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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br><br>This report briefly summarizes research on the following topics: game theory and energy; scheduling of large research and development programs; bimatrix games; cost/benefit analyses; measures of worth of weapons systems; hybrid primal algorithm; branch and round algorithm. A listing of papers prepared and published is included. |                                                                                          |                                                                      |

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MATHEMATICS OF OPERATIONS RESEARCH

FINAL REPORT

13 January 1978

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OFFICIEAL DEPARTMENT OF THE ARMY POSITION, UNLESS SO DESIGNATED  
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Guillermo Owen was on sabbatical leave for the year beginning mid-August 1975. The bibliography lists papers completed and either published or submitted, under the auspices of the ARO grant. Since his return he has had his research supported through NSF grant MCS-76-23865.

R. M. Thrall has had a number of research interests in the period covered by this grant. All of his work has been supported in part by Rice University; other sponsors in addition to ARO include the NASA Space Shuttle Program, NSF-RANN, The Energy Institute of the University of Houston, and the HEW-RSA (Rehabilitation Service Administration).

T.1. Game Theory and Energy. Initial sponsorship of this work was by NSF-RANN and Part 2 of the final report to NSF, "A Game Theoretic Analysis of Alternate Source Development," by J. Becker, G. Owen and R.M. Thrall initiates a study that has been carried on by Becker with ARO support. This paper was accepted for publication in Volume 24 of Management Science, but has been transferred for inclusion in a volume on energy which will be in the North-Holland/TIMS series: Studies in the Management Sciences. The work has two main thrusts, (a) to develop a game in extensive form which illustrates the potential of game theory in analyzing conflicts in energy related areas with obvious implications for energy policy formulation and, (b) to investigate the possibility of incorporating coalition formation into games in extensive form with special attention to dynamic features and to the stability of coalitions. (A copy of the final report to NSF has been sent to ARO and is available through NTIS as PB-260364, in paper back, \$5.00, and microfiche \$3.00.) Studies along these lines are still underway as a part of the dissertation research of J. Becker.

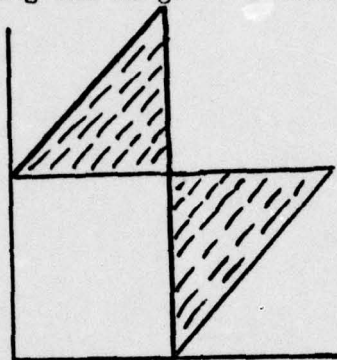
T.2. Scheduling of Large Research and Development Programs. This work was initially sponsored by NASA but the preparation of a technical paper "PLANET: A Simulation Approach to PERT," by K.W. Kennedy and R.M. Thrall was partially supported by this grant. (The standard 92 copies were transmitted to ARO.) This paper has since appeared in Computers and Operations Research, Vol. 4 (1976) pp. 313-325.

The central feature of this research is the development of PLANET (Program Length Analysis and Evaluation Technique). This system is a generalization of PERT which permits inclusion of arbitrary distributions for duration times of activities. This paper was presented (in a paper fair session) at the Philadelphia TIMS/ORSA meeting in April 1976. This presentation generated 26 requests for the report, (these have been fulfilled).

Following an expression of interest in PLANET by Mr. Banesh and others at the Rock Island Arsenal, ARO arranged for me to visit Rock Island in early April. A letter report dated 23 April 1976 discussing this visit was sent to Dr. J. Chandra of the ARO.

T.3. Bimatrix Games. While teaching a game theory course with Luce-Raiffa, Games and Decisions as the text, I noted an error in their analysis of a noncooperative bimatrix game (The Battle of the Sexes). Their Figure 1 on page 93 gave the incorrect bounding parabola for one portion of the space of payoffs. I assigned investigation of this topic

as a Master's thesis to Sergio Batiz, a graduate student attending Rice (with full support from the Mexican Government). He successfully solved the problem and expanded upon it. For this class the joint payoff space is non-convex with linear boundaries except for one quadratic edge. In one interesting special case the region consists of two triangles in a configuration resembling the wings of a butterfly! (see diagram)



Joint Payoff Space

**T.4. Benefit Cost Analysis.** In the last few years I have become interested in benefit/cost analysis, especially for the Research and Development project prioritization. Although none of this research has been sponsored by ARO, I believe the results are of potential interest to the Army and will cite several papers.

