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A SHORT TABLE OF
LANCHESTER-CLIFFORD-SCHLÄFLI FUNCTIONS

by
James G. Taylor
and
Gerald G. Brown

October 1977

NAVAL POSTGRADUATE SCHOOL
Monterey, California

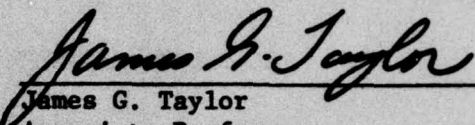
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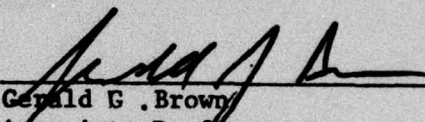
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This research has been partially supported by the U.S. Army Research Office, Durham, North Carolina with R&D Project No. 1L161102BH57-05 Math (funded under MIPR No. ARO 22-77) and partially by the Office of Naval Research.

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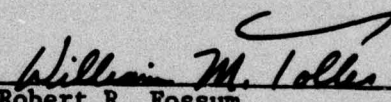

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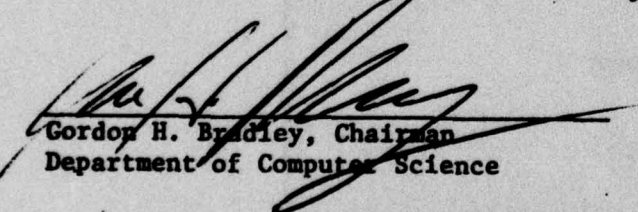

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) NPS55-77-424	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) A Short Table of Lanchester-Clifford-Schläfli Functions,	5. TYPE OF REPORT & PERIOD COVERED (9) Technical rept.,	
7. AUTHOR(s) (10) James G./Taylor Gerald G./Brown		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		8. CONTRACT OR GRANT NUMBER(s) (15) MIPR-AR0-22-77
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office Durham, North Carolina		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (16) 1L161102BH57 (17) 05
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE (11) Oct 1977
		13. NUMBER OF PAGES (12) 64p.
		16. SECURITY CLASS. (of this report) Unclassified
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		18a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Lanchester Theory of Combat Special Functions Combat Modelling Deterministic Combat Attrition Attrition Modelling Lanchester-Clifford-Schläfli Functions Combat Dynamics		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report contains a reduced set of tables of Lanchester-Clifford-Schläfli (LCS) functions. A companion report contains a more extensive (and currently the most extensive available) set of tables of the LCS functions. These functions may be used to analyze Lanchester-type combat between two homogeneous forces modelled by power attrition-rate coefficients with "no effect." Theoretical background for the LCS functions is given, as well as a narrative description of the physical circumstances under which the associated		

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Lanchester-type combat model may be expected to be applicable. Numerical examples are given to illustrate the use of the LCS functions for analyzing "aimed-fire" combat modelled by the power attrition-rate coefficients with "no offset." Our results and these tabulations allow one to study this particular variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

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This research was partially supported by the U. S. Army Research Office, Durham, R&D Project No. 1L161102BH57-05 Math (funded under MIPR No. ARO 22-77) and partially by the Office of Naval Research.

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1. Introduction

Lanchester-type* differential-equation combat models are an important tool for analyzing many important problems of military operations research. In such a combat model, a so-called attrition-rate coefficient represents the fire effectiveness of a particular weapon-system type against a particular target type, i.e. the weapon-system type's effective firepower against such a target. Time-dependent attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. Thus, we see that time-dependent attrition-rate coefficients are important (and, in fact, essential [4-6]) for the quantitative analysis of hypothetical combat.

Militarily realistic computer-based Lanchester-type models of quite complex military systems have been developed for almost the entire spectrum of combat operations, from combat between battalion-sized units to theater-level operations. Nevertheless, a simple combat model may yield a clearer understanding of significant interrelationships that are difficult to perceive in a more complex model, and such insights can subsequently provide valuable guidance for more detailed computerized investigations. In this report we consider such a simplified variable-coefficient Lanchester-type model of combat between two homogeneous forces.

For this variable-coefficient Lanchester-type model of combat between two homogeneous forces, different functional forms for the attrition-rate coefficients lead to different mathematical functions being involved in representing and computing the force-level trajectories. In a previous paper [5] we have discussed the plausibility of the hypothesis that except for the special case of a constant ratio of attrition-rate coefficients,

*So-called after pioneering work of F. W. Lanchester [3].

the solutions to such differential equations cannot be represented in term of "elementary" functions of analysis. Thus, new transcendental functions arise in the study of combat modelled with time-dependent attrition-rate coefficients. In particular, we have previously introduced [5-6] so-called Lanchester-Clifford-Schläfli (LCS) functions for analyzing combat modelled with power attrition-rate coefficients with "no offset" (see Section 3 below).

In the Appendix to this report is contained a reduced set of tables for the LCS functions: it contains tables of five-decimal-place values of the hyperbolic-like LCS functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ (see Section 4 below) for 11 fractional values of α (see Section 6 below). A companion report [8] contains the most extensive set of tables currently available. The main body of this report provides the theoretical and modelling background for the use of these tables. In particular, we examine a model of a constant-speed attack on a static defensive position and show how associated range-dependent kill rates give rise to time-dependent attrition-rate coefficients with "no offset." Numerical computations are presented to illustrate the use of the LCS functions for analyzing such "aimed-fire" combat. As a consequence of the availability of these tables, one can now study this variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

2. Variable-Coefficient Lanchester-Type Equations of Modern Warfare.

We consider combat between two homogeneous forces modelled by the following variable-coefficient Lanchester-type [3] (see [4,5]) equations of modern warfare

$$(2.1) \quad \begin{cases} \frac{dx}{dt} = -a(t)y & \text{with } x(0) = x_0, \\ \frac{dy}{dt} = -b(t)x & \text{with } y(0) = y_0, \end{cases} \quad (2.1)$$

where $t = 0$ denotes the time at which the battle begins, $x(t)$ and $y(t)$ denote the numbers of X and Y at time t , and $a(t)$ and $b(t)$ denote time-dependent Lanchester attrition-rate coefficients, which represent the effectiveness of each side's fire. These coefficients depend on variables such as force separation, tactical posture of targets, rate of target acquisition, firing rate, etc. (see [4-7] for further details). Variable attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. In any analysis of combat, moreover, we should use the above equations (2.1) only for x and $y > 0$ and, for example, set $dx/dt = 0$ when $x = 0$, since negative force levels have no physical meaning.

Mathematically, we assume that the attrition-rate coefficients $a(t)$ and $b(t)$ are defined, positive, and continuous for $t_0 < t < +\infty$ with $t_0 \leq 0$. We also assume that $a(t)$ and $b(t) \in L(t_0, T)$ for any finite $T \geq t_0$. We further take $a(t)$ and $b(t)$ to be given in the form

$$a(t) = k_a g(t), \quad \text{and} \quad b(t) = k_b h(t), \quad (2.2)$$

where k_a and k_b are positive constants chosen so that $a(t)/b(t) = k_a/k_b$ when $g(t) \equiv h(t)$. We introduce the combat-intensity parameter λ_I and the relative-fire-effectiveness parameter λ_R defined by

$$\lambda_I = \sqrt{k_a k_b}, \quad \text{and} \quad \lambda_R = k_a/k_b. \quad (2.3)$$

From our assumptions about $a(t)$ and $b(t)$, it follows that, for example, $a(t) \notin L(t_0, T)$ implies $\int_{t_0}^T a(t) dt = +\infty$.

The X force level as a function of time may be represented as [5,6]

$$x(t) = x_0 \{C_Y(0)C_X(t) - S_Y(0)S_X(t)\} - y_0 \sqrt{\lambda_R} \{C_X(0)S_X(t) - S_X(0)C_X(t)\}, \quad (2.4)$$

where the hyperbolic-like general Lanchester functions (GLF) $C_X(t)$ and $S_X(t)$ are linearly-independent solutions to the X force-level equation

$$\frac{d^2 x}{dt^2} - \left\{ \frac{1}{a(t)} \frac{da}{dt} \right\} \frac{dx}{dt} - a(t)b(t)x = 0, \quad (2.5)$$

with initial conditions

$$\begin{aligned} C_X(t_0) &= 1, & S_X(t_0) &= 0, \\ \{1/a(t_0)\} dC_X/dt(t_0) &= 0, & \{1/a(t_0)\} dS_X/dt(t_0) &= 1/\sqrt{\lambda_R}. \end{aligned} \quad (2.6)$$

Here t_0 denotes the largest finite time at which $a(t)$ or $b(t)$ ceases to be defined, positive, or continuous. The Y force level as a function of time is given by a similar expression, with $C_Y(t)$ and $S_Y(t)$ being analogously defined for the corresponding Y force-level equation.

It is sometimes convenient to introduce the new independent variable τ defined by

$$\tau = \int_{t_0}^t \sqrt{a(s)b(s)} ds. \quad (2.7)$$

It is readily seen that the transformation $\tau = \tau(t)$ is well defined and invertible. Let us denote $\tau(0)$ as τ_0 . We observe that $t_0 \leq 0$ implies that $\tau_0 \geq 0$. If we denote the "average intensity of combat" as $\sqrt{a(t)b(t)}$, then

$$\sqrt{a(t)b(t)} t = \left\{ \left(\frac{1}{t} \right) \int_0^t \sqrt{a(s)b(s)} ds \right\} t = \tau - \tau_0. \quad (2.8)$$

The substitution (2.7) transforms (2.5) into

$$\frac{d^2 x}{d\tau^2} - \left(\frac{1}{2} \right) \left\{ \frac{d}{d\tau} \ln R(\tau) \right\} \frac{dx}{d\tau} - x = 0, \quad (2.9)$$

with initial conditions

$$x(\tau_0) = x_0, \quad \text{and} \quad \{1/\sqrt{R(\tau_0)}\} dx/d\tau(\tau_0) = -y_0,$$

where $R(\tau) = a(t)/b(t)$.

3. Combat Modelled with Power Attrition-Rate Coefficients.

The above equations (2.1) basically apply to "aimed-fire" combat when target-acquisition times do not depend on the numbers of targets available (see [5,6] for further details). A large class of tactical situations of interest can be modelled with the following general power attrition-rate coefficients [5-7]

$$a(t) = k_a(t + C)^\mu, \quad \text{and} \quad b(t) = k_b(t + C + A)^\nu, \quad (3.1)$$

where A and $C \geq 0$. We will call A the offset parameter, since it allows us to model (with μ and $\nu \geq 0$) battles between opposing weapon systems with different maximum effective ranges (see [5,6]). We will call C the starting parameter, since it allows us to model (again, with μ and $\nu \geq 0$) battles that begin within the maximum effective ranges of the two opposing systems. We observe that for the general power attrition-rate coefficients (3.1) we have $t_0 = -C$, and μ and ν must be > -1 in order that $a(t)$ and $b(t) \in L(t_0, T)$.

The above nomenclature is motivated and possible applications of our work are indicated by considering S. Bonder's model of the constant-speed attack on a static defensive position (see [4-7] for further details)

$$\frac{dx}{dt} = -\alpha(r)y, \quad \text{and} \quad \frac{dy}{dt} = -\beta(r)x, \quad (3.2)$$

where r denotes the range between opposing forces, and $\alpha(r)$ and $\beta(r)$ denote range-dependent attrition-rate coefficients. Range is related to time by

$$r(t) = R_0 - vt, \quad (3.3)$$

where R_0 denotes the opening range of battle and $v > 0$ denotes the constant attack speed. For example, let us consider the constant-speed attack of a homogeneous Y force against the static defensive position of a homogeneous X force. Figure 1 diagrammatically portrays this situation.

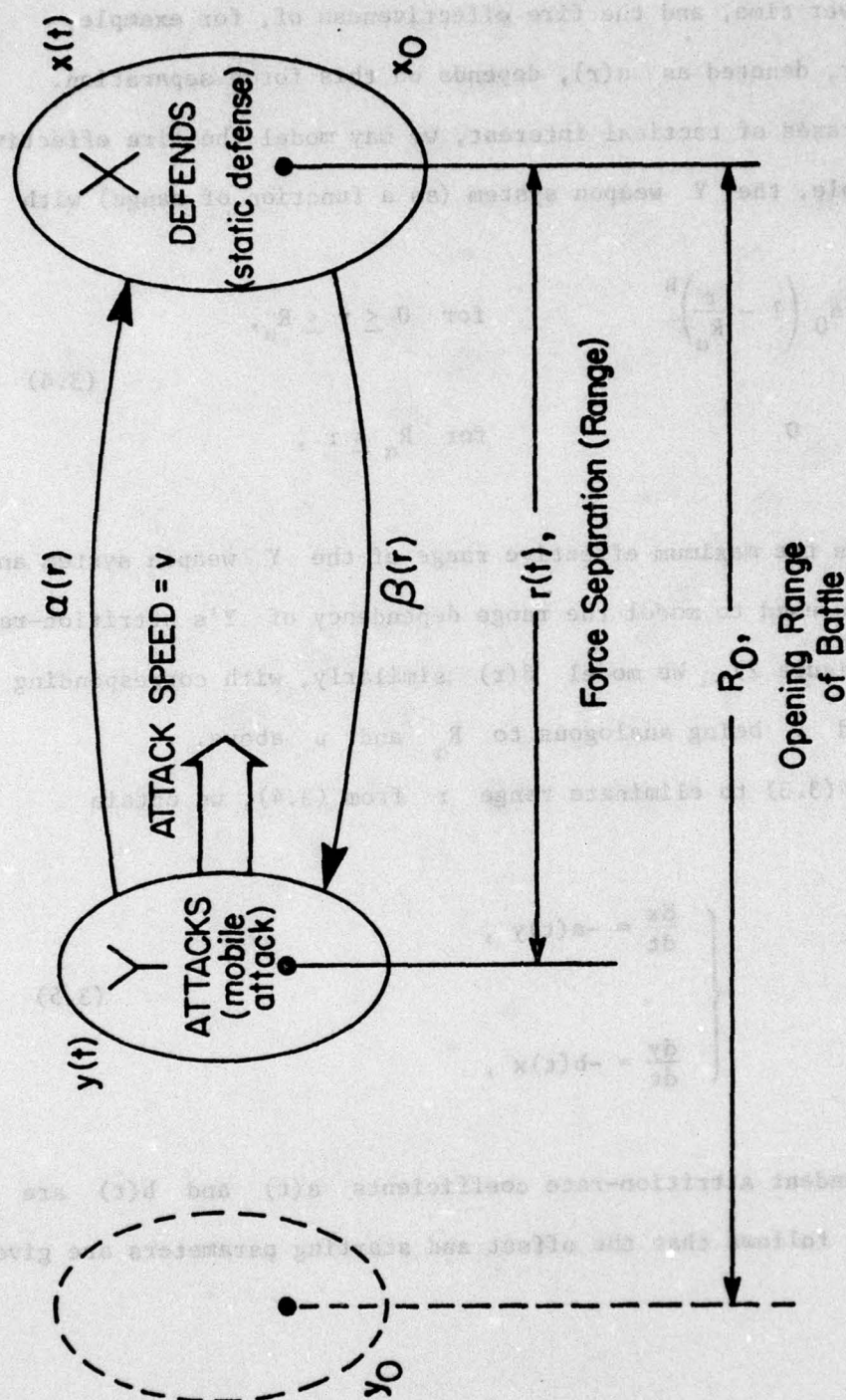


Figure 1. Diagram of Bonder's constant-speed attack model.
Force separation, $r(t)$, is given by $r(t) = R_0 - vt$.

The basic idea is that force separation, i.e. range between the opposing forces, changes over time, and the fire effectiveness of, for example, a single Y firer, denoted as $\alpha(r)$, depends on this force separation.

In many cases of tactical interest, we may model the fire effectiveness of, for example, the Y weapon system (as a function of range) with

$$\alpha(r) = \begin{cases} \alpha_0 \left(1 - \frac{r}{R_\alpha}\right)^\mu & \text{for } 0 \leq r \leq R_\alpha, \\ 0 & \text{for } R_\alpha \leq r, \end{cases} \quad (3.4)$$

where R_α denotes the maximum effective range of the Y weapon system and $\mu \geq 0$. Here μ is used to model the range dependency of Y's attrition-rate coefficient (see Figure 2). We model $\beta(r)$ similarly, with corresponding quantities R_β and ν being analogous to R_α and μ above.

If we use (3.3) to eliminate range r from (3.4), we obtain

$$\begin{cases} \frac{dx}{dt} = -a(t)y, \\ \frac{dy}{dt} = -b(t)x, \end{cases} \quad (3.5)$$

where the time-dependent attrition-rate coefficients $a(t)$ and $b(t)$ are given by (3.1). It follows that the offset and starting parameters are given by

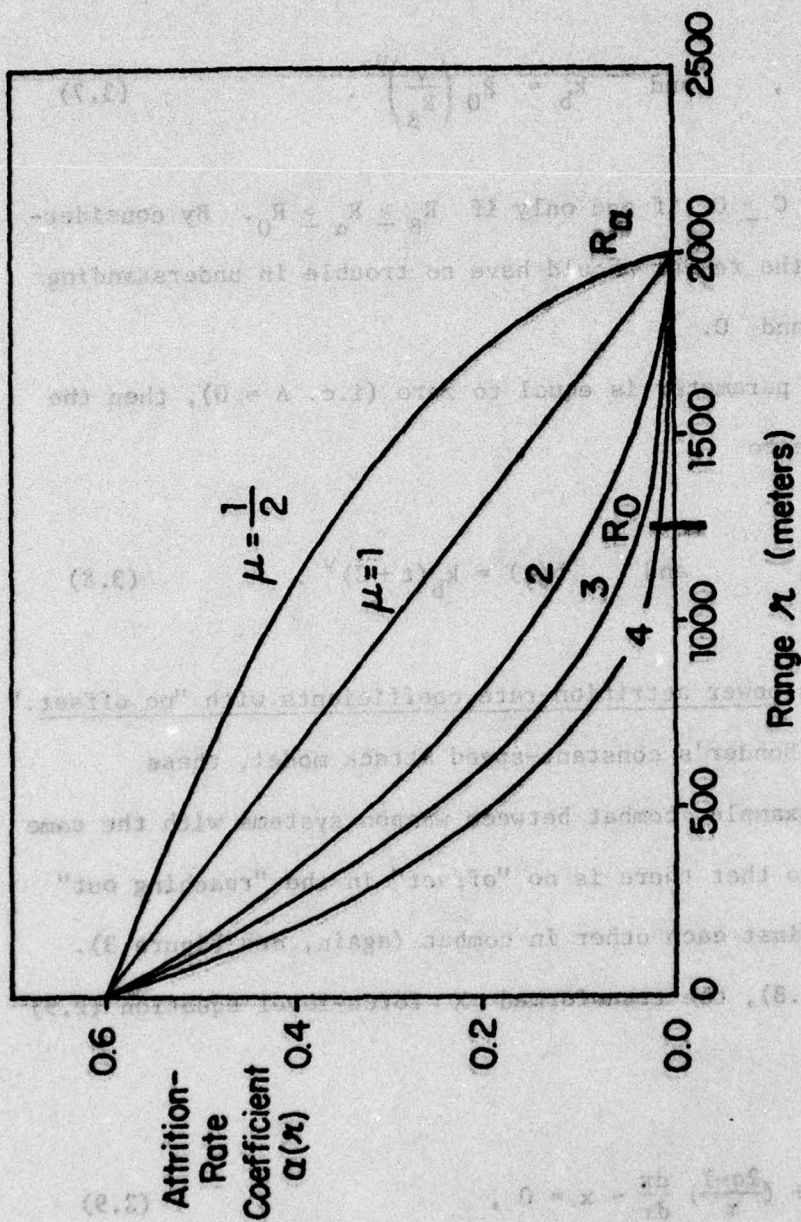


Figure 2. Dependence of Y's attrition-rate coefficient $\alpha(r)$ on the exponent μ with the maximum effective range of the weapon system and kill rate at zero range held constant. [NOTES: 1. The maximum effective range of the system is denoted as $R_\alpha = 2000$ meters. 2. $\alpha(0) = \alpha_0 = 0.6X$ casualties/(unit time $\times$ number of Y firers) denotes the weapon-system kill rate for Y at zero force separation (range). 3. The opening range of battle is denoted as $R_0 = 1250$ meters and (as shown) $R_0 < R_\alpha$.]

$$A = \left(\frac{R_\beta - R_\alpha}{v} \right), \quad \text{and} \quad C = \left(\frac{R_\alpha - R_0}{v} \right), \quad (3.6)$$

and that

$$k_a = \alpha_0 \left(\frac{v}{R_\alpha} \right)^\mu, \quad \text{and} \quad k_b = \beta_0 \left(\frac{v}{R_\beta} \right)^\nu. \quad (3.7)$$

We observe that A and $C \geq 0$ if and only if $R_\beta \geq R_\alpha \geq R_0$. By considering (3.6) and Figure 3, the reader should have no trouble in understanding our terminology for A and C .

