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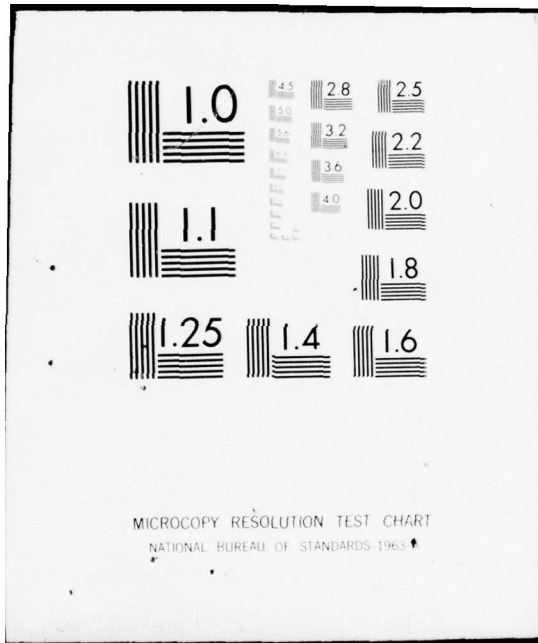
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OPTIMIZATION OF A TRANSPORTATION SYSTEM PLANNING PROBLEM,

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Final Report,

by

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Wilbert E. Wilhelm

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31 December 31, 1977

Approved for public release; distribution unlimited.

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ABSTRACT

This study investigated the problem of synthesizing a minimum cost transportation system plan to service forecast shipment requirements among a set of points in such a manner as to satisfy aggregate ship-time performance levels. Both commercial and dedicated modes may be used in the transportation plan; but the latter must be designed in detail including specification of vehicles to be used, the route each is to service, and arrival/departure time schedules. The problem, which is important to operations of the Air Force Logistics Command, requires prescription of a transport network capable of providing acceptable levels of service to the shipments which it accommodates.

Research objectives included: refinement of a modeling approach initiated during the 1976 USAF/ASEE Summer Faculty Program, development of solution approaches, and assessment of computation time necessary to solve problems of realistic size. Study results satisfy each objective.

A large-scale mixed, 01 integer, linear programming model of the planning problem is developed and simplified for solution by applying Benders' decomposition to yield two more simple, interacting subproblems. One of these, a linear program which assures ship-time performance, is amenable to efficient solution techniques (Generalized Upper Bounding and column generation) for which specialized algorithms are presented.

Several formulations of the other subproblem, which defines the dedicated mode network, were developed. The first requires enumeration of a set of

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feasible vehicle tours from which the dedicated mode may be designed. A special purpose, implicit enumeration algorithm applicable to this model is described.

Three additional formulations, each of which constructs vehicle tours directly, were investigated. One model, a linear program amenable to large-scale programming techniques, constructs tours via a column generation procedure and may be used in a branch-and-bound solution algorithm. An alternate model casts the problem as a modified material flow-circulation problem. Characteristics of this model which may be exploited to devise an efficient, implicit enumeration algorithm are described. Finally, approaches based on constraint aggregation and Generalized Lagrange Multipliers were investigated in the attempt to reduce the tour construction problem to a straightforward flow-circulation problem.

In all cases, the tactical, vehicle scheduling problem is treated subsequently using an available linear programming approach. Collectively, the results appear to offer the capability to solve transportation system planning problems of realistic size.

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CHAPTER I  
INTRODUCTION

The Air Force operates a complex logistics system which is composed of transportation, inventory, and repair subsystems. These components are related by the function each performs and through the management policies which guide system operations. The transportation subsystem is of particular importance, since it has a significant influence over the capital investment required for spare parts and since it provides a critical service necessary for maintaining acceptable numbers of weapons systems in operational condition. This study is directed to the design of the transportation system operated by the Air Force Logistics Command (AFLC). However, the mathematical structure of the problem is common to a variety of systems (including multi-modal transportation and other collection-distribution networks such as that operated by the postal service) and may be specialized for application in other contexts.

In order to optimize the entire logistics system, an optimal balance must be achieved amongst its operating components: transportation, inventory, and repair. If resupply time be reduced, fewer investment items need be purchased to provide equivalent logistics support to accomplish the prescribed Air Force mission. Since many investment items managed by the Air Force are very costly, it is important to be able to prescribe an optimal trade-off between inventory investment and transportation expense. Dr. A. Kovacs, a colleague

on the 1976 USAF/ASEE Summer Faculty Research Program, has examined the logistics system problem. He developed several models which describe essential features of the logistics system and showed how they might be used in a prescriptive model to define the optimal trade-off between transportation expense and inventory investment. In order to accomplish this overall objective, Dr. Kovacs' logistics system model requires the functional relationship of minimum transportation cost to service level. The Transportation System Planning Model, which is the topic of this research, can define that necessary functional relationship. Results of this study may therefore be used to optimize the transportation system plan as well as to provide guidance in making important decisions which integrate operations of the entire logistics system.

#### Problem Statement

The essential features of the transportation system planning problem are stated concisely below so that all components of the problem addressed are identified.

Given expected daily shipping requirements amongst all points in the logistics system categorized by the shipping times required and, perhaps, by characteristics which might be used in vehicle/mode assignment decisions (i. e., weight, volume, special handling needs).

Design an optimal, minimum total annual cost transportation system plan which defines specific routes by which shipments are to be

made, assures required shipping time for each material category, and determines the appropriate balance in the use of commercial modes and dedicated modes, which should be specified if they prove to be cost effective.

To provide the necessary flexibility, the dedicated modal network could (potentially) use a number of different types of aircraft and trucks. The optimal system design must specify dedicated modes in detail including the best types and numbers of vehicles to be used and designation of the route to be served by each vehicle. Characteristics of each vehicle such as weight/volume capacity, speed, endurance, and maintenance positioning requirements need be considered. The arrival/departure schedules of each vehicle at each point it services must be specified since this schedule is important to coordinating dock operations, to vehicle crew needs, and to service levels afforded material flows.

The system problem encompasses three important aspects of design: vehicle route synthesis, material flowtime control, and determination of vehicle schedules. The optimization model should permit detailed sensitivity analyses of important factors such as shipping time requirements and transportation priority scheme. The model should also facilitate evaluation of management policies such as the use of a single point to stock second-echelon inventories. Development of these post-optimality analysis procedures is not a part of this study,

but the need for such detailed evaluation was identified and used to guide development of the basic modeling capability.

A brief description of the operating procedures used in the current system is given in Appendix A. The description indicates the practical importance of each facet of the problem statement and its relationship to AFLC needs.

#### Summary of Approach and Report Outline

A model of the transportation system planning problem was developed during the 1976 USAF/ASEE Summer Faculty Program and presented in the proposal submitted to AFOSR which lead to this study. The original model, which is given in Appendix B of this report, incorporated the essential features of the transportation system planning problem but offers little opportunity to *develop efficient solution procedures because of the large number of integer variables and constraints in the formulation.* A significant improvement in formulation was developed and submitted to AFOSR while the research proposal was in review. The second model, which is listed in Appendix C, employed a modeling approach which allowed a significant reduction in the number of variables and constraints required to express the problem and offered a better opportunity to develop successful solution approaches.

Research during this study has provided further, significant improvements in model formulation, resulting in the forms presented in this report. Experience gained in developing the first two formulations led to the more recent version which is much more simple and which is amenable to efficient solution procedures which were developed during this study.

Solution strategy allows the vehicle scheduling component of the problem to be treated separately according to a method developed by a study conducted at the Air Force Institute of Technology (AFIT). The material flow and vehicle routing components are treated jointly in each of several solution procedures.

Given a network of tours which are traversed by vehicles in dedicated modes and knowledge of commercial modes available, the planning problem reduces to a multi-commodity material flow problem subject to the important ship time constraints. A model representing this flow problem is developed in Chapter III and efficient solution procedures are described. The model is a large-scale linear program which is amenable to Generalized Upper Bounding and column generation techniques. This model might be used to evaluate a proposed, dedicated mode network, or, perhaps, to design such a network heuristically. Most importantly, the model is used as a basic component of more comprehensive models which also incorporate the vehicle routing component.

Vehicles made available for use by the dedicated mode network are assumed to be defined by applicable operating characteristics such as carrying capacity, tour length limitations, and tour origin/termination points (which are dictated by vehicle maintenance-facility location). One approach, which was developed to provide an optimal transportation system plan relies upon the assumption that, at least in some applications, it is efficient to define a set of feasible vehicle tours from which the optimal dedicated mode network may be designed.

This approach, which is described in Chapter IV, may use either a limited set of tours which is likely to include the optimal solution, or the set of all feasible tours which may be enumerated rapidly for problems of limited size. A large-scale mixed, 01 integer, linear programming model is shown to include both the material flow and vehicle tour components. Efficient solution procedures are developed for this model applying Benders' decomposition to derive two interacting subproblems which invoke existing theory to determine optimal transportation system plans. One of the subproblems is (essentially) the same as the material flow model discussed in Chapter III and the other is amenable to efficient solution by a special purpose, implicit enumeration algorithm designed specifically for that purpose.

A third -- and more highly sophisticated approach -- was researched and is described in Chapter V. This approach relies upon the assumption that, in some applications, it may be best to employ a solution procedure which is capable of constructing vehicle tours, rather than merely selecting tours from among a set which is provided for use. Three strategies are described to implement this approach by which vehicle tours are constructed as an integral component of the solution procedure.

The first strategy leads to a large-scale, mixed integer linear programming model. The program is simplified by application of Benders' decomposition yielding the material flow (linear) program and an interacting integer problem which completely defines the vehicle network. This binary problem bears a close relationship to well known network problems of the flow circulation type.

Characteristics which allow efficient solution and techniques to exploit these characteristics in an implicit enumeration algorithm are discussed.

The second strategy gives rise to a model formulation which is amenable to efficient branch-and bound procedures. This strategy incorporates linear programming techniques such as Generalized Upper Bounding and column generation using a shortest path subproblem.

The third strategy examines the opportunity to construct vehicle tours using an efficient flow circulation algorithm. Techniques for aggregating constraints into the objective to yield a pure flow-circulation problem are evaluated. Generalized Lagrange multiplier techniques are also considered as a means of effecting this more simple structure.

A literature review which describes prior work in each of the three areas related in the transportation system planning problem appears in Chapter II. In particular, the vehicle scheduling approach developed in the AFIT study is described in that chapter.

A final chapter records conclusions from this study and outlines recommendations for continued research on the transportation system planning problem. A large number of variables interact in the planning problem and the notation required to represent the problem is lengthy. A complete listing of all notation used in the body of this report is therefore given in Appendix D for convenience.

## CHAPTER II

### LITERATURE REVIEW

A rather thorough search of the literature has revealed that there is little history on the stated problem which requires simultaneous optimization of a network design and control of multicommodity material flowtimes. There are, however, several recent studies which are of interest because of the approaches they take to solving related problems. There is also a well developed literature on each of the three main aspects of the planning problem--vehicle network design, multicommodity material flows through networks, and vehicle scheduling. Each of these areas is reviewed briefly in this section.

#### Related Problems

Two master's theses addressed to the design of the Logair route structure have recently been completed at the Air Force Institute of Technology (AFIT). Building on the experience gained by the Palmatier and Prescott (1975) study, McPherson and O'Hara (1976) formulate a limited version of the network flow design problem which minimizes a total cost function

$$\min \sum_{ij} (dC_{ij} + f) Z_{ij}$$

subject to range limitation on sortee length, material conservation constraints

$$\sum_{ij} X_{ijk} + R_{jk} = \sum_{l} X_{jlk}, \quad \forall j+k$$
$$i \neq j, i \neq k$$

vehicle capacity restrictions

$$\sum_k X_{ijk} \leq d_{ij} Z_{ij}; \quad \forall i, j, k$$

and vehicle conservation constraints

$$\sum_i Z_{ij} = \sum_i Z_{ji}$$

in which

$$X_{ijk} \geq 0; \quad Z_{ij} = 0, 1, \dots, n$$

$d$  = cost per mile

$C_{ij}$  = distance from point  $i$  to  $j$

$f$  = aircraft landing fee

$Z_{ij}$  = number of times arc  $ij$  is flown =  $0, 1, 2, \dots, n$

$X_{ijk}$  = tons shipped from  $i$  to  $j$  bound for destination  $k$

$d_{ij}$  = aircraft capacity on arc  $ij$

$R_{jk}$  = total requirements originating at  $j$  for  $k$

This model is a large scale, mixed integer linear program (MILP) which the authors solved using a standard, although sophisticated, algorithm available on the AFLC computer system. Three serious limitations arise in this formulation. First, optimal tours are not specified for vehicles. Secondly, transportation time for a particular shipment cannot be determined because the model considers only total cargo shipped out of each point rather than accounting for the route of each shipment. This omission will not allow a useful measure of service level to be developed. Finally, it took three hours of computation time (on a Honeywell 650 computer) to solve a twelve point problem, so applicability of this approach is limited to small network problems.

Three other studies are of interest because they successfully apply large scale programming techniques to related network design problems. Rao and Zoints (1968) studied the problem of minimizing the cost of transporting commodities amongst various ports by vessels which are routed according to model prescription. They employ a column generation scheme for which subproblems are solved by the efficient out-of-kilter algorithm. Computationally, the approach is efficient, but integer solutions do not result and connected vessel routes are not specified, so application of the model is limited.

Richardson (1976) formulated an airline routing design problem as a MILP problem and solved it by application of Benders' decomposition method. A standard linear programming "subproblem" and a 0, 1 integer "master" problem resulted and the latter was solved by a special purpose algorithm. Computation times for problems of realistic size (26 points) were modest (less than two minutes on a DEC-10/1055 computer).

Geoffrion and Graves (1974) studied a multicommodity distribution problem involving 14 manufacturing plants, 45 possible distribution center sites, 121 customer zones and 17 commodity classes. They solved the large scale MILP problem by a Benders' decomposition formulation which efficiently solved subproblems of the transportation type and a master 0, 1 problem using a specially designed, hybrid branch-and-bound/cutting plane approach. Computation times less than one minute using an IBM 360/91 computer were reported. The authors also present formidable arguments supporting their optimization approach since it allows comprehensive sensitivity studies and detailed analysis of managerial alternatives.

While the last three studies were not addressed specifically to the transportation system problem, they do indicate the computational success of certain large scale programming techniques in solving MILP models of related form. Other examples could be cited, but these should provide sufficient background for later discussion.

### Vehicle Network Design

The classical approach to designing a network through which vehicles transport goods is through the multiple traveling salesman formulation. This problem requires each of  $n$  points to be serviced once by only one of the  $m$  vehicles provided in the system; vehicles must collectively execute their itinerancy at minimum total cost. This problem may be expressed mathematically (see Taha (1971, Svestka (1973), and Miller, et al. (1960) ) as:

$$\min \sum_{ij} d_{ij} x_{ij} \quad [1]$$

subject to:

$$\sum_i X_{ij} = 1 \quad j=1, 2, \dots, n \quad [2]$$

$$\sum_j X_{ij} = 1 \quad i=1, 2, \dots, n \quad [3]$$

$$Y_i - Y_j + (n-m) X_{ij} \leq n-m-1 \quad [4]$$

in which  $d_{ij}$  is the cost of traversing the arc from  $i$  to  $j$  and  $X_{ij}$  is 1 if a vehicle travels over the arc between points  $i$  and  $j$  and "0" otherwise. The last constraint requires each vehicle tour to be a single, connected cycle in the network. Different formulations to invoke the connected-cycle requirement have given rise to a number of mathematical models which define the basic traveling salesman

problem. Golden, Magnanti, and Nguyen (1977) provide a rather complete taxonomy of integer formulations of this type.

Balinski and Quandt (1964) suggested an alternate type of program which is essentially a set partitioning problem:

$$\min \sum_{j=1}^n C_j X_j \quad [5]$$

s. t.

$$\sum_{j=1}^n a_{ij} X_j = 1; \quad i=1, 2, \dots, m \quad [6]$$

$$x_j = 0, 1; \quad j=1, 2, \dots, n \quad [7]$$

in which

$n$  is the total number of tours satisfying vehicle capacity constraints.

$a_{ij} = 1$  if delivery route  $j$  visits point  $i$ , 0 otherwise

$C_j =$  cost of delivery route  $j$

$X_j = 1$  if delivery route  $j$  is used, 0 otherwise.

The authors developed a column generation technique to solve problem [5] through [7]. The approach has apparently not been further refined by more recent research.

The (multiple-) vehicle routing problem has received a great deal of attention and a number of solution approaches have been developed. Three of these general approaches are of interest with respect to the transportation system planning problem.

The subtour elimination method solves the classical assignment problem consisting of equations [1], [2], and [3] by an efficient linear programming code.

Zero-one solutions result (since the problem structure is unimodular); but, in general, connected tours are not obtained. Branch and bound algorithms have been devised to eliminate subtours, assuring satisfaction of equation [4]. Svestka and Huckfeldt (1973) report favorable computational experience with this approach solving 60-city problems with mean run time of 80 seconds on a UNIVAC 1108 computer. However, their  $d_{ij}$  values were randomly generated and the resulting matrix of values was asymmetric ( $d_{ij} \neq d_{ji}$ ). Bellmore and Malone (1971) have shown that the more practical, symmetric case ( $d_{ij} = d_{ji}$ ) is more difficult to solve. In particular, the symmetric case requires an efficient technique to eliminate subtours of length two.

Little, et al. (1963) developed an efficient branch-and-bound algorithm for solving the traveling salesman problem. As discussed by Pierce (1969), this algorithm may be adapted to solve the multiple vehicle problem with additional constraints to reflect practical limitations such as vehicle capacity and time schedule. Problems with 25 cities may be solved in approximately 30 seconds on an IBM 7094 computer, although run time increases by a factor of 10 for each additional 7-10 cities. This particular algorithm is useful only for the asymmetric ( $d_{ij} \neq d_{ji}$ ) case. The transportation system problem assumes symmetric costs for a vehicle to travel between two points ( $d_{ij} = d_{ji}$ ). Approaches such as that developed by Little, et al. (1963) are, in general, ineffective and, for the most part, inappropriate in the case of symmetric distances. Complexities which characterize the symmetric case are stipulated in some detail by Steckhan (1970).

A variety of heuristic procedures have also been proposed. While they offer the advantage of relatively low computation times for large scale problems, they cannot guarantee an optimal solution. Historically, the savings approach initiated by Clarke and Wright (1964) has been the most thoroughly researched. The fundamental concept used by the method is that an arc is added to a vehicle tour if its inclusion would result in an overall cost saving. The technique was applied to single and multiple terminal delivery problems by Tillman (1968 and 1969), and embellishments to the basic approach were suggested by Tillman and Hering (1971), and Holmes and Parker (1976). Several studies--Gaskell (1967), Christofides and Eilon (1969), and Webb (1972)--evaluated the performance of the savings approach and found it to be acceptable in comparison to other procedures. Other, more sophisticated heuristics have been developed--Wren and Holiday (1972), Gillett and Miller (1974), and Lin and Kernighan (1973) --and shown to produce good results with reasonable computation time, even for large problems. Golden, Magnanti, and Nguyen (1977) combined the savings approach with efficient computational procedures and claim that the resulting algorithm is characterized by very low run times. It appears, however, that their algorithm terminates after quickly finding a "local minimum" (i. e., a solution which cannot be improved by making a simple, pairwise reallocation of points to vehicle routes).

Two additional approaches have been initiated in recent work. Held and Karp (1970), (1971) have shown that the traveling salesman problem may be solved by formulation of the appropriate subproblem which defines a minimum spanning tree which also forms a connected cycle including all points in the

network. Foster and Ryan (1976) incorporate characteristics of optimal solutions to routing problems (as defined by earlier work) in an efficient approach which solves the vehicle routing problem using linear programming algorithms adapted to handle special cases which arise in these problems.

A more detailed evaluation of vehicle routing problems is not presented here since they are related to the systems planning problem in a limited way. The problems are similar in the respect that both involve large scale 0, 1 integer programming complications. The research discussed seems to be that which is most closely related to the current problem. Excellent surveys of the work in this area have been provided by Bellmore and Nemhauser (1968); Pierce (1969); and Turner, Ghare, and Fourds (1974). A rather comprehensive bibliography of related studies is included in this report for easy reference.

#### Multicommodity Network Flows

Problems related to multicommodity flows in capacitated networks also encompass a large body of literature. Ford and Fulkerson (1958) initiated work in this area by developing an efficient algorithm to determine maximal multicommodity flows in capacitated networks. Gomory and Hu (1964) studied the problem of synthesizing a network to satisfy time varying requirements. Tomlin (1966) formulated the problem of satisfying multicommodity flow requirements in an existing, capacitated network at minimum total cost. He studied both node-arc and arc-chain formulations and indicated efficient, large scale programming techniques which might be applied to solve problems of realistic size.

Cremeans, Smith, and Tyndall (1970) reported an approach which provides significant insight into the proposed planning problem. The authors developed a minimum cost formulation of multicommodity flows (with preassigned origins and destinations) subject to capacity and resource constraints, and delivery requirements. Given a commercial/dedicated mode network design, minimum cost flow plans could be developed using this general approach. The possibility of solving the transportation system planning problem using this general approach is discussed more fully in Chapter III. Cremeans et al. utilize an arc chain incidence matrix which defines the arcs included in each possible chain (path) from origin to destination for all commodity flows. The arc-chain approach offers the advantage that shipping time can be calculated and included in the problem formulation, while the node-arc formulation examined by McPherson and O'Hara (1976) does not provide this important capability. Given the incidence matrix of a particular network design, the decision to be made in the optimization problem is simply how much flow to prescribe for each chain. The authors describe an efficient solution procedure which utilizes a shortest path algorithm in a column generation scheme. The paper reports "encouraging" results time-wise on problems involving 150 commodities, 1000 arcs, and 50 resources.

Weigel and Cremeans (1972) enhanced the initial model to allow more realistic treatment of practical considerations such as vehicles required per time period and node capacity constraints. A problem involving 290 arcs, 42 resources, and nine origin-destination pairs was solved on a CDC 6400 computer

in 30 minutes. The authors indicate that run time is sensitive to the particular problem formulation used.

#### Vehicle Scheduling

A variety of vehicle scheduling problems have appeared in the literature. Several types are closely related to the systems planning problem and are briefly outlined here to record similarities and to allow comparison of previous solution approaches with the one selected for application to the current problem.

Laderman, Gleiberman, and Egan (1966) studied the basic problem of allocating ships to (existing) routes using a linear programming (LP) formulation. Their objective was to satisfy requirements for transporting a single cargo considering vessel capacities and the times at which they would be available. Conley, et al. (1968) also used an LP model to minimize the total cost of shipping a single commodity through a rather large network. A more detailed problem involving several commodities, transshipment points, and two aircraft types was studied by Gould (1971). His work, which was sponsored by the Military Aircraft Command (MAC), developed an LP model to minimize the costs of moving aircraft over existing routes subject to operational constraints. Bennington (1970) was also able to formulate an LP model of a problem involving movement of military forces over an existing network to achieve a desired schedule of arrivals. He employed a node-chain formulation to describe flows and also included realistic details such as vehicle repositioning. Collectively, these studies represent the capability of LP models to prescribe material flows and vehicle schedules. The proposed systems planning problem requires prescription

of an optimal routing network in addition to flow and time considerations.

Several papers have been addressed to this more comprehensive problem.

Lampkin (1966) solved a problem involving prescription of four aspects of municipal bus service: route structure design, service frequency, detailed timetables, and bus schedules. The author simplified the problem for solution, dealing with different facets in a piecewise fashion. He used a heuristic procedure to specify a reasonable route structure.

Bellmore (1971) studied a similar problem of determining a time schedule and vessel routing which maximizes a utility function. A fleet of dissimilar tankers was employed to satisfy delivery requirements for a single commodity. Delivery dates were constrained by predetermined limits. The LP model was solved using Dantzig-Wolfe decomposition. A branch and bound algorithm (of the Land and Doig type) was devised to obtain final, integer results, since a fractional solution would, in general, be obtained from the LP solution.

A similar problem of selecting tankers with suitable characteristics and determining routes so that cargo is delivered within a specified time range at minimum total cost was researched by McKay (1974). Employing a pragmatic approach, he defined routes using a heuristic method (which was not discussed) and solved the remaining portion as an LP problem using a "selective" rounding procedure to obtain integer results.

These studies have been able to resolve network-flow-scheduling problems related to the transportation system problem. However, none would permit treatment of the transit time of individual shipments. This is a major limitation,

since shipping time is the prime measure of service level afforded by a multi-model transportation system. Additionally, the scheduling problem posed is more dynamic in nature. Delivery times to transshipment points cannot be pre-defined within useful limits. Rather, the schedules of all vehicles which service a transfer point should be co-ordinated to promote optimal service levels.

One study which addresses only the (dynamic) vehicle arrival/departure schedule was performed by Moberly and Gorychka (1976) at the AFIT. Designed to improve the efficiency of the current Logair system, the study minimizes shipping time for an established route structure. The objective function consists of terms of two types:  $W_{ij} (x_{ij} - y_{ij})$ , and  $W'_{ij} (x_{ij} - y_{ij})$ . Decision variables  $x_{ij}$  and  $y_{ij}$  represent vehicle departure and arrival times, respectively, of flight  $i$  at point  $j$ . The factor  $W_{ij}$  applied to ground time,  $(x_{ij} - y_{ij})$ , weights the objective function to represent the amount of cargo shipped through point  $j$  on vehicle  $i$ . Analogously, weighting factor  $W'_{ij}$  represents cargo transferred between flights at point  $j$ . Summing over all terms of this type, the objective function results in minimizing total weighted shipping time. Their LP model incorporates constraints which assure at least the necessary minimum ground time and flight time between points as well as relationships between flights involved in material transshipments. Numerical results show significant improvement over 1976 Logair schedules, which were designed by negotiations with the contractor.