The following three papers were prepared without any grant support: (1) Thrall, R.M., "Benefit/Cost Estimation, Alternatives, Requirements, Advantages and Disadvantages," Computer Applications in Health Care Delivery, Charles Weller, M.D., Ed. (1976) pp. 27-35. Stratton Inter-continental Medical Book Corporation, New York; (2) Cardus, David and Robert M. Thrall, "The Concept of Positive Health and the Planning of Health Care Systems," in Health System Modeling and the Information System for the Coordination of Research in Oncology, Proc. of the IIASA Biomedical Conference, Dec. 8-12, 1975, D.D. Venedictov. Ed., pp. 225-238. Also a revised version of this paper titled "Overview: Health and the Planning of Health Care Systems," appeared in Preventative Medicine, v. 6 (1977), pp. 134-142. (3) The paper, Thrall, Robert M. and David Cardus, "Benefit Cost Modeling in the Presence of Multiple Decision Criteria," also appeared in Proc. of the IIASA Biomedical Conference, Dec. 8-12, 1975, D.D. Venedictov ed., pp. 225-237. Thrall, Robert M., Archer McWhorter and Richard Sparkman, "A Benefit/Cost Model for Allocation of Universities' Research Funds," was prepared under the sponsorship of the University of Houston Energy Institute and has not yet been published.

Benefit-cost analysis is related to n-person game theory through the presence of multiple and sometimes conflicting objectives. Recent professional meetings have indicated a growing general interest in the subject of decision making under multiple objectives and it would seem that this problem must arise in Army planning. Research in this topic is now sponsored by HEW-RSA.

T.5. Measures of Worth of Weapons Systems. I have been interested in this topic for several years and have reported on it previously. During this past year I have been consulting with Mr. Herbert Cohen (US Army Concepts Analysis Agency) concerning some of his ideas for generalizing the eigenvalue approach (see Howes, David R., and Robert M. Thrall, "A Theory of Ideal Linear Weights for Heterogeneous Combat Forces," Naval Research Logistics Quarterly, v. 20 (1973) pp. 645-659). I visited CAA twice and held several telephone consultations on this topic. Mr. Cohen presented some of his results at the 1976 Toronto mathematical meetings.

A summary of R. D. Young's Integer Programming research accomplished under this ARO contract is given below.

1. Hybrid Primal Algorithm. This work abstracted from existing primal methods a subset of procedures called the partitioning primal algorithm and showed that this procedure organizes the solution of an integer program into a sequence of solutions of subproblems.

The main result developed in [ 1 ] was that arbitrary methods could be used to solve the subproblems generated by the partitioning primal algorithm. As a consequence, the class of finite primal methods was extended to include, for example, methods that employed combinatorial methods for subproblem solution.

The preceding development also facilitated analysis of the cause of observed inefficiency in primal methods. Inefficiency might stem from (i) the methods used to solve the subproblems generated by the PPA or from (ii) the way the given problem is factored into subproblems by the PPA. An initial investigation of this problem was undertaken in [ 2 ], there it was demonstrated that certain methods of implementing the PPA lead to sequences of subproblems that are apparently quite inefficient. Thus it is very probably not sufficient to find good methods for solving the subproblems to achieve computational success with a primal method. It is necessary to focus attention on the search for good (or best) methods of implementing the partitioning primal algorithm and thereby achieve an efficient factorization of the given problem into a sequence of subproblems.

2. Branch and Round Algorithm. The core concept for this algorithm is location of an integer solution by "rounding" a central point in a polyhedron that contains all feasible points with better objective function value than the best known integer solution. This idea was used as the basis of successful heuristic procedure by Hillier [ 3 ]. In [ 4 ] a finite enumerative algorithm is developed from the principle. Work is in progress to investigate the efficiency of the general method and to investigate special applications to particular problem structures.

3. Geraldine Myers has developed a new general solution method for solution of problems of integer programming over the cone. The method has unique, interesting characteristics and may prove effective against problems that are difficult for existing methods, [5].

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Multilinear Extensions and the Banzhaf Value, Naval Research Logistics Quarterly, 22, 741-752, 1975.

Accepted for publication, but not yet in print:

Computation of Equilibrium Price and Quantity Vectors, to appear in Convex Analysis, Springer Verlag, June 1977.

Values of Market Games without Side Payments, submitted to Mathematics of Operations Research, 1976

A Game-Theoretic Analysis of Alternate Energy Source Development, to appear in Management Science, 1978. (with Thrall and Becker)

Publications cont'd

R. M. Thrall

1973 A Theory of Ideal Linear Weights for Heterogeneous Combat Forces, Naval Logistics Quarterly, V. 20, No. 4, 1973 (with D. Howes).

1976 PLANET - see copy

R. D. Young

1975 The Eclectic Primal Algorithm: A Method that Employs Decomposition and Hybrid Subproblem Solution Techniques, Mathematical Programming, 9, 294-312 (1975).