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Phase Report
December 1977

SPATIAL BEHAVIOR OF WIDEBAND EMITTERS

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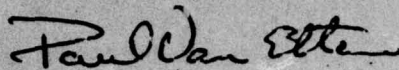
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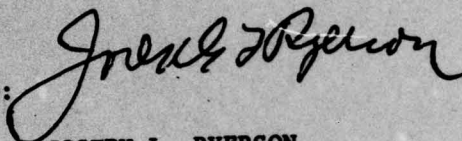
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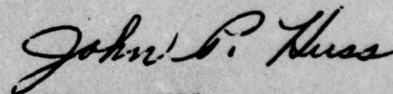
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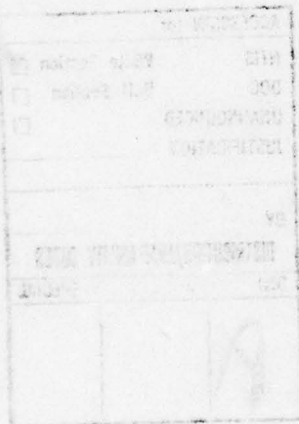
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TABLE OF CONTENTS

	<u>Page</u>
Preface-----	1
Acknowledgement-----	iii
Table of Contents-----	iv
I. Introduction-----	1
II. Linear Array with Matched Filter-----	2
III. Transient Chebyshev Array Response to Wideband Signal and Noise-----	3
IV. Expected Spatial Response to Bandlimited White Noise-----	5
V. Null-Steering Capability in a Wideband Jamming Environment-----	8
VI. Simultaneous Placement of Two Nulls-----	11
VII. Conclusions-----	12
References-----	14
Figures-----	15



I. INTRODUCTION

In modern radar applications one frequently encounters large arrays and signal spectra which are a significant fraction of the center frequency. Because electromagnetic waves travel with a finite velocity, the pattern characteristics of an array with huge dimensions are not established instantaneously and array behavior in the transient state becomes of importance [1]. The choice of signal waveform depends on the requirements of accuracy, ambiguity, and resolution. A desirable waveform consists of a single pulse with a large bandwidth-pulse width product. Frequency-modulation pulse compression is a means of achieving the resolution and accuracy of a short pulse and the detection capability of a long pulse [2]. However, it appears that the response to broadband signals incident on an array designed for a single-frequency operation is not widely understood and information on array characteristics in the transient state is generally not available. An objective of this project is to study the transient response to broadband signals and noise for a large array equipped with a matched filter.

In studying array response to noise, it is necessary to assume certain noise characteristics. Since the response depends on the noise sample, it is important to examine the expected (statistical average) pattern. A second objective of this project is to derive an expression for the expected power pattern of an array for bandlimited white noise and to obtain such a pattern for a typical array.

For applications in a jamming environment it is often desired that an array has the capability of steering a pattern null in a specified direction. This is not easily done for an array of wideband emitters. A third objective

of this project is to investigate the feasibility of null-steering for the suppression of wideband jamming noise. Possible schemes for creating two or more nulls are also considered.

II. LINEAR ARRAY WITH MATCHED FILTER

Consider a linear array of $2N$ isotropic elements spaced at distances $\pm x_1, \pm x_2, \dots, \pm x_N$ from the array center 0 , as shown in Fig. 1. The array is in a receiving mode with amplitude weighting factors I_n , $n = \pm 1, \pm 2, \dots, \pm N$. The incoming signal $s(t)$ is assumed to have a rectangular frequency spectrum centered at ω_0 with a bandwidth $2\gamma\omega_0$ and the transfer function, $H(j\omega)$, of the matched filter is the complex conjugate of signal spectrum $S(j\omega)$. The time delays, τ_n are provided to steer the mainbeam direction of the array pattern. If the signal is a pulse with a linear frequency modulation, we have

$$H(j\omega) = S^*(j\omega) = \text{Rect} \left(\frac{\omega - \omega_0}{2\gamma\omega_0} \right) e^{-j\alpha(\omega)} \quad (1)$$

