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ESTIMATION OF THE OPERATING CHARACTERISTICS OF

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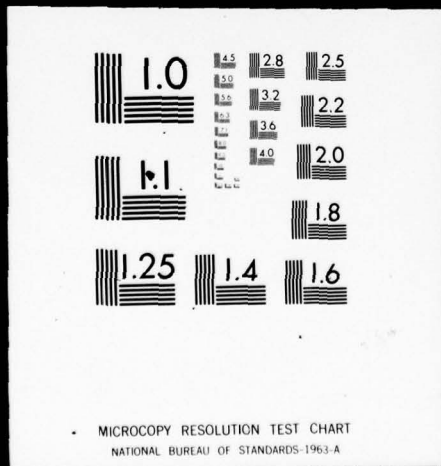
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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE

CATEGORIES I: INTRODUCTION TO THE TWO-PARAMETER BETA METHOD

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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE
CATEGORIES I: INTRODUCTION TO THE TWO-PARAMETER BETA METHOD

ABSTRACT

Following the Normal Approximation Method, in this study, the Two-Parameter Beta Method is developed and introduced, as a method of estimating the operating characteristics of a test item without assuming any prior model. The maximum likelihood estimate, $\hat{\theta}$, and its asymptotic property such that its conditional distribution, given ability θ , is $N(\theta, I(\theta)^{-1})$, where $I(\theta)$ is the test information function, are fully utilized. Data are the same set of simulated data calibrated and used in a previous study in which the Normal Approximation Method was introduced. They are of 500 hypothetical subjects whose ability levels are located at 100 equally distanced points of θ between -2.475 and 2.475 inclusive, with the interval length of 0.05 and five subjects at each point; their maximum likelihood estimates were estimated on 35 graded items; and a response pattern of each subject for 10 binary items was calibrated on the normal ogive model. The method of moments is adopted in the present study to approximate the probability density function of $\hat{\theta}$, using polynomials of degree 3 and 4. The conditional moments of θ , given $\hat{\theta}$, are derived from theory and computed for each $\hat{\theta}$. An approximation is made for the conditional distribution of θ , given $\hat{\theta}$, by a Beta distribution using the method of moments, with two a priori set parameters and the other two estimated parameters from the first two conditional moments of θ , given $\hat{\theta}$. Five scores of θ are calibrated following this approximated Beta distribution, which are denoted by $\tilde{\theta}$. The set of frequency ratios of $\tilde{\theta}$ for the group of subjects who have answered correctly

to each of the ten binary items to the total group of subjects is taken as the estimated item characteristic function of that binary item. The least square principle is adopted in estimating two parameters in the normal ogive model for each item.

The research was conducted at the principal investigator's laboratory, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants at various times include Michael K. Smith, Merle C. Steelman, Yeh Ching-Chuan and Robert L. Trestman, and an undergraduate student, Philip S. Livingston.

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I. Introduction

A method of estimating the operating characteristics of a test item without assuming any prior model was proposed by Samejima (Samejima, 1977b). Its features will be characterized as follows.

- (1) We have a set of test items measuring the uni-dimensional latent trait, or ability, whose operating characteristics are known. For convenience, hereafter we shall call it the Old Test.
- (2) The maximum likelihood estimates of a group of examinees have been obtained on the above set of test items.
- (3) To the same group of examinees a new item, or items, has been administered.
- (4) For the range of latent trait in which all the examinees are located, the test information function of the Old Test is large enough that we can approximate the conditional distribution of the maximum likelihood estimate, $\hat{\theta}$, given latent trait θ , by $N(\theta, I(\theta)^{-1})$, where $I(\theta)$ indicates the test information function. Furthermore, the values of $I(\theta)$ are approximately constant for the range of θ and, therefore, $I(\theta)^{-1}$ is replaced by σ^2 .
- (5) The bivariate normal distribution is used as the approximation for the distribution of estimate $\hat{\theta}$ and the error ϵ , which is the discrepancy of the estimate from its true value of θ , for each subgroup of examinees who share the same item score of a new item.

- (6) The regression coefficients of ϵ on θ , and the conditional variance of ϵ , given $\hat{\theta}$, are computed for each item score group of examinees under the assumption given in (5), and five error scores are calibrated for each $\hat{\theta}$ by means of the Monte Carlo method.
- (7) These error scores are subtracted from the respective maximum likelihood estimates, and the resulting scores are denoted by $\tilde{\theta}$. The frequency distribution of $\tilde{\theta}$ for each score group is obtained, using small equal-width intervals, and each frequency is divided by the sum of the frequencies of all the item score groups for that interval.

In the present paper, another method will be proposed in a similar setting. The difference lies in (5) and (6), and, instead of using the bivariate normal approximation for each score group, we consider the conditional distribution of ϵ , given $\hat{\theta}$, and approximate it by a Beta distribution. For convenience, hereafter the previous method will be called the Normal Approximation Method and the present method will be called the 2-Parameter Beta Method.

II. Conditional Moments of Error, Given the Estimate

Let λ be any estimator of latent trait θ , and η be the error of estimation such that

$$(2.1) \quad \lambda = \theta + \eta .$$

We assume that the conditional distribution of η , given θ , is the normal distribution, $N(0, \sigma^2)$. It should be noted that the above conditions are sufficient for the statistical independence of θ and η , and we obtain

$$(2.2) \quad E(\lambda) = E(\theta) ,$$

regardless of the distribution of θ , provided that $E(\theta)$ exists. Let $\psi(\lambda|\theta)$ denote the conditional probability density function of λ , given θ , which is the normal density function, $n(\theta, \sigma^2)$. Thus the first through fourth derivatives of $\psi(\lambda|\theta)$ with respect to λ can be written as follows

$$(2.3) \quad \frac{\partial}{\partial \lambda} \psi(\lambda|\theta) = -\psi(\lambda|\theta) \sigma^{-2} (\lambda - \theta) .$$

$$(2.4) \quad \frac{\partial^2}{\partial \lambda^2} \psi(\lambda|\theta) = \psi(\lambda|\theta) \sigma^{-2} [\sigma^{-2} (\lambda - \theta)^2 - 1] .$$

$$(2.5) \quad \frac{\partial^3}{\partial \lambda^3} \psi(\lambda|\theta) = 3\psi(\lambda|\theta) \sigma^{-4} (\lambda - \theta) - \psi(\lambda|\theta) \sigma^{-6} (\lambda - \theta)^3 .$$

$$(2.6) \quad \frac{\partial^4}{\partial \lambda^4} \psi(\lambda|\theta) = 3\psi(\lambda|\theta) \sigma^{-4} - 6\psi(\lambda|\theta) \sigma^{-6} (\lambda - \theta)^2 + \psi(\lambda|\theta) \sigma^{-8} (\lambda - \theta)^4 .$$

From these results, following the respective integration procedures, we obtain for the first through fourth conditional moments of η about the origin, given λ ,

$$(2.7) \quad E(\eta|\lambda) = -\sigma^2 \left[\frac{d}{d\lambda} g(\lambda) \right] [g(\lambda)]^{-1} ,$$

$$(2.8) \quad E(\eta^2|\lambda) = \sigma^4 \left[\frac{d^2}{d\lambda^2} g(\lambda) \right] [g(\lambda)]^{-1} + \sigma^2 ,$$

$$(2.9) \quad E(\eta^3|\lambda) = 3\sigma^2 E(\eta|\lambda) - \sigma^6 \left[\frac{d^3}{d\lambda^3} g(\lambda) \right] [g(\lambda)]^{-1} ,$$

$$(2.10) \quad E(\eta^4|\lambda) = 6\sigma^2 E(\eta^2|\lambda) + \sigma^8 \left[\frac{d^4}{d\lambda^4} g(\lambda) \right] [g(\lambda)]^{-1} - 3\sigma^4 ,$$

where $g(\lambda)$ is the probability density function of λ , which is given by

$$(2.11) \quad g(\lambda) = \int_{-\infty}^{\infty} f(\theta) \psi(\lambda|\theta) d\theta ,$$

and is assumed to be four times differentiable.

Note that the right hand sides of (2.7) through (2.10) solely consist of σ^2 , $g(\lambda)$ and its derivatives. Thus these conditional moments are observable, if we can fit an appropriate function for $g(\lambda)$ based on our raw data, or frequency distribution, of the estimate λ . Once these moments about the origin have been computed, it is easy to compute the corresponding moments about any arbitrary values (e.g., Elderton and Johnson, 1969, page 17).

III. Method of Moments to Graduate the Set of Observations

The method of moments (Elderton and Johnson, 1969) has been developed for graduating the observed frequency distribution using the observed moments of up to a certain degree, assuming a specified function. As the result, we obtain a probability density function which has the same values of these moments and the specified functional formula.

If, for example, we assume Pearson's system of frequency curves, we need the first four observed moments. From these moments, we can find out the value of the criterion κ , defined by

$$(3.1) \quad \kappa = \beta_1 (\beta_2 + 3)^2 [4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)]^{-1},$$

where β_1 and β_2 are given in terms of the second through fourth moments about the mean of the frequency distribution, μ_2 , μ_3 and μ_4 , such that

$$(3.2) \quad \beta_1 = \mu_3^2 \mu_2^{-3}$$

and

$$(3.3) \quad \beta_2 = \mu_4 \mu_2^{-2}.$$

If, for instance, κ turned out to be negative and finite, then the distribution will be of Pearson's Type I; if it is positive and less than unity, then the distribution will be of Pearson's Type IV; and so on.

It is warned by Elderton and Johnson that we should avoid

using high moments, for the higher the moment the more liable is it to error. Although this is a legitimate warning, this consideration will force us to adopt a relatively simple type of distribution, which does not allow varieties of possible curves. To give an example, Pearson's system only requires the first four moments, but its curves for the probability density function are, at most, uni-modal, except for the ones which have U shapes. We must, therefore, balance the two opposing factors, i.e., to avoid the error and to allow the variety in shape, and search for a happy medium.

One solution for this problem is to use a polynomial for the probability density function. If we use the first three moments, for instance, then we will obtain a polynomial of degree 3, which is expressed as

$$(3.4) \quad g(\lambda) = \alpha + \beta\lambda + \gamma\lambda^2 + \delta\lambda^3 .$$

These four coefficients are given by the four constants, a, b, c and d, such that

$$(3.5) \quad \begin{cases} a = [1.125\mu_0^*/R] - [1.875\mu_2^*/R^3] \\ b = [9.375\mu_1^*/R^3] - [13.125\mu_3^*/R^5] \\ c = [-1.875\mu_0^*/R^3] + [5.625\mu_2^*/R^5] \\ d = [-13.125\mu_1^*/R^5] + [21.875\mu_3^*/R^7] \end{cases} ,$$

through

$$(3.6) \quad \begin{cases} \alpha = a - bM + cM^2 - dM^3 \\ \beta = b - 2cM + 3dM^2 \\ \gamma = c - 3dM \\ \delta = d, \end{cases}$$

where μ_r^* is the r-th moment about the midpoint M of the range of λ , whose length is $2R$. If we add the fourth moment, then we will obtain a polynomial of degree 4, with the additional term, $v\lambda^4$ on the right hand side of (3.4). In this case, the five coefficients are determined by the five constants, a, b, c, d and e, such that

$$(3.7) \quad \begin{cases} a = [1.7578125\mu_0^*/R] - [8.203125\mu_2^*/R^3] + [7.3828125\mu_4^*/R^5] \\ b = [9.375\mu_1^*/R^3] - [13.125\mu_3^*/R^5] \\ c = [-8.203125\mu_0^*/R^3] + [68.90625\mu_2^*/R^5] - [73.828125\mu_4^*/R^7] \\ d = [-13.125\mu_1^*/R^5] + [21.875\mu_3^*/R^7] \\ e = [7.3828125\mu_0^*/R^5] - [73.828125\mu_2^*/R^7] + [86.1328125\mu_4^*/R^9] \end{cases}$$

through

$$(3.8) \quad \begin{cases} \alpha = a - bM + cM^2 - dM^3 + eM^4 \\ \beta = b - 2cM + 3dM^2 - 4eM^3 \\ \gamma = c - 3dM + 6eM^2 \\ \delta = d - 4eM \\ v = e. \end{cases}$$

An advantage of using a polynomial is that it allows more varieties of curves than other functions obtainable by using the same number of moments. It has a disadvantage, however, in that for some subsets of the variable we may obtain negative values for the probability density function, since a polynomial is not a type of function defined for the probability density.

The method of moments described above can also be applied for a set of observations, instead of its frequency distribution. In fact, in this case, we can preserve more detailed information which may be lost through the process of categorizing the observations into the frequency distribution. Neither do we have to adjust the values of moments by Sheppard's correction, and so on. It should also be noted that the method is applicable even when no set of observations is available, but the estimated values of moments are given.

This method is readily adoptable in estimating $g(\lambda)$, the probability density function of λ , which is essential in obtaining the conditional moments of error η , given λ , as the equations (2.7) through (2.10) show. Besides, together with σ^2 , we can also use the method for specifying the conditional distribution of η , given λ , from these conditional moments.

If we assume a Beta distribution for the conditional distribution of η , given λ , whose probability density function, $\xi(\eta|\lambda)$, is given by

$$(3.9) \quad \xi(\eta|\lambda) = [B(p, q)]^{-1} (\eta - a)^{p-1} (b - \eta)^{q-1} (b - a)^{-(p+q-1)},$$

the four parameters, a , b , p and q , can be estimated from the four conditional moments of η , given λ . When the two parameters, a and b , are known, then the estimation is much simpler, using only the first two conditional moments. In this case, we have

$$(3.10) \quad p = M_1^2 (1 - M_1) M_2^{-1} - M_1$$

and

$$(3.11) \quad q = M_1 (1 - M_1)^2 M_2^{-1} - (1 - M_1),$$

where M_1 and M_2 are given by

$$(3.12) \quad M_1 = \{E(\eta|\lambda) - a\} (b - a)^{-1}$$

and

$$(3.13) \quad M_2 = E\{(\eta - E(\eta|\lambda))^2|\lambda\} (b - a)^{-2},$$

respectively.

IV. Maximum Likelihood Estimate and Normal Approximation

When the number of test items is substantially large and its test information is high enough for the range of θ that we are interested in, the conditional distribution of the maximum likelihood estimate $\hat{\theta}$, given θ , is approximately $N(\theta, I(\theta)^{-1})$, where $I(\theta)$ is the test information function (Samejima, 1975, 1977b). To accept this approximation, however, we should be careful enough to check its fit in one way or another. A relatively simple Monte Carlo method will take care of this for an actual test as well as for a hypothetical test (Samejima, 1977b). When the test information function assumes a constant value throughout the range of θ of our interest, the variance $I(\theta)^{-1}$ becomes constant, and can be denoted by σ^2 . Thus we can replace η by ϵ and λ by $\hat{\theta}$ in equations (2.8) through (2.11), to obtain the conditional moments of ϵ about the origin, given $\hat{\theta}$.

In this setting, therefore, we can adopt the method of moments both for approximating the probability density function of $\hat{\theta}$ and for approximating the conditional distribution of ϵ , given $\hat{\theta}$. The greatest merit of this approach will be that we can do so without any assumption for $f(\theta)$, i.e., the probability density function of θ .

V. Data

The data used in the present study are the same simulated data that were used in the previous study (Samejima, 1977b). The group of examinees consists of 500 hypothetical subjects, whose ability levels are located at 100 equally distanced points between -2.475 and 2.475 inclusive, with the interval length of 0.05 and five subjects at each point. The Old Test is the set of 35 graded response items with, uniformly, three item score categories, whose test information function assumes approximately 21.63 for the range of θ , -3.0 through 3.0. Ten binary test items are used as new items, whose item characteristic functions are to be estimated. The model used here is the normal ogive model, for both the Old Test and the new items. Each of the 500 subjects has two response patterns, i.e., one on the Old Test and the other on the set of the 10 binary items. The two parameters of these ten binary items, a_g and b_g , are shown in Tables 1 and 2 in a later section.

In the previous study, (1) the response pattern of each of the 500 examinees was calibrated by the Monte Carlo method, (2) the maximum likelihood estimate was obtained for each subject on the Old Test, and (3) the response pattern of the set of new binary items was calibrated. Here we use the same data.

VI. Method

Following the rationale described in previous sections, the probability density function, $g(\hat{\theta})$, is estimated by the method of moments based on the raw data, i.e., the 500 maximum likelihood estimates. In so doing, at most, the first four moments are used, assuming: 1) the Pearson's system, 2) a polynomial of degree 3, and 3) a polynomial of degree 4. Then the conditional moments of ϵ about the origin, given $\hat{\theta}$, are computed by means of equations (2.7) through (2.10).

The conditional moments of ϵ about the origin, given $\hat{\theta}$, are computed using equations (2.7) through (2.10), and the first two moments are used to specify the conditional distribution of ϵ , given $\hat{\theta}$. For each conditional distribution, a Beta distribution, whose probability density function is given in section 3, is assumed, and the two parameters, a and b , are more or less arbitrarily assigned. From these two values and the first two moments, the other two parameters, p and q , are computed through (3.10) and (3.11).

Following the conditional distribution thus specified, five error scores are calibrated for each $\hat{\theta}$ by the Monte Carlo Method, and are subtracted from $\hat{\theta}$ to provide us with the five scores, $\tilde{\theta}$. The frequency distribution of $\tilde{\theta}$ for each of the two score groups of a new binary item is obtained, using equal length intervals, and the ratio of the frequency for the "success" group in each interval to the corresponding total frequency is treated as the estimated value of the item characteristic function at the midpoint of the interval.

The simple unweighted least square method is used to evaluate the result, by obtaining the estimates of the two parameters in the normal ogive model, as was used previously (Samejima, 1977b). The model specifies the item characteristic function, $P_g(\theta)$, for item g in the form

$$(6.1) \quad P_g(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a (\theta - b_g) e^{-\frac{u^2}{2}} du,$$

where a_g is the discrimination parameter and b_g is the difficulty parameter. Let $\tilde{P}_g(\theta_j)$ be the estimated value of the item characteristic function for the midpoint of the j -th interval, and define ζ_{gj} such that

$$(6.2) \quad \tilde{P}_g(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\zeta_{gj}} e^{-\frac{u^2}{2}} du.$$

Thus following the least square principle, we define Q by

$$(6.3) \quad 2Q = \sum_{j=1}^m (\zeta_{gj} - a_g(\theta_j - b_g))^2,$$

and differentiating Q with respect to a_g and b_g and setting the results equal to zero, we obtain

$$(6.4) \quad \frac{\partial Q}{\partial a_g} = \sum_{j=1}^m (\zeta_{gj} - a_g(\theta_j - b_g))(-\theta_j + b_g) = 0$$

and

$$(6.5) \quad \frac{\partial Q}{\partial b_g} = \sum_{j=1}^m (\zeta_{gj} - a_g(\theta_j - b_g)) a_g = 0.$$

The above equations lead us to the estimates of a_g and b_g such that

$$(6.6) \quad \hat{a}_g = \text{Cov.}(\zeta_{gj}, \theta_j) (\text{Var.}(\theta_j))^{-1}$$

and

$$(6.7) \quad \hat{b}_g = \bar{\theta} - (\text{Cov.}(\zeta_{gj}, \theta_j))^{-1} \text{Var.}(\theta_j) \bar{\zeta}_{gj},$$

where $\bar{\theta}$ and $\bar{\zeta}_{gj}$ are the means of θ_j and ζ_{gj} respectively.

These estimates, \hat{a}_g and \hat{b}_g , are to be compared with their respective parameters, a_g and b_g .

VII. Results

The resulting frequency distribution of the 500 maximum likelihood estimates is shown in Figures 7-1 through 7-3 , using 0.25 as the interval length. The first moment about the origin, or the mean, of the maximum likelihood estimate, $\hat{\theta}$, turned out to be -0.00577 , and the second through fourth moments about the mean are 2.14824 , -0.01465 and 8.65145 respectively. The value of the criterion κ , which is obtained by (3.1), is -0.00000762 , i.e., approximately zero. For this reason, the distribution in Pearson's system should be of Type II, i.e., a special case of the Beta distribution, whose probability density function is given by

$$(7.1) \quad [B(p, p)]^{-1} (\hat{\theta} - a)^{p-1} (b - \hat{\theta})^{p-1} (b - a)^{2p-1} ,$$

which is symmetric and whose mode and mean are both $(a + b)/2$. The three parameter values, p , a and b , are estimated from the mean and the variance of $\hat{\theta}$ and its fourth moment (Elderton and Johnson, 1969; Johnson and Kotz, 1970) in such a way that

$$(7.2) \quad \hat{p} = 3(\beta_2 - 1)/2(3 - \beta_2) ,$$

$$(7.3) \quad \hat{a} = E(\hat{\theta}) - (2 \text{ Var. } (\hat{\theta}) \beta_2)^{1/2} (3 - \beta_2)^{-1/2}$$

and

$$(7.4) \quad \hat{b} = E(\hat{\theta}) + (2 \text{ Var. } (\hat{\theta}) \beta_2)^{1/2} (3 - \beta_2)^{-1/2} ,$$

where β_2 is defined by (3.3) . These values turned out to be 1.16581 , -2.68111 and 2.66947 respectively. Figure 7-1 presents the

probability density function of the Pearson's Type II distribution using the above estimated parameters, together with the frequency distribution of $\hat{\theta}$ mentioned earlier.

The result of fitting a polynomial for the probability density function of $\hat{\theta}$ gave the estimated coefficients:

$$(7.5) \quad \begin{cases} \hat{\alpha} = 0.22416 \\ \hat{\beta} = -0.00351 \\ \hat{\gamma} = -0.01873 \\ \hat{\delta} = 0.00095 \end{cases}$$

for degree 3 , and for degree 4 we have

$$(7.6) \quad \begin{cases} \hat{\alpha} = 0.19620 \\ \hat{\beta} = 0.00238 \\ \hat{\gamma} = 0.01319 \\ \hat{\delta} = -0.00062 \\ \hat{\nu} = -0.00427 \end{cases} .$$

Figures 7-2 and 7-3 present these two estimated probability density functions in a similar manner as Figure 7-1. These three results will be more clearly illustrated if we draw the curves of their distribution functions, and those of the cumulative frequency distribution of the maximum likelihood estimate $\hat{\theta}$. Figure 7-4 through 7-6 present these results, in the same order as Figure 7-1 through 7-3.

It is clear from these figures that the discrepancy between the frequency distribution and the theoretical density

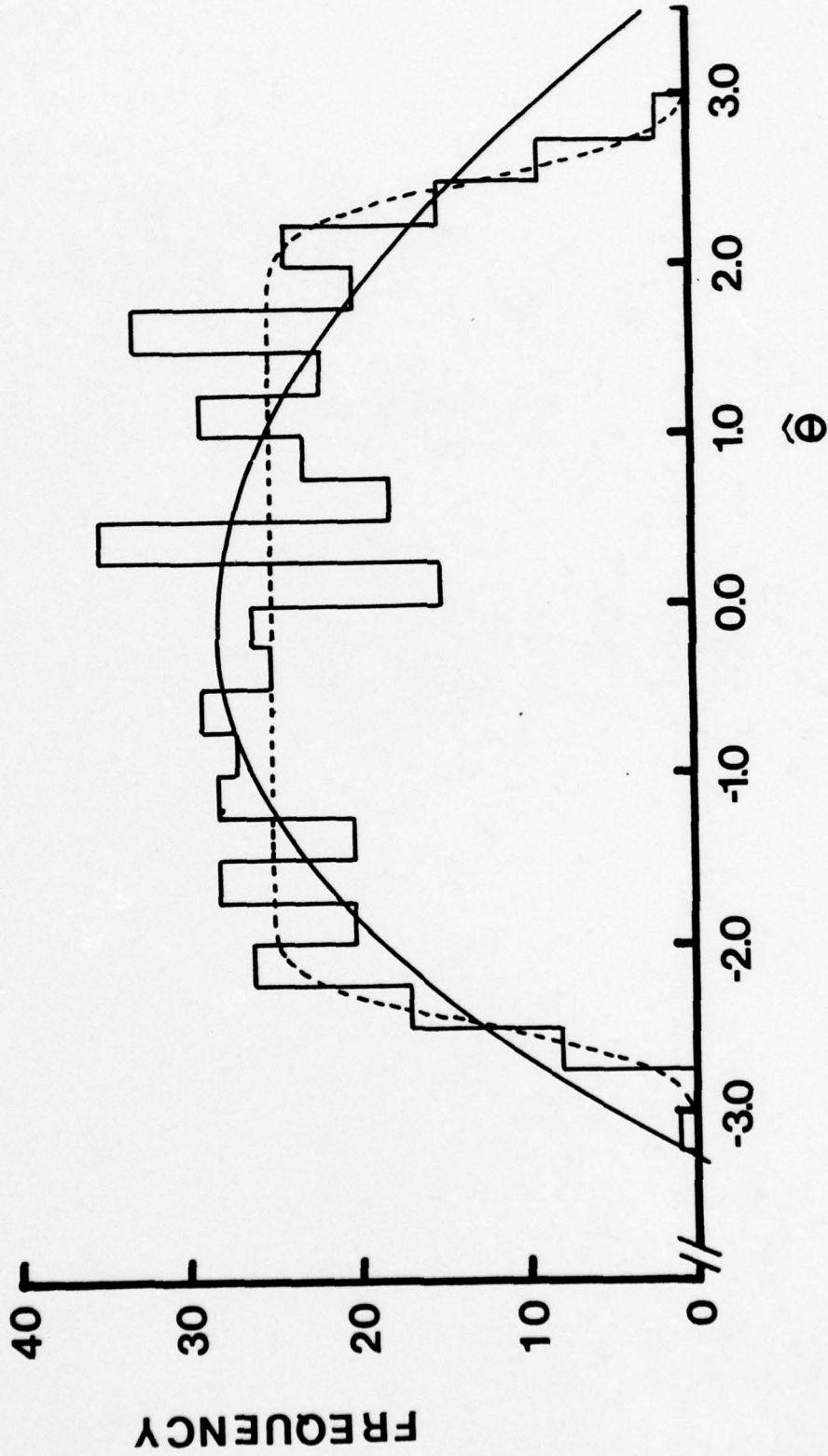


FIGURE 7-2
The frequency distribution of the maximum likelihood estimate (histogram), its graduated polynomial of degree 3 (solid curve) and the theoretical density of the maximum likelihood estimate (dotted curve).

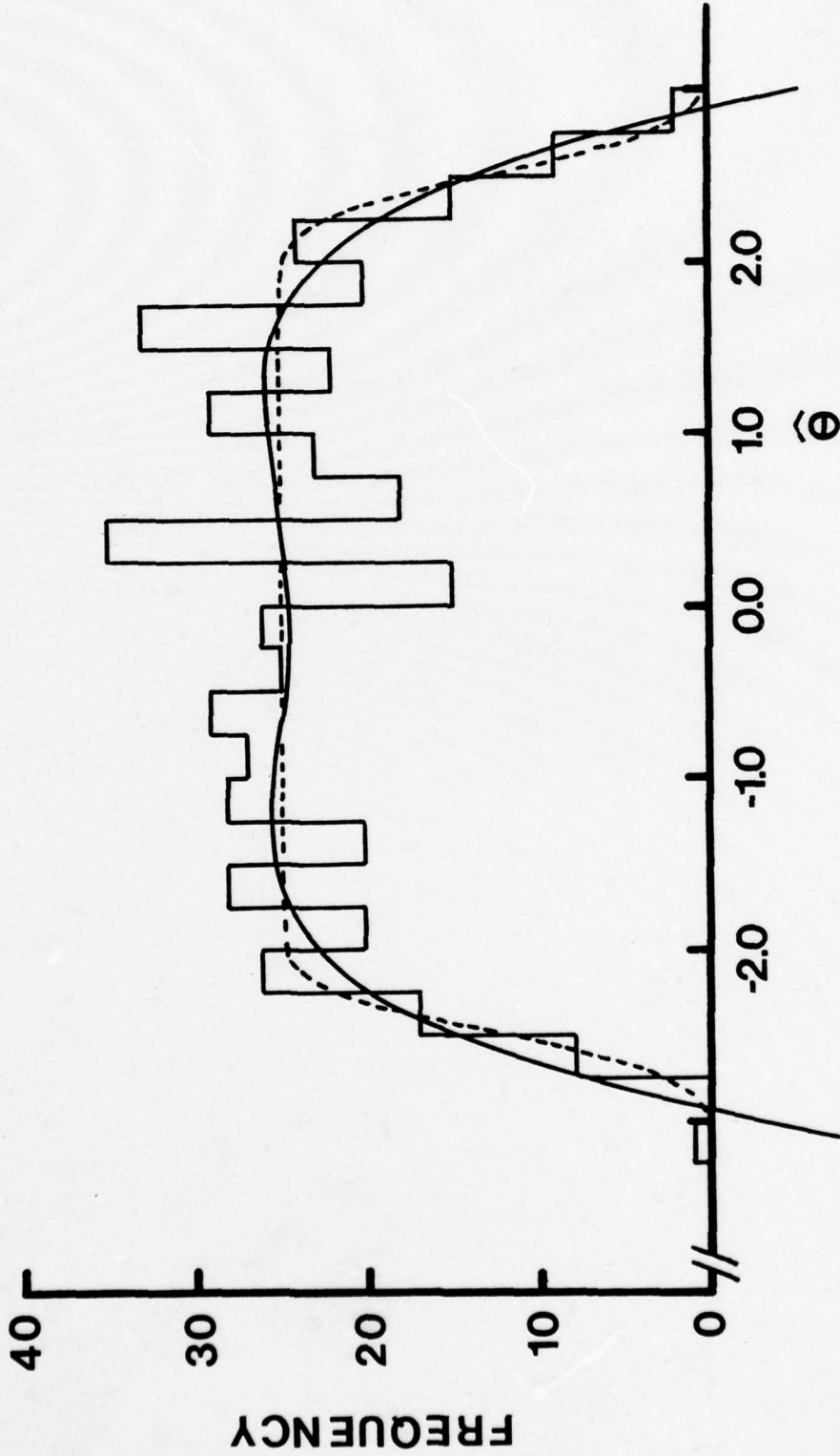


FIGURE 7-3

The frequency distribution of the maximum likelihood estimate (histogram), its graduated polynomial of degree 4 (solid curve) and the theoretical density of the maximum likelihood estimate (dotted curve).

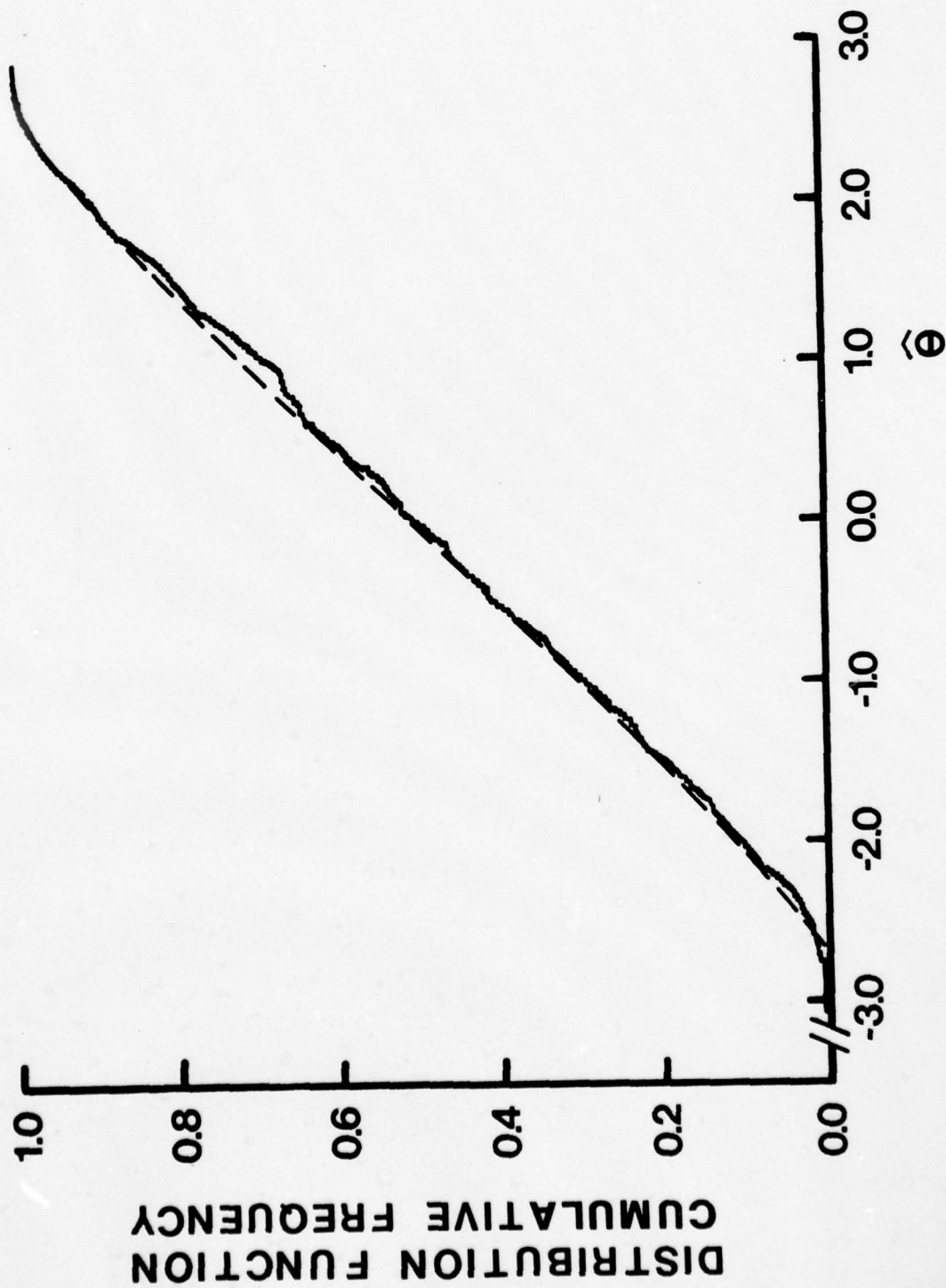


FIGURE 7-4

The distribution function obtained from the graduated Pearson's Type II function (dashed curve) and the cumulative frequency distribution of the maximum likelihood estimate (solid curve).

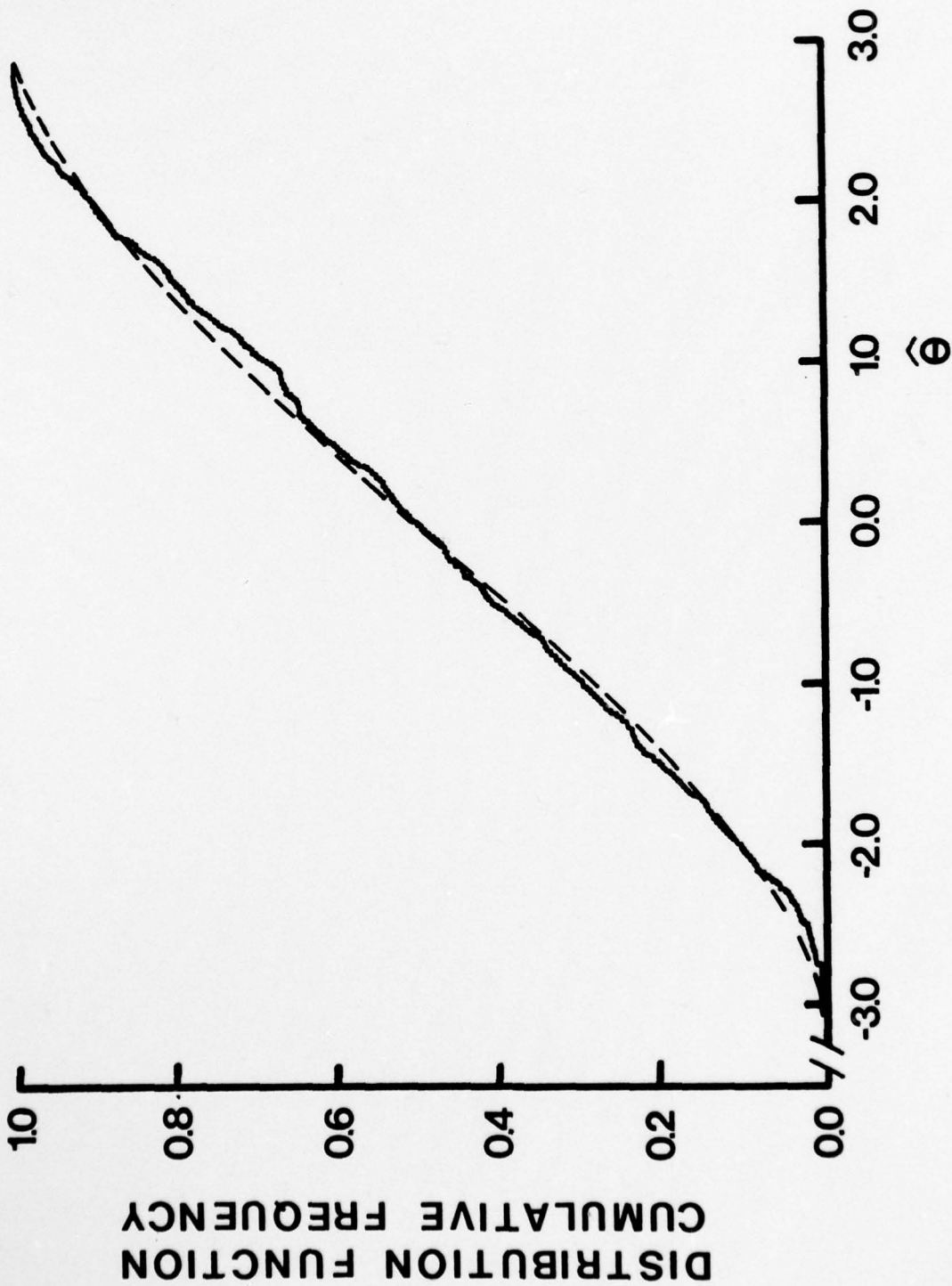


FIGURE 7-5
The distribution function obtained from the graduated polynomial of degree 3 (dashed curve) and the cumulative frequency distribution of the maximum likelihood estimate (solid curve).

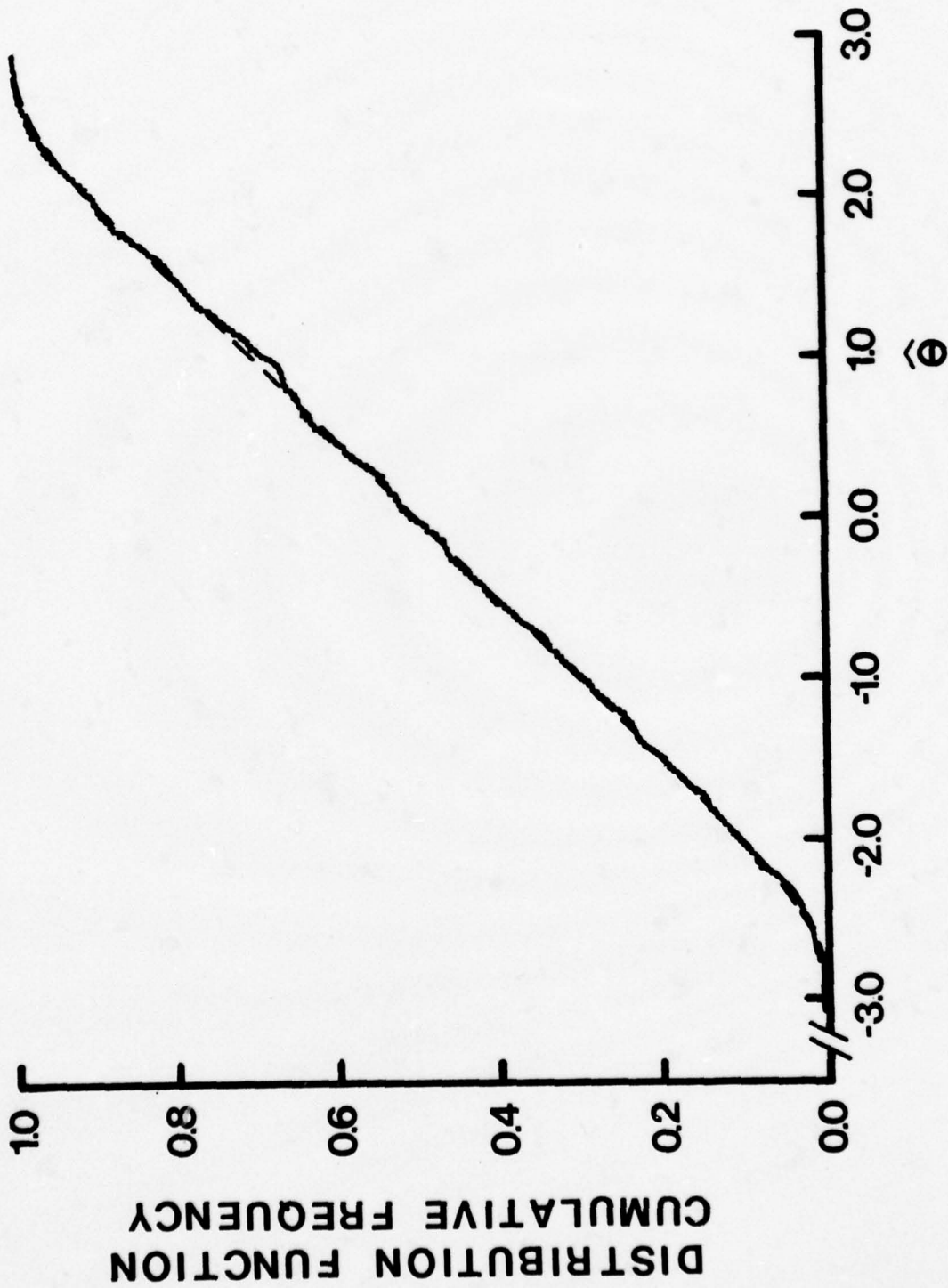


FIGURE 7-6
The distribution function obtained from the graduated polynomial of degree 4 (dashed curve) and the cumulative frequency distribution of the maximum likelihood estimate (solid curve).

is relatively large. This fact is, of course, due to the sampling fluctuation in the process of item score calibration for each item of the Old Test, on which the maximum likelihood estimate was derived. Since each function was graduated on our raw data, there is some conspicuous difference between the graduated curve and the curve for the theoretical density, especially in Degree 3 Case.

