.8618	0.7469	0.11486	0.7244	13.328	13.328
2	0	99	32	0.1678	0.0135	0.15430	0.1523	91.976	91.976
2	0	99	64	0.1802	0.0138	0.16633	0.1666	92.315	92.315
2	4	4	32	2.2398	0.6742	1.56561	1.8277	69.899	69.899
2	4	4	64	2.0322	0.9258	1.10649	1.9808	54.447	54.447
2	4	99	32	0.2696	0.0	0.26957	0.4920	100.000	100.000
2	4	99	64	0.3340	0.0	0.33397	0.4649	100.000	100.000
8	0	4	32	1.6231	1.5546	0.06851	1.2069	4.221	4.221
8	0	4	64	1.8043	1.8469	-0.04255	1.2381	2.358	2.358
8	0	99	32	0.1980	0.0135	0.18454	0.2151	93.201	93.201
8	0	99	64	0.2082	0.0215	0.18668	0.2171	89.656	89.656
8	4	4	32	3.6077	1.4319	2.17581	3.7214	60.310	60.310
8	4	4	64	4.0402	2.2550	1.78522	3.5572	44.187	44.187
8	4	99	32	0.3521	0.0096	0.36175	0.6062	98.689	98.689
8	4	99	64	0.4078	0.0	0.40782	0.7171	100.000	100.000
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				19.1182	10.1837	8.93460	6.2602	46.733	46.733

TABLE A2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING RETROSPECTIVE SIMULATION
FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	0.1494	0.1342	0.01514	0.1175	10.138	10.138
2	0	4	64	0.1794	0.1788	0.00061	0.1855	0.339	0.339
2	0	99	32	0.0239	0.0069	0.01696	0.0316	71.009	71.009
2	0	99	64	0.0259	0.0065	0.01940	0.0334	74.795	74.795
2	4	4	32	0.2288	0.1431	0.08574	0.1873	37.472	37.472
2	4	4	64	0.2073	0.1488	0.05845	0.1921	28.198	28.198
2	4	99	32	0.0283	0.0	0.02826	0.0507	100.000	100.000
2	4	99	64	0.0351	0.0	0.03507	0.0481	100.000	100.000
8	0	4	32	0.1803	0.1873	-0.00696	0.0922	3.560	3.560
8	0	4	64	0.2000	0.2135	-0.01349	0.0941	6.746	6.746
8	0	99	32	0.0220	0.0042	0.01777	0.0291	80.769	80.769
8	0	99	64	0.0231	0.0050	0.01814	0.0291	78.388	78.388
8	4	4	32	0.2112	0.1527	0.05848	0.1864	27.692	27.692
8	4	4	64	0.2341	0.2011	0.03295	0.1671	14.074	14.074
8	4	99	32	0.0249	0.0008	0.02409	0.0403	96.906	96.906
8	4	99	64	0.0284	0.0	0.02838	0.0464	100.000	100.000
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1126	0.0864	0.02619	0.0288	23.251	23.251

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	0.1185	0.1108	0.00769	0.0904	6.494	6.494
2	0	4	64	0.0919	0.0835	0.00846	0.0696	9.205	9.205
2	0	99	32	0.1135	0.1108	0.00269	0.0844	2.373	2.373
2	0	99	64	0.0915	0.0823	0.00923	0.0779	10.084	10.084
2	4	4	32	0.0977	0.0958	0.00192	0.0797	1.969	1.969
2	4	4	64	0.0808	0.0773	0.00346	0.0672	4.286	4.286
2	4	99	32	0.0819	0.0823	-0.00038	0.0694	0.469	0.469
2	4	99	64	0.0635	0.0635	0.00038	0.0568	0.606	0.606
8	0	4	32	0.2623	0.2519	0.01038	0.0966	3.959	3.959
8	0	4	64	0.2008	0.1938	0.00692	0.0774	3.488	3.488
8	0	99	32	0.2527	0.2504	0.00231	0.0939	0.913	0.913
8	0	99	64	0.1942	0.1942	0.0	0.0738	0.0	0.0
8	4	4	32	0.2354	0.2288	0.00654	0.0909	2.778	2.778
8	4	4	64	0.1777	0.1754	0.00231	0.0696	1.299	1.299
8	4	99	32	0.2054	0.1969	0.00846	0.0916	4.120	4.120
8	4	99	64	0.1635	0.1546	0.00885	0.0778	5.412	5.412
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1519	0.1470	0.00495	0.0200	3.259	3.259

TABLE A3: SINGLE ITEM FORECASTS OF TOTAL COST USING RETROSPECTIVE SIMULATION

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0				REVISION INTERVAL = 13				
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF FIAS	PERCENTAGE ERROR
2	0	4	32	12.5569	12.2614	0.29543	4.9929	2.353
2	0	4	64	15.7999	15.0156	0.78429	6.5791	4.964
2	0	99	32	37.3637	22.1906	15.16576	15.5762	40.590
2	0	99	64	40.9674	24.6349	16.33299	18.4283	39.868
2	4	4	32	22.3079	15.7326	6.57538	7.2005	29.476
2	4	4	64	25.4481	19.2099	6.23854	7.9139	24.515
2	4	99	32	68.0734	41.3682	26.70518	46.1822	39.230
2	4	99	64	72.1237	39.9836	32.13974	44.5899	44.562
8	0	4	32	26.1009	25.0030	1.09791	6.1543	4.206
8	0	4	64	32.9436	32.7182	0.22554	8.1056	0.697
8	0	99	32	59.1978	41.3151	17.88274	21.0649	30.208
8	0	99	64	66.3079	48.4079	17.90004	22.2001	26.995
8	4	4	32	45.7158	33.0126	12.70343	12.1058	27.788
8	4	4	64	49.7073	40.1127	9.59461	11.6281	19.302
8	4	99	32	112.1910	76.3683	35.82608	55.7844	31.933
8	4	99	64	119.5264	77.7456	41.77965	66.9429	34.954
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				806.3311	565.0793	241.25125	117.4782	29.920

APPENDIX B

<u>Table</u>	<u>Description</u>	<u>Page</u>
B1	Single item forecasts of holding and backlog quantity using analytical approximations	B-1
B2	Single item forecasts of backlog and replenishment frequency using analytical approximations	B-2
B3	Single item forecasts of total cost using analytical approximations	B-3

TABLE 81: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING ANALYTICAL APPROXIMATIONS

