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ESTIMATION OF A CONTACT'S COURSE,
SPEED AND POSITION BASED ON
BEARINGS-ONLY INFORMATION FROM
TWO MOVING SENSORS WITH A
PROGRAM FOR AN HP-67/97 CALCULATOR

by

R. H. Shudde

November 1977

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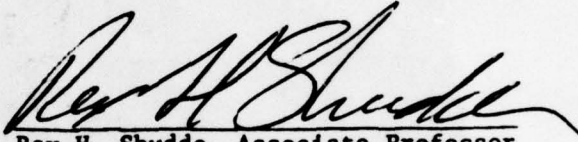
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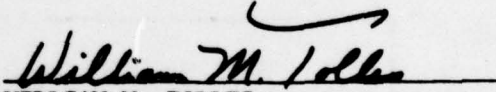
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This report was prepared by:


Rex H. Shudde, Associate Professor
Department of Operations Research

Reviewed by:


MICHAEL G. SOVEREIGN, CHAIRMAN
Department of Operations Research


WILLIAM M. TOLLES
Acting Dean of Research

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R. H. Shudde

Naval Postgraduate School
Monterey, Ca. 93940

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This report provides a procedure for estimating a contact's course, speed and position based on bearings-only data from two moving sensors. This report also contains a program for the HP-67/97 calculator to implement the procedure.

KEYWORDS:

Tracking
ASW
Calculator

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A. Problem Statement

Bearings-only data for a single target from two sensors which may be moving or stationary are available at two distinct times. The following quantities are required: an estimate of course, speed and position of the target at the latest time; an estimate of a future position of the target and/or an estimate of a point on the track of the target with a specified lead distance at a future time. The relative positions of the two sensors are assumed to be known at the time of each target bearing determination.

B. Operational Analysis

Two simultaneous bearings from two sensors at two distinct times and with known relative positions are used to estimate the course and speed of a target. The HP-67/97 program presented here was designed so that the data corresponding to the earliest time point is purged if data corresponding to a third time point is introduced. The relative position of the sensors may be updated when required. Thus the estimated target position, course and speed are continually updated as new information becomes available. No course smoothing is performed.

C. Computational Algorithm

1. Enter the course ψ_s and speed V_s of the primary sensor S_1 .
2. Enter the bearing ϕ and range ρ of the secondary sensor S_2 from the primary sensor S_1 at the time of the latest bearing observation.
3. Enter the time t_1 , the bearing of the target from S_1 , and the bearing of the target from S_2 . Output the target range from S_1 .
4. Repeat Step 3 or Steps 2 and 3 for a second time $t_2 > t_1$.
5. Compute and output:
 - a. The estimated course and speed of the target.
 - b. The bearing and range (n.mi.) of the target from S_1 at time t_2 .
 - c. The bearing and range (n.mi.) of the target from S_2 at time t_2 .
6. If required, enter a time $t_\ell > t_2$ at which a lead distance ℓ (n.mi.) is required. Then compute and output the target's predicted bearing and range from both S_1 and S_2 .
7. Repeat from Steps 1, 2, 3 or 4 as required.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1	Enter program card (both sides).			
2a	Set for HP-67 output (default) or		fA	none
b	set for HP-97 output.		fB	none
3	Enter course and speed of the primary sensor S_1 .	course ψ_s speed V_s	\uparrow A	course
4	Enter bearing and range of S_2 from S_1 at datum.	bearing ϕ range ρ	\uparrow B	bearing
5	Enter time t_1 of datum, bearing from S_1 , and bearing from S_2 . Optional: Display range from S_2 .	t_1 (HH.MM) θ_{11} (degrees) θ_{21} (degrees)	\uparrow \uparrow C	R_1 (n.mi.)
			R/S	r_1 (n.mi.)
6	Repeat Step 4 or 5 or Step 5 only for a second (subsequent) time, then proceed to Step 7.			
7a	Compute: Target course		D	ψ_T (degrees)
b	Target speed		(R/S)*	V_T (knots)
c	Display Sensor S_1 prompt.		(R/S)	1.
d	Target bearing from S_1 at latest time.		(R/S)	θ_{12} (degrees)
e	Target range from S_1 at latest time.		(R/S)	R_2 (n.mi.)
f	Display Sensor S_2 prompt.		(R/S)	2.
g	Target bearing from S_2 at latest time.		(R/S)	θ_{22} (degrees)
h	Target range from S_2 at latest time.		(R/S)	r_2 (n.mi.)