When the offset parameter is equal to zero (i.e. $A = 0$), then the coefficients (3.1) reduce to

$$a(t) = k_a (t+C)^\mu, \quad \text{and} \quad b(t) = k_b (t+C)^\nu. \quad (3.8)$$

We will refer to (3.8) as power attrition-rate coefficients with "no offset."

As we have seen above in Bonder's constant-speed attack model, these coefficients model, for example, combat between weapon systems with the same maximum effective range so that there is no "offset" in the "reaching out" of the weapon systems against each other in combat (again, see Figure 3).

For these coefficients (3.8), the transformed X force-level equation (2.9) becomes

$$\frac{d^2 x}{d\tau^2} + \left(\frac{2q-1}{\tau} \right) \frac{dx}{d\tau} - x = 0, \quad (3.9)$$

with initial conditions

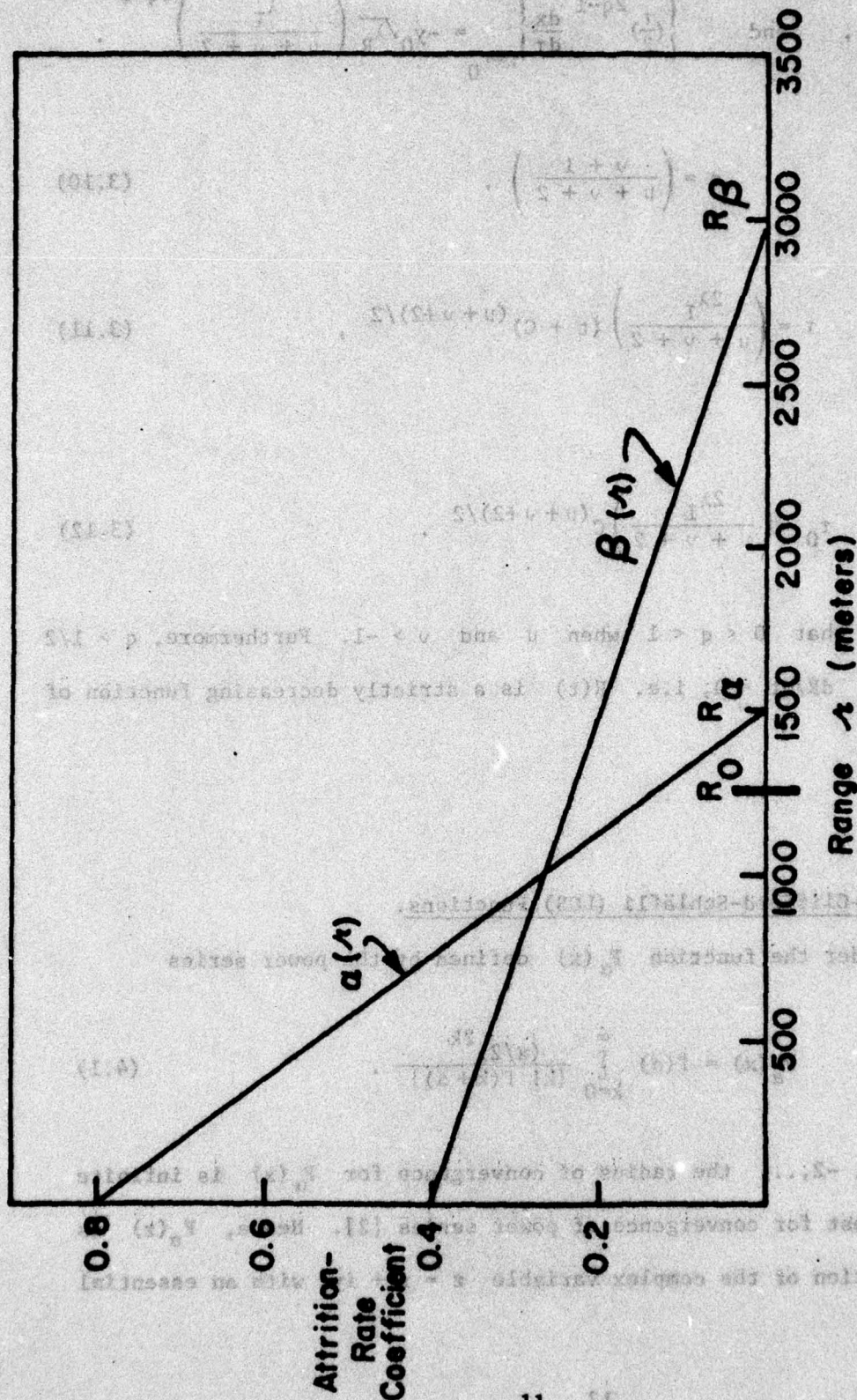


Figure 3. Explanation of the offset parameter A and the starting parameter C for power attrition-rate coefficients modelling constant-speed attack. [NOTES: 1. The maximum effective ranges of the X and Y weapon systems are denoted as R_α and R_β , respectively. 2. The opening range of battle is denoted as R_0 and (as shown) $R_0 < \min(R_\alpha, R_\beta)$. 3. The offset parameter is given by $A = (R_\beta - R_\alpha)/v$. 4. The starting parameter is given by $C = (R_\alpha - R_0)/v$.]

$$x(\tau_0) = x_0, \quad \text{and} \quad \left\{ \left(\frac{\tau}{2} \right)^{2q-1} \frac{dx}{d\tau} \right\}_{\tau=\tau_0} = -y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1}.$$

Here

$$q = \left(\frac{\nu + 1}{\mu + \nu + 2} \right), \quad (3.10)$$

$$\tau = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) (t + C)^{(\mu + \nu + 2)/2}, \quad (3.11)$$

and

$$\tau_0 = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) C^{(\mu + \nu + 2)/2}. \quad (3.12)$$

Let us observe that $0 < q < 1$ when μ and $\nu > -1$. Furthermore, $q > 1/2$ if and only if $dR/dt < 0$, i.e. $R(t)$ is a strictly decreasing function of time.

4. Lanchester-Clifford-Schlöfli (LCS) Functions.

Consider the function $F_\alpha(x)$ defined by the power series

$$F_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2k}}{\{k! \Gamma(k+\alpha)\}}. \quad (4.1)$$

For $\alpha \neq 0, -1, -2, \dots$ the radius of convergence for $F_\alpha(x)$ is infinite by the ratio test for convergence of power series [2]. Hence, $F_\alpha(z)$ is an entire function of the complex variable $z = x + iy$, with an essential

singularity at the point at infinity. Now consider the function $H_\alpha(x)$ defined by the infinite series

$$H_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2(k+\alpha)}}{\{k! \Gamma(k+\alpha+1)\}}. \quad (4.2)$$

Observing that

$$H_\alpha(x) = (1/\alpha)(x/2)^{2\alpha} F_{\alpha+1}(x), \quad (4.3)$$

we see that for $\alpha > 0$ the infinite series (4.2) is uniformly convergent on compact subsets of the complex plane. From (4.3) one can readily deduce the recursive relation

$$F_\alpha(x) = F_{\alpha+1}(x) + \left\{ \frac{(x/2)^2}{\alpha(\alpha+1)} \right\} F_{\alpha+2}(x). \quad (4.4)$$

We will call the functions $F_\alpha(x)$ and $H_\alpha(x)$ Lanchester-Clifford-Schlöfli (LCS) functions (see Note 10 on pp. 66-67 of [5]). Other properties are readily deduced and are given in Table I.

The function $F_\alpha(x)$ satisfies the linear second-order ordinary differential equation

$$\frac{d^2 F_\alpha}{dx^2} + \left(\frac{2\alpha-1}{x} \right) \frac{dF_\alpha}{dx} - F_\alpha = 0, \quad (4.5)$$

with initial conditions

Table I. Properties of the LCS Functions $F_\alpha(x)$ and $H_\alpha(x)$.

$$1. \quad dF_\alpha/dx = (x/2)^{1-2\alpha} H_\alpha(x)$$

$$2. \quad dH_\alpha/dx = (x/2)^{2\alpha-1} F_\alpha(x)$$

$$3. \quad F_\alpha(x)F_{1-\alpha}(x) - H_\alpha(x)H_{1-\alpha}(x) = 1 \quad \forall x$$

where α is not an integer (including zero)

$$4. \quad F_\alpha(x=0) = 1$$

$$5. \quad H_\alpha(x=0) = 0 \quad \text{for } \alpha > 0$$

$$6. \quad dF_\alpha/dx(x=0) = 0$$

$$7. \quad \{(x/2)^{1-2\alpha} dH_\alpha/dx\}_{x=0} = 1$$

$$8. \quad F_{1/2}(x) = \cosh x$$

$$9. \quad H_{1/2}(x) = \sinh x$$

(4.10)

$$F_\alpha(0) = 1, \quad \text{and} \quad \frac{dF_\alpha}{dx}(0) = 0,$$

while $H_\alpha(x)$ satisfies

$$\frac{d^2 H_\alpha}{dx^2} - \left(\frac{2\alpha-1}{x}\right) \frac{dH_\alpha}{dx} - H_\alpha = 0, \quad (4.6)$$

with initial conditions

$$H_\alpha(0) = 0, \quad \text{and} \quad \left\{ \left(\frac{x}{2}\right)^{1-2\alpha} \frac{dH_\alpha}{dx} \right\}_{x=0} = 1.$$

Thus, $\{F_\alpha, H_{1-\alpha}\}$ is a fundamental system of solutions to

$$\frac{d^2 F}{dx^2} + \left(\frac{2\alpha-1}{x}\right) \frac{dF}{dx} - F = 0, \quad (4.7)$$

with Wronskian $W(F_\alpha, H_{1-\alpha}) = (x/2)^{1-2\alpha}$. It follows that the GLF for the X and Y force-level equations for combat modelled with the attrition-rate coefficients (3.8) are given by

$$C_X(t) = F_q(\tau(t)), \quad S_X(t) = \left(\frac{\lambda_I}{\mu + v + 2}\right)^{2q-1} H_p(\tau(t)), \quad (4.8)$$

$$C_Y(t) = F_p(\tau(t)), \quad S_Y(t) = \left(\frac{\lambda_I}{\mu + v + 2}\right)^{1-2q} H_q(\tau(t)), \quad (4.9)$$

where $p = 1-q$. If we define

$$T_{\alpha}(x) = H_{1-\alpha}(x)/F_{\alpha}(x), \quad (4.10)$$

then

$$T_X(t) = \frac{S_X(t)}{C_X(t)} = \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \frac{H_p(\tau(t))}{F_q(\tau(t))}, \quad (4.11)$$

or

$$T_X(t) = \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} T_q(\tau(t)), \quad (4.12)$$

where $T_X(t)$ denotes a hyperbolic-like GLF, which corresponds to the hyperbolic tangent. Observing that for $\mu, \nu > -1$, $\lim_{\tau \rightarrow +\infty} \tau(t) = +\infty$, we see that $T_{\alpha}(x)$ is a strictly increasing function of x on the interval $[0, +\infty)$ and

$$0 \leq T_{\alpha}(x) < \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \quad \text{for } 0 \leq x < +\infty, \quad (4.13)$$

with

$$\lim_{x \rightarrow +\infty} T_{\alpha}(x) = \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}, \quad (4.14)$$

since by the results of Taylor and Comstock [7] the parity-condition parameter $Q^* = Q^*(\mu, \nu, C = 0)$ is given by

$$\lim_{t \rightarrow +\infty} T_X(t) = \frac{1}{Q^*(\mu, \nu, 0)} = \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \frac{\Gamma(p)}{\Gamma(q)}. \quad (4.15)$$

We recall that Taylor and Comstock [7] have introduced the so-called parity-condition parameter Q^* as the value (or range of such values) for the initial condition Q to the initial-value problem

$$\begin{cases} \frac{dE_X^-}{dt} = -\frac{1}{\sqrt{\lambda_R}} a(t) E_Y^- & \text{with } E_X^-(t_0) = 1, \\ \frac{dE_Y^-}{dt} = -\sqrt{\lambda_R} b(t) E_X^- & \text{with } E_Y^-(t_0) = Q, \end{cases} \quad (4.16)$$

such that $E_X^-(t; Q^*)$ and $E_Y^-(t; Q^*) > 0$ for all $t \geq t_0$. In other words, Q^* is the value of Q in (4.16) above such that neither E_X^- nor E_Y^- ever become zero. In this case, both $E_X^-(t; Q^*)$ and $E_Y^-(t; Q^*)$ are positive, strictly decreasing functions, similar to decreasing exponentials. Thus, we may call Q^* "the Y equivalent of an X force of unit strength," since the forces are "at parity," with neither force being annihilated in finite time. Taylor and Comstock have shown that for either $a(t) \notin L(0, +\infty)$ or $b(t) \notin L(0, +\infty)$, then Q^* is unique and given by

$$\lim_{t \rightarrow +\infty} \frac{S_X(t)}{C_X(t)} = \frac{1}{Q^*}. \quad (4.17)$$

The significance of the parity-condition parameter Q^* is that it allows us to predict force annihilation as the following theorem shows.

THEOREM 1 (Taylor and Comstock [7]): Assume that either $a(t) \notin L(0, +\infty)$ or $b(t) \notin L(0, +\infty)$. Then the X force will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \sqrt{\lambda_R} \left\{ \frac{C_X(0) - Q^* S_X(0)}{Q^* C_Y(0) - S_Y(0)} \right\}. \quad (4.18)$$

5. Use of LCS Functions for Analyzing Combat.

The Lanchester-Clifford-Schläfli (LCS) functions $F_\alpha(x)$ and $H_\alpha(x)$ are useful for analyzing "aimed-fire" combat (see Section 3 above) modelled with the power attrition-rate coefficients with "no offset" (3.8), which we rewrite here as

$$a(t) = k_a (t + C)^\mu, \quad \text{and} \quad b(t) = k_b (t + C)^\nu. \quad (5.1)$$

In other words, the LCS functions arise in solving the differential combat model (2.1) with attrition-rate coefficients (5.1). In order that both $a(t)$ and $b(t) \in L(t_0, T)$, we must have μ and $\nu > -1$. Military situations modelled by these equations have been discussed in Section 3 above, e.g. combat between two weapon systems with the same maximum effective range. For such combat, the LCS functions may be used to

- (1) compute force-level declines,
- (2) predict force annihilation,
- and (3) predict the time of force annihilation.

Let us now see how the LCS functions may be used to obtain the above information about force-level declines and force-annihilation prediction. According to (2.4), (4.8), and (4.9) above, the X force level is given by

$$x(t) = x_0 \{ F_p(\tau_0) F_q(\tau(t)) - H_q(\tau_0) H_p(\tau(t)) \} \\ - y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \{ F_q(\tau_0) H_p(\tau(t)) - H_p(\tau_0) F_q(\tau(t)) \}, \quad (5.2)$$

where q is given by (3.10), $p = 1-q$, and $\tau(t)$ is given by (3.11), which we rewrite as

$$\tau(t) = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) (t + C)^{(\mu + \nu + 2)/2}, \quad (5.3)$$

The time to annihilate the X force* is determined by $x(t_a^X) = 0$, and it follows that

$$T_q(\tau(t_a^X)) = \frac{x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0)}, \quad (5.4)$$

where from (4.10)

$$T_q(\tau(t)) = H_p(\tau(t)) / F_q(\tau(t)), \quad (5.5)$$

and we recall that $p + q = 1$. It follows that the time to annihilate X , t_a^X , is given by

* If we multiply the first equation of (2.1) by y , the second by x , add, and integrate, we obtain

$$x(t) y(t) = x_0 y_0 - \int_0^t \{a(s) y^2(s) + b(s) x^2(s)\} ds,$$

which shows that $x(t)$ and $y(t)$ can have at most one finite zero. Hence, if $x(t_a^X) = 0$, then we know that $y(t) > 0$ for all $t \geq 0$.

$$t_a^X = \tau^{-1} \left\{ T_q^{-1} \left[\frac{x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0)} \right] \right\}. \quad (5.6)$$

Taylor and Comstock [7] have shown that $T_q(\tau)$ is strictly increasing and satisfies (see also (4.12) above)

$$0 \leq T_q(\tau) < \Gamma(p)/\Gamma(q), \quad (5.7)$$

where $p = 1-q$. It follows that in order for X to be annihilated in finite time, the right-hand side of (5.4) must be less than $\Gamma(p)/\Gamma(q)$. Let us observe that for $t_0 = -C = 0$, (5.4) simplifies to

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0 \sqrt{\lambda_R}} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q}. \quad (5.8)$$

Thus, we have proved the following theorem concerning force-annihilation prediction.

THEOREM 2: Consider combat between two homogeneous forces modelled by (2.1) with power attrition-rate coefficients (5.1). Assume that μ and $\nu > -1$ and that the above equations hold for all time. Then the X force will be annihilated in finite time if and only if

$$\Gamma(q) \left\{ x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0) \right\} \\ < \Gamma(p) \left\{ x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0) \right\}, \quad (5.9)$$

where $q = (\nu + 1)/(\mu + \nu + 2)$ and $p = 1 - q$. For $t_0 = 0$ (i.e. $C = 0$ so that $\tau_0 = 0$), X will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)} \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p}. \quad (5.10)$$

6. Tabulation of LCS Functions.

This report contains a reduced set of tables of the Lanchester-Clifford-Schläfli functions. The Appendix contains tables of five-decimal-place values of the hyperbolic-like LCS functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for various values of the argument x , namely $x = 0.00$ (0.01) 2.00 (0.1) 10.0, and $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 3/7$, and $4/7$. These values of the index α correspond to $\mu, \nu = 0, 1, 2$, and 3 in (3.8) and allow one to analyze, for example, a basic spectrum of range capabilities for weapon systems in the constant-speed-attack model of Section 3. These tables have been calculated by the recursive means given in Section 8 of [5]. A more extensive tabulation (namely, for $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 2/7, 3/7, 4/7, 5/7, 4/9, 5/9, 3/11, 5/11, 6/11, 8/11, 5/13, 8/13, 5/17, 12/17, 5/21$, and $16/21$ corresponding to $\mu, \nu = 0, 1/4, 1/2, 1, 1\frac{1}{2}, 2, 3$)

is to be found in a companion report [8]. This companion report contains the most extensive set of tables of the Lanchester-Clifford-Schläfli functions currently available.

A representative tabulation of the hyperbolic-like LCS functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ is given in, for example, Tables 8A and 8B of the Appendix for $\alpha = 3/5$. The values of the argument x are the same as those used for the tabulation of the hyperbolic functions by Abramowitz and Stegun [1]. We observe from Table 8B and (4.13) that the limiting value of $T_{\alpha}(x)$ as $x \rightarrow +\infty$ (here $\alpha = 3/5$) is quickly reached, with three-decimal-place accuracy already attained for $x = 4.5$. Moreover, the use of these tables (specifically, Tables 8A and 8B of the Appendix) for combat analysis is illustrated in the next section.