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS:	AVERAGE	AVERAGE	ACTUAL VALUES	DISPERSION	PERCENTAGE		
		C(OUT)/C(IN)	ACTUAL VALUE	FORECAST VALUE	LESS FORECASTS	OF BIAS	ERROR		
2	0	4	5.7559	5.5454	0.2105	2.0967	3.656		
2	0	4	6.3302	6.1534	0.17672	2.7772	2.792		
2	0	99	17.3530	14.8840	2.46888	4.5225	14.227		
2	0	99	17.8625	15.6952	2.16731	4.5382	12.133		
2	4	4	10.5087	8.5186	1.99010	5.4016	18.938		
2	4	4	12.7705	9.4274	3.34308	6.4307	26.178		
2	4	99	38.3212	28.9884	9.33292	11.4743	24.354		
2	4	99	36.3076	25.7925	10.51562	13.0000	28.963		
8	0	4	11.3903	11.5292	-0.13880	2.2235	1.219		
8	0	4	12.6214	12.6465	-0.02502	2.8436	0.198		
8	0	99	32.1836	30.6001	1.58363	2.7634	4.921		
8	0	99	32.5880	31.7882	1.19975	3.5203	3.637		
8	4	4	23.6489	20.9221	2.72702	7.9766	11.531		
8	4	4	22.7877	19.7832	3.00465	8.3054	13.185		
8	4	99	71.6220	65.5094	6.11267	13.6658	8.535		
8	4	99	69.0973	63.3080	5.78939	13.7254	8.379		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :			421.5483	371.0913	50.45830	31.0669	11.970		

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS:	AVERAGE	AVERAGE	ACTUAL VALUES	DISPERSION	PERCENTAGE		
		C(OUT)/C(IN)	ACTUAL VALUE	FORECAST VALUE	LESS FORECASTS	OF BIAS	ERROR		
2	0	4	0.7741	1.0287	-0.25460	0.7938	32.892		
2	0	4	0.9083	1.0140	-0.10567	0.8832	11.633		
2	0	99	0.1693	0.0591	0.11021	0.1781	65.110		
2	0	99	0.1654	0.0574	0.10798	0.1613	65.303		
2	4	4	2.2826	1.1231	1.15954	2.0667	50.799		
2	4	4	1.9233	1.4043	0.51898	1.8996	26.984		
2	4	99	0.2563	0.0650	0.19128	0.4523	74.638		
2	4	99	0.3000	0.0637	0.23630	0.4538	78.771		
8	0	4	1.5735	1.7307	-0.15718	1.1850	9.989		
8	0	4	1.9292	1.7802	0.14903	1.3465	7.725		
8	0	99	0.1795	0.0824	0.09707	0.1980	54.090		
8	0	99	0.2100	0.0790	0.13102	0.2287	62.386		
8	4	4	3.3462	2.2890	1.05628	3.4010	31.566		
8	4	4	4.1545	2.4202	1.73438	3.9330	41.747		
8	4	99	0.3394	0.1006	0.23881	0.6427	70.365		
8	4	99	0.4062	0.1076	0.29855	0.7277	73.503		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :			18.9176	13.4057	5.51198	6.4066	29.137		

TABLE B2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING ANALYTICAL APPROXIMATIONS
FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C (OUT)/C (IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	0.1450	0.1680	-0.04257	0.0890	29.627	
2	0	4	64	0.1928	0.2266	-0.03379	0.1535	17.526	
2	0	99	32	0.0242	0.0351	-0.01093	0.0319	45.205	
2	0	99	64	0.0237	0.0258	-0.00208	0.0278	45.218	
2	4	4	32	0.2326	0.1274	0.10516	0.1316	9.940	
2	4	4	64	0.1961	0.1766	0.01949	0.1326	76.535	
2	4	99	32	0.0269	0.0475	-0.02058	0.0470	45.104	
2	4	99	64	0.0315	0.0457	-0.01422	0.0484	18.350	
8	0	4	32	0.1748	0.1428	0.03208	0.0563	6.304	
8	0	4	64	0.2138	0.2004	0.01348	0.0829	110.431	
8	0	99	32	0.0199	0.0420	-0.02202	0.0249	48.900	
8	0	99	64	0.0233	0.0347	-0.01141	0.0281	32.762	
8	4	4	32	0.1996	0.1342	0.06539	0.1554	26.045	
8	4	4	64	0.2356	0.1743	0.06137	0.1416	28.841	
8	4	99	32	0.0239	0.0170	0.00688	0.0397	19.117	
8	4	99	64	0.0281	0.0228	0.00538	0.0458	8.481	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1120	0.1025	0.00945	0.0222		8.125

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13									
MEAN	LEADTIME	ITEMS: C (OUT)/C (IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	PERCENTAGE ERROR
2	0	4	32	0.1235	0.1177	0.00577	0.0820	4.674	
2	0	4	64	0.0954	0.0867	0.00870	0.0719	9.121	
2	0	99	32	0.1115	0.1164	-0.00488	0.0817	4.374	
2	0	99	64	0.0919	0.0901	0.00177	0.0661	1.931	
2	4	4	32	0.0942	0.1015	-0.00730	0.0651	7.742	
2	4	4	64	0.0815	0.0814	0.00010	0.0609	0.119	
2	4	99	32	0.0804	0.1084	-0.02800	0.0650	34.827	
2	4	99	64	0.0615	0.0773	-0.01579	0.0576	25.663	
8	0	4	32	0.2596	0.2600	-0.00036	0.0805	0.137	
8	0	4	64	0.1973	0.1966	0.00071	0.0784	0.358	
8	0	99	32	0.2558	0.2578	-0.00199	0.0810	0.777	
8	0	99	64	0.1946	0.1952	-0.00060	0.0710	0.309	
8	4	4	32	0.2350	0.2409	-0.00590	0.0808	2.512	
8	4	4	64	0.1758	0.1801	-0.00431	0.0690	2.449	
8	4	99	32	0.2042	0.2338	-0.02954	0.1008	14.466	
8	4	99	64	0.1581	0.1763	-0.01823	0.0733	11.533	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1513	0.1575	-0.00624	0.0187		4.125

TABLE 83: SINGLE ITEM FORECASTS OF TOTAL COST USING ANALYTICAL APPROXIMATIONS
 FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0				REVISION INTERVAL = 13				
MEAN	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IM)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION CF EIAS	PERCENTAGE ERROR
2	0	4	32	12.8029	13.4262	-0.62334	5.2738	4.869
2	0	4	64	16.0681	15.7573	0.31086	7.3210	1.935
2	0	99	32	37.6790	24.4558	13.22353	18.2281	35.095
2	0	99	64	40.1153	27.1444	12.97130	17.3346	32.335
2	4	4	32	22.6585	16.2598	6.39476	7.9664	28.227
2	4	4	64	25.6819	20.2569	5.42522	8.0510	21.125
2	4	99	32	66.2654	38.8910	27.37402	43.5107	41.310
2	4	99	64	69.9445	37.0461	32.89818	42.3456	47.035
8	0	4	32	25.9917	26.7707	-0.77897	6.2528	2.597
8	0	4	64	32.9654	32.3492	0.61627	7.9287	1.869
8	0	99	32	58.1348	47.0088	11.13026	19.8089	19.146
8	0	99	64	66.2348	52.1022	14.13248	22.7541	21.337
8	4	4	32	44.5533	37.7902	6.76325	11.0078	15.180
8	4	4	64	50.6546	40.9879	9.66674	13.6830	19.084
8	4	99	32	111.7568	82.9472	28.80920	59.4302	25.778
8	4	99	64	119.4257	85.2462	34.17871	67.1747	28.619
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				800.9277	598.4351	202.49242	117.8735	25.282