Step	Instruction	Input	Key(s)	Output
8	Compute bearing and range at time t_l with lead distance l (n.mi.):	t_l (HH.MM) l (n.mi.)	↑ E	
a	Display Sensor S_1 prompt.			1.
b	Target bearing from S_1 at t_l .		(R/S)*	θ_{1l} (degrees)
c	Target range from S_1 at t_l .		(R/S)	R_l (n.mi.)
d	Display Sensor S_2 prompt.		(R/S)	2.
e	Target bearing from S_2 at t_l .		(R/S)	θ_{2l} (degrees)
f	Target range from S_2 at t_l .		(R/S)	r_l (n.mi.)
9	Repeat from Step 3 or from Step 6 as required.			

* Note: The (R/S) function is required when using the HP-67 mode. This output is automatically printed on the HP-97.

2. Sample Problem

- a. The Primary Sensor S_1 is traveling on a course of 210° at 10 knots.
- b. At the time of the first contact sensor, S_2 is 115° and 3.5 n.mi. from S_1 .
- c. At 1200 hours the first contact is at 245° from S_1 and 260° from S_2 . How far is the contact from S_1 and S_2 ?
(Ans.: 8 n.mi. from S_1 and 10 n.mi. from S_2 .)
- d. At the next time mark sensor S_2 is 100° and 5.0 n.mi. from S_1 .
- e. This next time mark is at 1230 hours with the contact at 160° from S_1 and 239° from S_2 .
- f. Estimate the course and speed of the contact.
(Ans.: 126° and 14 knots.)
- g. What is the bearing and range of the contact from S_1 at 1230 hours? (Ans.: 160° and 3 n.mi.)
From S_2 ? (Ans.: 239° and 4 n.mi.)
- h. Estimate the bearing and range of the contact from S_1 and S_2 at 1245 hours with a lead distance of 3.5 n.mi.
(S_1 Ans.: 137° and 10 n.mi.)
(S_2 Ans.: 164° and 7 n.mi.)

a.	210. ENT1 10. GSBA	ψ_s S_1 course V_s S_1 speed
b.	115. ENT1 3.5 GSBB	ϕ_1 Bearing of S_2 ρ_1 Range of S_2
c.	12.00 ENT1 245. ENT1 260. GSBC 8. *** R/S 10. ***	t_1 First contact time θ_{11} Target bearing from S_1 θ_{21} Target bearing from S_2 R_1 Est. range from S_1 r_1 Est. range from S_2
d.	100. ENT1 5. GSBB	ϕ_2 New bearing of S_2 ρ_2 New range of S_2
e.	12.30 ENT1 160. ENT1 239. GSBC	t_2 Second time mark θ_{12} Target bearing from S_1 θ_{22} Target bearing from S_2
f.	GSBD 125. *** 14. ***	Compute Est. target course Est. target speed (kts)
g.	1. *** 160. *** 3. *** 2. *** 239. *** 4. ***	S_1 : Target bearing Target range S_2 : Target bearing Target range
h.	12.45 ENT1 3.5 GSBE 1. *** 137. *** 10. *** 2. *** 164. *** 7. ***	Lead time (HH.MM) Lead distance (n.mi.) S_1 : Bearing Range S_2 : Bearing Range

3. Program Storage Allocation and Program Listing

Registers:

R0: X_2^T	S0: X_1^T	A: ρ_i
R1: Y_2^T	S1: Y_1^T	B: ϕ_i
R2: t_2	S2: t_1	C: V_S
R3: Δt	S3:	D: ψ_S
R4: θ_{12}	S4: Σx	E: V_T
R5: θ_{22}	S5: Used	I: ψ_T
R6: R_2	S6: Σy	
R7: r_2	S7: Used	
R8: t_ℓ	S8: Used	
R9: ℓ, R_ℓ and r_ℓ	S9: Used	

Initial Flag Status and Use:

0: OFF, HP67 or HP97 output	2: OFF, Used for t_ℓ option
1: OFF, Unused	3: OFF, Unused