In evaluating these three outcomes, it will be more helpful if we introduce the theoretical probability density function, $g(\hat{\theta})$. This is, in general, given by (2.11), replacing λ by $\hat{\theta}$. With our data, it is appropriate to assume a uniform distribution for the distribution of θ , and we can write

$$(7.7) \quad g(\hat{\theta}) = c (2\pi)^{-1/2} \sigma^{-1} \int_{\underline{\theta}}^{\bar{\theta}} \exp\{-(\hat{\theta} - \theta)^2/2\sigma^2\} d\theta \\ = c (2\pi)^{-1/2} \int_{(\underline{\theta}-\hat{\theta})/\sigma}^{(\bar{\theta}-\hat{\theta})/\sigma} \exp\{-t^2/2\} dt ,$$

defining

$$(7.8) \quad t = (\theta - \hat{\theta})\sigma^{-1} ,$$

where c is the constant probability density of θ for the closed interval $[\underline{\theta}, \bar{\theta}]$. With our data, $c = 0.2$, $\underline{\theta} = -2.5$ and $\bar{\theta} = 2.5$, and $\sigma = 0.215$, which is the square root of the inverse of the constant test information function, .21.63. The rightest hand side of (7.7) is shown in Figures 7-1 through 7-3 by dotted curves.

Comparing the two polynomials, it is obvious that the polynomial

of degree 4 provides us with the closer curves to both the frequency distribution of $\hat{\theta}$ and its theoretical probability density function, than the polynomial of degree 3. This fact is also observed in the comparison of the two distribution functions in Figures 7-5 and 7-6. This is an expected result, and there is no doubt that we should choose the polynomial of degree 4, if the choice should be made between these two.

As for the Pearson's Type II distribution, it should be noted that, at least, the symmetry of the theoretical probability density function is realized. Because of the restriction coming from the nature of the Beta distribution, however, the curve in Figure 7-1 is conspicuously different from the theoretical probability density function at both extreme values of $\hat{\theta}$. If we compare this with the curve of the polynomial of degree 4 given in Figure 7-3, we will find out this problem is substantially ameliorated in the latter. Also the comparison of Figures 7-4 and 7-6 convinces us that the fit of the polynomial of degree 4 is better than that of the Pearson's Type II function, in spite of the fact that both functions require the first through fourth moments in the parameter estimation.

From all these observations, it will be concluded that the polynomial of degree 4 provides us with the best fitted curve among the three, and that of degree 3 gives us the worst fitted one. A close observation of Figure 7-6 tells that a further increase in degree may not be necessary, considering that we must use the fifth moment of $\hat{\theta}$ to do that. (This will be confirmed by a later observation.)

For this reason, hereafter we use the polynomial of degree 4 as the estimated probability density function for the maximum likelihood estimate. We shall also use the polynomial of degree 3 in addition, to observe how much accuracy is lost by using a relatively poorly fitted function. For convenience, hereafter we shall call these two cases Degree 4 Case and Degree 3 Case respectively.

Using the estimated $g(\hat{\theta})$ in Degree 4 Case, the first through fourth conditional moments of the error ϵ , given $\hat{\theta}$, were computed for each value of the 500 maximum likelihood estimates through the formulas (2.8) through (2.11). As is expected from Figure 7-3, however, it was impossible to do so for one value of the maximum likelihood estimate, -3.0555, since the estimated probability density assumes a negative value. This looks like a deficiency in using a polynomial for the probability density function, since it is not originally developed as one. A close examination of Figure 7-1, however, will tell us that, if we use the Pearson's Type II distribution instead, we will have more than one value of the maximum likelihood estimate for which the estimated probability density function assumes negative values, although the function itself is a probability density function. In fact, if we adopt the Pearson's Type II distribution, then we will have ten values of the maximum likelihood estimates which are outside of the interval, (-2.68111, 2.66947), and, therefore, the estimated $g(\hat{\theta})$ assumes zero, a result that is worse than that of Degree 4 Case.

In addition to the above problem, in Degree 4 Case, several

negative values are observed for even conditional moments. For two cases where $\hat{\theta}$ is no greater than -2.7417 and for four cases where it is no less than 2.7137 , the second conditional moments about the mean turned out to be negative. In addition to these six cases, for four cases where $\hat{\theta}$ is no greater than -2.6723 and for three cases where it is no less than 2.6346 the fourth conditional moment about the mean turned out to be negative. We must exclude seven cases, therefore, if we use up to the second moments, and fourteen cases if we use up to the fourth moments, in Degree 4 Case. On the other hand, there is only one case, i.e., $\hat{\theta} = -3.0555$, in which the fourth conditional moments turned out to be negative, and all the 500 second conditional moments are positive, in Degree 3 Case.

It will be of theoretical propriety that we use the four conditional moments to graduate the conditional distribution of ϵ , given $\hat{\theta}$, assuming some appropriate functional formula. The above observation for Degree 4 Case makes us wonder, however, if it is worth trying, taking the risk of using higher moments which are liable to error. For this reason, in the present study, only the first two conditional moments for each maximum likelihood estimate were used. The seven cases, in which the second moments are negative, were excluded in Degree 4 Case, therefore, and the remaining 493 cases were used, whereas in Degree 3 Case all the 500 cases were used.

The selection of an appropriate functional formula for the conditional probability density function of ϵ , given $\hat{\theta}$, is rather difficult, if we use only the first two moments. It is desirable that

the function allows a variety of different curves, including simple ones as well as complicated ones, to simulate the unobserved probability functions. If, for instance, we use the normal density function, then the curve is always uni-modal and symmetric, which is not desirable. The polynomial of degree 2 also has the same problem, in addition to the possibility of producing negative values for some subset of the domain. In any case, it is difficult to avoid the uni-modality of the curves. The symmetry of the curves can be avoided, however, if we use the Beta density function, or Pearson's Types I and II, assuming appropriate sets of two parameters, i.e., a and b in (3.9). It is to our benefit that the Beta distribution includes a variety of different curves for its density function (e.g., Johnson and Kotz, 1970, page 44), such as straight lines, J-shape curves, U-shape curves, symmetric and non-symmetric uni-modal curves, etc. For this reason, in the present study, the Beta distribution was adopted for the conditional distribution of ϵ , given $\hat{\theta}$, with the assumed values of two parameters, a and b , for each maximum likelihood estimate. The question is how to determine the values of a and b , and it was answered more or less arbitrarily. Following the logic of interval estimation when the probability density function of θ is not known, for each value of the maximum likelihood estimate the value of θ , for which the probability of obtaining that value of the maximum likelihood estimate, or greater, is approximately 0.0054, is specified, and used as a , and the corresponding value of θ , for which the probability of obtaining that value of the maximum likelihood estimate, or less, is approximately

0.0054 , was used as b . Thus we have

$$(7.9) \quad \begin{cases} a_{\hat{\theta}} = \hat{\theta} - (0.215 \times 2.55) & \text{or} & a_{\hat{\theta}} = \hat{\theta} - 0.54825 \\ b_{\hat{\theta}} = \hat{\theta} + (0.215 \times 2.55) & \text{or} & b_{\hat{\theta}} = \hat{\theta} + 0.54825 . \end{cases}$$

The other two parameters, $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$, were estimated from the first two moments and $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$, through (3.10) and (3.11).

Following the previous study (Samejima, 1977b), five scores, $\tilde{\theta}$, are calibrated for each of the 500 maximum likelihood estimates, according to the Beta distribution thus specified. As the result, we obtained 2500 $\tilde{\theta}$ in Degree 3 Case, and 2465 $\tilde{\theta}$ in Degree 4 Case. Figures 7-7 and 7-8 present the frequency distributions of $\tilde{\theta}$ in Degree 3 and 4 Cases respectively, together with the distributions of θ . As was mentioned earlier, the five hundred values of the latent trait θ are placed between -2.475 and 2.475 inclusive, forming subgroups of five each at the interval length of 0.05, so as the frequency distribution the rectangle shown in Figures 7-7 for Degree 3 Case is the case. In Degree 4 Case, however, we had to exclude Subject 2 with $\theta = -2.425$ because of the negative estimated probability function of $\hat{\theta}$, and Subjects 99, 101, 201, 296, 299 and 300 because of their negative values of estimated second conditional moments about the mean, whose θ are 2.425 , -2.475 , -2.475 , 2.275 , 2.425 and 2.475 respectively. The histogram shown in Figure 7-8 differs slightly from the rectangle, therefore, at both ends of the interval of θ . The estimated conditional moments for each of the 500 maximum likelihood estimates, together with the values of β_1 , β_2 and the criterion κ , are given in Appendix I, as Table A-1-1 for Degree 3 Case and Table A-1-2 for Degree 4 Case respectively.

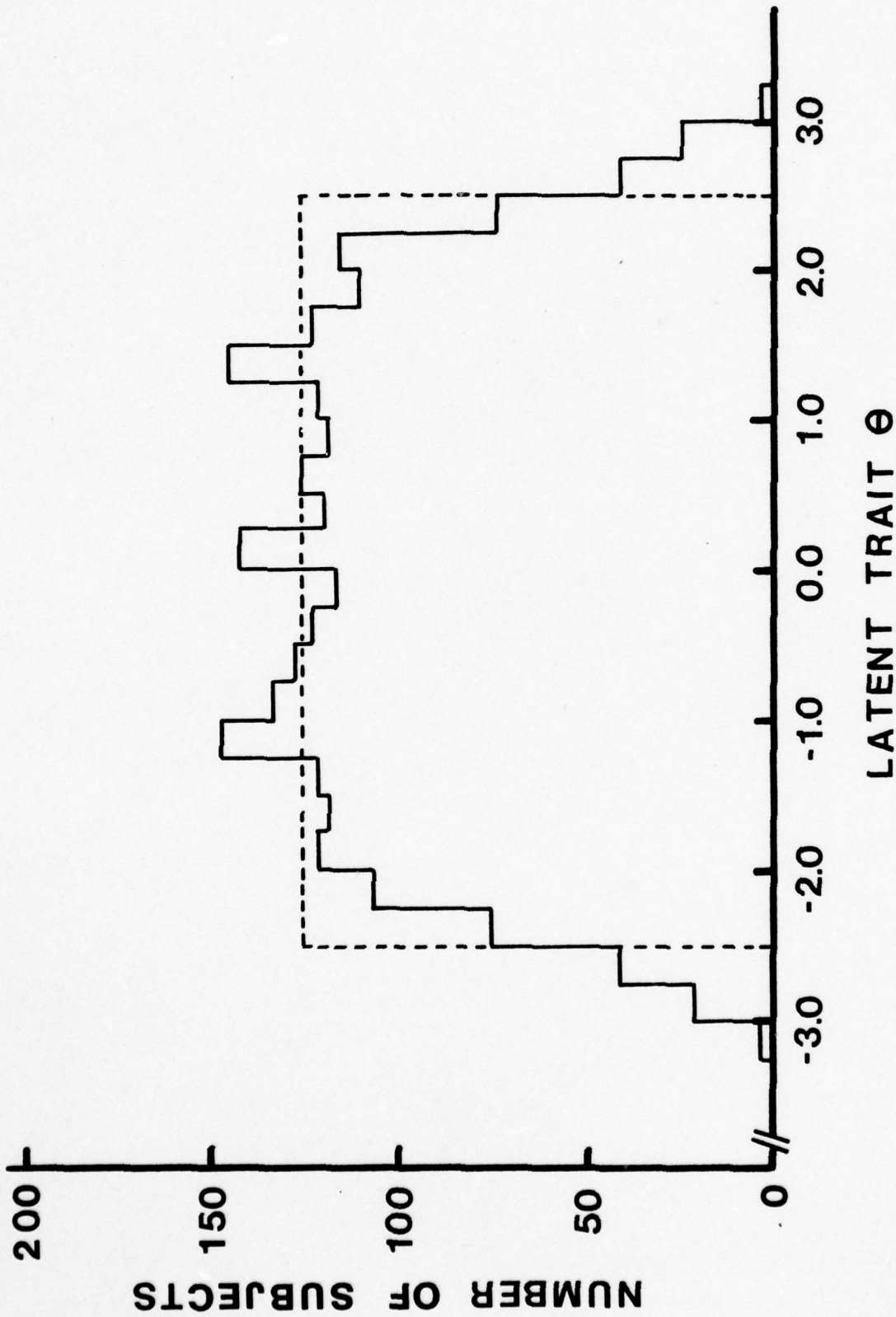


FIGURE 7-7

The frequency distributions of 2500 $\tilde{\theta}$ in Degree 3 Case (solid line) and of ability θ (dotted line). The latter is multiplied by five for comparison.

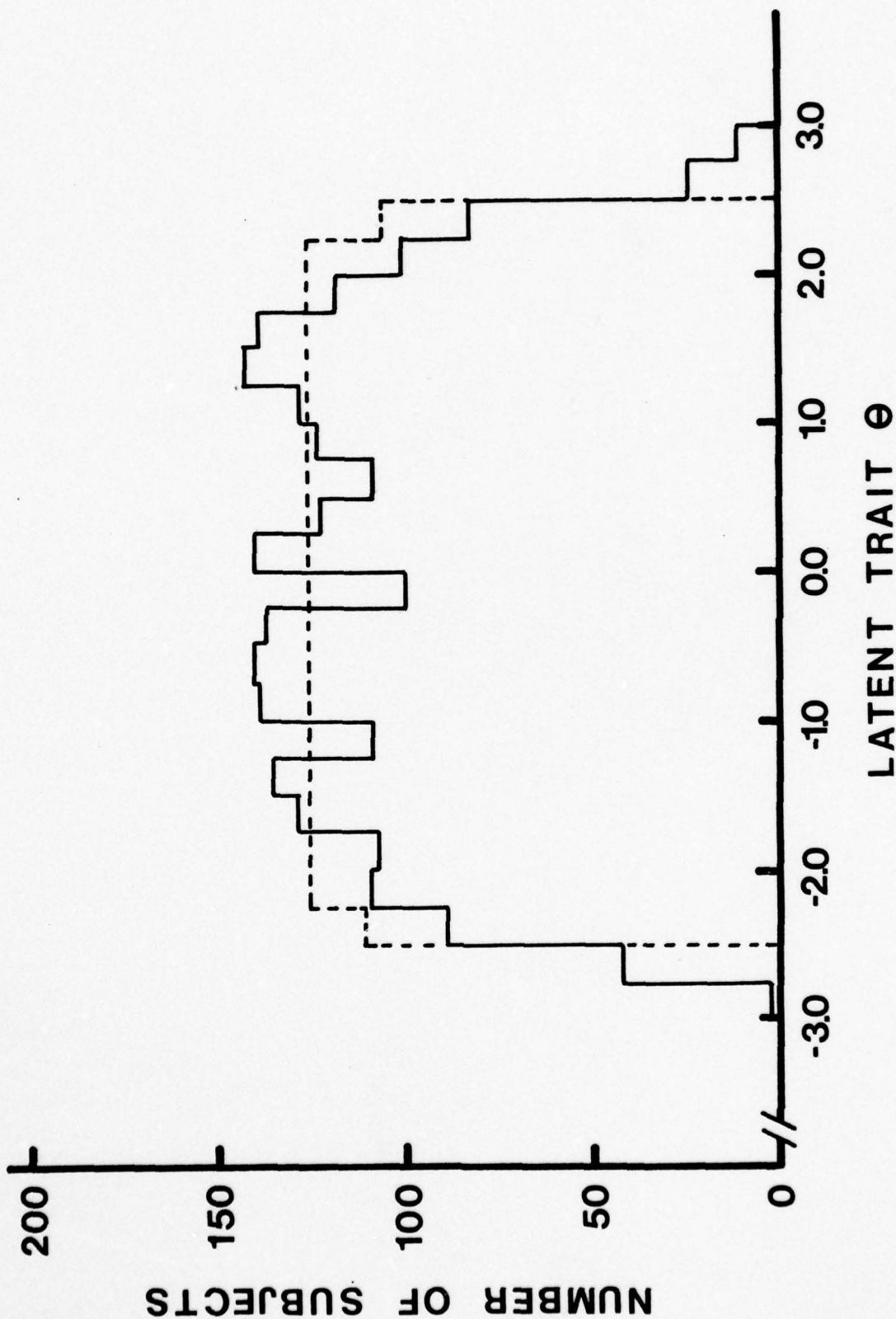


FIGURE 7-8

The frequency distributions of 2465 $\tilde{\theta}$ in Degree 4 Case (solid line) and of ability θ (dotted line). The latter is multiplied by five for comparison.

Comparing these two results, we can see that the fit of the frequency distribution of $\tilde{\theta}$ to that of θ is slightly better for Degree 4 Case than for Degree 3 Case, especially at both ends of the interval of θ . If we compare Figure 7-7 with the five histograms of $\tilde{\theta}$ obtained for Items 2, 4, 6, 8 and 10 by the Normal Approximation Method, however, we can see that the fit for Degree 3 Case is as good as those in these five cases (cf. Samejima, 1977b, Figure 7, page 187).

For each of the ten binary items, the 500 examinees are classified into two groups, i.e., the group of those who answered item g correctly, and that of those who responded incorrectly. This procedure also divides the 2500 or 2465 $\tilde{\theta}$ into two groups, with respect to each item. Thus the frequency distributions shown in Figures 7-7 and 7-8 are divided into two smaller frequency distributions respectively, and, for each interval, the frequency ratio of the frequency for the "correct" group to that for the total group provide us with the estimated value of the item Characteristic function of the item. Figure 7-9 presents these frequency ratios, together with the corresponding results obtained by the Normal Approximation Method (Samejima, 1977b), for each of the ten binary items. In these eleven graphs, the results obtained on 500 $\tilde{\theta}$ in the previous study are plotted with small hollow circles, those on 2500 $\tilde{\theta}$ and on 5000 $\tilde{\theta}$ are with large circles and small circles, each of which is put in a large hollow circle, respectively, and those obtained in the present study for Degree 3 and 4 Cases are with triangles and squares respectively. Except for the first group, the plots are solid in the range of (0.05, 0.95), and hollow otherwise.

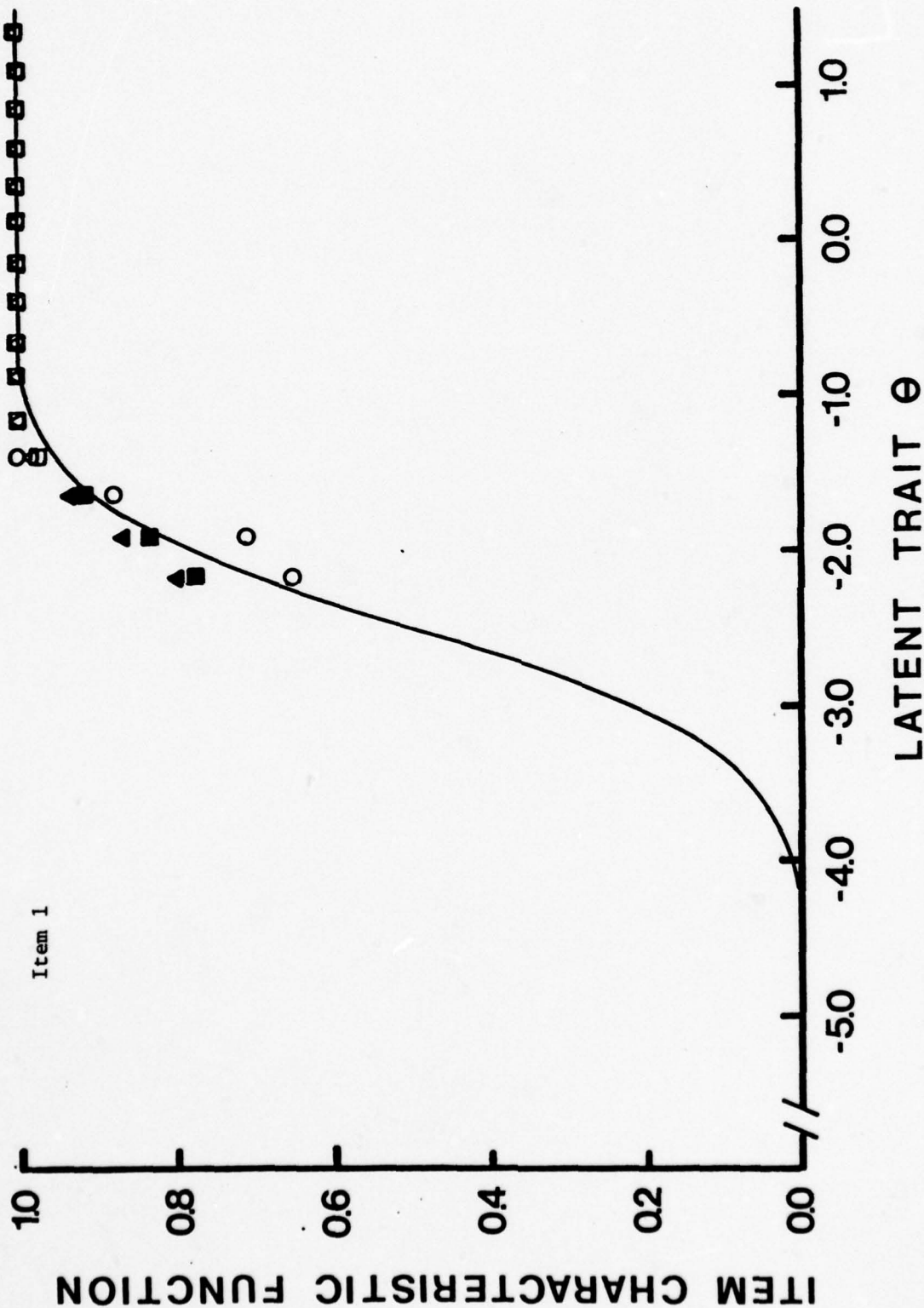


FIGURE 7-9

The true item characteristic function (curve), and the frequency ratio of those who answered the item correctly to the total frequency for each interval using: 2500 θ in Degree 3 Case (triangle), 2465 θ in Degree 4 Case (square), and 500 θ obtained by the Normal Approximation Method (circle).

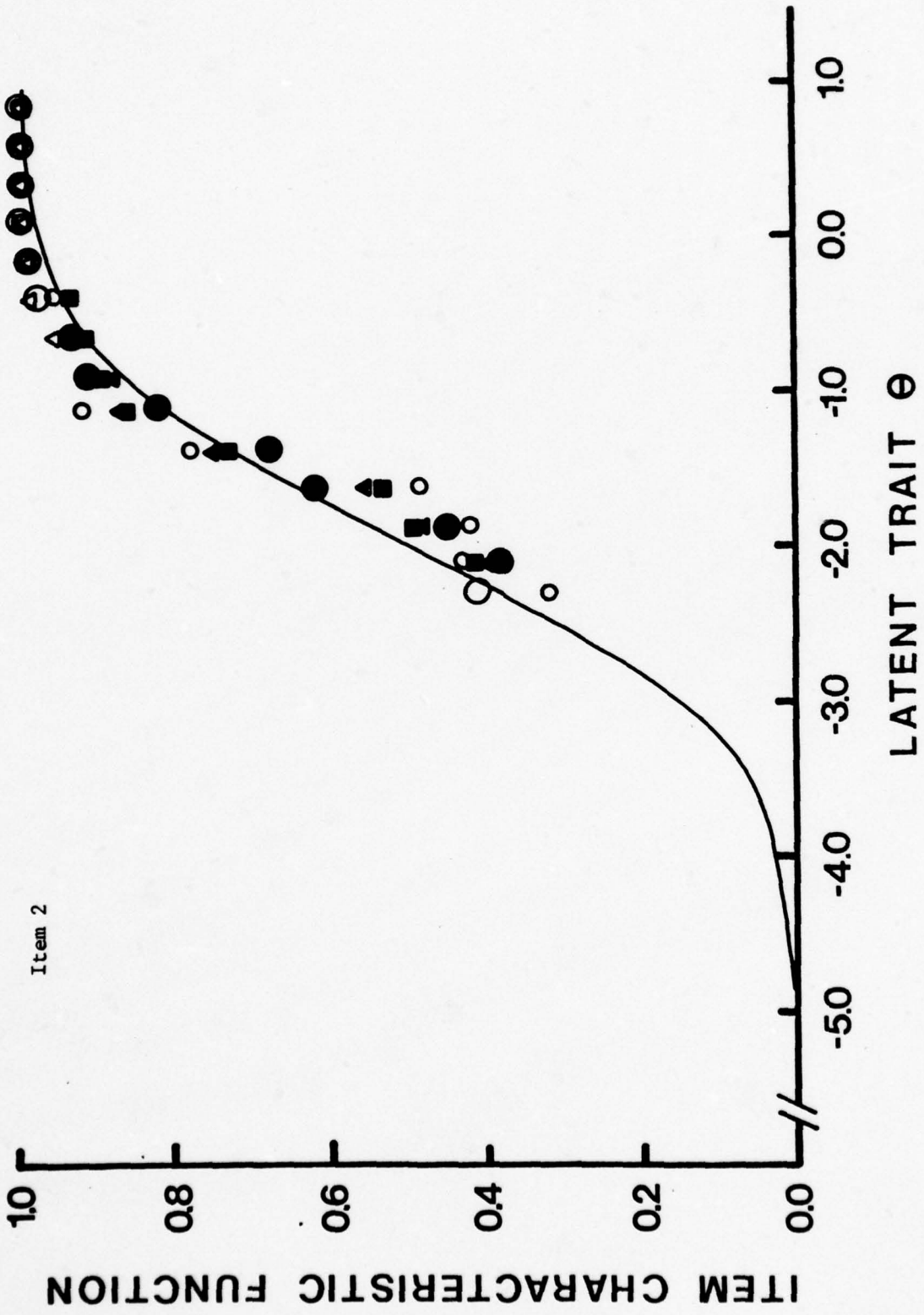
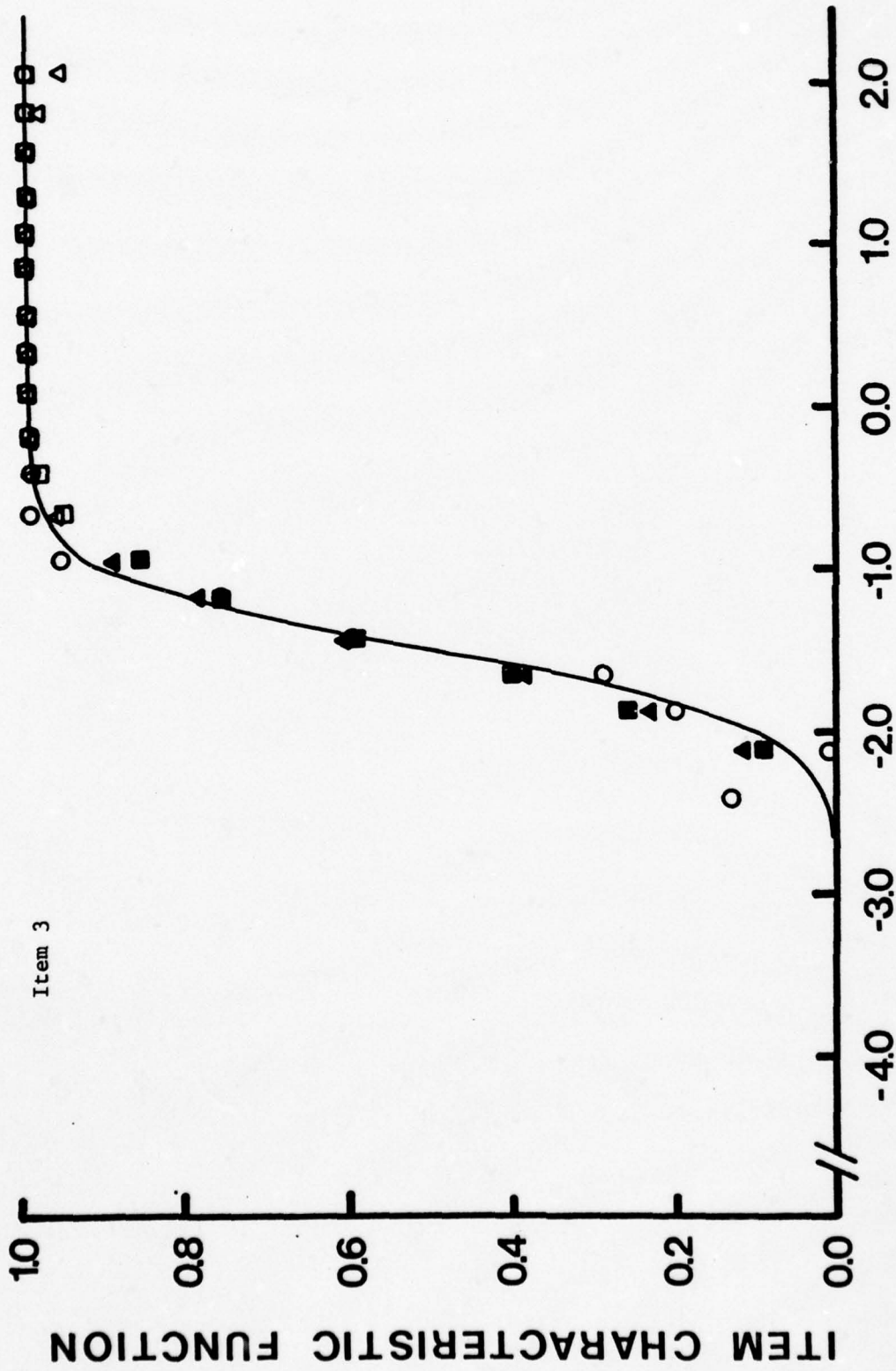


FIGURE 7-9

(Continued)

Addition: the result obtained by using 2500 $\tilde{\theta}$ in the Normal Approximation Method (large circle).



LATENT TRAIT θ

FIGURE 7-9

(Continued)

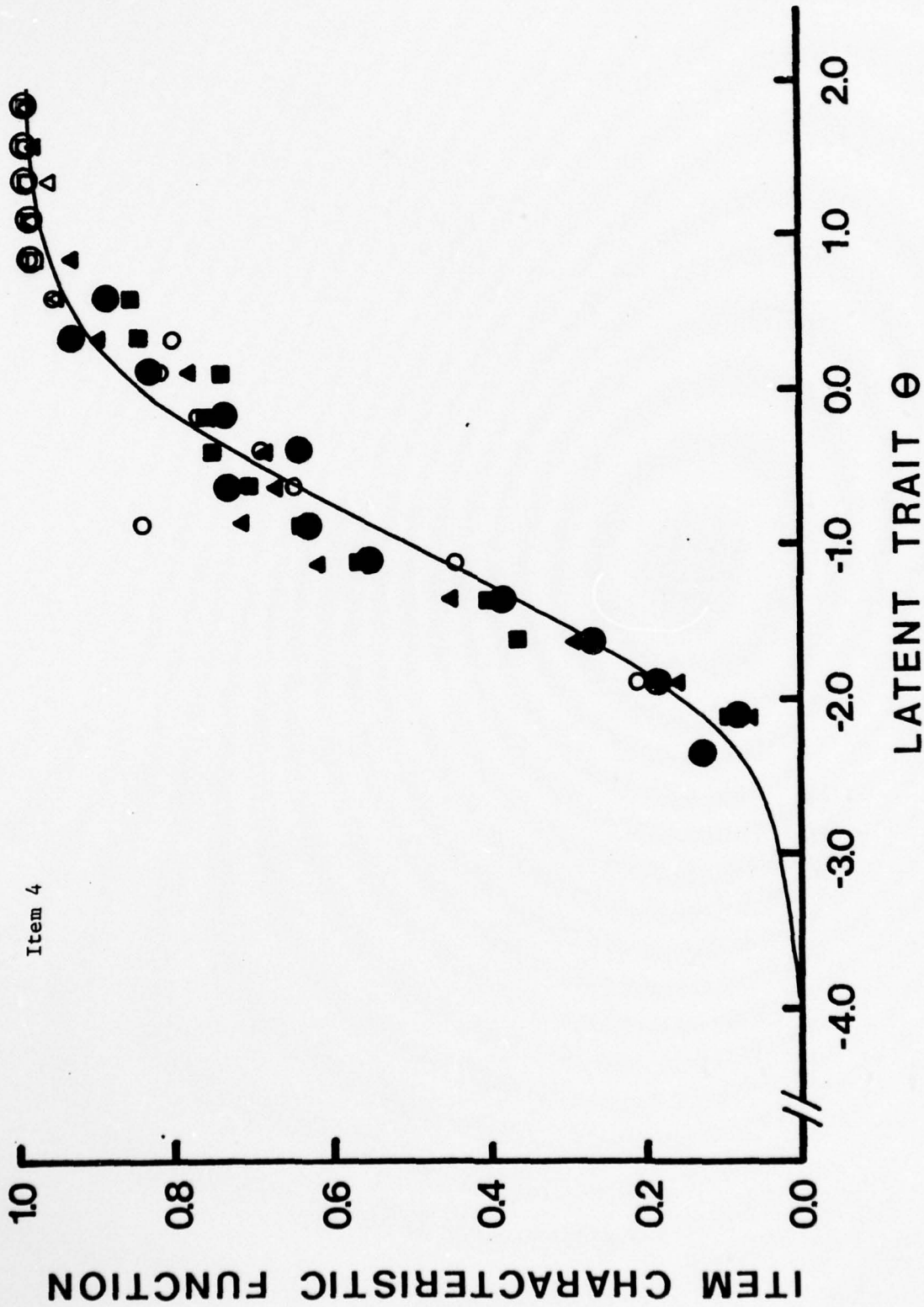


FIGURE 7-9

(Continued)

Addition: the same as Item 2 .

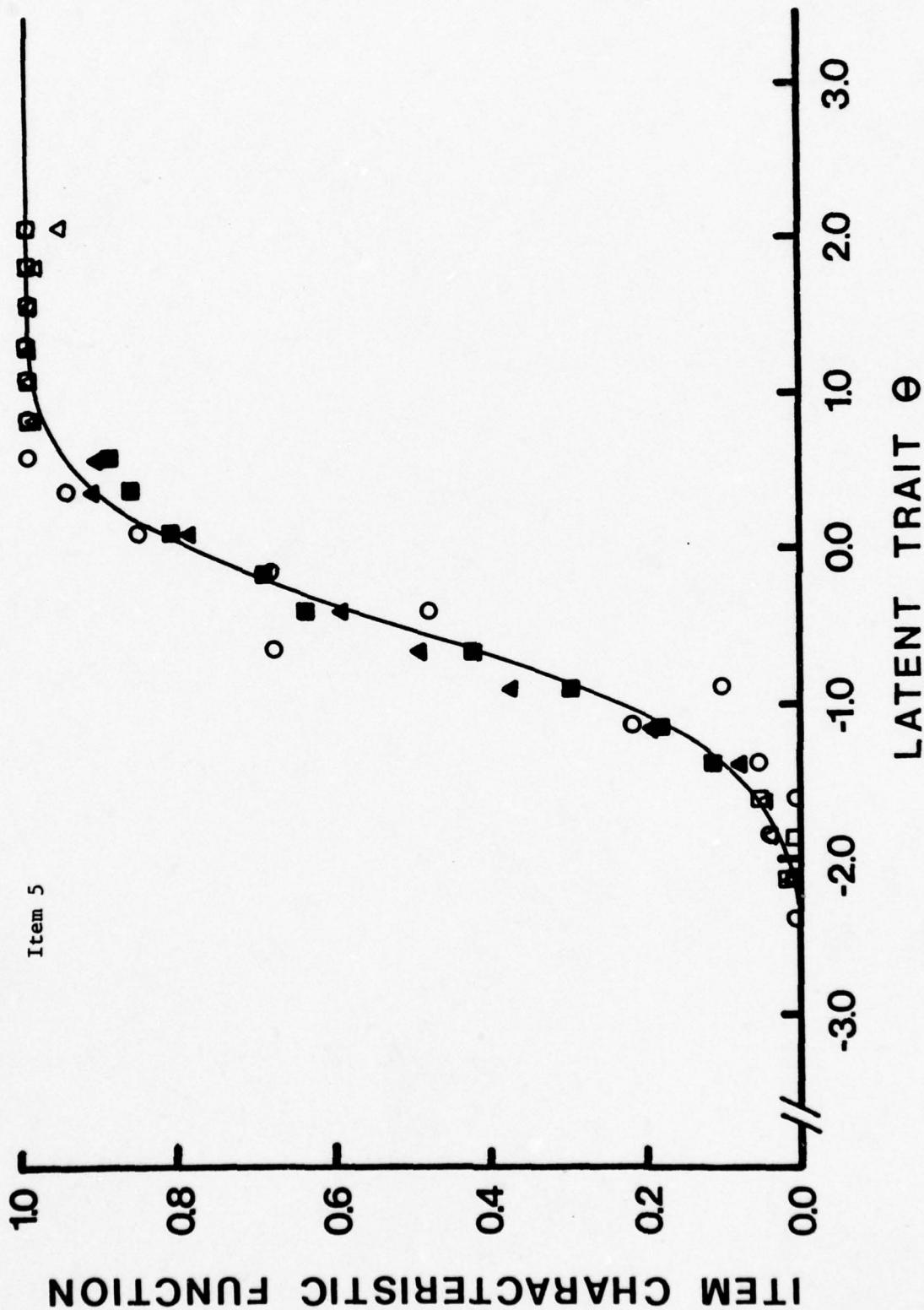


FIGURE 7-9

(Continued)

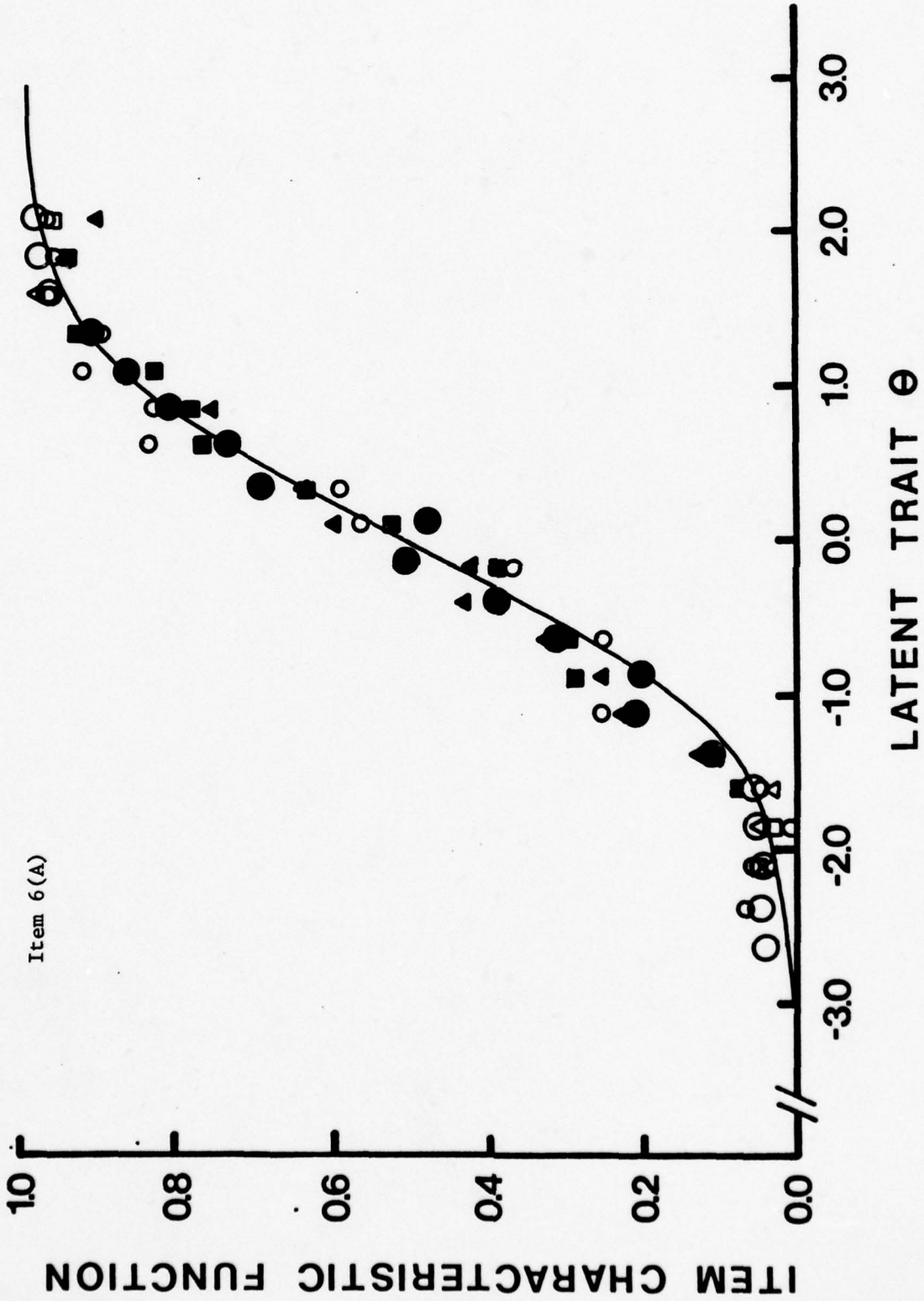


FIGURE 7-9

(Continued)

Addition: the same as Items 2 and 4.