7. Numerical Examples

In this section we examine a couple of numerical examples to show some of the insights that may be gained into the dynamics of combat between two homogeneous forces from our results (see also [6]). These examples illustrate the use of the LCS functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for analyzing "aimed-fire" combat modelled with the power attrition-rate coefficients with "no offset" (5.1). As in [4-7], we consider S. Bonder's model (3.2) for the constant-speed attack against a static defensive position. We will focus on the use of the LCS functions for predicting force annihilation, since the computing of force-level trajectories with Lanchester functions is adequately handled elsewhere (see [4-5]).

Let us accordingly consider the constant-speed attack of a homogeneous Y force against the static defensive position of a homogeneous X force (see Section 3 above for further modelling details, especially Figure 1). For our numerical computations, we assume that the fire effectiveness of the Y weapon system varies linearly with range, i.e.

$$\alpha(r) = \begin{cases} \alpha_0 \left(1 - \frac{r}{R_\alpha}\right) & \text{for } 0 \leq r \leq R_\alpha, \\ 0 & \text{for } R_\alpha \leq r, \end{cases} \quad (7.1)$$

and that the fire effectiveness of the X weapon system varies quadratically with range, i.e.

$$\beta(r) = \begin{cases} \beta_0 \left(1 - \frac{r}{R_\beta}\right)^2 & \text{for } 0 \leq r \leq R_\beta, \\ 0 & \text{for } R_\beta \leq r, \end{cases} \quad (7.2)$$

with $R_\alpha = R_\beta$, i.e. both weapon systems have the same maximum effective range. In other words, $\mu = 1$ in (3.4) and $\nu = 2$ for $\beta(r)$. We consider a battle modelled by the input data given in Table II. In terms of time as the independent variable, the attrition-rate coefficients (7.1) and (7.2) become via (3.3)

$$a(t) = k_a(t + C) \quad \text{and} \quad b(t) = k_b(t + C)^2, \quad (7.3)$$

Table II. Input Data for Numerical Examples

$$\mu = 1, \quad \nu = 2$$

$$\alpha_0 = 0.06 \text{ X casualties/minute/Y firer}$$

$$\beta_0 = 0.6 \text{ Y casualties/minute/X firer}$$

$$R_\alpha = R_\beta = 2000 \text{ meters}$$

$$v = 5 \text{ miles/hour}$$

where $R_\alpha = R_\beta$,

$$C = \frac{R_\alpha - R_0}{v}, \quad k_a = \frac{\alpha_0 v}{R_\alpha}, \quad \text{and} \quad k_b = \beta_0 \left(\frac{v}{R_\beta} \right)^2. \quad (7.4)$$

From the input data given in Table II, we compute the parameter values shown in Table III, since the transformed X force-level equation is given by (3.9) with $q = (v + 1)/(\mu + v + 2)$, $p = 1 - q$, $\mu = 1$, and $v = 2$. Thus, the X force level may be computed with $F_\alpha(\tau)$ and $H_{1-\alpha}(\tau)$ with $\alpha = q = 3/5$. Force-annihilation prediction involves the limiting value of $T_\alpha(\tau) = H_{1-\alpha}(\tau)/F_\alpha(\tau)$ as $\tau \rightarrow +\infty$. From Table 8B of the Appendix and Table III, we note the predicted agreement between $\Gamma(1-\alpha)/\Gamma(\alpha)$ and the limiting value of $T_\alpha(x)$ as $x \rightarrow +\infty$ [recall (4.13)] for $\alpha = q = 3/5$. We now consider two cases: (I) $R_0 = 2000$ meters, and (II) $R_0 = 1250$ meters.

When $R_0 = 2000$ meters (see Figure 3 of [4]), we have $C = 0$ and $\tau_0 = 0$. The maximum time that the battle can last is $t_{\max} = R_0/v = 14.91$ minutes, since at this time the attackers reach their final objective, i.e. the defender's position (again, see Figure 1). We now consider the qualitative behavior of the $\mu = 1$, $v = 2$ force-level trajectory shown in Figure 3 of [4]. Theorem 2 tells us that the X force can be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)} \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + v + 2} \right)^{q-p}, \quad (7.3)$$

where $q = 3/5$ and $p = 1 - q$. Using the numerical values in Table III, we compute from (7.3) that the X force can be annihilated in finite time if and only if

Table III. Parameter Values for Numerical Examples

$$k_a = 4.0233 \times 10^{-3} \text{ X casualties/(minute)}^u / \text{Y firer}$$

$$k_b = 2.6979 \times 10^{-3} \text{ Y casualties/(minute)}^v / \text{X firer}$$

$$p = 2/5, \quad q = 3/5$$

$$\Gamma(p)/\Gamma(q) = 1.48951$$

$$A = 0$$

$$\frac{x_0}{y_0} < 0.420. \quad (7.4)$$

When the X force can be annihilated, its annihilation time is given by (5.8), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0 \sqrt{\lambda_R}} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q}, \quad (7.5)$$

where

$$\tau(t) = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) t^{(\mu+\nu+2)/2}. \quad (7.6)$$

Thus, for the numerical values given in Table III, the time of annihilation of the X force is given by

$$T_q(\tau(t_a^X)) = 3.544 \frac{x_0}{y_0}, \quad (7.7)$$

with $q = 3/5$. We will now illustrate further computations for $x_0 = 10$ and $y_0 = 30$. From (7.4) we see that the X force can be annihilated in finite time (but we must verify that $t_a^X \leq t_{\max}$). In this case (7.7) becomes

$$T_q(\tau(t_a^X)) = 1.18122. \quad (7.8)$$

We must now determine $\tau(t_a^X)$ such that $\tau(t_a^X) = T_q^{-1}(1.18122)$ by using interpolation methods and Tables 8A and 8B. From Table 8A, we find

$$T_q(\tau) = 1.18172 \quad \text{for} \quad = 1.01$$

$$T_q(\tau) = 1.17630 \quad \text{for} \quad = 1.00$$

so that using linear interpolation, we obtain

$$\tau(t_a^X) = 1.009, \quad (7.9)$$

whence use of (7.6) yields

$$t_a^X = 14.24 \text{ minutes}, \quad (7.10)$$

which is less than $t_{\max} = 14.91$ minutes so that the defending X force is indeed annihilated before the attacking Y force reaches its final objective. Since $r(t) = R_0 - vt$, we find that force separation at the instant of annihilation of the X force is

$$r_a^X = 89.8 \text{ meters}. \quad (7.11)$$

Further results may be computed in a similar fashion and are given in Table IV.

When $R_0 = 1250$ meters (see Figure 3 of [5]), we have $C = 5.5923$ minutes, $\tau_0 = 0.0975$, and $t_{\max} = R_0/v = 9.32$ minutes. In this case Theorem 2 tells us that the X force can be annihilated in finite time if and only if

Table IV. Annihilation of the X Force as a Function
of the Initial Force Ratio for $R_0 = 2000$ meters

(x_0/y_0)	t_a^X (minutes)	r_a^X (meters)
0.333	14.24	89.8
0.250	11.61	443.2
0.200	10.19	633.2

$$\frac{x_0}{y_0} < \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} \frac{\frac{\Gamma(p)}{\Gamma(q)} \left\{ F_q(\tau_0) - \frac{\Gamma(q)}{\Gamma(p)} H_p(\tau_0) \right\}}{\left\{ F_p(\tau_0) - \frac{\Gamma(p)}{\Gamma(q)} H_q(\tau_0) \right\}}, \quad (7.12)$$

with $q = 3/5$ and $p = 1-q$. Using linear interpolation, we obtain from Tables 7A and 8A of the Appendix that for the numerical values of Table III

$$F_p(\tau_0) = 1.006, \quad H_q(\tau_0) = 0.044, \quad (7.13)$$

$$F_q(\tau_0) = 1.004, \quad H_p(\tau_0) = 0.223,$$

so that (7.12) says that the X force can be annihilated if and only if

$$\frac{x_0}{y_0} < 0.382. \quad (7.14)$$

When the X force can be annihilated, its annihilation time is given by (5.4), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{\left\{ \frac{x_0}{y_0 \sqrt{\lambda_R}} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} F_p(\tau_0) + H_p(\tau_0) \right\}}{\left\{ F_q(\tau_0) + \frac{x_0}{y_0 \sqrt{\lambda_R}} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} H_q(\tau_0) \right\}}, \quad (7.15)$$

whence for the data of Table III

$$T_a(\tau(t_a^X)) = \frac{3.565u_0 + 0.223}{0.156u_0 + 1.004}, \quad (7.16)$$

where $u_0 = x_0/y_0$. Let us also record here that (3.11) yields

$$t = \left(\frac{(\mu + \nu + 2)\tau}{2\lambda_1} \right)^{2/(\mu+\nu+2)} - c. \quad (7.17)$$

We will again illustrate further computations for $x_0 = 10$ and $y_0 = 30$.

From (7.14) we see that the X force can be annihilated in finite time (but again we must investigate whether or not $t_a^X \leq t_{\max}$). In this case (7.16) becomes

$$T_q(\tau(t_a^X)) = 1.33651, \quad (7.18)$$

whence Table 8A of the Appendix and linear interpolation yield

$$\tau(t_a^X) = 1.397, \quad (7.19)$$

so that by (7.17)

$$t_a^X = 10.63 \text{ minutes}. \quad (7.20)$$

Since $t_{\max} = R_0/v = 9.32$ minutes $< t_a^X$, we see that the attacking Y force overruns the defender's position before annihilation of the X force occurs.

Thus, the battle ends with $x_f = x(t_f) > 0$ and $y_f > 0$ at $t_f = t_{\max} = 9.32$ minutes. Corresponding to $t_f = 9.32$ minutes is $\tau_f = 1.1318$, and then Table 8A of the Appendix yields

$$F_q(\tau_f = 1.1318) = 1.589, \quad H_p(1.1318) = 1.973, \quad (7.21)$$

whence via (2.4), (4.8), (4.9), and (7.13) we obtain

$$x_f = x(t_f) = x(r = 0) = 1.35. \quad (7.22)$$

Some further numerical results are given in Table V. Again, these parametric results should be contrasted with the single $\mu = 1$, $\nu \neq 2$ force-level trajectory shown in Figure 3 of [5].

8. Final Remarks

In the previous section above, we have seen how the LCS functions allow one to conveniently obtain much valuable information about the model (2.1) with power attrition-rate coefficients (3.8) without having to explicitly compute the entire force-level trajectories. Previously we were limited to computing only force-level trajectories (see [4-5]). With the availability of these tabulations of LCS functions (see the Appendix of this report and [8]), we can now tell who is going to be annihilated and when this event will happen without having to compute the trajectories. Not only did we answer questions about the qualitative behavior of the model (e.g. force annihilation) for specific values of, for example, initial force levels but also for a range of values of the initial force ratio (i.e. parametric analysis of model behavior).

Table V. Annihilation of the X Force as a Function
of the Initial Force Ratio for $R_0 = 1250$ meters

(x_0/y_0)	t_a^X (minutes)	r_a^X (meters)
0.333	10.63	_____†
0.250	7.56	235.9
0.200	6.17	422.8

$$^{\dagger}t_{\max} = 9.32 \text{ minutes and } x_f = x(r=0) = 1.35.$$

The results of this report may be used for other parametric analyses, e.g. parametric dependence of battle outcome on attrition-rate coefficients. Thus, the contents of this report allow one to develop important insights into the dynamics of combat between two homogeneous forces with temporal variations in fire effectiveness. With the availability of tabulations of the LCS functions, one can now analyze such combat modelled by the power attrition-rate coefficients (3.8) with somewhat the same facility as he can for the constant-coefficient case and thus aid in parametric analyses. For further discussions of the significance of such results for military operations research, the reader is directed to [6] and [7].

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APPENDIX: Tabulation of the LCS Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 3/7$, and $4/7$.

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x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$	x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$
0.0000	0.0000	0.0000	0.0000	1.0000	0.46308	1.7500	0.76159
0.0005	0.0000	0.0000	0.0000	1.0050	0.46308	1.7500	0.76159
0.0010	0.0000	0.0000	0.0000	1.0100	0.46308	1.7500	0.76159
0.0015	0.0000	0.0000	0.0000	1.0150	0.46308	1.7500	0.76159
0.0020	0.0000	0.0000	0.0000	1.0200	0.46308	1.7500	0.76159
0.0025	0.0000	0.0000	0.0000	1.0250	0.46308	1.7500	0.76159
0.0030	0.0000	0.0000	0.0000	1.0300	0.46308	1.7500	0.76159
0.0035	0.0000	0.0000	0.0000	1.0350	0.46308	1.7500	0.76159
0.0040	0.0000	0.0000	0.0000	1.0400	0.46308	1.7500	0.76159
0.0045	0.0000	0.0000	0.0000	1.0450	0.46308	1.7500	0.76159
0.0050	0.0000	0.0000	0.0000	1.0500	0.46308	1.7500	0.76159
0.0055	0.0000	0.0000	0.0000	1.0550	0.46308	1.7500	0.76159
0.0060	0.0000	0.0000	0.0000	1.0600	0.46308	1.7500	0.76159
0.0065	0.0000	0.0000	0.0000	1.0650	0.46308	1.7500	0.76159
0.0070	0.0000	0.0000	0.0000	1.0700	0.46308	1.7500	0.76159
0.0075	0.0000	0.0000	0.0000	1.0750	0.46308	1.7500	0.76159
0.0080	0.0000	0.0000	0.0000	1.0800	0.46308	1.7500	0.76159
0.0085	0.0000	0.0000	0.0000	1.0850	0.46308	1.7500	0.76159
0.0090	0.0000	0.0000	0.0000	1.0900	0.46308	1.7500	0.76159
0.0095	0.0000	0.0000	0.0000	1.0950	0.46308	1.7500	0.76159
0.0100	0.0000	0.0000	0.0000	1.1000	0.46308	1.7500	0.76159
0.0105	0.0000	0.0000	0.0000	1.1050	0.46308	1.7500	0.76159
0.0110	0.0000	0.0000	0.0000	1.1100	0.46308	1.7500	0.76159
0.0115	0.0000	0.0000	0.0000	1.1150	0.46308	1.7500	0.76159
0.0120	0.0000	0.0000	0.0000	1.1200	0.46308	1.7500	0.76159
0.0125	0.0000	0.0000	0.0000	1.1250	0.46308	1.7500	0.76159
0.0130	0.0000	0.0000	0.0000	1.1300	0.46308	1.7500	0.76159
0.0135	0.0000	0.0000	0.0000	1.1350	0.46308	1.7500	0.76159
0.0140	0.0000	0.0000	0.0000	1.1400	0.46308	1.7500	0.76159
0.0145	0.0000	0.0000	0.0000	1.1450	0.46308	1.7500	0.76159
0.0150	0.0000	0.0000	0.0000	1.1500	0.46308	1.7500	0.76159
0.0155	0.0000	0.0000	0.0000	1.1550	0.46308	1.7500	0.76159
0.0160	0.0000	0.0000	0.0000	1.1600	0.46308	1.7500	0.76159
0.0165	0.0000	0.0000	0.0000	1.1650	0.46308	1.7500	0.76159
0.0170	0.0000	0.0000	0.0000	1.1700	0.46308	1.7500	0.76159
0.0175	0.0000	0.0000	0.0000	1.1750	0.46308	1.7500	0.76159
0.0180	0.0000	0.0000	0.0000	1.1800	0.46308	1.7500	0.76159
0.0185	0.0000	0.0000	0.0000	1.1850	0.46308	1.7500	0.76159
0.0190	0.0000	0.0000	0.0000	1.1900	0.46308	1.7500	0.76159
0.0195	0.0000	0.0000	0.0000	1.1950	0.46308	1.7500	0.76159
0.0200	0.0000	0.0000	0.0000	1.2000	0.46308	1.7500	0.76159
0.0205	0.0000	0.0000	0.0000	1.2050	0.46308	1.7500	0.76159
0.0210	0.0000	0.0000	0.0000	1.2100	0.46308	1.7500	0.76159
0.0215	0.0000	0.0000	0.0000	1.2150	0.46308	1.7500	0.76159
0.0220	0.0000	0.0000	0.0000	1.2200	0.46308	1.7500	0.76159
0.0225	0.0000	0.0000	0.0000	1.2250	0.46308	1.7500	0.76159
0.0230	0.0000	0.0000	0.0000	1.2300	0.46308	1.7500	0.76159
0.0235	0.0000	0.0000	0.0000	1.2350	0.46308	1.7500	0.76159
0.0240	0.0000	0.0000	0.0000	1.2400	0.46308	1.7500	0.76159
0.0245	0.0000	0.0000	0.0000	1.2450	0.46308	1.7500	0.76159
0.0250	0.0000	0.0000	0.0000	1.2500	0.46308	1.7500	0.76159
0.0255	0.0000	0.0000	0.0000	1.2550	0.46308	1.7500	0.76159
0.0260	0.0000	0.0000	0.0000	1.2600	0.46308	1.7500	0.76159
0.0265	0.0000	0.0000	0.0000	1.2650	0.46308	1.7500	0.76159
0.0270	0.0000	0.0000	0.0000	1.2700	0.46308	1.7500	0.76159
0.0275	0.0000	0.0000	0.0000	1.2750	0.46308	1.7500	0.76159
0.0280	0.0000	0.0000	0.0000	1.2800	0.46308	1.7500	0.76159
0.0285	0.0000	0.0000	0.0000	1.2850	0.46308	1.7500	0.76159
0.0290	0.0000	0.0000	0.0000	1.2900	0.46308	1.7500	0.76159
0.0295	0.0000	0.0000	0.0000	1.2950	0.46308	1.7500	0.76159
0.0300	0.0000	0.0000	0.0000	1.3000	0.46308	1.7500	0.76159
0.0305	0.0000	0.0000	0.0000	1.3050	0.46308	1.7500	0.76159
0.0310	0.0000	0.0000	0.0000	1.3100	0.46308	1.7500	0.76159
0.0315	0.0000	0.0000	0.0000	1.3150	0.46308	1.7500	0.76159
0.0320	0.0000	0.0000	0.0000	1.3200	0.46308	1.7500	0.76159
0.0325	0.0000	0.0000	0.0000	1.3250	0.46308	1.7500	0.76159
0.0330	0.0000	0.0000	0.0000	1.3300	0.46308	1.7500	0.76159
0.0335	0.0000	0.0000	0.0000	1.3350	0.46308	1.7500	0.76159
0.0340	0.0000	0.0000	0.0000	1.3400	0.46308	1.7500	0.76159
0.0345	0.0000	0.0000	0.0000	1.3450	0.46308	1.7500	0.76159
0.0350	0.0000	0.0000	0.0000	1.3500	0.46308	1.7500	0.76159
0.0355	0.0000	0.0000	0.0000	1.3550	0.46308	1.7500	0.76159
0.0360	0.0000	0.0000	0.0000	1.3600	0.46308	1.7500	0.76159
0.0365	0.0000	0.0000	0.0000	1.3650	0.46308	1.7500	0.76159
0.0370	0.0000	0.0000	0.0000	1.3700	0.46308	1.7500	0.76159
0.0375	0.0000	0.0000	0.0000	1.3750	0.46308	1.7500	0.76159
0.0380	0.0000	0.0000	0.0000	1.3800	0.46308	1.7500	0.76159
0.0385	0.0000	0.0000	0.0000	1.3850	0.46308	1.7500	0.76159
0.0390	0.0000	0.0000	0.0000	1.3900	0.46308	1.7500	0.76159
0.0395	0.0000	0.0000	0.0000	1.3950	0.46308	1.7500	0.76159
0.0400	0.0000	0.0000	0.0000	1.4000	0.46308	1.7500	0.76159
0.0405	0.0000	0.0000	0.0000	1.4050	0.46308	1.7500	0.76159
0.0410	0.0000	0.0000	0.0000	1.4100	0.46308	1.7500	0.76159
0.0415	0.0000	0.0000	0.0000	1.4150	0.46308	1.7500	0.76159
0.0420	0.0000	0.0000	0.0000	1.4200	0.46308	1.7500	0.76159
0.0425	0.0000	0.0000	0.0000	1.4250	0.46308	1.7500	0.76159
0.0430	0.0000	0.0000	0.0000	1.4300	0.46308	1.7500	0.76159
0.0435	0.0000	0.0000	0.0000	1.4350	0.46308	1.7500	0.76159
0.0440	0.0000	0.0000	0.0000	1.4400	0.46308	1.7500	0.76159
0.0445	0.0000	0.0000	0.0000	1.4450	0.46308	1.7500	0.76159
0.0450	0.0000	0.0000	0.0000	1.4500	0.46308	1.7500	0.76159
0.0455	0.0000	0.0000	0.0000	1.4550	0.46308	1.7500	0.76159
0.0460	0.0000	0.0000	0.0000	1.4600	0.46308	1.7500	0.76159
0.0465	0.0000	0.0000	0.0000	1.4650	0.46308	1.7500	0.76159
0.0470	0.0000	0.0000	0.0000	1.4700	0.46308	1.7500	0.76159
0.0475	0.0000	0.0000	0.0000	1.4750	0.46308	1.7500	0.76159
0.0480	0.0000	0.0000	0.0000	1.4800	0.46308	1.7500	0.76159
0.0485	0.0000	0.0000	0.0000	1.4850	0.46308	1.7500	0.76159
0.0490	0.0000	0.0000	0.0000	1.4900	0.46308	1.7500	0.76159
0.0495	0.0000	0.0000	0.0000	1.4950	0.46308	1.7500	0.76159
0.0500	0.0000	0.0000	0.0000	1.5000	0.46308	1.7500	0.76159

TABLE 1A. Lanchester-Clifford-Schlöfli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and

$$T_{\alpha}(x) \text{ for } \alpha = 1/2 \text{ and } x \text{ from } 0.00 \text{ to } 1.50.$$

[illegible]

TABLE 1B. Lanchester-Clifford-Schlöfli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and

$T_\alpha(x)$ for $\alpha = 1/2$ and x from 1.50 to 10.0.