APPENDIX C

<u>Table</u>	<u>Description</u>	<u>Page</u>
C1	Single item forecasts of holding and backlog quantity using fixed policy multiplicative approximations	C-1
C2	Single item forecasts of backlog and replenishment frequency using fixed policy multiplicative approximations	C-2
C3	Single item forecasts of total cost using fixed policy multiplicative approximations	C-3

TABLE C1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

ITEMS:	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS:		VARIANCE TO MEAN RATIO = 9.0		REVISION INTERVAL = 13		PERCENTAGE ERROR
	LEADTIME	C(OUT)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	
2	0	4	5.759	5.5773	0.17862	2.2125	3.103
2	0	4	6.3302	5.4444	0.88571	2.6984	13.992
2	0	99	17.3530	17.4110	-0.05797	6.9868	0.334
2	0	99	17.8625	16.9782	0.88437	5.6403	4.951
2	4	4	10.5087	8.3657	2.14300	5.6455	20.393
2	4	4	12.7705	9.7645	3.00595	6.4004	23.538
2	4	99	38.3212	30.4619	7.85931	16.8573	20.509
2	4	99	36.3076	26.8648	9.44326	14.5717	26.009
8	0	4	11.3903	11.6792	-0.28884	2.5709	2.536
8	0	4	12.6214	11.8176	0.80386	2.8494	6.369
8	0	99	32.1836	35.7039	-3.52029	9.5785	10.938
8	0	99	32.9880	34.7650	-1.77701	6.8766	5.387
8	4	4	23.6489	20.6733	2.97582	8.5521	12.583
8	4	4	22.7877	20.6147	2.17315	8.3902	9.537
8	4	99	71.6220	69.8931	2.12900	39.7019	2.973
8	4	99	69.0973	66.8083	2.28910	23.5112	3.313
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :			421.5483	392.4224	29.12698	55.5739	6.910

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

ITEMS:	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS:		VARIANCE TO MEAN RATIO = 9.0		REVISION INTERVAL = 13		PERCENTAGE ERROR
	LEADTIME	C(OUT)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	
2	0	4	0.7741	0.7620	0.01210	0.5532	1.563
2	0	4	0.9083	0.6591	0.24920	0.6200	27.435
2	0	99	0.1693	0.0600	0.12929	0.1670	76.381
2	0	99	0.1654	0.0500	0.11531	0.1588	69.735
2	4	4	2.2826	1.1294	1.15324	1.8906	50.523
2	4	4	1.9233	1.4173	0.50598	1.7667	26.308
2	4	99	0.2563	0.0334	0.22288	0.4403	86.969
2	4	99	0.3000	0.0581	0.24188	0.4543	80.633
8	0	4	1.5735	1.1802	0.39335	0.9410	24.998
8	0	4	1.9292	0.9398	0.98945	0.9975	51.288
8	0	99	0.1795	0.0657	0.11375	0.1819	63.385
8	0	99	0.2100	0.0818	0.12819	0.2231	61.040
8	4	4	3.3462	2.2334	1.11281	3.0942	33.256
8	4	4	4.1545	2.4835	1.67109	3.8777	40.223
8	4	99	0.3394	0.0217	0.31771	0.6512	93.615
8	4	99	0.4062	0.1008	0.30542	0.7254	75.194
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :			18.9176	11.2561	7.66165	5.6940	40.500

TABLE C2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1450	0.1889	-0.04386	0.0966	30.241
2	0	4	64	0.1928	0.1889	0.00392	0.1527	2.033
2	0	99	32	0.0242	0.0090	0.01515	0.0235	62.677
2	0	99	64	0.0237	0.0090	0.01467	0.0211	61.923
2	4	4	32	0.2326	0.1889	0.04368	0.1561	18.783
2	4	4	64	0.1961	0.1889	0.00724	0.1408	3.694
2	4	99	32	0.0269	0.0090	0.01786	0.0444	66.439
2	4	99	64	0.0315	0.0090	0.02254	0.0453	71.516
8	0	4	32	0.1748	0.1889	-0.01405	0.0715	8.034
8	0	4	64	0.2138	0.1889	0.02496	0.0925	11.674
8	0	99	32	0.0199	0.0090	0.01092	0.0189	54.751
8	0	99	64	0.0233	0.0090	0.01431	0.0225	61.334
8	4	4	32	0.1996	0.1889	0.01072	0.1457	5.372
8	4	4	64	0.2355	0.1889	0.04675	0.1575	19.639
8	4	99	32	0.0239	0.0090	0.01483	0.0408	62.170
8	4	99	64	0.0281	0.0090	0.01912	0.0454	67.944
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :								
				0.1120	0.0989	0.01305	0.0239	11.651

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1235	0.1213	0.00219	0.0821	1.770
2	0	4	64	0.0954	0.0888	0.00654	0.0718	6.860
2	0	99	32	0.1115	0.1204	-0.00889	0.0807	7.970
2	0	99	64	0.0919	0.0918	0.00014	0.0659	0.154
2	4	4	32	0.0942	0.1040	-0.00977	0.0647	10.368
2	4	4	64	0.0815	0.0834	-0.00185	0.0608	2.270
2	4	99	32	0.0804	0.1107	-0.03029	0.0658	37.685
2	4	99	64	0.0615	0.0794	-0.01786	0.0576	29.028
8	0	4	32	0.2596	0.2633	-0.00367	0.0842	1.413
8	0	4	64	0.1973	0.1971	0.00022	0.0792	0.109
8	0	99	32	0.2558	0.2598	-0.00403	0.0822	1.574
8	0	99	64	0.1946	0.1943	0.00036	0.0711	0.185
8	4	4	32	0.2350	0.2410	-0.00600	0.0803	2.533
8	4	4	64	0.1758	0.1794	-0.00366	0.0702	2.080
8	4	99	32	0.2042	0.2351	-0.03086	0.0996	15.109
8	4	99	64	0.1581	0.1763	-0.01827	0.0730	11.556
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :								
				0.1513	0.1591	-0.00786	0.0188	5.193