Vehicle scheduling contributes to total system effectiveness only by allowing some transshipments to be made expeditiously. The ability to exploit 'optimum' schedules and actually achieve potential ship time improvements relies upon material handling capabilities which allow shipments to be transshipped expeditiously.

In many cases, material handling requirements impose transshipment delays on the order of several days. Therefore it appears that vehicle scheduling represents a marginal opportunity to improve total system effectiveness (particularly with respect to the problems encountered by the Air Force Logistics Command which were used as a basis for identifying the transportation system planning problem). It is therefore assumed in this study that marginal improvements made possible by vehicle scheduling may be achieved by implementing the Moberly/Gorychka (1976) once 'optimal' vehicle routes and material flows have been defined.

This assumption reduces the transportation system problem to two components. Each of these is a classical problem type which has been thoroughly researched as indicated by the literature review. However, the primary difficulty encountered by most existing approaches (particularly those used in vehicle routing) is that there are no procedures available to allow the necessary interaction between the two problem types. This interaction is necessary so that, for example, vehicle routes may be redesigned to provide improved material flow capabilities. This study initiates research on this topic and indicates approaches which do provide such capability.

## CHAPTER III

### A MODEL FOR EVALUATING ALTERNATIVES

The model presented in this chapter might be applied in cases in which management needs a means of evaluating a particular transportation system plan devised from experience or, perhaps, from a considered review of historical trends in an existing transportation system. Certain operating systems may be so complex that it may not be possible to develop a realistic model to determine an optimal plan. In some cases, the system complexity may impose a mathematical structure for which no practical solution method is available. In other cases, management may like to retain the decision making function they have performed historically, yet need a tool to provide an objective measure of the economic implications of their plan and to assess the ship time performance afforded by the overall transportation system.

In addition to providing measures of the effectiveness of a particular plan, the model might be used in the decision making process to select the preferred plan from a set of feasible alternatives. Alternately, the model might point out certain weaknesses of a proposed plan so that marginal improvements could be identified and incorporated in the system design stage.

Finally, and perhaps most importantly, the model is actually a basic component of more complex, optimizing models which are discussed in later chapters. As in the heuristic applications, the model is used to evaluate the particular system designs in the more advanced model forms.

The model is described in the first section of this chapter. Efficient solution procedures are discussed in detail in the second section.

#### Model Description

Given a network of vehicle tours traversed by components of the dedicated mode(s), the transportation system planning problem reduces to a multicommodity material flow problem. In actuality, it is not possible to state a set of necessary and sufficient conditions for the dedicated mode network aside from the system objective of minimizing total costs subject to ship time and operating constraints. The utility of a particular network design cannot be evaluated until it is complete. There are no criteria, other than the system criteria, by which this network can be designed. In contrast, in the more simple, *multiple vehicle-routing type problems*, the network of vehicle tours may be defined with respect to criteria which require one vehicle to visit each point each day, or, perhaps, which provide sufficient vehicle capacity to satisfy the needs of each point. The transportation system problem has no such criteria for the vehicle network. Rather, the primary design criteria is to minimize total system cost. Since commercial modes are also available for use, the material flow problem must determine the manner in which the dedicated mode network is to be used, and, as a consequence, provide measures of system cost and performance.

The material flow problem must determine an optimal policy of using both dedicated and commercial shipping modes. In order to satisfy the important measure of ship time performance, it is necessary for the flow problem to

completely define the paths from source,  $s$ , to destination,  $d$ , for shipments made. These paths, which are also specific to the type of material,  $m$ , being shipped, are called flow chains. Once the flow chain paths are defined, the amount of material to be shipped across the flow chain must be determined.

Figure 1 presents the multicommodity material flow model, which is very similar to the one proposed by Cremeans, et al. (1970). Total system cost is composed of the dedicated mode cost plus that of the commercial modes as stated by the objective function, equation [8].

The model is a linear program in which the decision variables are scaled to indicate the portion of some  $msd$  shipment requirement,  $f_{msd}$ , which is accommodated by a specific flow chain. As stated, the model allows direct  $s$  to  $d$  shipments on either commercial air,  $S_{msd}^{rA}$ , or commercial truck,  $S_{msd}^{rT}$ . It is assumed that a feasible, albeit costly, solution is always possible through the exclusive use of commercial air. Provision of the more economical, yet more time consuming, commercial truck mode allows obvious trade-offs to be considered in system design. Additional commercial modes and/or complicated shipping paths which use both commercial and dedicated modes could be easily incorporated in this type of model.

Equation [9] invokes ship time standards which are the primary measure of system performance. Constraints of this form could be included for each type of material, for specific  $msd$  combinations, and/or for all shipments in an overall system performance requirement.

The total amount of material transshipped through each point in the network may be constrained by material handling capability and/or available

$$\text{Min. } f_{\text{msd}} \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right] \quad [8]$$

Subject to:

Ship Time Standards

$$\sum_{\ell \in \mu(m)} \tau_{\ell}^{(t)} f_{\text{msd}} P_{\ell}^{(t)} + \sum_{\text{msd}} f_{\text{msd}} (\tau_{\text{msd}}^{\text{A}} S_{\text{msd}}^{\text{rA}} + \tau_{\text{msd}}^{\text{T}} S_{\text{msd}}^{\text{rT}}) \leq \Gamma_m \quad m=1, 2, \dots, M \quad [9]$$

Transshipment Point Capacity

$$\sum_{\ell=1}^L t_{i\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq \bar{b}_i \quad i=1, 2, \dots, I \quad [10]$$

Vehicle/Arc Capacity

$$\sum_{\ell=1}^L a_{q\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq b_q \quad q=1, 2, \dots, Q \quad [11]$$

Ship Requirements

$$\sum_{\ell=1}^L q_{\text{msd},\ell} P_{\ell}^{(t)} + S_{\text{msd}}^{\text{rA}} + S_{\text{msd}}^{\text{rT}} = 1.0 \quad \forall \text{msd} \quad [12]$$

Non-negativity Constraints

$$P_{\ell}^{(t)}, S_{\text{msd}}^{\text{rA}}, S_{\text{msd}}^{\text{rT}} \geq 0 \quad \forall \ell, \text{msd} \quad [13]$$

Figure 1. --The Multicommodity Material Flow Problem

warehouse space. Equation [10] incorporates this type of practical restriction on material flows. In fact, management may wish to allow transshipments at only a few points in the network for administrative reasons. It should be noted, however, that the opportunity to transship among vehicles greatly enhances the carrying capacity of the dedicated mode.

Commercial modes are assumed to have infinite capacity, but vehicles in the dedicated mode provide limited capacity. The amount of material shipped on each arc traversed by a vehicle in the dedicated mode must, therefore, observe this capacity restraint as imposed by equation [11]. Vehicles in the network provided would tend to be used extensively since the flow problem incurs no cost for use of the dedicated mode ( $C_e = 0$  is assumed).

Ship requirements,  $f_{msd}$ , among all msd combinations are assumed to be known and must be satisfied by the system plan. Equation [12] invokes this operating requirement.

In some applications, there may be additional operating constraints which need be imposed. However, the basic characteristics of problems of this type are included in this model. One closely allied problem which may be encountered in practical situations is the case in which limited funds are provided to operate a transportation system and the best possible ship time performance must be obtained within the funding limitation. This problem could be solved using the material flow model and solution procedure by incorporating two minor modifications. The first would redefine the objective in order to minimize  $\Gamma_M$  (or, perhaps, some weighted sum of the  $\Gamma_m$ ). The second would incorporate the cost function, not as the objective, but as an operating constraint.

As formulated, the flow problem is a large-scale linear program. The structure of this program is analyzed in the next section and an efficient solution procedure is described.

#### Solution Procedure

The material flow problem is expressed in matrix notation in Figure 2. Matrix dimensions are stated in Appendix E. Two particular characteristics of the model are evident in this formulation. First, ship requirements impose a large number of constraints in practical problems for which there are a large number of points and/or material categories in the system. Secondly, there are an extremely large number of columns, each of which represents one possible flow chain or shipment route for some msd combination in the dedicated mode network. Fortunately, efficient computational devices are available to simplify each of these complexities.

#### Generalized Upper Bounding

All of the msd ship requirement constraints may be treated implicitly by the Generalized Upper Bounding (GUB) procedure, which is described in detail by Lasdon (1970), pages 324-340. The procedure incorporates the Revised Simplex method of solving a linear program. Since equation [12] stipulates that the sum of all flows for each msd combination must equal one, there must be at least one flow for each combination. Each basic feasible solution, therefore, has at least one column associated with each msd ship requirement. One basic column for each msd is defined as the "key column" for the msd set and other

Min  $P_o$

Subject to:

$$IS^S + \tau_f^A S^{rA} + \tau_f^T S^{rT} + \tau_f P = \Gamma$$

$$IS^t + \eta_f P = \bar{b}$$

$$IS^a + A_f P = \underline{b}$$

$$P_o - C_f^r A S^{rA} - C_f^r T S^{rT} = 0$$

$$IR^r + IS^{rA} + IS^{rT} + G P = 1$$

Associated Dual Variable
$\alpha$
$\beta$
$\gamma$
$d_o$
$\delta$

Figure 2. --The Multicommodity Material Flow Problem  
Matrix Notation

columns, if any, for that set are expressed as a function of the key column. This approach allows the ship requirements to be handled implicitly so that a working basis consisting only of nonkey, basic columns is sufficient for representing a basic feasible solution. Since the inverse of the working basis,  $B^{-1}$ , need be updated at each iteration of the Revised Simplex method, computation time is greatly enhanced by using the smaller, working basis which consists of only  $(M + I + Q + 1)$  rows and columns.

The GUB procedure also provides an efficient method to determine the initial basic feasible solution. Initially,  $S_m^s = \Gamma_m$ ,  $S_i^t = \bar{b}_i$ ,  $S_q^a = b_q$ , and  $R_{msd}^r = 1.0$ . A Phase I procedure is needed to replace the artificial variables,  $R_{msd}^r$ . This replacement is readily accomplished, since it was assumed that the commercial air mode is capable of delivering all shipments in such a way that all ship time standards are satisfied. Since the  $R_{msd}^r$  are initially defined as the key variables (columns) and there are no nonkey members in the working basis, the replacement may be accomplished merely by redefining the key variables to be the  $S_{msd}^{rA}$ . The working basis need not be updated in this process, conserving run time. Should commercial air not provide a feasible solution in some application, the standard "Big M" method should be applied in a routine Phase I process.

The basic feasible solution which results from the suggested Phase I procedure is very costly, since all shipments are accommodated by premium transportation. A second stage in the solution process solves the material flow problem using only commercial modes. The value of this solution,  $B_0$ ,

represents an upper bound on the total system cost, since dedicated modes would be specified only if they can reduce total cost below  $B_0$ .

In reporting computational experience, Lasdon (1970) states that the Generalized Upper Bounding procedure may decrease computer run time by a factor of 10 (in comparison to the straightforward Revised Simplex method). A problem with 2813 variables, the equivalent of 780 msd combinations, and 39 rows/columns in  $B^{-1}$  was solved on an IBM 7094 computer in 15 minutes. Run times of this order of magnitude are certainly necessary to successful application of the material flow model.

#### Column Generation

The large number of columns associated with material flow chains-- paths from  $s$  to  $d$  in the dedicated network--may be treated implicitly by a column generation procedure. This approach does not require explicit enumeration of all possible flow chains in the network; rather, flow chains are defined by solving a set of shortest-path subproblems to identify the column to enter the basis at each iteration of the Revised Simplex method.

This basic approach was first suggested by Ford and Fulkerson (1958). Cremeans, Smith, and Tyndall (1970) and Weigel and Cremeans (1972) report successful application of the approach in solving large-scale, multicommodity network flow problems. The most recent paper reports run time on the order of 20 minutes to solve a problem with 341 rows on a CDC 6400 computer. Since the Generalized Upper Bounding procedure is also being applied in the current research, the computational burden associated with the large number of rows in

that example would also be ameliorated using the approach discussed here.

The column generation procedure is described in detail in the remainder of this section. A typical column in  $P, K_{\ell}$ , includes ship time, transshipment point load, flow chain designation, and msd set membership information:

$$K_{\ell} = [ (0 \dots \tau_{\ell}^{(t)} f_{msd} \dots 0 \tau_{\ell}^{(t)} f_{msd}) (t_{1\ell} f_{msd} \dots t_{I\ell} f_{msd}) (a_{1\ell} f_{msd} \dots a_{Q\ell} f_{msd}) (0) (g_{1\ell} \dots g_{msd, \ell}) ]^T \quad [14]$$

If the columns are defined explicitly, the  $\tau_{\ell}^{(t)}$ ,  $t_{i\ell}$ ,  $a_{q\ell}$ , and  $g_{n\ell}$  need be specified for each flow chain:

- $\tau_{\ell}^{(t)}$  = ship time for material type  $m$  on flow chain  $\ell$
- $t_{i\ell}$  = 1 if flow chain  $\ell$  transships at point  $i$ , 0 otherwise
- $a_{q\ell}$  = 1 if flow chain  $\ell$  traverses vehicle arc  $q$ , 0 otherwise
- $g_{n\ell}$  = 1 if flow chain  $\ell$  is associated with  $msd = n$ , 0 otherwise.

Since the columns are to be generated, each of these must be treated as an unknown decision variable in the subproblems which define flow chains.

At each iteration of the Revised Simplex method, each row has an associated Simplex multiplier calculated from  $C_B B^{-1}$ . ( $C_B$  is the vector of objective function coefficients for basic variables arranged as the columns of  $B$ .) The vector of simplex multipliers is defined as

$$C_B B^{-1} = [ (\alpha_1 \alpha_2 \dots \alpha_M) (\beta_1 \beta_2 \dots \beta_I) (\gamma_1 \gamma_2 \dots \gamma_Q) (d_0) (\delta_1 \delta_2 \dots \delta_{msd}) ]^T \quad [15]$$

Since the objective is to minimize, the column,  $j$ , which maximizes " $(Z_j - C_j) > 0$ " enters solution at each iteration. Using the formulation in Figure 2,  $C_B$  is a vector of zeroes, except the last element is one. The Simplex multipliers defined by  $C_B B^{-1}$  are therefore the elements in the last row of  $B^{-1}$  at each iteration. The column criterion,

$$\max_{\mathbf{l}} \left\{ C_B B^{-1} K_{\mathbf{l}} - C_{\mathbf{l}} \right\} > 0$$

may be formulated using equations [14] and [15]:

$$\max_{\mathbf{l}} \left[ (\alpha_m + \alpha_M) (\tau_{\mathbf{l}}^{(t)} f_{\text{msd}}) + \sum_{i=1}^I \beta_i (t_{i\mathbf{l}} f_{\text{msd}}) + \sum_{q=1}^Q \gamma_q (a_{q\mathbf{l}} f_{\text{msd}}) + (0) d_0 + \sum_{j=1}^{\text{msd}} \delta_j (g_{j\mathbf{l}}) - C_{\mathbf{l}} \right] > 0 \quad [16]$$

Ship time on flow chain  $\mathbf{l}$  may be expressed in detail as:

$$\tau_{\mathbf{l}}^{(t)} = O_{sm} + \sum_{q=1}^Q T_q^t a_{q\mathbf{l}} + \sum_{i=1}^I T_{im}^S t_{i\mathbf{l}} + \sum_{i=1}^I T_{i\mathbf{l}}^e X_{i\mathbf{l}} + U_{dm} \quad [17]$$

Since each flow chain is related to some specific msd combination, let

$$\bar{\delta}_{\mathbf{l}} = \sum_{j=1}^{\text{msd}} \delta_j (g_{j\mathbf{l}}) \quad [18]$$

The approach allows a slack variable ( $S^s, S^t, S^a$ ) to enter into solution if its corresponding Simplex Multiplier ( $\alpha_m, \beta_i, \gamma_q$ ) is positive. All  $\alpha_m, \beta_i,$  and  $\gamma_q$  are therefore negative in this column generation procedure.  $\bar{\delta}_{\mathbf{l}}$  may be either positive or negative, although if it is nonpositive, column  $\mathbf{l}$  could not enter solution, since it could not "price out" at a positive value. Combining equations [16], [17], and [18], using absolute value signs, and defining

$$|\bar{\alpha}_l| = |\alpha_m| + |\alpha_M| ,$$

the criterion to determine the entering column is:

$$\begin{aligned} \max_l \left\{ \bar{\delta}_l - f_{msd} \left[ \sum_{i=1}^I (|\bar{\alpha}_l| T_{im}^s + |\beta_i|) t_{il} \right. \right. \\ + \sum_{q=1}^Q (|\bar{\alpha}_l| T_q^t + |\gamma_q|) a_{ql} \\ + \sum_{i=1}^I (|\bar{\alpha}_l| T_{il}^e) X_{il} \\ \left. \left. + |\bar{\alpha}_l| (O_{sm} + U_{dm}) \right] - C_l \right\} > 0 \end{aligned}$$

$$t_{il}, a_{ql}, X_{il} = 0, 1 \text{ and form a connected path from } s \text{ to } d. \quad [19]$$

The value of this criterion for each msd combination may be found by solving a shortest path problem. Figure 3 depicts a typical subproblem of this type. Two vehicles are provided in the dedicated mode network. Vehicle A traverses the route which visits points 1, 2, and 3 in that order; and vehicle B services the route consisting of points 5, 4, and 2. Overnight ship time delays are represented by the arcs between nodes 1A and 1A' as well as between 5B and 5B'. Transshipments may occur at point 2. Arc 'distances' or costs are labeled according to equation [19] components. Suppose a shipment of material m begins at source  $s = 1$  and is destined for  $d = 4$ . Arcs from a 'dummy' source, S, to  $s = 1$  and from  $d = 4$  to 'dummy' destination, D, account for the constants related to the msd combination. Dotted arcs representing transshipments are assigned 'distances' according to the coefficients of  $t_{il}$ , solid lines showing the

EXAMPLE VEHICLE NETWORK

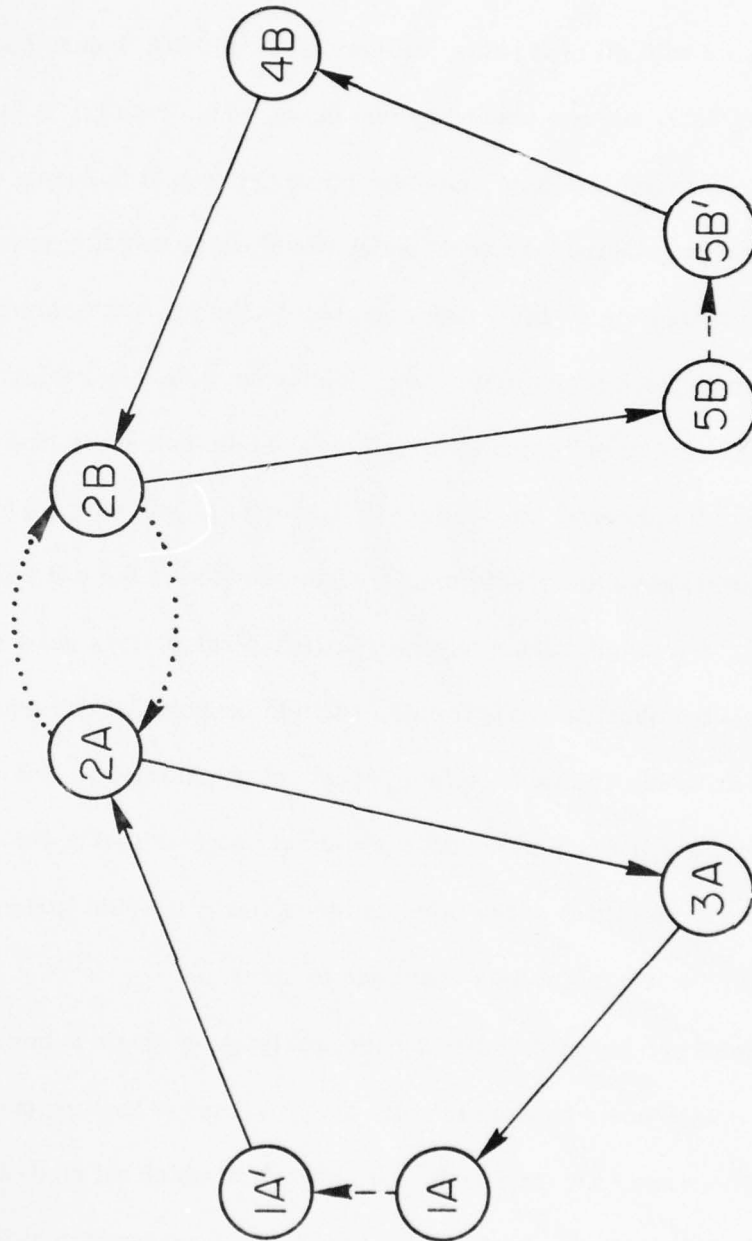


Figure 3

arcs traversed by vehicles are assigned the 'distances' given by the  $a_{q\ell}$  coefficients. The shortest path from S to D would be the set of arcs connecting points S-1A'-2A-2B-5B-5B'-4B-D in that order. The path includes 'distance' constants associated with all msd paths, utilizes both vehicles, incurs one overnight ship time delay and one transshipment delay. This path is, in fact, the only feasible one in this example since any other path would include a subcycle (a visit to some point more than once) which would unnecessarily increase ship time and require vehicle capacity which might be used for other shipments.

At each Revised Simplex iteration, a shortest path subproblem need be solved for each msd combination to determine the best column to enter. Given the path defined for the msd, the column criterion is given by equation [19]. The column which gives the maximum positive criterion is the one to enter at this iteration. If a column has a negative criterion value, it is not a candidate to enter; and if the shortest paths for all msd yield nonpositive criteria, the current solution is optimal since no column could enter and improve the current solution. A column already in solution would not be generated again by this procedure since (by definition of the Simplex Multipliers) it would 'price out' at zero and would therefore not be a candidate to enter.

In lieu of solving msd shortest path problems, a single subproblem appropriately defined could be used to solve 2, 3, ..., msd of the original subproblems simultaneously. Figure 4 depicts a network in which all msd subproblems could be solved as a single shortest path problem. A mathematical statement of this single problem is:

AGGREGATED SHORTEST PATH NETWORK  
FOR COLUMN GENERATION

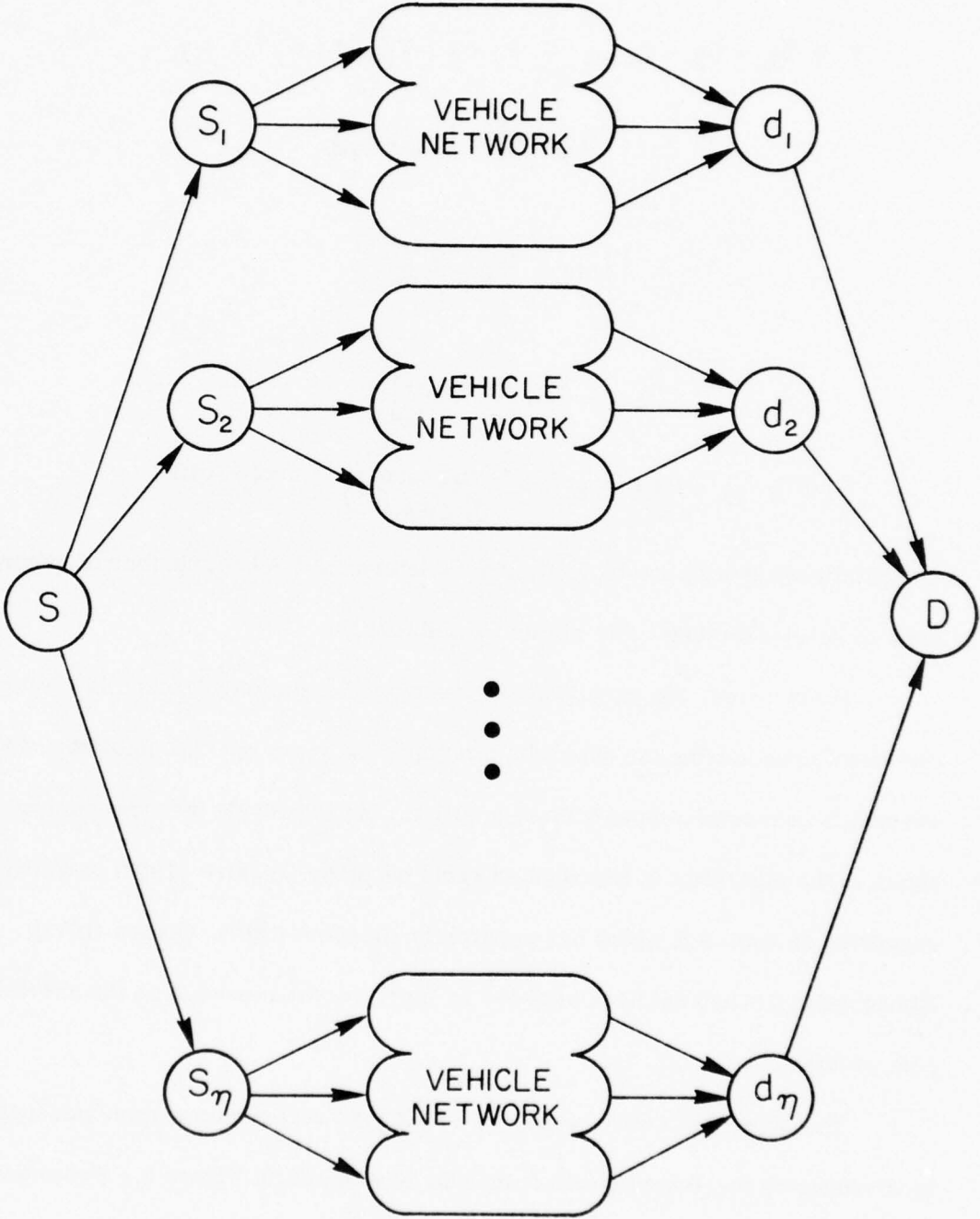


Figure 4

Min Z

Subject to:

$$\begin{aligned}
 Z \cong & \bar{\delta}_l - C_l - f_{msd} \left[ \sum_{i=1}^I (|\bar{\alpha}_l| T_{im}^s + |\beta_i|) t_{il} \right. \\
 & + \sum_{q=1}^Q (|\bar{\alpha}_l| T_q^t + |\gamma_q| a_{ql}) \\
 & + \sum_{i=1}^I (|\bar{\alpha}_l| T_{il}^e) X_{il} \\
 & \left. + |\bar{\alpha}_l| (O_{sm} + U_{dm}) \right]
 \end{aligned}$$

$Z \geq 0, t_{il}, a_{ql}, X_{il} = 0, 1$  and form a connected path from s to d.