TABLE 2B. Lanchester-Clifford-Schlafli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for $\alpha = 1/3$ and x from 1.50 to 10.0.

$\alpha = 2/3$

x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
0.0000	1.00000	0.00000	0.00000	0.5000	1.00000	0.00000	0.00000	1.0000	1.00000	0.00000	0.00000
0.0005	1.00004	0.00004	0.00004	0.5005	1.00004	0.00004	0.00004	1.0005	1.00004	0.00004	0.00004
0.0010	1.00015	0.00015	0.00015	0.5010	1.00015	0.00015	0.00015	1.0010	1.00015	0.00015	0.00015
0.0015	1.00034	0.00034	0.00034	0.5015	1.00034	0.00034	0.00034	1.0015	1.00034	0.00034	0.00034
0.0020	1.00060	0.00060	0.00060	0.5020	1.00060	0.00060	0.00060	1.0020	1.00060	0.00060	0.00060
0.0025	1.00094	0.00094	0.00094	0.5025	1.00094	0.00094	0.00094	1.0025	1.00094	0.00094	0.00094
0.0030	1.00135	0.00135	0.00135	0.5030	1.00135	0.00135	0.00135	1.0030	1.00135	0.00135	0.00135
0.0035	1.00184	0.00184	0.00184	0.5035	1.00184	0.00184	0.00184	1.0035	1.00184	0.00184	0.00184
0.0040	1.00240	0.00240	0.00240	0.5040	1.00240	0.00240	0.00240	1.0040	1.00240	0.00240	0.00240
0.0045	1.00304	0.00304	0.00304	0.5045	1.00304	0.00304	0.00304	1.0045	1.00304	0.00304	0.00304
0.0050	1.00375	0.00375	0.00375	0.5050	1.00375	0.00375	0.00375	1.0050	1.00375	0.00375	0.00375
0.0055	1.00451	0.00451	0.00451	0.5055	1.00451	0.00451	0.00451	1.0055	1.00451	0.00451	0.00451
0.0060	1.00532	0.00532	0.00532	0.5060	1.00532	0.00532	0.00532	1.0060	1.00532	0.00532	0.00532
0.0065	1.00618	0.00618	0.00618	0.5065	1.00618	0.00618	0.00618	1.0065	1.00618	0.00618	0.00618
0.0070	1.00709	0.00709	0.00709	0.5070	1.00709	0.00709	0.00709	1.0070	1.00709	0.00709	0.00709
0.0075	1.00805	0.00805	0.00805	0.5075	1.00805	0.00805	0.00805	1.0075	1.00805	0.00805	0.00805
0.0080	1.00906	0.00906	0.00906	0.5080	1.00906	0.00906	0.00906	1.0080	1.00906	0.00906	0.00906
0.0085	1.01012	0.01012	0.01012	0.5085	1.01012	0.01012	0.01012	1.0085	1.01012	0.01012	0.01012
0.0090	1.01123	0.01123	0.01123	0.5090	1.01123	0.01123	0.01123	1.0090	1.01123	0.01123	0.01123
0.0095	1.01239	0.01239	0.01239	0.5095	1.01239	0.01239	0.01239	1.0095	1.01239	0.01239	0.01239
0.0100	1.01360	0.01360	0.01360	0.5100	1.01360	0.01360	0.01360	1.0100	1.01360	0.01360	0.01360
0.0105	1.01486	0.01486	0.01486	0.5105	1.01486	0.01486	0.01486	1.0105	1.01486	0.01486	0.01486
0.0110	1.01617	0.01617	0.01617	0.5110	1.01617	0.01617	0.01617	1.0110	1.01617	0.01617	0.01617
0.0115	1.01753	0.01753	0.01753	0.5115	1.01753	0.01753	0.01753	1.0115	1.01753	0.01753	0.01753
0.0120	1.01894	0.01894	0.01894	0.5120	1.01894	0.01894	0.01894	1.0120	1.01894	0.01894	0.01894
0.0125	1.02040	0.02040	0.02040	0.5125	1.02040	0.02040	0.02040	1.0125	1.02040	0.02040	0.02040
0.0130	1.02191	0.02191	0.02191	0.5130	1.02191	0.02191	0.02191	1.0130	1.02191	0.02191	0.02191
0.0135	1.02347	0.02347	0.02347	0.5135	1.02347	0.02347	0.02347	1.0135	1.02347	0.02347	0.02347
0.0140	1.02508	0.02508	0.02508	0.5140	1.02508	0.02508	0.02508	1.0140	1.02508	0.02508	0.02508
0.0145	1.02674	0.02674	0.02674	0.5145	1.02674	0.02674	0.02674	1.0145	1.02674	0.02674	0.02674
0.0150	1.02845	0.02845	0.02845	0.5150	1.02845	0.02845	0.02845	1.0150	1.02845	0.02845	0.02845
0.0155	1.03021	0.03021	0.03021	0.5155	1.03021	0.03021	0.03021	1.0155	1.03021	0.03021	0.03021
0.0160	1.03202	0.03202	0.03202	0.5160	1.03202	0.03202	0.03202	1.0160	1.03202	0.03202	0.03202
0.0165	1.03388	0.03388	0.03388	0.5165	1.03388	0.03388	0.03388	1.0165	1.03388	0.03388	0.03388
0.0170	1.03579	0.03579	0.03579	0.5170	1.03579	0.03579	0.03579	1.0170	1.03579	0.03579	0.03579
0.0175	1.03775	0.03775	0.03775	0.5175	1.03775	0.03775	0.03775	1.0175	1.03775	0.03775	0.03775
0.0180	1.03976	0.03976	0.03976	0.5180	1.03976	0.03976	0.03976	1.0180	1.03976	0.03976	0.03976
0.0185	1.04182	0.04182	0.04182	0.5185	1.04182	0.04182	0.04182	1.0185	1.04182	0.04182	0.04182
0.0190	1.04393	0.04393	0.04393	0.5190	1.04393	0.04393	0.04393	1.0190	1.04393	0.04393	0.04393
0.0195	1.04609	0.04609	0.04609	0.5195	1.04609	0.04609	0.04609	1.0195	1.04609	0.04609	0.04609
0.0200	1.04830	0.04830	0.04830	0.5200	1.04830	0.04830	0.04830	1.0200	1.04830	0.04830	0.04830
0.0205	1.05056	0.05056	0.05056	0.5205	1.05056	0.05056	0.05056	1.0205	1.05056	0.05056	0.05056
0.0210	1.05287	0.05287	0.05287	0.5210	1.05287	0.05287	0.05287	1.0210	1.05287	0.05287	0.05287
0.0215	1.05523	0.05523	0.05523	0.5215	1.05523	0.05523	0.05523	1.0215	1.05523	0.05523	0.05523
0.0220	1.05764	0.05764	0.05764	0.5220	1.05764	0.05764	0.05764	1.0220	1.05764	0.05764	0.05764
0.0225	1.06010	0.06010	0.06010	0.5225	1.06010	0.06010	0.06010	1.0225	1.06010	0.06010	0.06010
0.0230	1.06261	0.06261	0.06261	0.5230	1.06261	0.06261	0.06261	1.0230	1.06261	0.06261	0.06261
0.0235	1.06517	0.06517	0.06517	0.5235	1.06517	0.06517	0.06517	1.0235	1.06517	0.06517	0.06517
0.0240	1.06778	0.06778	0.06778	0.5240	1.06778	0.06778	0.06778	1.0240	1.06778	0.06778	0.06778
0.0245	1.07044	0.07044	0.07044	0.5245	1.07044	0.07044	0.07044	1.0245	1.07044	0.07044	0.07044
0.0250	1.07315	0.07315	0.07315	0.5250	1.07315	0.07315	0.07315	1.0250	1.07315	0.07315	0.07315
0.0255	1.07591	0.07591	0.07591	0.5255	1.07591	0.07591	0.07591	1.0255	1.07591	0.07591	0.07591
0.0260	1.07872	0.07872	0.07872	0.5260	1.07872	0.07872	0.07872	1.0260	1.07872	0.07872	0.07872
0.0265	1.08158	0.08158	0.08158	0.5265	1.08158	0.08158	0.08158	1.0265	1.08158	0.08158	0.08158
0.0270	1.08449	0.08449	0.08449	0.5270	1.08449	0.08449	0.08449	1.0270	1.08449	0.08449	0.08449
0.0275	1.08745	0.08745	0.08745	0.5275	1.08745	0.08745	0.08745	1.0275	1.08745	0.08745	0.08745
0.0280	1.09046	0.09046	0.09046	0.5280	1.09046	0.09046	0.09046	1.0280	1.09046	0.09046	0.09046
0.0285	1.09352	0.09352	0.09352	0.5285	1.09352	0.09352	0.09352	1.0285	1.09352	0.09352	0.09352
0.0290	1.09663	0.09663	0.09663	0.5290	1.09663	0.09663	0.09663	1.0290	1.09663	0.09663	0.09663
0.0295	1.09979	0.09979	0.09979	0.5295	1.09979	0.09979	0.09979	1.0295	1.09979	0.09979	0.09979
0.0300	1.10300	0.10300	0.10300	0.5300	1.10300	0.10300	0.10300	1.0300	1.10300	0.10300	0.10300
0.0305	1.10626	0.10626	0.10626	0.5305	1.10626	0.10626	0.10626	1.0305	1.10626	0.10626	0.10626
0.0310	1.10957	0.10957	0.10957	0.5310	1.10957	0.10957	0.10957	1.0310	1.10957	0.10957	0.10957
0.0315	1.11293	0.11293	0.11293	0.5315	1.11293	0.11293	0.11293	1.0315	1.11293	0.11293	0.11293
0.0320	1.11634	0.11634	0.11634	0.5320	1.11634	0.11634	0.11634	1.0320	1.11634	0.11634	0.11634
0.0325	1.11980	0.11980	0.11980	0.5325	1.11980	0.11980	0.11980	1.0325	1.11980	0.11980	0.11980
0.0330	1.12331	0.12331	0.12331	0.5330	1.12331	0.12331	0.12331	1.0330	1.12331	0.12331	0.12331
0.0335	1.12687	0.12687	0.12687	0.5335	1.12687	0.12687	0.12687	1.0335	1.12687	0.12687	0.12687
0.0340	1.13048	0.13048	0.13048	0.5340	1.13048	0.13048	0.13048	1.0340	1.13048	0.13048	0.13048
0.0345	1.13414	0.13414	0.13414	0.5345	1.13414	0.13414	0.13414	1.0345	1.13414	0.13414	0.13414
0.0350	1.13785	0.13785	0.13785	0.5350	1.13785	0.13785	0.13785	1.0350	1.13785	0.13785	0.13785
0.0355	1.14161	0.14161	0.14161	0.5355	1.14161	0.14161	0.14161	1.0355	1.14161	0.14161	0.14161
0.0360	1.14542	0.14542	0.14542	0.5360	1.14542	0.14542	0.14542	1.0360	1.14542	0.14542	0.14542
0.0365	1.14928	0.14928	0.14928	0.5365	1.14928	0.14928	0.14928	1.0365	1.14928	0.14928	0.14928
0.0370	1.15319	0.15319	0.15319	0.5370	1.15319	0.15319	0.15319	1.0370	1.15319	0.15319	0.15319
0.0375	1.15715	0.15715	0.15715	0.5375	1.15715	0.15715	0.15715	1.0375	1.15715	0.15715	0.15715
0.0380	1.16116	0.16116	0.16116	0.5380	1.16116	0.16116	0.16116	1.0380	1.16116	0.16116	0.16116
0.0385	1.16522	0.16522	0.16522	0.5385	1.16522	0.16522	0.16522	1.0385	1.16522	0.16522	0.16522
0.0390	1.16933	0.16933	0.16933	0.5390	1.16933	0.16933	0.16933	1.0390	1.16933	0.16933	0.16933
0.0395	1.17349	0.17349	0.17349	0.5395	1.17349	0.17349	0.17349	1.0395	1.17349	0.17349	0.17349
0.0400	1.17770	0.17770	0.17770	0.5400	1.17770	0.17770	0.17770	1.0400	1.17770	0.17770	0.17770
0.0405	1.18196	0.18196	0.18196	0.5405	1.18196	0.18196	0.18196	1.0405	1.18196	0.18196	0.18196
0.0410	1.18627	0.18627	0.18627	0.5410	1.18627	0.18627	0.18627	1.0410	1.18627	0.18627	0.18627
0.0415	1.19063	0.1									

$\alpha = 1/4$ [illegible]

TABLE 4A. Lanchester-Clifford-Schl\"afli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and

$$T_{\alpha}(x) \text{ for } \alpha = 1/4 \text{ and } x \text{ from } 0.00 \text{ to } 1.50.$$

[illegible]

TABLE 4B. Lanchester-Clifford-Schlafli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for $\alpha = 1/4$ and x from 1.50 to 10.0.

x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
0.51234	1.87907	5.22942	2.71299	2.0	2.70370	7.98950	2.90047	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51235	1.88270	5.23150	2.71623	2.0	2.70371	7.98951	2.90048	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51236	1.88634	5.23358	2.71947	2.0	2.70372	7.98952	2.90049	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51237	1.88998	5.23566	2.72271	2.0	2.70373	7.98953	2.90050	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51238	1.89362	5.23774	2.72595	2.0	2.70374	7.98954	2.90051	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51239	1.89726	5.23982	2.72919	2.0	2.70375	7.98955	2.90052	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51240	1.90090	5.24190	2.73243	2.0	2.70376	7.98956	2.90053	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51241	1.90454	5.24398	2.73567	2.0	2.70377	7.98957	2.90054	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51242	1.90818	5.24606	2.73891	2.0	2.70378	7.98958	2.90055	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51243	1.91182	5.24814	2.74215	2.0	2.70379	7.98959	2.90056	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51244	1.91546	5.25022	2.74539	2.0	2.70380	7.98960	2.90057	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51245	1.91910	5.25230	2.74863	2.0	2.70381	7.98961	2.90058	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51246	1.92274	5.25438	2.75187	2.0	2.70382	7.98962	2.90059	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51247	1.92638	5.25646	2.75511	2.0	2.70383	7.98963	2.90060	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51248	1.93002	5.25854	2.75835	2.0	2.70384	7.98964	2.90061	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51249	1.93366	5.26062	2.76159	2.0	2.70385	7.98965	2.90062	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51250	1.93730	5.26270	2.76483	2.0	2.70386	7.98966	2.90063	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51251	1.94094	5.26478	2.76807	2.0	2.70387	7.98967	2.90064	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51252	1.94458	5.26686	2.77131	2.0	2.70388	7.98968	2.90065	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51253	1.94822	5.26894	2.77455	2.0	2.70389	7.98969	2.90066	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51254	1.95186	5.27102	2.77779	2.0	2.70390	7.98970	2.90067	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51255	1.95550	5.27310	2.78103	2.0	2.70391	7.98971	2.90068	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51256	1.95914	5.27518	2.78427	2.0	2.70392	7.98972	2.90069	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51257	1.96278	5.27726	2.78751	2.0	2.70393	7.98973	2.90070	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51258	1.96642	5.27934	2.79075	2.0	2.70394	7.98974	2.90071	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51259	1.97006	5.28142	2.79399	2.0	2.70395	7.98975	2.90072	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51260	1.97370	5.28350	2.79723	2.0	2.70396	7.98976	2.90073	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51261	1.97734	5.28558	2.80047	2.0	2.70397	7.98977	2.90074	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51262	1.98098	5.28766	2.80371	2.0	2.70398	7.98978	2.90075	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51263	1.98462	5.28974	2.80695	2.0	2.70399	7.98979	2.90076	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51264	1.98826	5.29182	2.81019	2.0	2.70400	7.98980	2.90077	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51265	1.99190	5.29390	2.81343	2.0	2.70401	7.98981	2.90078	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51266	1.99554	5.29598	2.81667	2.0	2.70402	7.98982	2.90079	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51267	1.99918	5.29806	2.81991	2.0	2.70403	7.98983	2.90080	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51268	2.00282	5.29999	2.82315	2.0	2.70404	7.98984	2.90081	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51269	2.00646	5.30197	2.82639	2.0	2.70405	7.98985	2.90082	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51270	2.01010	5.30395	2.82963	2.0	2.70406	7.98986	2.90083	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51271	2.01374	5.30593	2.83287	2.0	2.70407	7.98987	2.90084	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51272	2.01738	5.30791	2.83611	2.0	2.70408	7.98988	2.90085	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51273	2.02102	5.30989	2.83935	2.0	2.70409	7.98989	2.90086	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51274	2.02466	5.31187	2.84259	2.0	2.70410	7.98990	2.90087	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51275	2.02830	5.31385	2.84583	2.0	2.70411	7.98991	2.90088	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51276	2.03194	5.31583	2.84907	2.0	2.70412	7.98992	2.90089	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51277	2.03558	5.31781	2.85231	2.0	2.70413	7.98993	2.90090	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51278	2.03922	5.31979	2.85555	2.0	2.70414	7.98994	2.90091	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51279	2.04286	5.32177	2.85879	2.0	2.70415	7.98995	2.90092	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51280	2.04650	5.32375	2.86203	2.0	2.70416	7.98996	2.90093	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51281	2.05014	5.32573	2.86527	2.0	2.70417	7.98997	2.90094	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51282	2.05378	5.32771	2.86851	2.0	2.70418	7.98998	2.90095	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51283	2.05742	5.32969	2.87175	2.0	2.70419	7.98999	2.90096	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51284	2.06106	5.33167	2.87499	2.0	2.70420	7.99000	2.90097	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51285	2.06470	5.33365	2.87823	2.0	2.70421	7.99001	2.90098	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51286	2.06834	5.33563	2.88147	2.0	2.70422	7.99002	2.90099	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51287	2.07198	5.33761	2.88471	2.0	2.70423	7.99003	2.90100	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51288	2.07562	5.33959	2.88795	2.0	2.70424	7.99004	2.90101	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51289	2.07926	5.34157	2.89119	2.0	2.70425	7.99005	2.90102	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51290	2.08290	5.34355	2.89443	2.0	2.70426	7.99006	2.90103	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51291	2.08654	5.34553	2.89767	2.0	2.70427	7.99007	2.90104	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51292	2.09018	5.34751	2.90091	2.0	2.70428	7.99008	2.90105	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51293	2.09382	5.34949	2.90415	2.0	2.70429	7.99009	2.90106	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51294	2.09746	5.35147	2.90739	2.0	2.70430	7.99010	2.90107	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51295	2.10110	5.35345	2.91063	2.0	2.70431	7.99011	2.90108	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51296	2.10474	5.35543	2.91387	2.0	2.70432	7.99012	2.90109	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51297	2.10838	5.35741	2.91711	2.0	2.70433	7.99013	2.90110	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000
0.51298	2.11202	5.35939	2.92035	2.0	2.70434	7.99014	2.90111	0.0	0.00000	0.00000	0.00000	2.0	0.00000	0.00000	0.00000