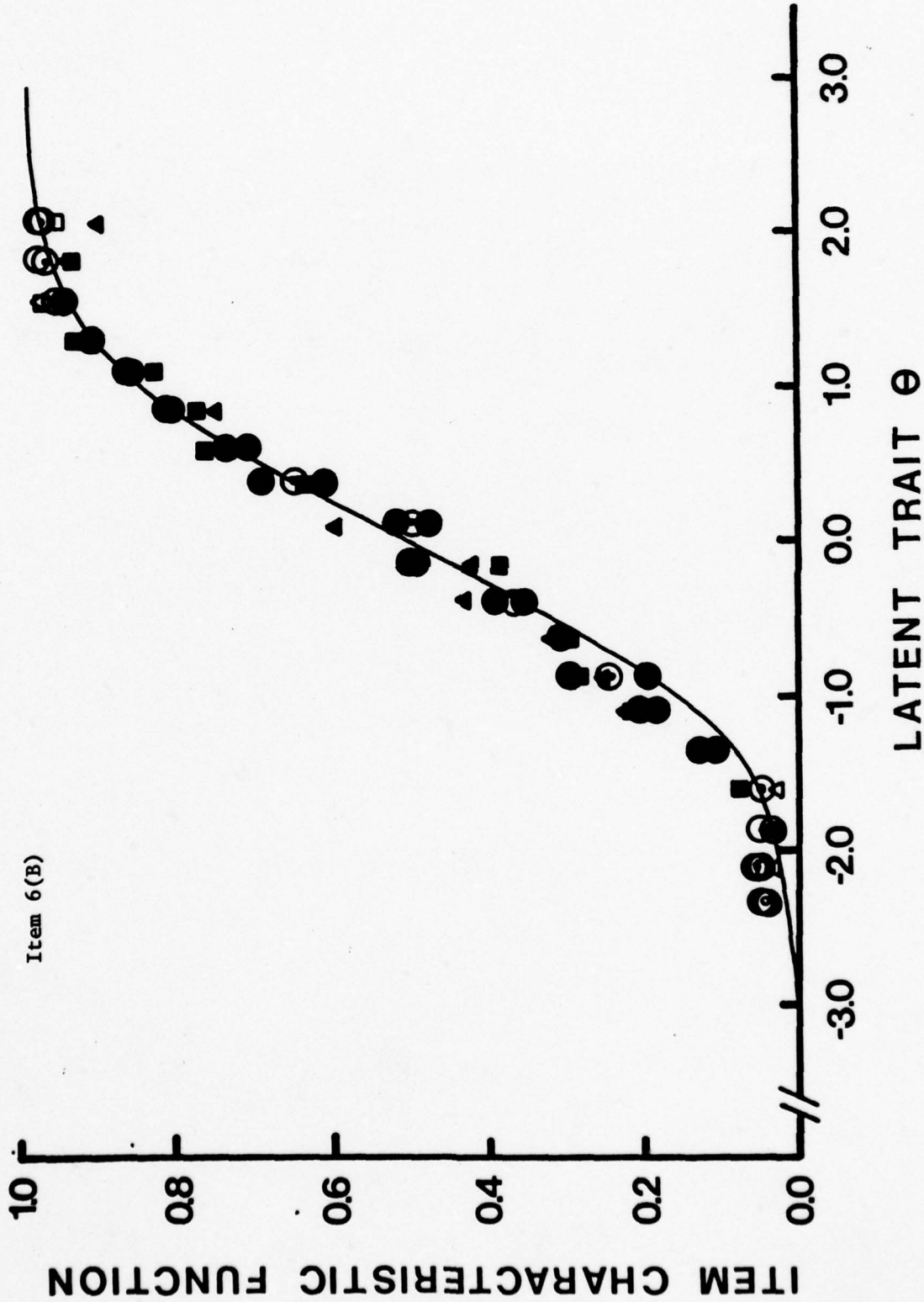


FIGURE 7-9

(Continued)

Exclusion: Normal Approximation Method, both the 500 $\hat{\theta}$ and 2500 $\hat{\theta}$ cases.
Inclusion: Normal Approximation Method, the other 2500 $\hat{\theta}$ and 5000 $\hat{\theta}$ cases.

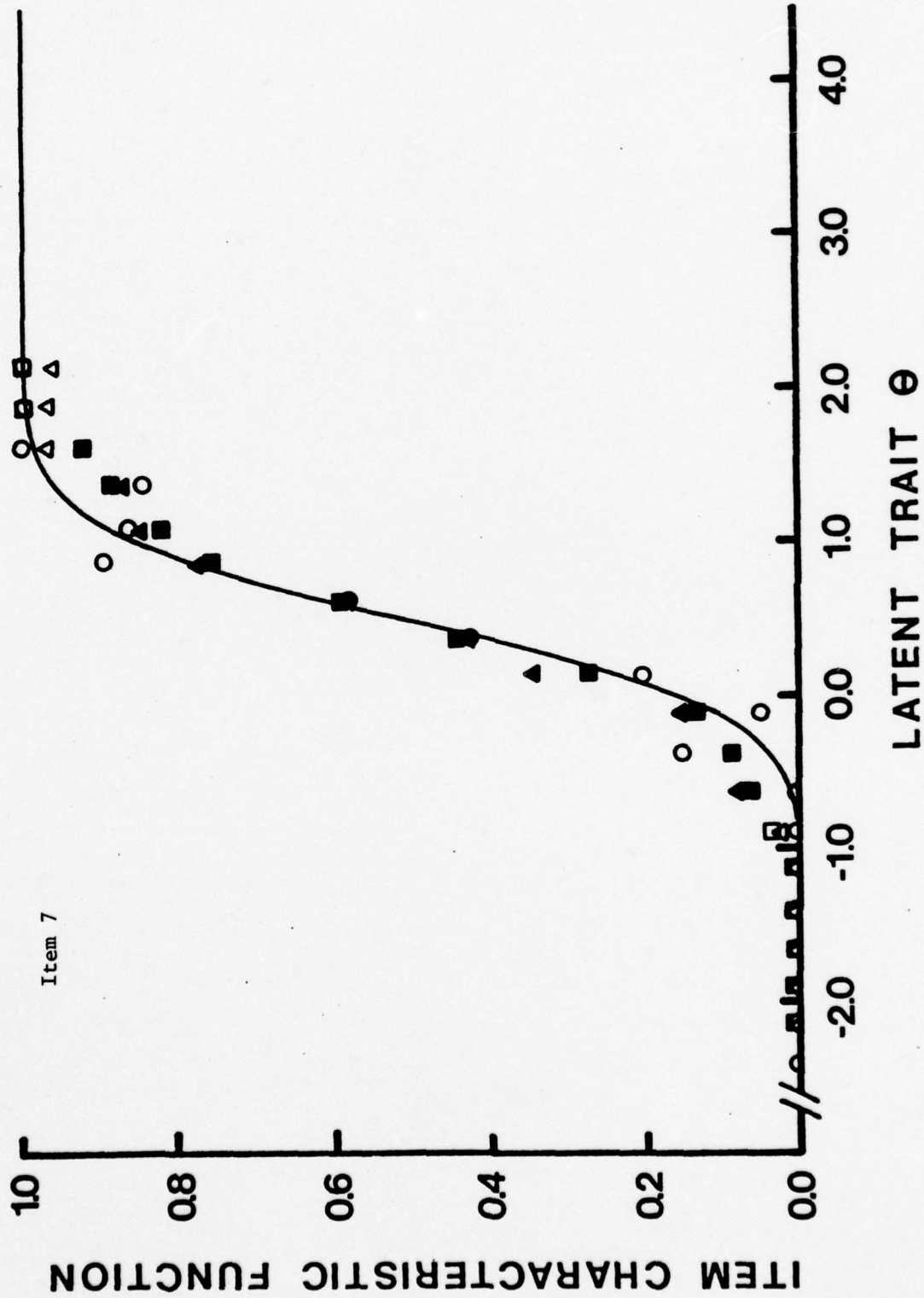


FIGURE 7-9
(Continued)

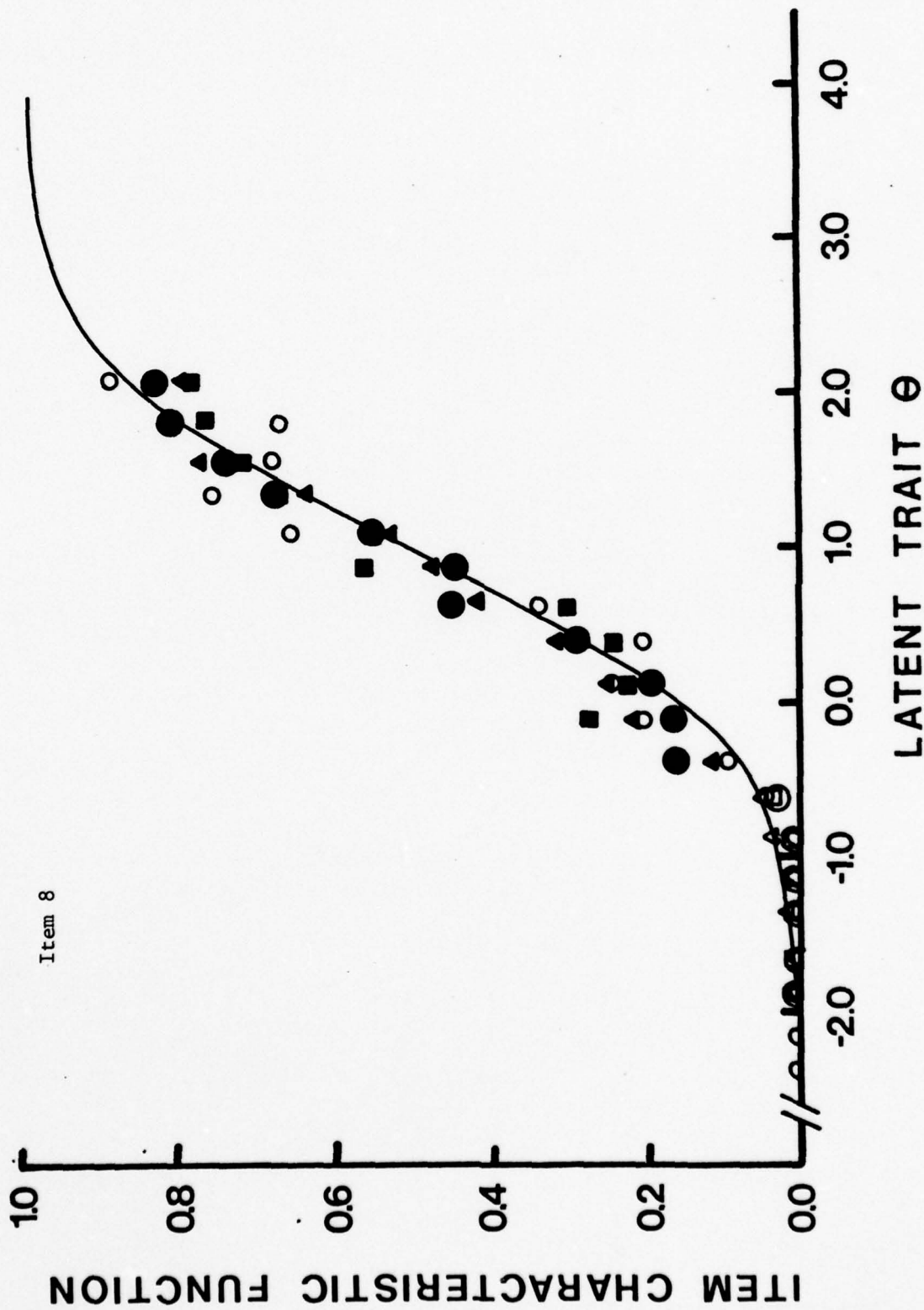


FIGURE 7-9

(Continued)

Addition: the same as Items 2, 4 and 6(A).

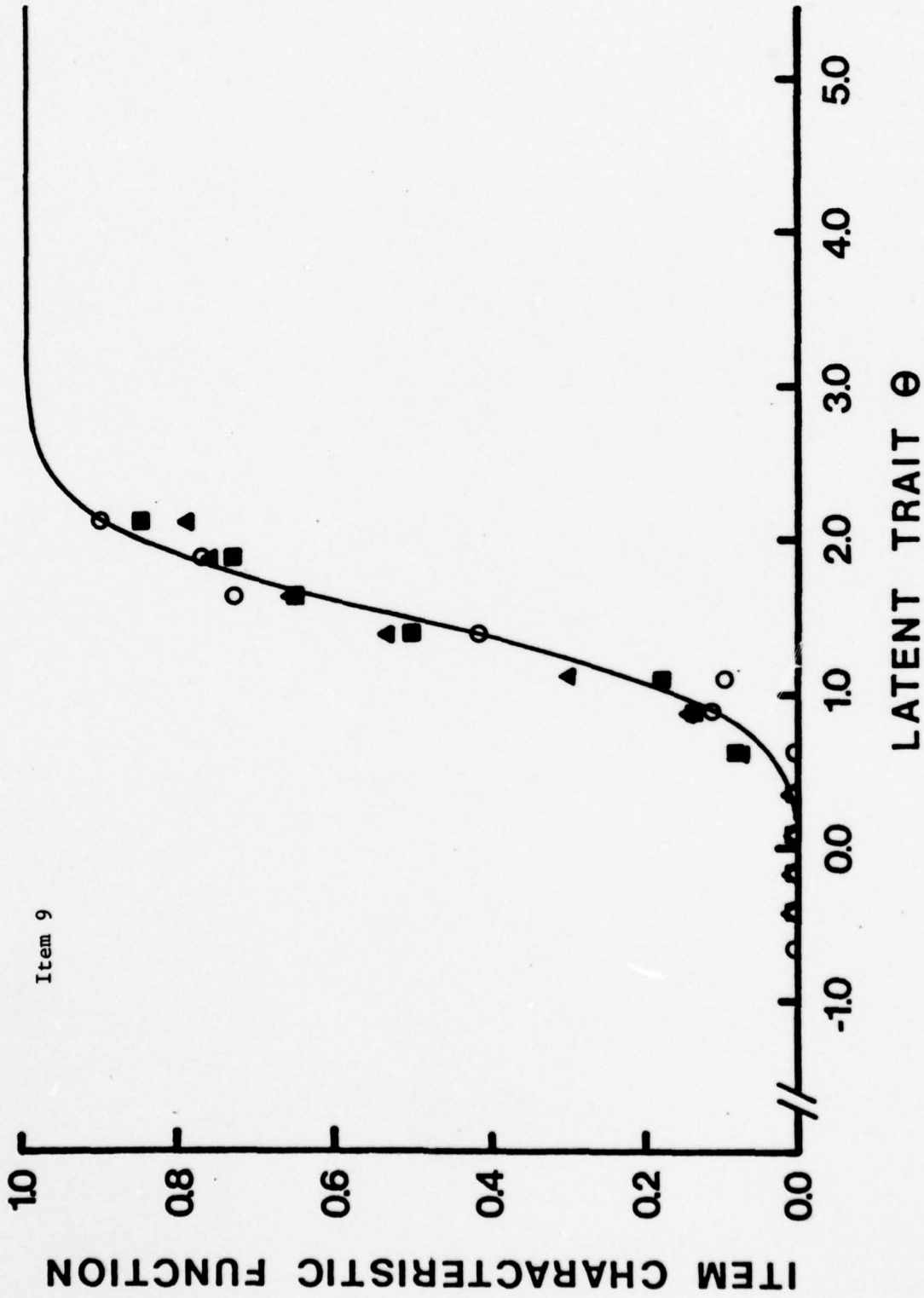
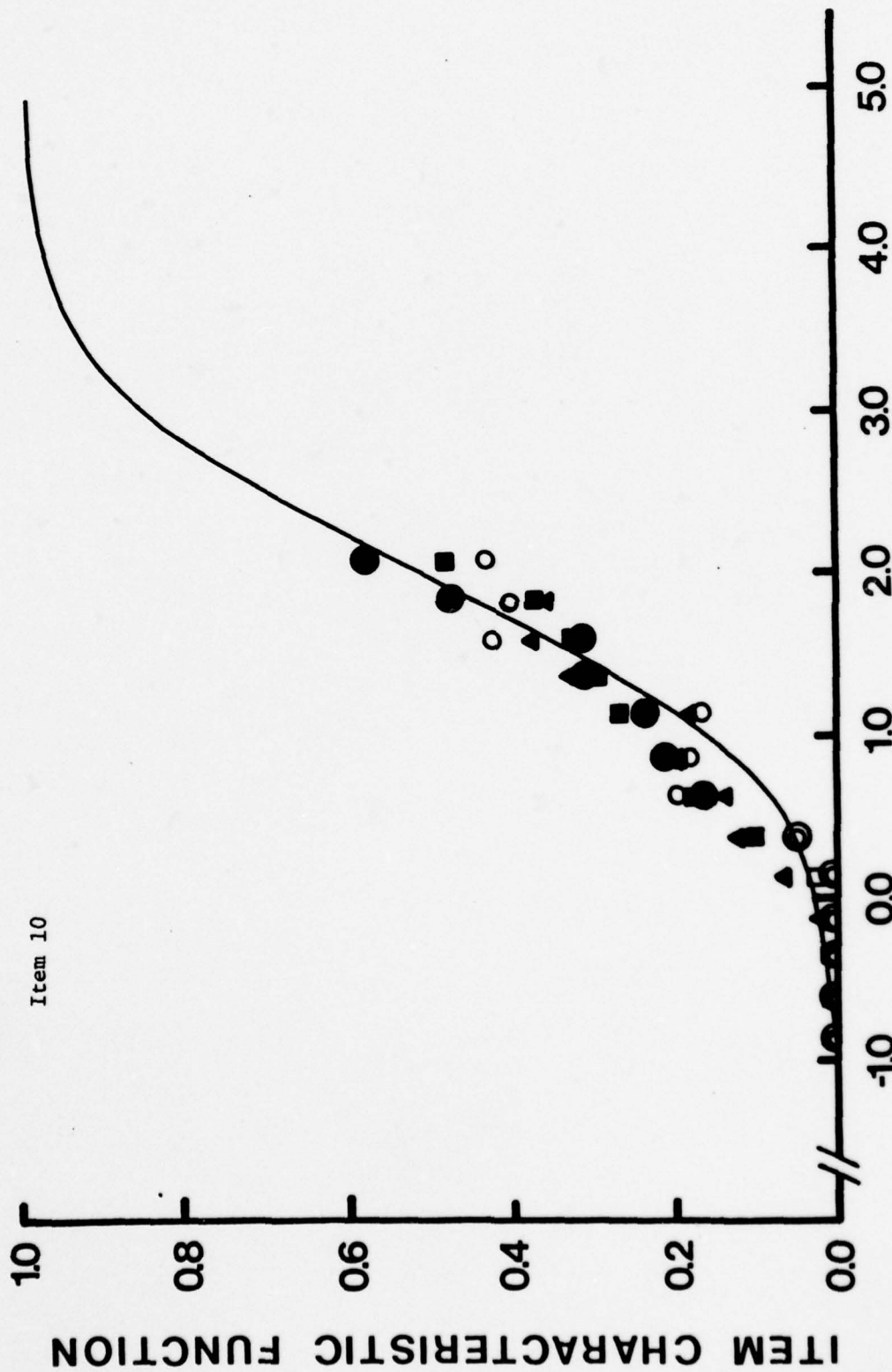


FIGURE 7-9

(Continued)



LATENT TRAIT Θ

FIGURE 7-9

(Continued)

Addition: the same as Items 2, 4, 6(A) and 8.

The numbers of hypothetical subjects who belong to the "correct" and "incorrect" groups for each of the ten binary items are given in Appendix II as Table A-2-1 . Each frequency in this table multiplied by 5 makes the frequency of $\tilde{\theta}$ for that category, for Degree 3 Case. In Degree 4 Case, each frequency added to the negative number shown in the brackets and then multiplied by 5 makes the frequency of $\tilde{\theta}$ for that category.

The estimates of the two parameters of the normal ogive model, \hat{a}_g and \hat{b}_g , were obtained through the method described in section 6. These values are shown in Tables 7-1 and 7-2, together with the true parameter values. In adopting the above method, those frequency ratios which are less than 0.05 or greater than 0.95 are excluded. As the result, the numbers of intervals used range from 3 to 14, and these numbers are also presented in Tables 7-1 and 7-2. Similar computations were made for both \hat{a}_g and \hat{b}_g , by changing the cutting points of the frequency ratio to 1) 0.15 and 0.85, 2) 0.10 and 0.90, and 3) 0.01 and 0.99. These results are presented in Appendix III as Tables A-3-1 and A-3-2, together with those obtained by the Normal Approximation Method.

TABLE 7-1
 The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using the Frequency Ratios between 0.05 and 0.95

ITEM	METHOD	TRUE a_g	DGR. 3		DGR. 4		(N = 500)		(N = 2500)		(N = 5000)	
							NORMAL	NORMAL	NORMAL	NORMAL	NORMAL	NORMAL
1		1.5	1.288 (3)	1.354 (3)	0.602 (2)							
2		1.0	1.315 (6)	1.128 (8)	1.381 (7)	1.301 (7)						
3		2.5	2.000 (6)	1.938 (6)	2.227 (4)							
4		1.0	0.926 (12)	0.812 (12)	0.807 (10)	0.959 (12)						
5		1.5	1.364 (9)	1.320 (9)	1.668 (5)							
6		1.0	0.787 (14)	0.890 (14)	0.951 (11)	0.936 (12)	0.919 (13)					
7		2.0	1.451 (9)	1.446 (10)	1.348 (6)							
8		1.0	0.842 (11)	0.775 (11)	0.880 (10)	0.888 (11)						
9		2.0	1.593 (7)	1.721 (7)	2.264 (6)							
10		1.0	0.773 (9)	0.616 (8)	0.606 (7)	0.751 (7)						

The number of intervals used in each estimation is shown in parentheses.

TABLE 7-2

The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using the Frequency Ratios between 0.05 and 0.95

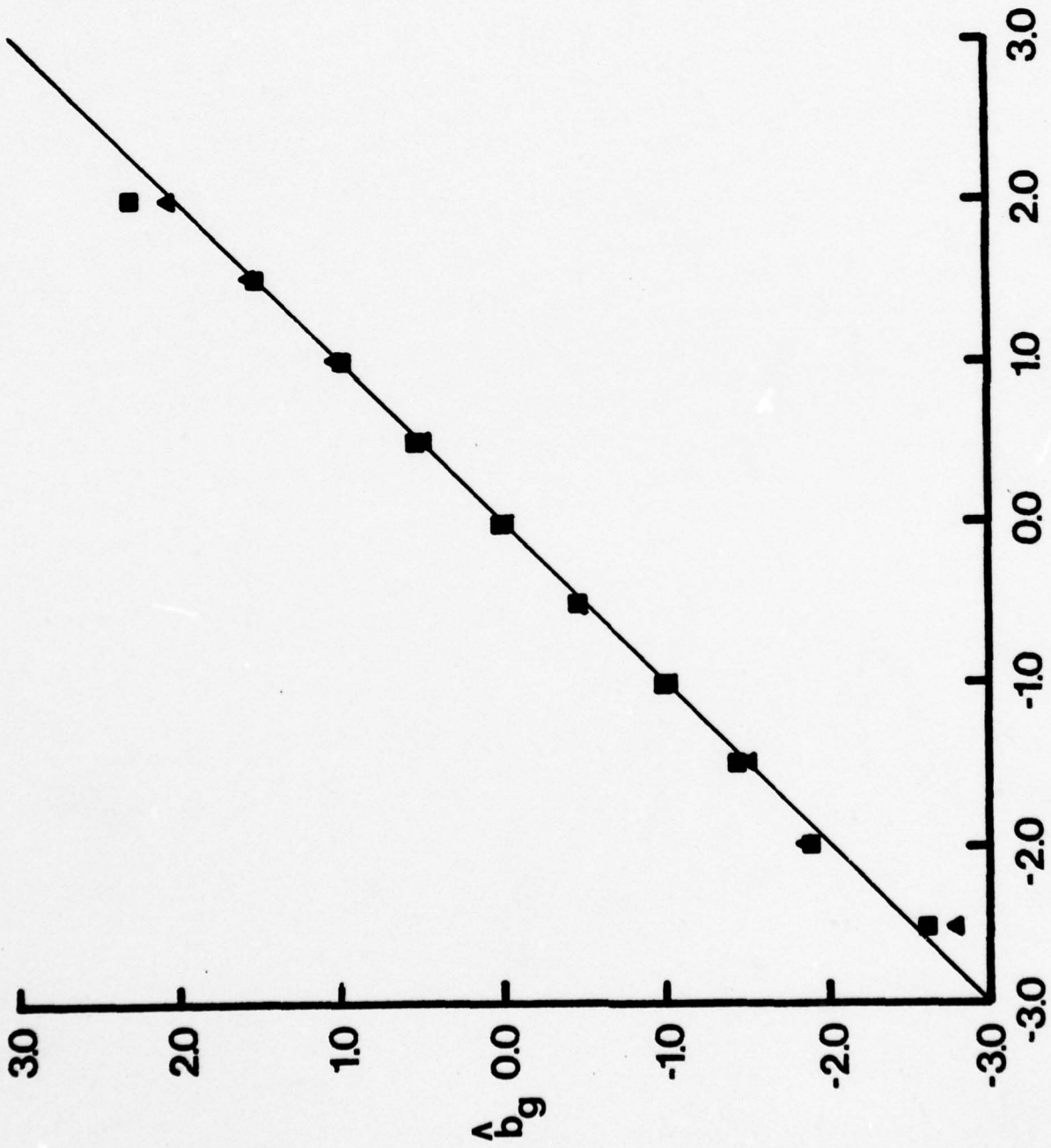
ITEM	METHOD					
	TRUE b_g	DGR. 3	DGR. 4	(N = 500) NORMAL	(N = 2500) NORMAL	(N = 5000) NORMAL
1	-2.5	-2.770 (3)	-2.643 (3)	-3.015 (2)		
2	-2.0	-1.856 (6)	-1.888 (8)	-1.857 (7)	-1.831 (7)	
3	-1.5	-1.502 (6)	-1.474 (6)	-1.445 (4)		
4	-1.0	-1.004 (12)	-1.001 (12)	-1.064 (10)	-0.971 (12)	
5	-0.5	-0.495 (9)	-0.469 (9)	-0.509 (5)		
6	0.0	-0.068 (14)	-0.051 (14)	-0.062 (11)	-0.048 (12)	-0.056 (12)
7	0.5	0.476 (9)	0.530 (10)	0.520 (6)		
8	1.0	0.932 (11)	0.970 (11)	1.012 (10)	0.953 (11)	
9	1.5	1.464 (7)	1.493 (7)	1.512 (6)		
10	2.0	2.076 (9)	2.303 (8)	2.285 (7)	2.031 (7)	

The number of intervals used in each estimation is shown in parentheses.

VIII. Further Observations and Analyses of the Results

The results presented in the preceding section indicate that, in the present study, the 2-Parameter Beta Method has worked as well as the Normal Approximation Method. There is no distinct indication, however, that the present method is better than the previous one, although it contains more mathematical sophistication. We can say that, in general, the $\tilde{P}_g(\theta)$'s obtained by the 2-Parameter Beta Method show better fits to the theoretical $P_g(\theta)$'s than do those obtained by the Normal Approximation Method using 500 $\tilde{\theta}$'s, but they are just as good as those obtained by the Normal Approximation Method using 2500 $\tilde{\theta}$'s (cf. Figure 7-9).

As for the estimates of the two parameters, a_g and b_g , it is observed that, in general, the estimates of the difficulty parameter, b_g , are close enough to the true parameter values. Figure 8-1 presents these estimates plotted against the true parameter values for both Degree 3 and 4 Cases. We can see that all these plots are, at least, reasonably close to the line intercepting the origin with an angle of 45 degrees from the abscissa. We notice, however, that the estimates of the discrimination parameters tend to be less than the true parameter values. Figure 8-2 presents the estimates of a_g for the ten items plotted against the true parameter values, and shows this tendency clearly. The same has been observed in the results of the Normal Approximation Method, and this inaccuracy in estimating the discrimination parameter is a subject we should work on in the future. Unlike the Normal Approximation Method, the 2-Parameter Beta Method leaves much



TRUE PARAMETER VALUES

FIGURE 8-1
Estimated difficulty parameters in Degree 3 Case (triangles) and in Degree 4 Case (squares) plotted against the true parameter values.

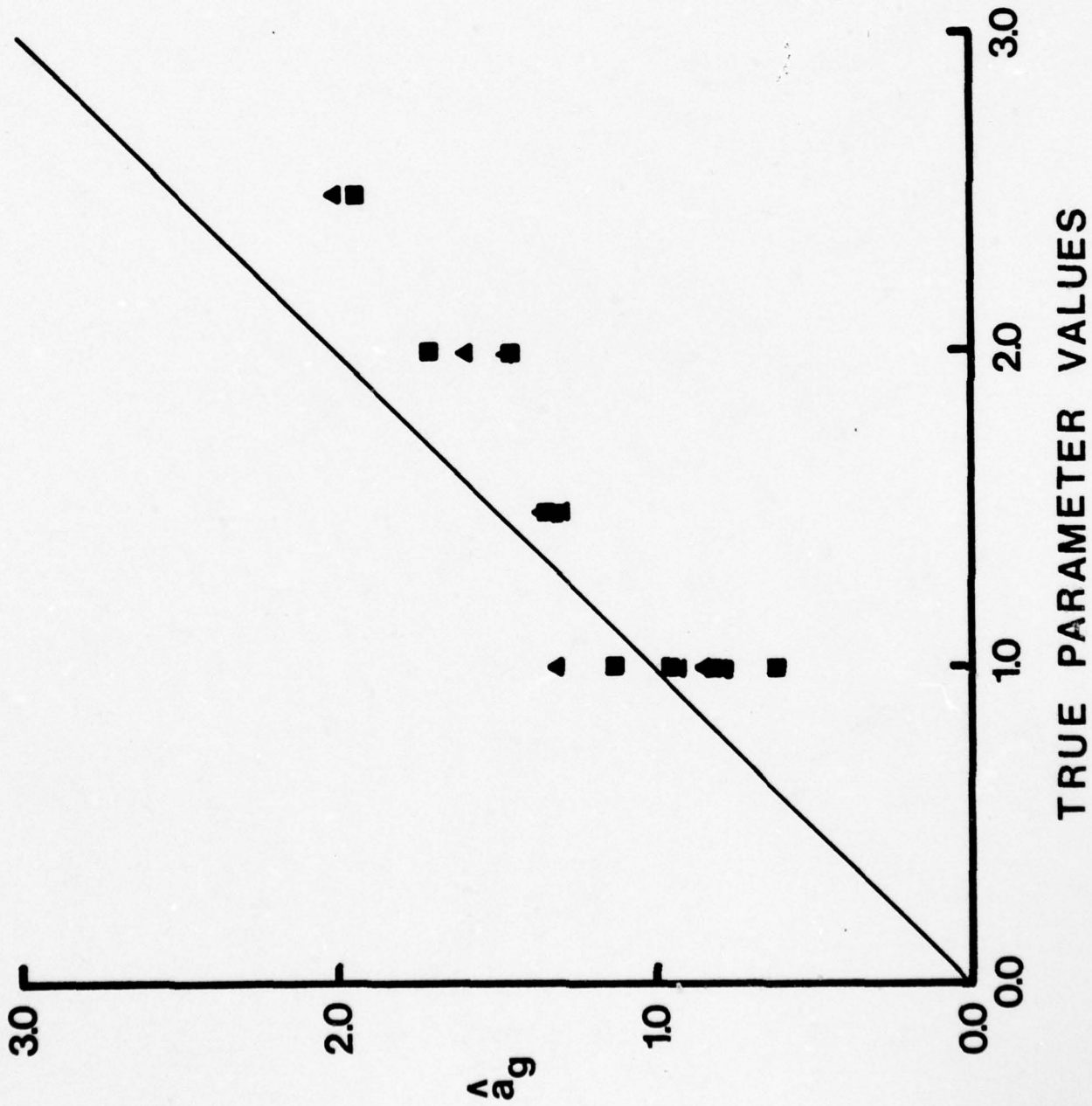


FIGURE 8-2

Estimated discrimination parameters in Degree 3 Case (triangles) and in Degree 4 Case (squares) plotted against the true parameter values.

room for improvement in accuracy, including the adjustment of the apriori set parameters, $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$. In the present study, they are set equal to $(\hat{\theta} \pm 2.55\sigma)$, and this may be too wide an interval. Further investigation will be done in a later study.

There is evidence which debates the above possibility and will explain the reason for the inaccuracy in estimating a_g , however. Table 8-1 presents the results of the discrimination parameter estimation obtained directly from the frequency ratios of the true θ , by the same method using the twenty equal length intervals with the total frequency of 25 for each interval. We can see that for nine items the estimated discrimination parameters are less than the true parameter values, and these values are very close to the estimates obtained in Degree 3 and 4 Cases. If we consider this fact, we must say that the 2-Parameter Beta Method, as well as the Normal Approximation Method, has worked very well. In the same table, the results obtained directly from the frequency ratios of the 500 maximum likelihood estimates are also presented, for two situations where all the 20 intervals are used and where only 16 intervals with the total frequency greater than, or equal to, 20 are used. As is expected from theory (Samejima, 1977b), for most items the estimates of the discrimination parameter are less than those obtained directly from the frequency ratios of the true values of θ . Similar estimates obtained by changing the cutting points of the frequency ratio to each of the three different sets, 0.15 and 0.85, 0.10 and 0.90, and 0.01 and 0.99, are presented in Appendix IV as Table A-4-1. The corresponding tables for the

TABLE 8-1

The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained Directly from the Frequency Ratios between 0.05 and 0.95 of the True θ , and from Those of the Maximum Likelihood Estimates Using 20 and 16 Intervals Respectively

ITEM	METHOD			
	TRUE a_g	\hat{a}_g from θ 0.05- 0.95	\hat{a}_g from MLE (20 points) 0.05- 0.95	\hat{a}_g from MLE (16 points) 0.05- 0.95
1	1.5	2.250 ₃	0.876 ₄	1.206 ₃
2	1.0	0.980	1.137	1.195
3	2.5	2.369	1.973	2.111
4	1.0	0.862	0.794	0.897
5	1.5	1.327	1.297	1.222
6	1.0	0.778	0.835	0.915
7	2.0	1.532	1.428	1.382
8	1.0	0.943	0.785	0.749
9	2.0	1.889	1.810	1.869
10	1.0	0.727	0.820	0.534

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE 8-2

The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained Directly from the Frequency Ratios between 0.05 and 0.95 of the True θ , and from Those of the Maximum Likelihood Estimates Using 20 and 16 Intervals Respectively

ITEM	METHOD			
	TRUE b_g	\hat{b}_g from θ 0.05- 0.95	\hat{b}_g from MLE (20 points) 0.05- 0.95	\hat{b}_g from MLE (16 points) 0.05- 0.95
1	-2.5	-2.411 ₃	-3.010 ₄	-2.631 ₃
2	-2.0	-2.023	-1.953	-1.913
3	-1.5	-1.478	-1.495	-1.482
4	-1.0	-0.957	-1.050	-1.119
5	-0.5	-0.442	-0.492	-0.445
6	0.0	0.001	-0.062	-0.112
7	0.5	0.577	0.518	0.580
8	1.0	0.969	0.959	0.905
9	1.5	1.514	1.511	1.507
10	2.0	2.117	2.012	2.495

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

difficulty parameter b_g are presented as Table 8-2 and Table A-4-2 in Appendix IV respectively.

Figure 8-3 presents these frequency ratios of θ and those of the maximum likelihood estimate. The comparison of these ten graphs with those in Figure 7-9 makes it obvious that the frequency ratios are by no means closer to the theoretical curves than those obtained by the 2-Parameter Beta Method, or by the Normal Approximation Method.

From these observations, it will be concluded that the 2-Parameter Beta Method has proved itself to be useful in estimating the operating characteristics of item response categories. It has also been shown that there is little evidence to support Degree 4 Case in preference to Degree 3 Case, regardless of the distinct difference between the two graduated curves (cf. Figures 7-2 and 7-3), at least, in the present study. This does not encourage us to use a polynomial of a higher degree to graduate the raw data of maximum likelihood estimates in the present study. In fact, the method of moments to fit a polynomial of degree 5 to $g(\lambda)$ requires up to the fifth moment, and the coefficients in the form

$$(8.1) \quad \hat{g}(\lambda) = \alpha + \beta\lambda + \gamma\lambda^2 + \delta\lambda^3 + \nu\lambda^4 + \zeta\lambda^5$$

are computed by

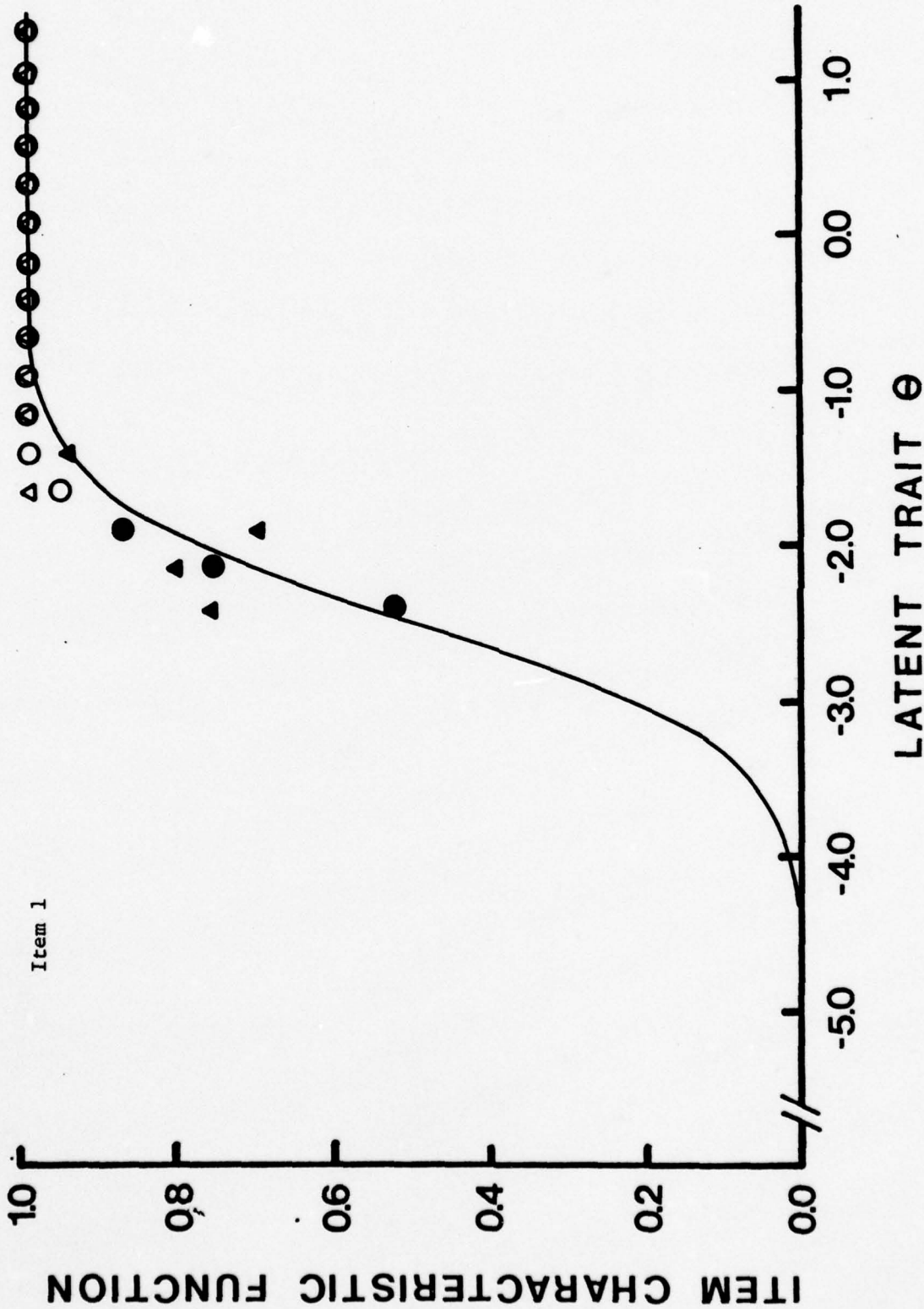


FIGURE 8-3

The true item characteristic function (curve), and the frequency ratio of those who answered the item correctly to the total frequency for each interval of θ (circle) and the corresponding frequency ratio of the maximum likelihood estimate (triangle); solid figures are within (0.05, 0.95) of the frequency ratio.