TABLE C3: SINGLE ITEM FORECASTS OF TOTAL COST USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS
FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

YEAR	LEADTIME	ITEMS: C(OUT)/C(IN)	C(PIN)/C(IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0		REVISION INTERVAL = 13		PERCENTAGE ERROR
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	
2	0	4	32	12.8029	12.5059	0.29694	4.1409	2.319
2	0	4	64	16.0681	13.7668	2.30128	5.8724	14.322
2	0	99	32	37.6790	25.2223	12.45655	19.7112	33.061
2	0	99	64	40.1153	27.8063	12.30938	18.5789	30.685
2	4	4	32	22.6545	16.2113	6.44329	6.9392	28.441
2	4	4	64	25.6819	20.7707	4.91142	7.4228	19.124
2	4	99	32	66.2654	37.3097	28.95544	44.7346	43.696
2	4	99	64	69.9445	37.6980	32.24626	44.3359	46.103
8	0	4	32	25.9917	24.8247	1.16719	3.9230	4.491
8	0	4	64	32.9654	28.1901	4.77534	6.0677	14.486
8	0	99	32	58.1348	50.5224	7.61253	22.1837	13.095
8	0	99	64	66.2348	55.2975	10.93728	24.4578	16.513
8	4	4	32	44.5533	37.3183	7.23502	8.8645	16.239
8	4	4	64	50.6546	42.0312	8.62354	11.7686	17.024
8	4	99	32	111.7568	79.1610	32.59503	62.0410	29.166
8	4	99	64	119.4257	88.0688	31.35599	70.6379	26.256
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				800.9277	596.7041	204.22289	122.6940	25.498

APPENDIX D

<u>Table</u>	<u>Description</u>	<u>Page</u>
D1	Single item forecasts of holding and backlog quantity using statistical policy multiplicative approximations	D-1
D2	Single item forecasts of backlog and replenishment frequency using statistical policy multiplicative approximations	D-2
D3	Single item forecasts of total cost using statistical policy multiplicative approximations	D-3

TABLE D1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS
FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR		
2	0	4	32	5.7559	5.6788	0.07704	2.1358	1.338		
2	0	4	64	6.3302	5.8169	0.51323	2.6508	8.108		
2	0	99	32	17.3530	17.1024	0.25055	5.3486	1.444		
2	0	99	64	17.8625	18.2989	-0.43634	5.2488	2.443		
2	4	4	32	10.5087	10.2823	0.22646	6.4027	2.152		
2	4	4	64	12.7705	11.8109	0.95963	6.7264	7.514		
2	4	99	32	38.3212	33.6644	4.65674	14.3411	12.152		
2	4	99	64	36.3076	33.9444	2.36329	14.5234	6.509		
8	0	4	32	11.3903	10.9809	0.40946	2.6335	3.595		
8	0	4	64	12.6214	11.6424	0.97903	3.2144	7.757		
8	0	99	32	32.1836	32.7755	-0.59198	7.2200	1.839		
8	0	99	64	32.9880	34.6410	-1.65285	7.3618	5.010		
8	4	4	32	23.6489	21.5945	2.05455	9.9554	8.688		
8	4	4	64	22.8877	22.8054	-0.08232	10.4080	0.077		
8	4	99	32	71.6220	64.0373	7.58482	24.7646	10.590		
8	4	99	64	69.0973	68.2749	0.82252	23.0494	1.190		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				421.5483	403.3503	18.19858	45.2346	8.317		

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR		
2	0	4	32	0.7741	1.1072	-0.33317	0.4871	43.043		
2	0	4	64	0.9083	0.9283	-0.02001	0.5858	2.203		
2	0	99	32	0.1693	0.1221	0.04717	0.1747	27.869		
2	0	99	64	0.1654	0.1335	0.03187	0.1580	19.272		
2	4	4	32	2.2826	2.1841	0.09853	1.9808	4.316		
2	4	4	64	1.9233	2.4783	-0.55508	1.7934	28.861		
2	4	99	32	0.2563	0.1991	0.05722	0.4607	22.328		
2	4	99	64	0.3000	0.2211	0.07885	0.4669	26.286		
8	0	4	32	1.7196	1.7196	-0.04607	0.7381	9.283		
8	0	4	64	1.9292	1.2268	0.70243	0.8535	36.410		
8	0	99	32	0.1795	0.2119	-0.03247	0.1880	18.092		
8	0	99	64	0.2100	0.2232	-0.01322	0.2194	6.296		
8	4	4	32	4.0898	4.0898	-0.74364	3.2055	22.223		
8	4	4	64	4.1545	4.1749	-0.02034	3.7115	0.490		
8	4	99	32	0.3394	0.3518	-0.01239	0.6479	3.650		
8	4	99	64	0.4062	0.4079	-0.00172	0.7364	0.824		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				18.9176	19.7797	-0.86204	5.8800	4.557		

TABLE D2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS
FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0			REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	
2	0	4	32	0.1450	0.2129	-0.06787	0.0966	46.802	
2	0	4	64	0.1928	0.2129	-0.02010	0.1527	10.424	
2	0	99	32	0.0242	0.0246	-0.00044	0.0235	1.800	
2	0	99	64	0.0237	0.0246	-0.00091	0.0211	3.856	
2	4	4	32	0.2326	0.2129	0.01966	0.1561	8.456	
2	4	4	64	0.1961	0.2129	-0.01677	0.1408	8.552	
2	4	99	32	0.0269	0.0246	0.00228	0.0844	8.463	
2	4	99	64	0.0315	0.0245	-0.00703	0.0453	22.309	
8	0	4	32	0.1748	0.2129	-0.03806	0.0715	21.771	
8	0	4	64	0.2138	0.2129	0.00095	0.0925	0.443	
8	0	99	32	0.0199	0.0246	-0.00467	0.0189	23.418	
8	0	99	64	0.0233	0.0246	-0.00127	0.0225	5.463	
8	4	4	32	0.1996	0.2129	-0.01330	0.1457	6.661	
8	4	4	64	0.2356	0.2129	0.02273	0.1575	9.646	
8	4	99	32	0.0239	0.0246	-0.00076	0.0408	3.182	
8	4	99	64	0.0281	0.0246	0.00354	0.0454	12.567	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1120	0.1187	-0.00675	0.0239	6.025	