In Eq. (1)

$$\text{Rect}(y) = \begin{cases} 1, & -\frac{1}{2} < y < \frac{1}{2} \\ 0, & \text{elsewhere} \end{cases} \quad (2)$$

and

$$\alpha(\omega) = \pi D(\omega - \omega_0)^2 / (2\gamma\omega_0)^2 \quad (3)$$

where D = pulse-compression ratio. The space-time response of the signal at the output of the matched filter is [3]

$$s_o(\theta, t) = \sum_{n=-N}^N I_n \cos(\omega_0 t - x_n u) \frac{\sin \gamma(\omega_0 t - x_n u)}{\gamma(\omega_0 t - x_n u)} \quad (4)$$

where

$$\begin{aligned}
u &= \frac{\omega_0}{c} (\sin \theta - \sin \theta_0) \\
&= \omega_0 \left(\frac{1}{c} \sin \theta - \frac{r_n}{x_n} \right)
\end{aligned} \tag{5}$$

θ_0 in Eq. (5) denotes the mainbeam direction of the array. When the amplitude weighting factors, I_n , are given, Eq. (4) enables us to examine the pattern characteristics as a function of t .

III. TRANSIENT CHEBYSHEV-ARRAY RESPONSE TO WIDEBAND SIGNAL AND NOISE

In order to study the effect of broadband signal and noise on the side-lobe structure of an array, it is convenient to examine the response of a Chebyshev array which has equal sidelobes for single-frequency CW operation. For a Chebyshev array with a half-wavelength spacing, we have [4]

$$I_n = \sum_{\ell=1}^N T_{2N-1} \{ W_1 \cos[(\ell - \frac{1}{2})\pi/N] \} \cos[(n - \frac{1}{2})(\ell - \frac{1}{2})\pi/N] \tag{6}$$

where the Chebyshev polynomial

$$T_{2N-1}(W) = \begin{cases} \cosh[(2N-1)\cosh^{-1}W], & W > 1 \\ \cos[2N-1)\cos^{-1}W], & W \leq 1 \end{cases} \tag{7}$$

$$W_1 = \cosh \left(\frac{1}{2N-1} \cosh^{-1}R \right) \tag{8}$$

and R is the mainbeam-to-sidelobe ratio. The expression for the amplitude weighting factor, I_n , in Eq. (6) will be used in Eq. (4) to calculate the space-time signal response of a matched-filter equipped Chebyshev array.

In practical applications it is important to examine not only the space-time response to the desired signal but also that of intentional jamming. Let the normalized jamming signal be bandlimited white noise

denoted by

$$n(t) = \sum_{k=-K}^K c_k \cos (\omega_k t + \phi_k) \quad (9)$$

where

$$\omega_k = \omega_0 (1 + k\gamma/K) \quad (10)$$

$$c_k = \sqrt{a_k^2 + b_k^2} / \sqrt{\sum_{k=-K}^K c_k^2} \quad (11)$$

$\{a_k^2, b_k^2\}$ are random numbers uniformly distributed between 0 and 1, and

$\{\phi_k\}$ represents a random phase uniformly distributed between 0 and 2π .

Owing to the assumed linearity of the matched filter, the amplitudes and phases of the noise components at the filter output will also be random with a uniform distribution. The space-time response of the array to the band-limited white noise is

$$n_o(\theta, t) = \sum_{n=-N}^N I_n \sum_{k=-K}^K c_k \cos (\omega_k t - x_n u + \phi_k) \quad (12)$$

The transient patterns with many values of $\omega_0 t$ for a half-wavelength spaced Chebyshev array responding to a pulse with a linear frequency modulation as well as to bandlimited white noise have been calculated for the following parameters: $2N = 40$, $\gamma = 0.1$ (20-percent bandwidth), and $\theta_0 = 0$. The array is designed to have -60 dB sidelobes at ω_0 and the weighting factors are:

$I_1 = 1.000$	$I_2 = 0.984$	$I_3 = 0.953$	$I_4 = 0.908$
$I_5 = 0.851$	$I_6 = 0.784$	$I_7 = 0.709$	$I_8 = 0.630$
$I_9 = 0.550$	$I_{10} = 0.469$	$I_{11} = 0.392$	$I_{12} = 0.320$
$I_{13} = 0.255$	$I_{14} = 0.197$	$I_{15} = 0.147$	$I_{16} = 0.106$
$I_{17} = 0.073$	$I_{18} = 0.047$	$I_{19} = 0.028$	$I_{20} = 0.020$

The random numbers $\{a_k, b_k, \phi_k\}$ for the noise are generated by the RANDU subroutine and the noise response is normalized with respect to ΣI_n .

The computed results for values of $\omega_0 t$ in steps of 2.513 rad from 0 to 10π (π/γ) and in steps of 8 rad up to 50π show that in no case did the highest sidelobe go above the designed -60 dB level. Two sets of typical patterns responding to the signal and the noise at $\omega_0 t = 0$ and $\omega_0 t = -25.13$ rad are shown in Figs. 2 and 3 respectively. Note that at $\omega_0 t = -25.13$ rad the maximum signal itself is down about 11 dB from its value at $\omega_0 t = 0$. Signal amplitudes have been normalized with respect to the pulse-compression ratio; hence they could be many orders of magnitude larger than noise amplitudes.

IV. EXPECTED SPATIAL RESPONSE TO BANDLIMITED WHITE NOISE

We now derive the expression for the expected power pattern of the array shown in Fig. 1 in response to the normalized bandlimited white noise represented by Eq. (9). Substituting Eq. (10) in Eq. (9), we can rewrite $n(t)$ as

$$n(t) = n_c(t) \cos \omega_0 t - n_s(t) \sin \omega_0 t \quad (13)$$

where

$$n_c(t) = \sum_{k=-K}^K c_k \cos (k\Delta\omega t + \phi_k) \quad (14)$$

$$n_s(t) = \sum_{k=-K}^K c_k \sin (k\Delta\omega t + \phi_k) \quad (15)$$

and

$$\Delta\omega = \gamma\omega_0/K \quad (16)$$

The following properties of the statistical averages are noted:

$$\langle n_c(t) \rangle = 0 \quad (17)$$

$$\langle n_s(t) \rangle = 0 \quad (18)$$

$$\langle n(t) \rangle = 0 \quad (19)$$

The space-time array response to the noise is

$$\begin{aligned} n_o(\theta, t) = & \sum_{n=-N}^N I_n \{ n_c(t-\tau_n) \cos \omega_o(t-\tau_n) \\ & - n_s(t-\tau_n) \sin \omega_o(t-\tau_n) \} \end{aligned} \quad (20)$$

The expected value of $n_o(\theta, t)$ vanishes; that is,

$$\begin{aligned} \langle n_o(\theta, t) \rangle = & \sum_{n=-N}^N I_n \{ \langle n_c(t-\tau_n) \rangle \cos \omega_o(t-\tau_n) \\ & - \langle n_s(t-\tau_n) \rangle \sin \omega_o(t-\tau_n) \} \\ = & 0 \end{aligned} \quad (21)$$

in view of Eqs. (17) and (18) and stationarity.