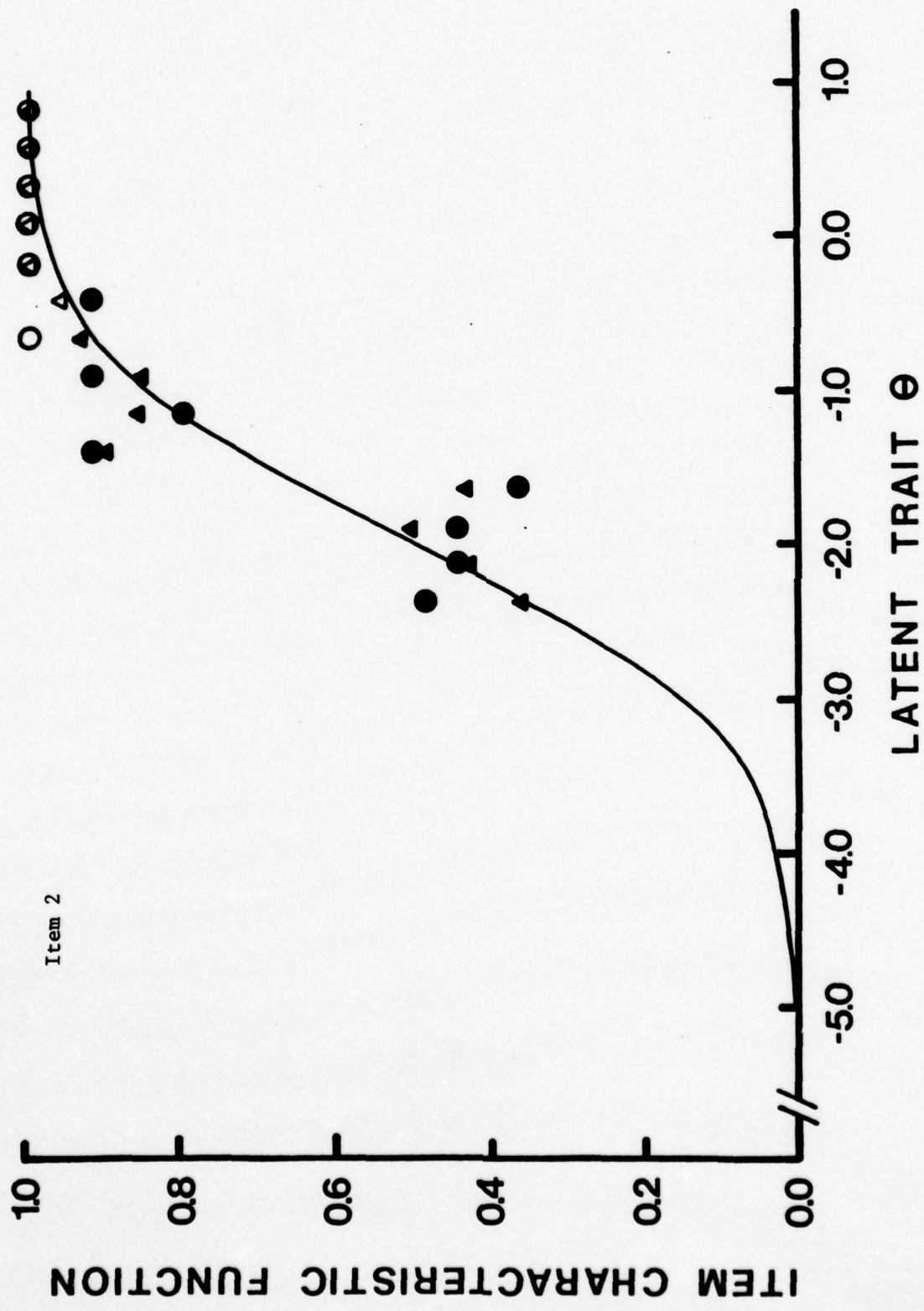


FIGURE 8-3

(Continued)

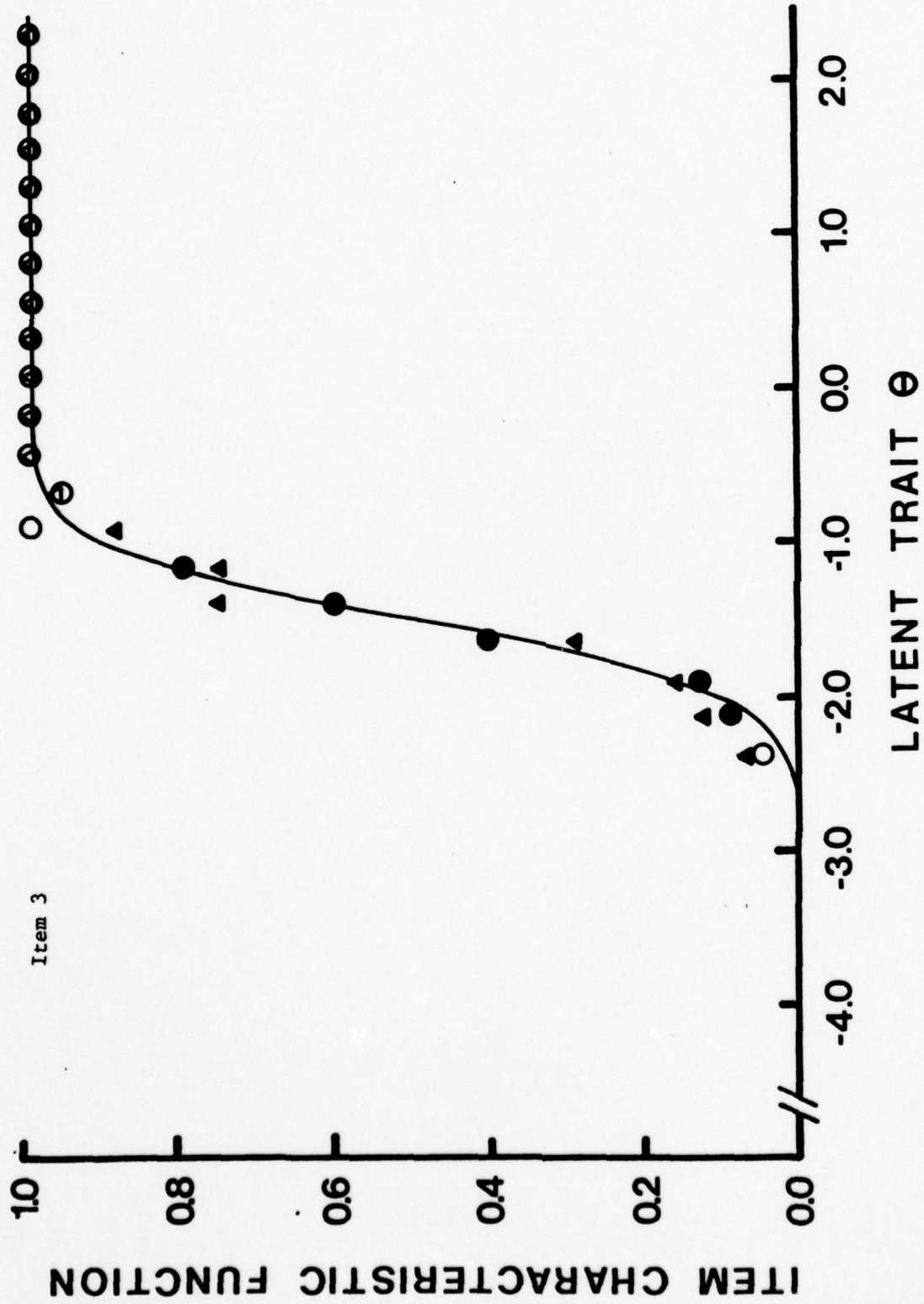


FIGURE 8-3

(Continued)

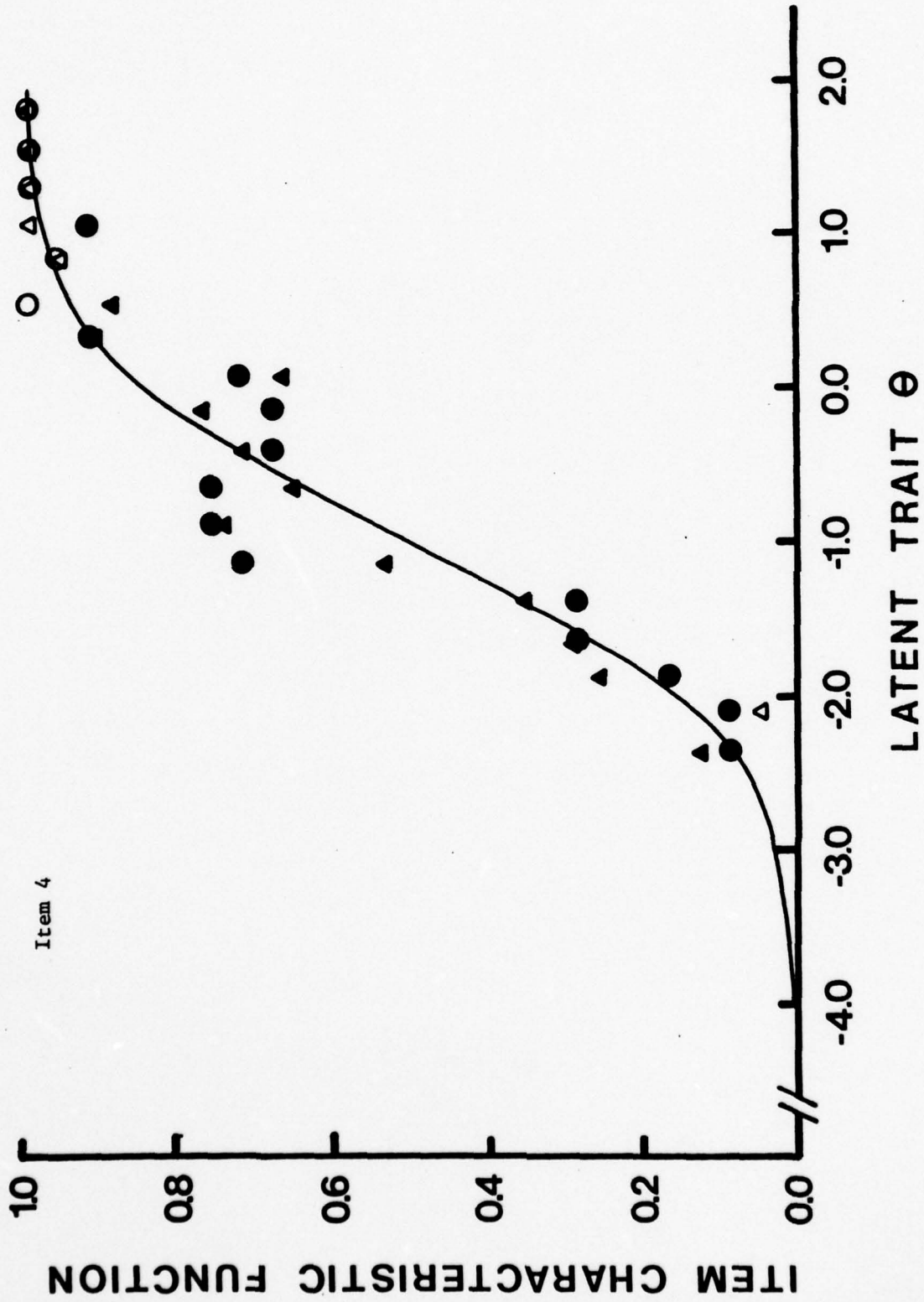


FIGURE 8-3

(Continued)

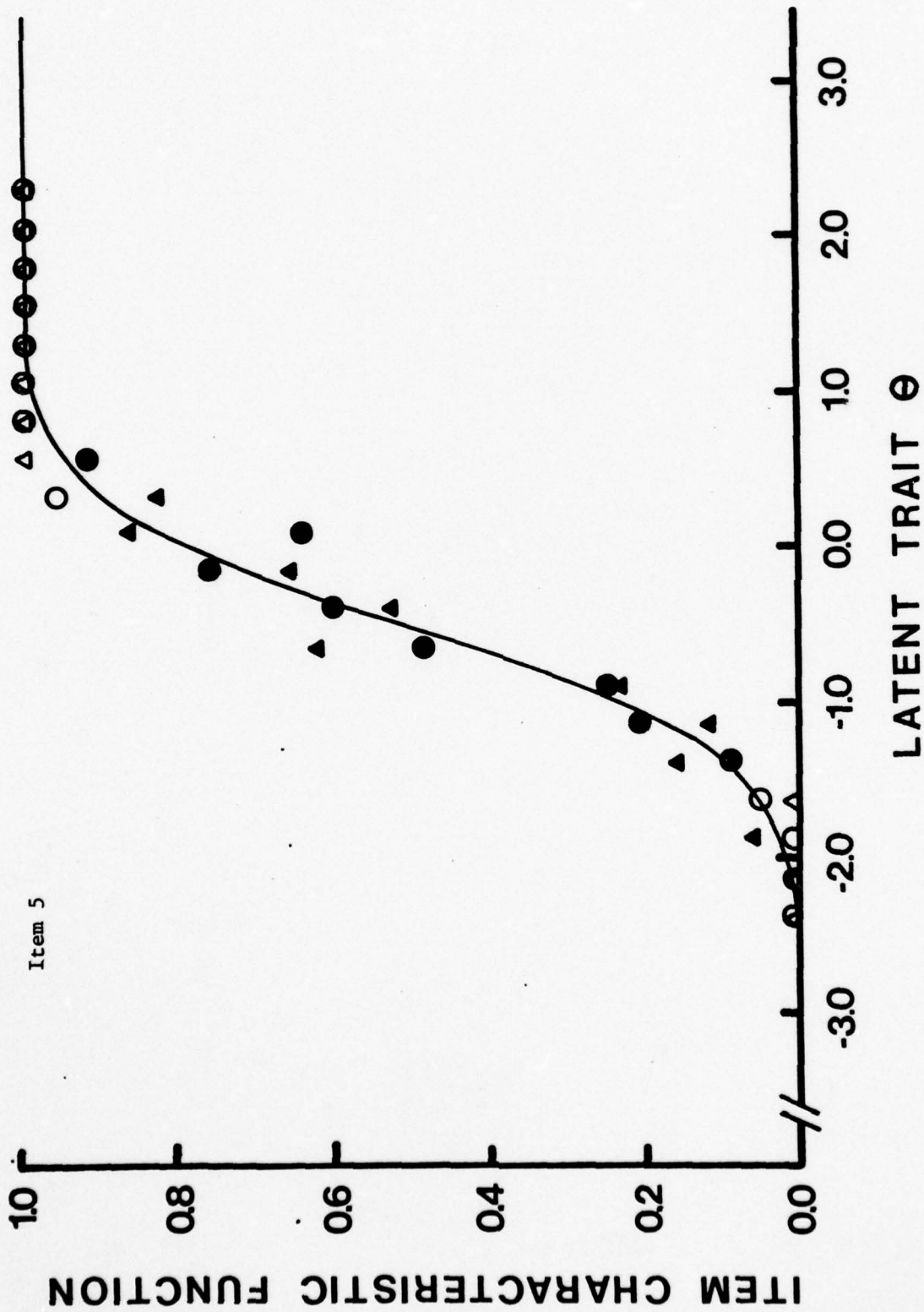


FIGURE 8-3

(Continued)

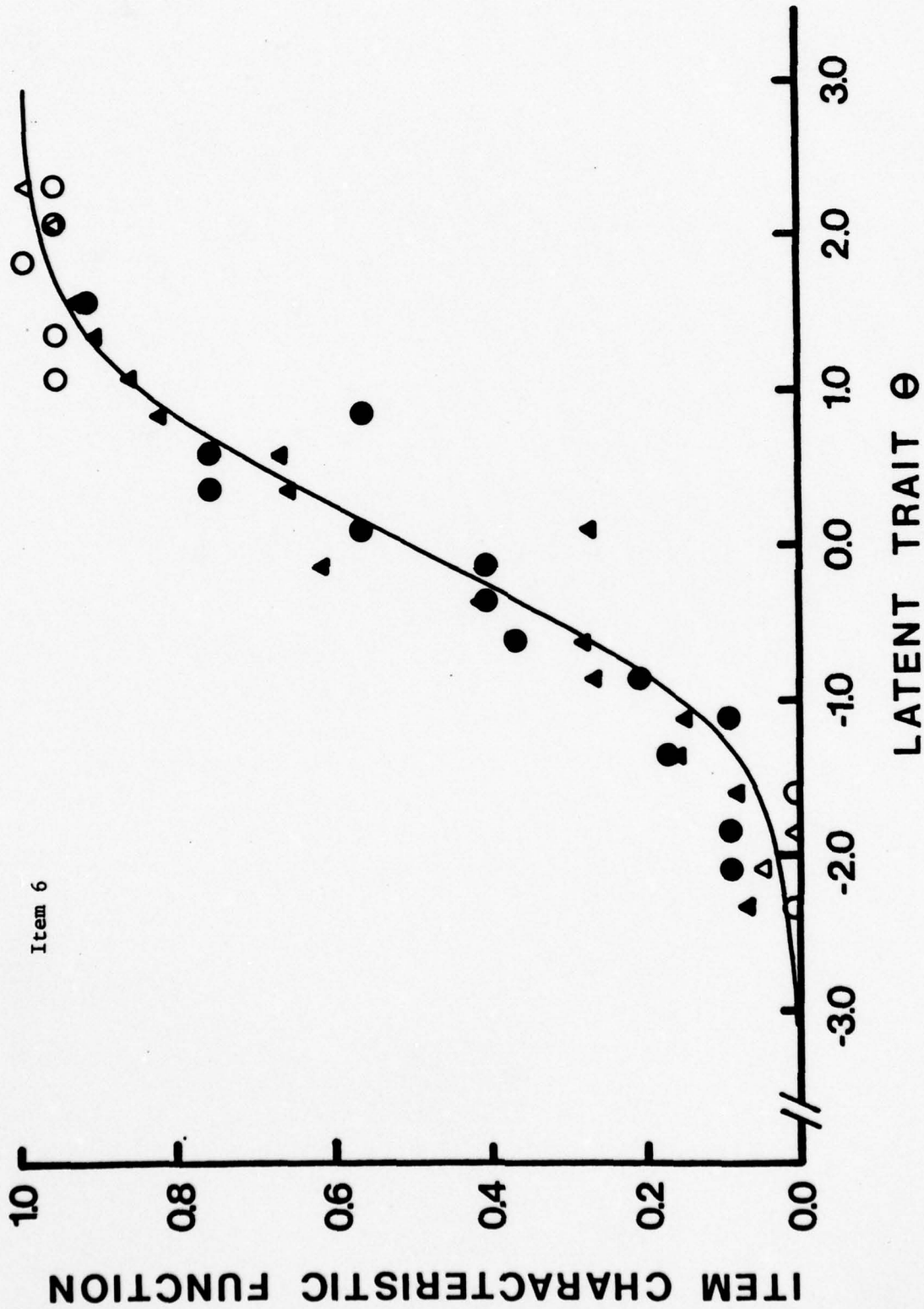


FIGURE 8-3
(Continued)

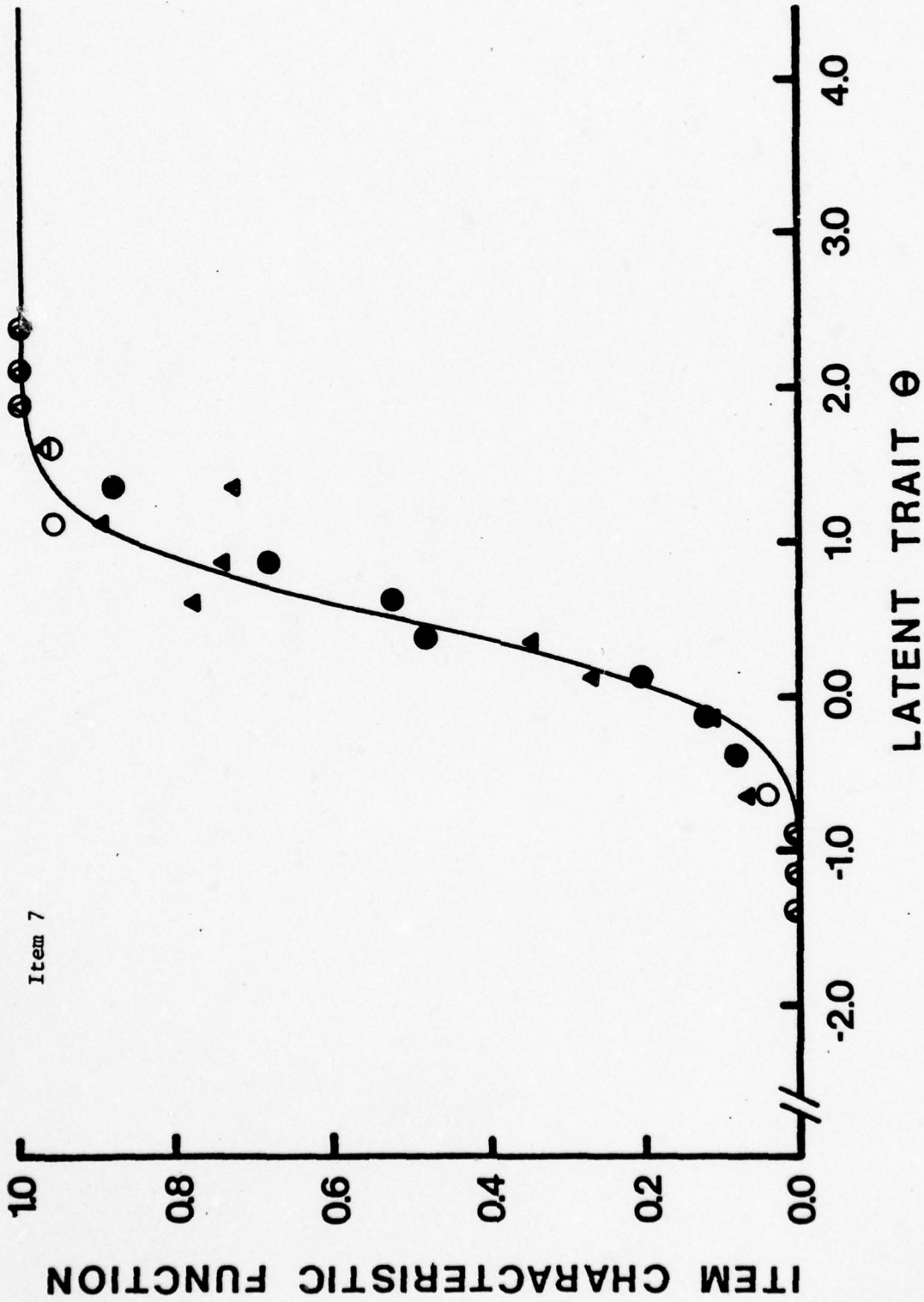


FIGURE 8-3

(Continued)

$$(8.2) \left\{ \begin{array}{l} \alpha = a - bM + cM^2 - dM^3 + eM^4 - fM^5 \\ \beta = b - 2cM + 3dM^2 - 4eM^3 + 5fM^4 \\ \gamma = c - 3dM + 6eM^2 - 10fM^3 \\ \delta = d - 4eM + 10fM^2 \\ \nu = e - 5fM \\ \zeta = f, \end{array} \right.$$

where

$$(8.3) \left\{ \begin{array}{l} a = [1.7578125\mu_0^*/R] - [8.203125\mu_2^*/R^3] + [7.3828125\mu_4^*/R^5] \\ b = [28.7109375\mu_1^*/R^3] - [103.359375\mu_3^*/R^5] + [81.2109375\mu_5^*/R^7] \\ c = [-8.203125\mu_0^*/R^3] + [68.90625\mu_2^*/R^5] - [73.828125\mu_4^*/R^7] \\ d = [-103.359375\mu_1^*/R^5] + [442.96875\mu_3^*/R^7] - [378.984375\mu_5^*/R^9] \\ e = [7.3828125\mu_0^*/R^5] - [73.828125\mu_2^*/R^7] + [86.1328125\mu_4^*/R^9] \\ f = [81.2109375\mu_1^*/R^7] - [378.984375\mu_3^*/R^9] + [341.0859375\mu_5^*/R^{11}] \end{array} \right.$$

With our raw data of the 500 maximum likelihood estimates, these values turned out to be:

$$(8.4) \left\{ \begin{array}{ll} \hat{\alpha} = 0.19539 & (0.19620) \\ \hat{\beta} = -0.00638 & (0.00238) \\ \hat{\gamma} = 0.01449 & (0.01319) \\ \hat{\delta} = 0.00405 & (-0.00062) \\ \hat{\nu} = -0.00449 & (-0.00427) \\ \hat{\zeta} = -0.00048 & . \end{array} \right.$$

The values in the brackets in (8.4) are corresponding estimates for the polynomial of degree 4, which were introduced in section 7. We

notice that, if we take three places below the decimal point, these two sets of coefficients are almost identical. Figure 8-4 shows this polynomial of degree 5, and it is, indeed, very similar to the polynomial of degree 4, which was presented as Figure 7-3 in the preceding section. In spite of this result, however, we should be cautious about the adoption of a relatively simple polynomial in preference to polynomials of higher degrees. As is shown in Figures 7-1, 7-2, 7-3 and 8-4, the theoretical probability function of the maximum likelihood estimate in the present study has a relatively simple curve, but we should not expect this is always the case. A polynomial of a higher degree will be appreciated if this theoretical function is more complicated, like one having three modal points, and so on. As was warned earlier, however, the adoption of a polynomial of a higher degree requires us to use moments of higher degrees, which are liable to error. To solve this problem, it may be advisable to divide the total group of subjects into several subgroups, and fit a polynomial of a relatively low degree to each subset of the maximum likelihood estimates. For the purpose of illustration, Figure 8-5 presents the results obtained by using five subgroups, each of which consists of 100 subjects, and polynomials of degree 3 and 4 respectively. Although these sets of curves are too complicated to use, they give us the idea that it is possible to graduate our raw data fairly accurately, without using polynomials of higher degrees. The estimated coefficients of these five polynomials are

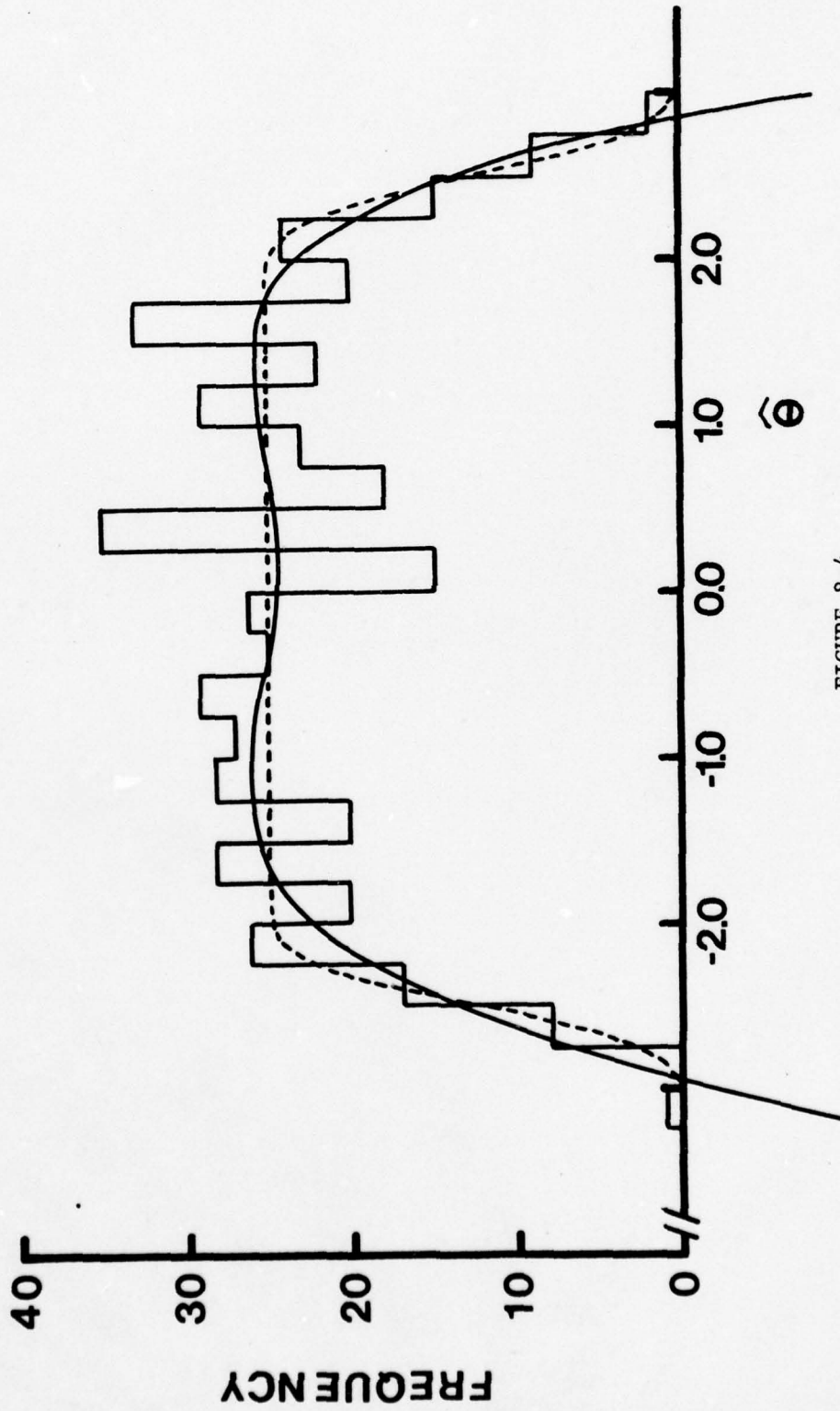


FIGURE 8-4

The frequency distribution of the maximum likelihood estimate (histogram), its graduated polynomial of degree 5 (solid curve) and the theoretical density of the maximum likelihood estimate (dotted curve).

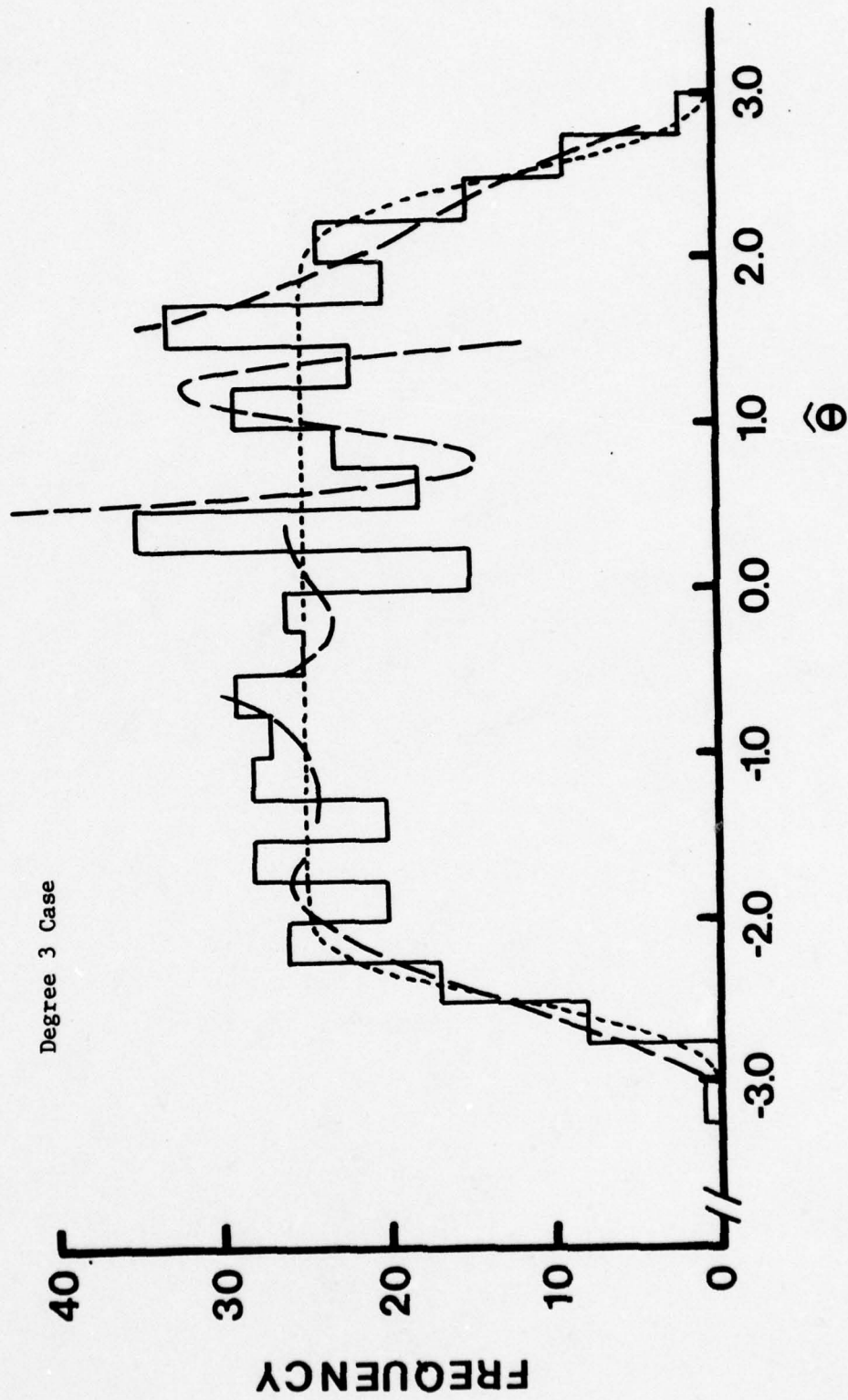


FIGURE 8-5

The five polynomials graduated for five separate intervals (dashed curve), the frequency distribution of the maximum likelihood estimate (histogram) and the theoretical density of the maximum likelihood estimate (dotted curve).