$\alpha = 1/5$

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
0.00000	1.00000	0.00000	0.00000	0.50	1.32072	0.45534	0.10661	1.00	2.32244	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.51	1.33762	0.45534	0.10661	1.01	2.34109	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.52	1.35451	0.45534	0.10661	1.02	2.35974	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.53	1.37140	0.45534	0.10661	1.03	2.37839	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.54	1.38829	0.45534	0.10661	1.04	2.39704	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.55	1.40518	0.45534	0.10661	1.05	2.41569	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.56	1.42207	0.45534	0.10661	1.06	2.43434	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.57	1.43896	0.45534	0.10661	1.07	2.45299	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.58	1.45585	0.45534	0.10661	1.08	2.47164	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.59	1.47274	0.45534	0.10661	1.09	2.49029	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.60	1.48963	0.45534	0.10661	1.10	2.50894	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.61	1.50652	0.45534	0.10661	1.11	2.52759	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.62	1.52341	0.45534	0.10661	1.12	2.54624	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.63	1.54030	0.45534	0.10661	1.13	2.56489	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.64	1.55719	0.45534	0.10661	1.14	2.58354	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.65	1.57408	0.45534	0.10661	1.15	2.60219	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.66	1.59097	0.45534	0.10661	1.16	2.62084	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.67	1.60786	0.45534	0.10661	1.17	2.63949	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.68	1.62475	0.45534	0.10661	1.18	2.65814	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.69	1.64164	0.45534	0.10661	1.19	2.67679	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.70	1.65853	0.45534	0.10661	1.20	2.69544	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.71	1.67542	0.45534	0.10661	1.21	2.71409	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.72	1.69231	0.45534	0.10661	1.22	2.73274	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.73	1.70920	0.45534	0.10661	1.23	2.75139	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.74	1.72609	0.45534	0.10661	1.24	2.76999	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.75	1.74298	0.45534	0.10661	1.25	2.78864	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.76	1.75987	0.45534	0.10661	1.26	2.80729	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.77	1.77676	0.45534	0.10661	1.27	2.82594	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.78	1.79365	0.45534	0.10661	1.28	2.84459	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.79	1.81054	0.45534	0.10661	1.29	2.86324	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.80	1.82743	0.45534	0.10661	1.30	2.88189	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.81	1.84432	0.45534	0.10661	1.31	2.90054	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.82	1.86121	0.45534	0.10661	1.32	2.91919	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.83	1.87810	0.45534	0.10661	1.33	2.93784	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.84	1.89499	0.45534	0.10661	1.34	2.95649	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.85	1.91188	0.45534	0.10661	1.35	2.97514	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.86	1.92877	0.45534	0.10661	1.36	2.99379	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.87	1.94566	0.45534	0.10661	1.37	3.01244	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.88	1.96255	0.45534	0.10661	1.38	3.03109	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.89	1.97944	0.45534	0.10661	1.39	3.04974	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.90	1.99633	0.45534	0.10661	1.40	3.06839	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.91	2.01322	0.45534	0.10661	1.41	3.08704	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.92	2.03011	0.45534	0.10661	1.42	3.10569	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.93	2.04700	0.45534	0.10661	1.43	3.12434	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.94	2.06389	0.45534	0.10661	1.44	3.14299	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.95	2.08078	0.45534	0.10661	1.45	3.16164	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.96	2.09767	0.45534	0.10661	1.46	3.18029	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.97	2.11456	0.45534	0.10661	1.47	3.19894	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.98	2.13145	0.45534	0.10661	1.48	3.21759	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	0.99	2.14834	0.45534	0.10661	1.49	3.23624	0.42223	0.19198
0.00000	1.00000	0.00000	0.00000	1.00	2.16523	0.45534	0.10661	1.50	3.25489	0.42223	0.19198

TABLE 6A. Lanchester-Clifford-Schlafli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for $\alpha = 1/5$ and x from 0.00 to 1.50.

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$\alpha = 1/5$

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
1.00000	4.33040	1.06142	0.23429	2.00000	8.42496	3.79232	0.24088	3.00000	12.59071	10.74225	0.25360
1.00000	4.58200	1.07440	0.23469	2.00000	8.67000	3.79232	0.24088	3.00000	12.84225	10.74225	0.25360
1.00000	4.70999	1.08140	0.23509	2.00000	8.91520	3.79232	0.24088	3.00000	13.09471	10.74225	0.25360
1.00000	4.83399	1.08840	0.23549	2.00000	9.16040	3.79232	0.24088	3.00000	13.34717	10.74225	0.25360
1.00000	4.95799	1.09540	0.23589	2.00000	9.40560	3.79232	0.24088	3.00000	13.59963	10.74225	0.25360
1.00000	5.08199	1.10240	0.23629	2.00000	9.65080	3.79232	0.24088	3.00000	13.85209	10.74225	0.25360
1.00000	5.20599	1.10940	0.23669	2.00000	9.89600	3.79232	0.24088	3.00000	14.10455	10.74225	0.25360
1.00000	5.32999	1.11640	0.23709	2.00000	10.14120	3.79232	0.24088	3.00000	14.35701	10.74225	0.25360
1.00000	5.45399	1.12340	0.23749	2.00000	10.38640	3.79232	0.24088	3.00000	14.60947	10.74225	0.25360
1.00000	5.57799	1.13040	0.23789	2.00000	10.63160	3.79232	0.24088	3.00000	14.86193	10.74225	0.25360
1.00000	5.70199	1.13740	0.23829	2.00000	10.87680	3.79232	0.24088	3.00000	15.11439	10.74225	0.25360
1.00000	5.82599	1.14440	0.23869	2.00000	11.12200	3.79232	0.24088	3.00000	15.36685	10.74225	0.25360
1.00000	5.94999	1.15140	0.23909	2.00000	11.36720	3.79232	0.24088	3.00000	15.61931	10.74225	0.25360
1.00000	6.07399	1.15840	0.23949	2.00000	11.61240	3.79232	0.24088	3.00000	15.87177	10.74225	0.25360
1.00000	6.19799	1.16540	0.23989	2.00000	11.85760	3.79232	0.24088	3.00000	16.12423	10.74225	0.25360
1.00000	6.32199	1.17240	0.24029	2.00000	12.10280	3.79232	0.24088	3.00000	16.37669	10.74225	0.25360
1.00000	6.44599	1.17940	0.24069	2.00000	12.34800	3.79232	0.24088	3.00000	16.62915	10.74225	0.25360
1.00000	6.56999	1.18640	0.24109	2.00000	12.59320	3.79232	0.24088	3.00000	16.88161	10.74225	0.25360
1.00000	6.69399	1.19340	0.24149	2.00000	12.83840	3.79232	0.24088	3.00000	17.13407	10.74225	0.25360
1.00000	6.81799	1.20040	0.24189	2.00000	13.08360	3.79232	0.24088	3.00000	17.38653	10.74225	0.25360
1.00000	6.94199	1.20740	0.24229	2.00000	13.32880	3.79232	0.24088	3.00000	17.63899	10.74225	0.25360
1.00000	7.06599	1.21440	0.24269	2.00000	13.57400	3.79232	0.24088	3.00000	17.89145	10.74225	0.25360
1.00000	7.18999	1.22140	0.24309	2.00000	13.81920	3.79232	0.24088	3.00000	18.14391	10.74225	0.25360
1.00000	7.31399	1.22840	0.24349	2.00000	14.06440	3.79232	0.24088	3.00000	18.39637	10.74225	0.25360
1.00000	7.43799	1.23540	0.24389	2.00000	14.30960	3.79232	0.24088	3.00000	18.64883	10.74225	0.25360
1.00000	7.56199	1.24240	0.24429	2.00000	14.55480	3.79232	0.24088	3.00000	18.90129	10.74225	0.25360
1.00000	7.68599	1.24940	0.24469	2.00000	14.80000	3.79232	0.24088	3.00000	19.15375	10.74225	0.25360
1.00000	7.80999	1.25640	0.24509	2.00000	15.04520	3.79232	0.24088	3.00000	19.40621	10.74225	0.25360
1.00000	7.93399	1.26340	0.24549	2.00000	15.29040	3.79232	0.24088	3.00000	19.65867	10.74225	0.25360
1.00000	8.05799	1.27040	0.24589	2.00000	15.53560	3.79232	0.24088	3.00000	19.91113	10.74225	0.25360
1.00000	8.18199	1.27740	0.24629	2.00000	15.78080	3.79232	0.24088	3.00000	20.16359	10.74225	0.25360
1.00000	8.30599	1.28440	0.24669	2.00000	16.02600	3.79232	0.24088	3.00000	20.41605	10.74225	0.25360
1.00000	8.42999	1.29140	0.24709	2.00000	16.27120	3.79232	0.24088	3.00000	20.66851	10.74225	0.25360
1.00000	8.55399	1.29840	0.24749	2.00000	16.51640	3.79232	0.24088	3.00000	20.92097	10.74225	0.25360
1.00000	8.67799	1.30540	0.24789	2.00000	16.76160	3.79232	0.24088	3.00000	21.17343	10.74225	0.25360
1.00000	8.80199	1.31240	0.24829	2.00000	17.00680	3.79232	0.24088	3.00000	21.42589	10.74225	0.25360
1.00000	8.92599	1.31940	0.24869	2.00000	17.25200	3.79232	0.24088	3.00000	21.67835	10.74225	0.25360
1.00000	9.04999	1.32640	0.24909	2.00000	17.49720	3.79232	0.24088	3.00000	21.93081	10.74225	0.25360
1.00000	9.17399	1.33340	0.24949	2.00000	17.74240	3.79232	0.24088	3.00000	22.18327	10.74225	0.25360
1.00000	9.29799	1.34040	0.24989	2.00000	17.98760	3.79232	0.24088	3.00000	22.43573	10.74225	0.25360
1.00000	9.42199	1.34740	0.25029	2.00000	18.23280	3.79232	0.24088	3.00000	22.68819	10.74225	0.25360
1.00000	9.54599	1.35440	0.25069	2.00000	18.47800	3.79232	0.24088	3.00000	22.94065	10.74225	0.25360
1.00000	9.66999	1.36140	0.25109	2.00000	18.72320	3.79232	0.24088	3.00000	23.19311	10.74225	0.25360
1.00000	9.79399	1.36840	0.25149	2.00000	18.96840	3.79232	0.24088	3.00000	23.44557	10.74225	0.25360
1.00000	9.91799	1.37540	0.25189	2.00000	19.21360	3.79232	0.24088	3.00000	23.69803	10.74225	0.25360
1.00000	10.04199	1.38240	0.25229	2.00000	19.45880	3.79232	0.24088	3.00000	23.95049	10.74225	0.25360
1.00000	10.16599	1.38940	0.25269	2.00000	19.70400	3.79232	0.24088	3.00000	24.20295	10.74225	0.25360
1.00000	10.28999	1.39640	0.25309	2.00000	19.94920	3.79232	0.24088	3.00000	24.45541	10.74225	0.25360
1.00000	10.41399	1.40340	0.25349	2.00000	20.19440	3.79232	0.24088	3.00000	24.70787	10.74225	0.25360
1.00000	10.53799	1.41040	0.25389	2.00000	20.43960	3.79232	0.24088	3.00000	24.96033	10.74225	0.25360
1.00000	10.66199	1.41740	0.25429	2.00000	20.68480	3.79232	0.24088	3.00000	25.21279	10.74225	0.25360
1.00000	10.78599	1.42440	0.25469	2.00000	20.93000	3.79232	0.24088	3.00000	25.46525	10.74225	0.25360
1.00000	10.90999	1.43140	0.25509	2.00000	21.17520	3.79232	0.24088	3.00000	25.71771	10.74225	0.25360
1.00000	11.03399	1.43840	0.25549	2.00000	21.42040	3.79232	0.24088	3.00000	25.97017	10.74225	0.25360
1.00000	11.15799	1.44540	0.25589	2.00000	21.66560	3.79232	0.24088	3.00000	26.22263	10.74225	0.25360
1.00000	11.28199	1.45240	0.25629	2.00000	21.91080	3.79232	0.24088	3.00000	26.47509	10.74225	0.25360
1.00000	11.40599	1.45940	0.25669	2.00000	22.15600	3.79232	0.24088	3.00000	26.72755	10.74225	0.25360
1.00000	11.52999	1.46640	0.25709	2.00000	22.40120	3.79232	0.24088	3.00000	26.98001	10.74225	0.25360
1.00000	11.65399	1.47340	0.25749	2.00000	22.64640	3.79232	0.24088	3.00000	27.23247	10.74225	0.25360
1.00000	11.77799	1.48040	0.25789	2.00000	22.89160	3.79232	0.24088	3.00000	27.48493	10.74225	0.25360
1.00000	11.90199	1.48740	0.25829	2.00000	23.13680	3.79232	0.24088	3.00000	27.73739	10.74225	0.25360
1.00000	12.02599	1.49440	0.25869	2.00000	23.38200	3.79232	0.24088	3.00000	27.98985	10.74225	0.25360
1.00000	12.14999	1.50140	0.25909	2.00000	23.62720	3.79232	0.24088	3.00000	28.24231	10.74225	0.25360
1.00000	12.27399	1.50840	0.25949	2.00000	23.87240	3.79232	0.24088	3.00000	28.49477	10.74225	0.25360
1.00000	12.39799	1.51540	0.25989	2.00000	24.11760	3.79232	0.24088	3.00000	28.74723	10.74225	0.25360
1.00000	12.52199	1.52240	0.26029	2.00000	24.36280	3.79232	0.24088	3.00000	28.99969	10.74225	0.25360
1.00000	12.64599	1.52940	0.26069	2.00000	24.60800	3.79232	0.24088	3.00000	29.25215	10.74225	0.25360
1.00000	12.76999	1.53640	0.26109	2.00000	24.85320	3.79232	0.24088	3.00000	29.50461	10.74225	0.25360
1.00000	12.89399	1.54340	0.26149	2.00000	25.09840	3.79232	0.24088	3.00000	29.75707	10.74225	0.25360
1.00000	13.01799	1.55040	0.26189	2.00000	25.34360	3.79232	0.24088	3.00000	30.00953	10.74225	0.25360
1.00000	13.14199	1.55740	0.26229	2.00000	25.58880	3.79232	0.24088	3.00000	30.26199	10.74225	0.25360
1.00000	13.26599	1.56440	0.26269	2.00000	25.83400	3.79232	0.24088	3.00000	30.51445	10.74225	0.25360
1.00000	13.38999	1.57140	0.26309	2.00000	26.07920	3.79232	0.24088	3.00000	30.76691	10.74225	0.25360
1.00000	13.51399	1.57840	0.26349	2.00000	26.32440	3.79232	0.24088	3.00000	31.01937	10.74225	0.25360
1.00000	13.63799	1.58540	0.26389	2.00000	26.56960	3.79232	0.24088	3.00000	31.27183	10.74225	0.25360
1.00000	13.76199	1.59240	0.26429	2.00000	26.81480	3.79232	0.24088	3.00000	31.52429	10.74225	0.25360
1.00000	13.88599	1.59940	0.26469	2.00000	27.06000	3.79232	0.24088	3.00000	31.77675	10.74225	0.25360
1.00000	14.00999	1.60640	0.26509	2.00000	27.30520	3.79232	0.24088	3.00000	32.02921	10.74225	0.25360
1.00000	14.13399	1.61340	0.26549	2.00000	27.55040	3.79232	0.24088	3.00000	32.28167	10.74225	0