Computational experience is necessary to determine the best number of subproblems to solve simultaneously in this formulation.

In any event, the usefulness of this overall solution approach is largely dependent upon solving the subproblem(s) efficiently. A special algorithm was devised to conserve computer storage space, and to provide efficient computation times. The algorithm is based on an early paper by Dijkstra (1959) in which he suggested an approach which has apparently (Dreyfus (1967), Golden (1976), Elmaghraby (1970) ) not been bettered by more recent research on the shortest path problem.

The algorithm solves a set,  $\eta$ , of the msd subproblems simultaneously by discovering the shortest path from S to D as shown in Figure 4. Processing steps operate on the nodes N in the set of subnetworks, placing them on list A if

the shortest path from S to N has been discovered, on list B if some path from S to N has been defined, and on list C if no path from S to N has yet been defined. To begin, all nodes for  $msd \subseteq \eta$  are on list C. The source nodes, s, are removed and assigned to list A and all nodes accessible from the source nodes are placed on list B. Entries on list B are ordered low to high with respect to the distance from S to N. At each iteration, the first entry on list B, N, is removed and placed on list A since the shortest path from S to N is now evident--any other path from S to N would necessarily be longer. Each node J which is accessible from node N is then checked to see if the path through N to J is better than any identified previously. List B and, if necessary, list C are updated accordingly. The shortest path from S to D is known when any of the  $d_{msd}$  for  $msd \subseteq \eta$  reaches the top of list B. The column criterion may then be calculated to determine if this flow chain should become the next entering column. If no flow chain which satisfied equation [19] is defined in this procedure, the current solution is optimal for the material flow problem.

In addition to using the Dijkstra approach as the underlying mechanism in the shortest path algorithm, several additional features are incorporated to promote efficiency. Arc 'distances' need not be calculated until needed, so the 'distances' of some number of arcs may never be calculated. Nodes and arcs need not be added to represent transshipments and overnight vehicle terminal delays as suggested by the conceptual presentation in Figure 3. Rather, these factors may be treated without expanding network size. Finally, an interval bisection procedure may be used to reorder nodes on list B. This procedure is

expected to be efficient in solving large problems. A detailed statement of the algorithm follows.

Step 0: Initialization

- A. Initially, all nodes,  $I$ , for  $\text{msd}$  combinations  $\subseteq \eta$  are on list  $C$  with  $D(I) = 0$ . A vector of data for each vehicle arc defines the beginning node,  $I$ ; the ending node,  $J$ ; the associated vehicle,  $k$ ; the number of the successor arc traversed by vehicle  $k$ ,  $\text{NSUC}$ ; the number of transshipments possible at point  $J$  and the successor vehicle arc numbers for each transshipment.

Step 1: Originating Flow Chains

- A. For each  $\text{msd} \subseteq \eta$

Remove the source node,  $s$ , from list  $C$ . Calculate

$$D(s) = -\bar{\delta}_l - C_l + f_{\text{msd}} \mid \bar{\alpha}_{\text{msd}} \mid O_{sm}$$

If  $D(s) \geq 0$ , drop this  $\text{msd}$  combination since it could not provide the entering column. Otherwise, add node  $s$  to list  $A$ , since  $D(s)$  is the shortest length path from  $S$  to  $s$ . Process each arc  $q$  which emanates from node  $s$  in subnetwork  $\text{msd}$  according to step 1/B.

- B. For each arc emanating from  $s$  in  $\text{msd} \subseteq \eta$

Calculate the distance along arc  $q$  and at its end point,  $J$ ,

$$\bar{D}_q = ( \mid \bar{\alpha}_{\text{msd}} \mid T_q^t + \mid \gamma_q \mid )$$

If point  $J$  is the destination,  $d_{\text{msd}}$ , add  $\mid \bar{\alpha}_{\text{msd}} \mid U_{Jm}$  to  $\bar{D}_q$  to account for unload time at the destination. If  $J \neq d_{\text{msd}}$ , and  $J$  is the termination

point for the  $k$  tour, add  $|\bar{\alpha}_{\text{msd}}| T_{i\ell}^e$  to  $\bar{D}_q$  to account for overnight ship time delay before next-day movement. Calculate distance measure

$$D(J) = D(s) + f_{\text{msd}} \bar{D}_q.$$

If  $D(J) \geq 0$ , drop this potential path, since it would not be able to provide the shortest (nonpositive) path from  $S$  to  $J$ . Otherwise, remove  $J$  from list  $C$  and place it on list  $B$  in order of increasing  $D(\cdot)$  for the nodes on list  $B$ . Record the  $\text{msd}$  subnetwork associated with this node ( $\text{NET}$ ) and the predecessor of node  $J$  on list  $A$ ,  $\text{PRED}(J) = s$ .

C. After all source nodes for  $\text{msd} \subseteq \eta$  and the vehicle arcs which emanate from them have been processed, go to step 2.

Step 2: General Iterative Process

A. If list  $B$  is empty, stop; the entering column cannot be from the set  $\eta$ . Otherwise, remove the first entry from list  $B$ , node  $N$ . The shortest path from  $S$  to  $N$  is now known to be the stored value,  $D(N)$ , the predecessor node on this path is  $\text{PRED}(N)$ , the applicable  $\text{msd}$  subnetwork is the stored value,  $\text{NET}$ . Add node  $N$  to list  $A$ . If  $N$  is the destination,  $d_{\text{msd}}$ , go to step 4. If there is no successor arc,  $\text{NSUC}$ , for the vehicle which brought the flow chain to  $N$ , go to step 3. Otherwise, go to step 2/B.

B. The successor arc  $q$  for vehicle  $k$  begins at point  $N$  and ends at point  $J$ .

Determine the 'distance' along arc  $q$  and at its end point:

$$\bar{D}_q = (|\bar{\alpha}_{\text{msd}}| T_q^t + |\gamma_q|).$$

As in step 1/B, update  $\bar{D}_q$  if point J is the destination,  $d_{msd}$ , or, if  $J \neq d_{msd}$  and J is the termination point for vehicle k.

Calculate distance measure

$$D(J) = D(N) + \bar{D}_q.$$

If  $D(J) \geq 0$ , drop this potential path to J and go to step 3.

C. If node J in this msd subnetwork is on list C, remove it and order it on list B using measure  $D(J)$ . Record the msd network (NET) for J and set  $PRED(J) = N$ . Go to step 3.

D. If node J in this msd subnetwork is already on list B and if  $D(J)$  calculated above is greater than the stored distance from S to J,  $D(J)$ , go to step 3. Otherwise, a shorter path from S to J has been found. Give  $D(J)$  its new value calculated above, reorder J on list B, and record  $PRED(J) = N$ . Go to step 3.

### Step 3: Transshipments

A. If there can be no transshipments at node J, return to step 2/A.

Otherwise process each of the vehicle arcs which transship out of node J according to the following substeps.

B. The flow chain leaves vehicle k at point J, incurs delay  $T_{Jm}^s$ , and departs J on vehicle k' on arc q bound for point J'.

Determine the distance along arc q and at its end point:

$$\bar{D}_q = (|\bar{\alpha}_q| T_{Jm}^s + \beta_J) + (|\bar{\alpha}_q| T_q^t + |\gamma_q|)$$

Update  $\bar{D}_q$  for  $U_{Jm}$  and  $T_{Jl}^e$  (if appropriate) as in step 2/B.

Calculate distance measure

$$D(J') = D(J) + \bar{D}_q .$$

If  $D(J') \geq 0$ , drop this potential path to  $J'$  .

Otherwise, process node  $J'$  using  $D(J')$  according to steps 2/C and 2/D.

C. Return to step 2/A after all possible transshipments out of  $J$  have been examined.

Step 4: Destination Node  $d_{msd}$  has been reached.

- A. Node  $d_{msd}$  was just removed from list B in step 2/A, the shortest path from S to any destination for the  $msd \subseteq \eta$  has thus been identified (the algorithm generates paths from S to nodes in increasing distance so no path to another destination could be better than the one just discovered). Trace the path from  $d_{msd}$  back to S using the PRED(N) data. Accumulate the total ship time along this flow chain as defined by equation [17], and also accumulate the total column criterion as defined by equation [19]. If the column criterion is greater than the current best, record the new column data as defined in equation [19].
- B. If all  $msd$  have been included in some subproblem, stop. (Either the best entering column has been defined or it has been shown that no column will enter and the current solution is optimal). Otherwise, increment the set  $\eta$  to include a new set of the  $msd$  combinations and go to step 0.

## CHAPTER IV

### AN OPTIMIZING APPROACH

A hypothesized economic relationship between the cost of the dedicated mode vehicle network and the total system cost is depicted in Figure 5. If no dedicated mode be provided, the minimum cost shipping plan using commercial modes exclusively is  $B_0$  as described in the previous chapter. Provision of a 'small' dedicated mode network may actually increase total system expenses, since vehicles in the dedicated mode may not be used to capacity. As the scale of the dedicated mode is increased, more transshipments become possible and material flow is facilitated. A variety of network designs may be possible for any particular level of expense. However, each design may give rise to different material flow characteristics and, hence, to different system cost. This phenomenon is reflected by the band in which total costs are hypothesized to lie. As the dedicated mode is made larger and larger, capacity is available to transport most shipments and transshipment opportunities are present to promote smooth flow. Continued increases in scale at that point bring about little, if any, system improvement; and, ultimately, excess capacity beyond that needed is indicated. It is expected that a rather broad range of network designs would provide total system cost near the optimum. The relative locations of points to be served in combination with other important features evident in a particular application may allow an analyst to determine a solution within this

# SYSTEM COST RELATIONSHIP

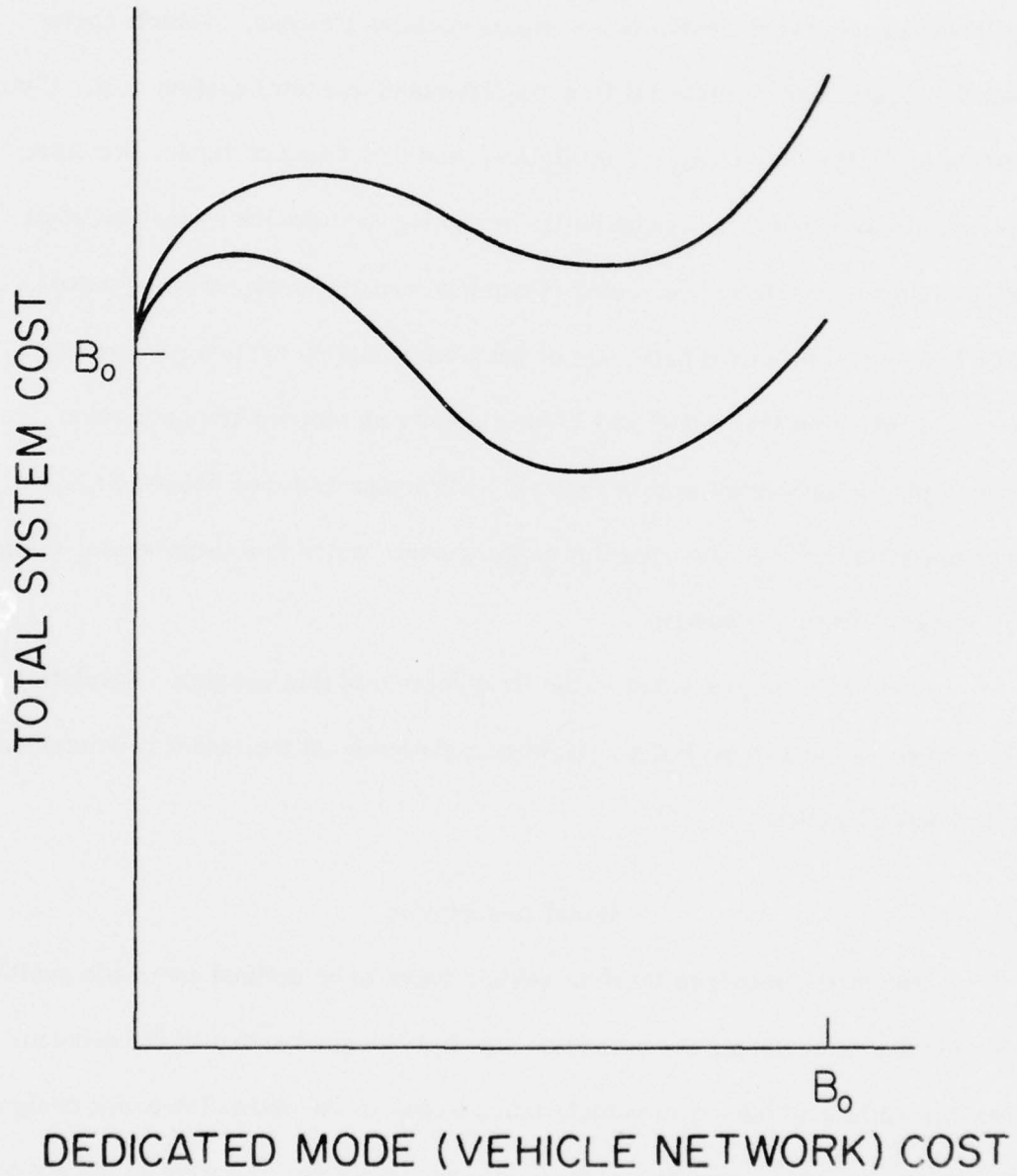


Figure 5

broad range using only the heuristic procedure described in the previous chapter.

However, in other applications it may be difficult to design a good network with heuristics and a method to determine an optimal solution is necessary. Synthesizing a network design is a complex decision process. Vehicle tours must be determined by material flow requirements and total system cost. Characteristics of the less complex, multiple-vehicle routing problems--providing one vehicle to service each point daily, designing the network so that the total travel distance is minimized, using cloverleaf route geometries--may have little bearing on practical problems of the transportation system planning type.

A mathematical model which does specify an optimal transportation system plan is presented in this chapter. All design features described in the problem statement are incorporated in the model, which is a large scale, mixed 0,1 integer, linear program.

The model is presented in the first section of this chapter. A solution algorithm designed to exploit the particular structure of the model is described in the second section.

#### Model Description

The model requires feasible vehicle tours to be defined and made available for use in designing the network. The 0,1 integer portion of the problem then determines which set of vehicle tours to use in the optimal network design. The optimality of the resulting system plan is, of course, relative to the set of tours provided. However, this may not be particularly restrictive, since

experience should be a good indicator of the types of vehicle tours most likely to service system needs efficiently and since a large number of tours could be made available for possible use by the design process. For problems with a small number of points, all feasible tours could be enumerated using a simple algorithm described later. At worst, this approach allows the analyst to evaluate a set of design combinations far in excess of the number possible through heuristic or manual procedures.

A detailed statement of the model appears in Figure 6. The objective function combines the cost of material flow and the expense of vehicle tours used in the dedicated mode. Tour cost,  $C_{rk}$ , may reflect a number of components including, for example, a fixed investment-type expense, a cost to travel between each of the points on the tour, and costs to service each point. Equations [21], [22], and [24], which provide constraints on ship time, transshipment point loading, and ship requirements are the same as equations [9], [10], and [12] in the material flow model of Figure 1.

The remainder of the model accounts for the network design process. Equation [23] defines vehicle arc capacity, but differs from equation [11] in the material flow model in one important respect. It is assumed in this model that each vehicle provided establishes a set of constraints of the form in equation [23]. Arcs not used in a particular network design cannot be used for material flow and have right hand side equal to zero. Each arc,  $q$ , is defined by a vector of information including arc beginning and ending points, the specific vehicle involved, and whether this is the first or second time the vehicle travels from the

Problem P1

$$\begin{aligned} \text{Min } F = f_{\text{msd}} & \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right] \\ & + \sum_{k=1}^K \sum_{r \subseteq T(k)} C_{rk} T_{rk} \end{aligned} \quad [20]$$

Subject to:

Ship Time Standards

$$\sum_{\ell \in \mu(m)} \tau_{\ell}^{(t)} f_{\text{msd}} P_{\ell}^{(t)} + \sum_{\text{msd}} f_{\text{msd}} (\tau_{\text{msd}}^{\text{A}} S_{\text{msd}}^{\text{rA}} + \tau_{\text{msd}}^{\text{T}} S_{\text{msd}}^{\text{rT}}) \leq \Gamma_m; \quad [21]$$

$m=1, 2, \dots, M$

Transshipment Point Capacity

$$\sum_{\ell=1}^L t_{i\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq \bar{b}_i; \quad i=1, 2, \dots, I \quad [22]$$

Vehicle Arc Capacity

$$\sum_{\ell=1}^L a_{q\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq b_q \quad \sum_{r \subseteq T_q(k)} T_{rk}; \quad q=1, 2, \dots, Q \quad [23]$$

Ship Requirements

$$\sum_{\ell=1}^L g_{\text{msd},\ell} P_{\ell}^{(t)} + S_{\text{msd}}^{\text{rA}} + S_{\text{msd}}^{\text{rT}} = 1.0; \quad \forall \text{msd} \quad [24]$$

Vehicle Assignment to One Tour Only

$$\sum_{r \subseteq T(k)} T_{rk} \leq 1; \quad k=1, 2, \dots, K \quad [25]$$

Figure 6.--The Transportation System Planning Model

Complementary Vehicle Relationships

$$\sum_{r \subseteq T(k)} T_{rk} + \sum_{r \subseteq \bar{T}(k)} T_{rk}^- = 1 \quad ; \quad k=1, 2, \dots, K \quad [26]$$

Vehicle Network Constraints

As required in a specific application to restrict, for example, the total vehicle capacity provided, the total number of vehicle visits to a point, or to require a return tour supplementing vehicle paths. [27]

Non-negativity and Integer Requirements

$$P_{\ell}^{(t)}, S_{msd}^{rA}, S_{msd}^{rT} \geq 0 \quad \forall \ell, msd$$
$$T_{rk} = 0, 1 \quad \forall k, r \subseteq T(k) \quad [28]$$

Figure 6. --Continued

beginning to ending point. This level of detail is necessary to assure that no vehicle is overloaded on any arc.

The remaining constraints relate to vehicle tours. Equation [25] requires each vehicle to be used in at most one tour in a network design. Should the null tour (the vehicle actually does nothing) be defined for a vehicle, equation [25] should be stated as a strict equality, indicating that each vehicle must have some tour in solution.

Equation [26] provides a means of improving solution times in some cases by defining vehicles in predetermined pairs to be complementary to each other. This device permits several possibilities. First, the null tour for vehicle  $k$  could be formulated as the only tour allowed for  $\bar{k}$ , providing a means of invoking the relationship noted above. Secondly, this type of constraint could distinguish between tour paths and tour routes traversed by a single vehicle. For example, vehicle  $k$  may originate at Los Angeles, visit several points and terminate at New York. The set of all such paths could be viewed as the possible tours for  $k$ . An alternate network strategy might call for  $k$  to traverse a route, traveling eastward, say, to Dallas and subsequently terminating at its point of origin, Los Angeles, to complete the route. In actuality, there may be only one vehicle and the complementarily constraint indicates that only one type of tour is feasible. Additionally, the constraint may improve solution time by providing a logical means of branching.

Should a tour path be defined, other tour paths also need be defined to return the vehicle from its termination point the next day. Otherwise, one point

may be a 'sink' at which all vehicles terminate and therefore originate the next day. This and other constraints which management might impose on network design in any particular application are represented by the note, equation [27].

Each of the defined tours is assumed to satisfy all constraints imposed by management as well as any limitations which result from vehicle operating characteristics. Examples of such constraints are: the number of points visited by a vehicle; the tour length in time and/or distance stipulated by vehicle capabilities, or, perhaps, crew requirements; and the number of times a vehicle visits a particular point. Furthermore, it is assumed that each vehicle traverses its assigned tour during the course of a day. Routes are traversed daily and in the case of paths, vehicles are redefined and depart from their termination point the next day. Alternate assumptions concerning tour duration and frequency could be incorporated in the model if necessary.

Even though the model may be stated using only a few types of constraints, it does represent a large scale mathematical programming problem. Available mixed 0,1 integer, linear programming codes might be applied to solve problems on limited scope. But this approach is expected to find limited success unless the GUB and column generation procedures described in the previous chapter can be applied. An alternate procedure which does allow use of these efficient techniques results from application of the concepts of Benders' decomposition. These concepts were originally developed by Benders (1962) and were later extended by Geoffrion (1972). The technique is well summarized by Lasdon (1970). Benders' decomposition allows the system model in Figure 6 to be re-expressed

as two interacting models. One of these problems is defined as Problem LP in Figure 7; and the other, Problem 01, is stated in Figure 8. Each of these problems is more simple than the original problem, and efficient solution procedures may be devised to exploit the special structure of each providing efficient computation capability.

Problem LP is essentially the same as the multicommodity material flow problem in Figure 1. LP accepts a vehicle network (specified by 01) and determines the least-cost shipping plan using both dedicated and commercial modes. In turn, LP provides information to 01 by adding a constraint which essentially gives an indication of the relative utility of the vehicle network. The solution procedure continues iteratively in this fashion until the optimal solution is determined. Convergence is guaranteed by the theory of Benders' decomposition, but a large number of iterations may be required. However, the literature records successful application of the method to a variety of problems (see, for example, Geoffrion and Graves (1974) and Richardson (1976) ). Considering these reports of computational experience, the decomposition approach is expected to provide an efficient solution procedure for the transportation system planning problem.

The right-hand sides of constraints represented by equation [32] are zero if the arc is not used in a given vehicle network and equal to  $b_q$  if arc  $q$  is, in fact, defined in the network. At each  $\kappa$ -iteration, equation [32] need be defined only for those arcs in the vehicle network defined by Problem 01. Zero arc capacity merely indicates that no material flow chain may use the arc. Excluding

Problem LP:

$$\text{Min } P_o = f_{\text{msd}} \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right] \quad [29]$$

Subject to:

		Associated Dual Variable
<u>Ship Time Standards</u>		
$\sum_{\ell \in \mu(m)} \tau_{\ell}^{(t)} f_{\text{msd}} P_{\ell}^{(t)} + \sum_{\text{msd}} f_{\text{msd}} (\tau_{\text{msd}}^{\text{A}} S_{\text{msd}}^{\text{rA}} + \tau_{\text{msd}}^{\text{T}} S_{\text{msd}}^{\text{rT}}) \leq \Gamma_m \quad m=1, 2, \dots, M$	$\alpha_m$	[30]

Transshipment Point Capacity

$$\sum_{\ell=1}^L t_{i\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq \bar{b}_i \quad ; \quad i=1, 2, \dots, I \quad \beta_i \quad [31]$$

Vehicle/Arc Capacity

$$\sum_{\ell=1}^L a_{q\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq b_q \left[ \sum_{r \subseteq T_q(k)} T_{rk} \right] \quad ; \quad q=1, 2, \dots, Q \quad \gamma_q \quad [32]$$

Ship Requirements

$$\sum_{\ell=1}^L g_{\text{msd},\ell} P_{\ell}^{(t)} + S_{\text{msd}}^{\text{rA}} + S_{\text{msd}}^{\text{rT}} = 1.0 \quad ; \quad \forall \text{msd} \quad \delta_{\text{msd}} \quad [33]$$

Non-negativity Requirements

$$P_{\ell}^{(t)}, S_{\text{msd}}^{\text{rA}}, S_{\text{msd}}^{\text{rT}} \geq 0 \quad \forall \ell, \text{msd} \quad [34]$$

Figure 7. -- Problem LP

it from the problem obviously accomplishes the same result. As shown in Appendix F, the dual variables associated with these undefined arcs must equal zero whether or not they are included in the formulation of LP.