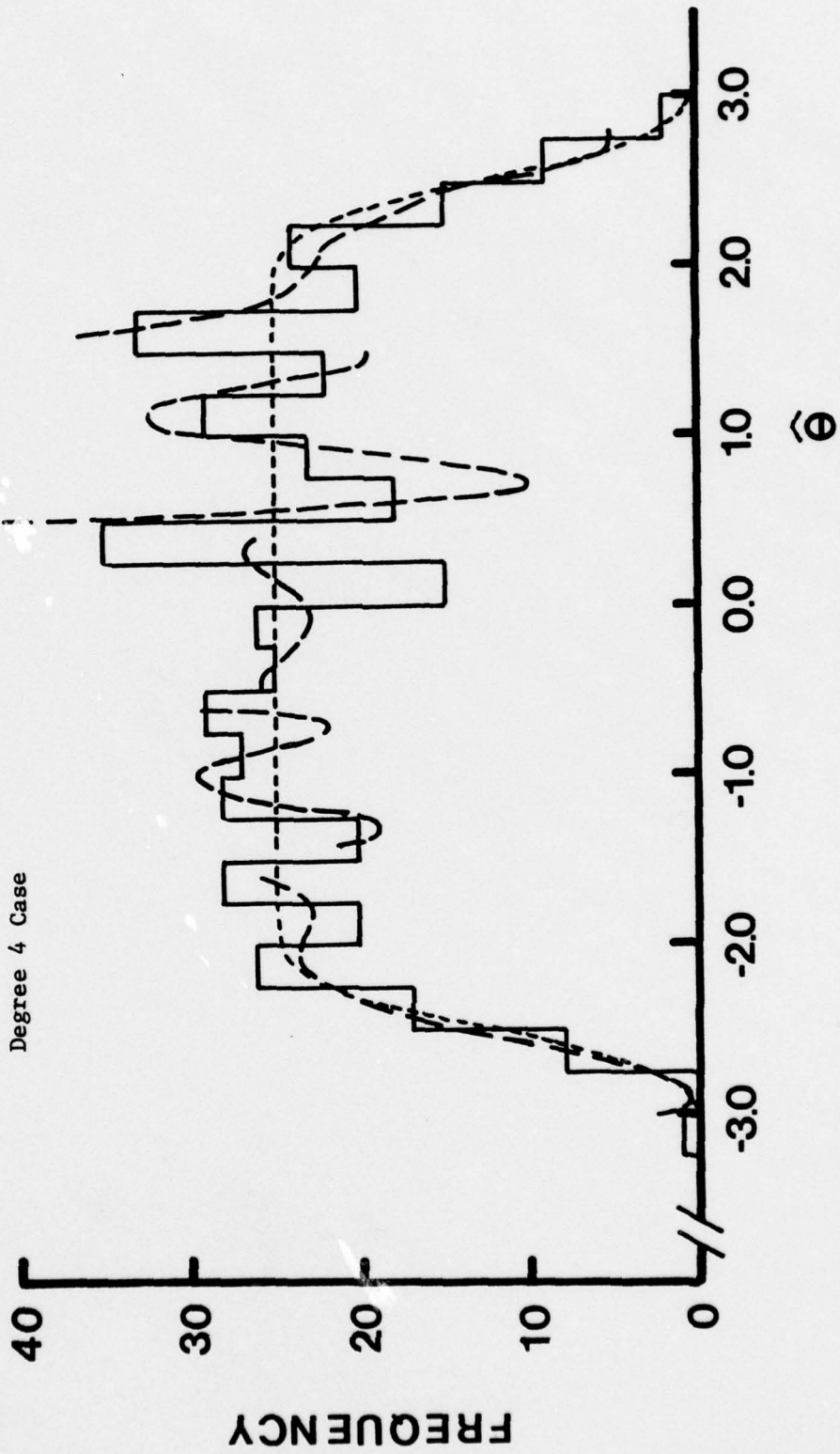


FIGURE 8-5
(Continued)

$$(8.5) \left\{ \begin{array}{l} \hat{\alpha} : -1.17587 \quad 0.56491 \quad 0.19259 \quad 2.67749 \quad 2.00465 \\ \hat{\beta} : -1.91141 \quad 0.91174 \quad 0.05255 \quad -8.27070 \quad -2.16327 \\ \hat{\gamma} : -0.82751 \quad 0.74415 \quad 0.07557 \quad 8.59277 \quad 0.88624 \\ \hat{\delta} : -0.10701 \quad 0.20091 \quad -0.25630 \quad -2.81832 \quad -0.13030 \end{array} \right.$$

for the polynomials of degree 3, and

$$(8.6) \left\{ \begin{array}{l} \hat{\alpha} : 15.22161 \quad 7.81272 \quad 0.18580 \quad 8.70223 \quad 25.44446 \\ \hat{\beta} : 28.35231 \quad 32.93790 \quad 0.06613 \quad -35.55154 \quad -46.44038 \\ \hat{\gamma} : 19.74585 \quad 51.74863 \quad 0.34740 \quad 52.73598 \quad 31.81450 \\ \hat{\delta} : 6.00195 \quad 34.99721 \quad -0.38289 \quad -33.19569 \quad -9.60191 \\ \hat{\nu} : 0.66909 \quad 8.61039 \quad -1.28133 \quad 7.53819 \quad 1.07356 \end{array} \right.$$

for those of degree 4. The intervals of the maximum likelihood estimate used for the five subgroups are $[-3.0555, -1.5096]$, $[-1.4941, -0.5265]$, $[-0.5265, 0.4771]$, $[0.4852, 1.5297]$ and $[1.5395, 2.8718]$, respectively. Figure 8-6 presents similar results obtained by using two subgroups, each of which has 250 subjects, for polynomials of degrees 3 and 4 respectively. The estimated coefficients are

$$(8.7) \left\{ \begin{array}{l} \hat{\alpha} : 0.20878 \quad 0.21750 \\ \hat{\beta} : 0.03405 \quad -0.11653 \\ \hat{\gamma} : 0.05388 \quad 0.13965 \\ \hat{\delta} : 0.02235 \quad -0.04411 \end{array} \right.$$

for the two polynomials of degree 3, and

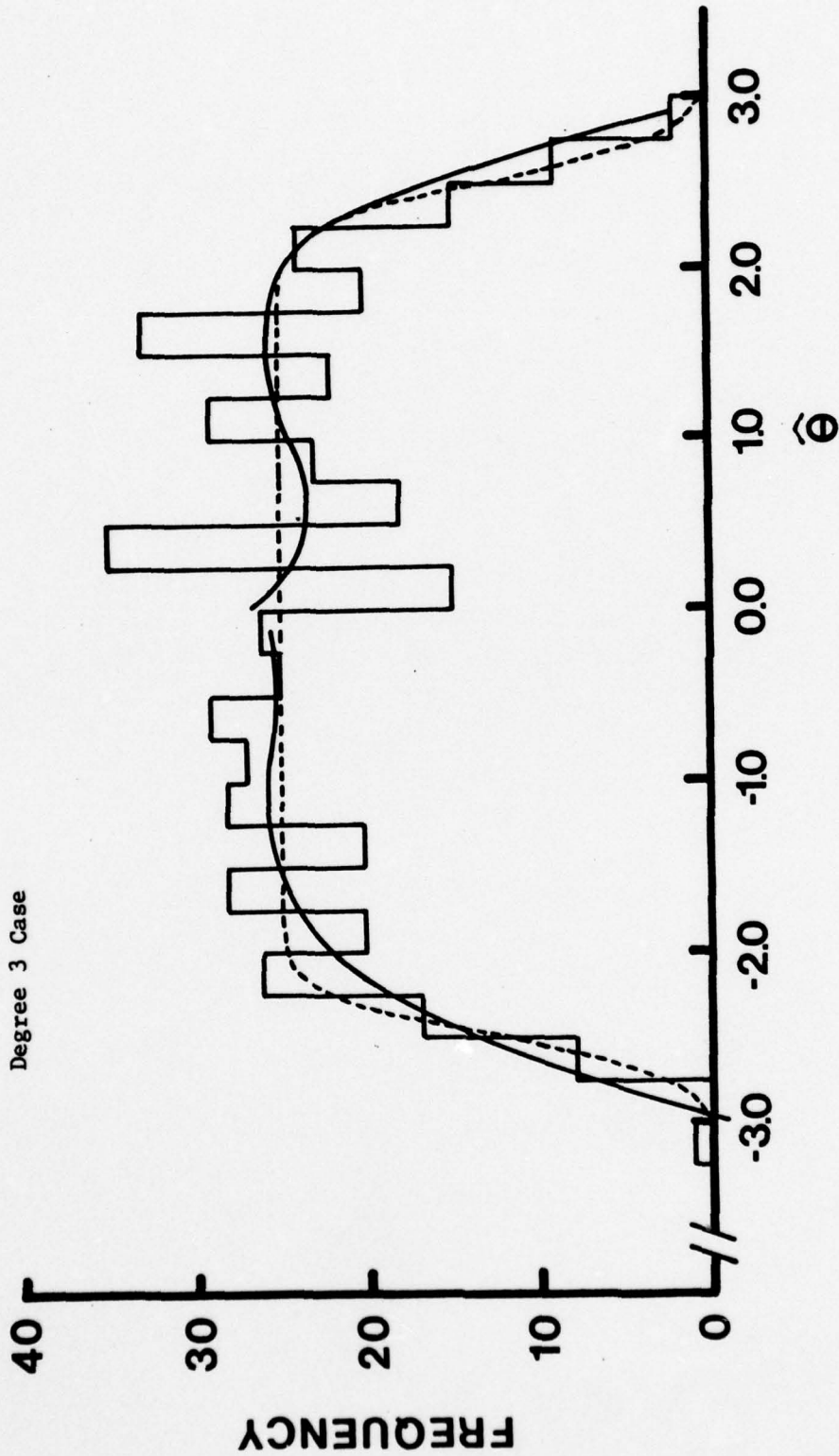


FIGURE 8-6

The two polynomials graduated for two separate intervals (solid curve), the frequency distribution of the maximum likelihood estimate (histogram) and the theoretical density of the maximum likelihood estimate (dotted curve).

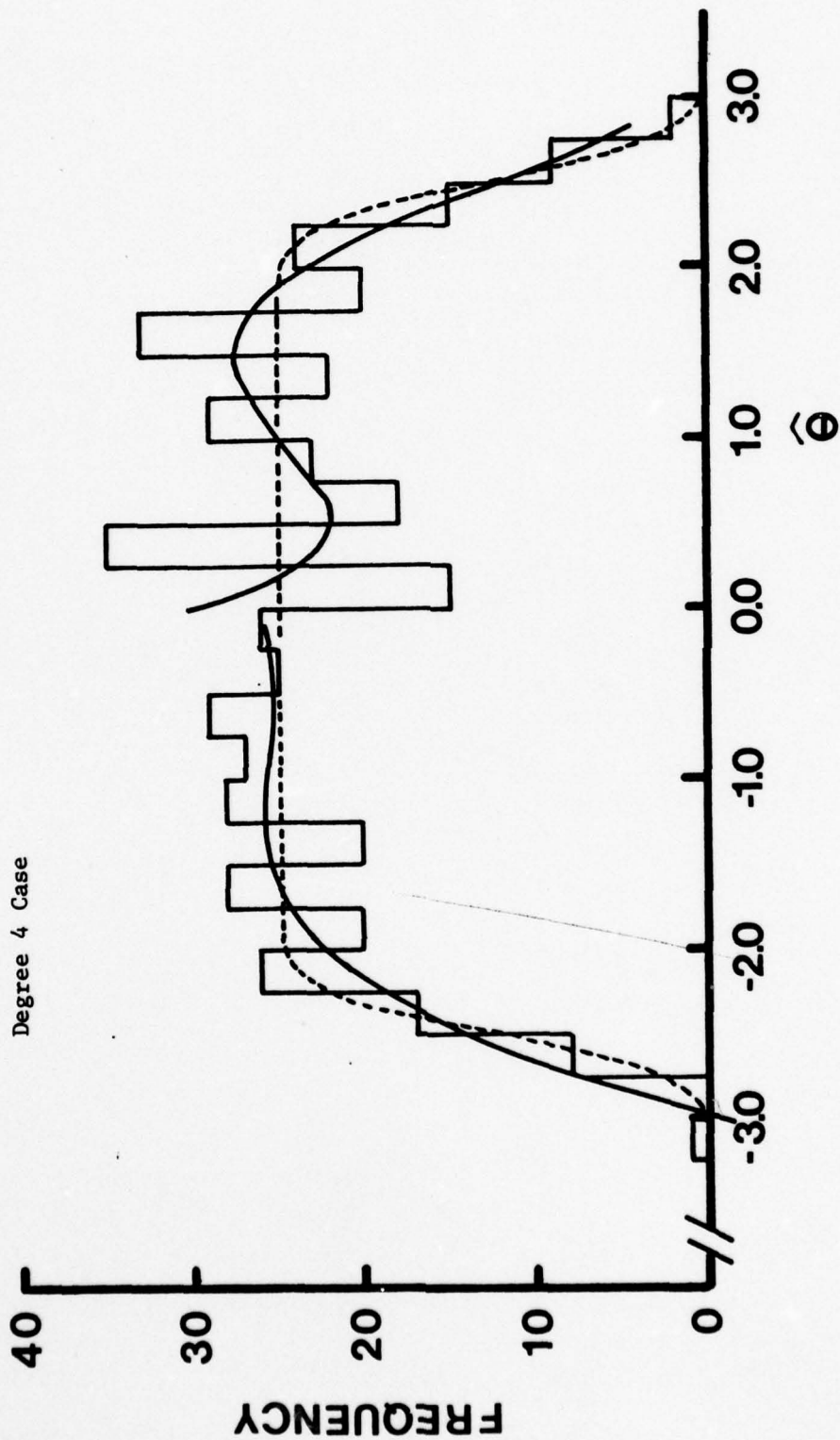


FIGURE 8-6
(Continued)

$$(8.8) \left\{ \begin{array}{ll} \hat{\alpha} : & 0.20953 \quad 0.24402 \\ \hat{\beta} : & 0.03869 \quad -0.31223 \\ \hat{\gamma} : & 0.06056 \quad 0.45212 \\ \hat{\delta} : & 0.02572 \quad -0.21494 \\ \hat{\nu} : & 0.00055 \quad 0.02991 \end{array} \right.$$

for those of degree 4. The intervals of the maximum likelihood estimate for the two subgroups are $[-3.0555, -0.0200]$ and $[-0.0159, 2.8718]$ respectively. It is interesting to note that the two sets of results are much more similar to each other than those obtained without dividing the total set of raw data, and especially for the first subgroup the two polynomials are very close.

The value of the criterion κ , which was introduced in section 3, was calculated for each of the 500 maximum likelihood estimates, for both Degree 3 and 4 Cases. The result shows that, in Degree 3 Case, 318 conditional distributions of θ , given the maximum likelihood estimate $\hat{\theta}$, belong to Pearson's Type I, and 181 are normal distributions, with one case for which the fourth conditional moment turned out to be negative and, therefore, no distribution is specified; in Degree 4 Case 432 conditional distributions are of Type I and 54 are of Type II, with the exceptions of 13 cases for which the fourth and/or the second conditional moments are negative and one case for which the estimated probability density itself is negative.

IX. Discussion and Conclusion

The 2-Parameter Beta Method was proposed and adopted to estimate the item characteristic functions of ten hypothetical binary items in the simulation study, and the results are compared with those obtained by the Normal Approximation Method, and others. It can be concluded that the method proved to be useful. On the other hand, more intensive research is desired to further improve the accuracy of the estimation. Unlike the Normal Approximation Method, it is very probable that such an effort will be rewarded, by investigating an optimal set of two fixed parameters, $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$, and so on.

As was presented at the end of the preceding section, the criterion K shows that the majority of the conditional distributions of θ , given the maximum likelihood estimate $\hat{\theta}$, are of Pearson's Type I or II in both Degree 3 and 4 Cases, although it is interesting to note that about 36 percent of them in Degree 3 Case are normal distributions. It is expected, therefore, that the accuracy of estimation increases if we approximate these conditional distributions by Beta distributions without fixing any parameters, though this will cost more numerical elaboration. This will be attempted in a later study, as well as further intensive research on the 2-Parameter Beta Method.

It is also desirable to use different sets of data in our future studies. In particular, in the present study, the value of σ is as small as 0.215, and it will be worthwhile to use other simulation data with a larger value of σ . This will easily be done by using simulated

tailored testing data in which a fixed amount of test information is used as the criterion for terminating the presentation of new items (Samejima, 1977a, 1977d), rather than using a hypothetical paper-and-pencil test, since the whole procedure in such a tailored testing situation can be considered as producing a weak parallel test to each subject (Samejima, 1977c).

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APPENDICES

TABLE A-1-1

The Estimated Conditional Moments of θ , Given the Maximum Likelihood Estimate, β_1 , β_2 and the Criterion κ for the 500 Hypothetical Subjects, in Degree 3 Case

SUBJECT	MLE		MEAN		MOMENTS ABOUT MEAN		BETA1	BETA2	CRITERION	TYPE	ID
	MLE	MEAN	MOMENTS ABOUT MEAN	MOMENTS ABOUT MEAN							
1	-2.6985	-2.62923	0.03993	-0.00098	0.00455	0.015	2.854	-0.034	1		
2	-3.0555	-2.86235	0.00536	-0.01649	-0.00991	1770.746	0.00000	0.00000	9		
3	-2.6723	-2.63610	0.36246	-0.00086	0.00472	0.011	2.881	-0.031	3		
4	-2.2061	-2.63544	0.03378	-0.00102	0.00450	0.017	2.845	-0.035	4		
5	-2.3634	-2.32293	0.34364	-0.00025	0.00566	0.001	2.980	-0.014	5		
6	-2.2448	-2.21022	0.04418	-0.00017	0.00583	0.000	2.988	-0.011	6		
7	-2.2048	-2.21192	0.04433	-0.00016	0.00587	0.000	2.991	-0.010	7		
8	-1.9855	-1.96017	0.04589	-0.00009	0.00604	0.000	2.995	-0.007	8		
9	-1.8243	-1.80511	0.04515	-0.00006	0.00611	0.000	2.997	-0.005	9		
10	-2.2219	-2.18831	0.36427	-0.00116	0.00586	0.000	2.989	-0.010	10		
11	-2.0980	-2.06913	0.04664	-0.00012	0.00596	0.000	2.993	-0.008	11		
12	-2.2277	-2.18716	0.04427	-0.00016	0.00586	0.000	2.989	-0.010	12		
13	-2.1519	-2.12110	0.04649	-0.00016	0.00592	0.000	2.992	-0.008	13		
14	-2.1213	-2.09162	0.04658	-0.00012	0.00595	0.000	2.992	-0.008	14		
15	-1.7199	-1.70104	0.04528	-0.00005	0.00614	0.000	2.998	-0.004	15		
16	-1.6149	-1.59807	0.04538	-0.00004	0.00617	0.000	2.998	-0.004	16		
17	-1.6669	-1.64910	0.04533	-0.00007	0.00616	0.000	2.997	-0.006	17		
18	-1.8711	-1.84883	0.04508	-0.00007	0.00609	0.000	2.997	-0.006	18		
19	-1.6616	-1.64370	0.04534	-0.00004	0.00616	0.000	2.998	-0.004	19		
20	-1.4638	-1.44954	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	20		
21	-1.1387	-1.12892	0.04567	-0.00002	0.00625	0.000	2.998	-0.002	21		
22	-1.4259	-1.41223	0.04552	-0.00003	0.00621	0.000	2.999	-0.003	22		
23	-1.4398	-1.42591	0.04551	-0.00003	0.00621	0.000	2.999	-0.003	23		
24	-1.4632	-1.44895	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	24		
25	-1.2066	-1.19598	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	25		
26	-0.7593	-0.75360	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	26		
27	-1.4183	-1.40474	0.04553	-0.00003	0.00622	0.000	2.999	-0.003	27		
28	-0.7163	-0.71101	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	28		
29	-1.2171	-1.20634	0.04563	-0.00002	0.00625	0.000	2.999	-0.002	29		
30	-1.2159	-1.20516	0.04563	-0.00002	0.00625	0.000	2.999	-0.002	30		
31	-1.1601	-1.15006	0.04566	-0.00002	0.00625	0.000	2.999	-0.002	31		
32	-0.7801	-0.77420	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	32		
33	-0.9941	-0.98599	0.04572	-0.00001	0.00628	0.000	3.000	-0.001	33		
34	-0.9045	-0.89735	0.04574	-0.00001	0.00628	0.000	3.000	-0.001	34		
35	-0.9941	-0.98599	0.04572	-0.00001	0.00628	0.000	3.000	-0.001	35		
36	-1.8498	-1.84321	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	36		
37	-0.3724	-0.37015	0.04584	-0.00000	0.00630	0.000	3.000	-0.000	37		
38	-1.2274	-1.21651	0.04563	-0.00002	0.00624	0.000	3.000	-0.001	38		
39	-0.6985	-0.69518	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	39		
40	-0.4130	-0.40690	0.04583	-0.00001	0.00630	0.000	3.000	-0.001	40		
41	-1.4277	-1.41805	0.04583	-0.00001	0.00630	0.000	3.000	-0.001	41		
42	-0.3684	-0.36619	0.04584	-0.00001	0.00630	0.000	3.000	-0.001	42		
43	-1.0266	-1.02567	0.04587	-0.00001	0.00631	0.000	3.000	-0.001	43		
44	-0.2486	-0.24717	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	44		
45	-0.6129	-0.61031	0.04586	-0.00000	0.00630	0.000	3.000	-0.001	45		
46	-0.2501	-0.24885	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	46		
47	-0.0822	-0.08228	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	47		
48	-1.0794	-1.07951	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	48		
49	-0.3038	-0.30025	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	49		
50	-0.7066	-0.70727	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	50		
51	0.1203	0.11865	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	51		
52	0.3786	0.37497	0.04587	0.00000	0.00631	0.000	3.000	-0.000	52		
53	-1.1431	-1.14128	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	53		

54	0.4957	0.49117	0.04586	0.00000	0.00000	0.00000	0.00000	3.000	-0.000	8	54
55	0.0174	0.01674	0.04587	-0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	55
56	-0.3158	-0.31652	0.04587	-0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	56
57	-0.0261	-0.02662	0.04587	-0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	57
58	0.3387	0.30561	0.04587	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	58
59	0.2803	0.27243	0.04587	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	59
60	0.6487	0.64296	0.04586	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	60
61	0.5905	0.58522	0.04586	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	61
62	0.5322	0.52761	0.04586	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	62
63	0.3723	0.36872	0.04587	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	63
64	0.9578	0.94949	0.04582	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	64
65	0.2702	0.26740	0.04587	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	65
66	1.0166	1.00777	0.04581	0.00001	0.00630	0.00630	0.000	3.000	-0.001	8	66
67	1.1270	1.11716	0.04579	0.00001	0.00629	0.00629	0.000	3.000	-0.001	8	67
68	0.9847	0.98426	0.04582	0.00001	0.00630	0.00630	0.000	3.000	-0.001	8	68
69	0.3363	0.33359	0.04587	0.00000	0.00631	0.00631	0.000	3.000	-0.000	8	69
70	1.2520	1.24095	0.04577	0.00001	0.00628	0.00628	0.000	3.000	-0.001	8	70
71	1.1924	1.18489	0.04578	0.00001	0.00629	0.00629	0.000	3.000	-0.001	8	71
72	1.1850	1.17461	0.04578	0.00001	0.00629	0.00629	0.000	3.000	-0.001	8	72
73	1.2626	1.24947	0.04576	0.00001	0.00628	0.00628	0.000	3.000	-0.001	8	73
74	0.9636	0.95553	0.04582	0.00001	0.00630	0.00630	0.000	3.000	-0.001	8	74
75	1.2318	1.22395	0.04577	0.00001	0.00628	0.00628	0.000	3.000	-0.001	8	75
76	0.9962	0.98755	0.04582	0.00001	0.00630	0.00630	0.000	3.000	-0.001	8	76
77	1.5287	1.51567	0.04568	0.00002	0.00626	0.00626	0.000	2.999	-0.002	8	77
78	1.6301	1.61485	0.04564	0.00002	0.00625	0.00625	0.000	2.999	-0.002	8	78
79	1.3639	1.35171	0.04574	0.00001	0.00627	0.00627	0.000	3.000	-0.001	8	79
80	1.6155	1.60343	0.04565	0.00002	0.00625	0.00625	0.000	2.999	-0.002	8	80
81	1.7958	1.77833	0.04556	0.00003	0.00623	0.00623	0.000	2.999	-0.003	8	81
82	1.3451	1.33311	0.04574	0.00001	0.00627	0.00627	0.000	3.000	-0.001	8	82
83	1.3637	1.35151	0.04574	0.00001	0.00627	0.00627	0.000	3.000	-0.001	8	83
84	1.7627	1.74570	0.04558	0.00002	0.00623	0.00623	0.000	2.999	-0.003	8	84
85	1.3833	1.37090	0.04573	0.00001	0.00627	0.00627	0.000	3.000	-0.002	8	85
86	1.6049	1.59194	0.04565	0.00002	0.00625	0.00625	0.000	2.999	-0.002	8	86
87	1.6430	1.62759	0.04564	0.00002	0.00625	0.00625	0.000	2.999	-0.002	8	87
88	1.7798	1.76265	0.04557	0.00003	0.00623	0.00623	0.000	2.999	-0.003	8	88
89	1.7438	1.73100	0.04559	0.00002	0.00623	0.00623	0.000	2.999	-0.003	8	89
90	1.8884	1.86955	0.04551	0.00003	0.00621	0.00621	0.000	2.999	-0.003	8	90
91	1.8339	1.81587	0.04554	0.00003	0.00622	0.00622	0.000	2.999	-0.003	8	91
92	2.1047	2.08211	0.04533	0.00004	0.00616	0.00616	0.000	2.998	-0.004	8	92
93	2.0506	2.02932	0.04538	0.00004	0.00618	0.00618	0.000	2.998	-0.004	8	93
94	2.2720	2.24585	0.04514	0.00006	0.00611	0.00611	0.000	2.997	-0.005	8	94
95	2.1136	2.09383	0.04532	0.00005	0.00616	0.00616	0.000	2.998	-0.004	8	95
96	2.3570	2.32874	0.04501	0.00008	0.00607	0.00607	0.000	2.996	-0.006	8	96
97	2.2527	2.22700	0.04517	0.00006	0.00611	0.00611	0.000	2.997	-0.005	8	97
98	2.6483	2.61049	0.04432	0.00016	0.00587	0.00587	0.000	2.990	-0.010	8	98
99	2.7142	2.67351	0.04408	0.00019	0.00580	0.00580	0.000	2.987	-0.011	8	99
100	2.5177	2.48471	0.04469	0.00011	0.00598	0.00598	0.000	2.994	-0.010	8	100
101	2.7417	2.66575	0.04389	-0.00014	0.00623	0.00623	0.000	2.991	-0.010	8	101
102	2.5074	2.45744	0.04263	-0.00042	0.00538	0.00538	0.000	2.999	-0.019	8	102
103	2.0847	2.05628	0.04467	-0.00011	0.00597	0.00597	0.000	2.993	-0.008	8	103
104	2.5488	2.51365	0.04199	-0.00034	0.00519	0.00519	0.000	2.991	-0.023	8	104
105	2.4672	2.42021	0.04296	-0.00036	0.00548	0.00548	0.000	2.967	-0.017	8	105
106	2.0780	2.04981	0.04469	-0.00011	0.00598	0.00598	0.000	2.994	-0.008	8	106
107	2.1210	2.09133	0.04458	-0.00012	0.00595	0.00595	0.000	2.992	-0.008	8	107
108	2.3210	2.28279	0.04386	-0.00022	0.00574	0.00574	0.000	2.993	-0.012	8	108
109	2.4760	2.42839	0.04289	-0.00037	0.00546	0.00546	0.000	2.965	-0.018	8	109
110	2.2402	2.20582	0.04420	-0.00017	0.00535	0.00535	0.000	2.988	-0.010	8	110
111	1.0710	1.04873	0.04538	-0.00037	0.00619	0.00619	0.000	2.997	-0.026	8	111
112	1.5181	1.50296	0.04546	-0.00003	0.00620	0.00620	0.000	2.999	-0.003	8	112
113	2.1842	2.15215	0.04439	-0.00015	0.00589	0.00589	0.000	2.990	-0.029	8	113
114	2.3336	2.29774	0.04485	-0.00009	0.00603	0.00603	0.000	2.993	-0.007	8	114

115	-1.8711	-1.84883	0.04508	-0.00007	0.00609	0.000	2.997	-0.006	1	115
116	-1.9525	-1.92811	0.04595	-0.00003	0.00605	0.000	2.998	-0.006	1	116
117	-1.5243	-1.50906	0.04565	-0.00003	0.00619	0.000	2.999	-0.006	1	117
118	-2.2070	-2.17632	0.04632	-0.00009	0.00587	0.000	2.993	-0.007	1	118
119	-1.9657	-1.94533	0.04592	-0.00005	0.00604	0.000	2.995	-0.007	1	119
120	-1.6604	-1.64272	0.04534	-0.00006	0.00616	0.000	2.998	-0.007	1	120
121	-1.5796	-1.56460	0.04546	-0.00003	0.00620	0.000	2.999	-0.003	1	121
122	-1.3340	-1.32167	0.04557	-0.00003	0.00623	0.000	2.999	-0.003	1	122
123	-1.6439	-1.62359	0.04535	-0.00004	0.00617	0.000	2.998	-0.004	1	123
124	-1.5794	-1.56321	0.04551	-0.00004	0.00618	0.000	2.998	-0.004	1	124
125	-1.2659	-1.25845	0.04561	-0.00002	0.00625	0.000	2.997	-0.002	1	125
126	-1.1130	-1.10056	0.04568	-0.00002	0.00626	0.000	2.999	-0.002	1	126
127	-1.4331	-1.41932	0.04552	-0.00003	0.00621	0.000	2.999	-0.003	1	127
128	-1.8681	-1.86132	0.04575	-0.00001	0.00628	0.000	3.000	-0.001	1	128
129	-1.2574	-1.24632	0.04561	-0.00002	0.00624	0.000	2.999	-0.002	1	129
130	-1.1185	-1.10896	0.04567	-0.00002	0.00626	0.000	2.999	-0.002	1	130
131	-1.2182	-1.20743	0.04563	-0.00002	0.00625	0.000	2.999	-0.002	1	131
132	-0.7801	-0.77420	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	1	132
133	-0.7678	-0.76232	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	1	133
134	-0.6672	-0.66237	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	1	134
135	-0.5666	-0.56267	0.04581	-0.00001	0.00631	0.000	3.000	-0.001	1	135
136	-0.8923	-0.88527	0.04575	-0.00001	0.00628	0.000	3.000	-0.001	1	136
137	-0.6673	-0.66247	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	1	137
138	-1.2851	-1.28357	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	138
139	-0.8356	-0.82915	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	1	139
140	-0.6247	-0.62326	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	1	140
141	-0.5783	-0.57428	0.04581	-0.00001	0.00630	0.000	3.000	-0.001	1	141
142	-0.5285	-0.52294	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	142
143	-0.3456	-0.34597	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	143
144	-0.2494	-0.24816	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	144
145	-0.5265	-0.52294	0.04582	-0.00001	0.00631	0.000	3.000	-0.001	1	145
146	-0.3574	-0.35528	0.04584	-0.00000	0.00630	0.000	3.000	-0.000	1	146
147	-0.3302	-0.33372	0.04584	-0.00000	0.00631	0.000	3.000	-0.000	1	147
148	-0.0793	-0.07941	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	148
149	-0.2675	-0.26611	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	149
150	-0.3724	-0.37115	0.04584	-0.00001	0.00630	0.000	3.000	-0.000	1	150
151	-0.0121	-0.01273	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	151
152	0.0094	0.00860	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	152
153	0.0175	0.01664	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	153
154	0.1762	0.17412	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	154
155	0.4514	0.44721	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	155
156	0.1939	0.19169	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	156
157	0.2167	0.21431	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	157
158	0.4598	0.45554	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	158
159	0.3748	0.37120	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	159
160	0.2594	0.25669	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	160
161	0.3928	0.38916	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	161
162	0.7691	0.76238	0.04584	-0.00000	0.00631	0.000	3.000	-0.000	1	162
163	0.9596	0.95127	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	163
164	0.7799	0.77309	0.04584	-0.00000	0.00630	0.000	3.000	-0.000	1	164
165	0.6535	0.64732	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	165
166	0.5028	0.49821	0.04584	-0.00000	0.00631	0.000	3.000	-0.000	1	166
167	0.3619	0.35284	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	1	167
168	0.6228	0.62223	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	168
169	1.1006	1.09100	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	1	169
170	0.4996	0.49504	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	170
171	0.8761	0.86763	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	171
172	1.7431	1.73175	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	1	172
173	1.6789	1.66333	0.04562	-0.00002	0.00624	0.000	2.999	-0.002	1	173
174	1.2554	1.24432	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	1	174
175	1.2148	1.20008	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	1	175

176	1.2985	1.28699	0.04575	0.00001	0.00628	0.000	3.000	-0.001	176
177	1.1964	1.19590	0.04578	0.00001	0.00629	0.000	3.000	-0.001	177
178	1.5437	1.54354	0.04568	0.00002	0.00624	0.000	2.999	-0.002	178
179	1.6001	1.58522	0.04568	0.00002	0.00625	0.000	2.999	-0.002	179
180	1.4832	1.46970	0.04570	0.00002	0.00624	0.000	2.999	-0.002	180
181	1.3832	1.37090	0.04573	0.00001	0.00627	0.000	3.000	-0.002	181
182	1.4991	1.48542	0.04542	0.00002	0.00626	0.000	2.999	-0.002	182
183	1.2496	1.23858	0.04577	0.00001	0.00628	0.000	3.000	-0.001	183
184	1.5137	1.49985	0.04569	0.00002	0.00624	0.000	2.999	-0.002	184
185	1.7626	1.74560	0.04558	0.00002	0.00623	0.000	2.999	-0.003	185
186	2.1396	2.11632	0.04530	0.00005	0.00615	0.000	2.998	-0.004	186
187	2.0455	2.02401	0.04539	0.00004	0.00618	0.000	2.998	-0.004	187
188	1.5776	1.56300	0.04567	0.00002	0.00625	0.000	2.999	-0.002	188
189	1.9978	1.97716	0.04543	0.00004	0.00619	0.000	2.998	-0.004	189
190	1.7912	1.77379	0.04557	0.00003	0.00623	0.000	2.999	-0.003	190
191	1.8954	1.87644	0.04550	0.00003	0.00621	0.000	2.999	-0.003	191
192	2.5017	2.46923	0.04473	0.00011	0.00599	0.000	2.994	-0.008	192
193	2.3212	2.30115	0.04541	0.00004	0.00618	0.000	2.998	-0.004	193
194	2.4563	2.42525	0.04483	0.00013	0.00602	0.000	2.995	-0.007	194
195	2.0269	2.00575	0.04540	0.00004	0.00618	0.000	2.998	-0.004	195
196	1.4284	1.40970	0.04551	0.00002	0.00621	0.000	2.999	-0.003	196
197	2.1021	2.07956	0.04534	0.00004	0.00616	0.000	2.998	-0.004	197
198	2.5188	2.48577	0.04469	0.00011	0.00598	0.000	2.994	-0.008	198
199	2.6483	2.61349	0.04432	0.00016	0.00587	0.000	2.990	-0.010	199
200	2.2211	2.19806	0.04520	0.00006	0.00612	0.000	2.997	-0.005	200
201	2.7417	2.66575	0.03891	-0.00124	0.00423	0.026	2.791	-0.040	201
202	2.4726	2.42523	0.04292	-0.00037	0.00546	0.002	2.866	-0.017	202
203	2.2251	2.19137	0.04426	-0.00017	0.00505	0.000	2.989	-0.010	203
204	2.4500	2.40420	0.04308	-0.00034	0.00551	0.001	2.970	-0.017	204
205	2.0743	2.04623	0.04470	-0.00011	0.00598	0.000	2.994	-0.011	205
206	2.4193	2.37551	0.04331	-0.00030	0.00530	0.001	2.974	-0.015	206
207	1.8821	1.85956	0.04507	-0.00007	0.00609	0.000	2.996	-0.006	207
208	2.2569	2.22178	0.04614	-0.00018	0.00582	0.000	2.987	-0.011	208
209	2.2230	2.18469	0.04385	-0.00022	0.00574	0.001	2.984	-0.012	209
210	2.3110	2.27330	0.04391	-0.00021	0.00575	0.001	2.984	-0.012	210
211	2.0102	1.98415	0.04484	-0.00009	0.00602	0.000	2.995	-0.002	211
212	1.7808	1.76064	0.04520	-0.00006	0.00612	0.000	2.997	-0.005	212
213	1.5281	1.51279	0.04545	-0.00004	0.00619	0.000	2.999	-0.003	213
214	1.4941	1.47936	0.04547	-0.00003	0.00620	0.000	2.999	-0.003	214
215	1.7347	1.71553	0.04526	-0.00005	0.00614	0.000	2.998	-0.005	215
216	1.8593	1.83732	0.04510	-0.00007	0.00610	0.000	2.997	-0.005	216
217	1.5973	1.58078	0.04538	-0.00004	0.00618	0.000	2.998	-0.004	217
218	1.6462	1.62879	0.04535	-0.00004	0.00617	0.000	2.998	-0.004	218
219	1.8372	1.81575	0.04513	-0.00006	0.00610	0.000	2.997	-0.005	219
220	1.9800	1.96094	0.04532	-0.00001	0.00627	0.000	2.999	-0.002	220
221	1.2141	1.20338	0.04563	-0.00002	0.00625	0.000	2.999	-0.002	221
222	1.5868	1.57068	0.04540	-0.00004	0.00618	0.000	2.998	-0.004	222
223	1.0664	1.05747	0.04569	-0.00002	0.00626	0.000	2.999	-0.002	223
224	1.4632	1.44895	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	224
225	0.8773	0.87043	0.04575	-0.00001	0.00628	0.000	3.000	-0.001	225
226	1.3658	1.35711	0.04570	-0.00002	0.00626	0.000	2.999	-0.002	226
227	1.0431	1.03446	0.04570	-0.00002	0.00626	0.000	2.999	-0.002	227
228	1.0445	1.03582	0.04570	-0.00002	0.00626	0.000	2.999	-0.002	228
229	1.9844	1.97630	0.04572	-0.00001	0.00627	0.000	2.999	-0.002	229
230	1.0650	1.05609	0.04569	-0.00002	0.00626	0.000	2.999	-0.002	230
231	1.2997	1.28785	0.04559	-0.00002	0.00623	0.000	2.999	-0.002	231
232	1.0195	1.01110	0.04571	-0.00002	0.00627	0.000	2.999	-0.002	232
233	1.0190	1.01061	0.04571	-0.00002	0.00627	0.000	2.999	-0.002	233
234	1.9931	1.98511	0.04572	-0.00001	0.00627	0.000	2.999	-0.002	234
235	0.6400	0.63542	0.04500	-0.00001	0.00629	0.000	3.000	-0.001	235
236	1.5268	1.52324	0.04582	-0.00001	0.00633	0.000	3.000	-0.001	236

237	-0.9279	-0.92050	0.04574	0.04574	-0.00001	0.00627	0.000	3.000	-0.001	1	237
238	-0.7372	-0.73171	0.04578	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	1	238
239	-0.2628	-0.26161	0.04585	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	239
240	-0.9212	-0.91387	0.04574	0.04574	-0.00001	0.00628	0.000	3.000	-0.001	1	240
241	-0.5486	-0.54484	0.04582	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	241
242	-0.3242	-0.32066	0.04582	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	242
243	-0.3831	-0.38076	0.04584	0.04584	-0.00001	0.00630	0.000	3.000	-0.000	1	243
244	-0.4587	-0.45572	0.04583	0.04583	-0.00001	0.00630	0.000	3.000	-0.001	1	244
245	-0.6053	-0.60278	0.04586	0.04586	-0.00001	0.00631	0.000	3.000	-0.001	1	245
246	-0.7330	-0.72755	0.04578	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	1	246
247	-0.5816	-0.57755	0.04581	0.04581	-0.00001	0.00630	0.000	3.000	-0.001	1	247
248	-0.1485	-0.14806	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	248
249	0.3000	0.29697	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	249
250	-0.3241	-0.32225	0.04585	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	250
251	-0.0602	-0.06066	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	251
252	-0.0379	-0.03833	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	252
253	-0.0027	-0.00340	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	253
254	0.2533	0.24766	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	254
255	0.2738	0.27098	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	255
256	-0.2573	-0.25600	0.04585	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	256
257	0.3729	0.36932	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	257
258	0.3538	0.35036	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	258
259	0.2557	0.25301	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	259
260	0.5325	0.52768	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	260
261	0.4771	0.47271	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	261
262	0.3478	0.34441	0.04587	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	262
263	0.8120	0.80493	0.04586	0.04586	-0.00001	0.00630	0.000	3.000	-0.000	1	263
264	0.7516	0.74513	0.04585	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	264
265	0.5313	0.52649	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	265
266	0.6121	0.60665	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	266
267	0.6142	0.60873	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	267
268	0.6197	0.61419	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	268
269	0.5114	0.50674	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	269
270	0.7755	0.76873	0.04586	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	270
271	1.1183	1.10854	0.04579	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	1	271
272	1.1850	1.17641	0.04578	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	1	272
273	1.1524	1.14232	0.04579	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	1	273
274	1.4642	1.45092	0.04571	0.04571	-0.00001	0.00627	0.000	3.000	-0.002	1	274
275	1.4238	1.41096	0.04572	0.04572	-0.00001	0.00627	0.000	3.000	-0.002	1	275
276	0.7889	0.78202	0.04584	0.04584	-0.00001	0.00631	0.000	3.000	-0.000	1	276
277	1.2766	1.26531	0.04576	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	1	277
278	1.7152	1.70812	0.04557	0.04557	-0.00003	0.00623	0.000	2.999	-0.003	1	278
279	1.3234	1.31163	0.04575	0.04575	-0.00001	0.00628	0.000	3.000	-0.001	1	279
280	1.4048	1.39217	0.04573	0.04573	-0.00001	0.00627	0.000	3.000	-0.002	1	280
281	1.4283	1.41307	0.04564	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	1	281
282	1.6234	1.60823	0.04565	0.04565	-0.00002	0.00625	0.000	2.999	-0.002	1	282
283	1.4361	1.42313	0.04572	0.04572	-0.00001	0.00627	0.000	2.999	-0.002	1	283
284	1.7061	1.68387	0.04561	0.04561	-0.00002	0.00624	0.000	2.999	-0.002	1	284
285	1.6615	1.64585	0.04563	0.04563	-0.00002	0.00624	0.000	2.999	-0.002	1	285
286	1.7056	1.68937	0.04561	0.04561	-0.00002	0.00624	0.000	2.999	-0.002	1	286
287	1.6222	1.60735	0.04565	0.04565	-0.00002	0.00625	0.000	2.999	-0.002	1	287
288	2.2391	2.21273	0.04518	0.04518	-0.00006	0.00612	0.000	2.997	-0.005	1	288
289	1.9000	1.88007	0.04550	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	1	289
290	1.6520	1.63736	0.04563	0.04563	-0.00002	0.00625	0.000	2.999	-0.002	1	290
291	2.0156	1.99465	0.04541	0.04541	-0.00004	0.00618	0.000	2.998	-0.004	1	291
292	2.0895	2.06730	0.04535	0.04535	-0.00004	0.00617	0.000	2.998	-0.004	1	292
293	2.3599	2.33156	0.04501	0.04501	-0.00008	0.00607	0.000	2.996	-0.008	1	293
294	2.3904	2.36902	0.04494	0.04494	-0.00008	0.00605	0.000	2.996	-0.008	1	294
295	2.7494	2.72784	0.04508	0.04508	-0.00010	0.00610	0.000	2.998	-0.010	1	295
296	2.7137	2.67303	0.04480	0.04480	-0.00019	0.00580	0.000	2.987	-0.019	1	296
297	1.8640	1.82671	0.04553	0.04553	-0.00003	0.00622	0.000	2.999	-0.003	1	297