Display Status: DSP0, FIX, DEG

User Control Keys:

A: $\psi_S \uparrow V_S$	a: HP-67 output mode
B: $\phi_i \uparrow \rho_i$	b: HP-97 output mode
C: $t_i \uparrow \theta_{1i} \uparrow \theta_{2i}$	c. Unused
D: Compute ψ_T, V_T and position	d: Unused
E: $t_\ell \uparrow \ell$	e. Unused

001	*LBLA	21 11	Primary Sensor S ₁	039	SIN	41	Compute range from S ₁ or S ₂ using bearing data.
002	STOC	35 17	Store speed V _s	040	RCL5	36 05	
003	R↓	-21	Store course ψ _s	041	RCL4	36 04	Computation of target course and speed
004	STOD	35 14	S ₂ position	042	-	-45	
005	RTN	24	Store range ρ _i	043	SIN	41	t ₂ - t ₁ = Δt Error if Δt < 0
006	*LBLB	21 12	Store bearing φ _i	044	÷	-24	
007	STOA	35 11	Time and Bearing Input	045	RCLA	36 11	Store -R ₁
008	R↓	-31	Store θ _{2i}	046	x	-35	
009	STOB	35 12	Compute and store r _i	047	RTN	24	Compute R ₂ - R ₁
010	RTN	24	Compute and store R _i	048	*LBLD	21 14	
011	*LBLE	21 13	Compute and store	049	RCLZ	36 56	Compute V _s Δt
012	P↔S	16 51	X _i	050	Σ-	16 56	
013	STOE	35 05	and Y _i	051	P↔S	16 51	Compute V _T Δt and V _T
014	R↓	-31	Display R _i	052	RCL1	36 01	
015	STO4	35 04	Display r _i	053	RCL0	36 00	Compute V _T Δt and V _T
016	R↓	-31	Subroutine	054	RCL2	36 02	
017	HMS→	16 36		055	P↔S	16 51	
018	STO2	35 02		056	RCL2	36 02	
019	RCL4	36 04		057	-	-45	
020	GSB9	23 09		058	CHS	-22	
021	STO7	35 07		059	X<0?	16 45	
022	RCL5	36 05		060	GT00	22 00	
023	GSB9	23 09		061	STO3	35 03	
024	STO6	35 06		062	R↓	-31	
025	RCL4	36 04		063	Σ-	16 56	
026	*Y	-41		064	RCL1	36 01	
027	R	44		065	RCL0	36 00	
028	STO0	35 00		066	Σ+	56	
029	R↓	-31		067	RCLD	36 14	
030	STO1	35 01		068	RCLC	36 17	
031	RCL6	36 06		069	RCL3	36 03	
032	R/S	51		070	x	-35	
033	RCL7	36 07		071	R	44	
034	R/S	51		072	Σ+	56	
035	GT00	22 00		073	RCLΣ	36 56	
036	*LBL9	21 09		074	P	34	
037	RCLB	36 12		075	RCL3	36 03	
038	-	-45		076	÷	-24	