The objective function of the dual of LP is

$$\max \sum_{m=1}^M \Gamma_m \alpha_m + \sum_{i=1}^I \bar{b}_i \beta_i + \sum_{q=1}^Q b_q \left[ \sum_{r \subseteq T_q(k)} T_{rk} \right] \gamma_q + \sum_{msd} \delta_{msd}. \quad [35]$$

In most applications, the dual of the linear programming problem would be solved rather than the primal, since the set of feasible solutions in the dual is not affected by the values assigned to the binary variables. The advantage of using the dual is that the solution found at the last  $\kappa$ -iteration may be used as the initial basic feasible solution for the current  $\kappa$ -iteration. Only the dual objective function coefficients change as a function of the binary variables as noted by equation [35]. However, in the current application, it is recommended that the primal, LP, be used to take advantage of several possibilities to reduce computation time: 1) only constraints with nonzero right-hand sides need be defined in equation [32] at each iteration, 2) the GUB method may be applied to the primal, 3) the column generation scheme described in the previous chapter may be applied to the primal problem. Since an efficient Phase I procedure is available, use of the primal appears preferable to use of the dual in this application.

It is well known that the optimal solution to LP has the same value as the optimal solution to its dual problem. Thus, for optimal values of all variables:

$$f_{\text{msd}} \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right]$$

$$= \sum_{m=1}^M \Gamma_m \alpha_m + \sum_{i=1}^I \bar{b}_i \beta_i + \sum_{q=1}^Q b_q \left[ \sum_{r \subseteq T_q(k)} T_{rk} \right] \gamma_q + \sum_{\text{msd}} \delta_{\text{msd}} \cdot [36]$$

Considering the directions of the inequalities in the constraints of problem P1, it is seen that  $\alpha$ ,  $\beta$ , and  $\gamma$  are nonpositive and  $\delta$  is unrestricted in sign. The value of the dual objective function is, however, nonnegative as noted by equation [36].

At each  $\kappa$ -iteration, the dual objective function is combined with the expression for vehicle tour cost to form an additional Benders' cut (or constraint) in Problem 01:

$$Z \geq \sum_{k=1}^K \sum_{r \subseteq T(k)} C_{rk} T_{rk} + \sum_{m=1}^M \Gamma_m \alpha_{m,\kappa} + \sum_{i=1}^I \bar{b}_i \beta_{i,\kappa}$$

$$+ \sum_{q=1}^Q b_q \left[ \sum_{r \subseteq T_q(k)} T_{rk} \right] \gamma_{q\kappa} + \sum_{\text{msd}} \delta_{\text{msd},\kappa} \cdot [37]$$

By defining the non-negative constant  $B_{\kappa}$  as

$$B_{\kappa} = \sum_{m=1}^M \Gamma_m \alpha_{m,\kappa} + \sum_{i=1}^I \bar{b}_i \beta_{i,\kappa} + \sum_{\text{msd}} \delta_{\text{msd},\kappa}, \quad [38]$$

equation [37] may be re-expressed as in equation [43].

Problem 01

$$\text{Min } Z \quad [39]$$

Subject to:

Vehicle Assignment to One Tour Only

$$\sum_{r \subseteq T(k)} T_{rk} \leq 1 \quad ; \quad k=1, 2, \dots, K \quad [40]$$

Complementary Vehicle Relationships

$$\sum_{r \subseteq T(k)} T_{rk} + \sum_{r \subseteq T(\bar{k})} T_{r\bar{k}} = 1 \quad ; \quad k=1, 2, \dots, K \quad [41]$$

Vehicle Network Constraints

As required in a specific application to restrict, for example, the total vehicle capacity provided, the total number of vehicle visits to a point, or to require a return tour supplementing vehicle paths. [42]

Benders' Cuts

$$Z \geq \sum_{k=1}^K \sum_{r \subseteq T(k)} \left[ C_{rk} - \sum_{q \subseteq T_q(k)} b_q | \gamma_{q\kappa} | \right] T_{rk} + B_\kappa \quad ; \quad \kappa=1, 2, \dots, \bar{\kappa} \quad [43]$$

Integer and Non-negativity Constraints

$$T_{rk} = 0, 1 \quad \forall k, r \subseteq T(k)$$

$$Z \geq 0 \quad [44]$$

Figure 8. -- Problem 01

The other constraints in Problem 01 are taken from Problem P1 and carry the same implications as discussed previously. The continuous variable,  $Z$ , is defined by the Benders' cut which places the highest cost assessment on a particular network design. Benders' cuts essentially place a value on a proposed network which is determined from network designs previously evaluated by Problem LP. In particular,  $\gamma_{q\kappa} = 0$  if the vehicle which traverses arc  $q$  is not loaded to capacity. The cuts, therefore, tend to feed back more information to 01 if a number of vehicle arcs are loaded to capacity. Fortunately, Problem LP attempts to load arcs to capacity since they are available at zero cost, while expenses are incurred for use of commercial modes. The cuts do not provide an indication of individual tours which might be effective. Rather, they place a value on the network, or complete set of vehicle tours. The effective cost of including an arc, which was loaded to capacity in a previous solution, is reduced by the  $\gamma_{q\kappa}$  terms. The  $\gamma_{q\kappa}$  indicate the utility of a vehicle arc with respect to the material flow subproblem and tend to identify arcs which may be used effectively by material flow if they are provided by solution of the network subproblem.

In a general formulation involving Benders' decomposition, a second type of constraint may have to be augmented to the integer problem. The function of the second type of cut is to assure that solutions provided by the integer problem allow the primal linear program to attain a feasible solution. These cuts are not required in the current formulation, since it is assumed that exclusive use of commercial air is always feasible. Thus LP may attain a feasible solution no matter what vehicle network is specified.

## Solution Procedure

Efficient procedures to solve Problem LP were described in Chapter III. Steps required to solve Problem 01 and the combination of 01 and LP are detailed in this section.

### Tour Provision

A set of feasible tours for each vehicle may be defined by the analyst and made available for use in Problem 01. This process would not be expected to be particularly difficult or time consuming, since tours could be readily defined by picking likely candidates, considering the relative locations of the points to be served and the material flow requirements between them. Tours need not be confined to limitations typically imposed in the multiple-vehicle routing problem. For example, a given vehicle tour might visit one point a number of times, or all vehicle tours may service one of the points to assure transshipment capability.

All feasible tours may be enumerated using the following algorithm:

#### Step 0: Initialization

Define the set of 0-length tours to be empty. Set  $n = 0$ . Go to step 1.

#### Step 1: Tour Definition

Define  $(n + 1)$ -length tours by augmenting the termination point,  $t$ , (for vehicle  $k$ ) to each of the  $n$ -length partial tours which do not have  $t$  as the last point. Check tour feasibility with respect to time and/or distance length, number of visits/point, etc. Record any feasible

$(n+1)$ -length tours. If  $(n+1) =$  maximum feasible tour length, stop.

Otherwise, go to step 2.

Step 2: Partial Tour Extension

Increment  $n$ . Define  $n$ -length partial tours by augmenting point  $i$  ( $i=1, 2, \dots, I$ ;  $i \neq$  last point on  $(n-1)$ -length tour) to each of the  $(n-1)$ -length partial tours. Check tour feasibility with respect to time and/or distance length, number of visits/point, etc. Drop any infeasible, partial tours from the list of  $n$ -length partial tours. Go to step 1.

Since this algorithm would define a large number of tours, it should be used only when each vehicle can service a low number of points and/or when feasibility constraints rule out a large number of potential tours. Of course, the algorithm could be used to define a set of tours from which the analyst may pick likely candidates to make available to Problem 01.

It is also recommended that the analyst define a 'good' vehicle network so that a relatively tight upper bound on the solution to P1 may be defined early. This should not be difficult assuming that the analyst has some familiarity with historical operations of the system under study or of similar systems.

Solution Bounds

Iterations may begin by solving LP using the initial network defined by the analyst. Useful bounds on the solution to P1 and on the value of  $Z$  may be defined at each iteration according to the theory of Benders' decomposition (Lasdon, 1970).

If at iteration  $\kappa$ ,  $(Z^\kappa, T^\kappa)$  solves Problem 01 and  $P_o$  is the optimal value of Problem LP given  $T^\kappa$ , then  $[T^\kappa, P^\kappa, (S^{rA})^\kappa, (S^{rT})^\kappa]$  is a feasible solution to the original Problem P1. The corresponding objective function value in P1 is

$$F^\kappa = \sum_{kr} C_{rk} T_{rk}^\kappa + P_o^\kappa.$$

This relationship may be used to define the upper bound on the value of  $Z$  for the next iteration:

$$UB^{\kappa+1} = \min \{ UB^\kappa, F^\kappa \}. \quad [45]$$

A lower bound on the value of  $Z$  for the next iteration is

$$LB^{\kappa+1} = Z^\kappa$$

since the value of  $Z$  is monotonically nondecreasing for successive  $\kappa$ -iterations.

Therefore, at the next iteration

$$LB^{\kappa+1} \leq Z^{\kappa+1} \leq UB^{\kappa+1},$$

and at the optimal solution

$$LB^\kappa = Z^\kappa = UB^\kappa.$$

The iterative procedure may be stopped when  $[T^\kappa, P^\kappa, (S^{rA})^\kappa, (S^{rT})^\kappa]$  provide a solution which is within  $\epsilon$  of optimality; i. e., stop when  $(UB^{\kappa+1} - LB^{\kappa+1}) \leq \epsilon$ . The true optimal solution is then known to be

somewhere between the lower and upper bounds. In problems which converge slowly, it may be prohibitively expensive to determine the optimum exactly. The ability to define  $\epsilon$  optimality may therefore be a useful characteristic of this approach.

#### An Algorithm for Problem 01

Since  $n_k$  tours are provided for vehicle,  $k$ , there are a total of

$$\bar{N} = \sum_{k=1}^K n_k$$

binary variables in addition to the one continuous variable,  $Z$ , in Problem 01. The total number of combinations of  $\bar{N}$  binary variables is  $2^{\bar{N}}$ . However, since each vehicle may use only one tour at level 1 in a feasible solution, there are only

$$\bar{\bar{N}} = \prod_{k=1}^K n_k$$

feasible combinations. If complementary vehicle relationships and other network constraints are imposed, the number of feasible solutions is less than  $\bar{\bar{N}}$ . Standard integer programming codes could be applied to solve Problem 01 but they would be forced to operate with the  $\bar{N}$  solution space rather than that of  $\bar{\bar{N}}$ . Additional binary variables would be required to re-express  $Z$  as a function of integer variables, further complicating the problem. A special purpose, implicit enumeration algorithm was developed to exploit the structure of Problem 01 and provide an efficient solution procedure. Since Problem 01 need be solved several times in the process of solving Problem P1, an efficient solution

procedure contributes to the overall capability of the decomposition approach to solve transportation system planning problems.

The special purpose algorithm operates in a manner similar to that of Balas's implicit enumeration strategy. Balas's (1965) strategy, in combination with the enumeration logic devised by Geoffrion (1966) (see Taha (1971) for a detailed description) forms the basis for a number of efficient integer programming algorithms (see, for example, Piper (1974) ). Balas's strategy discovers complete, feasible solutions which successively improve the objective function value. To obtain a complete solution, a partial solution,  $J_t$ , is iteratively augmented by assigning a binary value to some variable not already in  $J_t$ . The procedure starts with an 'optimal' but infeasible solution and attempts to reduce infeasibility at each iteration. If it is determined that no completion of  $J_t$  can be feasible or optimal (and after a complete solution is discovered), a new partial solution is defined by a special backtracking procedure and the search for an improved, complete solution continues. The strategy enumerates, either implicitly or explicitly, all feasible combinations of the variables so that it guarantees an optimal solution.

The algorithm developed during this study specializes Balas's strategy to promote efficiency in five significant ways:

1. Assigning one  $k$ -tour to level 1 in  $J_t$  requires all other  $k$ -tours as well as  $\bar{k}$ -tours to be assigned to level 0 to invoke equations [40] and [41]. Algorithm logic imposes these restrictions automatically, no feasibility checks (other than vehicle network constraints, if any) need be made so the algorithm essentially searches only for improved solutions.

2. The complementary vehicle relationships in equation [41] promote efficiency in a manner similar to the cross branching procedure described by Martin (1976).
3.  $Z$  is treated explicitly as a continuous variable so it is not necessary to expand the problem to express  $Z$  as a function of integer variables.
4. The single variable  $Z$  is used in lieu of the set of surplus variables employed by the Balas routine, reducing algorithm complexity.
5. Vehicle and vehicle-tour set memberships are defined with the objective of improving run time efficiency.

A detailed statement of the algorithm follows the discussion below which provides an overview of the logic involved.

Before beginning to solve Problem 01 at iteration  $\bar{k}$ , the 'current best solution' to 01,  $Z_c$ , is known to be  $UB^k$  as defined in equation [45] and only solutions which improve  $Z_c$  need be examined. The algorithm uses an efficient bookkeeping procedure to invoke equations [40] and [41] and determine which variables might best augment  $J_t$  at the  $j$ -iteration within 01. Before iteration number one, all tours are assigned to the set  $E_t$ , and all vehicles to set  $V^e$ , indicating that no tours have arcs associated with any negative  $\gamma$  values from equation [37]. Each tour in set  $E_t$  has the same coefficient,  $W_{rk\bar{k}}$ , in all Benders' cuts, so the minimum cost  $T_{rk}$  is the best  $k$ -tour among all  $k$ -tours in set  $E_t$ . This one  $k$ -tour, the 'representative', may be easily identified and is the only one from  $E_t$  which may enter solution at iteration  $\bar{k}$ . Any other  $T_{rk} \subseteq E_t$  would necessarily increase  $Z$  over the lowest attainable value using the representative tour.  $V^e$  is defined to reduce tour-membership checking time.

At each successive  $\kappa$ -iteration,  $T_{rk} \subseteq E_t$ , which employ any arc associated with a negative  $\gamma_{q\kappa}$  defined by LP are upgraded from  $E_t$  to  $N_t^c$ , the set of tours which are candidates for use in 01 during the  $\kappa$ -iteration. Representative tours are redefined if necessary.

At each  $j$ -iteration (within 01) tours eligible to augment the current partial solution,  $J_t$ , are selected from the list of candidates,  $N_t^c$ , by culling any tours which can be shown to be infeasible or non-optimal by Exclusion Test II. It is assumed that  $K'$  vehicles ( $K' \leq K$ , depending upon the number of complementarity constraints) must each be assigned a tour at level 1 to provide a complete, feasible network design. Tours which would violate network constraints may be excluded (assigned to set  $D_t$ ) immediately. Defining  $W'_{t\kappa}$  as

$$W'_{t\kappa} = \sum_{kr} W_{tk\kappa} T_{rk} \text{ for } T_{rk} \subseteq J_t,$$

the right-hand side of each equation [43], which results from the current partial solution is  $B_\kappa + W'_{t\kappa}$ . For each Benders' cut  $\kappa$ , the minimum cost addition to complete a network design may be determined knowing  $T_{rk} \subseteq J_t$ ,  $K'$ , and vehicle complementarity relationships. Each tour  $T_{rk} \subseteq N_t^c$  is tested in each cut,  $\kappa$ , using equation [46] to determine if its inclusion [plus the minimum  $(W_{rk\kappa}, W_{r\bar{k}\kappa})$  for  $(k, \bar{k})$  required to complete a network design] would yield a  $Z$  value greater than the current best,  $Z_c$ . Tours which could not improve  $Z_c$  even in this most favorable condition are excluded by assigning them to set  $D_t$  for this  $j$ -iteration. A second component of Exclusion Test II makes a similar check to determine if  $J_t$ ,  $T_{rk}$ , and the associated minimum cost network completion

would exceed the cost of using the commercial air mode exclusively,  $B_0$ , defined by Problem LP. Tours which could not yield a network cost less than  $B_0$  in this most favorable condition are also assigned to set  $D_t$  for this  $j$ -iteration, since they could not lead to a minimum (system) cost solution in general. In some applications, checking time may be reduced by identifying the 'best' tour of a given length (number of arcs) and checking conditions of step 1/B/2 and 1/B/3 for this tour first. If the best tour of the given length fails the test, all tours of the given length may be excluded immediately. Tours which pass the exclusion test are eligible to augment  $J_t$  at level 1.

The variable to enter solution is determined in step 2 by a heuristic procedure analogous to that used in the Balas routine. The parameter  $\alpha$  forms a linear combination (for each eligible  $T_{rk}$ ) of the sum of  $T_{rk}$  coefficients in all Benders' cuts and its maximum coefficient over all cuts. Computational experience should indicate values for  $\alpha$  which provide best run times.

If  $(K' - 1)$  vehicles are already assigned to level 1 in  $J_t$ , Exclusion Test II and step 2/A are not used. Rather, step 2/B is used to determine the feasible  $k$ -tour which minimizes the maximum right hand side of all Benders' cuts, yielding a  $Z$  value of  $Z_p$ . If  $Z_p < Z_c$ , an improved, complete solution has been discovered and is recorded. If an improved network cannot be defined, the backtracking procedure is used to define a new  $J_t$ .

$J_t$  is augmented in step 3 by adding the entering variable,  $T_{rk}$ , as the rightmost element in the vector as is done in the Balas procedure. It is (implicitly) understood that all other candidate  $T_{rk}$  (and  $T_{r\bar{k}}$ ) for  $k$  (and  $\bar{k}$ ) are

also augmented to  $J_t$  at level 0, between the old elements and the new addition. This is accomplished by updating the set membership of the vehicles involved and, later, by the logic of the backtracking step.

The backtracking procedure in step 4 is invoked after a complete solution has been discovered or when it is determined that no completion of  $J_t$  can improve  $Z_c$ . As in the Balas routine, backtracking sets the rightmost positive element of  $J_t$  to 0 (by multiplying the element by -1) and dropping negative elements (if any) to the right of this element. Set membership of  $k$  (and  $\bar{k}$ ) is updated appropriately to effect changes in the implicit elements of  $J_t$  at level 0.

The current best solution may be judged optimal in the backtracking procedure when all  $k$ -tours (for any  $k$ ) are included as 0 explicitly.<sup>1</sup> This condition indicates that all possible combinations of including all  $k$ -tours have been examined (implicitly or explicitly) and further reductions in  $Z_c$  could be attained only by excluding vehicle  $k$ , an infeasible alternative. If this  $k$  has a complementary vehicle,  $\bar{k}$  must be reassigned to the set of candidate vehicles,  $V_t^c$ , ( $k$  is assigned to a set  $\bar{V}_t$  of vehicles which cannot be candidates when a  $k$ -tour enters the solution at level 1) and the search continued until both  $k$  and  $\bar{k}$  (for some  $k$  and  $\bar{k}$ ) have all candidate tours explicitly included in  $J_t$  at level 0, indicating that all possible networks using either  $k$  or  $\bar{k}$  have been (implicitly or explicitly) examined.

---

<sup>1</sup>Specifically, if no other vehicle has a tour at level one to the left in  $J_t$ .

Step 0: Initialization

- A. Initialize iteration  $\bar{\kappa}$  by defining the 'current best solution' as

$$Z_c = UB^{\bar{\kappa}}$$

and set  $J_t = \emptyset$ ,  $V_t = \emptyset$ ,  $\bar{V}_t = \emptyset$ ,  $V_t^\phi = \emptyset$ ,  $N_t^c = (N - E_t)$ ,

$V^e = \{k \mid \text{all } T_{rk} \subseteq E_t\}$ ,  $V_t^c = (V - V^e)$ ,  $V^d = \emptyset$ ,  $j = 0$ .

Complete vehicle networks which improve  $Z_c$  are now sought.

- B. For the set of  $\gamma_{q\bar{\kappa}} < 0$  defined by LP at iteration  $(\bar{\kappa} - 1)$ , some of the arcs,  $q$ , may not have been associated with a negative  $\gamma$  value from a previous iteration. Remove all  $k$  tours from the set  $E_t$  which employ any of the  $q$  arcs with  $\gamma_{q\bar{\kappa}} < 0$ ,  $q \subseteq T_q(k)$ , and assign them to the set of candidate tours,  $N_t^c$ . If  $k$  had belonged to the set  $V^e$ , remove  $k$  from  $V^e$  and assign it to  $V_t^c$ . Go to step 1.

Step 1: Tour Exclusion

- A. Exclusion I

1. If  $E_t = \emptyset$ , go to step 1/B, Exclusion II.
2. For each vehicle  $k$  which has tours in  $E_t$ , determine the tour in  $E_t$  with the  $\min_r (C_{rk})$ . This is the best possible  $k$  tour in  $E_t$  and is called the representative  $k$ -tour. If all  $N$  tours  $\subseteq E_t$ , STOP, since the vehicle network defined by the representative tours is optimal (but not practical if its cost is greater than  $B_0$ ). Otherwise, place each representative  $k$ -tour in set  $N_t^c$ . If  $k \subseteq V^e$ , place  $k$  in set  $V_t^c$  since the representative is now a candidate. (If the null  $k$ -tour is defined, it will always be the representative  $k$ -tour.)

3. Set  $j = 1$  and go to step 2 since  $J_t = \emptyset$ .

B. Exclusion II

1. Increment  $j$  ( $j = j + 1$ ). If only one more vehicle tour is required to complete a vehicle network, go to step 2/B.

2. For each  $k \subseteq V_t^c$ , check all candidate tours ( $T_{rk} \subseteq N_t^c$ ) to determine if  $T_{rk}$  would violate a network constraint or if, for cut  $\kappa = 1, 2, \dots, \bar{\kappa}$

$$W_{rk\kappa} > Z_c - B_\kappa - W_{t\kappa} - \sum_{\substack{k' \subseteq V_t^c \\ k' \neq \bar{k} \\ T_{r\bar{k}} \subseteq N_t^c \text{ (if any)}}} \left[ \min_{T_{rk'} \subseteq N_t^c} \{ W_{rk'\kappa}, W_{r\bar{k}'\kappa} \} \right]. \quad [46]$$

If so, remove tour  $rk$  from  $N_t^c$  and assign it to the set of excluded tours,  $D_t$ , since including it in solution would either be infeasible or would increase  $Z$  over the current best value,  $Z_c$ . If this is the last  $k$ -tour  $\subseteq N_t^c$ , remove  $k$  from  $V_t^c$  and assign it to  $V^d$ .

3. For each  $k \subseteq V_t^c$  check all candidate tours ( $T_{rk} \subseteq N_t^c$ ) to determine if

$$C_{rk} > B_0 - \sum_{T_{rk} \subseteq J_t} C_{rk} T_{rk} - \sum_{\substack{k' \subseteq V_t^c \\ k' \neq \bar{k} \\ T_{r\bar{k}'} \subseteq N_t^c \text{ (if any)}}} \left[ \min_{T_{rk'} \subseteq N_t^c} \{ C_{rk'}, C_{r\bar{k}'} \} \right].$$

If so, remove tour  $rk$  from  $N_t^c$  and assign it to the set of excluded tours,  $D_t$ , since including it would increase  $Z$  over  $B_0$ . If this is the last  $k$ -tour  $\subseteq N_t^c$ , remove  $k$  from  $V_t^c$  and assign it to  $V^d$ .

4. The set of tours  $T_{rk} \subseteq N_t^c$  for vehicles  $k \subseteq V_t^c$  are eligible to augment  $J_t$  at level one. If  $N_t^c = \emptyset$ , no completion of  $J_t$  will improve  $Z_c$ ; set  $j = j+1$  and backtrack by going to step 4. Otherwise go to step 2.

Step 2: Determine variable to enter at level one. If only one tour is required to complete the vehicle network, go to step 2/B; otherwise, step 2/A.

A. Entering tour will not complete the vehicle network.

1. The tour to enter solution at this iteration is the tour,  $T_{rk}^*$ , for some  $k \subseteq V_t^c$  and some  $T_{rk} \subseteq N_t^c$  which provides the

$$\min_{T_{rk} \subseteq N_t^c} \left\{ \alpha \left[ \sum_{\kappa=1}^{\bar{\kappa}} W_{rk\kappa} \right] + (1 - \alpha) \left[ \max_{\kappa} W_{rk\kappa} \right] \right\}$$

in which  $\alpha$  is a parameter  $0 \leq \alpha \leq 1$ . If  $\alpha = 0$ , the tour which adds the least to any cut would enter; if  $\alpha = 1$ , the tour which has the smallest sum of cut coefficients would enter.

2. Go to step 3 to augment  $J_t$  with the tour  $T_{rk}^*$ .

B. One tour is required to complete the vehicle network.

1. For eligible vehicles ( $k \subseteq V_t^c$ ) select the tour  $T_{rk}^*$  from the  $T_{rk} \subseteq N_t^c$  which violates no network constraints and which provides the

$$\min Z_p = \min_{T_{rk} \subseteq N_t^c} \left\{ \max_{\kappa} \left[ (B_{\kappa} + W'_{t\kappa}) + W_{rk\kappa} \right] \right\}$$

2. If  $Z_p > Z_c$  or if no  $T_{rk}$  is feasible with respect to the network constraints, go to step 4 to backtrack since no completion of  $J_t$  can improve the value  $Z_c$ .

3. If  $Z_p \leq Z_c$ , go to step 3 to augment  $J_t$  with the tour  $T_{rk}^*$ .

Step 3: Augment  $J_t$  by adding  $T_{rk}^*$  at level one.