298	2.2188	2.19386	0.04521	0.00006	0.00613	0.000	2.997	-0.005	1	298
299	2.8718	2.82249	0.04323	0.00031	0.00516	0.001	2.973	-0.016	1	299
300	2.8564	2.80807	0.04335	0.00130	0.00559	0.001	2.875	-0.015	1	300
301	-2.2354	-2.20123	0.04422	-0.00017	0.00584	0.000	2.988	-0.010	1	301
302	-2.4875	-2.43905	0.04280	-0.00039	0.00563	-0.002	2.963	-0.018	1	302
303	-2.3129	-2.25111	0.04390	-0.00021	0.00575	0.000	2.984	-0.012	1	303
304	-2.0554	-2.02794	0.04474	-0.00011	0.00599	0.000	2.994	-0.008	1	304
305	-2.3517	-2.31187	0.04371	-0.00024	0.00569	0.001	2.981	-0.013	1	305
306	-2.2135	-2.18026	0.04430	-0.00016	0.00587	0.000	2.995	-0.010	1	306
307	-1.9910	-1.96551	0.04488	-0.00009	0.00623	-0.002	2.989	-0.007	1	307
308	-2.1423	-2.10298	0.04375	-0.00024	0.00571	0.001	2.982	-0.013	1	308
309	-1.8545	-1.83044	0.04485	-0.00008	0.00605	0.000	2.986	-0.004	1	309
310	-1.8350	-1.81360	0.04513	-0.00036	0.00610	0.000	2.997	-0.005	1	310
311	-1.8880	-1.86531	0.04506	-0.00007	0.00608	0.000	2.996	-0.006	1	311
312	-1.9420	-1.91714	0.04493	-0.00008	0.00635	0.000	2.996	-0.006	1	312
313	-2.0743	-2.04623	0.04470	-0.00011	0.00598	0.000	2.994	-0.008	1	313
314	-1.7152	-1.69643	0.04528	-0.00005	0.00615	0.000	2.998	-0.004	1	314
315	-1.7294	-1.71034	0.04526	-0.00005	0.00616	0.000	2.998	-0.004	1	315
316	-1.5985	-1.58197	0.04539	-0.00004	0.00618	0.000	2.998	-0.004	1	316
317	-1.6367	-1.61878	0.04536	-0.00004	0.00617	0.000	2.998	-0.004	1	317
318	-1.6461	-1.62868	0.04535	-0.00004	0.00617	0.000	2.998	-0.004	1	318
319	-1.7316	-1.71250	0.04526	-0.00005	0.00614	0.000	2.999	-0.002	1	319
320	-1.1918	-1.18136	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	1	320
321	-1.2207	-1.20182	0.04527	-0.00005	0.00614	0.000	2.998	-0.004	1	321
322	-1.2639	-1.25253	0.04561	-0.00002	0.00624	0.000	2.999	-0.002	1	322
323	-1.2403	-1.22924	0.04562	-0.00002	0.00624	0.000	2.999	-0.002	1	323
324	-1.1888	-1.17840	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	1	324
325	-1.8000	-1.78946	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	1	325
326	-1.2442	-1.23309	0.04562	-0.00002	0.00624	0.000	2.999	-0.002	1	326
327	-0.9590	-0.95127	0.04573	-0.00001	0.00627	0.000	3.000	-0.001	1	327
328	-1.4507	-1.43665	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	1	328
329	-0.9480	-0.94425	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	1	329
330	-1.7312	-1.72267	0.04573	-0.00002	0.00627	0.000	2.999	-0.002	1	330
331	-1.1020	-1.09266	0.04568	-0.00002	0.00626	0.000	2.999	-0.002	1	331
332	-0.6028	-0.59956	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	1	332
333	-0.8550	-0.84835	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	1	333
334	-1.1963	-1.19581	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	1	334
335	-0.9448	-0.93722	0.04573	-0.00001	0.00627	0.000	3.000	-0.002	1	335
336	-0.4164	-0.41378	0.04584	-0.00001	0.00630	0.000	3.000	-0.001	1	336
337	-0.7203	-0.71498	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	1	337
338	-0.6207	-0.61630	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	1	338
339	-1.6487	-1.64434	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	1	339
340	-0.3780	-0.37571	0.04584	-0.00001	0.00630	0.000	3.000	-0.000	1	340
341	-1.2519	-1.25064	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	341
342	-0.2764	-0.27494	0.04585	-0.00000	0.00631	0.000	3.000	-0.000	1	342
343	-0.5633	-0.55941	0.04581	-0.00001	0.00631	0.000	3.000	-0.001	1	343
344	-0.1961	-0.19529	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	344
345	-0.1153	-0.11513	0.04584	-0.00000	0.00631	0.000	3.000	-0.000	1	345
346	-0.5997	-0.59480	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	1	346
347	-0.0200	-0.02057	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	347
348	-1.3382	-1.33751	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	348
349	-0.1339	-0.13215	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	349
350	-0.3209	-0.31771	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	350
351	-0.1704	-0.16919	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	351
352	-0.0861	-0.08615	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	352
353	-0.7278	-0.72846	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	353
354	-0.1241	-0.12242	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	354
355	0.1339	0.13215	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	355
356	0.4085	0.40464	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	356
357	1.4957	1.49117	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	1	357
358	-0.4482	-0.44815	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	1	358

420	-1.4613	-1.44708	0.04550	-0.00003	0.00621	0.000	2.999	-0.003	420
421	-1.1329	-1.12319	0.04567	-0.00002	0.00626	0.000	2.999	-0.003	421
422	-1.4106	-1.39656	0.04553	-0.00003	0.00622	0.000	2.999	-0.003	422
423	-1.4090	-1.39558	0.04553	-0.00003	0.00622	0.000	2.999	-0.003	423
424	-1.3931	-1.37992	0.04554	-0.00003	0.00622	0.000	2.999	-0.003	424
425	-1.4632	-1.44895	0.04550	-0.00001	0.00621	0.000	2.999	-0.001	425
426	-0.8433	-0.83697	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	426
427	-0.7981	-0.79272	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	427
428	-1.0008	-0.99170	0.04569	-0.00002	0.00626	0.000	2.999	-0.002	428
429	-0.9684	-0.96057	0.04572	-0.00001	0.00627	0.000	3.000	-0.002	429
430	-1.2159	-1.20437	0.04560	-0.00002	0.00624	0.000	2.999	-0.002	430
431	-0.8068	-0.80026	0.04577	-0.00001	0.00628	0.000	3.000	-0.001	431
432	-0.8523	-0.84568	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	432
433	-0.5265	-0.52294	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	433
434	-0.8545	-0.84984	0.04576	-0.00001	0.00628	0.000	3.000	-0.001	434
435	-0.6267	-0.62026	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	435
436	-0.6904	-0.68536	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	436
437	-1.0214	-1.01621	0.04569	-0.00002	0.00626	0.000	2.999	-0.002	437
438	-0.7153	-0.71002	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	438
439	-0.6521	-0.64741	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	439
440	-0.3552	-0.35308	0.04586	-0.00001	0.00631	0.000	3.000	-0.001	440
441	-0.3564	-0.35428	0.04584	-0.00000	0.00620	0.000	3.000	-0.000	441
442	-0.7537	-0.74806	0.04578	-0.00001	0.00629	0.000	3.000	-0.001	442
443	-1.3969	-1.39487	0.04586	-0.00001	0.00631	0.000	3.000	-0.000	443
444	-0.3256	-0.32366	0.04584	-0.00000	0.00630	0.000	3.000	-0.000	444
445	-0.5265	-0.52294	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	445
446	-0.4507	-0.44719	0.04583	-0.00001	0.00630	0.000	3.000	-0.001	446
447	-0.1268	-0.12653	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	447
448	-0.1139	-0.11374	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	448
449	-0.5188	-0.51531	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	449
450	-0.1189	-0.11873	0.04586	-0.00001	0.00631	0.000	3.000	-0.000	450
451	-0.2843	-0.28140	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	451
452	-0.1038	-0.10372	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	452
453	-0.2876	-0.28467	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	453
454	-0.3023	-0.30156	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	454
455	-0.3094	-0.30630	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	455
456	-0.3255	-0.32228	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	456
457	-0.3686	-0.36505	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	457
458	-0.4051	-0.40127	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	458
459	-0.5366	-0.53175	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	459
460	-0.5714	-0.56627	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	460
461	-0.4852	-0.48375	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	461
462	-0.5275	-0.52272	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	462
463	-0.7849	-0.77805	0.04584	-0.00000	0.00630	0.000	3.000	-0.000	463
464	-0.9030	-0.89516	0.04583	-0.00001	0.00630	0.000	3.000	-0.001	464
465	-0.3711	-0.36753	0.04587	-0.00000	0.00631	0.000	3.000	-0.000	465
466	-0.6233	-0.619514	0.04586	-0.00000	0.00631	0.000	3.000	-0.000	466
467	-1.1268	-1.11706	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	467
468	-1.0962	-1.08664	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	468
469	-1.0306	-1.02164	0.04581	-0.00001	0.00630	0.000	3.000	-0.001	469
470	-1.3810	-1.38169	0.04580	-0.00001	0.00629	0.000	3.000	-0.001	470
471	-0.9842	-0.97365	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	471
472	-1.1549	-1.14480	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	472
473	-1.1215	-1.11265	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	473
474	-1.7252	-1.71871	0.04560	-0.00002	0.00624	0.000	2.999	-0.002	474
475	-0.9706	-0.96217	0.04582	-0.00001	0.00630	0.000	3.000	-0.001	475
476	-1.1506	-1.14356	0.04579	-0.00001	0.00629	0.000	3.000	-0.001	476
477	-1.0575	-1.04830	0.04581	-0.00001	0.00629	0.000	3.000	-0.001	477
478	-1.6490	-1.63351	0.04564	-0.00002	0.00625	0.000	2.999	-0.002	478
479	-1.4339	-1.41799	0.04572	-0.00001	0.00627	0.000	3.000	-0.002	479
480	-1.5395	-1.52535	0.04568	-0.00002	0.00626	0.000	2.999	-0.002	480

481	1.5407	1.52654	0.04568	0.00002	0.00626	0.000	2.999	-0.002	1	481
482	1.3486	1.33697	0.04574	0.00001	0.00628	0.000	3.000	-0.001	1	482
483	1.5545	1.54317	0.04567	0.00002	0.00626	0.000	2.999	-0.002	1	483
484	1.6315	1.61623	0.04564	0.00002	0.00625	0.000	2.999	-0.002	1	484
485	1.7366	1.71996	0.04559	0.00002	0.00623	0.000	2.999	-0.002	1	485
486	1.5668	1.55233	0.04567	0.00002	0.00626	0.000	2.999	-0.002	1	486
487	1.7759	1.75871	0.04557	0.00003	0.00623	0.000	2.999	-0.003	1	487
488	1.9916	1.97106	0.04543	0.00004	0.00619	0.000	2.999	-0.004	1	488
489	1.7693	1.73268	0.04559	0.00002	0.00623	0.000	2.999	-0.002	1	489
490	1.7555	1.73860	0.04558	0.00002	0.00623	0.000	2.999	-0.002	1	490
491	2.1161	2.15207	0.04526	0.00005	0.00614	0.000	2.998	-0.005	1	491
492	2.1943	2.16959	0.04524	0.00005	0.00613	0.000	2.998	-0.005	1	492
493	2.1953	2.17087	0.04524	0.00005	0.00613	0.000	2.998	-0.005	1	493
494	2.3440	2.31608	0.04503	0.00007	0.00608	0.000	2.996	-0.006	1	494
495	2.2866	2.26010	0.04512	0.00006	0.00611	0.000	2.997	-0.005	1	495
496	2.1341	2.11093	0.04530	0.00005	0.00615	0.000	2.998	-0.004	1	496
497	2.1692	2.14532	0.04527	0.00005	0.00614	0.000	2.998	-0.005	1	497
498	2.4361	2.42565	0.04467	0.00009	0.00603	0.000	2.995	-0.007	1	498
499	2.4271	2.39691	0.04469	0.00009	0.00604	0.000	2.995	-0.007	1	499
500	2.6346	2.59735	0.04437	0.00015	0.00589	0.000	2.990	-0.010	1	500

TYPE 1-7 : Pearson's Types

8 : Normal Distribution

9 : Undefined Due to Negative Even Moment(s)

10 : Undefined Due to Negative P.D.F.

TABLE A-1-2

The Estimated Conditional Moments of θ , Given the Maximum Likelihood Estimate, β_1 , β_2 and the Criterion κ for the 500 Hypothetical Subjects, in Degree 4 Case

CONDITIONAL MEAN OF THETA GIVEN ME AND PEASON CRITERION										
SUBJECT	ME	PE	MOMENTS ABOUT MEAN	MEAN	PEASON	MEAN	PEASON	MEAN	PEASON	ID
1	-2.6989	-2.53504	0.00931	-0.01613	-0.00707	247.500	0.00000000	0.000000	0.000000	9
2	-3.0555 GRADUATED P.D.E. ASSUMES A NEGATIVE VALUE									
3	-2.6723	-2.52758	0.01626	-0.01032	-0.00455	25.750	0.00000000	0.000000	0.000000	9
4	-2.7061	-2.53631	0.00773	-0.01947	-0.00933	696.604	0.00000000	0.000000	0.000000	9
5	-2.3674	-2.31189	0.03972	-0.03104	0.00469	0.017	2.863	-0.035	-0.035	4
6	-2.2448	-2.20715	0.04174	-0.03760	0.00511	0.035	2.933	-0.025	-0.025	5
7	-2.2048	-2.17081	0.04222	-0.03351	0.00525	0.003	2.948	-0.022	-0.022	6
8	-1.9895	-1.96610	0.04387	-0.00024	0.00376	0.001	2.983	-0.016	-0.016	7
9	-1.8263	-1.81378	0.04454	-0.00016	0.00593	0.000	2.991	-0.011	-0.011	8
10	-2.2219	-2.18660	0.04203	-0.00954	0.00520	0.004	2.962	-0.023	-0.023	9
11	-2.0980	-2.07208	0.04318	-0.03034	0.00554	0.031	2.971	-0.017	-0.017	10
12	-2.2207	-2.18530	0.04204	-0.03054	0.00520	0.004	2.962	-0.023	-0.023	11
13	-2.1519	-2.12219	0.04274	-0.00041	0.00541	0.002	2.964	-0.020	-0.020	12
14	-2.1213	-2.09381	0.04300	-0.03037	0.00549	0.002	2.967	-0.018	-0.018	13
15	-1.7199	-1.71088	0.04485	-0.03112	0.00632	0.000	2.994	-0.010	-0.010	14
16	-1.6149	-1.60870	0.04511	-0.00010	0.00610	0.000	2.984	-0.013	-0.013	15
17	-1.6669	-1.65938	0.04499	-0.03011	0.00606	0.000	2.995	-0.010	-0.010	16
18	-1.8711	-1.85687	0.04438	-0.00017	0.00599	0.000	2.990	-0.012	-0.012	17
19	-1.6614	-1.65402	0.04500	-0.03011	0.00637	0.000	2.993	-0.010	-0.010	18
20	-1.4638	-1.46076	0.04540	-0.00008	0.00618	0.000	2.997	-0.009	-0.009	19
21	-1.1387	-1.13980	0.04585	-0.03005	0.00630	0.000	2.999	-0.009	-0.009	20
22	-1.4259	-1.42332	0.04546	-0.00007	0.00619	0.000	2.998	-0.009	-0.009	21
23	-1.4308	-1.43718	0.04544	-0.03008	0.00619	0.000	2.998	-0.009	-0.009	22
24	-1.4632	-1.46017	0.04540	-0.00006	0.00618	0.000	2.997	-0.009	-0.009	23
25	-1.2066	-1.20709	0.04577	-0.00006	0.00628	0.000	2.998	-0.009	-0.009	24
26	-0.7593	-0.76189	0.04621	-0.03004	0.00641	0.000	2.999	-0.009	-0.009	25
27	-1.4183	-1.41604	0.04547	-0.00007	0.00620	0.000	2.998	-0.009	-0.009	26
28	-0.7163	-0.71888	0.04625	-0.03003	0.00641	0.000	2.999	-0.009	-0.009	27
29	-1.2171	-1.21748	0.04575	-0.03005	0.00628	0.000	2.999	-0.009	-0.009	28
30	-1.2159	-1.21629	0.04576	-0.03006	0.00628	0.000	2.999	-0.009	-0.009	29
31	-1.1601	-1.16102	0.04582	-0.03005	0.00630	0.000	2.999	-0.009	-0.009	30
32	-0.7801	-0.78268	0.04620	-0.00004	0.00640	0.000	2.999	-0.009	-0.009	31
33	-0.9941	-0.99612	0.04600	-0.03005	0.00635	0.000	2.999	-0.008	-0.008	32
34	-0.9045	-0.90687	0.04609	-0.03004	0.00637	0.000	2.999	-0.008	-0.008	33
35	-0.9941	-0.99612	0.04600	-0.03005	0.00635	0.000	2.999	-0.008	-0.008	34
36	-0.8498	-0.85230	0.04614	-0.00004	0.00630	0.000	2.999	-0.007	-0.007	35
37	-0.3324	-0.33490	0.04645	-0.03002	0.00646	0.000	2.999	-0.008	-0.008	36
38	-1.2274	-1.22767	0.04574	-0.03006	0.00627	0.000	2.999	-0.009	-0.009	37
39	-0.4985	-0.50063	0.04639	-0.03002	0.00645	0.000	2.999	-0.009	-0.009	38
40	-0.4130	-0.41503	0.04640	-0.00002	0.00646	0.000	2.999	-0.009	-0.009	39
41	-0.4277	-0.42252	0.04643	-0.03002	0.00646	0.000	2.999	-0.009	-0.009	40
42	-0.3684	-0.36998	0.04645	-0.03002	0.00647	0.000	2.999	-0.009	-0.009	41
43	-0.0266	-0.02733	0.04651	0.03000	0.00649	0.000	2.999	-0.009	-0.009	42
44	-0.2484	-0.24935	0.04649	-0.03001	0.00648	0.000	2.999	-0.009	-0.009	43
45	-0.4129	-0.41469	0.04643	-0.00002	0.00646	0.000	2.999	-0.009	-0.009	44
46	-0.2501	-0.25106	0.04649	-0.03001	0.00648	0.000	2.999	-0.009	-0.009	45
47	-0.0822	-0.08215	0.04651	-0.00000	0.00649	0.000	2.999	-0.009	-0.009	46
48	-0.0794	-0.07933	0.04651	-0.03000	0.00649	0.000	2.999	-0.009	-0.009	47
49	-0.3038	-0.30607	0.04644	-0.00002	0.00644	0.000	2.999	-0.009	-0.009	48
50	-0.1766	-0.17608	0.04651	0.03000	0.00649	0.000	2.999	-0.009	-0.009	49
51	0.1204	0.12154	0.04640	0.03001	0.00648	0.000	2.999	-0.009	-0.009	50

52	0.3786	0.38120	0.04641	0.00002	0.00646	0.030	2.999	-0.002	1	51
53	0.1431	0.14452	0.04649	0.00001	0.00648	0.000	2.999	-0.000	2	52
54	0.4997	0.49868	0.04634	0.00003	0.00644	0.000	2.999	-0.000	1	53
55	0.0176	0.01827	0.04651	0.00000	0.00649	0.000	2.999	-0.000	2	54
56	-0.0159	-0.01564	0.04651	0.00000	0.00649	0.000	2.999	-0.000	2	55
57	-0.0261	-0.02570	0.04651	0.00000	0.00649	0.000	2.999	-0.000	2	56
58	0.3087	0.31100	0.04644	0.00000	0.00647	0.000	2.999	-0.001	1	57
59	0.2803	0.28246	0.04645	0.00002	0.00647	0.000	2.999	-0.001	1	58
60	0.6487	0.65191	0.04624	0.00003	0.00641	0.000	2.999	-0.005	1	59
61	0.5905	0.59366	0.04628	0.00003	0.00642	0.000	2.999	-0.004	1	60
62	0.5022	0.50520	0.04634	0.00003	0.00644	0.000	2.999	-0.003	1	61
63	0.3723	0.37487	0.04641	0.00002	0.00646	0.000	2.999	-0.003	1	62
64	0.9578	0.96325	0.04597	0.00005	0.00636	0.000	2.999	-0.009	1	63
65	0.2102	0.21231	0.04645	0.00005	0.00632	0.000	2.999	-0.001	2	64
66	1.0166	1.01868	0.04591	0.00005	0.00632	0.000	2.999	-0.009	1	65
67	1.1270	1.12818	0.04578	0.00005	0.00629	0.000	2.999	-0.009	1	66
68	0.9547	0.95633	0.04593	0.00005	0.00633	0.000	2.999	-0.008	1	67
69	0.3369	0.33932	0.04643	0.00002	0.00646	0.000	2.999	-0.001	1	68
70	1.2520	1.25177	0.04562	0.00006	0.00624	0.000	2.998	-0.009	1	69
71	1.1924	1.19284	0.04570	0.00006	0.00624	0.000	2.998	-0.009	1	70
72	1.1850	1.18358	0.04571	0.00005	0.00627	0.000	2.999	-0.009	1	71
73	1.2606	1.26026	0.04561	0.00006	0.00624	0.000	2.998	-0.009	1	72
74	0.9439	0.94431	0.04594	0.00005	0.00624	0.000	2.998	-0.009	1	73
75	1.2318	1.23183	0.04565	0.00006	0.00625	0.000	2.998	-0.009	1	74
76	0.9862	0.98642	0.04593	0.00005	0.00633	0.000	2.999	-0.009	1	75
77	1.5297	1.52457	0.04517	0.00009	0.00611	0.000	2.998	-0.009	1	76
78	1.6301	1.62243	0.04494	0.00011	0.00605	0.000	2.995	-0.010	1	77
79	1.3639	1.36202	0.04546	0.00007	0.00620	0.000	2.998	-0.009	1	78
80	1.6155	1.60823	0.04498	0.00011	0.00606	0.000	2.995	-0.011	1	79
81	1.7958	1.78272	0.04466	0.00016	0.00591	0.000	2.991	-0.011	1	80
82	1.3451	1.34353	0.04549	0.00007	0.00620	0.000	2.998	-0.009	1	81
83	1.3637	1.36182	0.04546	0.00007	0.00620	0.000	2.998	-0.009	1	82
84	1.7627	1.75085	0.04457	0.00015	0.00594	0.000	2.992	-0.011	1	83
85	1.3833	1.38109	0.04543	0.00007	0.00619	0.000	2.998	-0.009	1	84
86	1.6049	1.59986	0.04500	0.00011	0.00604	0.000	2.995	-0.010	1	85
87	1.6430	1.63497	0.04491	0.00012	0.00604	0.000	2.995	-0.010	1	86
88	1.7798	1.76742	0.04451	0.00016	0.00583	0.000	2.991	-0.011	1	87
89	1.7478	1.73667	0.04462	0.00014	0.00594	0.000	2.992	-0.011	1	88
90	1.8884	1.87143	0.04409	0.00021	0.00581	0.000	2.986	-0.013	1	89
91	1.8339	1.81931	0.04432	0.00018	0.00587	0.000	2.989	-0.012	1	90
92	2.1047	2.07491	0.04272	0.00041	0.00540	0.000	2.961	-0.020	1	91
93	2.0506	2.02465	0.04316	0.00034	0.00553	0.000	2.971	-0.017	1	92
94	2.2720	2.22610	0.04057	0.00084	0.00475	0.000	2.887	-0.031	1	93
95	2.1136	2.08313	0.04264	0.00063	0.00538	0.000	2.959	-0.020	1	94
96	2.3570	2.29907	0.03865	0.00130	0.00414	0.029	2.775	-0.041	1	95
97	2.2327	2.20909	0.04091	0.00077	0.00486	0.009	2.902	-0.029	1	96
98	2.6483	2.48592	0.03989	0.00139	0.00378	1.96.890	2.902	*****	9	97
99	2.7142	2.48078	0.02212	0.00356	0.00356	*****	2.902	*****	9	98
100	2.5177	2.42219	0.03080	0.00380	0.00144	0.493	1.516	-0.123	1	99
101	-2.7417	-2.53629	-0.00831	-0.02539	-0.01756	*****	*****	*****	9	100
102	-2.5074	-2.42892	0.09470	-0.00245	0.00283	0.163	2.350	-0.066	1	101
103	-2.7847	-2.05965	0.04328	-0.00033	0.00557	0.001	2.973	-0.017	1	102
104	-2.5688	-2.47251	0.03060	-0.00387	0.00136	0.523	1.457	-0.131	1	103
105	-2.4672	-2.39790	0.03657	-0.00188	0.00346	0.072	2.570	-0.254	1	104
106	-2.0780	-2.05337	0.04332	-0.00032	0.00558	0.001	2.974	-0.017	1	105
107	-2.1210	-2.09353	0.04300	-0.00037	0.00559	0.001	2.967	-0.018	1	106
108	-2.3210	-2.27505	0.04058	-0.00084	0.00475	0.011	2.887	-0.031	1	107
109	-2.4760	-2.40482	0.03620	-0.00198	0.00334	0.083	2.549	-0.056	1	108
110	-1.8710	-1.85677	0.04438	-0.00031	0.00513	0.005	2.935	-0.024	1	109
111	-1.8710	-1.85677	0.04438	-0.00031	0.00513	0.005	2.935	-0.024	1	110
112	-1.5181	-1.51404	0.04530	-0.00009	0.00615	0.000	2.997	-0.009	1	111

174	1.2554	1.25513	0.04562	0.33306	0.30624	0.000	2.998	-0.009	1	173
175	1.2188	1.21899	0.04567	0.00006	0.00622	0.000	2.998	-0.009	1	174
176	1.2985	1.29763	0.04556	0.00707	0.00622	0.000	2.998	-0.009	1	175
177	1.1964	1.19685	0.04570	0.33306	0.33626	0.000	2.999	-0.009	1	176
178	1.5407	1.53532	0.04514	0.00310	0.00611	0.000	2.996	-0.009	1	177
179	1.6001	1.59324	0.04501	0.30011	0.00607	0.000	2.995	-0.010	1	178
180	1.4832	1.47909	0.04526	0.00009	0.00614	0.000	2.997	-0.009	1	179
181	1.3833	1.38108	0.04543	0.33307	0.33619	0.000	2.998	-0.009	1	180
182	1.4991	1.49465	0.04523	0.00009	0.00613	0.000	2.997	-0.009	1	181
183	1.2496	1.24940	0.04563	0.00006	0.00624	0.000	2.998	-0.009	1	182
184	1.5137	1.50893	0.04520	0.00009	0.33612	0.000	2.997	-0.009	1	183
185	1.7426	1.75075	0.04457	0.00015	0.00594	0.000	2.992	-0.011	1	184
186	2.1396	2.10705	0.04238	0.33347	0.33530	0.001	2.952	-0.021	1	185
187	2.0455	2.01989	0.04320	0.03334	0.00554	0.001	2.932	-0.017	1	186
188	1.5776	1.57132	0.04506	0.00029	0.00564	0.001	2.978	-0.016	1	187
189	1.9978	1.97514	0.04352	0.00029	0.00564	0.001	2.978	-0.016	1	188
190	1.7912	1.77830	0.04548	0.00016	0.00592	0.000	2.991	-0.011	1	189
191	1.8954	1.87811	0.04406	0.33321	0.33580	0.001	2.986	-0.013	1	190
192	2.5017	2.41131	0.03203	0.03335	0.00189	0.341	1.845	-0.394	1	191
193	2.0212	1.99714	0.04337	0.33331	0.00559	0.001	2.975	-0.016	1	192
194	2.4563	2.37840	0.03483	0.30241	0.00287	0.137	2.369	-0.365	1	193
195	2.0269	2.00248	0.04333	0.00332	0.00558	0.001	2.974	-0.017	1	194
196	1.8784	1.86189	0.04514	0.00020	0.00582	0.000	2.987	-0.013	1	195
197	2.1021	2.07251	0.04274	0.03241	0.00541	0.002	2.962	-0.019	1	196
198	2.5188	2.42292	0.05071	0.33381	0.33140	0.507	1.490	-0.127	1	197
199	2.6483	2.48592	0.00289	0.33379	0.00738	1.98	0.806	0.000	9	198
200	2.2231	2.18274	0.04137	0.03067	0.33530	0.000	2.920	-0.027	1	199
201	2.7417	2.53629	-0.00831	-0.02539	-0.31756	0.000	0.000	0.000	9	200
202	2.4726	2.40215	0.03635	-0.00194	0.00339	0.074	2.565	-0.055	1	201
203	2.2251	2.18930	0.04199	0.33055	0.33518	0.004	2.941	-0.023	1	202
204	2.4500	2.38416	0.03723	0.03169	0.00368	0.055	2.657	-0.350	1	203
205	2.0733	2.04890	0.04335	-0.03331	0.00559	0.001	2.974	-0.017	1	204
206	2.4193	2.35908	0.03826	0.00141	0.00502	0.035	2.745	-0.044	1	205
207	1.8821	1.86742	0.04018	0.00018	0.00588	0.000	2.989	-0.012	1	206
208	2.2569	2.21805	0.04158	-0.33363	0.00506	0.005	2.927	-0.025	1	207
209	2.3230	2.27681	0.04054	-0.33385	0.33474	0.311	2.885	-0.031	1	208
210	2.3110	2.26626	0.04075	-0.00080	0.00481	0.009	2.895	-0.330	1	209
211	2.0102	1.98951	0.04374	-0.00024	0.33370	0.001	2.981	-0.015	1	210
212	1.7808	1.76987	0.04468	0.00014	0.00597	0.000	2.993	-0.011	1	211
213	1.5281	1.52383	0.04528	-0.33309	0.00614	0.000	2.997	-0.009	1	212
214	1.4941	1.48050	0.04534	-0.33308	0.33616	0.000	2.997	-0.009	1	213
215	1.7347	1.72524	0.04481	-0.33313	0.00601	0.000	2.994	-0.010	1	214
216	1.8593	1.84553	0.04442	-0.33317	0.33590	0.000	2.990	-0.012	1	215
217	1.5973	1.59152	0.04514	-0.33310	0.00611	0.000	2.996	-0.010	1	216
218	1.6462	1.63922	0.04504	-0.00311	0.00608	0.000	2.995	-0.010	1	217
219	1.8372	1.82428	0.04450	-0.00016	0.00592	0.000	2.991	-0.011	1	218
220	2.9890	2.99105	0.04601	-0.00005	0.00635	0.000	2.999	-0.008	1	219
221	1.2161	1.21451	0.04576	-0.00306	0.33628	0.000	2.999	-0.009	1	220
222	1.5868	1.58126	0.04517	0.00310	0.00611	0.000	2.996	-0.010	1	221
223	1.0664	1.06802	0.04593	-0.33335	0.33633	0.302	2.939	-0.009	1	222
224	1.4632	1.46717	0.04540	-0.33308	0.00618	0.000	2.997	-0.009	1	223
225	0.8773	0.87974	0.04612	-0.33304	0.33638	0.000	2.999	-0.007	1	224
226	1.0458	1.04755	0.04595	-0.00005	0.33633	0.000	2.999	-0.009	1	225
227	1.0431	1.04486	0.04595	-0.33305	0.00633	0.000	2.999	-0.009	1	226
228	1.0445	1.04626	0.04595	-0.33305	0.33633	0.300	2.999	-0.009	1	227
229	0.9844	0.98467	0.04601	-0.00005	0.00635	0.000	2.999	-0.008	1	228
230	1.0650	1.06663	0.04593	-0.00005	0.00633	0.000	2.999	-0.009	1	229
231	1.2997	1.29915	0.04565	-0.00006	0.00625	0.000	2.998	-0.009	1	230
232	1.0195	1.02160	0.04598	-0.00005	0.33634	0.000	2.999	-0.008	1	231
233	1.0190	1.02090	0.04598	-0.00005	0.33634	0.333	2.999	-0.008	1	232
234	0.9931	0.99513	0.04601	-0.00005	0.00635	0.000	2.999	-0.008	1	233

235	-0.6400	-0.64250	0.04630	-0.00003	0.00663	0.000	2.999	-0.004	1	234
236	-0.5268	-0.52902	0.04637	-0.00304	0.00645	0.000	2.999	-0.002	1	235
237	-0.9279	-0.93019	0.04607	-0.00304	0.00645	0.000	2.999	-0.008	1	236
238	-0.7372	-0.73919	0.04623	-0.00004	0.00641	0.000	2.999	-0.005	1	237
239	-0.2628	-0.26372	0.04649	-0.00301	0.00648	0.000	2.999	-0.000	2	238
240	-0.9212	-0.92352	0.04608	-0.00004	0.00637	0.000	2.999	-0.007	1	239
241	-0.5486	-0.55089	0.04636	-0.00303	0.00645	0.000	2.999	-0.003	1	240
242	-0.5242	-0.52661	0.04637	-0.00303	0.00645	0.000	2.999	-0.002	1	241
243	-0.3831	-0.38475	0.04644	-0.00302	0.00647	0.000	2.999	-0.001	1	242
244	-0.4587	-0.46068	0.04641	-0.00302	0.00646	0.000	2.999	-0.002	1	243
245	-0.4053	-0.40705	0.04643	-0.00002	0.00647	0.000	2.999	-0.001	1	244
246	-0.7330	-0.73559	0.04624	-0.00306	0.00641	0.000	2.999	-0.005	1	245
247	-0.5816	-0.58398	0.04634	-0.00003	0.00645	0.000	2.999	-0.003	1	246
248	-0.1485	-0.14884	0.04651	-0.00301	0.00649	0.000	2.999	-0.000	2	247
249	0.3000	0.30226	0.04644	0.00302	0.00647	0.000	2.999	-0.001	2	248
250	-0.3241	-0.32546	0.04646	-0.00002	0.00647	0.000	2.999	-0.001	2	249
251	-0.0602	-0.06001	0.04651	-0.00300	0.00649	0.000	2.999	-0.000	2	250
252	-0.0379	-0.03758	0.04651	0.00000	0.00649	0.000	2.999	-0.000	2	251
253	-0.0027	-0.00216	0.04651	0.00300	0.00647	0.000	2.999	-0.000	2	252
254	-0.2503	-0.25231	0.04646	0.00002	0.00647	0.000	2.999	-0.001	2	253
255	0.2738	0.27593	0.04645	0.00002	0.00647	0.000	2.999	-0.001	1	254
256	-0.2573	-0.25830	0.04648	-0.00301	0.00648	0.000	2.999	-0.000	2	255
257	-0.3728	-0.37547	0.04641	-0.00002	0.00646	0.000	2.999	-0.002	1	256
258	-0.3538	-0.35630	0.04642	-0.00302	0.00646	0.000	2.999	-0.002	1	257
259	0.2557	0.25774	0.04646	0.00002	0.00647	0.000	2.999	-0.001	2	258
260	0.5325	0.53557	0.04632	0.00303	0.00643	0.000	2.999	-0.003	1	259
261	0.4771	0.48003	0.04635	0.00003	0.00644	0.000	2.999	-0.003	1	260
262	0.3478	0.35027	0.04642	0.00002	0.00646	0.000	2.999	-0.002	1	261
263	0.8120	0.81503	0.04611	0.00306	0.00641	0.000	2.999	-0.007	1	262
264	0.7516	0.75475	0.04616	0.00304	0.00639	0.000	2.999	-0.006	1	263
265	0.5313	0.53436	0.04632	0.00303	0.00644	0.000	2.999	-0.003	1	264
266	0.6121	0.61523	0.04627	0.00003	0.00642	0.000	2.999	-0.002	1	265
267	0.6142	0.61739	0.04627	0.00003	0.00642	0.000	2.999	-0.005	1	266
268	0.6197	0.62289	0.04626	0.00003	0.00642	0.000	2.999	-0.005	1	267
269	0.5114	0.51442	0.04633	-0.00303	0.00646	0.000	2.999	-0.003	1	268
270	0.7755	0.77861	0.04614	0.00304	0.00638	0.000	2.999	-0.007	1	269
271	1.1183	1.11956	0.04579	0.00305	0.00629	0.000	2.999	-0.009	1	270
272	1.1850	1.18558	0.04571	0.00306	0.00627	0.000	2.999	-0.009	1	271
273	1.1524	1.15333	0.04575	0.00306	0.00628	0.000	2.999	-0.009	1	272
274	1.4642	1.46048	0.04529	0.00008	0.00615	0.000	2.997	-0.009	1	273
275	1.4238	1.42087	0.04536	0.00308	0.00617	0.000	2.997	-0.009	1	274
276	0.7889	0.79199	0.04613	0.00306	0.00638	0.000	2.999	-0.007	1	275
277	1.2766	1.27404	0.04559	0.00306	0.00623	0.000	2.998	-0.009	1	276
278	1.7753	1.76299	0.04453	0.00315	0.00593	0.000	2.991	-0.011	1	277
279	1.3234	1.32216	0.04552	0.00307	0.00621	0.000	2.998	-0.009	1	278
280	1.4048	1.40222	0.04539	0.00308	0.00618	0.000	2.998	-0.009	1	279
281	1.6283	1.62068	0.04495	0.00011	0.00605	0.000	2.995	-0.010	1	280
282	1.6234	1.61592	0.04496	0.00311	0.00605	0.000	2.995	-0.010	1	281
283	1.4361	1.43293	0.04534	0.00008	0.00616	0.000	2.997	-0.009	1	282
284	1.7061	1.69617	0.04474	0.00313	0.00599	0.000	2.993	-0.010	1	283
285	1.6615	1.65294	0.04486	0.00312	0.00603	0.000	2.994	-0.010	1	284
286	1.7056	1.69568	0.04474	0.00313	0.00599	0.000	2.993	-0.010	1	285
287	1.6222	1.61475	0.04496	0.00311	0.00605	0.000	2.995	-0.010	1	286
288	2.2381	2.19613	0.04114	0.00072	0.00607	0.000	2.911	-0.028	1	287
289	1.9003	1.88249	0.04434	0.00321	0.00579	0.000	2.986	-0.013	1	288
290	1.6528	1.64459	0.04489	0.00312	0.00603	0.000	2.995	-0.010	1	289
291	2.0156	1.99188	0.04340	0.00330	0.00561	0.000	2.976	-0.016	1	290
292	2.0895	2.06085	0.04285	0.00339	0.00544	0.000	2.964	-0.019	1	291
293	2.3590	2.30149	0.03857	0.00132	0.00462	0.000	2.769	-0.062	1	292
294	2.3984	2.33311	0.03733	0.00165	0.00372	0.000	2.667	-0.049	1	293
295	2.0494	2.02353	0.04317	0.00034	0.00354	0.000	2.971	-0.017	1	294