The output noise power can be found from Eq. (20).

$$\begin{aligned} P_n = & |n_o(\theta, t)|^2 \\ = & \sum_{n=-N}^N \sum_{m=-N}^N I_n I_m \{ n_c(t-\tau_n) n_c(t-\tau_m) \cos \omega_o(t-\tau_n) \cos \omega_o(t-\tau_m) \\ & + n_s(t-\tau_n) n_s(t-\tau_m) \sin \omega_o(t-\tau_n) \sin \omega_o(t-\tau_m) \\ & - n_c(t-\tau_n) n_s(t-\tau_m) \cos \omega_o(t-\tau_n) \sin \omega_o(t-\tau_m) \\ & - n_s(t-\tau_n) n_c(t-\tau_m) \sin \omega_o(t-\tau_n) \cos \omega_o(t-\tau_m) \} \end{aligned} \quad (22)$$

The expected value of P_n , or the average noise power is

$$\begin{aligned} \langle P_n \rangle = & \sum_{n=-N}^N \sum_{m=-N}^N I_n I_m \{ R_{cc}(\tau_n - \tau_m) \cos \omega_o(\tau_n - \tau_m) \\ & - R_{cs}(\tau_n - \tau_m) \sin \omega_o(\tau_n - \tau_m) \} \end{aligned} \quad (23)$$

where the autocorrelation function $R_{cc}(\tau_n - \tau_m)$ is

$$\begin{aligned} R_{cc}(\tau_n - \tau_m) &= \langle n_c(t - \tau_n) n_c(t - \tau_m) \rangle \\ &= R_{cc}(\tau_n - \tau_m) = \langle n_s(t - \tau_n) n_s(t - \tau_m) \rangle \end{aligned} \quad (24)$$

and the cross-correlation function $R_{cs}(\tau_n - \tau_m)$ is

$$\begin{aligned} R_{cs}(\tau_n - \tau_m) &= \langle n_c(t - \tau_n) n_s(t - \tau_m) \rangle \\ &= - R_{cs}(\tau_n - \tau_m) = - \langle n_s(t - \tau_n) n_c(t - \tau_m) \rangle \end{aligned} \quad (25)$$

For bandlimited white noise with a power density spectrum

$$S_n(f) = 1, \quad \text{for } (1-\gamma)f_o < f < (1+\gamma)f_o \quad (26)$$

we have

$$\begin{aligned} R_{cc}(\tau) &= 2 \int_{(1-\gamma)f_o}^{(1+\gamma)f_o} \cos 2\pi(f-f_o)\tau df \\ &= \frac{2\gamma\omega_o}{\pi} \left(\frac{\sin \gamma\omega_o\tau}{\gamma\omega_o\tau} \right) \end{aligned} \quad (27)$$

and

$$R_{cs}(\tau) = 2 \int_{(1-\gamma)f_o}^{(1+\gamma)f_o} \sin 2\pi(f-f_o)\tau df = 0 \quad (28)$$

Substitution of Eqs. (27) and (28) in Eq. (23) yields

$$\langle P_n \rangle = \frac{2\gamma\omega_o}{\pi} \sum_{n=-N}^N \sum_{m=-N}^N I_n I_m \left[\frac{\sin \gamma\omega_o(\tau_n - \tau_m)}{\gamma\omega_o(\tau_n - \tau_m)} \right] \cos \omega_o(\tau_n - \tau_m) \quad (29)$$

In terms of the spatial parameter u defined in Eq. (5) we obtain

$$\langle P_n \rangle = \frac{2\gamma\omega_0}{\pi} \sum_{n=-N}^N \sum_{m=-N}^N I_n I_m \left[\frac{\sin \gamma (x_n - x_m)u}{\gamma (x_n - x_m)u} \right] \cos(x_n - x_m)u \quad (30)$$

The average noise power pattern $\langle P_n \rangle$ of the 40-element Chebyshev array in Section III (designed to have -60 dB sidelobes at ω_0) has been calculated for white noise with $\gamma = 0.10$ (20% bandwidth) and $\gamma = 0.20$ (40% bandwidth). These are shown in Fig. 4. It is seen that the average sidelobe level is everywhere below -60 dB.