$\alpha = 2/5$

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
0.0000	1.00000	0.00000	0.00000	0.5000	1.15977	0.38226	0.28804	0.4000	1.69739	0.89737	0.51153
0.0005	1.00006	0.00009	0.00029	0.5005	1.16037	0.38266	0.28809	0.4005	1.70084	0.89779	0.51164
0.0010	1.00011	0.00018	0.00053	0.5010	1.16097	0.38303	0.28814	0.4010	1.70428	0.89821	0.51175
0.0015	1.00016	0.00027	0.00077	0.5015	1.16157	0.38340	0.28819	0.4015	1.70772	0.89863	0.51186
0.0020	1.00021	0.00036	0.00101	0.5020	1.16217	0.38377	0.28824	0.4020	1.71116	0.89905	0.51197
0.0025	1.00026	0.00045	0.00125	0.5025	1.16277	0.38414	0.28829	0.4025	1.71460	0.89947	0.51208
0.0030	1.00031	0.00054	0.00149	0.5030	1.16337	0.38451	0.28834	0.4030	1.71804	0.89989	0.51219
0.0035	1.00036	0.00063	0.00173	0.5035	1.16397	0.38488	0.28839	0.4035	1.72148	0.90031	0.51230
0.0040	1.00041	0.00072	0.00197	0.5040	1.16457	0.38525	0.28844	0.4040	1.72492	0.90073	0.51241
0.0045	1.00046	0.00081	0.00221	0.5045	1.16517	0.38562	0.28849	0.4045	1.72836	0.90115	0.51252
0.0050	1.00051	0.00090	0.00245	0.5050	1.16577	0.38599	0.28854	0.4050	1.73180	0.90157	0.51263
0.0055	1.00056	0.00099	0.00269	0.5055	1.16637	0.38636	0.28859	0.4055	1.73524	0.90199	0.51274
0.0060	1.00061	0.00108	0.00293	0.5060	1.16697	0.38673	0.28864	0.4060	1.73868	0.90241	0.51285
0.0065	1.00066	0.00117	0.00317	0.5065	1.16757	0.38710	0.28869	0.4065	1.74212	0.90283	0.51296
0.0070	1.00071	0.00126	0.00341	0.5070	1.16817	0.38747	0.28874	0.4070	1.74556	0.90325	0.51307
0.0075	1.00076	0.00135	0.00365	0.5075	1.16877	0.38784	0.28879	0.4075	1.74900	0.90367	0.51318
0.0080	1.00081	0.00144	0.00389	0.5080	1.16937	0.38821	0.28884	0.4080	1.75244	0.90409	0.51329
0.0085	1.00086	0.00153	0.00413	0.5085	1.16997	0.38858	0.28889	0.4085	1.75588	0.90451	0.51340
0.0090	1.00091	0.00162	0.00437	0.5090	1.17057	0.38895	0.28894	0.4090	1.75932	0.90493	0.51351
0.0095	1.00096	0.00171	0.00461	0.5095	1.17117	0.38932	0.28899	0.4095	1.76276	0.90535	0.51362
0.0100	1.00101	0.00180	0.00485	0.5100	1.17177	0.38969	0.28904	0.5100	1.76620	0.90577	0.51373
0.0105	1.00106	0.00189	0.00509	0.5105	1.17237	0.39006	0.28909	0.5105	1.76964	0.90619	0.51384
0.0110	1.00111	0.00198	0.00533	0.5110	1.17297	0.39043	0.28914	0.5110	1.77308	0.90661	0.51395
0.0115	1.00116	0.00207	0.00557	0.5115	1.17357	0.39080	0.28919	0.5115	1.77652	0.90703	0.51406
0.0120	1.00121	0.00216	0.00581	0.5120	1.17417	0.39117	0.28924	0.5120	1.77996	0.90745	0.51417
0.0125	1.00126	0.00225	0.00605	0.5125	1.17477	0.39154	0.28929	0.5125	1.78340	0.90787	0.51428
0.0130	1.00131	0.00234	0.00629	0.5130	1.17537	0.39191	0.28934	0.5130	1.78684	0.90829	0.51439
0.0135	1.00136	0.00243	0.00653	0.5135	1.17597	0.39228	0.28939	0.5135	1.79028	0.90871	0.51450
0.0140	1.00141	0.00252	0.00677	0.5140	1.17657	0.39265	0.28944	0.5140	1.79372	0.90913	0.51461
0.0145	1.00146	0.00261	0.00701	0.5145	1.17717	0.39302	0.28949	0.5145	1.79716	0.90955	0.51472
0.0150	1.00151	0.00270	0.00725	0.5150	1.17777	0.39339	0.28954	0.5150	1.80060	0.90997	0.51483
0.0155	1.00156	0.00279	0.00749	0.5155	1.17837	0.39376	0.28959	0.5155	1.80404	0.91039	0.51494
0.0160	1.00161	0.00288	0.00773	0.5160	1.17897	0.39413	0.28964	0.5160	1.80748	0.91081	0.51505
0.0165	1.00166	0.00297	0.00797	0.5165	1.17957	0.39450	0.28969	0.5165	1.81092	0.91123	0.51516
0.0170	1.00171	0.00306	0.00821	0.5170	1.18017	0.39487	0.28974	0.5170	1.81436	0.91165	0.51527
0.0175	1.00176	0.00315	0.00845	0.5175	1.18077	0.39524	0.28979	0.5175	1.81780	0.91207	0.51538
0.0180	1.00181	0.00324	0.00869	0.5180	1.18137	0.39561	0.28984	0.5180	1.82124	0.91249	0.51549
0.0185	1.00186	0.00333	0.00893	0.5185	1.18197	0.39598	0.28989	0.5185	1.82468	0.91291	0.51560
0.0190	1.00191	0.00342	0.00917	0.5190	1.18257	0.39635	0.28994	0.5190	1.82812	0.91333	0.51571
0.0195	1.00196	0.00351	0.00941	0.5195	1.18317	0.39672	0.28999	0.5195	1.83156	0.91375	0.51582
0.0200	1.00201	0.00360	0.00965	0.5200	1.18377	0.39709	0.29004	0.5200	1.83500	0.91417	0.51593
0.0205	1.00206	0.00369	0.00989	0.5205	1.18437	0.39746	0.29009	0.5205	1.83844	0.91459	0.51604
0.0210	1.00211	0.00378	0.01013	0.5210	1.18497	0.39783	0.29014	0.5210	1.84188	0.91501	0.51615
0.0215	1.00216	0.00387	0.01037	0.5215	1.18557	0.39820	0.29019	0.5215	1.84532	0.91543	0.51626
0.0220	1.00221	0.00396	0.01061	0.5220	1.18617	0.39857	0.29024	0.5220	1.84876	0.91585	0.51637
0.0225	1.00226	0.00405	0.01085	0.5225	1.18677	0.39894	0.29029	0.5225	1.85220	0.91627	0.51648
0.0230	1.00231	0.00414	0.01109	0.5230	1.18737	0.39931	0.29034	0.5230	1.85564	0.91669	0.51659
0.0235	1.00236	0.00423	0.01133	0.5235	1.18797	0.39968	0.29039	0.5235	1.85908	0.91711	0.51670
0.0240	1.00241	0.00432	0.01157	0.5240	1.18857	0.40005	0.29044	0.5240	1.86252	0.91753	0.51681
0.0245	1.00246	0.00441	0.01181	0.5245	1.18917	0.40042	0.29049	0.5245	1.86596	0.91795	0.51692
0.0250	1.00251	0.00450	0.01205	0.5250	1.18977	0.40079	0.29054	0.5250	1.86940	0.91837	0.51703
0.0255	1.00256	0.00459	0.01229	0.5255	1.19037	0.40116	0.29059	0.5255	1.87284	0.91879	0.51714
0.0260	1.00261	0.00468	0.01253	0.5260	1.19097	0.40153	0.29064	0.5260	1.87628	0.91921	0.51725
0.0265	1.00266	0.00477	0.01277	0.5265	1.19157	0.40190	0.29069	0.5265	1.87972	0.91963	0.51736
0.0270	1.00271	0.00486	0.01301	0.5270	1.19217	0.40227	0.29074	0.5270	1.88316	0.92005	0.51747
0.0275	1.00276	0.00495	0.01325	0.5275	1.19277	0.40264	0.29079	0.5275	1.88660	0.92047	0.51758
0.0280	1.00281	0.00504	0.01349	0.5280	1.19337	0.40301	0.29084	0.5280	1.89004	0.92089	0.51769
0.0285	1.00286	0.00513	0.01373	0.5285	1.19397	0.40338	0.29089	0.5285	1.89348	0.92131	0.51780
0.0290	1.00291	0.00522	0.01397	0.5290	1.19457	0.40375	0.29094	0.5290	1.89692	0.92173	0.51791
0.0295	1.00296	0.00531	0.01421	0.5295	1.19517	0.40412	0.29099	0.5295	1.90036	0.92215	0.51802
0.0300	1.00301	0.00540	0.01445	0.5300	1.19577	0.40449	0.29104	0.5300	1.90380	0.92257	0.51813
0.0305	1.00306	0.00549	0.01469	0.5305	1.19637	0.40486	0.29109	0.5305	1.90724	0.92299	0.51824
0.0310	1.00311	0.00558	0.01493	0.5310	1.19697	0.40523	0.29114	0.5310	1.91068	0.92341	0.51835
0.0315	1.00316	0.00567	0.01517	0.5315	1.19757	0.40560	0.29119	0.5315	1.91412	0.92383	0.51846
0.0320	1.00321	0.00576	0.01541	0.5320	1.19817	0.40597	0.29124	0.5320	1.91756	0.92425	0.51857
0.0325	1.00326	0.00585	0.01565	0.5325	1.19877	0.40634	0.29129	0.5325	1.92100	0.92467	0.51868
0.0330	1.00331	0.00594	0.01589	0.5330	1.19937	0.40671	0.29134	0.5330	1.92444	0.92509	0.51879
0.0335	1.00336	0.00603	0.01613	0.5335	1.19997	0.40708	0.29139	0.5335	1.92788	0.92551	0.51890
0.0340	1.00341	0.00612	0.01637	0.5340	1.20057	0.40745	0.29144	0.5340	1.93132	0.92593	0.51901
0.0345	1.00346	0.00621	0.01661	0.5345	1.20117	0.40782	0.29149	0.5345	1.93476	0.92635	0.51912
0.0350	1.00351	0.00630	0.01685	0.5350	1.20177	0.40819	0.29154	0.5350	1.93820	0.92677	0.51923
0.0355	1.00356	0.00639	0.01709	0.5355	1.20237	0.40856	0.29159	0.5355	1.94164	0.92719	0.51934
0.0360	1.00361	0.00648	0.01733	0.5360	1.20297	0.40893	0.29164	0.5360	1.94508	0.92761	0.51945
0.0365	1.00366	0.00657	0.01757	0.5365	1.20357	0.40930	0.29169	0.5365	1.94852	0.92803	0.51956
0.0370	1.00371	0.00666	0.01781	0.5370	1.20417	0.40967	0.29174	0.5370	1.95196	0.92845	0.51967
0.0375	1.00376	0.00675	0.01805	0.5375	1.20477	0.41004	0.29179	0.5375	1.95540	0.92887	0.51978
0.0380	1.00381	0.00684	0.01829	0.5380	1.20537	0.41041	0.29184	0.5380	1.95884	0.92929	0.51989
0.0385	1.00386	0.00693	0.01853	0.5385	1.20597	0.41078	0.29189	0.5385	1.96228	0.92971	0.51999
0.0390	1.00391	0.00702	0.01877	0.5390	1.20657	0.41115	0.29194	0.5390	1.96572	0.93013	0.52010
0.0395	1.00396	0.00711	0.01901	0.5395	1.20717	0.41152	0.29199	0.5395	1.96916	0.93055	0.52021
0.0400	1.00401	0.00720	0.01925	0.5400	1.20777	0.41189	0.29204	0.5400	1.97260	0.93097	0.52032
0.0405	1.00406	0.00729	0.01949	0.5405	1.20837	0.41226	0.29209	0.5405	1.97604	0.93139	0.52043
0.0410	1.00411	0.00738	0.01973	0.5410	1.20897	0.41263	0.29214	0.5410	1.97948	0.93181	0.52054
0.0415	1.00416	0.00747	0.01997	0.5415	1.20957	0.41300	0.29219	0.5415	1.98		

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$\alpha = 2/5$

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
1.50	2.71176	1.66228	0.60561	2.0	5.52641	2.9225	0.6492	6.0	278.92057	187.25497	0.67136
1.51	2.71313	1.66228	0.60569	2.1	5.52758	2.9235	0.64935	6.1	278.92057	187.25497	0.67136
1.52	2.71450	1.66228	0.60576	2.2	5.52875	2.9245	0.64948	6.2	278.92057	187.25497	0.67136
1.53	2.71587	1.66228	0.60583	2.3	5.52992	2.9255	0.64961	6.3	278.92057	187.25497	0.67136
1.54	2.71724	1.66228	0.60590	2.4	5.53109	2.9265	0.64974	6.4	278.92057	187.25497	0.67136
1.55	2.71861	1.66228	0.60597	2.5	5.53226	2.9275	0.64987	6.5	278.92057	187.25497	0.67136
1.56	2.72000	1.66228	0.60604	2.6	5.53343	2.9285	0.64999	6.6	278.92057	187.25497	0.67136
1.57	2.72139	1.66228	0.60611	2.7	5.53460	2.9295	0.65012	6.7	278.92057	187.25497	0.67136
1.58	2.72278	1.66228	0.60618	2.8	5.53577	2.9305	0.65025	6.8	278.92057	187.25497	0.67136
1.59	2.72417	1.66228	0.60625	2.9	5.53694	2.9315	0.65038	6.9	278.92057	187.25497	0.67136
1.60	2.72556	1.66228	0.60632	3.0	5.53811	2.9325	0.65051	7.0	278.92057	187.25497	0.67136
1.61	2.72695	1.66228	0.60639	3.1	5.53928	2.9335	0.65064	7.1	278.92057	187.25497	0.67136
1.62	2.72834	1.66228	0.60646	3.2	5.54045	2.9345	0.65077	7.2	278.92057	187.25497	0.67136
1.63	2.72973	1.66228	0.60653	3.3	5.54162	2.9355	0.65090	7.3	278.92057	187.25497	0.67136
1.64	2.73112	1.66228	0.60660	3.4	5.54279	2.9365	0.65103	7.4	278.92057	187.25497	0.67136
1.65	2.73251	1.66228	0.60667	3.5	5.54396	2.9375	0.65116	7.5	278.92057	187.25497	0.67136
1.66	2.73390	1.66228	0.60674	3.6	5.54513	2.9385	0.65129	7.6	278.92057	187.25497	0.67136
1.67	2.73529	1.66228	0.60681	3.7	5.54630	2.9395	0.65142	7.7	278.92057	187.25497	0.67136
1.68	2.73668	1.66228	0.60688	3.8	5.54747	2.9405	0.65155	7.8	278.92057	187.25497	0.67136
1.69	2.73807	1.66228	0.60695	3.9	5.54864	2.9415	0.65168	7.9	278.92057	187.25497	0.67136
1.70	2.73946	1.66228	0.60702	4.0	5.54981	2.9425	0.65181	8.0	278.92057	187.25497	0.67136
1.71	2.74085	1.66228	0.60709	4.1	5.55098	2.9435	0.65194	8.1	278.92057	187.25497	0.67136
1.72	2.74224	1.66228	0.60716	4.2	5.55215	2.9445	0.65207	8.2	278.92057	187.25497	0.67136
1.73	2.74363	1.66228	0.60723	4.3	5.55332	2.9455	0.65220	8.3	278.92057	187.25497	0.67136
1.74	2.74502	1.66228	0.60730	4.4	5.55449	2.9465	0.65233	8.4	278.92057	187.25497	0.67136
1.75	2.74641	1.66228	0.60737	4.5	5.55566	2.9475	0.65246	8.5	278.92057	187.25497	0.67136
1.76	2.74780	1.66228	0.60744	4.6	5.55683	2.9485	0.65259	8.6	278.92057	187.25497	0.67136
1.77	2.74919	1.66228	0.60751	4.7	5.55799	2.9495	0.65272	8.7	278.92057	187.25497	0.67136
1.78	2.75058	1.66228	0.60758	4.8	5.55916	2.9505	0.65285	8.8	278.92057	187.25497	0.67136
1.79	2.75197	1.66228	0.60765	4.9	5.56033	2.9515	0.65298	8.9	278.92057	187.25497	0.67136
1.80	2.75336	1.66228	0.60772	5.0	5.56150	2.9525	0.65311	9.0	278.92057	187.25497	0.67136
1.81	2.75475	1.66228	0.60779	5.1	5.56267	2.9535	0.65324	9.1	278.92057	187.25497	0.67136
1.82	2.75614	1.66228	0.60786	5.2	5.56384	2.9545	0.65337	9.2	278.92057	187.25497	0.67136
1.83	2.75753	1.66228	0.60793	5.3	5.56501	2.9555	0.65350	9.3	278.92057	187.25497	0.67136
1.84	2.75892	1.66228	0.60800	5.4	5.56618	2.9565	0.65363	9.4	278.92057	187.25497	0.67136
1.85	2.76031	1.66228	0.60807	5.5	5.56735	2.9575	0.65376	9.5	278.92057	187.25497	0.67136
1.86	2.76170	1.66228	0.60814	5.6	5.56852	2.9585	0.65389	9.6	278.92057	187.25497	0.67136
1.87	2.76309	1.66228	0.60821	5.7	5.56969	2.9595	0.65402	9.7	278.92057	187.25497	0.67136
1.88	2.76448	1.66228	0.60828	5.8	5.57086	2.9605	0.65415	9.8	278.92057	187.25497	0.67136
1.89	2.76587	1.66228	0.60835	5.9	5.57203	2.9615	0.65428	9.9	278.92057	187.25497	0.67136
1.90	2.76726	1.66228	0.60842	6.0	5.57320	2.9625	0.65441	10.0	278.92057	187.25497	0.67136
1.91	2.76865	1.66228	0.60849								
1.92	2.77004	1.66228	0.60856								
1.93	2.77143	1.66228	0.60863								
1.94	2.77282	1.66228	0.60870								
1.95	2.77421	1.66228	0.60877								
1.96	2.77560	1.66228	0.60884								
1.97	2.77699	1.66228	0.60891								
1.98	2.77838	1.66228	0.60898								
1.99	2.77977	1.66228	0.60905								
2.00	2.78116	1.66228	0.60912								

TABLE 7B. Lanchester-Clifford-Schláflil Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and

$T_{\alpha}(x)$ for $\alpha = 2/5$ and x from 1.50 to 10.0.

$\alpha = 3/5$

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$	x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$	x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
0.0	1.00000	0.0	0.0	0.50	1.10622	0.86199	0.77922	1.0	1.45028	1.70397	1.17637
0.01	1.00004	0.03607	0.03607	0.51	1.10622	0.86199	0.77922	1.01	1.45028	1.70397	1.17637
0.02	1.00017	0.07214	0.07214	0.52	1.10622	0.86199	0.77922	1.02	1.45028	1.70397	1.17637
0.03	1.00039	0.10821	0.10821	0.53	1.10622	0.86199	0.77922	1.03	1.45028	1.70397	1.17637
0.04	1.00067	0.14428	0.14428	0.54	1.10622	0.86199	0.77922	1.04	1.45028	1.70397	1.17637
0.05	1.00100	0.18035	0.18035	0.55	1.10622	0.86199	0.77922	1.05	1.45028	1.70397	1.17637
0.06	1.00139	0.21642	0.21642	0.56	1.10622	0.86199	0.77922	1.06	1.45028	1.70397	1.17637
0.07	1.00184	0.25249	0.25249	0.57	1.10622	0.86199	0.77922	1.07	1.45028	1.70397	1.17637
0.08	1.00235	0.28856	0.28856	0.58	1.10622	0.86199	0.77922	1.08	1.45028	1.70397	1.17637
0.09	1.00292	0.32463	0.32463	0.59	1.10622	0.86199	0.77922	1.09	1.45028	1.70397	1.17637
0.10	1.00355	0.36070	0.36070	0.60	1.10622	0.86199	0.77922	1.10	1.45028	1.70397	1.17637
0.11	1.00424	0.39677	0.39677	0.61	1.10622	0.86199	0.77922	1.11	1.45028	1.70397	1.17637
0.12	1.00499	0.43284	0.43284	0.62	1.10622	0.86199	0.77922	1.12	1.45028	1.70397	1.17637
0.13	1.00579	0.46891	0.46891	0.63	1.10622	0.86199	0.77922	1.13	1.45028	1.70397	1.17637
0.14	1.00664	0.50498	0.50498	0.64	1.10622	0.86199	0.77922	1.14	1.45028	1.70397	1.17637
0.15	1.00754	0.54105	0.54105	0.65	1.10622	0.86199	0.77922	1.15	1.45028	1.70397	1.17637
0.16	1.00849	0.57712	0.57712	0.66	1.10622	0.86199	0.77922	1.16	1.45028	1.70397	1.17637
0.17	1.00949	0.61319	0.61319	0.67	1.10622	0.86199	0.77922	1.17	1.45028	1.70397	1.17637
0.18	1.01054	0.64926	0.64926	0.68	1.10622	0.86199	0.77922	1.18	1.45028	1.70397	1.17637
0.19	1.01164	0.68533	0.68533	0.69	1.10622	0.86199	0.77922	1.19	1.45028	1.70397	1.17637
0.20	1.01279	0.72140	0.72140	0.70	1.10622	0.86199	0.77922	1.20	1.45028	1.70397	1.17637
0.21	1.01399	0.75747	0.75747	0.71	1.10622	0.86199	0.77922	1.21	1.45028	1.70397	1.17637
0.22	1.01524	0.79354	0.79354	0.72	1.10622	0.86199	0.77922	1.22	1.45028	1.70397	1.17637
0.23	1.01654	0.82961	0.82961	0.73	1.10622	0.86199	0.77922	1.23	1.45028	1.70397	1.17637
0.24	1.01789	0.86568	0.86568	0.74	1.10622	0.86199	0.77922	1.24	1.45028	1.70397	1.17637
0.25	1.01929	0.90175	0.90175	0.75	1.10622	0.86199	0.77922	1.25	1.45028	1.70397	1.17637
0.26	1.02074	0.93782	0.93782	0.76	1.10622	0.86199	0.77922	1.26	1.45028	1.70397	1.17637
0.27	1.02224	0.97389	0.97389	0.77	1.10622	0.86199	0.77922	1.27	1.45028	1.70397	1.17637
0.28	1.02379	1.00996	1.00996	0.78	1.10622	0.86199	0.77922	1.28	1.45028	1.70397	1.17637
0.29	1.02539	1.04603	1.04603	0.79	1.10622	0.86199	0.77922	1.29	1.45028	1.70397	1.17637
0.30	1.02704	1.08210	1.08210	0.80	1.10622	0.86199	0.77922	1.30	1.45028	1.70397	1.17637
0.31	1.02874	1.11817	1.11817	0.81	1.10622	0.86199	0.77922	1.31	1.45028	1.70397	1.17637
0.32	1.03049	1.15424	1.15424	0.82	1.10622	0.86199	0.77922	1.32	1.45028	1.70397	1.17637
0.33	1.03229	1.19031	1.19031	0.83	1.10622	0.86199	0.77922	1.33	1.45028	1.70397	1.17637
0.34	1.03414	1.22638	1.22638	0.84	1.10622	0.86199	0.77922	1.34	1.45028	1.70397	1.17637
0.35	1.03604	1.26245	1.26245	0.85	1.10622	0.86199	0.77922	1.35	1.45028	1.70397	1.17637
0.36	1.03799	1.29852	1.29852	0.86	1.10622	0.86199	0.77922	1.36	1.45028	1.70397	1.17637
0.37	1.03999	1.33459	1.33459	0.87	1.10622	0.86199	0.77922	1.37	1.45028	1.70397	1.17637
0.38	1.04204	1.37066	1.37066	0.88	1.10622	0.86199	0.77922	1.38	1.45028	1.70397	1.17637
0.39	1.04414	1.40673	1.40673	0.89	1.10622	0.86199	0.77922	1.39	1.45028	1.70397	1.17637
0.40	1.04629	1.44280	1.44280	0.90	1.10622	0.86199	0.77922	1.40	1.45028	1.70397	1.17637
0.41	1.04849	1.47887	1.47887	0.91	1.10622	0.86199	0.77922	1.41	1.45028	1.70397	1.17637
0.42	1.05074	1.51494	1.51494	0.92	1.10622	0.86199	0.77922	1.42	1.45028	1.70397	1.17637
0.43	1.05304	1.55101	1.55101	0.93	1.10622	0.86199	0.77922	1.43	1.45028	1.70397	1.17637
0.44	1.05539	1.58708	1.58708	0.94	1.10622	0.86199	0.77922	1.44	1.45028	1.70397	1.17637
0.45	1.05779	1.62315	1.62315	0.95	1.10622	0.86199	0.77922	1.45	1.45028	1.70397	1.17637
0.46	1.06024	1.65922	1.65922	0.96	1.10622	0.86199	0.77922	1.46	1.45028	1.70397	1.17637
0.47	1.06274	1.69529	1.69529	0.97	1.10622	0.86199	0.77922	1.47	1.45028	1.70397	1.17637
0.48	1.06529	1.73136	1.73136	0.98	1.10622	0.86199	0.77922	1.48	1.45028	1.70397	1.17637
0.49	1.06789	1.76743	1.76743	0.99	1.10622	0.86199	0.77922	1.49	1.45028	1.70397	1.17637
0.50	1.07054	1.80350	1.80350	1.00	1.10622	0.86199	0.77922	1.50	1.45028	1.70397	1.17637