077	STOE	35 15	115	$\Sigma+$	56	$t_\ell - t_2$ Error if $t_\ell - t_2 < 0$ $\uparrow V_T(t_\ell - t_2)$ ℓ added Add to $\uparrow R_2$
078	XZY	-41.	116	RCL1	36 46	
079	GSB7	23 07	117	RCL2	36 15	
080	STO1	35 46	118	RCL8	36 08	
081	SPC	16-11	119	RCL2	36 02	
082	SPC	16-11	120	-	-45	
083	GSB6	23 0E	121	X<0?	16-45	
084	RCL2	36 15	122	GT00	22 00	
085	GSB6	23 0E	123	X	-35	
086	SPC	16-11	124	RCL9	36 09	
087	1	01	125	+	-55	
088	GSB6	23 0E	126	+R	44	
089	RCL4	36 04	127	$\Sigma+$	56	
090	GSB6	23 0E	128	*LBL1	21 01	
091	RCL6	36 06	129	RCLX	36 56	
092	GSB6	23 0E	130	+P	34	
093	SPC	16-11	131	STO9	35 09	
094	2	02	132	XZY	-41	
095	GSB6	23 0E	133	GSB7	23 07	
096	RCL5	36 05	134	GSB6	23 0E	
097	GSB6	23 0E	135	RCL9	36 09	
098	RCL7	36 07	136	GSB6	23 0E	
099	GSB6	23 0E	137	F2?	16 23 02	
100	R/S	51	138	GT02	22 02	
101	GT00	22 00	139	R/S	51	
102	*LBL2	21 15	140	GT00	22 00	
103	SF2	16 21 02	141	*LBL2	21 02	
104	STO9	35 09	142	SPC	16-11	
105	R4	-31	143	2	02	
106	HMS+	16 3E	144	GSB6	23 0E	
107	STO8	35 08	145	RCL8	36 12	
108	SPC	16-11	146	RCLA	36 11	
109	1	01	147	+R	44	
110	GSB6	23 0E	148	$\Sigma-$	16 56	
111	RCLX	36 56	149	GT01	22 01	
112	$\Sigma-$	16 5E	150	*LBL7	21 07	
113	RCL1	36 01	151	X<0?	16-45	
114	RCL0	36 00	152	X=0?	16-43	
115	Store V_T					
116	Store ψ_T					
117	Double space HP-97					
118	Display ψ_T					
119	Display V_T					
120	Display 1. (S_1)					
121	Display bearing from S_1					
122	Display range from S_1					
123	Display 2. (S_2)					
124	Display bearing from S_2					
125	Display bearing from S_1					
126	Error					
127	Lead time and lead distance					
128	Store ℓ					
129	Store t_ℓ					
130	Set for S_1 and display					
131	Clear Σ_x and Σ_y					
132	Store $\uparrow R_2$					
133	Subroutine.					

153	RTN	24	Add 360° to negative bearings
154	3	03	
155	6	06	
156	0	00	
157	+	-55	
158	RTN	24	
159	#LBL6	21 06	
160	F0?	16 23 00	
161	PRTX	-14	
162	F0?	16 23 00	
163	RTN	24	HP-67 display and HP-97 print routine
164	R/S	51	
165	RTN	24	
166	#LBLa	21 16 11	
167	CF0	16 22 00	
168	RTN	24	
169	#LBLb	21 16 12	
170	SF0	16 21 00	
171	RTN	24	
172	R/S	51	
Set for HP-67 display mode			
Set for Hp-97 print mode.			

E. Geometric Analysis

1. Static Geometry

Let $\vec{R}_i = (\theta_{1i}, R_i)$ denote the bearing and range of the target from the reference (primary) sensor S_1 at time t_i , and let $\vec{r}_i = (\theta_{2i}, r_i)$ denote the bearing and range of the target from the secondary sensor S_2 at time t_i , $i = 1, 2$, where $t_1 < t_2$. Let $\vec{\rho}_i = (\phi_i, \rho_i)$ denote the bearing and range of S_2 from S_1 at time t_i . The static geometry for some fixed time t_i is depicted in Figure 1.

From Figure 1 we see that

$$\vec{R}_i = \vec{\rho}_i + \vec{r}_i . \quad (1)$$

By equating the rectangular components of Equation (1) we have

$$R_i \cos \theta_{1i} = \rho \cos \phi + r_i \cos \theta_{2i} \quad (2a)$$

and

$$R_i \sin \theta_{1i} = \rho \sin \phi + r_i \sin \theta_{2i} . \quad (2b)$$

Equations (2) are two equations in the two unknown ranges R_i and r_i . Solving this system of equations we obtain

$$R_i = \rho_i \frac{\sin(\theta_{2i} - \phi_i)}{\sin(\theta_{2i} - \theta_{1i})} \quad \text{for any } i, \quad (3)$$