V. NULL-STEERING CAPABILITY IN A WIDEBAND JAMMING ENVIRONMENT

An important advantage of an array antenna system over a reflector antenna is the former's ability to create a pattern null in a specified direction. This null-steering ability is particularly of value in a jamming environment. Adaptive control-loop networks can be designed to steer nulls automatically onto jamming sources or other origins of interference on a real-time basis so that the probability of detecting the desired signals is enhanced [5]. We shall now consider the feasibility of using an array of doublet antenna elements for steering pattern nulls in order to suppress a wide-band jamming noise.

Refer to Fig. 5 where an array with $2N$ doublets is shown. Each doublet consists of two elements spaced a distance d apart. The output of one of the elements is delayed by an amount τ_0 before being subtracted from that of the other. The regular time-delay and amplitude-adjusting network (τ_n and I_n , $n = \pm 1, \pm 2, \dots, \pm N$) follows the combined output of the doublets.

The transfer function, $H(j\omega)$, of the matched filter is matched to the desired signal. The additional filter $H_{\tau_0}(j\omega)$ is provided to compensate for the distorting effect due to the doublets.

$$H_{\tau_0}(j\omega) = 1 - e^{j\omega\tau_0} \quad (31)$$

The relation between τ_0 and the angle θ'_J at which a pattern null is desired is

$$\tau_0 = \frac{d}{c} \cos \theta'_J \quad (32)$$

Note that θ'_J measured from the array line. With the doublet arrangement shown in Fig. 5 the array pattern will no longer be symmetrical with respect to the array center. A jamming noise impinging on the array from the direction of the angle θ'_J will, under ideal conditions, yield no output because it will be cancelled at the output of the doublets. On the other hand, the array retains its designed characteristics. The null direction θ'_J can be changed by changing τ_0 .

If the matched filter is designed to match a linear FM signal, its normalized impulse response is

$$h(t) = \text{Rect}\left(\frac{t}{T}\right) \cos(\omega_0 t - bt^2) \quad (33)$$

where

$$b = (\gamma\omega_0)^2 / \pi D \quad (34)$$

The array response to a jamming impulse noise

$$n_J(t) = \delta(t) \quad (35)$$

is

$$E_O(u, t) = \sum_{n=-N}^N I_n \left\{ h\left(t - \frac{x_n u'}{\omega_0}\right) - h\left(t - \frac{x_n u' + \Delta}{\omega_0}\right) - h\left(t - \frac{x_n u'}{\omega_0} + \tau_0\right) + h\left(t - \frac{x_n u' + \Delta}{\omega_0} + \tau_0\right) \right\} \quad (35)$$

where

$$u' = \frac{\omega_0}{c} (\cos \theta' - \cos \theta'_0) \quad (37)$$

$$\begin{aligned} \Delta &= \frac{d}{c} \omega_0 (\cos \theta' - \cos \tau_J) \\ &= \omega_0 \left(\frac{d}{c} \cos \theta' - \tau_0 \right) \end{aligned} \quad (38)$$

The transient pattern response of a Chebyshev array consisting of 40 doublets has been computed for a number of $\omega_0 t$ values. The Chebyshev array is designed to have the following parameters:

Total number of elements	80
Number of doublets	40
Element spacing	0.4λ
Matched filter, $H(j\omega)$	Eq. (1)
Sidelobes	-60 dB
Pulse compression ratio	100
Bandwidth	20%
Mainbeam direction, θ'_0	90°
Null direction, θ'_J	40°

In Fig. 6 are plotted two patterns at $\omega_0 t = 0$. The pattern in solid line represents the response to an impulse noise with τ_0 adjusted for $\theta'_J = 40^\circ$ according to Eq. (32). The deep null at $\theta'_J = 40^\circ$ is plainly visible. The response in the sidelobe region is everywhere below the designed -60 dB level, and is especially low in the neighborhood of θ'_J . When the doublet arrangement is not used, the response of the conventional 40-element Chebyshev array with $d = 0.8\lambda$ is indicated by crosses. The pattern is now symmetrical with respect to the broadside ($\theta'_0 = 90^\circ$) direction and the sidelobes are mostly

at the -60 dB level.