A. Updates

1. Assign  $T_{rk}^*$  as the rightmost element of  $J_t$ .
2. Update  $W'_{tk} \forall k$  and  $C'_t$ .
3. Remove  $k$  from  $V_t^c$  and assign it to  $V_t$ . If  $\bar{k} \subseteq V_t^c$  or if  $\bar{k} \subseteq V^d$ , remove  $\bar{k}$  and assign it to  $\bar{V}_t$ . Remove  $T_{rk}^*$  from  $N_t^c$ .
4. Assign all  $k \subseteq V^d$  to  $V_t^c$  and set  $V^d = \emptyset$ .  
Assign all  $T_{rk} \subseteq D_t$  to  $N_t^c$  and set  $D_t = \emptyset$ .

B. Continue implicit enumeration.

1. If the vehicle network is not complete, go to step 1/B.
2. If the vehicle network is complete,  $Z_p$  will be  $\leq Z_c$ . Record the solution and set  $Z_c = Z_p$ . Go to step 4 to backtrack.

Step 4: Backtrack

- A. Find the rightmost positive tour in the vector  $J_t$ ,  $T_{rk}^R$ . If there are negative tours to its right, delete them from  $J_t$  and reassign them to  $N_t^c$ . If the associated  $k$  had belonged to  $V_t^\phi$ , reassign it to  $V_t^c$ .
- B. If there are candidate  $k$ -tours not in  $J_t$ , go to step 4/C. If all candidate  $k$ -tours are in  $J_t$  and if there are no tours at level one to the left of  $T_{rk}^R$ , go to step 4/D. If all candidate  $k$ -tours are in  $J_t$  and if there is some tour at level one to the left of  $T_{rk}^R$ , multiply  $T_{rk}^R$  by  $-1$  and go to step 4/A.

- C. Backtrack. Make  $T_{rk}^R$  negative to specify that the tour is included in  $J_t$  at level zero. Remove  $k$  from  $V_t$  and assign it to  $V_t^c$ . If  $\bar{k} \subseteq \bar{V}_t$ , remove it and assign it to  $V_t^c$ . Go to step 1/B to begin another iteration.
- D. Optimality Conditions. If  $k$  has no complementary vehicle, STOP; since the optimal solution has been identified (all  $k$  tours have been fathomed). If  $\bar{k} \subseteq V_t^\phi$  also; STOP, and, again, the optimal solution has been identified. If  $\bar{k} \not\subseteq V_t^c$ , assign  $\bar{k}$  to  $V_t^c$ , remove  $k$  from  $V_t$  and assign it to  $V_t^\phi$ , make  $T_{rk}^R$  negative to specify that the tour is included in  $J_t$  at level zero, and go to step 1/B to begin another iteration.

## CHAPTER V

### TOUR CONSTRUCTION

In many applications, particularly in cases involving a large number of points, it may be difficult to predefine a limited number of tours to make available to Problem 01. In fact, in most applications, it may be preferable to construct tours using an appropriate model formulation and solution algorithm. Possibilities for tour construction are examined in this chapter.

A basic requirement of any tour construction approach is the definition of some network over which vehicle tours may be determined. Figures 9 and 10 depict characteristics of one type of network which may be used to define tours. A simple case involving two complementary vehicles and a set of points is shown in Figure 9. Each actual point,  $i$ , appears in each vehicle subnetwork coded as point  $ik$ . Should more than one visit be allowed to point  $i$  in subnetwork  $k$ , a number of 'dummy'  $k$ -points need be defined to represent the possibility. Figure 10 indicates a device used in the formulation to constrain the number of visits to each  $ik$  point to be at most one. In general, the network to define tours may be formulated as a shortest path problem, or, as shown in Figure 9, as a flow circulation problem. Since the number of vehicles required to define a network is one ( $K' = 1$ ) in this example, one unit of 'flow' circulates through the network describing either a  $k$  or a  $\bar{k}$ -tour from dummy origin,  $\hat{O}$ , to dummy termination,  $\hat{T}$ , and returning to  $\hat{O}$  along the return arc  $(\hat{T}, \hat{O})$ . The network

FLOW CIRCULATION NETWORK FOR VEHICLE TOUR CONSTRUCTION

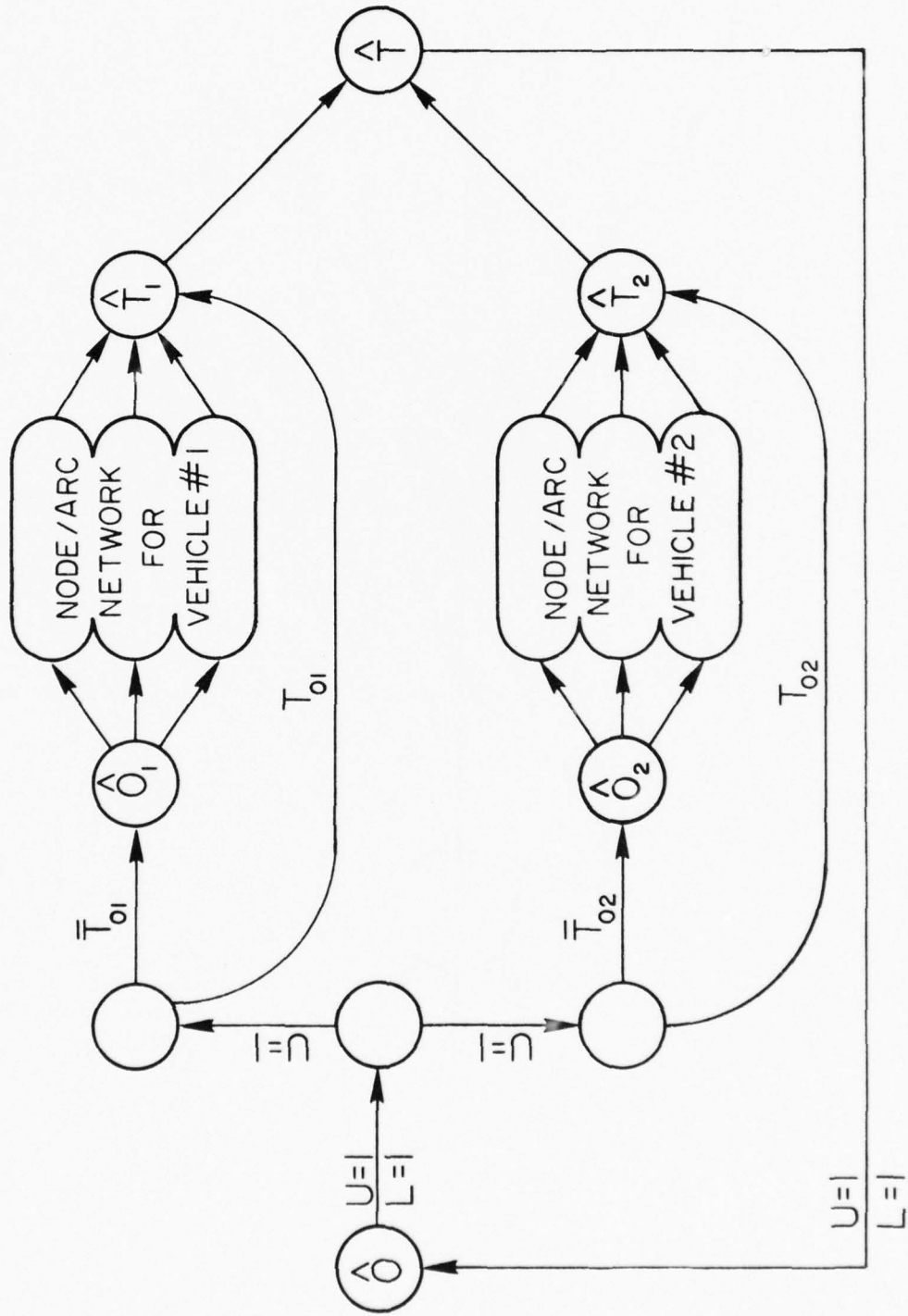


Figure 9

FLOW RESTRICTION THRU POINT  $\bar{i}$

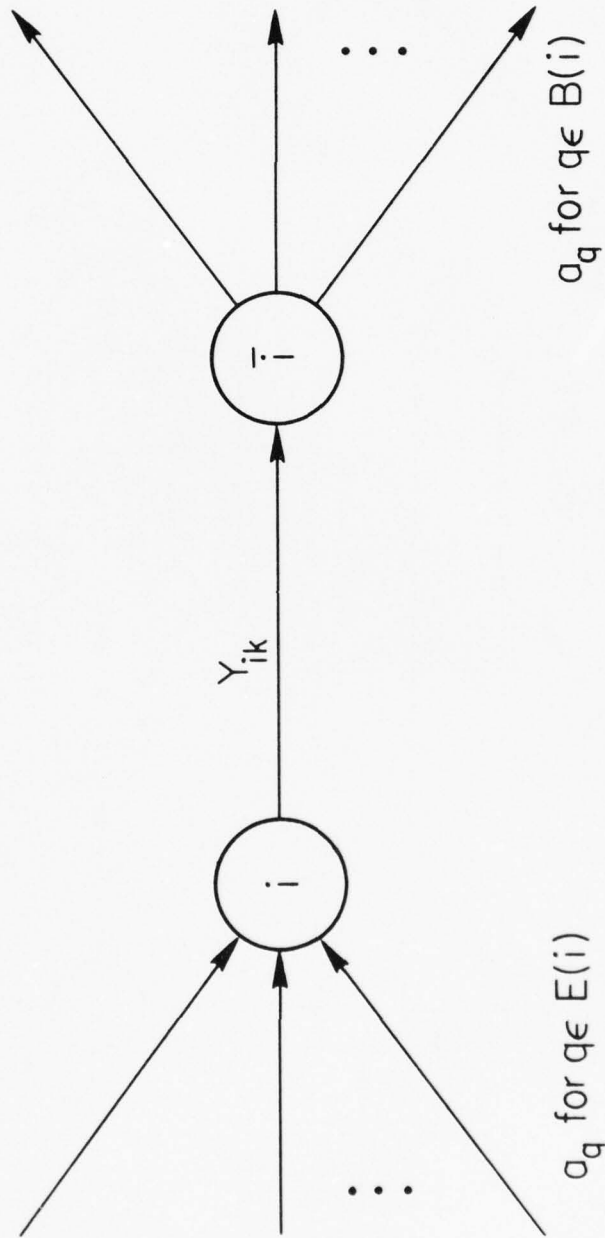


Figure 10

concept may allow a variety of logical network constraints to be defined to model actual requirements in a particular application.

The distinction between the multiple-vehicle routing problem and the transportation system planning problem is made evident in the flow-circulation network. In the former problem type, each (actual)  $i$ -point is represented in each subnetwork by a single  $ik$ -point and each  $i$ -point must be visited once and only once in all  $K'$  subnetworks combined. If only one vehicle be used, a traveling salesman problem must be solved to determine the minimum cost flow (path) from  $\hat{O}$  to  $\hat{T}$  which is constrained to visit all  $ik$ -points. In the transportation system planning problem, not all  $ik$ -points need be visited by a feasible  $k$ -tour, and some  $i$ -points may be visited more than once. Hence, the minimum cost flow path from  $\hat{O}_k$  to  $\hat{T}_k$  is sought, and the points visited on the tour are not predetermined.

Three strategies by which the tour construction approach may be implemented are examined in this chapter. An implicit enumeration approach is described first. An alternate formulation permits use of efficient LP procedures in a branch-and-bound strategy discussed in the second section. Finally a strategy to construct tours by solving a simple flow circulation problem is considered.

#### An Implicit Enumeration Approach

The primary (binary) variable in any tour construction approach must be one defined as  $\bar{a}_q$ :

$$\bar{a}_q = \begin{cases} 1 & \text{if vehicle } k \text{ traverses the arc from point } i \text{ to point } j \\ & (i, j, \text{ and } k \text{ are implicitly related by the index } q), \\ 0 & \text{otherwise.} \end{cases}$$

This variable bears a close relationship to the binary variables used in Problem P1 since

$$\bar{a}_q = \sum_{r \subseteq T_q(k)} T_{rk} \quad [47]$$

for the appropriate  $k$ . The cost of a vehicle tour may be defined in detail using the new variables:

$$C_{rk} = \sum_{q \subseteq T_q(k)} \bar{C}_q \bar{a}_q + \hat{C}_k (1 - T_{ok})$$

in which the costs of traveling from  $i$  to  $j$  and of servicing point  $j$  are included in  $\bar{C}_q$  and fixed, investment-type expenses for vehicle  $k$  are represented by  $\hat{C}_k$ .

Combining the flow circulation concept and the new cost function, the transportation system planning problem may be formulated as shown in Figure 11. Problem P2 is equivalent to Problem P1, which was examined in the previous chapter.

Material flow constraints given in equations [49] to [52] are the same as those used in Problem P1 with one exception. The right hand side of each vehicle arc capacity constraint (equation [51]) is now defined with respect to  $\bar{a}_q$  rather than  $T_{rk}$  variables using the relationship noted in equation [47] above.

The remaining constraints--equations [53] through [59]--impose the flow circulation concept to construct tours. Practical limitations on tour length are represented by equation [53]. In some applications, more complicated constraints may be required, for example, to place lower limits on tour length as well, or to allow tour length to fall in one of several feasible ranges. Equations [54] and [55] allow each  $ik$ -point to be visited at most once, imposing the form shown in

Problem P2:

$$\begin{aligned} \text{Min} \quad & f_{\text{msd}} \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right] \\ & + \sum_{q=1}^Q \bar{C}_q \bar{a}_q + \sum_{k=1}^K \hat{C}_k (1 - T_{\text{ok}}) \end{aligned} \quad [48]$$

Subject to:

Ship Time Standards

$$\sum_{\ell \in \mu(m)} \tau_{\ell}^{(t)} f_{\text{msd}} P_{\ell}^{(t)} + \sum_{\text{msd}} f_{\text{msd}} (\tau_{\text{msd}}^{\text{A}} S_{\text{msd}}^{\text{rA}} + \tau_{\text{msd}}^{\text{T}} S_{\text{msd}}^{\text{rT}}) \leq \tau_m; \quad m=1, 2, \dots, M \quad [49]$$

Transshipment Point Capacity

$$\sum_{\ell=1}^L t_{i\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq \bar{b}_i; \quad i=1, 2, \dots, I \quad [50]$$

Vehicle/arc Capacity

$$\sum_{\ell=1}^L a_{q\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq b_q \bar{a}_q; \quad q=1, 2, \dots, Q \quad [51]$$

Ship Requirements

$$\sum_{\ell=1}^L g_{\text{msd},\ell} P_{\ell}^{(t)} + S_{\text{msd}}^{\text{rA}} + S_{\text{msd}}^{\text{rT}} = 1.0; \quad \forall \text{msd} \quad [52]$$

Tour Time/Distance Length Limit

$$\sum_{q \in T_q(k)} d_q \bar{a}_q \leq \bar{d}_k; \quad k=1, 2, \dots, K \quad [53]$$

Figure 11. --Alternate Formulation of the Transportation System Planning Model

Problem P2: (Continued)

Flow Into Point  $k_i$

$$\sum_{q \in E_i(k)} \bar{a}_q - Y_{ik} = 0 ; \quad \forall ik \quad [54]$$

Flow Out of Point  $k_i$

$$Y_{ik} - \sum_{q \in B_i(k)} \bar{a}_q = 0 ; \quad \forall ik \quad [55]$$

Complementary Vehicle Relationships

$$T_{ok} + T_{ok}^- = 1 ; \quad \text{for appropriate } k, \bar{k} \quad [56]$$

Vehicle Network Constraints

As required in a specific application to define the most appropriate network flow problem, and to impose the logical structure of the network; i. e.,

$$T_{ok} - \sum_{q \in B_O(k)} \bar{a}_q = 0 ; \quad k=1, 2, \dots, K \quad [57]$$

Lower/Upper Bounds on Flows

$$\begin{aligned} 0 &\leq \bar{a}_q \leq 1; \quad \forall q \\ 0 &\leq Y_{ik} \leq 1; \quad \forall ik \\ 0 &\leq T_{ok} \leq 1; \quad \forall k \end{aligned} \quad [58]$$

Non-negativity and Integer Constraints

$$\begin{aligned} P_{\mathcal{L}}^{(t)}, S_{msd}^{rA}, S_{msd}^{rT} &\geq 0 ; \quad \forall \mathcal{L}, msd \\ \bar{a}_q, Y_{ik}, T_{ok} &= 0, 1 ; \quad \forall q, k \end{aligned} \quad [59]$$

Figure 11 (Continued)

Figure 10. If an arc incident to point  $ik$  is placed on the  $k$ -tour, these constraints assure that some arc leaving point  $ik$  will also be placed on the  $k$ -tour. A connected path from  $\hat{O}_k$  to  $\hat{T}_k$  will therefore be defined. No subtours allowing flow to circulate amongst a set of  $k$ -points not on this path will be permitted, since that additional flow would only increase costs and would therefore not be optimal. (This type of problem is encountered by the subtour elimination approach to the traveling salesman problem and is discussed in Chapter II.)

Complementary vehicle relationships may also be used in this formulation as shown by equation [56]. Additional network constraints important in a particular application may also be added according to the ingenuity of the analyst. In particular, logic required to use the flow circulation strategy must be invoked by constraints such as equation [57], which essentially allows the variable  $T_{ok}$  to be defined, indicating whether or not the null  $k$ -tour is used in solution. Lower and upper bounds for flows on each arc in the conceptual network are stated in equation [58], and correspond to the concepts described earlier.

Problem P2 is also a large-scale, mixed 01 integer, linear programming problem which may be reduced to two more simple, interacting problems by applying the concepts of Benders' decomposition. The resulting linear program, Problem LP', is stated in Figure 12. LP' is the material flow component and is the same as Problem LP with the exception that the right hand sides of the vehicle arc capacity constraints (equation [63]) are now stated in terms of the  $\bar{a}_q$  instead of the  $T_{rk}$ . A particular network design is communicated to LP' by the set of values assigned to the binary variables  $\bar{a}_q$ . In turn, LP' feeds back

Problem LP':

$$\text{Min } f_{\text{msd}} \left[ \sum_{\ell=1}^L C_{\ell} P_{\ell}^{(t)} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rA}} S_{\text{msd}}^{\text{rA}} + \sum_{\text{msd}} C_{\text{msd}}^{\text{rT}} S_{\text{msd}}^{\text{rT}} \right] \quad [60]$$

Subject to:

Ship Time Standards

$$\sum_{\ell \in \mu(m)} \tau_{\ell}^{(t)} f_{\text{msd}} P_{\ell}^{(t)} + \sum_{\text{msd}} f_{\text{msd}} (\tau_{\text{msd}}^{\text{A}} S_{\text{msd}}^{\text{rA}} + \tau_{\text{msd}}^{\text{T}} S_{\text{msd}}^{\text{rT}}) \leq \Gamma_m; m=1, 2, \dots, M \quad [61]$$

Transshipment Point Capacity

$$\sum_{\ell=1}^L t_{i\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq \bar{b}_i; i=1, 2, \dots, I \quad [62]$$

Vehicle/arc Capacity

$$\sum_{\ell=1}^L a_{q\ell} f_{\text{msd}} P_{\ell}^{(t)} \leq b_q [\bar{a}_q]; q=1, 2, \dots, Q \quad [63]$$

Ship Requirements

$$\sum_{\ell=1}^L g_{\text{msd},\ell} P_{\ell}^{(t)} + S_{\text{msd}}^{\text{rA}} + S_{\text{msd}}^{\text{rT}} = 1.0; \forall \text{msd} \quad [64]$$

Non-negativity Constraints

$$P_{\ell}^{(t)}, S_{\text{msd}}^{\text{rA}}, S_{\text{msd}}^{\text{rT}} \geq 0; \forall \ell, \text{msd}$$

Associated Dual Variable	
$\alpha_m$	[61]
$\beta_i$	[62]
$\gamma_q$	[63]
$\delta_{\text{msd}}$	[64]

Figure 12. -- Problem LP'

information concerning the efficacy of the network design by adding a constraint to Problem MP using the optimal values of the associated dual variables.

The interacting integer program, Problem MP, which consists of the flow circulation constraints augmented by the Benders' cuts, is stated in Figure 13. As in Problem 01, the  $\gamma_{q\kappa}$  are non-positive and  $B_{\kappa}$  (as defined by equation [38]) is non-negative. Since a feasible solution to LP' is guaranteed by the assumption that exclusive use of commercial air transportation is feasible, only cuts of the form in equation [66] are necessary.

Problem MP makes evident a network structure which may be used to develop an efficient, implicit enumeration algorithm based on the Balas-type strategy. As in the algorithm developed to solve Problem 01, the theory of Benders' decomposition provides an initial upper bound on the solution to MP. Furthermore, most constraints may be invoked by algorithm logic rather than time consuming search; the variable Z may be used to evaluate the optimality of a solution, treating it as a continuous variable and circumventing the need to add a surplus variable to each Benders' cut. Tour length constraints may be invoked easily. Branching may be accomplished according to the set of vehicle arcs which depart from the last point on a partial tour, which may be constructed by beginning at point  $\hat{O}$  in Figure 9 and progressing toward point  $\hat{T}$  in each vehicle subnetwork.

Problem MP

$$\text{Min } Z \quad [65]$$

Subject to:

Benders' Cuts

$$Z \geq \sum_{q=1}^Q (\bar{C}_q - b_q | \gamma_{q\kappa} | ) a_q + \sum_{k=1}^K \hat{C}_k (1 - T_{ok}) + B_\kappa; \quad \kappa = 1, 2, \dots, \bar{\kappa} \quad [66]$$

Tour Time Distance Length Limit

$$\sum_{q \in T_q(k)} d_q \bar{a}_q \leq \bar{d}_k ; \quad k=1, 2, \dots, K \quad [67]$$

Flow Into Point ki

$$\sum_{q \in E_i(k)} \bar{a}_q - Y_{ik} = 0 ; \quad \forall ik \quad [68]$$

Flow Out of Point ki

$$Y_{ik} - \sum_{q \in B_i(k)} \bar{a}_q = 0 ; \quad \forall ik \quad [69]$$

Complementary Vehicle Relationships

$$T_{ok} + T_{o\bar{k}} = 1 ; \quad \text{for appropriate } k, \bar{k} \quad [70]$$

Vehicle Network Constraints

As required in a specific application to define the most appropriate network flow problem, and to impose the logical structure of the network; i. e.,

$$T_{ok} - \sum_{q \in B_O(k)} \bar{a}_q = 0 ; \quad k=1, 2, \dots, K \quad [71]$$

Figure 13. --Problem MP

Problem MP: Continued

Lower Upper Bounds on Flows

$$0 \leq \bar{a}_q \leq 1; \quad \forall q$$

$$0 \leq Y_{ik} \leq 1; \quad \forall ik$$

$$0 \leq T_{ok} \leq 1; \quad \forall k \quad [72]$$

Integer Constraints

$$\bar{a}_q, Y_{ik}, T_{ok} = 0, 1; \quad \forall q, k, ik \quad [73]$$

Figure 13. --Continued

### A Branch-and-Bound Approach

The column generation technique developed in Chapter III to solve the material flow problem is, from one viewpoint, merely one way to define the elements in the column associated with the next (continuous) variable to enter solution in the linear program. Another view of the procedure reveals that it is an efficient way of assigning values to a large number of binary variables:  $a_{q\ell}$ ,  $t_{i\ell}$ , and  $X_{i\ell}$ . Since this set of binary values must satisfy a set of feasibility conditions, the column generation procedure solves an integer program in the course of determining the level at which a continuous variable will enter solution in a linear program. With this idea in mind, it seems that an efficient tour generation procedure might be developed by formulating an appropriate model and using simple subproblems (such as shortest path problems) to generate tours without solving a large integer program directly. One model formulated in the attempt to effect this strategy appears in matrix form in Figure 14.

Constraints of the type in equation [75] impose vehicle network logic-type constraints, complementary vehicle relationships, and tour length limits. Equation [76] constrains each tour to visit each point at most once (multiple visits could be permitted by defining another 'dummy' point to represent, for example, the second visit to one point). The entire model represents the integer subproblem resulting from Benders' decomposition and equation [77] imposes the Benders' cuts. Finally, equation [78] states that each vehicle may traverse at most one tour. Relaxing the requirement that the  $T_{rk}$  need be binary, the problem becomes a linear program which has exactly the structure of the material flow problem.

$$\text{Min } Z \quad [74]$$

Subject to:

$$IS^{\mathcal{L}} \quad + \quad \Lambda T = \mathcal{L} \quad [75]$$

$$IS^P \quad + \quad \bar{A} T = 1 \quad [76]$$

$$IR^B \quad - IS^B \quad + 1 \cdot Z \quad + [c-by] T = B \quad [77]$$

$$R^K \quad + \quad \Sigma T = 1 \quad [78]$$

Figure 14. -- Tour Generating Linear Program

Equations [78] are of the Generalized Upper Bounding type, logic constraints are analogous to the ship time standards, equations [76] are essentially the same as the vehicle arc capacities, and Benders' cuts correspond to transshipment point capacity limitations. The GUB and column generation procedures described in Chapter III could be applied directly to this formulation. Tours would be generated as desired; i. e., as a by product of determining the entering variable. The difficulty posed by this formulation is that the program could assign some level to  $T_{rk}$  between 0 and 1 so that several k-tours might be in the optimal solution of the linear programming problem simultaneously, each at some fractional value. Such a solution would have no practical meaning.