and

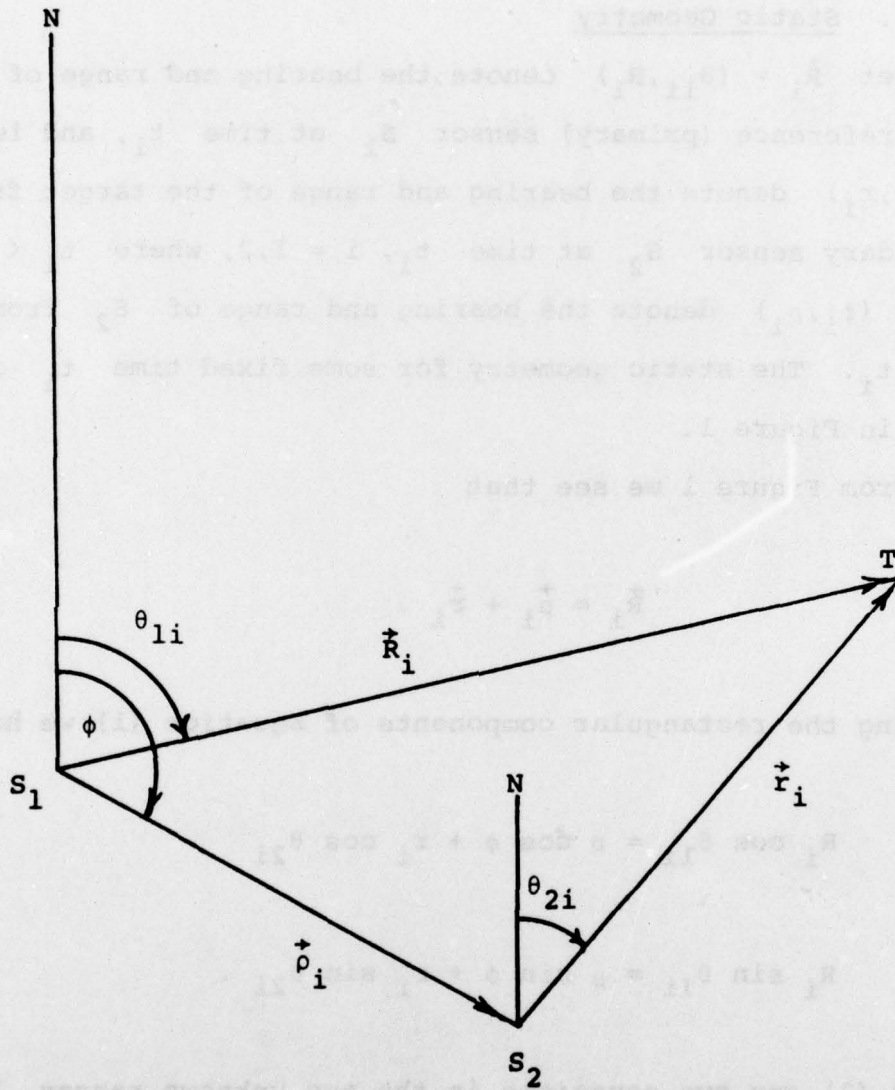


FIGURE 1. The Relative Sensor and Target Geometry at Time t_i .

$$r_i = \rho_i \frac{\sin(\theta_{1i} - \phi_i)}{\sin(\theta_{2i} - \theta_{1i})} \quad \text{for any } i . \quad (4)$$

At any time t_i the target range R_i from sensor S_1 and the target range r_i from sensor S_2 may be computed from Equations (3) and (4), respectively. Thus \vec{R}_i and \vec{r}_i are determined at any time t_i .

2. Dynamic Geometry

Let $\vec{V}_s = (\psi_s, V_s)$ denote the course and speed of the primary sensor S_1 , and let $\vec{V}_T = (\psi_T, V_T)$ denote the unknown course and speed of the target. Let $\Delta t = t_2 - t_1 > 0$ be the time between first and second observations of the target. The absolute motion of sensors and the target is depicted in Figure 2. From Figure 2 it is evident that one of the many vectorial relationships is

$$\vec{R}_1 + \vec{V}_T \Delta t = \vec{V}_s \Delta t + \vec{R}_2 . \quad (5)$$

The target course and speed vector \vec{V}_T is then found to be

$$\vec{V}_T = \vec{V}_s + \frac{1}{\Delta t} (\vec{R}_2 - \vec{R}_1) . \quad (6)$$