The patterns for the same array appears to be worst at $\omega_0 t = -1520.797$ rad. The response to an impulse noise for the array with 40 doublets and τ_0 adjusted for $\theta'_J = 40^\circ$ is shown in solid line in Fig. 7. Note that the response rises to -30 dB at some angles. However, the deep null at $\theta'_J = 40^\circ$ is still in evidence. The pattern for the 40-element array without the null-steering capability is plotted as crosses. They follow the general trend of the solid pattern. It may be concluded that the doublet arrangement is effective for null-steering in a wideband jamming environment and that the array pattern for impulse noise varies over a wide range during the transient state.

IV. SIMULTANEOUS PLACEMENT OF TWO NULLS

The concept of using doublets to obtain a steerable null in a wideband operation can be extended to create two or more nulls in specified directions. For two pattern nulls we can use a "doublet of doublets" as a basic array element. By this we mean that the basic array element is a subarray of two doublets. Each doublet is adjusted with a proper time delay τ_{01} to place a pattern null at an angle θ'_{J1} , and an additional time delay τ_{02} is inserted in one of the doublets to create a second null at an angle θ'_{J2} . This is depicted in Fig. 8(a), where

$$\tau_{01} = \frac{d_1}{c} \cos \theta'_{J1} \quad (39)$$

$$\tau_{02} = \frac{d_2}{c} \cos \theta'_{J2} \quad (40)$$

By the principle of pattern multiplication, the subarray will have two nulls at θ'_{J1} and θ'_{J2} .

Instead of the doublet of doublets in Fig. 8(a), the quadruplet shown in Fig. 8(b) can be used as a basic array element. That the quadruplet is equivalent to the doublet of doublets is obvious from the figures. The quadruplet arrangement has the advantage of needing only one, instead of three, summing devices. When the quadruplets are used in place of the doublets in an array as shown in Fig. 5 the directions of the pattern nulls θ'_{J1} and θ'_{J2} can be steered independently by adjusting the time delays τ_{01} and τ_{02} . The transfer function of the additional filter for compensating the distorting effect of the quadruplet must now be

$$\begin{aligned} H_{\tau_0}(j\omega) &= 1 - e^{j\omega\tau_{01}} - e^{j\omega\tau_{02}} + e^{j\omega(\tau_{01} + \tau_{02})} \\ &= (1 - e^{j\omega\tau_{01}})(1 - e^{j\omega\tau_{02}}) \end{aligned} \quad (41)$$

The direction of the mainbeam of the array can be steered by adjusting the time delays τ_n ($n = \pm 1, \pm 2, \dots, \pm N$) in the usual manner.

VII. CONCLUSIONS

In the study of the transient response to a linear FM signal and band-limited white noise for a Chebyshev array equipped with a matched filter, it is found that the sidelobes are lower than the designed level for single-frequency CW operation. This result may appear surprising. However, we recall that the sidelobes and nulls are the consequences of the coherent additions and subtractions, respectively, of the responses of the array elements and that a wideband signal or noise is a superposition of different frequency components. A frequency component within the operating band but different from that for which the array is designed would not cause much change in the mainbeam response; but, in the sidelobe region, there would be

a smearing effect, filling out the nulls and reducing the sidelobes. Hence the normalized pattern response to a band of frequencies may well have lower sidelobes. The frequency-dependent character of the array elements is not considered in this study.

Array response to noise depends on the noise sample used for computation. An expression for the expected (statistical average) pattern of an array for bandlimited white noise has been derived and the expression used to obtain the expected patterns for white noise with 20% and 40% bandwidths. The sidelobes of these patterns again do not exceed the designed level.