296	2.7137	2.48102	-0.02173	0.03534	-0.02619	0.00000	0.00000	0.00000	0.00000	9	295
297	1.8449	1.82985	0.04428	0.00018	0.00586	0.000	0.000	2.989	-0.012	1	296
298	2.2188	2.17889	0.04143	0.03066	0.00502	0.006	0.006	2.922	-0.026	1	297
299	2.8718	0.51402	-5.64267	27.13669	-98.55305	0.00000	0.00000	0.00000	0.00000	9	298
300	2.8564	1.55472	-1.72029	6.65923	-0.89286	0.00000	0.00000	0.00000	0.00000	9	299
301	-2.2354	-2.19865	0.04186	-0.00214	0.00317	0.004	0.004	2.937	-0.024	1	300
302	-2.4875	-2.41377	0.03569	-0.00016	0.00317	0.000	0.000	2.987	-0.059	1	301
303	-2.3129	-2.26793	0.04072	-0.00081	0.00580	0.010	0.010	2.893	-0.030	1	302
304	-2.0554	-2.03215	0.04347	-0.00030	0.00563	0.001	0.001	2.977	-0.016	1	303
305	-2.3517	-2.30181	0.03997	-0.00098	0.00457	0.015	0.015	2.857	-0.034	1	304
306	-2.2133	-2.17815	0.04212	-0.00352	0.00523	0.004	0.004	2.945	-0.023	1	305
307	-1.9910	-1.97132	0.04384	-0.00324	0.00513	0.001	0.001	2.983	-0.014	1	306
308	-2.3423	-2.29366	0.04017	-0.00093	0.00633	0.013	0.013	2.861	-0.033	1	307
309	-1.9549	-1.93701	0.04402	-0.00022	0.00519	0.001	0.001	2.985	-0.013	1	308
310	-1.8350	-1.82216	0.04451	-0.00016	0.00592	0.000	0.000	2.991	-0.011	1	309
311	-1.8880	-1.87398	0.04431	-0.00018	0.00587	0.000	0.000	2.989	-0.012	1	310
312	-1.9620	-1.94377	0.04399	-0.00022	0.00578	0.001	0.001	2.985	-0.014	1	311
313	-2.0743	-2.04990	0.04335	-0.00031	0.00559	0.001	0.001	2.974	-0.017	1	312
314	-1.7152	-1.70632	0.04487	-0.00012	0.00603	0.000	0.000	2.994	-0.010	1	313
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316	-1.5985	-1.59269	0.04514	-0.00010	0.00611	0.000	0.000	2.996	-0.010	1	315
317	-1.6360	-1.62928	0.04506	-0.00010	0.00608	0.000	0.000	2.996	-0.010	1	316
318	-1.6461	-1.63912	0.04504	-0.00011	0.00608	0.000	0.000	2.995	-0.010	1	317
319	-1.7316	-1.72223	0.04482	-0.00013	0.00602	0.000	0.000	2.994	-0.010	1	318
320	-1.1918	-1.19243	0.04579	-0.00006	0.00629	0.000	0.000	2.999	-0.009	1	319
321	-1.7237	-1.71166	0.04485	-0.00012	0.00622	0.000	0.000	2.994	-0.010	1	320
322	-1.2639	-1.26377	0.04569	-0.00005	0.00626	0.000	0.000	2.998	-0.009	1	321
323	-1.2403	-1.24044	0.04573	-0.00006	0.00627	0.000	0.000	2.999	-0.009	1	322
324	-1.1888	-1.18946	0.04579	-0.00006	0.00629	0.000	0.000	2.999	-0.009	1	323
325	-1.2000	-1.20055	0.04578	-0.00006	0.00628	0.000	0.000	2.999	-0.009	1	324
326	-1.2442	-1.24430	0.04572	-0.00006	0.00627	0.000	0.000	2.999	-0.009	1	325
327	-0.9590	-0.96118	0.04604	-0.00004	0.00636	0.000	0.000	2.999	-0.008	1	326
328	-1.4507	-1.44790	0.04542	-0.00008	0.00618	0.000	0.000	2.998	-0.009	1	327
329	-0.5480	-0.55029	0.04636	-0.00003	0.00645	0.000	0.000	2.999	-0.003	1	328
330	-1.0312	-1.03303	0.04587	-0.00005	0.00634	0.002	0.002	2.998	-0.008	1	329
331	-1.1020	-1.10338	0.04589	-0.00005	0.00632	0.000	0.000	2.999	-0.009	1	330
332	-0.6028	-0.60523	0.04633	-0.00003	0.00644	0.000	0.000	2.999	-0.003	1	331
333	-0.8550	-0.85759	0.04614	-0.00004	0.00638	0.000	0.000	2.999	-0.007	1	332
334	-1.1963	-1.19689	0.04578	-0.00006	0.00628	0.000	0.000	2.999	-0.009	1	333
335	-0.9448	-0.94703	0.04605	-0.00004	0.00636	0.000	0.000	2.999	-0.008	1	334
336	-0.4164	-0.41820	0.04643	-0.00002	0.00646	0.000	0.000	2.999	-0.001	1	335
337	-0.7203	-0.72288	0.04624	-0.00003	0.00641	0.000	0.000	2.999	-0.005	1	336
338	-0.6207	-0.62316	0.04631	-0.00003	0.00643	0.000	0.000	2.999	-0.004	1	337
339	-0.6687	-0.65121	0.04630	-0.00003	0.00643	0.000	0.000	2.999	-0.004	1	338
340	-0.3780	-0.37963	0.04644	-0.00002	0.00647	0.000	0.000	2.999	-0.001	1	339
341	-0.2519	-0.25287	0.04649	-0.00001	0.00648	0.000	0.000	2.999	-0.000	2	340
342	-0.2754	-0.27750	0.04648	-0.00001	0.00648	0.000	0.000	2.999	-0.001	2	341
343	-0.5633	-0.56563	0.04635	-0.00003	0.00644	0.000	0.000	2.999	-0.003	1	342
344	-0.1961	-0.19674	0.04650	-0.00001	0.00648	0.000	0.000	2.999	-0.003	1	343
345	-0.1153	-0.11546	0.04651	-0.00001	0.00649	0.000	0.000	2.999	-0.002	2	344
346	-0.5990	-0.60142	0.04633	-0.00003	0.00644	0.000	0.000	2.999	-0.003	1	345
347	-0.0200	-0.01956	0.04651	-0.00000	0.00649	0.000	0.000	2.999	-0.000	2	346
348	-0.0083	-0.00891	0.04651	-0.00000	0.00649	0.000	0.000	2.999	-0.000	2	347
349	-0.1339	-0.13527	0.04649	-0.00001	0.00647	0.000	0.000	2.999	-0.000	2	348
350	-0.3209	-0.32325	0.04643	-0.00002	0.00647	0.000	0.000	2.999	-0.001	1	349
351	-0.1704	-0.17282	0.04650	-0.00001	0.00648	0.000	0.000	2.999	-0.002	2	350
352	-0.0961	-0.09608	0.04651	-0.00000	0.00649	0.000	0.000	2.999	-0.000	2	351
353	-0.0078	-0.00729	0.04651	-0.00000	0.00649	0.000	0.000	2.999	-0.000	2	352
354	-0.1241	-0.12541	0.04650	-0.00001	0.00648	0.000	0.000	2.999	-0.000	2	353
355	0.1339	0.13527	0.04649	0.00001	0.00648	0.000	0.000	2.999	-0.000	2	354
356	0.4085	0.41121	0.04639	0.00002	0.00645	0.000	0.000	2.999	-0.002	1	355

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TENNESSEE UNIV KNOXVILLE DEPT OF PSYCHOLOGY
ESTIMATION OF THE OPERATING CHARACTERISTICS OF
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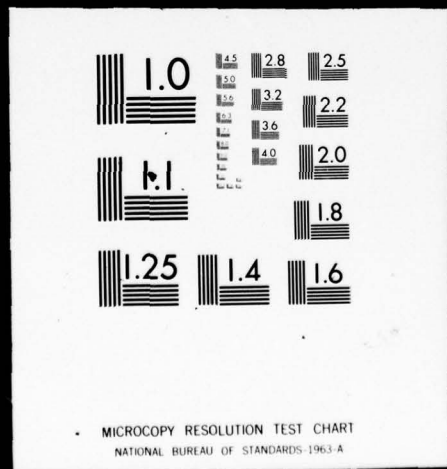
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357	0.4957	0.49868	0.04634	0.00003	0.00644	0.0000	2.999	-0.003	1	356
358	0.4852	0.48815	0.04635	0.00000	0.00644	0.0000	2.999	-0.003	1	357
359	0.0264	0.02712	0.04651	0.00000	0.00649	0.0000	2.999	-0.000	2	358
360	0.3398	0.34224	0.04651	0.00002	0.00646	0.0000	2.999	-0.001	1	359
361	0.4357	0.43890	0.04625	0.00003	0.00642	0.0000	2.999	-0.005	1	360
362	0.7735	0.77662	0.04614	0.00005	0.00638	0.0000	2.999	-0.007	1	361
363	0.9640	0.96641	0.04596	0.00005	0.00634	0.0000	2.999	-0.009	1	362
364	0.8954	0.89815	0.04603	0.00004	0.00633	0.0000	2.999	-0.008	1	363
365	0.9962	0.99842	0.04593	0.00005	0.00633	0.0000	2.999	-0.009	1	364
366	0.9715	0.97387	0.04595	0.00005	0.00633	0.0000	2.999	-0.009	1	365
367	1.0124	1.01421	0.04591	0.00005	0.00632	0.0000	2.999	-0.009	1	366
368	0.9493	0.95179	0.04598	0.00005	0.00634	0.0000	2.999	-0.009	1	367
369	1.1307	1.13184	0.04578	0.00006	0.00628	0.0000	2.999	-0.009	1	368
370	1.2872	1.28649	0.04557	0.00007	0.00623	0.0000	2.999	-0.009	1	369
371	0.7787	0.78181	0.04614	0.00006	0.00638	0.0000	2.999	-0.007	1	370
372	1.1387	1.13976	0.04577	0.00006	0.00628	0.0000	2.999	-0.009	1	371
373	1.1531	1.15402	0.04575	0.00006	0.00628	0.0000	2.999	-0.009	1	372
374	0.6400	0.64321	0.04625	0.00003	0.00661	0.0000	2.999	-0.005	1	373
375	1.2318	1.23183	0.04565	0.00006	0.00625	0.0000	2.998	-0.009	1	374
376	1.2114	1.21176	0.04567	0.00006	0.00625	0.0000	2.999	-0.009	1	375
377	1.2115	1.21177	0.04568	0.00006	0.00626	0.0000	2.999	-0.009	1	376
378	1.6506	1.64235	0.04489	0.00012	0.00603	0.0000	2.995	-0.010	1	377
379	1.2575	1.25720	0.04562	0.00005	0.00624	0.0000	2.998	-0.009	1	378
380	1.6966	1.68697	0.04477	0.00013	0.00600	0.0000	2.994	-0.010	1	379
381	1.5110	1.50629	0.04520	0.00009	0.00612	0.0000	2.997	-0.009	1	380
382	1.7324	1.72160	0.04467	0.00014	0.00597	0.0000	2.993	-0.011	1	381
383	1.2537	1.25345	0.04562	0.00006	0.00624	0.0000	2.998	-0.009	1	382
384	1.8591	1.84365	0.04422	0.00019	0.00584	0.0000	2.988	-0.012	1	383
385	1.6743	1.66729	0.04482	0.00012	0.00602	0.0000	2.996	-0.010	1	384
386	1.8943	1.87706	0.04407	0.00021	0.00580	0.0000	2.986	-0.013	1	385
387	2.0121	1.98859	0.04343	0.00030	0.00561	0.0000	2.976	-0.016	1	386
388	1.7472	1.73589	0.04462	0.00014	0.00596	0.0000	2.992	-0.011	1	387
389	1.9966	1.97401	0.04352	0.00029	0.00564	0.0001	2.978	-0.016	1	388
390	1.9401	1.92061	0.04384	0.00024	0.00573	0.0001	2.983	-0.014	1	389
391	2.1021	2.07251	0.04274	0.00041	0.00541	0.0002	2.962	-0.019	1	390
392	2.3436	2.28764	0.03902	0.00121	0.00626	0.0005	2.800	-0.039	1	391
393	2.1727	2.13721	0.04232	0.00054	0.00520	0.0004	2.942	-0.023	1	392
394	2.0497	2.02381	0.04317	0.00036	0.00554	0.0001	2.971	-0.017	1	393
395	2.3841	2.32149	0.03782	0.00152	0.00388	0.0003	2.711	-0.046	1	394
396	2.2645	2.21950	0.04071	0.00081	0.00479	0.0001	2.893	-0.030	1	395
397	2.5133	2.41924	0.03115	0.00347	0.00157	0.0004	1.616	-0.112	1	396
398	2.6535	2.37629	0.03497	0.00236	0.00292	0.0003	2.390	-0.064	1	397
399	2.1959	2.15829	0.04174	0.00059	0.00511	0.0005	2.933	-0.025	1	398
400	2.2563	2.21227	0.03085	0.00378	0.00284	0.0009	2.859	-0.029	1	399
401	2.7057	2.53625	0.00714	0.01539	-0.00296	651.545	*****	*****	9	400
402	2.0113	1.99055	0.04374	0.00026	0.00570	0.0001	2.981	-0.015	1	401
403	2.2235	2.18785	0.04201	0.00055	0.00519	0.0004	2.941	-0.023	1	402
404	2.3713	2.31867	0.03954	0.003108	0.00443	0.0019	2.833	-0.036	1	403
405	2.2352	2.19847	0.04187	0.00057	0.00515	0.0004	2.937	-0.024	1	404
406	2.3525	2.30250	0.03996	0.00398	0.00256	0.0015	2.856	-0.034	1	405
407	2.2197	2.18439	0.04205	0.00035	0.00520	0.0004	2.943	-0.023	1	406
408	2.0740	2.04962	0.04335	0.00031	0.00529	0.0001	2.974	-0.017	1	407
409	2.3153	2.27004	0.04068	0.00382	0.00478	0.0010	2.891	-0.030	1	408
410	1.6141	1.60792	0.04511	0.00010	0.00610	0.0000	2.996	-0.010	1	409
411	1.7420	1.73232	0.04479	0.00013	0.00601	0.0000	2.994	-0.010	1	410
412	1.9591	1.94601	0.04500	0.00032	0.00578	0.0001	2.985	-0.013	1	411
413	1.8879	1.87298	0.04431	0.00018	0.00587	0.0000	2.989	-0.012	1	412
414	1.7470	1.73038	0.04480	0.00013	0.00601	0.0000	2.994	-0.010	1	413
415	1.7248	1.71574	0.04486	0.00012	0.00602	0.0000	2.994	-0.010	1	414
416	1.7641	1.75372	0.04473	0.00014	0.00602	0.0000	2.994	-0.011	1	415
417	1.5728	1.56758	0.04519	0.00009	0.00612	0.0000	2.996	-0.010	1	416

418	-1.8603	-1.86649	0.04442	-0.00017	0.00597	0.000	2.990	-0.012	417
419	-1.5334	-1.54860	0.04523	-0.00000	0.00000	0.000	2.997	-0.009	418
420	-1.4613	-1.45831	0.04540	-0.00000	0.00000	0.000	2.999	-0.009	419
421	-1.1329	-1.13404	0.04586	-0.00000	0.00000	0.000	2.999	-0.009	420
422	-1.1104	-1.10827	0.04548	-0.00000	0.00000	0.000	2.998	-0.009	421
423	-1.4390	-1.40689	0.04549	-0.00000	0.00000	0.000	2.998	-0.009	422
424	-1.3931	-1.39124	0.04551	-0.00000	0.00000	0.000	2.998	-0.009	423
425	-1.6432	-1.64017	0.04540	-0.00000	0.00000	0.000	2.997	-0.009	424
426	-0.8435	-0.84601	0.04615	-0.00004	0.00639	0.000	2.999	-0.006	425
427	-0.7981	-0.80067	0.04618	-0.00004	0.00640	0.000	2.999	-0.006	426
428	-1.0808	-1.08233	0.04591	-0.00005	0.00632	0.000	2.998	-0.009	427
429	-0.9684	-0.97054	0.04603	-0.00005	0.00635	0.000	2.999	-0.008	428
430	-1.2759	-1.27563	0.04568	-0.00006	0.00626	0.000	2.998	-0.009	429
431	-0.8084	-0.80836	0.04618	-0.00005	0.00639	0.000	2.999	-0.006	430
432	-0.8923	-0.89479	0.04614	-0.00004	0.00638	0.000	2.999	-0.007	431
433	-0.5265	-0.52872	0.04637	-0.00003	0.00645	0.000	2.999	-0.007	432
434	-0.8555	-0.85898	0.04613	-0.00004	0.00638	0.000	2.999	-0.007	433
435	-0.6247	-0.62117	0.04631	-0.00003	0.00643	0.000	2.999	-0.004	434
436	-0.6904	-0.69296	0.04627	-0.00003	0.00642	0.000	2.999	-0.004	435
437	-1.0714	-1.07299	0.04592	-0.00005	0.00632	0.000	2.999	-0.009	436
438	-0.7153	-0.71788	0.04625	-0.00003	0.00641	0.000	2.999	-0.005	437
439	-0.6521	-0.65462	0.04629	-0.00003	0.00643	0.000	2.999	-0.004	438
440	-0.3552	-0.35672	0.04655	-0.00002	0.00657	0.000	2.999	-0.001	439
441	-0.3564	-0.35792	0.04655	-0.00002	0.00657	0.000	2.999	-0.001	440
442	-0.7337	-0.73629	0.04622	-0.00004	0.00641	0.000	2.999	-0.005	441
443	-0.0969	-0.09694	0.04651	-0.00000	0.00644	0.000	2.999	-0.005	442
444	-0.3356	-0.33702	0.04646	-0.00002	0.00647	0.000	2.999	-0.001	443
445	-0.5265	-0.52872	0.04637	-0.00003	0.00645	0.000	2.999	-0.002	444
446	-0.4507	-0.45265	0.04641	-0.00002	0.00644	0.000	2.999	-0.002	445
447	-0.1268	-0.12703	0.04651	-0.00000	0.00649	0.000	2.999	-0.000	446
448	-0.1129	-0.11405	0.04651	-0.00000	0.00649	0.000	2.999	-0.000	447
449	-0.5188	-0.52100	0.04638	-0.00002	0.00645	0.000	2.999	-0.002	448
450	-0.1189	-0.11908	0.04651	-0.00000	0.00649	0.000	2.999	-0.000	449
451	0.2843	0.28648	0.04645	0.00002	0.00647	0.000	2.999	-0.001	450
452	-0.1338	-0.13388	0.04651	-0.00000	0.00649	0.000	2.999	-0.000	451
453	0.2876	0.28980	0.04645	0.00002	0.00647	0.000	2.999	-0.001	452
454	0.0023	0.00288	0.04651	0.00000	0.00649	0.000	2.999	-0.000	453
455	0.3096	0.31170	0.04644	0.00002	0.00647	0.000	2.999	-0.001	454
456	0.3255	0.32787	0.04643	0.00002	0.00647	0.000	2.999	-0.001	455
457	0.3646	0.37116	0.04641	0.00002	0.00646	0.000	2.999	-0.002	456
458	0.5051	0.50780	0.04639	0.00002	0.00646	0.000	2.999	-0.002	457
459	0.5366	0.53968	0.04632	0.00003	0.00643	0.000	2.999	-0.004	458
460	0.5714	0.57454	0.04630	0.00003	0.00643	0.000	2.999	-0.004	459
461	0.4852	0.48815	0.04635	0.00003	0.00644	0.000	2.999	-0.003	460
462	0.5275	0.53056	0.04632	0.00003	0.00644	0.000	2.999	-0.003	461
463	0.7849	0.78800	0.04613	0.00004	0.00638	0.000	2.999	-0.007	462
464	0.9030	0.90512	0.04602	0.00004	0.00635	0.000	2.999	-0.008	463
465	0.3711	0.37367	0.04641	0.00002	0.00646	0.000	2.999	-0.002	464
466	0.6005	0.60367	0.04627	0.00003	0.00642	0.000	2.999	-0.004	465
467	1.1269	1.12808	0.04578	0.00008	0.00629	0.000	2.999	-0.009	466
468	1.0962	1.09766	0.04582	0.00005	0.00630	0.000	2.999	-0.009	467
469	1.0306	1.03258	0.04589	0.00005	0.00632	0.000	2.999	-0.009	468
470	1.3910	1.39251	0.04582	0.00005	0.00633	0.000	2.999	-0.009	469
471	0.9842	0.98649	0.04594	0.00005	0.00633	0.000	2.999	-0.009	470
472	1.1549	1.15580	0.04575	0.00006	0.00628	0.000	2.999	-0.009	471
473	1.1275	1.12867	0.04578	0.00006	0.00628	0.000	2.999	-0.009	472
474	1.7252	1.71464	0.04469	0.00014	0.00598	0.000	2.999	-0.011	473
475	0.9706	0.97298	0.04596	0.00005	0.00633	0.000	2.999	-0.009	474
476	1.1506	1.15156	0.04575	0.00006	0.00628	0.000	2.999	-0.009	475
477	1.0575	1.05928	0.04586	0.00005	0.00631	0.000	2.999	-0.009	476
478	1.6490	1.64080	0.04490	0.00012	0.00604	0.000	2.995	-0.010	477

479	1.4309	1.42703	0.04535	0.00008	0.00816	0.000	2.997	-0.009	1	478
480	1.5395	1.53414	0.05215	0.00010	0.00811	0.000	2.996	-0.009	1	479
481	1.5407	1.53532	0.04514	0.00010	0.00811	0.000	2.996	-0.009	1	480
482	1.3486	1.34697	0.04548	0.00007	0.00820	0.000	2.998	-0.009	1	481
483	1.5545	1.54879	0.04511	0.00010	0.00810	0.000	2.996	-0.009	1	482
484	1.6315	1.62380	0.04494	0.00011	0.00805	0.000	2.995	-0.010	1	483
485	1.7366	1.72566	0.04465	0.00014	0.00807	0.000	2.993	-0.011	1	484
486	1.5668	1.56038	0.04508	0.00010	0.00808	0.000	2.996	-0.010	1	485
487	1.7759	1.76357	0.04453	0.00015	0.00793	0.000	2.991	-0.011	1	486
488	1.9916	1.96930	0.04355	0.00028	0.00765	0.001	2.978	-0.015	1	487
489	1.7483	1.73792	0.04461	0.00015	0.00784	0.001	2.992	-0.011	1	488
490	1.7555	1.74390	0.04459	0.00015	0.00795	0.000	2.992	-0.011	1	489
491	2.1761	2.16036	0.04198	0.00055	0.00518	0.006	2.961	-0.023	1	490
492	2.1940	2.15657	0.04176	0.00059	0.00512	0.005	2.934	-0.024	1	491
493	2.1953	2.15775	0.04175	0.00059	0.00511	0.005	2.933	-0.025	1	492
494	2.3440	2.28815	0.03900	0.00121	0.00426	0.025	2.799	-0.039	1	493
495	2.2866	2.23887	0.04030	0.00090	0.00467	0.012	2.874	-0.032	1	494
496	2.1361	2.10200	0.04244	0.00046	0.00532	0.003	2.954	-0.021	1	495
497	2.1692	2.13409	0.04206	0.00053	0.00521	0.004	2.943	-0.023	1	496
498	2.4361	2.36296	0.03581	0.00210	0.00321	0.096	2.502	-0.028	1	497
499	2.4271	2.35595	0.03621	0.00198	0.00334	0.083	2.550	-0.056	1	498
500	2.6346	2.48242	0.01365	0.01170	-0.00576	53.796	#####	#####	9	499

TYPE 1-7 : Pearson's Types

8 : Normal Distribution

9 : Undefined Due to Negative Even Moment(s)

10 : Undefined Due to Negative P.D.F.

TABLE A-2-1

The Number of Hypothetical Subjects in Each of the Success and Failure Groups of Each Binary Item in Degree 3 Case, and the Negative Number (in Parentheses) to be Added to Each Frequency to Give the Corresponding Number of Subjects in Degree 4 Case

Binary Item	Failure Subgroup	Success Subgroup
1	22 (-3)	478 (-4)
2	68 (-1)	432 (-6)
3	100 (-3)	400 (-4)
4	150 (-3)	350 (-4)
5	202 (-3)	298 (-4)
6	246 (-3)	254 (-4)
7	302 (-3)	198 (-4)
8	345 (-3)	155 (-4)
9	399 (-3)	101 (-4)
10	429 (-4)	71 (-3)

TABLE A-3-1
 The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using Four Different Sets of Cutting Points of Frequency Ratio Respectively

METHOD ITEM	TRUE σ_g	DGR. 3				DGR. 4				Normal Approximation N = 500			
		0.15-	0.10-	0.05-	0.01-	0.15-	0.10-	0.05-	0.01-	0.15-	0.10-	0.05-	0.01-
		0.85	0.90	0.96	0.99	0.85	0.90	0.95	0.99	0.85	0.90	0.95	0.99
1	1.5	---	---	1.288 ₃	0.185 ₅	---	---	1.354 ₃	1.558 ₄	---	---	---	---
2	1.0	1.234 ₄	1.315	1.315	0.549	1.057 ₄	1.282	1.128	1.128	1.218 ₄	1.218 ₄	1.381	1.295
3	2.5	2.076 ₄	2.000	2.000	0.621	1.816 ₄	1.786 ₅	1.938	1.969	2.227 ₄	2.227 ₄	2.227 ₄	2.641 ₅
4	1.0	0.836	0.836	0.926	0.766	0.790	0.721	0.812	0.812	0.807	0.807	0.807	0.882
5	1.5	1.281	1.281	1.364	1.010	1.471	1.320	1.320	1.321	1.292 ₃	1.325 ₄	1.668 ₅	1.564
6	1.0	0.782	0.839	0.787	0.909	0.826	0.849	0.890	0.923	0.868	0.892	0.951	0.923
7	2.0	1.530 ₄	1.492	1.451	1.356	1.559 ₅	1.557	1.446	1.404	1.389 ₄	1.348	1.348	1.539
8	1.0	0.815	0.842	0.842	0.929	0.775	0.775	0.775	0.877	0.834	0.834	0.880	0.945
9	2.0	1.330 ₅	1.536	1.593	1.593	1.826 ₅	1.811	1.721	1.721	1.931 ₃	2.100 ₄	2.264	2.264
10	1.0	0.819	0.765	0.773	0.863	0.556	0.556	0.616	0.765	0.606	0.606	0.606	0.796

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE A-3-1
(Continued)

ITEM	METHOD	Normal Approximation N = 2500					Normal Approximation N = 5000				
		0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99		0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	
2	1.0	1.229 _s	1.229 _s	1.301	1.341		0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	
4	1.0	0.886	0.826	0.959	0.959						
6	1.0	0.918	0.922	0.936	0.959		0.878	0.910	0.919	0.937	
8	1.0	0.888	0.888	0.888	0.980						
10	1.0	0.751	0.751	0.751	0.895						

TABLE A-3-2

The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using Four Different Sets of Cutting Points of Frequency Ratio Respectively

METHOD	TRUE b	DGR. 3				DGR. 4				Normal Approximation N = 500			
		0.15	0.10-	0.05-	0.01-	0.15-	0.10-	0.05-	0.01-	0.15-	0.10-	0.05-	0.01-
1	-2.5	---	---	-2.770 ₃	-8.923 ₅	---	---	-2.643 ₃	-2.554 ₄	---	---	---	---
2	-2.0	-1.845 ₄	-1.856	-1.856	-2.701	-1.855 ₄	-1.861	-1.888	-1.888	-1.821 ₄	-1.821 ₄	-1.857	-1.873
3	-1.5	-1.497 ₄	-1.502	-1.502	-1.624	-1.499 ₄	-1.497 ₅	-1.474	-1.478	-1.445 ₄	-1.445 ₄	-1.445 ₄	-1.473 ₅
4	-1.0	-1.097	-1.097	-1.004	-1.031	-1.106	-1.101	-1.001	-1.001	-1.064	-1.064	-1.064	-1.069
5	-0.5	-0.536	-0.536	-0.495	-0.352	-0.503	-0.469	-0.469	-0.466	-0.659 ₃	-0.664 ₄	-0.509 ₅	-0.501
6	0.0	-0.068	-0.078	-0.068	-0.027	-0.049	-0.038	-0.051	-0.038	-0.014	-0.018	-0.062	-0.058
7	0.5	0.444 ₄	0.482	0.476	0.510	0.491 ₅	0.521	0.530	0.520	0.597 ₄	0.520	0.520	0.588
8	1.0	0.917	0.932	0.932	0.963	0.970	0.970	0.970	1.008	1.001	1.001	1.012	1.020
9	1.5	1.403 ₅	1.455	1.464	1.464	1.506 ₅	1.504	1.493	1.493	1.434 ₃	1.453 ₄	1.512	1.512
10	2.0	2.052	2.082	2.076	2.011	2.375	2.375	2.303	2.154	2.285	2.285	2.285	2.127

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE A-4-1

The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained Directly from the Frequency Ratios of True θ , and from Those of the Maximum Likelihood Estimates Using 20 and 16 Intervals Respectively, for Four Different Sets of Cutting Points of Frequency Ratio

METHOD ITEM	TRUE θ	θ				MLE (20 intervals)				MLE (16 intervals)			
		0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99
1	1.5	---	2.250 ₃	2.250 ₃	2.228 ₄	-0.394 ₃	-0.394 ₃	0.876 ₄	0.876 ₄	---	---	1.206 ₃	1.206 ₃
2	1.0	0.646 ₅	0.646 ₅	0.980	0.980	0.315 ₄	1.153	1.137	1.125	0.028 ₃	1.241	1.195	1.166
3	2.5	2.190 ₃	2.623 ₄	2.369 ₅	2.114	2.549 ₄	2.111	1.973	2.007	2.549 ₄	2.111	2.111	2.115
4	1.0	0.779	0.779	0.862	0.879	0.705	0.758	0.794	0.903	0.882	0.882	0.897	1.011
5	1.5	1.205	1.205	1.327	1.480	1.161	1.375	1.297	1.297	1.161	1.284	1.222	1.222
6	1.0	0.734	0.734	0.778	0.836	0.794	0.855	0.835	0.831	0.901	0.915	0.915	0.878
7	2.0	1.611 ₄	1.565	1.532	1.654	1.056 ₅	1.461	1.428	1.493	1.009 ₃	1.486 ₅	1.382	1.476
8	1.0	0.922	0.840	0.943	0.999	0.785	0.785	0.785	0.785	0.749	0.749	0.749	0.749
9	2.0	1.447 ₄	2.092 ₅	1.889	1.889	1.494 ₅	1.494 ₅	1.810	1.810	1.494 ₅	1.494 ₅	1.869	1.869
10	1.0	0.692	0.727	0.727	0.875	0.870	0.820	0.820	0.820	0.459	0.534	0.534	0.534

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE A-4-2

The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained Directly from the Frequency Ratios of True θ , and from Those of the Maximum Likelihood Estimates Using 20 and 16 Intervals Respectively, for Four Different Sets of Cutting Points of Frequency Ratio

METHOD	TRUE θ	θ				MLE (20 intervals)				MLE (16 intervals)			
		0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	-2.5	---	-2.411 ₃	-2.411 ₃	-2.413 ₄	-0.336 ₃	-0.336 ₃	-3.010 ₄	-3.010 ₄	---	---	-2.631 ₃	-2.631 ₃
2	-2.0	-1.866 ₅	-1.866 ₅	-2.023	-2.023	-1.403 ₄	-1.952	-1.953	-1.955	2.567 ₃	-1.906	-1.913	-1.920
3	-1.5	-1.503 ₃	-1.468 ₄	-1.478 ₅	-1.472	-1.475 ₄	-1.482	-1.495	-1.501	-1.475 ₄	-1.482	-1.482	-1.482
4	-1.0	-1.068	-1.068	-0.957	-0.966	-1.073	-1.030	-1.050	-0.956	-1.117	-1.117	-1.119	-0.999
5	-0.5	-0.461	-0.461	-0.442	-0.481	-0.486	-0.479 ₁	-0.492	-0.492	-0.486	-0.440	-0.445	-0.445
6	0.0	-0.003	-0.003	0.001	-0.101	-0.001	-0.012	-0.062	-0.052	-0.162	-0.130	-0.112	-0.095
7	0.5	0.558 ₄	0.582	0.577	0.502	0.489 ₅	0.523	0.518	0.498	0.597 ₃	0.603 ₅	0.580	0.540
8	1.0	1.032	1.020	0.969	0.931	0.959	0.959	0.959	0.959	0.905	0.905	0.905	0.905
9	1.5	1.379 ₄	1.532 ₅	1.514	1.514	1.386 ₅	1.386 ₅	1.511	1.511	1.386 ₅	1.386 ₅	1.507	1.507
10	2.0	2.016	2.117	2.117	2.060	1.998	2.012	2.012	2.012	2.630	2.495	2.495	2.495

The number of intervals used in estimation is shown as a subscript when it is less than 6.

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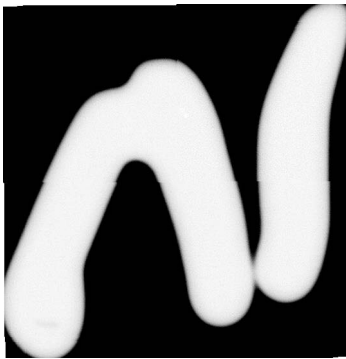
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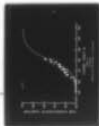
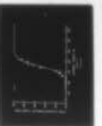
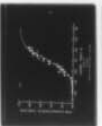
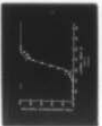
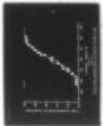
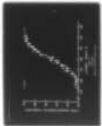
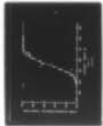
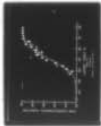
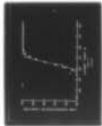
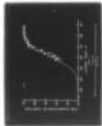
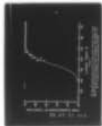
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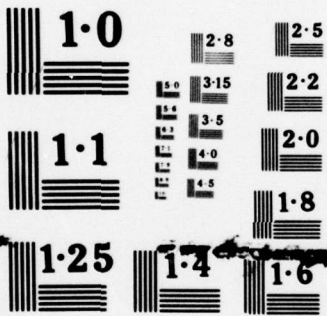
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ERRATA

We have found certain errors in RESEARCH REPORT 77-1,
AD A049 618 .

Kindly insert the attached pages into their proper
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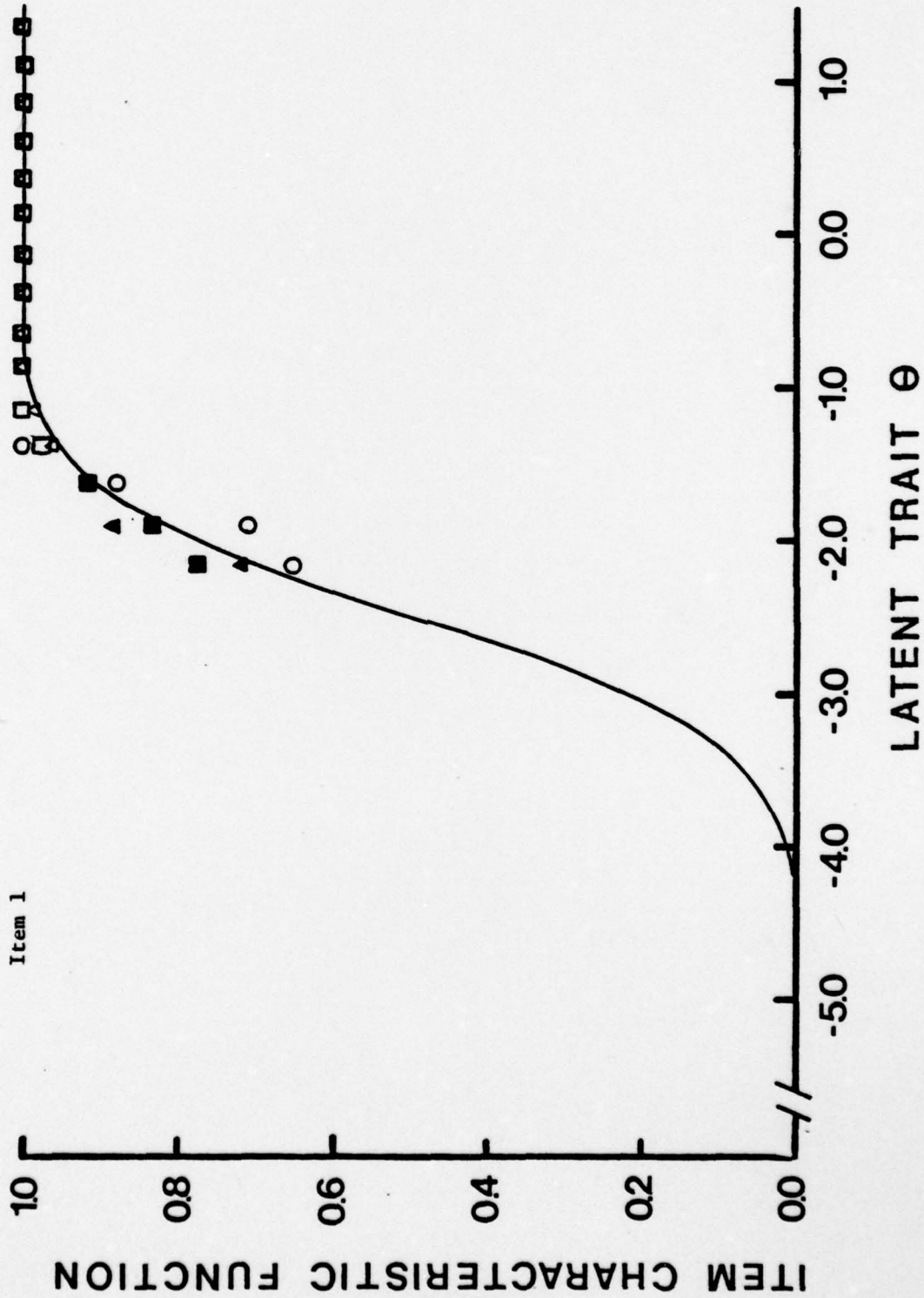
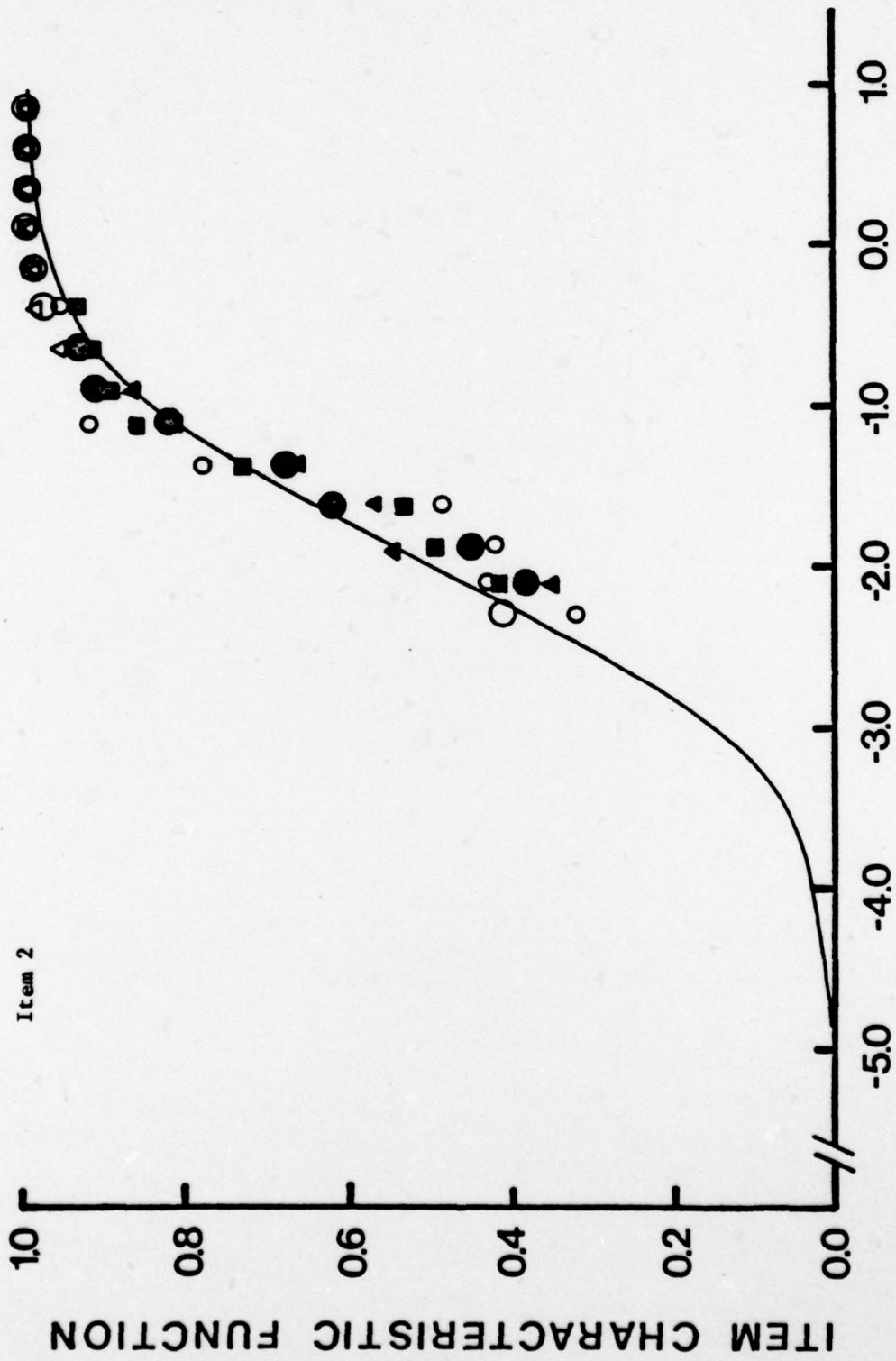


FIGURE 7-9

The true item characteristic function (curve), and the frequency ratio of those who answered the item correctly to the total frequency for each interval using: 2500 θ in Degree 3 Case (triangle), 2465 θ in Degree 4 Case (square), and 500 θ obtained by the Normal Approximation Method (circle).