However, this model and approach would be efficient timewise. The relaxed version of integer problems is commonly used to give some indication of the solution; or, at least, to provide a bound for the optimum value of the objective. Binary values would not be guaranteed to result from the linear program since the constraint matrix is not unimodular (unimodularity is shown to be a sufficient condition for integer solutions to result from application of the linear programming procedure by Hu (1969) ). However, computational experience might show that integer solutions are obtained routinely. Alternately, solutions which might be rounded (say, by some heuristic procedure) to 'very good' integer solutions might be attained.

In fact, it is possible to improve computational efficiency even further by application of an additional large-scale programming technique. Blocks of constraints each representing limitations on a specific vehicle may be identified

in the model presented in Figure 14. The set of Benders' cuts and certain logical relationships which relate several vehicles may be viewed as coordinating constraints relating vehicle tours. This structure may be solved efficiently by Dantzig-Wolfe decomposition, or by an extended version of the GUB method which treats such block-diagonal forms (see Lasdon, 1970). Both of these methods would maintain a  $B^{-1}$  for each vehicle block, rather than for the entire constraint set and would thereby improve storage and run time efficiency. At each iteration, a new tour for a single vehicle, rather than a set of tours for all  $K'$  vehicles, would be constructed by the column generating procedure. In any event, the approach might be used to generate tours used in the first few  $\kappa$ -iterations to develop a set of Benders' cuts which, upon application of the integer programming algorithm, might lead to an optimal solution rather quickly.

Alternately, and preferably, the linear integer program in Figure 14 could be solved using a branch-and-bound procedure such as that devised by Land and Doig (1960), and Dakin (1965). Forrest, Hirst, and Tomlin (1974) describe efficient techniques for solving large-scale mixed, integer linear problems of this type. The branch-and-bound strategy, in combination with the efficient procedures applicable to the linear program, appears to offer an efficient approach for solving this integer problem directly.

#### Tour Construction Via Flow Circulation

Problem MP provides an (apparently) compact model for solution by implicit enumeration since it identifies feasibility conditions which might be

checked rapidly. In addition, the model suggests the opportunity to construct tours using very efficient network flow subproblems.

The set of feasible solutions to Problem MP (and hence the optimal solution) is a subset of the feasible solutions to the straightforward network flow problem (i. e.,  $(\bar{a}_q \underline{T}_{ok}) \subseteq \underline{A}$ ) defined by equations [68], [69], and [72]. A procedure which searches efficiently among these extreme points would therefore appear to be well worth evaluating. Several possibilities exist.

A modest amount of research which develops procedures for invoking constraints by formulating them as components of the objective functions has been reported in the literature: Anthonisse (1973), Bradley (1971), Glover and Woolsey (1972), Kendall and Zoints (1977). The procedures all remove an equation from the constraint set, multiply it by some (numerically large) integer and incorporate the resulting function into the objective. Some of these procedures operate on constraints two at a time, a policy which could be implemented effectively for the current problem, since Benders' cuts are added one at a time. The integer multipliers must satisfy certain conditions (particularly that they be relatively prime with respect to each other) to assure that the underlying constraints are, in fact, satisfied by the optimal solution to the substituted problem. The procedures are distinguished by the conditions each establishes for selecting specific values for the integer multipliers. The difficulty which none of these procedures has overcome is that the multipliers must increase in numerical value for each successive constraint incorporated into the objective.

Consequently, the multipliers must be assigned enormous--in fact computation-ally prohibitive--values when more than a few constraints are involved.

This approach would be useful if the optimal solution to Problem P2 could be identified after augmenting only a few Benders' cuts to MP (in the absence of tour constraints such as equations [53] ). It is expected that the number of cuts required to identify the optimum in problems of modest size would be on the order of 10-15. This is still more than the four-to-six constraints which may be handled effectively by available procedures of this type. However, this tactic would be successful for solving MP at each of the first few  $\kappa$ -iterations. After relegating constraints to the objective function, Problem MP reduces to a simple network flow problem for which a number of very efficient algorithms are available (for example, see Fulkerson (1961), Durbin and Kroenke (1967), and Bradley, Brown, and Graves (1976) ).

An alternate approach would apply the Lagrange multiplier technique to incorporate equations [66] and [67] into the objective function. Everett (1963) published the first application of Lagrange multipliers in solving integer programs. Brooks and Geoffrion (1966) developed an efficient means of implementing Everett's approach using linear programming. Nemhauser and Widhelm (1971) formalized and extended this work. Others, including Geoffrion (1974) and Shapiro (1971), have contributed to the development of this approach to integer programming.

Application of the Brooks/Geoffrion approach to Problem MP yields Problem BG in Figure 15. The objective of BG is to minimize a convex combination (see equation [79] ) of the Z (objective values in MP) which result from

Problem BG:

$$\text{Min } \sum_{j=1}^{\bar{J}} (Z^j) \lambda_j \quad [79]$$

Subject to:

$$\sum_{j=1}^{\bar{J}} \lambda_j = 1 \quad \mu_0 \quad [80]$$

$$\sum_{j=1}^{\bar{J}} \left[ Z^i - \sum_{q=1}^Q (\bar{C}_q - b_q |\gamma_{q\kappa}|) \bar{a}_q^j - \sum_{k=1}^K \hat{C}_k (1 - T_{ok}^j) \right] \lambda_j \geq B_\kappa ; \quad \mu_\kappa \quad [81]$$

$\kappa=1, 2, \dots, \bar{K}$

$$\sum_{j=1}^{\bar{J}} \left[ \sum_{q \in T_q(k)} d_q \bar{a}_q^j \right] \lambda_j \leq \bar{d}_k ; \quad k=1, 2, \dots, K \quad \bar{\mu}_k \quad [82]$$

$$\lambda_j \geq 0 ; \quad j=1, 2, \dots, \bar{J} \quad [83]$$

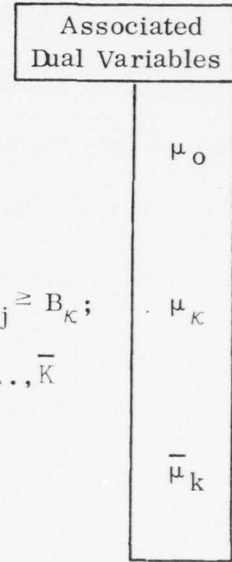
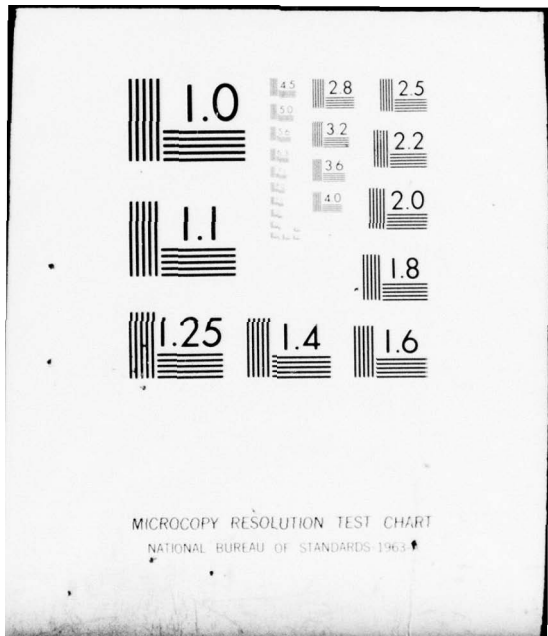


Figure 15. -- Problem BG





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each of the  $\bar{J}$  feasible extreme points (vehicle networks) of MP; that is for each  $(\bar{a}_q, \bar{T}_{ok}) \subseteq \underline{A}$ , where the set  $\underline{A}$  is defined by equations [68] through [73]. Equations [81] and [82] assure that the  $\lambda_j$  multipliers generate solutions which are feasible with respect to equations [66] and [67] in MP. BG is a linear program in the continuous  $\lambda_j$  variables. The purpose of BG is to generate optimal values of the Lagrange multipliers for MP. These multipliers are, in fact, the set of  $\mu_o$ ,  $\mu_\kappa$ , and  $\bar{\mu}_k$  variables which are the Simplex multipliers in BG.

The relationship of this approach to the current line of investigation is evident upon considering the criterion which determines the entering column at each iteration of the Simplex method applied to BG. The criterion to identify the best column to enter solution ( $\min_j "C_j - Z_j" < 0$ ) is

$$\min \left\{ Z^j - \left[ \mu_o + \sum_{\kappa=1}^{\bar{\kappa}} \left[ Z^j - \sum_{q=1}^Q (\bar{C}_q - b_q | \gamma_{q\kappa} | ) \bar{a}_q^j - \sum_{k=1}^K \hat{C}_k (1 - T_{ok}^j) \right] \mu_\kappa + \sum_{k=1}^K \left[ \sum_{q \subseteq T_q(k)} d_q \bar{a}_q^j \right] \bar{\mu}_k \right] \right\} < 0$$

which may be expressed for closer scrutiny as

$$\min \left\{ Z^j \left( 1 - \sum_{\kappa=1}^{\bar{\kappa}} \mu_\kappa \right) + \sum_{q=1}^Q \left[ (\bar{C}_q - b_q | \gamma_{q\kappa} | ) \mu_\kappa - d_q \bar{\mu}_k \right] \bar{a}_q^j + \sum_{k=1}^K \hat{C}_k (1 - T_{ok}^j) \mu_\kappa - \mu_o \right\} < 0 \quad [84]$$

This criterion evaluates the  $j^{\text{th}}$  extreme point,  $(\bar{a}_q^j, \bar{T}_{ok}^j) \subseteq \underline{A}$ , as a candidate to enter solution to the linear program, BG, given the current values of the Simplex multipliers:  $\mu_o$ ,  $\mu_\kappa$ , and  $\bar{\mu}_k$ . The constant term,  $\mu_o$ , and all

terms involving  $\bar{a}_q^j$  and  $T_{ok}^j$  may be viewed as defining the objective function for a network flow problem defined on the set of extreme points,  $\underline{A}$ . Overlooking one difficulty which is discussed below, this approach would solve MP very efficiently by solving BG, a linear program, using a network flow algorithm to generate entering columns as in equation [84]. At the optimal solution to BG, optimal values of the Lagrange multipliers ( $\mu_o$ ,  $\mu_\kappa$ , and  $\mu_k$ ) are known. The vehicle network generated by these multipliers is the optimal solution to MP at this  $\kappa$ -iteration.

The difficulty which hampers this tactic is the term involving  $Z^j$  in equation [84]. As noted in equation [66],  $Z^j$  is determined as a function of the  $(\bar{a}_q^j, T_{ok}^j)$  vector evaluated by all  $\bar{\kappa}$  Benders' cuts. It is therefore not a term which may be related simply to a single arc in the network flow problem as all other terms in equation [84] may be. There is, therefore, apparently no simple network flow subproblem which may be used in the required column generation procedure. The summation involving the  $\mu_\kappa$  could be restricted to equal 1.0 and cause the  $Z^j$  term to be eliminated so that the network flow subproblem could be used as a column generator. However, research efforts have not identified conditions which support this as a legitimate restriction.

A second difficulty may be encountered using the Lagrange multiplier approach. Problem BG may have an optimal solution and therefore a set of associated, 'optimal' Lagrange multipliers. But these multipliers might not generate the optimal vehicle network for Problem MP. This 'gap' phenomenon is incurred when it is not possible for the Lagrange multipliers to generate the

right hand sides (resources) of the associated constraints. In such cases, the approach fails to define the optimal solution to MP. The literature does not record computational experience which would allow one to develop an expectation of the likelihood of the occurrence of 'gaps' in the current problem. Nemhauser and Widhelm (1971) discuss this problem at length and suggest a means of detecting gaps. Their approach is more sophisticated than the Brooks/Geoffrion method applied above, but it introduces nonlinear relationships which would complicate solution of the transportation system planning problem.

In any event, BG could be used to solve MP if the criterion in equation [84] could be defined by an efficient procedure. In the absence of an optimizing procedure, the formulation could be the basis of a number of heuristic procedures which could be devised to define an objective for the network flow subproblem. Since problems of this type may be solved very efficiently, even heuristics which require solution of a large number of network flow problems (in identifying the best MP solution at the  $\kappa$ -iteration) would be practical and, perhaps preferable to the implicit enumeration procedure described in the previous subsection. A heuristic procedure which identifies 'good' solutions may be useful in generating 'good' Benders' cuts using LP'. That is, even though the network supplied to LP' is not truly optimal in MP at the current  $\kappa$ -iteration, it would allow LP' to furnish additional information to MP (in the form of the Benders' cut added at the next iteration) to progress toward the optimal solution. The optimizing routine using the implicit enumeration strategy may then be

required for the last few iterations. Total run time may be reduced appreciably, but computational experience is necessary to estimate computation time required for problems of realistic size.

CHAPTER VI  
CONCLUSIONS AND RECOMMENDATIONS  
FOR FUTURE RESEARCH

Conclusions

The transportation system planning models developed during this study appear to be capable of incorporating a number of considerations important to the design of actual systems. In general, this study contributes to early work on the class of problems which require a network to be synthesized to satisfy, in an optimal manner, a set of conditions concerning the network operation. A large number of scheduling and vehicle routing type problems may be shown to be members of this class.

In particular, the formulations presented in this report are capable of imposing important operating requirements concerning ship time performance which may be used to evaluate the potential effectiveness of a transportation plan in a large-scale logistics network. In addition, the approaches developed are capable of solving problems which involve symmetric distance matrices, a condition which precludes the use of most procedures designed to solve the more simple, multiple-vehicle routing problem.

It is expected that the material flow problem described in Chapter III will be useful in a number of practical applications, even though it does not provide a mathematically optimal system design. This expectation is based on the observation that much of the recent work in operations research is directed

toward providing tools of analysis which may be used to augment human judgment rather than replace it with inflexible requirements imposed by mathematical necessity. In complex logistics systems, it may be difficult, if not impossible, to make explicit all constraints and operating considerations important to the ultimate system design. Use of the material flow model permits this type of flexibility while providing an efficient solution procedure based on concepts known to be efficient computationally in prior studies.

The optimizing method presented in Chapter IV provides the capability to determine a mathematically optimal solution in problems of modest size, and may be used to indicate a reference system design which might be further modified as desired by managerial judgment. The approach does not appear to be severely restricted by the need to identify likely tours for inclusion in system design, since it is expected that such tours could be readily identified in most practical cases. The algorithm which enumerates all feasible tours extends capability to solve all problems of modest size. The proposed solution approach which includes use of Benders' decomposition and an implicit enumeration algorithm based on the Balasian approach is expected to be efficient, since both concepts have proven computationally attractive in numerous prior applications.

The greatest potential for algorithm autonomy is presented by the tour construction approaches described in Chapter V. Identification of the material flow-circulation formulation of the network design subproblem provides a basis for making feasibility checks and could be used to devise an efficient, implicit enumeration solution strategy. Several approaches (constraint aggregation and

Generalized Lagrange Multipliers) which might be applied to simplify the network design subproblem to a straightforward flow-circulation problem were investigated but do not appear to offer general computational advantages. An alternate model by which tours could be constructed using linear programming routines in a branch-and-bound algorithm provides a viable approach. Continued study and evaluation of this line of investigation appears to offer the potential for determining mathematically optimal solutions to planning problems of considerable size.

#### Recommendations for Future Research

The most pressing needs at this time are to computerize the algorithms developed during this study and to evaluate their effectiveness in solving actual problems. A great deal of work in this direction has already been accomplished, but the limitations imposed by the current minigrant have precluded complete implementation of the computer program under development. However, it is believed that the objectives of the minigrant were fully accomplished. Expectations of computational efficiency (based on the experience with concepts proven successful in many other applications) appear sufficient to conclude that the approaches developed for the transportation system problem will also be found satisfactory *runtime*wise. An outline of the existing computer program and its capabilities is given in Appendix G.

In most practical applications it is important, not only to provide an optimal solution, but also to be able to answer a variety of 'what if' questions in a sensitivity analysis designed to promote understanding of system operation.

The approaches developed during this study are readily amenable to such extensions. A full range of such capabilities should be included in any program used in an actual design situation.

Capability to formulate the complex systems planning problem and apply efficient, large scale programming techniques should stimulate study of a range of related and extended problems. For example, the related problem which requires minimization of (weighted) ship time subject to a total system cost restriction involves a minor change in model formulation and implementation of a modest number of changes in the solution procedure. This problem is also important to the Logistics Command which must ultimately provide the best level of service possible within the funding level appropriated by Congress. Extended problems of this general type might involve, for example, assessment of the ALC stocking-point location policy, or evaluation of the location of vehicle-maintenance facilities.

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Appendix A

A Summary of Current AFLC  
Operating Procedures  
(from the minigrant proposal)

The Air Force expends over forty million dollars annually to operate a dedicated air transport network called Logair and an additional thirty million dollars for transportation on commercial modes. Even though most of this expense is incurred to provide premium (air) transportation, there is no guarantee that the Order and Ship Times (O & ST) experienced are the shortest possible. While shipping time may not constitute the bulk of O & ST, it does represent the portion which is most difficult to optimize from a systems viewpoint. Transportation has a significant impact on the logistics system not only through direct costs but also from the inventory investment and operational readiness ramifications of the service level provided.

Material is transported amongst CONUS installations using commercial modes (such as UPS, Air Freight, and truck) and the extensive air transport network, Logair, which is contracted especially for Air Force needs. Decisions to ship via a commercial mode are made on an ad hoc basis within general guidelines provided by the Air Force policy. A shipment is eligible for premium transportation on Logair if criteria specified by management policy are satisfied.

The inventory system, which gives rise to transportation requirements, is composed of two echelons: the first level is distributed amongst user bases and the second level for each item is located at a single point. Five Air Logistics Centers

(ALC) in the CONUS provide world-wide support at this second echelon. Each item is stocked at only one ALC. Furthermore, depot level repair for an item is located at the same ALC. Several general types of shipments result from this inventory structure.

Investment items probably constitute the bulk of shipments which require premium transportation on Logair. By definition, these items are so costly that repair is economically justified in lieu of replacement upon failure. An investment item may be repaired at the local base or, if necessary, returned to the depot at the appropriate ALC for maintenance. In the latter case, the base would requisition a replacement from the ALC so that the base mission would not be impaired by the reduction in the number of items on hand. This procedure corresponds to an (s-1,s) inventory policy. On the average, some number of items will be in the shipping pipeline from base to ALC and vice versa. Reductions in shipping time therefore have a proportional influence on the number of spares required to support a weapons system. Since investment items are expensive, premium transportation is justified to provide expedited shipping times in order to reduce total system cost.

Less expensive items are controlled by an economic order quantity (EOQ) inventory policy and are not repaired upon failure. Base stock is replenished by placing an EOQ requisition on the appropriate ALC. Management policy provides

sufficient safety stock to accommodate longer shipping times so that more economical ground transport on commercial modes may be used. However, random demand patterns and other factors may make it necessary to upgrade the urgency of a shipment, making it eligible for air transportation.

A variety of factors may precipitate critical situations in which a weapons system would become inoperative (NORS for example) if highly expedited transportation is not provided. In this sense, the transportation system is relied upon to compensate for complications which might arise from other aspects of the logistics system. Lateral support capability (base to base resupply) represents an example of such flexibility afforded by Logair.

Additionally, special shipments are made on Logair since the dedicated mode provides certain flexibilities not typically provided by commercial modes. Classified shipments, explosive and hazardous material, and outsize cargo may be shipped on Logair for economic and managerial reasons.

In all cases, the individual requisitioner assigns a transportation priority to the shipment. This priority directs transportation personnel to provide either ground or air service and specifies a standard time within which delivery must be made.

Shipping time is therefore the primary measure of performance for the transportation system. But costs must be controlled

at reasonable levels. In general, the cost of shipping decreases as the quantity shipped increases. For example, a full truck load (FTL) may be transported at a rate per ton-mile which is significantly less than that which would be incurred by several less than truck load (LTL) shipments. Additionally, FTL shipments may provide better service time since cargo is not transferred amongst vehicles.

An optimal transportation plan should evaluate economics which might result from accumulating individual shipments at certain points in the network in order to achieve overall cost reductions from increasing vehicle utilization and promoting use of vehicles of suitable capacities. While service level may be improved under certain conditions, a time penalty for diverting shipments to permit aggregation would be incurred in most cases. The optimal plan must therefore achieve a balance between shipping time performance and system cost.

Vehicles which operate on a fixed schedule in a dedicated mode incur total costs which may be independent of utilization. Therefore, once a dedicated mode is established, shipments should be consolidated to achieve maximum vehicle utilization and to avoid additional costs from shipping by other modes. The Logair cost structure includes mileage charges and a fee for each landing so the cost is independent of cargo carried. Non-air eligible cargo may be carried at times to improve aircraft utilization and avoid extra costs of shipping the material on a commercial mode.

The Logair network is designed with the objective of providing the service level advantages of a dedicated mode. The network is specified by AFLC personnel to accommodate the requirements of CONUS installations. Requirements are forecast by each installation and submitted for AFLC use. The current design process does not explicitly consider shipping time requirements; only total transportation capacity needs are evaluated.

The network, as historically designed, consists of trunk and feeder routes. Efficient, high capacity aircraft are used on the trunk lines to transport aggregated shipments amongst the five ALC's, Wright-Patterson Air Force Base, and six ports of debarkation from which shipments are transshipped to overseas installations. Feeder routes, which are serviced by smaller aircraft, connect ALC's to bases. Current management policy permits only one Logair flight to service a base each day. Each of the bases in the network may actually represent the collective needs of a number of installations in its vicinity, so shipments may require trucking to ultimate destinations.

AFLC personnel designate the aircraft to be used and the route each will service by applying a heuristic, manual design procedure. Since aircraft require extensive routine maintenance, routes are carefully designed considering the locations of contractor maintenance facilities. The length of each route must consider aircraft flight capability as

well as federal regulations regarding crew working times. Long routes are feasible but increase costs since crews must be changed.

The final aspect of system design, specifying arrival/ departure times for each aircraft at each point serviced, is currently negotiated with the contractor. Prime considerations in this process are the times at which bases provide ground crews to service Logair flights and flight crew needs as represented by the contractor.

Management policies by which the system operates appear attractive intuitively, but the cost implications which they entail have not been well defined quantitatively. Current heuristic procedures do not allow the trade-offs between shipping times and costs to be made explicit for management review. Succinctly, a systems analysis is necessary to prescribe the optimal commercial/dedicated mode transportation system and permit thorough evaluation of service level requirements and management policies.

Appendix B

The Transportation System Planning Model

Developed during the 1976 USAF/ASEE

Summer Faculty Program

(Excerpted from the minigrant proposal)

$A'_l$  = Administrative cost of establishing material flow chain  $l$

$A_{(ijk)l_m}$  = Proportion of  $f_{mid}$  flowing on arc  $(ijk)$  of chain  $l_m$  (DV)\*

$C_k$  = Weight (volume) capacity of vehicle  $k$

$C'_{l_m}(F_{l_m})$  = Cost of shipping  $F_{l_m}$  amount of material  $m$  on chain  $l$

$d$  = Destination point for material shipment

$d'_{ijk}$  = Cost of vehicle  $k$  traversing arc  $ij$

$D_{ijk}$  = Distance on arc  $ij$  for vehicle  $k$

$\bar{D}_k, \underline{D}_k$  = Upper, lower bounds on total route (or path) distance (time) for vehicle  $k$

$\bar{D}_k, \underline{D}_k$  = Upper, lower bounds on distance (time) for each arc traversed by vehicle  $k$

$f_{mid}$  = Required weight (volume) of material  $m$  to be shipped from  $i$  to  $d$

$F_{l_m}$  = Weight (volume) flow of material  $m$  on chain  $l$  (DV)\*

$i$  = Index on points (ALC's and BASES) in the system

$j$  = Index on points (ALC's and BASES) in the system

$k$  = Index on vehicles (aircraft, trucks, commercial modes) available for use

$l$  = Index on transport chains connecting material flow origins and destinations

$L_{ik}$  = Cost for vehicle  $k$  to service point  $i$

- $\mathcal{L}_{mid}$  = Set of chains connecting origin  $i$  to destination  $d$   
for material  $m$
- $m$  = Index on material types (priority, weight, cost, etc.)
- $n$  = Number of a specific arc in a vehicle route (path)  
or material chain
- $\emptyset$  = Point of origin for a vehicle
- $P_{\ell m}$  = Portion of material  $m$  (for a specific OD pair)  
flowing on chain  $\ell$  (DV)\*
- $q_{ik}$  = 1 if vehicle  $k$  serves point  $i$  and "0" otherwise (DV)\*
- $s$  = Source or origin of a material flow chain
- $t$  = Termination point for a vehicle route (path)
- $T'_{(ijk)m}$  = Time required for material  $m$  to traverse arc  $(ijk)$
- $T_{\ell m}$  = Time required for material  $m$  to traverse chain  $\ell$
- $u_k$  = 1 if vehicle  $k$  is utilized and "0" otherwise (DV)\*
- $U_{\ell m}$  = Urgency (or priority) factor associated with  
material  $m$  on chain  $\ell$
- $v_{\ell m}$  = 1 if chain  $\ell m$  is used and "0" otherwise (DV)\*
- $V_k$  = Fixed cost for using vehicle  $k$
- $V_{mid}$  = Number of chains considered for mid requirements
- $X_{ijkn}$  = 1 if vehicle  $k$  traverses arc  $(ij)$   $n^{\text{th}}$  in its  
route (path), "0" otherwise (DV)\*
- $Y_{(ijk)\ell mn}$  = 1 if arc  $(ijk)$  is  $n^{\text{th}}$  in chain  $\ell$  for material  
 $m$ , "0" otherwise (DV)\*

$\alpha_k$  = Factor to represent compartmentalized capacity  
of vehicles

$\tau_m$  = Maximum standard shipping time permitted for  
material m

$\bar{\tau}_{std}$  = Standard mean shipping time for the system

---

(DV)\* Implies a decision variable in the program.