TABLE 8A. Lanchester-Clifford-Schlafli Functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ for $\alpha = 3/5$ and x from 0.00 to 1.50.

[illegible]

$\alpha = 4/5$

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$
0.0000	1.00000	0.00000	0.00000	0.0000	1.07999	3.02345	2.00081	1.0000	1.3486	4.2677	3.4461
0.0005	1.00003	0.00001	0.00001	0.0005	1.08240	3.02345	2.00081	1.0005	1.3486	4.2677	3.4461
0.0010	1.00006	0.00002	0.00002	0.0010	1.08480	3.02345	2.00081	1.0010	1.3486	4.2677	3.4461
0.0015	1.00009	0.00003	0.00003	0.0015	1.08720	3.02345	2.00081	1.0015	1.3486	4.2677	3.4461
0.0020	1.00012	0.00004	0.00004	0.0020	1.08960	3.02345	2.00081	1.0020	1.3486	4.2677	3.4461
0.0025	1.00015	0.00005	0.00005	0.0025	1.09200	3.02345	2.00081	1.0025	1.3486	4.2677	3.4461
0.0030	1.00018	0.00006	0.00006	0.0030	1.09440	3.02345	2.00081	1.0030	1.3486	4.2677	3.4461
0.0035	1.00021	0.00007	0.00007	0.0035	1.09680	3.02345	2.00081	1.0035	1.3486	4.2677	3.4461
0.0040	1.00024	0.00008	0.00008	0.0040	1.09920	3.02345	2.00081	1.0040	1.3486	4.2677	3.4461
0.0045	1.00027	0.00009	0.00009	0.0045	1.10160	3.02345	2.00081	1.0045	1.3486	4.2677	3.4461
0.0050	1.00030	0.00010	0.00010	0.0050	1.10400	3.02345	2.00081	1.0050	1.3486	4.2677	3.4461
0.0055	1.00033	0.00011	0.00011	0.0055	1.10640	3.02345	2.00081	1.0055	1.3486	4.2677	3.4461
0.0060	1.00036	0.00012	0.00012	0.0060	1.10880	3.02345	2.00081	1.0060	1.3486	4.2677	3.4461
0.0065	1.00039	0.00013	0.00013	0.0065	1.11120	3.02345	2.00081	1.0065	1.3486	4.2677	3.4461
0.0070	1.00042	0.00014	0.00014	0.0070	1.11360	3.02345	2.00081	1.0070	1.3486	4.2677	3.4461
0.0075	1.00045	0.00015	0.00015	0.0075	1.11600	3.02345	2.00081	1.0075	1.3486	4.2677	3.4461
0.0080	1.00048	0.00016	0.00016	0.0080	1.11840	3.02345	2.00081	1.0080	1.3486	4.2677	3.4461
0.0085	1.00051	0.00017	0.00017	0.0085	1.12080	3.02345	2.00081	1.0085	1.3486	4.2677	3.4461
0.0090	1.00054	0.00018	0.00018	0.0090	1.12320	3.02345	2.00081	1.0090	1.3486	4.2677	3.4461
0.0095	1.00057	0.00019	0.00019	0.0095	1.12560	3.02345	2.00081	1.0095	1.3486	4.2677	3.4461
0.0100	1.00060	0.00020	0.00020	0.0100	1.12800	3.02345	2.00081	1.0100	1.3486	4.2677	3.4461
0.0105	1.00063	0.00021	0.00021	0.0105	1.13040	3.02345	2.00081	1.0105	1.3486	4.2677	3.4461
0.0110	1.00066	0.00022	0.00022	0.0110	1.13280	3.02345	2.00081	1.0110	1.3486	4.2677	3.4461
0.0115	1.00069	0.00023	0.00023	0.0115	1.13520	3.02345	2.00081	1.0115	1.3486	4.2677	3.4461
0.0120	1.00072	0.00024	0.00024	0.0120	1.13760	3.02345	2.00081	1.0120	1.3486	4.2677	3.4461
0.0125	1.00075	0.00025	0.00025	0.0125	1.14000	3.02345	2.00081	1.0125	1.3486	4.2677	3.4461
0.0130	1.00078	0.00026	0.00026	0.0130	1.14240	3.02345	2.00081	1.0130	1.3486	4.2677	3.4461
0.0135	1.00081	0.00027	0.00027	0.0135	1.14480	3.02345	2.00081	1.0135	1.3486	4.2677	3.4461
0.0140	1.00084	0.00028	0.00028	0.0140	1.14720	3.02345	2.00081	1.0140	1.3486	4.2677	3.4461
0.0145	1.00087	0.00029	0.00029	0.0145	1.14960	3.02345	2.00081	1.0145	1.3486	4.2677	3.4461
0.0150	1.00090	0.00030	0.00030	0.0150	1.15200	3.02345	2.00081	1.0150	1.3486	4.2677	3.4461
0.0155	1.00093	0.00031	0.00031	0.0155	1.15440	3.02345	2.00081	1.0155	1.3486	4.2677	3.4461
0.0160	1.00096	0.00032	0.00032	0.0160	1.15680	3.02345	2.00081	1.0160	1.3486	4.2677	3.4461
0.0165	1.00099	0.00033	0.00033	0.0165	1.15920	3.02345	2.00081	1.0165	1.3486	4.2677	3.4461
0.0170	1.00102	0.00034	0.00034	0.0170	1.16160	3.02345	2.00081	1.0170	1.3486	4.2677	3.4461
0.0175	1.00105	0.00035	0.00035	0.0175	1.16400	3.02345	2.00081	1.0175	1.3486	4.2677	3.4461
0.0180	1.00108	0.00036	0.00036	0.0180	1.16640	3.02345	2.00081	1.0180	1.3486	4.2677	3.4461
0.0185	1.00111	0.00037	0.00037	0.0185	1.16880	3.02345	2.00081	1.0185	1.3486	4.2677	3.4461
0.0190	1.00114	0.00038	0.00038	0.0190	1.17120	3.02345	2.00081	1.0190	1.3486	4.2677	3.4461
0.0195	1.00117	0.00039	0.00039	0.0195	1.17360	3.02345	2.00081	1.0195	1.3486	4.2677	3.4461
0.0200	1.00120	0.00040	0.00040	0.0200	1.17600	3.02345	2.00081	1.0200	1.3486	4.2677	3.4461
0.0205	1.00123	0.00041	0.00041	0.0205	1.17840	3.02345	2.00081	1.0205	1.3486	4.2677	3.4461
0.0210	1.00126	0.00042	0.00042	0.0210	1.18080	3.02345	2.00081	1.0210	1.3486	4.2677	3.4461
0.0215	1.00129	0.00043	0.00043	0.0215	1.18320	3.02345	2.00081	1.0215	1.3486	4.2677	3.4461
0.0220	1.00132	0.00044	0.00044	0.0220	1.18560	3.02345	2.00081	1.0220	1.3486	4.2677	3.4461
0.0225	1.00135	0.00045	0.00045	0.0225	1.18800	3.02345	2.00081	1.0225	1.3486	4.2677	3.4461
0.0230	1.00138	0.00046	0.00046	0.0230	1.19040	3.02345	2.00081	1.0230	1.3486	4.2677	3.4461
0.0235	1.00141	0.00047	0.00047	0.0235	1.19280	3.02345	2.00081	1.0235	1.3486	4.2677	3.4461
0.0240	1.00144	0.00048	0.00048	0.0240	1.19520	3.02345	2.00081	1.0240	1.3486	4.2677	3.4461
0.0245	1.00147	0.00049	0.00049	0.0245	1.19760	3.02345	2.00081	1.0245	1.3486	4.2677	3.4461
0.0250	1.00150	0.00050	0.00050	0.0250	1.20000	3.02345	2.00081	1.0250	1.3486	4.2677	3.4461
0.0255	1.00153	0.00051	0.00051	0.0255	1.20240	3.02345	2.00081	1.0255	1.3486	4.2677	3.4461
0.0260	1.00156	0.00052	0.00052	0.0260	1.20480	3.02345	2.00081	1.0260	1.3486	4.2677	3.4461
0.0265	1.00159	0.00053	0.00053	0.0265	1.20720	3.02345	2.00081	1.0265	1.3486	4.2677	3.4461
0.0270	1.00162	0.00054	0.00054	0.0270	1.20960	3.02345	2.00081	1.0270	1.3486	4.2677	3.4461
0.0275	1.00165	0.00055	0.00055	0.0275	1.21200	3.02345	2.00081	1.0275	1.3486	4.2677	3.4461
0.0280	1.00168	0.00056	0.00056	0.0280	1.21440	3.02345	2.00081	1.0280	1.3486	4.2677	3.4461
0.0285	1.00171	0.00057	0.00057	0.0285	1.21680	3.02345	2.00081	1.0285	1.3486	4.2677	3.4461
0.0290	1.00174	0.00058	0.00058	0.0290	1.21920	3.02345	2.00081	1.0290	1.3486	4.2677	3.4461
0.0295	1.00177	0.00059	0.00059	0.0295	1.22160	3.02345	2.00081	1.0295	1.3486	4.2677	3.4461
0.0300	1.00180	0.00060	0.00060	0.0300	1.22400	3.02345	2.00081	1.0300	1.3486	4.2677	3.4461
0.0305	1.00183	0.00061	0.00061	0.0305	1.22640	3.02345	2.00081	1.0305	1.3486	4.2677	3.4461
0.0310	1.00186	0.00062	0.00062	0.0310	1.22880	3.02345	2.00081	1.0310	1.3486	4.2677	3.4461
0.0315	1.00189	0.00063	0.00063	0.0315	1.23120	3.02345	2.00081	1.0315	1.3486	4.2677	3.4461
0.0320	1.00192	0.00064	0.00064	0.0320	1.23360	3.02345	2.00081	1.0320	1.3486	4.2677	3.4461
0.0325	1.00195	0.00065	0.00065	0.0325	1.23600	3.02345	2.00081	1.0325	1.3486	4.2677	3.4461
0.0330	1.00198	0.00066	0.00066	0.0330	1.23840	3.02345	2.00081	1.0330	1.3486	4.2677	3.4461
0.0335	1.00201	0.00067	0.00067	0.0335	1.24080	3.02345	2.00081	1.0335	1.3486	4.2677	3.4461
0.0340	1.00204	0.00068	0.00068	0.0340	1.24320	3.02345	2.00081	1.0340	1.3486	4.2677	3.4461
0.0345	1.00207	0.00069	0.00069	0.0345	1.24560	3.02345	2.00081	1.0345	1.3486	4.2677	3.4461
0.0350	1.00210	0.00070	0.00070	0.0350	1.24800	3.02345	2.00081	1.0350	1.3486	4.2677	3.4461
0.0355	1.00213	0.00071	0.00071	0.0355	1.25040	3.02345	2.00081	1.0355	1.3486	4.2677	3.4461
0.0360	1.00216	0.00072	0.00072	0.0360	1.25280	3.02345	2.00081	1.0360	1.3486	4.2677	3.4461
0.0365	1.00219	0.00073	0.00073	0.0365	1.25520	3.02345	2.00081	1.0365	1.3486	4.2677	3.4461
0.0370	1.00222	0.00074	0.00074	0.0370	1.25760	3.02345	2.00081	1.0370	1.3486	4.2677	3.4461
0.0375	1.00225	0.00075	0.00075	0.0375	1.26000	3.02345	2.00081	1.0375	1.3486	4.2677	3.4461
0.0380	1.00228	0.00076	0.00076	0.0380	1.26240	3.02345	2.00081	1.0380	1.3486	4.2677	3.4461
0.0385	1.00231	0.00077	0.00077	0.0385	1.26480	3.02345	2.00081	1.0385	1.3486	4.2677	3.4461
0.0390	1.00234	0.00078	0.00078	0.0390	1.26720	3.02345	2.00081	1.0390	1.3486	4.2677	3.4461
0.0395	1.00237	0.00079	0.00079	0.0395	1.26960	3.02345	2.00081	1.0395	1.3486	4.2677	3.4461
0.0400	1.00240	0.00080	0.00080	0.0400	1.27200	3.02345	2.00081	1.0400	1.3486	4.2677	3.4461
0.0405	1.00243	0.00081	0.00081	0.0405	1.27440	3.02345	2.00081	1.0405	1.3486	4.2677	3.4461
0.0410	1.00246	0.00082	0.00082	0.0410	1.27680	3.02345	2.00081	1.0410	1.3486	4.2677	3.4461
0.0415	1.00249	0.00083	0.00083	0.0415	1.27920	3.02345	2.00081	1.0415	1.3486	4.2677	3.4461
0.0420	1.00252	0.00084	0.00084	0.0420	1.28160	3.02345	2.00081	1.0420	1.3486	4.2677	