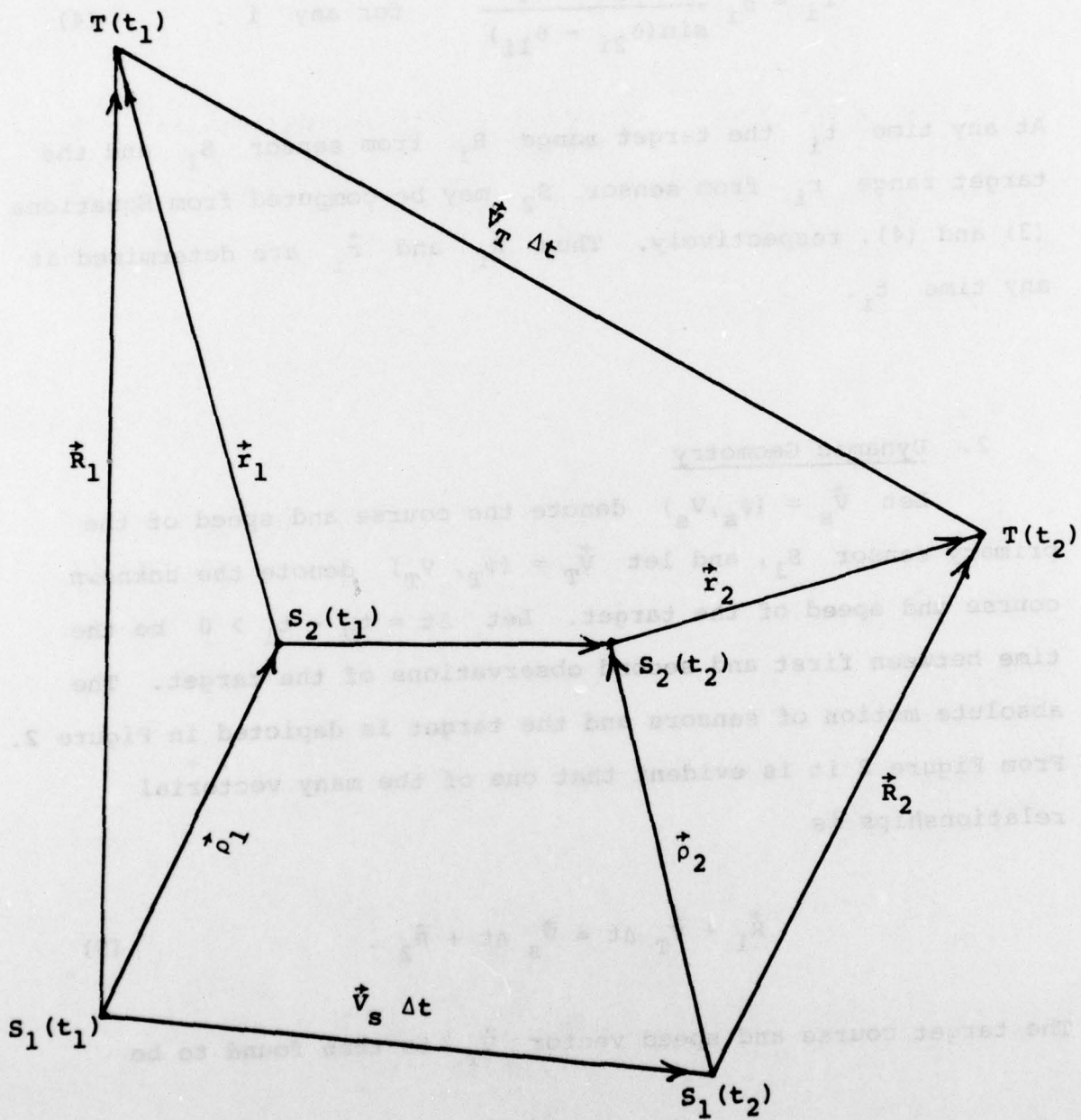


FIGURE 2. Motion of Sensors S_1 and S_2 and of the Target T from Time t_1 to Time t_2 .