In the feasibility study of null-steering for the suppression of a bandlimited noise, the use of doublets has been found to be effective. The direction of the null can be changed by adjusting the time delay without affecting the direction of the mainbeam. For the simultaneous placement of two nulls, quadruplets may be used. Further investigations on alternative methods of placing nulls in desired directions and on array response to both signals and various types of noise would be important and fruitful.

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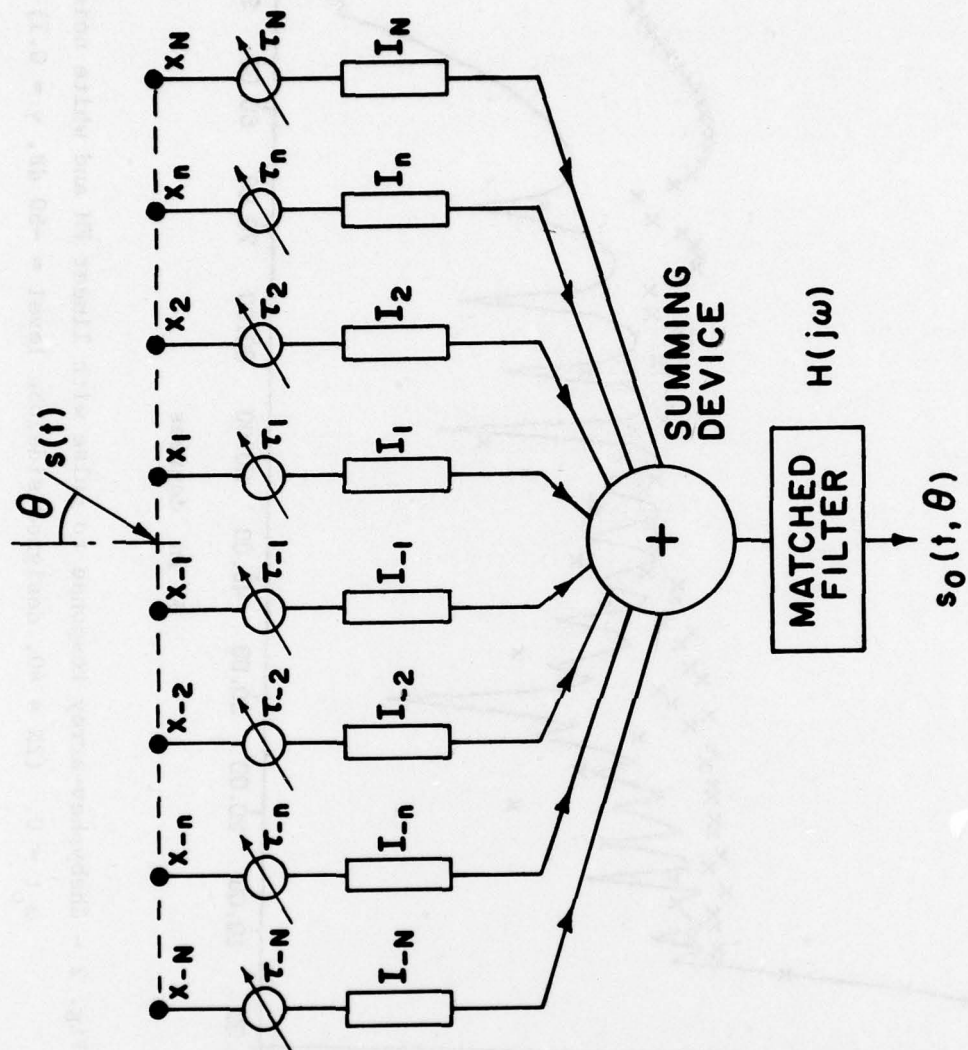