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	$\alpha = 4/5$
0.0000	1.82062	6.8873	3.35017	0.0000	2.44139	10.2188	3.8656	0.0000	97.1599	382.0954	3.94325	
0.0001	1.83330	6.88281	3.35422	0.0001	2.44139	10.2188	3.8656	0.0001	97.1599	382.0954	3.94325	
0.0002	1.84598	6.87830	3.35827	0.0002	2.44139	10.2188	3.8656	0.0002	97.1599	382.0954	3.94325	
0.0003	1.85866	6.87379	3.36232	0.0003	2.44139	10.2188	3.8656	0.0003	97.1599	382.0954	3.94325	
0.0004	1.87134	6.86928	3.36637	0.0004	2.44139	10.2188	3.8656	0.0004	97.1599	382.0954	3.94325	
0.0005	1.88402	6.86477	3.37042	0.0005	2.44139	10.2188	3.8656	0.0005	97.1599	382.0954	3.94325	
0.0006	1.89670	6.86026	3.37447	0.0006	2.44139	10.2188	3.8656	0.0006	97.1599	382.0954	3.94325	
0.0007	1.90938	6.85575	3.37852	0.0007	2.44139	10.2188	3.8656	0.0007	97.1599	382.0954	3.94325	
0.0008	1.92206	6.85124	3.38257	0.0008	2.44139	10.2188	3.8656	0.0008	97.1599	382.0954	3.94325	
0.0009	1.93474	6.84673	3.38662	0.0009	2.44139	10.2188	3.8656	0.0009	97.1599	382.0954	3.94325	
0.0010	1.94742	6.84222	3.39067	0.0010	2.44139	10.2188	3.8656	0.0010	97.1599	382.0954	3.94325	
0.0011	1.96010	6.83771	3.39472	0.0011	2.44139	10.2188	3.8656	0.0011	97.1599	382.0954	3.94325	
0.0012	1.97278	6.83320	3.39877	0.0012	2.44139	10.2188	3.8656	0.0012	97.1599	382.0954	3.94325	
0.0013	1.98546	6.82869	3.40282	0.0013	2.44139	10.2188	3.8656	0.0013	97.1599	382.0954	3.94325	
0.0014	1.99814	6.82418	3.40687	0.0014	2.44139	10.2188	3.8656	0.0014	97.1599	382.0954	3.94325	
0.0015	2.01082	6.81967	3.41092	0.0015	2.44139	10.2188	3.8656	0.0015	97.1599	382.0954	3.94325	
0.0016	2.02350	6.81516	3.41497	0.0016	2.44139	10.2188	3.8656	0.0016	97.1599	382.0954	3.94325	
0.0017	2.03618	6.81065	3.41902	0.0017	2.44139	10.2188	3.8656	0.0017	97.1599	382.0954	3.94325	
0.0018	2.04886	6.80614	3.42307	0.0018	2.44139	10.2188	3.8656	0.0018	97.1599	382.0954	3.94325	
0.0019	2.06154	6.80163	3.42712	0.0019	2.44139	10.2188	3.8656	0.0019	97.1599	382.0954	3.94325	
0.0020	2.07422	6.79712	3.43117	0.0020	2.44139	10.2188	3.8656	0.0020	97.1599	382.0954	3.94325	
0.0021	2.08690	6.79261	3.43522	0.0021	2.44139	10.2188	3.8656	0.0021	97.1599	382.0954	3.94325	
0.0022	2.09958	6.78810	3.43927	0.0022	2.44139	10.2188	3.8656	0.0022	97.1599	382.0954	3.94325	
0.0023	2.11226	6.78359	3.44332	0.0023	2.44139	10.2188	3.8656	0.0023	97.1599	382.0954	3.94325	
0.0024	2.12494	6.77908	3.44737	0.0024	2.44139	10.2188	3.8656	0.0024	97.1599	382.0954	3.94325	
0.0025	2.13762	6.77457	3.45142	0.0025	2.44139	10.2188	3.8656	0.0025	97.1599	382.0954	3.94325	
0.0026	2.15030	6.77006	3.45547	0.0026	2.44139	10.2188	3.8656	0.0026	97.1599	382.0954	3.94325	
0.0027	2.16298	6.76555	3.45952	0.0027	2.44139	10.2188	3.8656	0.0027	97.1599	382.0954	3.94325	
0.0028	2.17566	6.76104	3.46357	0.0028	2.44139	10.2188	3.8656	0.0028	97.1599	382.0954	3.94325	
0.0029	2.18834	6.75653	3.46762	0.0029	2.44139	10.2188	3.8656	0.0029	97.1599	382.0954	3.94325	
0.0030	2.20102	6.75202	3.47167	0.0030	2.44139	10.2188	3.8656	0.0030	97.1599	382.0954	3.94325	
0.0031	2.21370	6.74751	3.47572	0.0031	2.44139	10.2188	3.8656	0.0031	97.1599	382.0954	3.94325	
0.0032	2.22638	6.74300	3.47977	0.0032	2.44139	10.2188	3.8656	0.0032	97.1599	382.0954	3.94325	
0.0033	2.23906	6.73849	3.48382	0.0033	2.44139	10.2188	3.8656	0.0033	97.1599	382.0954	3.94325	
0.0034	2.25174	6.73398	3.48787	0.0034	2.44139	10.2188	3.8656	0.0034	97.1599	382.0954	3.94325	
0.0035	2.26442	6.72947	3.49192	0.0035	2.44139	10.2188	3.8656	0.0035	97.1599	382.0954	3.94325	
0.0036	2.27710	6.72496	3.49597	0.0036	2.44139	10.2188	3.8656	0.0036	97.1599	382.0954	3.94325	
0.0037	2.28978	6.72045	3.50002	0.0037	2.44139	10.2188	3.8656	0.0037	97.1599	382.0954	3.94325	
0.0038	2.30246	6.71594	3.50407	0.0038	2.44139	10.2188	3.8656	0.0038	97.1599	382.0954	3.94325	
0.0039	2.31514	6.71143	3.50812	0.0039	2.44139	10.2188	3.8656	0.0039	97.1599	382.0954	3.94325	
0.0040	2.32782	6.70692	3.51217	0.0040	2.44139	10.2188	3.8656	0.0040	97.1599	382.0954	3.94325	
0.0041	2.34050	6.70241	3.51622	0.0041	2.44139	10.2188	3.8656	0.0041	97.1599	382.0954	3.94325	
0.0042	2.35318	6.69790	3.52027	0.0042	2.44139	10.2188	3.8656	0.0042	97.1599	382.0954	3.94325	
0.0043	2.36586	6.69339	3.52432	0.0043	2.44139	10.2188	3.8656	0.0043	97.1599	382.0954	3.94325	
0.0044	2.37854	6.68888	3.52837	0.0044	2.44139	10.2188	3.8656	0.0044	97.1599	382.0954	3.94325	
0.0045	2.39122	6.68437	3.53242	0.0045	2.44139	10.2188	3.8656	0.0045	97.1599	382.0954	3.94325	
0.0046	2.40390	6.67986	3.53647	0.0046	2.44139	10.2188	3.8656	0.0046	97.1599	382.0954	3.94325	
0.0047	2.41658	6.67535	3.54052	0.0047	2.44139	10.2188	3.8656	0.0047	97.1599	382.0954	3.94325	
0.0048	2.42926	6.67084	3.54457	0.0048	2.44139	10.2188	3.8656	0.0048	97.1599	382.0954	3.94325	
0.0049	2.44194	6.66633	3.54862	0.0049	2.44139	10.2188	3.8656	0.0049	97.1599	382.0954	3.94325	
0.0050	2.45462	6.66182	3.55267	0.0050	2.44139	10.2188	3.8656	0.0050	97.1599	382.0954	3.94325	
0.0051	2.46730	6.65731	3.55672	0.0051	2.44139	10.2188	3.8656	0.0051	97.1599	382.0954	3.94325	
0.0052	2.47998	6.65280	3.56077	0.0052	2.44139	10.2188	3.8656	0.0052	97.1599	382.0954	3.94325	
0.0053	2.49266	6.64829	3.56482	0.0053	2.44139	10.2188	3.8656	0.0053	97.1599	382.0954	3.94325	
0.0054	2.50534	6.64378	3.56887	0.0054	2.44139	10.2188	3.8656	0.0054	97.1599	382.0954	3.94325	
0.0055	2.51802	6.63927	3.57292	0.0055	2.44139	10.2188	3.8656	0.0055	97.1599	382.0954	3.94325	
0.0056	2.53070	6.63476	3.57697	0.0056	2.44139	10.2188	3.8656	0.0056	97.1599	382.0954	3.94325	
0.0057	2.54338	6.63025	3.58102	0.0057	2.44139	10.2188	3.8656	0.0057	97.1599	382.0954	3.94325	
0.0058	2.55606	6.62574	3.58507	0.0058	2.44139	10.2188	3.8656	0.0058	97.1599	382.0954	3.94325	
0.0059	2.56874	6.62123	3.58912	0.0059	2.44139	10.2188	3.8656	0.0059	97.1599	382.0954	3.94325	
0.0060	2.58142	6.61672	3.59317	0.0060	2.44139	10.2188	3.8656	0.0060	97.1599	382.0954	3.94325	
0.0061	2.59410	6.61221	3.59722	0.0061	2.44139	10.2188	3.8656	0.0061	97.1599	382.0954	3.94325	
0.0062	2.60678	6.60770	3.60127	0.0062	2.44139	10.2188	3.8656	0.0062	97.1599	382.0954	3.94325	
0.0063	2.61946	6.60319	3.60532	0.0063	2.44139	10.2188	3.8656	0.0063	97.1599	382.0954	3.94325	
0.0064	2.63214	6.59868	3.60937	0.0064	2.44139	10.2188	3.8656	0.0064	97.1599	382.0954	3.94325	
0.0065	2.64482	6.59417	3.61342	0.0065	2.44139	10.2188	3.8656	0.0065	97.1599	382.0954	3.94325	
0.0066	2.65750	6.58966	3.61747	0.0066	2.44139	10.2188	3.8656	0.0066	97.1599	382.0954	3.94325	
0.0067	2.67018	6.58515	3.62152	0.0067	2.44139	10.2188	3.8656	0.0067	97.1599	382.0954	3.94325	
0.0068	2.68286	6.58064	3.62557	0.0068	2.44139	10.2188	3.8656	0.0068	97.1599	382.0954	3.94325	
0.0069	2.69554	6.57613	3.62962	0.0069	2.44139	10.2188	3.8656	0.0069	97.1599	382.0954	3.94325	
0.0070	2.70822	6.57162	3.63367	0.0070	2.44139	10.2188	3.8656	0.0070	97.1599	382.0954	3.94325	
0.0071	2.72090	6.56711	3.63772	0.0071	2.44139	10.2188	3.8656	0.0071	97.1599	382.0954	3.94325	
0.0072	2.73358	6.56260	3.64177	0.0072	2.44139	10.2188	3.8656	0.0072	97.1599	382.0954	3.94325	
0.0073	2.74626	6.55809	3.64582	0.0073	2.44139	10.2188	3.8656	0.0073	97.1599	382.0954	3.94325	
0.0074	2.75894	6.55358	3.64987	0.0074	2.44139	10.2188	3.8656	0.0074	97.1599	382.0954	3.94325	
0.0075	2.77162	6.54907	3.65392	0.0075	2.44139	10.2188	3.8656	0.0075	97.1599	382.0954	3.94325	
0.0076	2.78430	6.54456	3.65797	0.0076	2.44139	10.2188	3.8656	0.0076	97.1599	382.0954	3.94325	
0.0077	2.79698	6.54005	3.66202	0.0077	2.44139	10.2188	3.8656	0.0077	97.1599	382.0954	3.94325	
0.0078	2.80966	6.53554	3.66607	0.0078	2.44139	10.2188	3.8656	0.0078	97.1599	382.0954	3.94325	
0.0079	2.82234	6.53103	3.67012	0.0079	2.44139	10.2188	3.8656	0.0079	97.1599	382.0954	3.94325	
0.0080	2.83502	6.52652	3.67417									

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$	x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$	x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$	$\alpha = 3/7$
0.1000	2.59169	1.73254	0.68007	0.1000	4.71007	3.10173	0.71627	6.4	352.71948	190.50872	0.75384	
0.1000	2.61109	1.73350	0.68199	0.1000	4.72997	3.10270	0.71724	6.4	353.09247	190.50872	0.75384	
0.1000	2.63052	1.73446	0.68391	0.1000	4.74987	3.10367	0.71821	6.4	353.46546	190.50872	0.75384	
0.1000	2.64995	1.73542	0.68583	0.1000	4.76977	3.10464	0.71918	6.4	353.83845	190.50872	0.75384	
0.1000	2.66938	1.73638	0.68775	0.1000	4.78967	3.10561	0.72015	6.4	354.21144	190.50872	0.75384	
0.1000	2.68881	1.73734	0.68967	0.1000	4.80957	3.10658	0.72112	6.4	354.58443	190.50872	0.75384	
0.1000	2.70824	1.73830	0.69159	0.1000	4.82947	3.10755	0.72209	6.4	354.95742	190.50872	0.75384	
0.1000	2.72767	1.73926	0.69351	0.1000	4.84937	3.10852	0.72306	6.4	355.33041	190.50872	0.75384	
0.1000	2.74710	1.74022	0.69543	0.1000	4.86927	3.10949	0.72403	6.4	355.70340	190.50872	0.75384	
0.1000	2.76653	1.74118	0.69735	0.1000	4.88917	3.11046	0.72500	6.4	356.07639	190.50872	0.75384	
0.1000	2.78596	1.74214	0.69927	0.1000	4.90907	3.11143	0.72597	6.4	356.44938	190.50872	0.75384	
0.1000	2.80539	1.74310	0.70119	0.1000	4.92897	3.11240	0.72694	6.4	356.82237	190.50872	0.75384	
0.1000	2.82482	1.74406	0.70311	0.1000	4.94887	3.11337	0.72791	6.4	357.19536	190.50872	0.75384	
0.1000	2.84425	1.74502	0.70503	0.1000	4.96877	3.11434	0.72888	6.4	357.56835	190.50872	0.75384	
0.1000	2.86368	1.74598	0.70695	0.1000	4.98867	3.11531	0.72985	6.4	357.94134	190.50872	0.75384	
0.1000	2.88311	1.74694	0.70887	0.1000	5.00857	3.11628	0.73082	6.4	358.31433	190.50872	0.75384	
0.1000	2.90254	1.74790	0.71079	0.1000	5.02847	3.11725	0.73179	6.4	358.68732	190.50872	0.75384	
0.1000	2.92197	1.74886	0.71271	0.1000	5.04837	3.11822	0.73276	6.4	359.06031	190.50872	0.75384	
0.1000	2.94140	1.74982	0.71463	0.1000	5.06827	3.11919	0.73373	6.4	359.43330	190.50872	0.75384	
0.1000	2.96083	1.75078	0.71655	0.1000	5.08817	3.12016	0.73470	6.4	359.80629	190.50872	0.75384	
0.1000	2.98026	1.75174	0.71847	0.1000	5.10807	3.12113	0.73567	6.4	360.17928	190.50872	0.75384	
0.1000	2.99969	1.75270	0.72039	0.1000	5.12797	3.12210	0.73664	6.4	360.55227	190.50872	0.75384	
0.1000	3.01912	1.75366	0.72231	0.1000	5.14787	3.12307	0.73761	6.4	360.92526	190.50872	0.75384	
0.1000	3.03855	1.75462	0.72423	0.1000	5.16777	3.12404	0.73858	6.4	361.29825	190.50872	0.75384	
0.1000	3.05798	1.75558	0.72615	0.1000	5.18767	3.12501	0.73955	6.4	361.67124	190.50872	0.75384	
0.1000	3.07741	1.75654	0.72807	0.1000	5.20757	3.12598	0.74052	6.4	362.04423	190.50872	0.75384	
0.1000	3.09684	1.75750	0.72999	0.1000	5.22747	3.12695	0.74149	6.4	362.41722	190.50872	0.75384	
0.1000	3.11627	1.75846	0.73191	0.1000	5.24737	3.12792	0.74246	6.4	362.79021	190.50872	0.75384	
0.1000	3.13570	1.75942	0.73383	0.1000	5.26727	3.12889	0.74343	6.4	363.16320	190.50872	0.75384	
0.1000	3.15513	1.76038	0.73575	0.1000	5.28717	3.12986	0.74440	6.4	363.53619	190.50872	0.75384	
0.1000	3.17456	1.76134	0.73767	0.1000	5.30707	3.13083	0.74537	6.4	363.90918	190.50872	0.75384	
0.1000	3.19399	1.76230	0.73959	0.1000	5.32697	3.13180	0.74634	6.4	364.28217	190.50872	0.75384	
0.1000	3.21342	1.76326	0.74151	0.1000	5.34687	3.13277	0.74731	6.4	364.65516	190.50872	0.75384	
0.1000	3.23285	1.76422	0.74343	0.1000	5.36677	3.13374	0.74828	6.4	365.02815	190.50872	0.75384	
0.1000	3.25228	1.76518	0.74535	0.1000	5.38667	3.13471	0.74925	6.4	365.40114	190.50872	0.75384	
0.1000	3.27171	1.76614	0.74727	0.1000	5.40657	3.13568	0.75022	6.4	365.77413	190.50872	0.75384	
0.1000	3.29114	1.76710	0.74919	0.1000	5.42647	3.13665	0.75119	6.4	366.14712	190.50872	0.75384	
0.1000	3.31057	1.76806	0.75111	0.1000	5.44637	3.13762	0.75216	6.4	366.52011	190.50872	0.75384	
0.1000	3.32999	1.76902	0.75303	0.1000	5.46627	3.13859	0.75313	6.4	366.89310	190.50872	0.75384	
0.1000	3.34942	1.77000	0.75495	0.1000	5.48617	3.13956	0.75410	6.4	367.26609	190.50872	0.75384	
0.1000	3.36885	1.77096	0.75687	0.1000	5.50607	3.14053	0.75507	6.4	367.63908	190.50872	0.75384	
0.1000	3.38828	1.77192	0.75879	0.1000	5.52597	3.14150	0.75604	6.4	368.01207	190.50872	0.75384	
0.1000	3.40771	1.77288	0.76071	0.1000	5.54587	3.14247	0.75701	6.4	368.38506	190.50872	0.75384	
0.1000	3.42714	1.77384	0.76263	0.1000	5.56577	3.14344	0.75798	6.4	368.75805	190.50872	0.75384	
0.1000	3.44657	1.77480	0.76455	0.1000	5.58567	3.14441	0.75895	6.4	369.13104	190.50872	0.75384	
0.1000	3.46599	1.77576	0.76647	0.1000	5.60557	3.14538	0.75992	6.4	369.50403	190.50872	0.75384	
0.1000	3.48542	1.77672	0.76839	0.1000	5.62547	3.14635	0.76089	6.4	369.87702	190.50872	0.75384	
0.1000	3.50485	1.77768	0.77031	0.1000	5.64537	3.14732	0.76186	6.4	370.25001	190.50872	0.75384	
0.1000	3.52428	1.77864	0.77223	0.1000	5.66527	3.14829	0.76283	6.4	370.62300	190.50872	0.75384	
0.1000	3.54371	1.77960	0.77415	0.1000	5.68517	3.14926	0.76380	6.4	370.99600	190.50872	0.75384	
0.1000	3.56314	1.78056	0.77607	0.1000	5.70507	3.15023	0.76477	6.4	371.36900	190.50872	0.75384	
0.1000	3.58257	1.78152	0.77799	0.1000	5.72497	3.15120	0.76574	6.4	371.74200	190.50872	0.75384	
0.1000	3.60199	1.78248	0.77991	0.1000	5.74487	3.15217	0.76671	6.4	372.11500	190.50872	0.75384	
0.1000	3.62142	1.78344	0.78183	0.1000	5.76477	3.15314	0.76768	6.4	372.48800	190.50872	0.75384	
0.1000	3.64085	1.78440	0.78375	0.1000	5.78467	3.15411	0.76865	6.4	372.86100	190.50872	0.75384	
0.1000	3.66028	1.78536	0.78567	0.1000	5.80457	3.15508	0.76962	6.4	373.23400	190.50872	0.75384	
0.1000	3.67971	1.78632	0.78759	0.1000	5.82447	3.15605	0.77059	6.4	373.60700	190.50872	0.75384	
0.1000	3.69914	1.78728	0.78951	0.1000	5.84437	3.15702	0.77156	6.4	373.98000	190.50872	0.75384	
0.1000	3.71857	1.78824	0.79143	0.1000	5.86427	3.15799	0.77253	6.4	374.35300	190.50872	0.75384	
0.1000	3.73799	1.78920	0.79335	0.1000	5.88417	3.15896	0.77350	6.4	374.72600	190.50872	0.75384	
0.1000	3.75742	1.79016	0.79527	0.1000	5.90407	3.15993	0.77447	6.4	375.09900	190.50872	0.75384	
0.1000	3.77685	1.79112	0.79719	0.1000	5.92397	3.16090	0.77544	6.4	375.47200	190.50872	0.75384	
0.1000	3.79628	1.79208	0.79911	0.1000	5.94387	3.16187	0.77641	6.4	375.84500	190.50872	0.75384	
0.1000	3.81571	1.79304	0.80103	0.1000	5.96377	3.16284	0.77738	6.4	376.21800	190.50872	0.75384	
0.1000	3.83514	1.79400	0.80295	0.1000	5.98367	3.16381	0.77835	6.4	376.59100	190.50872	0.75384	
0.1000	3.85457	1.79496	0.80487	0.1000	6.00357	3.16478	0.77932	6.4	376.96400	190.50872	0.75384	
0.1000	3.87399	1.79592	0.80679	0.1000	6.02347	3.16575	0.78029	6.4	377.33700	190.50872	0.75384	
0.1000	3.89342	1.79688	0.80871	0.1000	6.04337	3.16672	0.78126	6.4	377.71000	190.50872	0.75384	
0.1000	3.91285	1.79784	0.81063	0.1000	6.06327	3.16769	0.78223	6.4	378.08300	190.50872	0.75384	
0.1000	3.93228	1.79880	0.81255	0.1000	6.08317	3.16866	0.78320	6.4	378.45600	190.50872	0.75384	
0.1000	3.95171	1.79976	0.81447	0.1000	6.10307	3.16963	0.78417	6.4	378.82900	190.50872	0.75384	
0.1000	3.97114	1.80072	0.81639	0.1000	6.12297	3.17060	0.78514	6.4	379.20200	190.50872	0.75384	
0.1000	3.99057	1.80168	0.81831	0.1000	6.14287	3.17157	0.78611	6.4	379.57500	190.50872	0.75384	
0.1000	4.00999	1.80264	0.82023	0.1000	6.16277	3.17254	0.78708	6.4	379.94800	190.50872	0.75384	
0.1000	4.02942	1.80360	0.82215	0.1000	6.18267	3.17351	0.78805	6.4	380.32100	190.50872	0.75384	
0.1000	4.04885	1.80456	0.82407	0.1000	6.20257	3.17448	0.78902	6.4	380.69400	190.50872	0.75384	
0.1000	4.06828	1.80552	0.82599	0.1000	6.22247	3.17545	0.79000	6.4	381.06700	190.50872	0.75384	
0.1000	4.08771	1.80648	0.82791	0.1000	6.24237	3.17642	0.79097	6.4	381.44000	190.50872	0.75384	
0.1000	4.10714	1.80744	0.82983	0.1000	6.26227	3.17739	0.79194	6.4	381.81300	190.50872	0.75384	
0.1000	4.12657	1.80840	0.83175	0.1000	6.28217	3.17836	0.79291	6.4	382.18600	190.50872	0.75384	
0.1000	4.14599	1.80936	0.83367									

Q = 4/7

[illegible]