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0			REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR	
2	0	4	32	0.1235	0.1140	0.00949	0.0864	7.683	
2	0	4	64	0.0954	0.0843	0.01112	0.0762	11.658	
2	0	99	32	0.1115	0.1134	-0.00188	0.0822	1.688	
2	0	99	64	0.0919	0.0883	0.00366	0.0677	3.979	
2	4	4	32	0.0942	0.0867	0.00755	0.0658	8.018	
2	4	4	64	0.0815	0.0738	0.00776	0.0603	9.522	
2	4	99	32	0.0804	0.0955	-0.01510	0.0653	16.788	
2	4	99	64	0.0615	0.0694	-0.00782	0.0564	12.711	
8	0	4	32	0.2596	0.2601	-0.00045	0.0883	0.172	
8	0	4	64	0.1973	0.2005	-0.00316	0.0831	1.604	
8	0	99	32	0.2558	0.2555	0.00026	0.0856	0.100	
8	0	99	64	0.1946	0.1961	-0.00145	0.0726	0.746	
8	4	4	32	0.2350	0.2191	0.01588	0.0793	6.759	
8	4	4	64	0.1758	0.1681	0.00765	0.0706	4.351	
8	4	99	32	0.2042	0.2135	-0.00931	0.0967	4.560	
8	4	99	64	0.1581	0.1657	-0.00762	0.0713	4.819	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1513	0.1502	0.00104	0.0191	6.685	

TABLE D3: SINGLE ITEM FORECASTS OF TOTAL COST USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS
 FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

HEAD	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS:		VARIANCE TO MEAN RATIO = 9.0	REVISION INTERVAL = 13	PERCENTAGE ERROR		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE			ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	12.8029	13.7550	-0.95213	3.7940	7.437		
2	0	4	64	16.0681	14.9232	1.14490	5.7769	7.125		
2	0	99	32	37.6790	32.8188	4.86028	16.4559	12.899		
2	0	99	64	40.1153	37.1629	2.95261	16.0914	7.360		
2	4	4	32	22.6545	21.7922	0.86235	6.3744	3.807		
2	4	4	64	25.6819	26.4457	-0.76383	6.9708	2.978		
2	4	4	32	66.2654	56.4266	9.83840	41.7071	14.847		
2	4	99	64	69.9445	60.2751	9.66928	42.1199	13.824		
8	0	4	32	25.9917	26.1810	-0.18912	3.5938	0.728		
8	0	4	64	32.9654	29.3793	3.58614	5.8048	10.878		
8	0	99	32	58.1348	61.9332	-3.79816	15.9836	6.533		
8	0	99	64	66.2348	69.2897	-3.05477	15.8692	4.612		
8	4	4	32	44.5533	48.9652	-4.41178	7.9433	0.928		
8	4	4	64	50.6546	50.2642	0.39043	11.2720	0.771		
8	4	99	32	111.7568	105.6966	6.06023	53.6820	5.423		
8	4	99	64	119.4257	119.2611	0.16459	62.4974	0.138		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				800.9277	770.5688	30.35957	108.8806	3.791		

APPENDIX E

<u>Table</u>	<u>Description</u>	<u>Page</u>
E1	Single item forecasts of holding and backlog quantity using future demand generation (negative binomial)	E-1
E2	Single item forecasts of backlog and replenishment frequency using future demand generation (negative binomial)	E-2
E3	Single item forecasts of total cost using future demand generation (negative binomial)	E-3

TABLE E1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS:		C (FIX)/C (IH)	C (FIX)/C (IIN)	ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
		C (OUT)/C (IH)	C (FIX)/C (IIN)							
2	0	4	4	32	5.6002	5.6413	-0.0412	2.1186	0.734	
2	0	4	4	64	6.4698	6.5008	-0.0310	2.5820	0.481	
2	0	99	99	32	17.1250	17.2047	-0.07974	2.6047	0.466	
2	0	99	99	64	17.8211	17.8211	-0.54910	3.4539	3.179	
2	4	4	4	32	10.2225	10.4106	-0.18811	5.4354	1.840	
2	4	4	4	64	12.1502	11.4993	0.65087	6.2964	5.357	
2	4	99	99	32	38.7644	39.9697	-1.20554	9.8884	3.110	
2	4	99	99	64	34.9989	36.5015	-1.50269	8.3503	4.294	
6	0	4	4	32	11.2149	10.8057	0.40927	1.9508	3.649	
6	0	4	4	64	12.8777	12.4983	0.37937	2.5126	2.946	
6	0	99	99	32	31.5095	31.6278	-0.11827	2.9722	0.375	
6	0	99	99	64	33.2635	33.6685	-0.40508	3.5754	1.218	
6	4	4	4	32	23.7531	21.8*43	1.93897	10.6034	8.163	
6	4	4	4	64	22.1750	20.5423	1.63281	6.7839	7.363	
6	4	99	99	32	70.7571	71.3073	-0.55024	13.6889	0.778	
6	4	99	99	64	68.6904	69.3022	-0.61172	14.6271	0.891	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :					416.8440	417.1147	-0.27141	29.6862	0.065	

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS:		C (FIX)/C (IH)	C (FIX)/C (IIN)	ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
		C (OUT)/C (IH)	C (FIX)/C (IIN)							
2	0	4	4	32	0.7915	0.6711	0.12041	0.6144	15.214	
2	0	4	4	64	0.8618	0.7093	0.15250	0.6533	17.696	
2	0	99	99	32	0.1678	0.0603	0.10748	0.1918	64.064	
2	0	99	99	64	0.1802	0.0517	0.12850	0.1907	71.316	
2	4	4	4	32	2.2398	0.9914	1.24842	2.0390	55.738	
2	4	4	4	64	2.0322	1.2637	0.76856	2.1021	37.819	
2	4	99	99	32	0.2696	0.0209	0.24871	0.5020	92.260	
2	4	99	99	64	0.3340	0.0254	0.30854	0.4654	92.385	
6	0	4	4	32	1.6231	1.5603	0.06278	1.1676	3.668	
6	0	4	4	64	1.8043	1.8041	0.00019	1.1163	0.010	
6	0	99	99	32	0.1980	0.0905	0.10752	0.2500	54.304	
6	0	99	99	64	0.2082	0.0911	0.11716	0.2540	56.269	
6	4	4	4	32	3.6077	2.3110	1.29667	4.0301	35.942	
6	4	4	4	64	4.0402	2.9266	1.11357	3.6636	27.562	
6	4	99	99	32	0.3521	0.0386	0.31358	0.6265	89.050	
6	4	99	99	64	0.4078	0.0570	0.35085	0.7472	86.030	
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :					19.1182	12.6729	6.44542	6.5783	33.714	