5 3 0 1 2 7 0 8 2

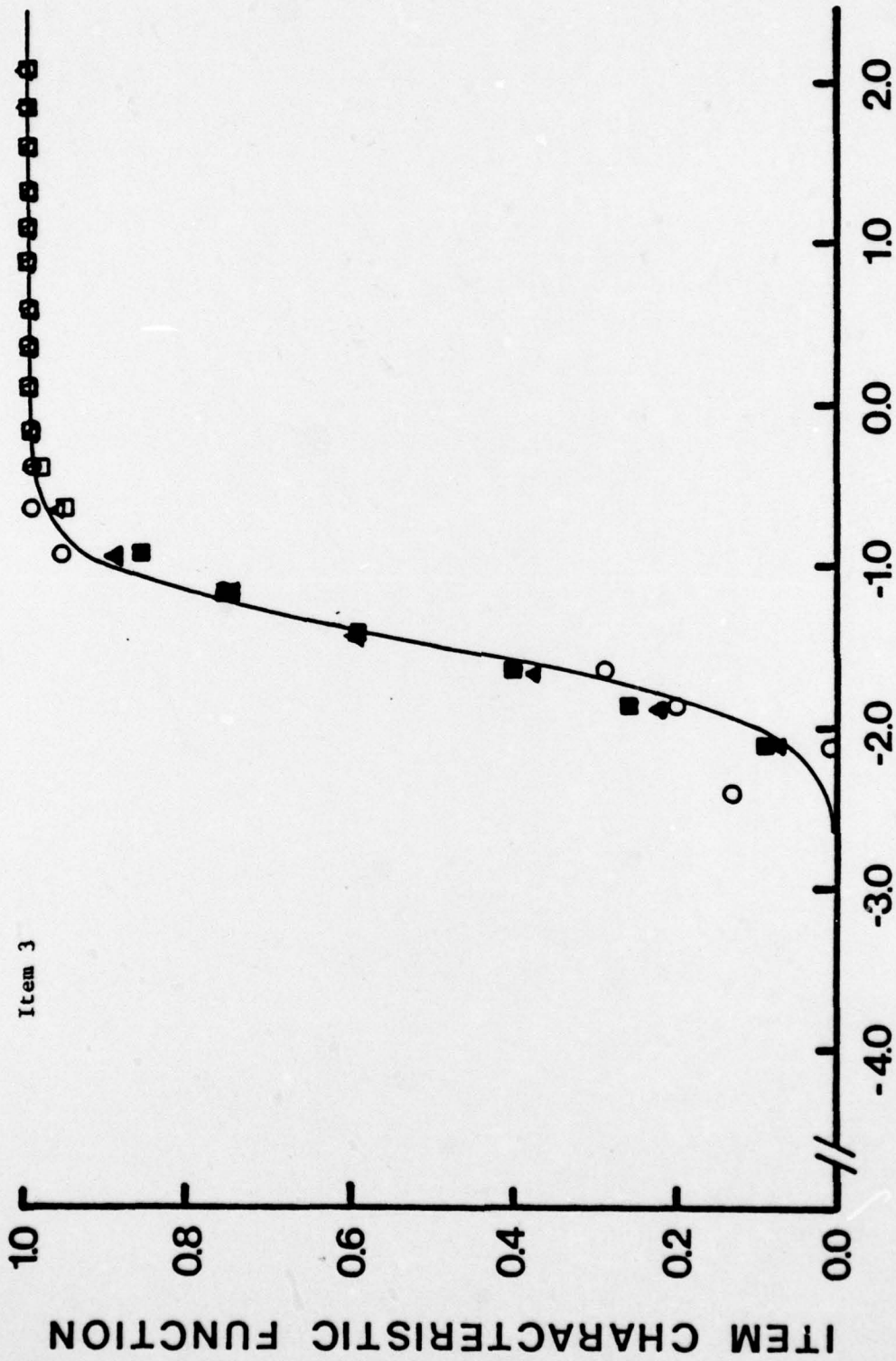


LATENT TRAIT θ

FIGURE 7-9

(Continued)

Addition: the result obtained by using 2500 $\tilde{\theta}$ in the Normal Approximation Method (large circle).



LATENT TRAIT θ

FIGURE 7-9

(Continued)

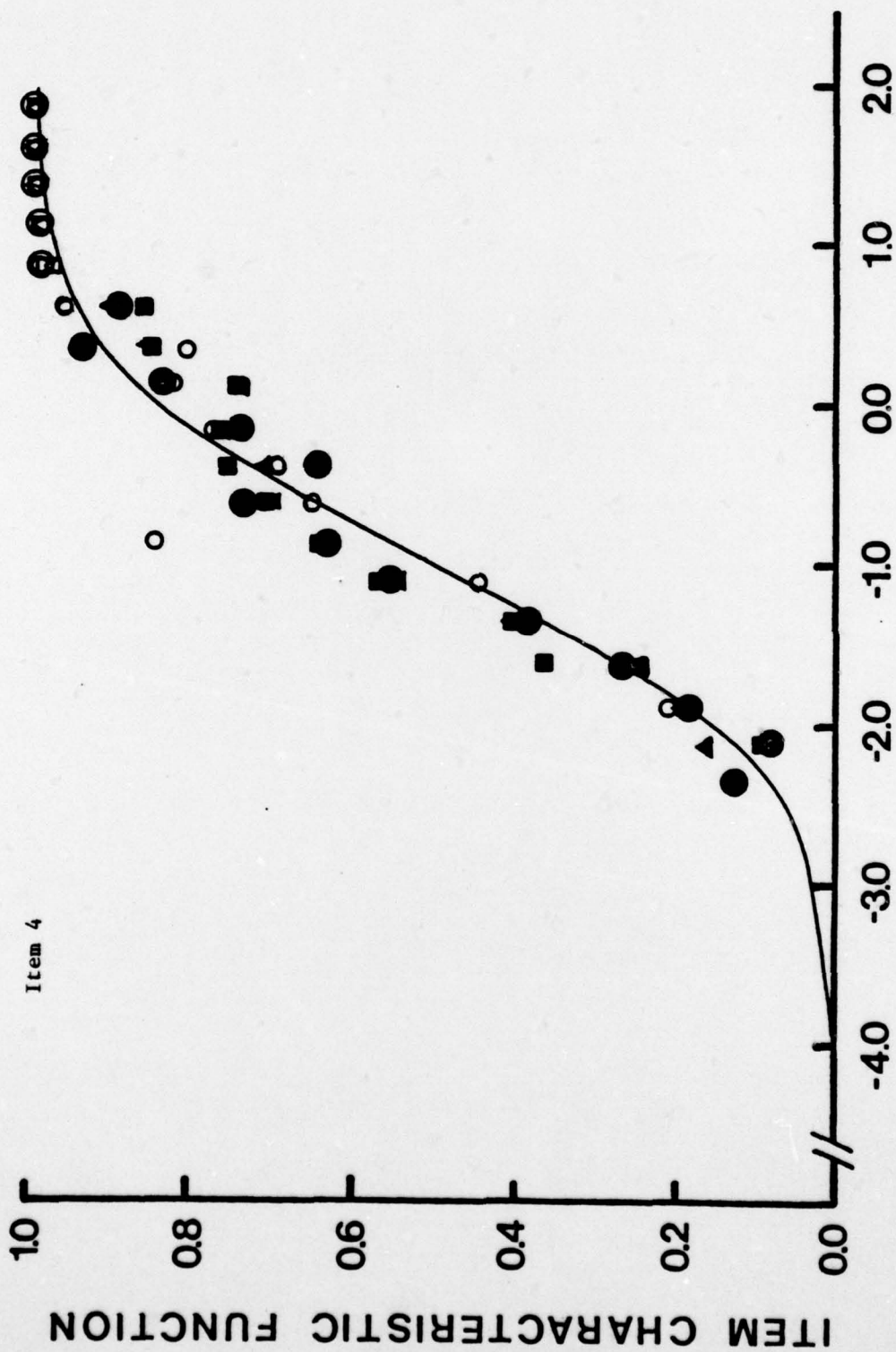


FIGURE 7-9
(Continued)

Addition: the same as Item 2 .

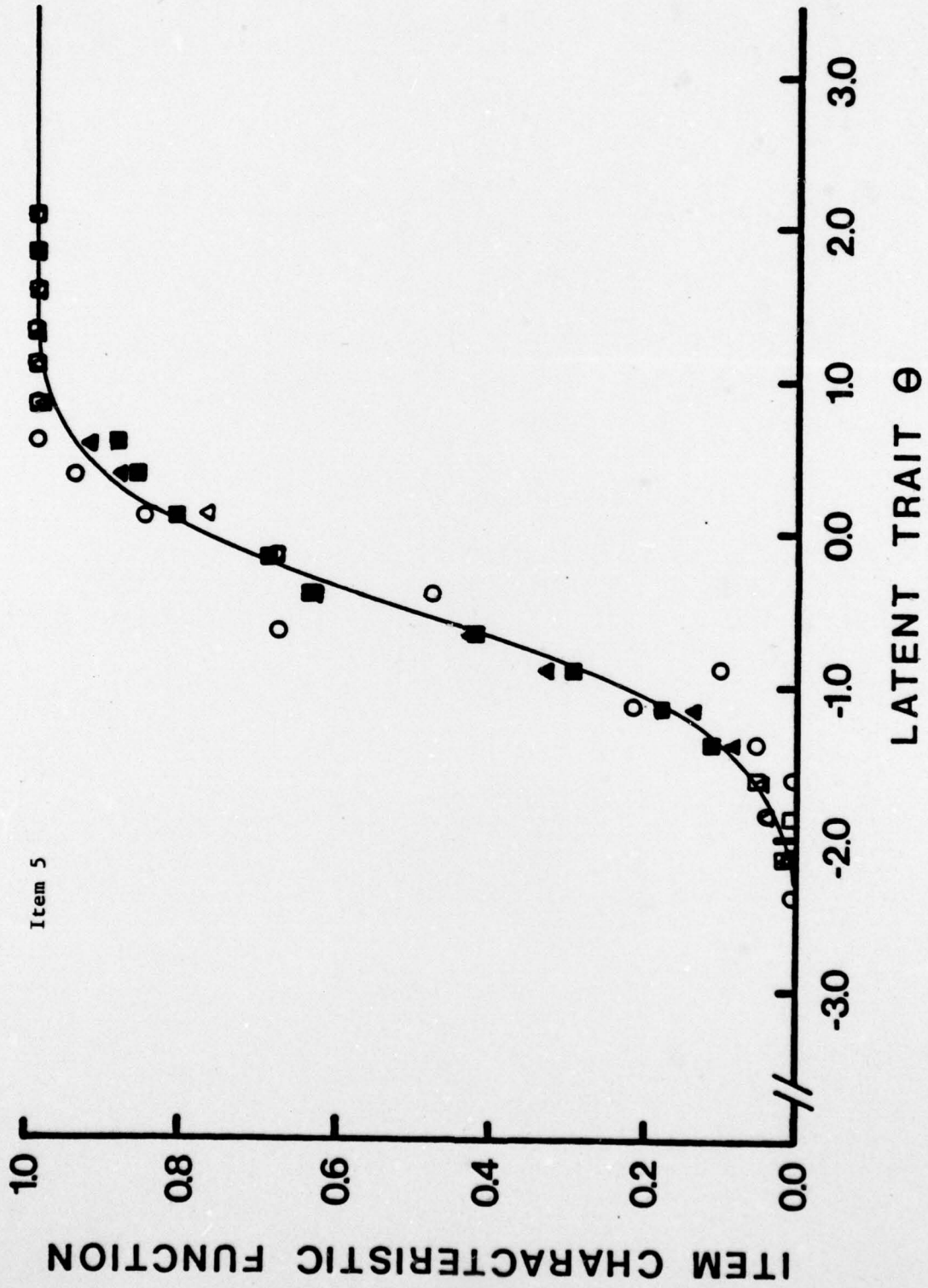
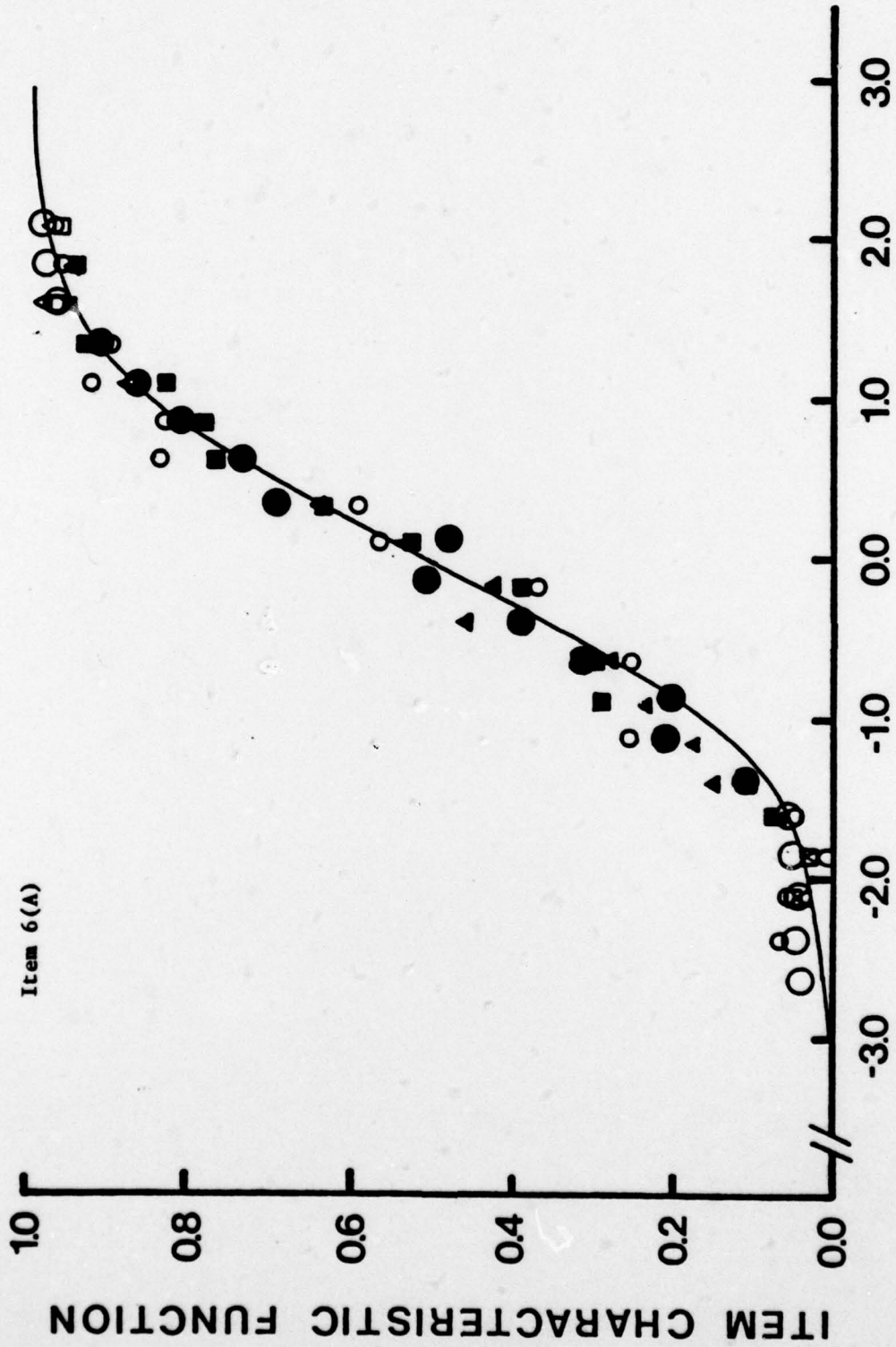


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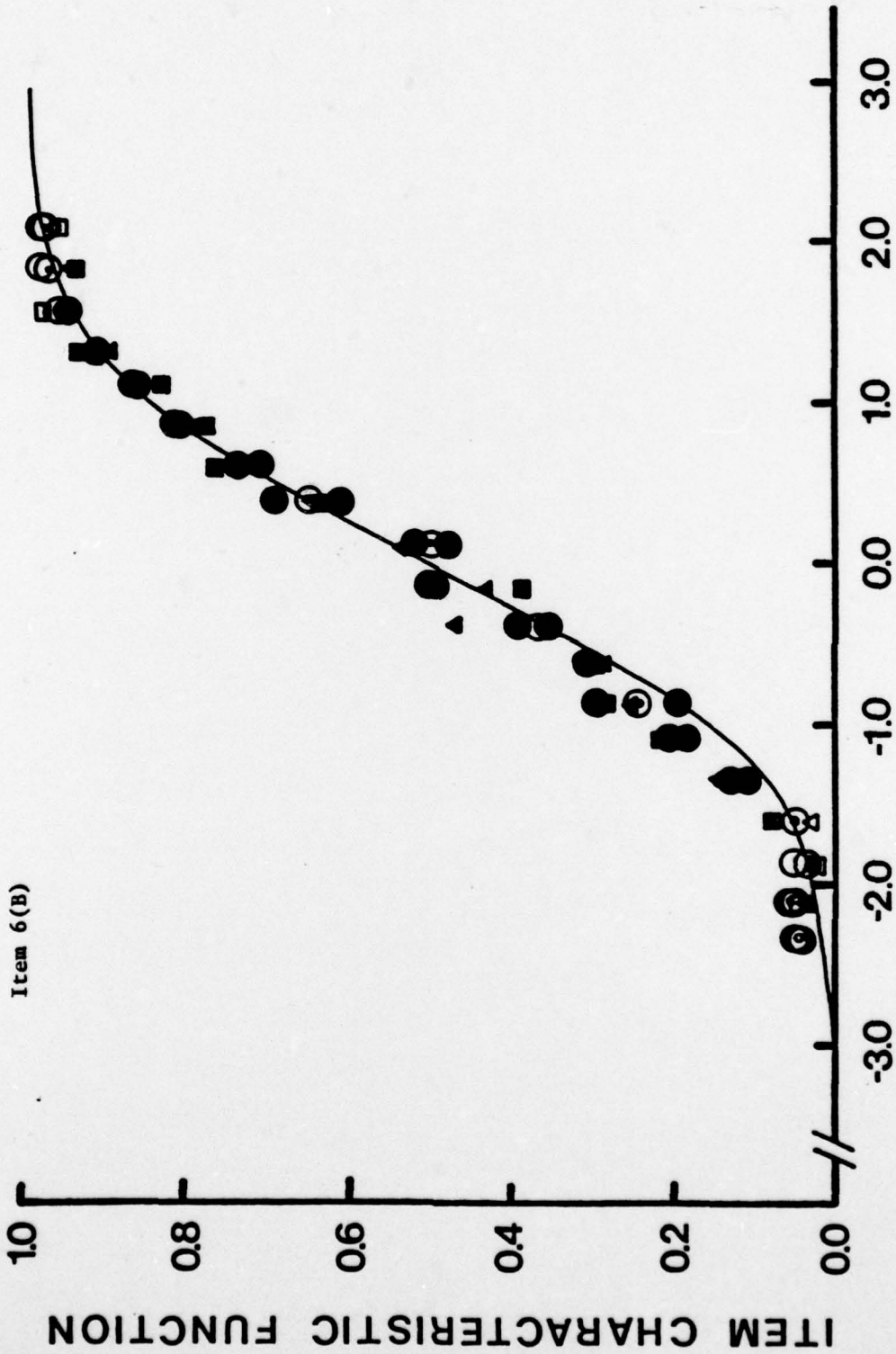
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LATENT TRAIT Θ

FIGURE 7-9
(Continued)

Addition: the same as Items 2 and 4.



LATENT TRAIT θ

FIGURE 7-9

(Continued)

Exclusion: Normal Approximation Method, both the 500 $\tilde{\theta}$ and 2500 $\tilde{\theta}$ cases.
Inclusion: Normal Approximation Method, the other 2500 $\tilde{\theta}$ and 5000 $\tilde{\theta}$ cases.

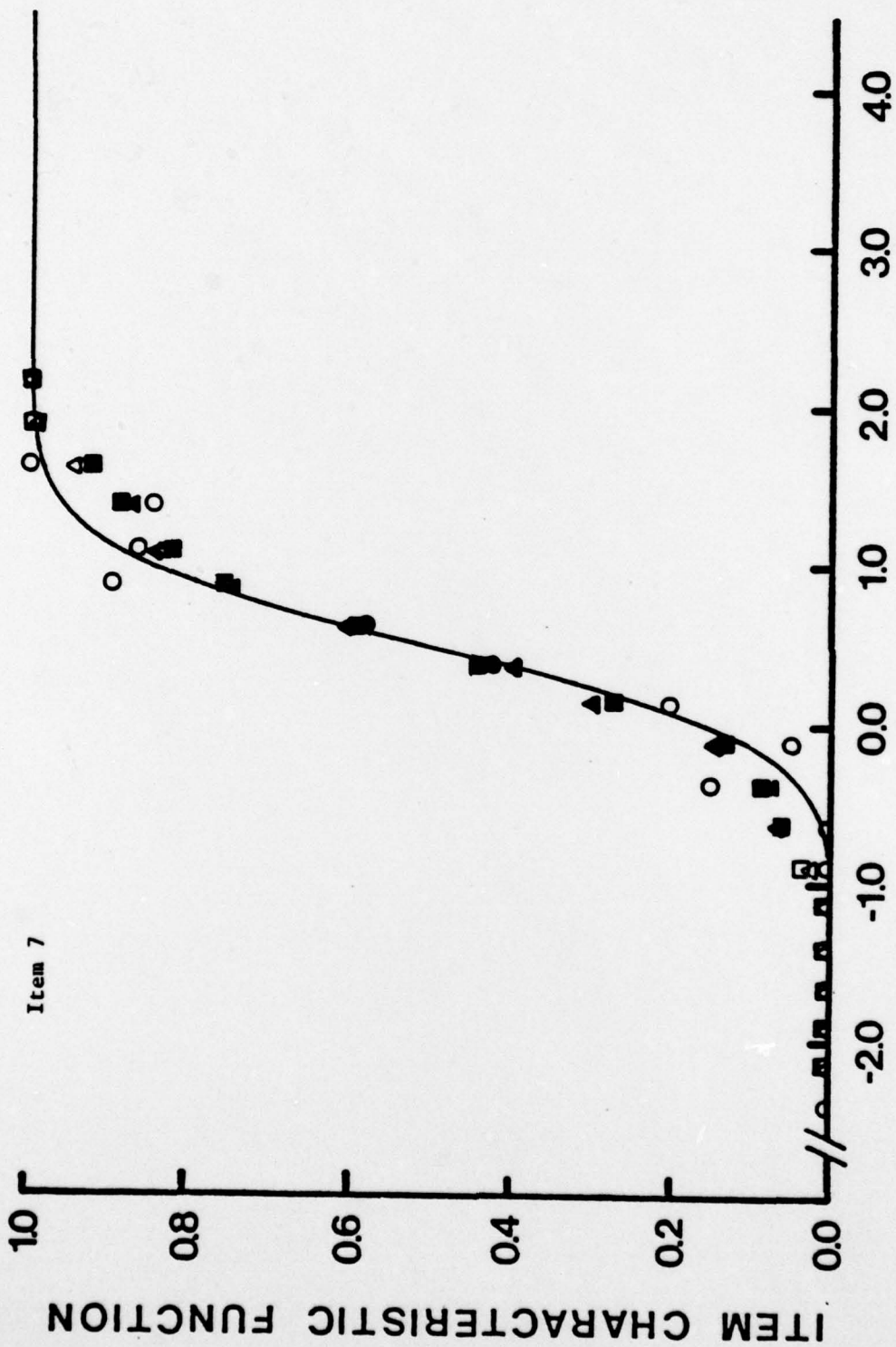
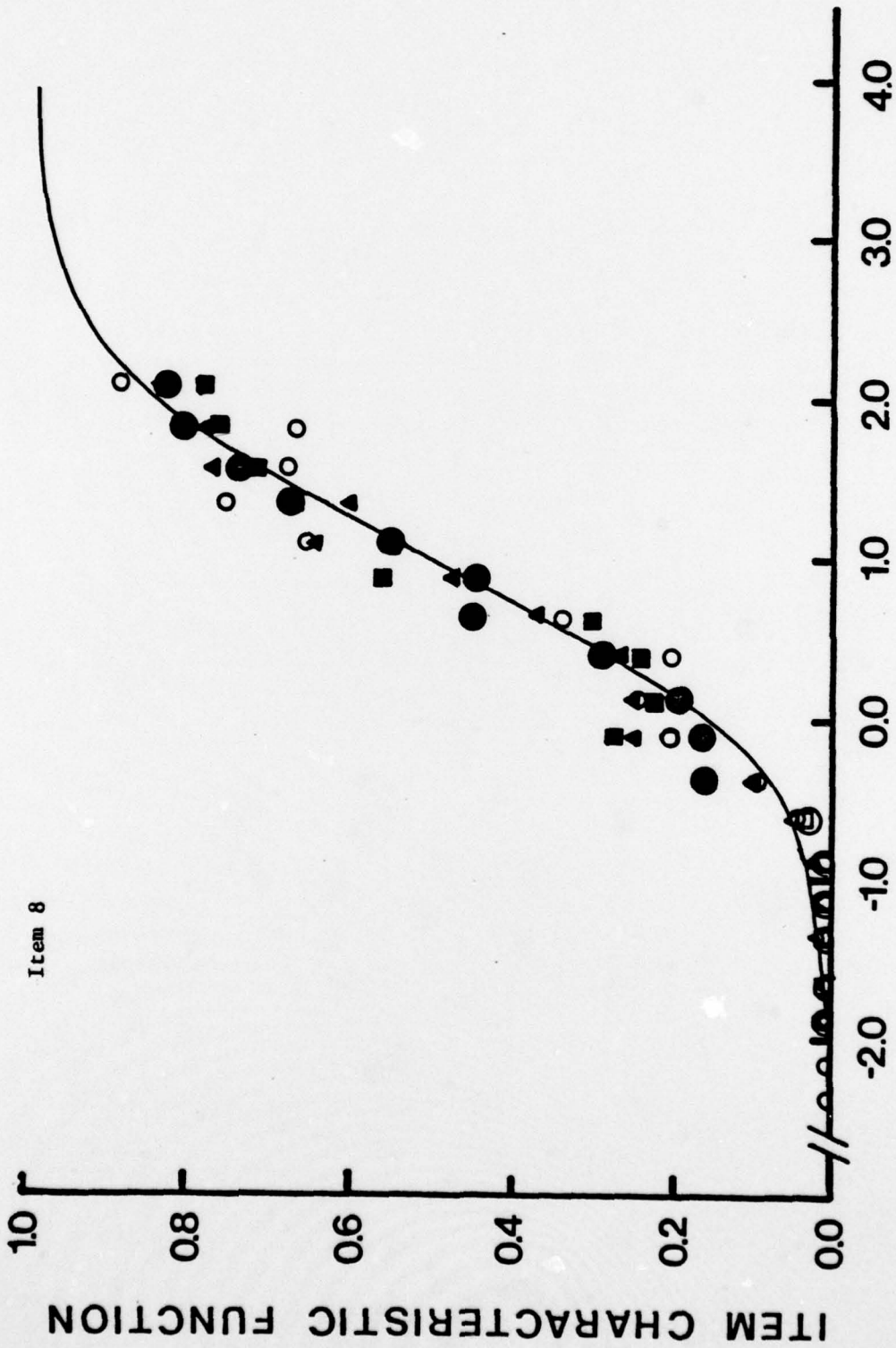


FIGURE 7-9

(Continued)



LATENT TRAIT θ

FIGURE 7-9

(Continued)

Addition: the same as Items 2, 4 and 6(A).

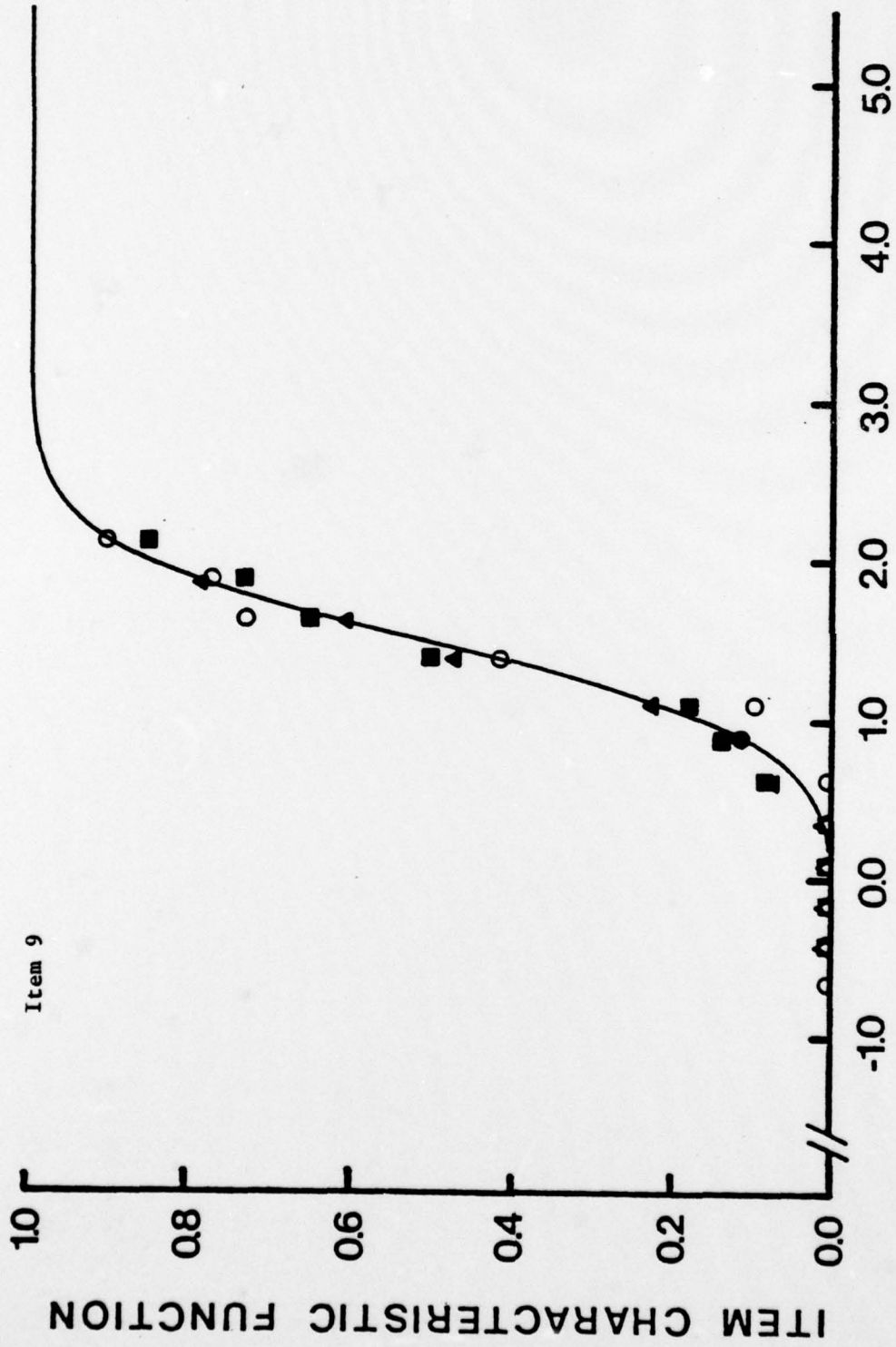
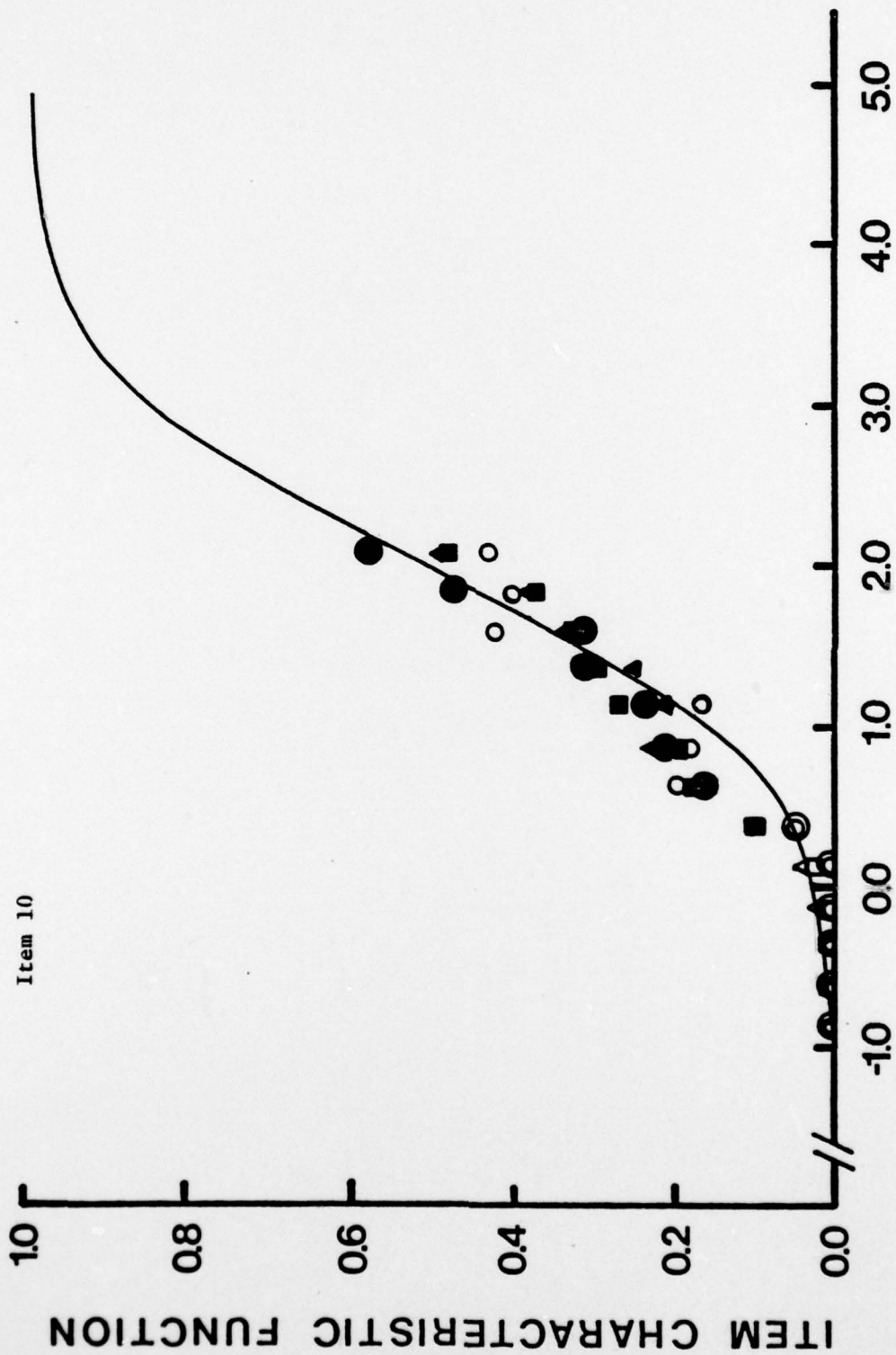


FIGURE 7-9

(Continued)



LATENT TRAIT Θ

FIGURE 7-9

(Continued)

Addition: the same as Items 2, 4, 6(A) and 8.

TABLE 7-1
 The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using the Frequency Ratios between 0.05 and 0.95

ITEM	METHOD	TRUE a_g	(N = 500)		(N = 2500)		(N = 5000)	
			DGR. 3	DGR. 4	NORMAL	NORMAL	NORMAL	NORMAL
1		1.5	1.495 (3)	1.354 (3)	0.602 (2)			
2		1.0	1.230 (7)	1.128 (8)	1.381 (7)	1.301 (7)		
3		2.5	2.108 (6)	1.938 (6)	2.227 (4)			
4		1.0	0.824 (12)	0.812 (12)	0.807 (10)	0.959 (12)		
5		1.5	1.442 (9)	1.320 (9)	1.668 (5)			
6		1.0	0.905 (14)	0.890 (14)	0.951 (11)	0.936 (12)	0.919 (12)	
7		2.0	1.481 (9)	1.446 (10)	1.348 (6)			
8		1.0	0.861 (11)	0.775 (11)	0.880 (10)	0.888 (11)		
9		2.0	1.768 (7)	1.721 (7)	2.264 (6)			
10		1.0	0.657 (8)	0.616 (8)	0.606 (7)	0.751 (7)		

The number of intervals used in each estimation is shown in parentheses.

TABLE 7-2
 The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using the Frequency Ratios between 0.05 and 0.95

ITEM	METHOD	TRUE b_g	DGR. 3		DGR. 4		(N = 500)		(N = 2500)		(N = 5000)	
							NORMAL	NORMAL	NORMAL	NORMAL	NORMAL	NORMAL
1		-2.5	-2.555 (3)	-2.643 (3)	-3.015 (2)							
2		-2.0	-1.841 (7)	-1.888 (8)	-1.857 (7)	-1.831 (7)						
3		-1.5	-1.459 (6)	-1.474 (6)	-1.445 (4)							
4		-1.0	-1.037 (12)	-1.001 (12)	-1.064 (10)	-0.971 (12)						
5		-0.5	-0.471 (9)	-0.469 (9)	-0.509 (5)							
6		0.0	-0.073 (14)	-0.051 (14)	-0.062 (11)	-0.048 (12)	-0.071 (13)					
7		0.5	0.509 (9)	0.530 (10)	0.520 (6)							
8		1.0	0.955 (11)	0.970 (11)	1.012 (10)	0.953 (11)						
9		1.5	1.474 (7)	1.493 (7)	1.512 (6)							
10		2.0	2.238 (8)	2.303 (8)	2.285 (7)	2.031 (7)						

The number of intervals used in each estimation is shown in parentheses.

TABLE A-3-1

The Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using Four Different Sets of Cutting Points of Frequency Ratio Respectively

METHOD ITEM	TRUE a_g	DGR. 3				DGR. 4				Normal Approximation N = 500			
		0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99	0.15- 0.85	0.10- 0.90	0.05- 0.95	0.01- 0.99
1	1.5	---	2.325 ₂	1.495 ₃	1.580 ₅	---	---	1.354 ₃	1.558 ₄	---	---	---	---
2	1.0	1.195 ₅	1.203	1.230	1.185	1.057 ₄	1.282	1.128 ₄	1.128 ₄	1.218	1.218	1.381	1.295
3	2.5	2.020 ₄	1.985 ₅	2.108	2.151	1.816 ₄	1.786 ₅	1.938	1.969	2.227 ₄	2.227 ₄	2.227 ₄	2.641 ₅
4	1.0	0.854	0.827	0.824	0.857	0.790	0.721	0.812	0.812	0.807	0.807	0.807	0.882
5	1.5	1.193 ₅	1.424	1.442	1.387	1.471	1.320	1.320	1.321	1.292 ₃	1.325 ₄	1.668 ₅	1.564
6	1.0	0.863	0.878	0.905	0.903	0.826	0.849	0.890	0.923	0.868	0.892	0.951	0.923
7	2.0	1.570 ₅	1.509	1.481	1.473	1.559 ₅	1.557	1.446	1.404	1.389 ₄	1.348	1.348	1.539
8	1.0	0.810	0.810	0.861	0.983	0.775	0.775	0.775	0.877	0.834	0.834	0.880	0.945
9	2.0	1.987 ₄	1.841	1.768	1.773	1.826 ₅	1.811	1.721	1.721	1.931 ₃	2.100 ₄	2.264	2.264
10	1.0	0.599	0.599	0.657	0.851	0.556	0.556	0.616	0.765	0.606	0.606	0.606	0.796

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE A-3-2

The Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Following the Normal Ogive Model, in Degree 3 and 4 Cases, Together with Those Obtained by the Normal Approximation Method, Using Four Different Sets of Cutting Points of Frequency Ratio Respectively

METHOD ITEM	TRUE b _g	DGR. 3				DGR. 4				Normal Approximation N = 500			
		0.15	0.10	0.05	0.01	0.15	0.10	0.05	0.01	0.15	0.10	0.05	0.01
		0.85	0.90	0.95	0.99	0.85	0.90	0.95	0.99	0.85	0.90	0.95	0.99
1	-2.5	---	-2.372 ₂	-2.555 ₃	-2.523 ₅	---	---	-2.643 ₃	-2.554 ₄	---	---	---	---
2	-2.0	-1.844 ₅	-1.844	-1.841	-1.856	-1.855 ₄	-1.861	-1.888	-1.888	-1.821 ₄	-1.821 ₄	-1.857	-1.873
3	-1.5	-1.477 ₄	-1.475 ₅	-1.459	-1.464	-1.499 ₄	-1.497 ₅	-1.474	-1.478	-1.445 ₄	-1.445 ₄	-1.445 ₄	-1.473 ₅
4	-1.0	-1.049	-1.038	-1.037	-1.044	-1.106	-1.101	-1.001	-1.001	-1.064	-1.064	-1.064	-1.069
5	-0.5	-0.538 ₅	-0.486	-0.471	-0.490	-0.503	-0.469	-0.469	-0.466	-0.659 ₃	-0.664 ₄	-0.509 ₅	-0.501
6	0.0	-0.046	-0.078	-0.073	-0.043	-0.049	-0.038	-0.051	-0.038	-0.014	-0.018	-0.062	-0.058
7	0.5	0.477 ₅	0.512	0.509	0.507	0.491 ₅	0.521	0.530	0.520	0.597 ₄	0.520	0.520	0.588
8	1.0	0.929	0.929	0.955	0.992	0.970	0.970	0.970	1.008	1.001	1.001	1.012	1.020
9	1.5	1.459 ₄	1.483	1.474	1.474	1.506 ₅	1.504	1.493	1.493	1.434 ₃	1.453 ₄	1.512	1.512
10	2.0	2.298	2.298	2.238	2.071	2.375	2.375	2.303	2.154	2.285	2.285	2.285	2.127

The number of intervals used in estimation is shown as a subscript when it is less than 6.