3. Lead Distance Geometry

If, at some time t_ℓ ($t_\ell > t_2$), it is desired to lead the target on its track by a distance ℓ , then the bearing $\theta_{1\ell}$ and range R_ℓ to this position from the primary sensor S_1 is obtained by converting the vector $[\psi_T, V_T(t_\ell - t_2) + \ell]$ to rectangular coordinates and adding it to the rectangular form of the position vector \vec{R}_2 (see Figure 3). The resulting vector is then converted to polar coordinates to obtain the vector $(\theta_{1\ell}, R_\ell)$. The predicted bearing and range \vec{r}_ℓ of the target from the secondary sensor S_2 is computed from

$$\vec{r}_\ell = \vec{R}_\ell - \vec{\rho} \quad (7)$$

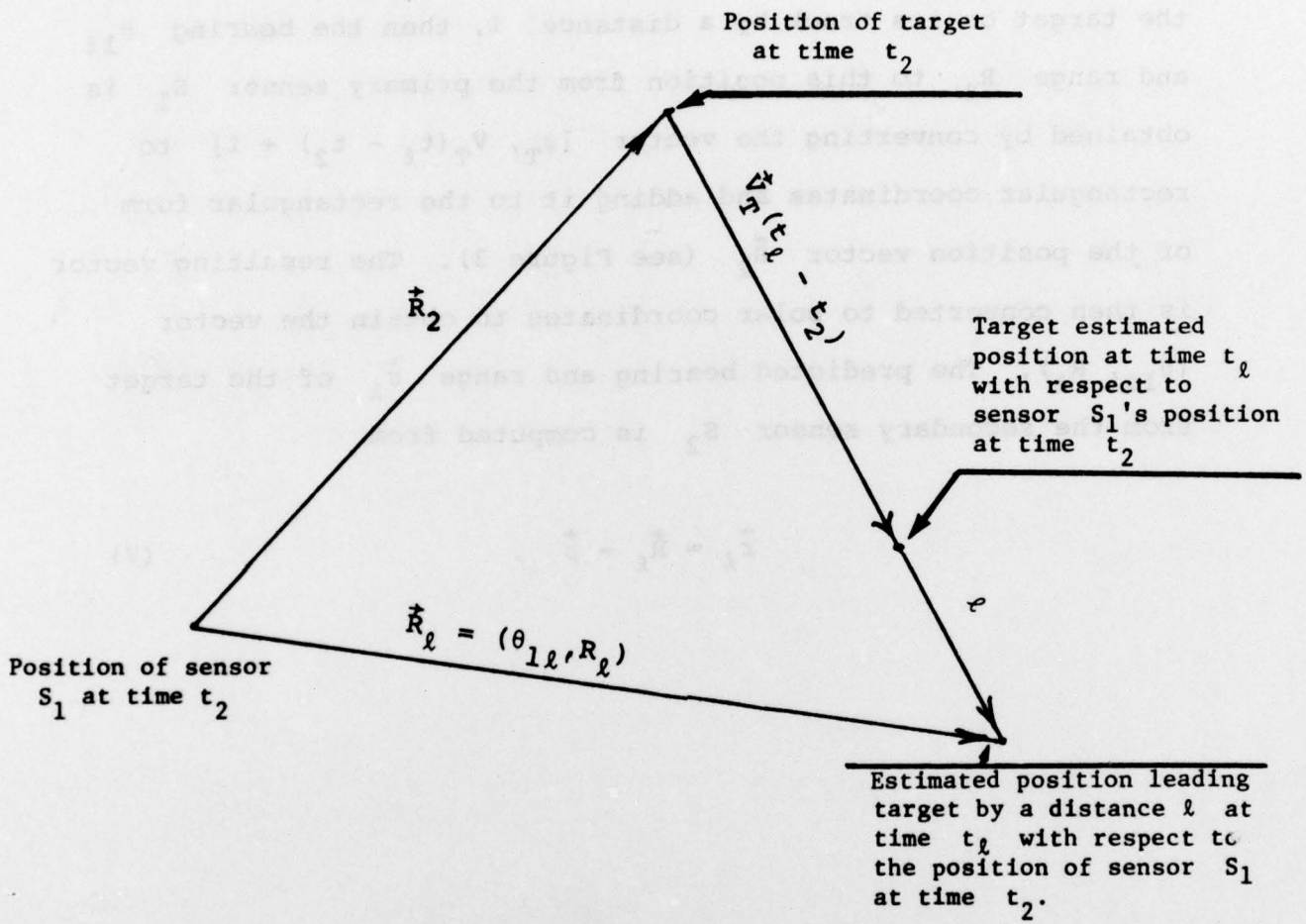


FIGURE 3. Target Lead Distance Geometry.