TABLE E2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1494	0.1511	-0.00168	0.0961	1.128
2	0	4	64	0.1794	0.1754	0.00402	0.1411	2.241
2	0	99	32	0.0239	0.0102	0.01369	0.0270	57.319
2	0	99	64	0.0259	0.0099	0.01609	0.0264	62.007
2	4	4	32	0.2288	0.1371	0.09170	0.1720	40.077
2	4	4	64	0.2073	0.1615	0.04576	0.1692	22.075
2	4	99	32	0.0283	0.0033	0.02499	0.0516	88.432
2	4	99	64	0.0351	0.0040	0.03103	0.0477	88.485
8	0	4	32	0.1803	0.1789	0.00140	0.0767	0.779
8	0	4	64	0.2000	0.2061	-0.00609	0.0817	3.043
8	0	99	32	0.0220	0.0113	0.01070	0.0244	48.646
8	0	99	64	0.0231	0.0108	0.01232	0.0242	53.244
8	4	4	32	0.2112	0.1632	0.04799	0.1786	22.729
8	4	4	64	0.2341	0.1985	0.03564	0.1514	15.224
8	4	99	32	0.0249	0.0035	0.02140	0.0412	86.077
8	4	99	64	0.0284	0.0062	0.02222	0.0486	78.315
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1126	0.0894	0.02320	0.0255	20.599

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1185	0.1144	0.00409	0.0908	3.450
2	0	4	64	0.0519	0.0841	0.00784	0.0654	8.525
2	0	99	32	0.1135	0.1099	0.00356	0.0796	3.136
2	0	99	64	0.0915	0.0826	0.00889	0.0736	9.716
2	4	4	32	0.0977	0.0919	0.00577	0.0769	5.906
2	4	4	64	0.0808	0.0783	0.00250	0.0609	3.095
2	4	99	32	0.0819	0.0885	-0.00654	0.0645	7.981
2	4	99	64	0.0635	0.0622	0.00130	0.0499	2.045
8	0	4	32	0.2623	0.2557	0.00659	0.0979	2.511
8	0	4	64	0.2008	0.1921	0.00870	0.0768	4.334
8	0	99	32	0.2527	0.2505	0.00221	0.0905	0.875
8	0	99	64	0.1942	0.1899	0.00428	0.0709	2.203
8	4	4	32	0.2354	0.2260	0.00937	0.0922	3.983
8	4	4	64	0.1777	0.1736	0.00413	0.0667	2.327
8	4	99	32	0.2054	0.2054	0.00303	0.0870	1.475
8	4	99	64	0.1635	0.1547	0.00880	0.0732	5.382
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1519	0.1473	0.00866	0.0193	3.065

TABLE E3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM										
NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(PIX)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR		
2	0	4	32	12.5569	11.9856	0.57130	4.6172	4.550		
2	0	4	64	15.7999	14.7195	1.08045	5.8695	6.838		
2	0	99	32	37.3637	26.6897	10.67403	19.2508	28.569		
2	0	99	64	40.9674	28.2263	12.74109	20.4562	31.101		
2	4	4	32	22.3079	17.3179	4.99010	7.3599	22.369		
2	4	4	64	25.4481	21.5633	3.88505	8.2780	15.267		
2	4	99	32	68.0734	44.8661	23.20738	46.7514	34.092		
2	4	99	64	72.1237	42.9976	29.12570	44.0360	40.383		
8	0	4	32	26.1009	25.2297	0.87119	6.0037	3.338		
8	0	4	64	32.9436	32.0066	0.93705	7.4942	2.844		
8	0	99	32	59.1978	48.6003	10.59739	24.7636	17.902		
8	0	99	64	66.3079	54.8358	11.46825	25.6698	17.295		
8	4	4	32	45.7158	38.2902	7.42567	11.9220	16.243		
8	4	4	64	49.7073	43.3556	6.35165	11.5746	12.778		
8	4	99	32	112.1910	81.5998	30.59058	56.1924	27.267		
8	4	99	64	119.5264	84.8407	34.68488	69.1812	29.019		
				806.3311	617.1274	189.20229	121.1508	23.465		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :										

REVISION INTERVAL = 13

ACTUAL VALUES LESS FORECASTS

DISPERSION OF BIAS

PERCENTAGE ERROR

806.3311

617.1274

189.20229

121.1508

23.465

APPENDIX F

<u>Table</u>	<u>Description</u>	<u>Page</u>
F1	Single item forecasts of holding and backlog quantity using future demand generation with variable means and variances	F-1
F2	Single item forecasts of backlog and replenishment frequency using future demand generation with variable means and variances	F-2
F3	Single item forecasts of total cost using future demand generation with variable means and variances	F-3

TABLE F1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES
 FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(LIN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0		REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF REAS	PERCENTAGE ERROR
2	0	4	32	5.6002	5.5730	0.02724	2.1835	0.486
2	0	4	64	6.4698	6.4620	0.00779	2.7491	0.120
2	0	99	32	17.1250	17.2877	-0.16272	2.9415	0.950
2	0	99	64	18.0381	18.0381	-0.76617	3.6471	4.436
2	4	4	32	10.2225	10.7114	-0.48893	5.4090	4.783
2	4	4	64	12.1502	12.0587	0.09150	6.5922	0.753
2	4	4	32	38.7644	40.2291	-1.46485	9.5856	3.779
2	4	99	64	34.9989	37.1003	-2.10157	8.5277	6.005
8	0	4	32	11.2149	10.8108	0.40408	2.2844	3.603
8	0	4	64	12.8777	12.6509	0.22678	2.6476	1.761
8	0	99	32	31.5095	31.9251	-0.41563	3.5126	1.319
8	0	99	64	33.2635	33.8071	-0.54368	4.0642	1.634
8	4	4	32	23.7531	22.7546	0.99864	11.2741	4.204
8	4	4	64	22.1750	22.1771	-0.00211	9.5267	0.010
8	4	99	32	70.7571	71.8123	-1.05524	14.7176	1.491
8	4	99	64	68.6904	69.9754	-1.28484	14.8564	1.870
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				416.8440	423.3733	-6.52970	31.1105	1.566

F-1

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(LIN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0		REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF REAS	PERCENTAGE ERROR
2	0	4	32	0.7915	0.6612	0.13022	0.5827	16.453
2	0	4	64	0.8618	0.7068	0.15500	0.6817	17.986
2	0	99	32	0.1678	0.0537	0.11406	0.1828	67.990
2	0	99	64	0.1802	0.0401	0.14008	0.1696	77.747
2	4	4	32	2.2398	1.2320	1.00789	2.0103	44.999
2	4	4	64	2.0322	1.5546	0.47765	2.3146	23.504
2	4	99	32	0.2696	0.0383	0.23130	0.5270	85.804
2	4	99	64	0.3340	0.0311	0.30287	0.4800	90.686
8	0	4	32	1.6231	1.5445	0.07855	1.1286	4.840
8	0	4	64	1.8043	1.8839	-0.07962	1.2703	4.413
8	0	99	32	0.1980	0.1022	0.09579	0.2685	48.379
8	0	99	64	0.2082	0.1015	0.10668	0.2607	51.236
8	4	4	32	3.6077	3.2352	0.37245	4.5580	10.324
8	4	4	64	4.0402	3.5259	0.51424	4.2948	12.728
8	4	99	32	0.3521	0.0476	0.30449	0.6383	86.470
8	4	99	64	0.4078	0.0889	0.31886	0.7578	78.191
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				19.1182	18.8477	4.27053	7.3486	22.337