Transportation System Model

$$\begin{aligned} \min \quad & \sum_{\ell m} C'_{\ell m} (F_{\ell m}) + \sum_{ijkn} d'_{ijk} X_{ijkn} + \sum_{ik} L_{ik} \sum_{jn} X_{jikn} \\ & + \sum_k V_k u_k + \sum_{\ell m} A'_{\ell m} v_{\ell m} \end{aligned} \quad [23]$$

subject to:

I. The Dedicated Mode Network

$$\sum_j X_{\emptyset jkl} = u_k ; \forall_k, \text{ vehicle } k \text{ leaves its point of origin } \emptyset \text{ if used} \quad [6]$$

$$\sum_{jn} X_{jtkn} = u_k ; \forall_k, \text{ vehicle } k \text{ terminates at point } t \text{ if used} \quad [7]$$

$$\sum_{ij} X_{ijkn} \leq u_k ; \forall_{kn}, \text{ vehicle } k \text{ traverses its } n^{\text{th}} \text{ arc at most once} \quad [8]$$

$$\sum_n X_{ijkn} \leq u_k ; \forall_{ij,k}, \text{ vehicle } k \text{ traverses arc } ij \text{ at most once (optional)} \quad [9]$$

$$\sum_i X_{ijkn} = \sum_i X_{jik(n+1)} ; \forall_{jkn, j \neq t, j \neq \emptyset}, \text{ tour } k \text{ is connected} \quad [10]$$

$$\underline{D}_k \leq \sum_{ijn} D_{ijk} X_{ijkn} \leq \bar{D}_k ; \forall_k, \text{ tour } k \text{ distance (time) limits} \quad [11]$$

$$\underline{D}_k \leq \sum_n D_{ijk} X_{ijkn} \leq \bar{D}_k ; \forall_{ij,k}, \text{ distance (time) limits for arcs on tour } k \quad [12]$$

[Linear constraints in  $u_k$  giving vehicle relationships]

## II. Multicommodity Material Flow

$$\sum_{l \in \mathcal{L}_{mid}} F_{lm} = f_{mid} ; \forall_{mid}, \text{ flow requirements are satisfied} \quad [13]$$

$$\sum_{l \in \mathcal{L}_{mid}} v_{lm} \leq \bar{v}_{mid} ; \forall_{mid}, \text{ number of } f_{mid} \text{ chains is limited} \quad [14]$$

$$\sum_{jk} Y_{(sjk)lm1} = v_{lm} ; \forall_{lm} \in \mathcal{L}_{mid}, \text{ chain } lm \text{ begins at point } s \quad [15]$$

$$\sum_{jkn} Y_{(jdk)lmn} = v_{lm} ; \forall_{lm} \in \mathcal{L}_{mid}, \text{ chain } lm \text{ ends at point } d \quad [16]$$

$$\sum_{ijk} Y_{(ijk)lmn} \leq v_{lm} ; \forall_{n, lm} \in \mathcal{L}_{mid}, \text{ chain } lm \text{ uses } n^{\text{th}} \text{ link at most once} \quad [17]$$

$$\sum_n Y_{(ijk)lmn} \leq v_{lm} ; \forall_{ij, lm} \in \mathcal{L}_{mid}, \text{ chain } lm \text{ uses arc } ij \text{ at most once (optional)} \quad [24]$$

$$\sum_{ik} Y_{(ijk)lmn} = \sum_{ik} Y_{(jik)lm(n+1)} ; \forall_{lm} \in \mathcal{L}_{mid}, j \neq t, j \neq \emptyset$$

chain  $lm$  is connected

[18]

$$\sum_n Y_{(ijk)lmn} \leq \sum_n X_{ijkn} ; \forall_{ijk, lm} \in \mathcal{L}_{mid}, \text{ flow link-vehicle arc relationship} \quad [19]$$

$$\sum_{lmn} Y_{(ijk)lmn} F_{lm} \leq \alpha_k C_k \sum_n X_{ijkn} ; \forall_{ij, k}, \text{ vehicle capacity} \quad [20]$$

### III. Level of Service

$$T_{lm} = \sum_{ijkn} Y_{(ijk)lmn} T'_{(ijk)m} \leq \bar{\tau}_m ; \forall l_m \in \mathcal{L}_{mid}, \text{ shipping} \\ \text{time on each flow chain} \quad [21]$$

$$(1 / \sum_{mid} f_{mid}) \sum_{lm} U_{lm} T_{lm} F_{lm} \leq \bar{\tau}_{std} ; \text{ system performance} \\ \text{measure} \quad [22]$$

### IV. Non-Negativity and 0,1 Constraints

$$F \geq 0 ; X, Y, u, v = 0, 1 \quad [25]$$

Appendix C

Second Model Formulation Developed in Fall, 1976

THE OHIO STATE UNIVERSITY

December 7, 1976

Dr. Ismail N. Shimi  
Program Manager  
Air Force Office of Scientific  
Research  
Mathematical and Information  
Science  
Bolling Air Force Base  
Washington, D.C. 20332

Dear Dr. Shimi:

I submitted a proposal (number 06017-55-00) entitled "Optimization of a Transportation System Planning Problem" to your minigrant program early this fall. The proposed research is a continuation of work initiated during the 1976 USAF/ASEE Summer Faculty Research Program held at the Logistics Command Headquarters at Wright-Patterson Air Force Base. The proposal presented a mathematical programming formulation of a complex logistics problem and recommended research to continue model development and examine potential solution approaches to evaluate computation time required to solve problems of practical size.

Continued thought on this problem has just recently yielded a much improved formulation of the transportation system problem; and, according to our telephone conversation of 12-7-76, I am forwarding the new formulation to you for review. While this may not yet represent the ultimate form, it is a significant improvement over the model in the proposal, and I believe it should be considered as a part of the proposal review process.

The new model contributes to the first step of the proposed work by providing a more tractable form and by making a number of feasibility-checking procedures obvious. The new model makes a significant reduction in the number of zero/one integer variables and should, therefore, reduce computation times required to solve problems of realistic size. A portion of the network design burden is switched from the master integer program to new variables Y and W which may be treated as continuous variables. In fact, specific values of Y and W are either zero or one and are determined directly by current values of the integer variables X and G in constraints (7), (8), (9), and (19). Values of dual variables

Dr. Ismail N. Shimi  
December 7, 1976  
Page two

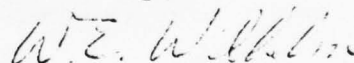
associated with these constraints may be determined directly (this portion of the problem is trivial and does not even require a linear programming solution) and provide feedback to improve the network design in the X and G variables.

Constraints in the X variables (equations 2-5) represent a special case transportation problem as do those constraints involving only the G variables (equations 12-15). These two sets of constraints are tied together by equation (18). This structure permits efficient feasibility checking, and, in fact, may lead to an efficient solution algorithm.

This formulation is unique and provides an efficient approach to modeling a class of problems in which a network must be designed as a component of the overall problem. Problems in scheduling, sequencing, and vehicle routing fall into this class and certain problems not amenable to previous approaches may be solvable using this decomposition approach. A formal research proposal to study the structure of this class of problems is being written and will be submitted to you in the near future. The transportation system problem is one specific, albeit complex, model in this class.

I would appreciate your considering the improved formulation as a factor in the review of the minigrant proposal submitted earlier.

Sincerely,



W. E. Wilhelm  
Assistant Professor

WEW:cm

cc: J. Abell, AFLC  
A. B. Bishop, OSU  
M. W. Woody, OSURF

## The Transportation Systems Problem

### Notation

$A'_\ell$	=	Administrative cost of material flow chain $\ell$
$A_{ijnkq}$	=	Cost of material flow chain $q\ell$ using arc $ijnk$
$C_k$	=	Weight (volume) capacity of vehicle $k$
$C'_\ell(F_\ell)$	=	Cost of shipping $F_\ell$ amount of material on chain $\ell$
$d$	=	Destination point for material shipment
$d'_{ijk}$	=	Cost of vehicle $k$ traversing arc $ij$
$D_{ijk}$	=	Distance on arc $ij$ for vehicle $k$
$\bar{D}_k, \underline{D}_k$	=	Upper, lower bounds on total route (or path) distance (time) for vehicle $k$
$\bar{D}_k, \underline{D}_k$	=	Upper, lower bounds on distance (time) for each arc traversed by vehicle $k$
$f_{msd}$	=	Required weight (volume) of material $m$ to be shipped from $S$ to $d$
$F_\ell$	=	Weight (volume) flow of material $m$ on chain $\ell$
$G_{iq\ell}$	=	1 if material flow chain $\ell$ departs from point $i$ as the $q^{\text{th}}$ link and 0 otherwise (DV*)
$i$	=	Index on points in the system
$j$	=	Index on points in the system
$k$	=	Index on vehicles (aircraft, trucks, commercial modes,) available for use
$K$	=	Number of vehicles available for use
$\ell$	=	Index on transport chains connecting material flow origins and destinations. Each $k$ is specific to some $f_{msd}$ .
$L_{ik}$	=	Cost for vehicle $k$ to service point $i$

- $\mathcal{L}_{msd}$  = Set of chains connecting source  $s$  to destination  $d$  for material  $m$
- $m$  = Index on material types (priority, weight, cost, etc.)
- $n$  = Number of a specific arc in a vehicle route (path)
- $N_k$  = Number of points which vehicle  $k$  may visit  $n=1,2,\dots, N_k$
- $[N_k]$  = set of specific  $N_k$  specific points which  $k$  may visit
- $\emptyset$  = Point of origin for a vehicle
- $\bar{P}_\ell$  = Portion of  $f_{msd}$  which flows on chain (DV)\*
- $P_{ink\ell}$  =  $\bar{P}_\ell$  on arcs  $(ink)$  used by chain  $\ell$  (DV)\*
- $q$  = Index on number of arcs in material flow chain
- $\bar{Q}_\ell$  = Maximum number of arcs management will permit in chain  $\ell$
- $[\bar{Q}_\ell]$  = Set of points which might be used in chain  $\ell$
- $Q$  = Large positive number
- $s$  = Source or origin of a material flow chain  $\ell$
- $t$  = Termination point for a vehicle route (path)
- $T_{ink\ell}$  = Time required to traverse arch  $(ink\ell)$
- $T_\ell$  = Time required for material  $m$  to traverse chain  $\ell$
- $U_{nk}$  = 1 if vehicle services  $n$  points, 0 otherwise (DV)\*
- $\bar{U}_\ell$  = Urgency (or priority) factor associated with chain  $\ell$
- $v_{\ell q}$  = 1 if chain  $\ell$  is composed of  $q$  arcs, 0 otherwise (DV)\*
- $V_k$  = Fixed cost for using vehicle  $k$
- $\bar{V}_{msd}$  = Number of chains considered for  $msd$  requirements
- $X_{ink}$  = 1 if point  $i$  is the  $n^{\text{th}}$  point in the route (path) for vehicle  $k$  0 otherwise (DV)

$Y_{ijk}$  = 1 if vehicle k travels from point i to point j  
0 otherwise (DV)\*

$W_{ijnkq\ell}$  = 1 if both G and X allow vehicle k and chain  $\ell$   
to traverse the arc from point i to point  
j, 0 otherwise (DV)\*

$\alpha_k$  = Factor to represent compartmentalized capacity  
of vehicles

$\tau_m$  = Maximum standard shipping time permitted for  
material m

$\bar{\tau}_{std}$  = Standard mean shipping time for the system

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(DV)\* Implies a decision variable in the program.

$$\begin{aligned}
\min \quad & \sum_{\ell} C'_{\ell} (\bar{P}_{\ell} f_{msd}) + \sum_{ijk} d'_{ijk} Y_{ijk} \\
& + \sum_{ik} L_{ik} \left( \sum_n X_{ink} \right) + \sum_k V_k (1 - U_{0k}) + \sum_{\ell} A'_{\ell} (1 - v_{0\ell}) \\
& + \sum_{ikl} A_{ijnkq\ell} W_{ijnkq\ell}
\end{aligned} \tag{1}$$

Subject to:

I. Dedicated Mode Network Constraints

$$\sum_{in} X_{ink} = \left( \sum_n n U_{nk} \right), \quad \forall k \tag{2}$$

$$\sum_{in} n X_{ink} = \frac{1}{2} \sum_n (n^2 + n) U_{nk}, \quad \forall k \tag{3}$$

$$X_{\emptyset 1k} = (1 - U_{0k}), \quad \forall k \tag{4}$$

$$n X_{tnk} = \left( \sum_n n U_{nk} \right) \quad \text{for } k \text{ which traverse a path;} \\
n = 2, 3, \dots, N_k \tag{5}$$

$$\sum_{n=0}^{N_k} U_{nk} = 1, \quad \forall k \tag{6}$$

$$X_{\emptyset 1k} + X_{j2k} - Y_{\emptyset jk} \leq 1 \quad \forall k; j \in [N_k], j \neq \emptyset \tag{7}$$

$$X_{ink} + X_{j,n+1,k} - Y_{ijk} \leq 1 \quad \forall k; i, j \in [N_k], i \neq j \\
n = 2, 3, \dots, N_k - 1 \tag{8}$$

$$-nX_{jnk} + \sum_n U_{nk} + Y_{j\emptyset, n+1} \geq 1 \quad \text{for } k \text{ which traverse a route;} \\ j \in [N_k], j \neq \emptyset; n=2, 3, \dots, N_k \quad (9)$$

$$D_k \leq \sum_{ij} D_{ijk} Y_{ijk} \leq \bar{D}_k, \quad \forall k \quad (10)$$

$$D_k \leq D_{ijk} Y_{ijk} \leq \bar{D}_k, \quad \forall k, i, j \in [N_k] \quad (11)$$

## II. Multicommodity Material Flow Chains

$$\sum_{iq} G_{iq\ell} = \sum_q q v_{q\ell} \quad \forall \ell \quad (12)$$

$$\sum_{iq} q G_{iq\ell} = \frac{1}{2} \sum_q (q^2 + q) v_{q\ell}, \quad \forall \ell \quad (13)$$

$$G_{s1\ell} = (1 - v_{0\ell}), \quad \forall \ell \quad (14)$$

$$q G_{dq\ell} = \sum_q q v_{q\ell}, \quad \forall \ell, q \quad (15)$$

$$\sum_{q=0} v_{q\ell} = 1, \quad \forall \ell \quad (16)$$

$$\sum_{\ell \in \mathcal{L}_{msd}} (1 - v_{0\ell}) \leq \bar{v}_{msd}, \quad \forall msd \quad (17)$$

$$G_{iq\ell} + G_{j,q+1,\ell} \leq \sum_{nk} (x_{ink} + x_{j,n+1,k}), \quad \forall ijq\ell \quad (18)$$

$$G_{iq\ell} + G_{j,q+1,\ell} + x_{ink} + x_{j,n+1,k} - w_{ijnkq\ell} \leq 3$$

$$w_{ijnkq\ell} \leq G_{iq\ell}; \quad w_{ijnkq\ell} \leq G_{j,q+1,\ell}; \quad w_{ijnkq\ell} \leq x_{ink};$$

$$w_{ijnkq\ell} \leq x_{j,n+1,k} \quad \forall ijql \quad nk \quad (19)$$

### III. Multicommodity Material Flow Constraints

$$\sum_{\ell \in \mathcal{L}_{msd}} \bar{p}_{\ell} = 1.0, \quad \forall msd \quad (20)$$

$$\sum_{nk} p_{snk\ell} \leq G_{s1\ell}, \quad \forall \ell \quad (21)$$

$$\delta_{ink\ell} \leq \sum_q w_{ijnkq\ell}, \quad \forall kn, i\ell \quad (22)$$

$$\sum_{kn} \delta_{ink\ell} \leq 1, \quad \forall i\ell \quad (23)$$

$$p_{ink\ell} \geq \bar{p}_{\ell} - Q(1 - \delta_{ink\ell}) \quad \forall i \quad nk \quad (24)$$

IV. Service Level Constraints

$$\sum_{\ell} f_{\text{msd}} P_{\text{ink}\ell} \leq \alpha_k C_k, \quad \forall \text{ink} \quad (25)$$

$$\sum_{\substack{\text{ink} \\ \ell \in \mathcal{L}_{\text{msd}}}} \bar{U}_{\ell} T_{\text{ink}\ell} f_{\text{msd}} P_{\text{ink}\ell} \leq \tau_{\text{msd}} f_{\text{msd}}, \quad \forall \text{msd} \quad (26)$$

$$\sum_{\text{ink}\ell} \bar{U}_{\ell} T_{\text{ink}\ell} f_{\text{msd}} P_{\text{ink}\ell} \leq \bar{\tau}_{\text{std}} \sum_{\text{msd}} f_{\text{msd}} \quad (27)$$

V. Non-negativity Constraint

$$X, G, \delta, U, v = 0, 1 \quad (28)$$

$$Y, W, \bar{P}, P \geq 0$$

The objective of this mathematical programming formulation (Equation 1) is to minimize total system cost which is composed of the cost of shipping on each material flow chain, the travel cost of vehicles used by dedicated modes, the cost of a vehicle to service a point, a fixed charge for each vehicle used, an administrative fee for each material flow chain established, and the cost of material flow on each arc in the transportation network.

The purpose of each constraint is described below:

<u>Constraint Equation Number</u>	<u>Affect of Constraint</u>
2	assures that each vehicle services the assigned number of points, ( $nU_{nk}$ ).
3	assures that vehicle tours are connected
4	assures that a vehicle tour begins at the predefined point of origin if the vehicle is used
5	requires a vehicle path to terminate at the predefined termination point
6	determines the number of points in the tour for each vehicle
7	defines the arc which a vehicle takes upon departing its predefined point of origin
8	defines the arcs used on the tour of vehicle k
9	defines the arc from the ( $nU_{nk}$ ) <sup>th</sup> point visited to the predefined point of origin for vehicles which traverse a route

<u>Constraint Equation Number</u>	<u>Affect of Constraint</u>
10	limits vehicle route/path distance/time to predetermined specification
11	limits vehicle arc distance/time to predetermined specification
12	assures that each material flow chain passes thru the assigned number of points, $(qV_{q\ell})$
13	assures that material flow chains are connected
14	requires a material flow chain to originate at the source point
15	requires a material flow chain to end at the destination point
16	determines the number of points in a material flow chain
17	assures that the actual number of material flow chains defined for an (mid) combination is within a limit set by management
18	assures that a material flow link from point i to point j is not defined unless some vehicle services that arc
19	defines the specific vehicles (k) which are available for use by the material flow link $(ijq\ell)$ from point i to point j
20	assures that all material for a given (mid) combination will be delivered to the destination
21	allows material to flow only in chains which are defined
22	the vehicle used by chain $\ell$ to depart point i must be one which traverses the correct arc (note: this constraint holds for fixed values of W and carries implications about off loading material on vehicles which visit a point or an arc more than once.)

Constraint Equation  
Number

Affect of Constraint

- |    |   |
|----|---|
| 23 | a material flow link will use at most one vehicle between points i and j  |
| 24 | assures that all vehicle arcs used by material flow chain $\ell$ carry the same material  |
| 25 | assures that vehicle capacity is not violated on any vehicle arc  |
| 26 | assures that an urgency/material weighted time average of shipping time for each (mid) combination is within limits set by management                                       |
| 27 | assures that an urgency/material weighted time average of shipping time over all material flow chains is within a standard set by management for overall system performance |

Appendix D

Summary of Notation Used in Report

- $\underline{A}$  = set of extreme points in flow circulation network defined in Problem MP
- $\bar{a}_q$  = 1 if vehicle  $k$  traverses arc  $q$  (from point  $i$  to point  $j$ ), 0 otherwise
- $a_{q\ell}$  = 1 if flow chain  $\ell$  uses arc  $q$  ( $ij$  and  $k$  implicit), 0 otherwise
- $b_q$  = (vehicle) capacity provided on arc  $q$   
 $q \Rightarrow$  specific points  $ij$  and vehicle  $k$   
 The index  $q$  also indicates the 1st or 2nd time arc  $ij$  is traversed by  $k$ .
- $\bar{b}_i$  = transshipment capacity at point  $i$
- $B^{-1}$  = inverse of matrix whose columns correspond to a basic feasible solution
- $B_O$  = minimum cost of making all material shipments on commercial modes subject to the ship time constraints
- $B_\kappa$  = non-negative constant in Benders' cut  $\kappa$  (see equation [38] )
- $B_i(k)$  = set of arcs beginning at point  $i$  for vehicle  $k$
- $C$  = cost coefficient for flow on chain  $\ell$
- $\hat{C}_k$  = fixed cost for using vehicle  $k$
- $\bar{C}_q$  = cost of traversing arc ( $ijk$ ) and of servicing point  $j$
- $C_{msd}^{rA}$  = cost/unit to ship material combination  $msd$  on commercial air
- $C_{msd}^{rT}$  = cost/unit to ship material combination  $msd$  on commercial truck
- $C_{rk}$  = cost of tour  $r$  for vehicle  $k$
- $C'_t = \sum_{rk} C_{rk} T_{rk}$ , for  $T_{rk} \subseteq J_t$
- $d$  = destination point for a material shipment
- $d_o$  = dual variable associated with constraint defining  $P_o$  in Figure 2
- $d_q$  = length in distance (or time) of arc  $q$
- $\bar{d}_k$  = maximum tour length for vehicle  $k$

- $D$  = dummy destination point  
 $D_t$  = set of tours excluded at step 1/B in solving Problem 01  
 $D(N)$  = 'distance' from source S to node N  
 $\bar{D}_q$  = 'distance' or length of arc q  
 $E_t$  = set of subscripts of tours which have no associated  $\gamma$  values  
 $E_i^{(k)}$  = set of arcs ending at point i for vehicle k  
 $f_{msd}$  = shipping requirements commodity m, from source s to destination d  
 $F$  = Problem P1 objective function value  
 $g_{n,\ell}$  = 1 if flow chain  $\ell$  accommodates combination  $msd = n$ , 0 otherwise  
 $i$  = 1, 2, ..., I points  
 $j$  = 1, 2, ..., I points  
 $J_t$  = vector of the  $T_{rk}$  variables that have been assigned a binary specification in the current partial solution in Problem 01  
 $\bar{J}$  = number of feasible vehicle network designs  
 $k$  = 1, 2, ..., K vehicles  
 $\bar{k}$  = vehicle which is the complement of vehicle k  
 $K_\ell$  = column representing flow chain  $\ell$   
 $K'$  = number of vehicle tours in a complete network  
 $\ell$  = 1, 2, ..., L flow chains  
 $m$  = 1, 2, ..., M commodities types (M may imply all types)  
 $n_k$  = number of tours defined in Problem 01 for vehicle k  
 $N_t^c$  = set of subscripts of the T variables which are candidates for improving the current partial solution in Problem 01  
 $O_{sm}$  = order and ship time at source s for material type m  
 $\hat{O}$  = 'dummy' point of origin for all vehicle tours  
 $\hat{O}_k$  = point of origin for vehicle k

- $P_{\ell}^{(t)}$  = portion of material of type  $t$ ,  $f_{msd}$ , shipped on flow chain  $\ell$   
 $\ell$  and  $t \Rightarrow$  a specific  $msd$  combination
- $P_o$  = Problem LP objective function value
- $q$  =  $1, 2, \dots, Q$  vehicle arcs ( $ijk$  implicit)
- $r$  = index for vehicle tours in Problem P1
- $R_{msd}^r$  = artificial variable for material flow requirement  $f_{msd}$
- $s$  = source point for a material shipment
- $S$  = dummy source point
- $S_q^a$  = slack variable associated with vehicle-arc constraint  $q$
- $S_m^s$  = slack variable associated with ship time constraint  $m$
- $S_i^t$  = slack variable associated with transshipment point  $i$  capacity
- $S_{msd}^{rA}$  = portion of  $f_{msd}$  shipped on commercial air
- $S_{msd}^{rT}$  = portion of  $f_{msd}$  shipped on commercial truck
- $t_{i\ell}$  = 1 if flow chain  $\ell$  transships at point  $i$ , 0 otherwise
- $T_q^t$  = shipment time on vehicle arc  $q$
- $T_{i\ell}^e$  = (overnight) ship time delay incurred if flow chain  $\ell$  is carried  
 through vehicle  $k$  termination point  $i$
- $T_{i,m}^s$  = transshipment time for material type  $m$  at point  $i$
- $T_{ok}$  = 1 if vehicle  $k$  is not used, 0 if it is used (the null tour) ( $\bar{T}_{ok} = 1 - T_{ok}$ )
- $\hat{T}$  = 'dummy' termination point for all vehicle tours
- $\hat{T}_k$  = termination point for vehicle  $k$
- $T$  = vector of  $T_{rk}$  values
- $T(k)$  = set of tours which vehicle  $k$  might traverse
- $T_q(k)$  = set of tours for vehicle  $k$  which traverse arc  $q$
- $T_{rk}$  = 1 if vehicle  $k$  traverses tour  $r$ , 0 otherwise