TABLE F2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES
 FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13										
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR		
2	0	4	32	0.1494	0.1511	-0.00178	0.0970	1.192		
2	0	4	64	0.1794	0.1733	0.00614	0.1476	3.420		
2	0	99	32	0.0239	0.0094	0.01446	0.0259	66.540		
2	0	99	64	0.0259	0.0084	0.01753	0.0252	67.567		
2	4	4	32	0.2288	0.1631	0.06569	0.1655	28.711		
2	4	4	64	0.2073	0.1818	0.02552	0.1767	12.311		
2	4	99	32	0.0283	0.0049	0.02340	0.0525	82.818		
2	4	99	64	0.0351	0.0058	0.02925	0.0492	83.412		
8	0	4	32	0.1803	0.1805	-0.00013	0.0771	0.074		
8	0	4	64	0.2000	0.2052	-0.00522	0.0877	2.610		
8	0	99	32	0.0220	0.0114	0.01056	0.0260	47.990		
8	0	99	64	0.0231	0.0107	0.01241	0.0237	53.660		
8	4	4	32	0.2112	0.1996	0.01160	0.1895	5.494		
8	4	4	64	0.2341	0.2145	0.01958	0.1674	8.365		
8	4	99	32	0.0249	0.0036	0.02126	0.0410	85.497		
8	4	99	64	0.0284	0.0075	0.02088	0.0481	73.571		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1126	0.0957	0.01695	0.0265	15.047		

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM										
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C (FIX)/C (IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR		
2	0	4	32	0.1185	0.1193	-0.00082	0.0886	0.690		
2	0	4	64	0.0919	0.0854	0.00649	0.0661	7.061		
2	0	99	32	0.1135	0.1155	-0.00207	0.0827	1.822		
2	0	99	64	0.0915	0.0844	0.00716	0.0736	7.826		
2	4	4	32	0.0977	0.0955	0.00216	0.0758	2.215		
2	4	4	64	0.0808	0.0800	0.00077	0.0602	0.952		
2	4	99	32	0.0819	0.0892	-0.00731	0.0629	8.920		
2	4	99	64	0.0635	0.0652	-0.00173	0.0515	2.727		
8	0	4	32	0.2623	0.2556	0.00668	0.0991	2.548		
8	0	4	64	0.2008	0.1926	0.00817	0.0788	4.071		
8	0	99	32	0.2527	0.2512	0.00149	0.0945	0.590		
8	0	99	64	0.1942	0.1923	0.00197	0.0711	1.015		
8	4	4	32	0.2354	0.2251	0.01029	0.0956	4.371		
8	4	4	64	0.1777	0.1702	0.00745	0.0701	4.194		
8	4	99	32	0.2054	0.2027	0.00269	0.0862	1.311		
8	4	99	64	0.1635	0.1540	0.00947	0.0751	5.794		
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1519	0.1486	0.00331	0.0195	2.175		

TABLE F3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES
 FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0										REVISION INTERVAL = 13	
MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(PH)/C(IN)	AVERAGE		AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS		DISPERSION CF EIAS	PERCENTAGE ERROR	
				ACTUAL VALUE	FORECAST VALUE		ACTUAL VALUES LESS FORECASTS	DISPERSION CF EIAS			
2	0	4	32	12.5569	12.0349	0.52198	4.3522	4.157			
2	0	4	64	15.7999	14.7567	1.04319	6.0107	6.602			
2	0	99	32	37.3637	26.3007	11.06342	18.1632	29.610			
2	0	99	64	40.9674	27.4072	13.56062	18.8490	33.101			
2	4	4	32	22.3079	18.6962	3.61180	7.4276	16.191			
2	4	4	64	25.4481	23.3969	2.05129	8.6748	8.061			
2	4	99	32	68.0734	46.8729	21.20052	49.1702	31.144			
2	4	99	64	72.1237	44.3519	27.77144	45.5972	38.505			
8	0	4	32	26.1009	25.1687	0.93215	5.7572	3.571			
8	0	4	64	32.9436	32.5123	0.43138	7.8071	1.309			
8	0	99	32	59.1978	50.0820	9.11561	26.7811	15.399			
8	0	99	64	66.3079	56.1636	10.14436	26.1362	15.299			
8	4	4	32	45.7158	42.8982	2.81763	13.8725	6.163			
8	4	4	64	49.7073	47.1755	2.53175	13.4428	5.093			
8	4	99	32	112.1910	83.0152	29.17517	57.7132	26.005			
8	4	99	64	119.5264	88.6358	30.88965	70.1502	25.843			
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				806.3311	639.4680	166.86191	128.4163	20.694			

APPENDIX G

<u>Table</u>	<u>Description</u>	<u>Page</u>
G1	Single item forecasts of holding and backlog quantity using future demand generation (gamma)	G-1
G2	Single item forecasts of backlog and replenishment frequency using future demand generation (gamma)	G-2
G3	Single item forecasts of total cost using future demand generation (gamma)	G-3

TABLE 61: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION (GAMMA)
FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

YEAR	LEADTIME	ITEMS: C(OUT)/C(IN)	C(PHX)/C(IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0			REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	5.6002	5.2350	0.36519	2.0269	2.0269	6.521
2	0	4	64	6.4698	6.1239	0.34584	2.5339	2.5339	5.346
2	0	99	32	17.1250	16.6912	0.43379	2.6750	2.6750	2.533
2	0	99	64	17.2720	17.4005	-0.12849	3.5594	3.5594	0.744
2	4	4	32	10.2225	9.7260	0.49645	5.0951	5.0951	4.857
2	4	4	64	12.1502	10.9911	1.15912	5.4447	5.4447	9.540
2	4	99	32	38.7644	38.9375	-0.17323	8.7272	8.7272	0.447
2	4	99	64	34.9989	35.1542	-0.15524	8.6762	8.6762	0.444
8	0	4	32	11.2149	10.3333	0.88162	2.5777	2.5777	7.861
8	0	4	64	12.8777	11.8272	1.05050	2.8801	2.8801	6.157
8	0	99	32	31.5095	30.8098	0.69983	3.7208	3.7208	2.221
8	0	99	64	33.2635	32.8495	0.41407	4.0390	4.0390	1.245
8	4	4	32	23.7531	19.2636	4.48965	13.3827	13.3827	18.901
8	4	4	64	22.1750	19.4881	2.68696	12.0868	12.0868	12.117
8	4	99	32	70.7571	68.3355	2.42164	18.1044	18.1044	3.422
8	4	99	64	68.6904	66.2595	2.43110	17.0393	17.0393	3.539
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				416.8440	399.4255	17.41884	35.0113	35.0113	4.779

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

YEAR	LEADTIME	ITEMS: C(OUT)/C(IN)	C(PHX)/C(IN)	NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0			REVISION INTERVAL = 13		
				AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.7915	0.7534	0.03809	0.5441	0.5441	4.812
2	0	4	64	0.8618	0.9023	-0.04048	0.6028	0.6028	4.637
2	0	99	32	0.1678	0.0863	0.08148	0.2081	0.2081	48.571
2	0	99	64	0.1802	0.0779	0.10229	0.1989	0.1989	56.771
2	4	4	32	2.2398	1.2357	1.00412	2.0353	2.0353	44.830
2	4	4	64	2.0322	1.6534	0.38079	2.4904	2.4904	18.738
2	4	99	32	0.2696	0.0831	0.18646	0.6021	0.6021	69.168
2	4	99	64	0.3340	0.1949	0.13910	0.5114	0.5114	41.649
8	0	4	32	1.6231	1.8031	-0.18000	1.0912	1.0912	11.090
8	0	4	64	1.8043	2.1655	-0.36122	1.3077	1.3077	20.019
8	0	99	32	0.1980	0.1272	0.07076	0.2271	0.2271	35.739
8	0	99	64	0.2082	0.1437	0.06450	0.2572	0.2572	30.974
8	4	4	32	3.6077	5.2336	-1.62587	5.4759	5.4759	45.066
8	4	4	64	4.0402	5.1108	-1.07063	4.9899	4.9899	26.500
8	4	99	32	0.3521	0.2098	0.14230	0.6620	0.6620	40.413
8	4	99	64	0.4078	0.3585	0.04937	1.3380	1.3380	12.105
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				19.1182	20.1372	-1.01895	8.5261	8.5261	5.330

TABLE G2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION (GAMMA)

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(P1X)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1494	0.2069	-0.05751	0.0996	36.502
2	0	4	64	0.1794	0.2379	-0.05843	0.1581	32.560
2	0	99	32	0.0239	0.0152	0.00869	0.0284	36.382
2	0	99	64	0.0259	0.0152	0.01071	0.0256	41.288
2	4	4	32	0.2288	0.1786	0.05020	0.1839	21.941
2	4	4	64	0.2073	0.1978	0.00945	0.1832	4.560
2	4	99	32	0.0283	0.0127	0.01557	0.0650	55.088
2	4	99	64	0.0351	0.0250	0.01003	0.0953	28.605
8	0	4	32	0.1803	0.2189	-0.03858	0.0995	21.391
8	0	4	64	0.2000	0.2495	-0.04953	0.1063	24.768
8	0	99	32	0.0220	0.0187	0.00331	0.0208	15.037
8	0	99	64	0.0231	0.0173	0.00579	0.0212	25.025
8	4	4	32	0.2112	0.3009	-0.08972	0.2249	42.486
8	4	4	64	0.2341	0.2931	-0.05901	0.2055	25.209
8	4	99	32	0.0249	0.0212	0.00367	0.0487	18.762
8	4	99	64	0.0284	0.0299	-0.00151	0.0788	5.309
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1126	0.1274	-0.01480	0.0307	13.144

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(P1X)/C(IN)	AVERAGE ACTUAL VALUE	AVERAGE FORECAST VALUE	ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
2	0	4	32	0.1185	0.1155	0.00300	0.0880	2.532
2	0	4	64	0.0919	0.0983	-0.00642	0.0830	6.987
2	0	99	32	0.1135	0.1117	0.00177	0.0855	1.559
2	0	99	64	0.0915	0.0869	0.00465	0.0732	5.084
2	4	4	32	0.0977	0.0948	0.00285	0.0781	2.913
2	4	4	64	0.0808	0.0777	0.00304	0.0613	3.762
2	4	99	32	0.0819	0.0876	-0.00565	0.0691	6.901
2	4	99	64	0.0635	0.0664	-0.00296	0.0574	4.667
8	0	4	32	0.2623	0.2701	-0.00781	0.1165	2.976
8	0	4	64	0.2008	0.2103	-0.00954	0.0921	4.751
8	0	99	32	0.2527	0.2670	-0.01427	0.1111	5.647
8	0	99	64	0.1942	0.2033	-0.00904	0.0790	4.653
8	4	4	32	0.2354	0.2478	-0.01242	0.1082	5.278
8	4	4	64	0.1777	0.1823	-0.00462	0.0810	2.597
8	4	99	32	0.2054	0.2139	-0.00854	0.1014	4.157
8	4	99	64	0.1635	0.1658	-0.00235	0.0837	1.435
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				0.1519	0.1562	-0.00427	0.0218	2.810

TABLE G3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION (GAMMA)

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

MEAN	LEADTIME	ITEMS: C(OUT)/C(IN)	C(FIX)/C(IN)	AVERAGE		ACTUAL VALUES LESS FORECASTS	DISPERSION OF BIAS	PERCENTAGE ERROR
				ACTUAL VALUE	FORECAST VALUE			
2	0	4	32	12.5569	11.9433	0.61354	4.2221	4.886
2	0	4	64	15.7999	16.0271	-0.22714	7.3254	1.438
2	0	99	32	37.3637	28.8066	8.55736	20.7087	22.903
2	0	99	64	80.9674	30.6716	10.29603	20.7147	25.132
2	4	4	32	22.3079	17.7040	4.60394	7.6644	20.638
2	4	4	64	25.4481	22.5715	2.87673	9.9720	11.304
2	4	99	32	68.0734	49.9683	18.10547	57.5825	26.597
2	4	99	64	72.1237	58.6978	13.42587	88.1035	18.615
8	0	4	32	26.1009	26.1890	-0.08823	5.6886	0.338
8	0	4	64	32.9436	33.9485	-1.00482	8.6387	3.050
8	0	99	32	59.1978	51.9490	7.24885	22.2945	12.245
8	0	99	64	66.3079	60.0873	6.22062	25.6957	9.381
8	4	4	32	45.7158	48.1272	-2.41131	16.6180	5.275
8	4	4	64	49.7073	51.5983	-1.89082	15.9227	3.804
8	4	99	32	112.1910	95.9547	16.23627	61.7128	14.472
8	4	99	64	119.5264	112.3581	7.16835	127.6945	5.997
TOTALS FOR THE ENTIRE 16-ITEM SYSTEM :				806.3311	716.6011	89.73062	184.7298	11.128