- $U$  = number of arcs on a tour  
 $U_{dm}$  = unload time incurred at shipment destination  $d$  for material type  $m$   
 $V$  = set of all vehicles provided  
 $V_t$  = set of subscripts of vehicles which have been assigned (one  $T_{rk} = 1$  in  $J_t$ ) in  $J_t$   
 $\bar{V}_t$  = set of subscripts of vehicles which are excluded because their complementary vehicles are in the set  $V_t$   
 $V_t^c$  = set of vehicles which are candidates for inclusion in solution at level one  
 $V^d$  = set of vehicles for which all tours  $\subseteq D_t$   
 $V^e$  = set of vehicles for which all tours  $\subseteq E_t$   
 $V_t^\phi$  = set of vehicles which have all tours at level zero in  $J_t$   
 $W_{rk\kappa} = C_{rk} - \sum_{q \in T_k(q)} b_q | \gamma_{q\kappa} |$  the coefficient of tour  $r$  for vehicle  $k$  in Benders' cut  $\kappa$   
 $W'_{t\kappa} = \sum_{kr} W_{rk\kappa} T_{rk}$  for  $T_{rk} \subseteq J_t$   
 $X_{i\ell} = 1$  if flow chain  $\ell$  is carried through vehicle  $k$  termination point  $i$ ,  
 $0$  otherwise  
 $Y_{ik}$  = amount of 'flow' through point  $i$  in subnetwork  $k$  in Problem P2  
 $Z$  = continuous variable in Benders' subproblems  
 $Z_c$  = current best  $Z$  value resulting from a complete solution  
 $Z_p$  =  $Z$  value which results from a completion of the current partial solution,  $J_t$   
 $\alpha$  = parameter  $0 \leq \alpha \leq 1$   
 $\alpha_m$  = dual variable associated with (material  $m$ ) ship time constraint  
 $|\bar{\alpha}_\ell| = |\bar{\alpha}_m| + |\bar{\alpha}_M|$

- $\beta_i$  = dual variable associated with transshipment capacity at point  $i$   
 $\gamma_q$  = dual variable associated with vehicle arc  $q$   
 $\Gamma_m$  = ship time standard for material type  $m$   
 $\delta_n$  = dual variable associated with  $n = \text{msd}$  ship requirement  
 $\bar{\delta}_\ell$  = see equation [18]  
 $\kappa$  = iteration number between Benders' subproblems  
 $\bar{\kappa}$  = current Benders' iteration number  
 $\lambda$  = continuous decision variable,  $0 \leq \lambda \leq 1$   
 $\mu(m)$  = set of flow chains accommodating material type  $m$   
 $\mu_0$  = dual variable associated with normalizing constraint  
 $\mu_\kappa$  = dual variable associated with Benders' cut  $\kappa$   
 $\bar{\mu}_k$  = dual variable associated with tour length constraint for vehicle  $k$   
 $\eta$  = set of vehicles used simultaneously in a shortest path problem used for column generation  
 $\tau_\ell^{(t)}$  = ship time on flow chain  $\ell$  ( $t$  implicit)  
 $\tau_{\text{msd}}^A$  = ship time for  $\text{msd}$  combination on commercial air  
 $\tau_{\text{msd}}^T$  = ship time for  $\text{msd}$  combination on commercial truck

Appendix E

Matrix Notation and Dimensions

<u>Matrix</u>	<u>Column Vector</u>	<u>Elements</u>	<u>Dimension(s)</u>	<u>Reference Figure</u>
$\bar{A}$		0, 1: to relate points to a tour	(# points, # tours)	14
$A_f$		$f_{msd} A_{q\ell}$	(L, Q)	2
	B	Benders' cut constants, $B_{\kappa}$	$\bar{\kappa}$	14
	$C^{rA}$	$C_{msd}^{rA}$	msd	2
	$C^{rT}$	$C_{msd}^{rT}$	msd	2
	$C_f^{rA}$	$f_{msd} C_{msd}^{rA}$	msd	2
	$C_f^{rT}$	$f_{msd} C_{msd}^{rT}$	msd	2
[C-by]		tour cost in Benders' cut, eq. [43]	# tours	14
G		$g_{n,\ell}$	(msd, L)	2
I		identity matrix	appropriate	2, 14
	$\mathcal{L}$	right hand side in network	logic constraints, eq. [42]	14
	P	$P_{\ell}^{(t)}$	L	2
	$R^B$	artificial variables, eq. [43]	$\bar{\kappa}$	14
	$R^K$	artificial variables, eq. [78]	K	14
	$R^r$	artificial variables, $R_{msd}^r$ , eq. [12]	msd	2
	$S^a$	$S_q^a$	Q	2
	$S^B$	surplus variables, eq. [43]	$\bar{\kappa}$	14
	$S^{\mathcal{L}}$	slack variables, eq. [42]	appropriate	14
	$S^P$	slack variables, eq. [40]	K	14
	$S^{rA}$	$S_{msd}^{rA}$	msd	2
	$S^{rT}$	$S_{msd}^{rT}$	msd	2

<u>Matrix</u>	<u>Column Vector</u>	<u>Elements</u>	<u>Dimension(s)</u>	<u>Reference Figure</u>
	$S^s$	$S_m^s$	M	2
	$S^t$	$S_i^t$	# transshipment points	2
	T	$T_{rk}$	# tours	14
	$\Gamma$	$\Gamma_m$	M	14
$\Lambda$		0, 1: relate tours to network constraints	(# network constraints, # tours)	14
$\eta_f$		0, 1: relate flow chains to transshipment points	(# transshipment points, L)	2
$\Sigma$		0, 1: relate tours to vehicles	(K, # tours)	14
$\tau_f$		$f_{msd} \tau_{\ell}^{(t)}$	(M, L)	2
$\tau_f^A$		$f_{msd} \tau_{msd}^A$	(M, msd)	2
$\tau_f^T$		$f_{msd} \tau_{msd}^T$	(M, msd)	2
	1	column vector of ones	appropriate	2, 14

Appendix F

Proof of an Assertion Used in Chapter IV

Assertion:  $\gamma_{q\kappa} = 0$  for arcs  $q$  not used by the vehicle network specified at iteration  $\kappa$  in LP.

Proof: Suppose the variables are renumbered so that the first  $M$  are in solution.

The  $q^{\text{th}}$  row would be of the form

$$A_f P \leq \phi$$

and the first  $M$  columns would contain zeroes, since no flow chain,  $\ell$ ,

which uses arc  $q$  may be used in a feasible solution. Currently,

$$B(P S^{rA} S^{rT})' = b, (P S^{rA} S^{rT})' = B^{-1}b, P_o = C_B(P S^{rA} S^{rT}),$$

$$(\alpha \beta \gamma d_o \delta) = C_B B^{-1}A.$$

Add row  $q$  to the problem and consider the possibility of adding an additional basic variable,  $P_q$ .

$$\begin{bmatrix} B & 0 \\ a_q, \dots, a_{qM} & 1 \end{bmatrix} \begin{pmatrix} P \\ S^{rA} \\ S^{rT} \\ P_q \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} P \\ S^{rA} \\ S^{rT} \\ P_q \end{pmatrix} = \begin{bmatrix} B & 0 \\ a_q, \dots, a_{qm} & 1 \end{bmatrix}^{-1} \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{bmatrix} B^{-1} & 0 \\ -(a_q, \dots, a_{qM})B^{-1} & 1 \end{bmatrix} \begin{pmatrix} b \\ 0 \end{pmatrix}$$

But  $(a_q, \dots, a_{qm}) = (0, \dots, 0)$ , so

$$(P S^{rA} S^{rT} P_q)' = \begin{pmatrix} B^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} B^{-1} b \\ 0 \end{pmatrix}$$

and dual variables are defined by

$$(C_B \ C_{Pq}) \begin{bmatrix} B^{-1} & 0 \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} C_B B^{-1} \\ C_{Pq} \end{pmatrix} = \begin{pmatrix} C_B B^{-1} \\ 0 \end{pmatrix}$$

since  $P_q$  is a slack with  $C_{Pq} = 0$

The dual variable associated with arc  $q$  is therefore zero as originally stated.

This argument may easily be extended to prove the assertion for any number of constraints not made explicit in LP.

Appendix G  
Computer Program

A computer program to test the solution procedures described in Chapters III and IV was written and is currently being debugged. The program includes a Revised Simplex algorithm with Generalized Upper Bounding which has been specialized to improve run time and reduce storage requirements. The column generation routine defined in Chapter III is included. Techniques such as 'checks for zeroes' and explicit storage of nonzero elements only are incorporated as is the use of double precision to moderate accumulation of numerical round-off errors. Routines which implement the tour selection procedure (the algorithm for Problem 01) have also been keypunched, along with the necessary input, output, and cross-communication routines. Programs to solve the two subproblems consist of 3233 cards (several input/output routines were duplicated to allow parallel debugging of the program halves).

The objective of promoting run time efficiency guided program design throughout. Other features were included to simplify implementation. Examples are the use of one-dimensional storage vectors to reduce run time and to allow the routines to be stored on disc. Dimensions of arrays need be defined only in the main program for each different application. In addition, the program was designed to allow easy testing, including provisions for inputting  $\eta$  and values  $\alpha$  (see Chapters III and IV, respectively) as data elements.

Emphasis was placed on pursuing the line of investigation reported in Chapter V in lieu of concentrating on the computer program. Investigation of tour construction ability was deemed more important, overall, in this development study. Additionally, this line of investigation was emphasized considering

the related research proposed as another project and under review by the USA F/OSR during most of this summer.

Variable definitions and program codes are included in this appendix for completeness. Faculty release time provided this fall quarter which is not used in report preparation will be applied to continue computer program development. However, the level of testing required should be accomplished on a project of longer duration and more concentrated effort.

LIST OF VARIABLES

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
ABARS(I)	entering column W/R to current basis (MSD, MTH)			MSD + MTH	
ALP(I, J)	a matrix for LP. S <sup>RA</sup> and S <sup>RT</sup> variables			3	2 MSD
ALPHA	user specified parameter $0 \leq \alpha \leq 1$				
AS(I)	entering column - associated key column			MTH	
ASI(I)	entering column			MTH	
BASIC(J)	variable associated with column J in B <sup>-1</sup> + # $\Rightarrow$ art., slack, P <sub>0</sub> or comm. mode - # $\Rightarrow$ a generated variable	X		MTH	
BBAR(I)	transshipment capacity at point I			INPOINT	
BENCON(K)	constant association with Benders' cut K		X		
BINV(I, J)	B <sup>-1</sup>		X	MTH	MTH
CELL	temporary accumulator in updating B <sup>-1</sup>		X		
CHAIN(I, J)	a <sub>ij</sub> values for generated column J MTH + 1 row $\Rightarrow$ NET			MTH + 1	MSD + MTH
COL(I)	incoming column vector for LP				
COLDEC(I)	ALPHA * COLSUM(I) + (1-ALPHA) * COLMAX(I)		X	(NMA TST + NTRANS + QUP + 1)	
COLSUM(I)	$\bar{k}$ ( $\sum_{k=1}^{\bar{k}} W_{Tkk}$ ) column sum in Benders' cuts			NTOUR	
COLMAX(I)	( $\max_k W_{Tkk}$ ) max column value in Benders' cuts			NTOUR	

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
COSTRA(J)	cost/unit for MSD combination J via comm. air			MSD	
COSTRT(J)	cost/unit for MSD combination J via comm. truck			MSD	
CQF(A)	cost of arc A (including servicing end point)			MXARC	
CSUM	sum of CTOUR(I) for $I \subseteq J_t$				
CTOUR(I)	cost of tour I			NTOUR	
CTMIN(I)	gives tour # for ranked CTOUR(*) (each KVEH low to high)	X		NTOUR	
CUMCST(N,J)	cum. cost from source to node J, subnetwork, N			(NNT * QUP)	
DBARS(I)	ABARS(I) with respect to current basis Eq (34)		X	MTH	
DUAL(I)	dual variable assoc. with constraint I $B^{-1}$ and $MU(J)$ (MTH, MSD) MTH row		X	MTH + MSD	
DUTIM(I,M)	destination unload time for material type M at destination point I			(NPOINT * NMA T)	
FMSD(N)	ship RQMT: Mat'l M, source S, Dest. D			MSD	
IALP(I,J)	ALP(I,J) entry in row I col. J	X		3	2 MSD
ICHNG	Code: 0 = no better SC, 1 = SHORT found improvement 2 = COLGEN acknowledges improvement				
INDCOL	number columns in node				
ISTVN(A)	beginning point for arc A	X		MXARC	
JNDVN(A)	ending point for arc A	X		MXARC	

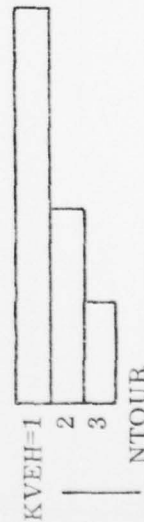
Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
JT(I)	record of tours assigned binary values in $J_t$ + # $\Rightarrow$ tour # = 1 in $J_t$ - # $\Rightarrow$ tour # = 0 in $J_t$	X		NTOUR	
KARCLO(N)	lowest # arc used by vehicle type N	X		NVEH	
KARCUP(N)	highest # arc used by vehicle type N	X		NVEH	
KASE	KASE = 0, 1, 21, 22, 23 (see pp. 331-333, Lasdon)				
KEY	1 if exiting variable is key, 0 if not				
KEYVAR(I)	key variable # for set I + $\Rightarrow$ var # $\leq$ NTH + 2(MSD) - $\Rightarrow$ 'generated' variable #	X		MSD	
KNET(I)	MSD combination associated with column I in $B^{-1}$	X		MTH	
KOLEX	column of exiting variable				
KPOS(I)	midpoint for interval of length I	X		KPSMAX	
KPSMAX	max # intervals in KPOS(*)	X			
KTRLO(KVEH)	lowest # tour for vehicle KVEH	X		NVEH	
KTRUP(KVEH)	highest # tour for vehicle KVEH	X		NVEH	
KVAR(I)	+ col. # if variable in $B^{-1}$ - col. # if variable is key 0 if variable is not in solution	X		MSD+MTH+2*MSD + (MSD+MTH)	
KVEH	vehicle identification				

Variable	Definition	Int * 2	Real*8	Max # rows	Max # col
LB	lower bound on the value of Z*				
LENGTH(I)	length of tour I in # arcs (input data is # points on tour ( 0 P <sub>1</sub> P <sub>2</sub> ... t ) )	X		MXLONG	
LISTB	Code: 0 ⇒ NSUC on LISTC, 1 ⇒ NSUC on LISTB				
MATL	material type				
MSD	number MSD combinations				
MTH	P <sub>0</sub> <sup>th</sup> row in B <sup>-1</sup> = NMA TST + NTRANS + QMAX + 1				
MXARC	total number arcs				
MXLONG	max number arcs used by any vehicle ( V tours)				
NALP	number of elements in each A column				
NDA	node (vehicle arc) just added to list A in short				
NET	particular MSD combination				
NETBST	subnetwork with best SC, the entering column				
NETEX	NET (MSD) for exiting variable				
NLISTB	number of entries on list B				
NMAT	number of material categories				
NMA TST	number of (material categories) ship time constraints NMA T + 1 ⇒ all				
NNT	number of subnetworks/shortest path problem				

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
NOCAND(KVEH, J)	$J = 1 \Rightarrow$ # candidate KVEH tours $\subseteq E_t$ $J = 2 \Rightarrow$ # KVEH tours = 0 in $J_t$ ( $T(I) = -4$ )	X		(KVEH x 2)	
NODE(I, J)	array which defines a vehicle network $J = 1$ arc begin point $J = 2$ arc end point $J = 3$ veh # $J = 4$ veh visit # $J = 5$ succ this veh $J = 6$ arc shipments possible $J = 7$ transshipment arcs $J = 8$	X		QMAX	8 + NTRN
NOE	number tours $\subseteq E_t$				
NOEK(KVEH)	KVEH tour representative in $E_t$ $+ \# =$ tour # (NTOUR + 1) $\Rightarrow$ old rep was deposited in PREP01 this ITERAT also initial value from READIN $0 \Rightarrow$ # KVEH tours $\subseteq E_t$ $- \# \Rightarrow$ rep was in $D_t$ last ITER01	X			NODE(I, INDCUI) = arc #
NOJ	number tours $\subseteq J_t$				
NOK	number vehicles assigned (with a tour = 1 in $J_t$ )				
NPOINT	number of points in the network				
NSUC	vehicle arc to successor node				
NTH	variable number of $P_0 = MSD + MTH$				
NTOUR	total number of tours defined in Problem 01				
NTRANS	number transshipment capacity constraints (0 or NPOINT)				

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
NTRN	max. number of transshipments at any point $\frac{2}{2(\text{NVEH} - 1)}$				
NTWRK(I, J)	network (NET) msd reference $\frac{J=1}{\text{NET}} \frac{J=2}{\text{Mat'I}} \frac{J=3}{\text{source}} \frac{J=4}{\text{dest.}}$ (msd) type point point	X		MSD	4
NTYPES	number of vehicle types				
NVAR	(NTH + 3(MSD) ) number variables not generated				
NVEH	number of vehicles				
NVTR	number vehicles required to complete network				
ORDER1(I)	data for entry in order I on list B vehicle arc end node Q: NDA	X		QUP * NNT	
ORDER2(I)	path number for ORDER1(I) entry	X		QUP * NNT	
OSTIME(I, M)	order and ship time for material type M at origin point I			(NPOINT x NMAT)	
P(I)	value of I <sup>th</sup> variable in the basis see KEYVAR( ) and BASIC( ) for variable number (MSD, MTH)		X	MSD + MTH	
PATH(I)	$\frac{I=1}{\text{NET}} \frac{Q=1}{\text{msd}} \left\  \begin{array}{l} \text{arc} \\ \text{code} \end{array} \right\  \text{Pred} \left\  \begin{array}{l} \text{arc} \\ \text{code} \end{array} \right\  \text{Pred} \left\  \begin{array}{l} \text{arc} \\ \text{code} \end{array} \right\  \text{Pred} \left\  \begin{array}{l} \text{arc} \\ \text{code} \end{array} \right\  \text{Pred}$ Code: -1 $\Rightarrow$ LISTA 0 $\Rightarrow$ LISTC # $\Rightarrow$ order on LISTB Pred: neg # $\Rightarrow$ transshipment from arc # 0 $\Rightarrow$ source point is predecessor pos # $\Rightarrow$ preceding arc#, same vehicle	X		NNT	1 + 2 * QUP

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
POINT(L)	sequence of points visited in a tour	X		MXLONG	
QMAX	number of vehicle arcs in current node	X			
QUP	maximum number of vehicle arcs in any LP solution	X			
SC	simplex optimizing criterion				
T(I)	set membership indicator for tours 2 representative tour from set $E_t$ 1 not used 0 eligible to augment $J_t$ -1 $\subseteq E_t$ (initial value) -2 $\subseteq D_t$ -3 code 2 found negligible at 260, $\subseteq D_t$ -4 tour = 0 in $J_t$ -5 tour = 1 in $J_t$ -6 $E_t$ representative set = 0 in $J_t$	X		NTOUR	
TAU(M)	ship/time constraint for mat'l. category M			NMATST	
TAUA(I)	comm. air ship time for NET = msd = I			MSD	
TAUT(I)	comm. truck ship time for NET = msd = I			MSD	
TENTER	number of entering tour	X			
THETA	simplex criterion to determine exiting variable		X		
TOUR(T,A)	tour/arc cross reference	X		(NTOUR x MXLONG)	



Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
TRVDST(K, I, J)	travel distance vehicle type K, point I to J			NPOINT *NTYPES	NPOINT
TRVTIM(K, I, J)	travel time for vehicle type K, from point I to point J			NPOINT *NTYPES	NPOINT
TSTIME(I, M)	transshipment time at point I, mat'l, M			NPOINT	NMAT
UB	upperbound on the value of Z*				
V(KVEH)	set membership indicator for vehicles + # $\Rightarrow$ k tour # = 1 in $J_t$ 0 $\Rightarrow$ vehicle eligible to augment $J_t$ -1 $\Rightarrow$ k excluded, since $k \subseteq V_t$ -2 $\Rightarrow$ k excluded, $k \subseteq V^d$ -3 $\Rightarrow$ $k \subseteq V_t^0$	X		NVEH	
VARC(A)	vehicle type associated with arc A	X		MXARC	
VAREX	variable number of exiting variable	X			
VARIN	variable number of incoming variable	X			
VBEST(KVEH)	current best V(KVEH) corresponding to ZC	X		NVEH	
VCAP(K)	capacity of vehicle K			NTYPES	
VEThor(KVEH)	either/or vehicle exclusion record 0 $\Rightarrow$ number either/or relationship for KVEH # $\Rightarrow$ related vehicle #	X		NVEH	
VORIG(K)	point of origin for vehicle K	X		NVEH	
VSPEED(K)	average mph for vehicle K			NTYPES	

Variable	Definition	Int * 2	Real * 8	Max # rows	Max # col
VTERM (K)	termination point for vehicle K	X		NVEH	
VTOUR(I)	vehicle associated with tour I	X		NTOUR	
VTYPE(K)	type vehicle for vehicle K	X		NVEH	
W(K, I)	$(C_{rk} - \sum_q b_q   \gamma_{qk}   )$ tour I cost in cut K			$(\bar{k} \times NTOUR)$	
WMIN(K, I)	tour number for ranked W(K, I) (each KVEH each cut, low to high)	X		$(\bar{k} \times NTOUR)$	
WSUM(K)	$\sum W_{rk\kappa}$ for tours at level 1 in $J_t$ in cut K			$\bar{k}$	
Y	1 if CTOUR(I) is specified, 0 if CTOUR(I) = $\sum CQF(A)$	X			
ZC	current best Z value for a complete 01 solution				
ZP	Z value associated with current partial 01 solution				

CODES FOR SUBROUTINE

MESGE (I1, I2, I3, I4, I5, I6, I7, I8, I9, I10, I11, I12, I13, I14, I15)

- I1 CURRENT LP SOLUTION  
KEYVAR(.), NET, P(.)  
BASIC(.), KNET(.), P(.)  
KVAR(.)
- I2 BINV(.)
- I3 ALP(.), IALP(.)
- I4 ENTERING GENERATED COLUMN COL(.)
- I5 CHAIN(.)
- I6 AS(.), DBARS(.), ABARS(.)
- I7 DUAL(.)
- I8 W(.), WMIN(.)
- I9 01 SOLUTION STATUS  
T(.), JT(.), NOEK(.), V(.), LB, ZC, UB
- I10 NODE(.)
- I11 CUMCST(.), PATH(.), ORDER1(.), ORDER2(.)
- I12 Reserved for Problem MP
- I13 "
- I14 "
- I15 CURRENT 01 DECISION CRITERIA  
WSUM(.), CTMIN(.), COLDEC(.), COLMAX(.), COLSUM(.)

ERROR CODES

<u>PROGRAM</u>	<u>SUBROUTINE</u>	<u>SUBROUTINE NAME</u>
2	1	READIN
2	2	PREPLP
2	3	RSGUB
2	4	COLGEN
2	5	SHORT
2	6	REORDR
2	7	PHILP
2	9	UPBINV
3	1	READIN
3	2	PREP01
3	3	IE01
3	4	PREPND

PROGRAM CODES

2 subroutines associated with problem LP

3 subroutines associated with problem 01

ROW AND VARIABLE INDICES FOR PROBLEM LP

ROW INDICES

		<u>NUMBER</u>
(1)	Ship time constraints	NMATST
(2)	Transshipment capacity constraints	NTRANS
(3)	Vehicle arc capacity constraints	QMAX
(4)	$P_0$ row defining the LP objective	1, the MTH row

VARIABLE INDICES

		<u>NUMBER</u>
(1)	Artificial variables for ship requirements	MSD
(2)	Slacks for ship time constraints	NMATST
(3)	Slacks for transshipment capacity constraints	NTRANS
(4)	Slacks for vehicle arc capacity constraints	QMAX
(5)	$P_0$ the LP objective function value	1, the NTH VARIABLE
(6)	Commercial air shipment for msd requirement	MSD
(7)	Commercial truck shipment for msd requirement	MSD
(8)	Generated columns	up to MSD + MTH - 1

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM		
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multicommodity material flow multiple vehicle routing transportation system planning logistics system planning	vehicle routing/scheduling network synthesis network design multimodal transportation network			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This study investigated the problem of synthesizing a minimum cost transportation system plan to service forecast shipment requirements among a set of points in such a manner as to satisfy aggregate ship-time performance levels. Both commercial and dedicated modes may be used in the transportation plan; but the later must be designed in detail including specification of vehicles to be used, the route each is to service, and arrival/departure time schedules. A large-scale, mixed integer, linear programming model of the planning				

## 20. Abstract

problem is developed and simplified for solution by applying Benders' decomposition to yield two more simple, interacting subproblems. One of these, a linear program which assures ship-time performance, is amenable to large-scale programming techniques for which specialized algorithms are stated.

Several formulations of the other subproblem, which defines the dedicated mode network, are described. The first designs the network using a set of feasible, vehicle tours. An implicit enumeration algorithm applicable to this model is described. Three additional formulations, each of which constructs vehicle tours directly, were developed and solution approaches for each are described.

The tactical, vehicle scheduling problem is treated subsequently. Collectively, study results offer capability to solve transportation system planning problems of realistic size.

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