Fig. 1 - Linear array in receiving mode with matched filter

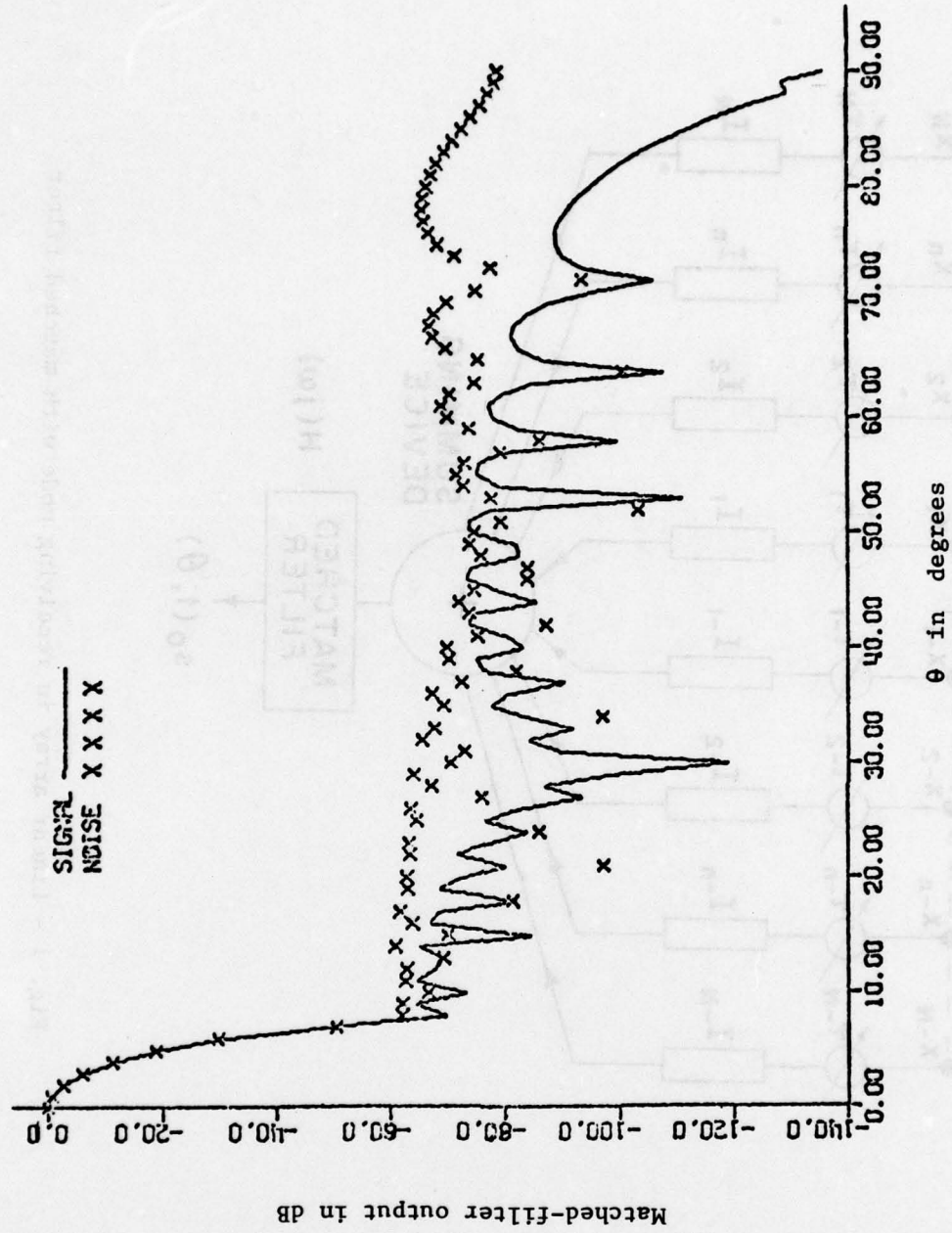


Fig. 2 - Chebyshev-array response to pulse with linear FM and white noise,
 $\omega_0 t = 0$. ($2N = 40$, designed sidelobe level = -60 dB, $\gamma = 0.1$)

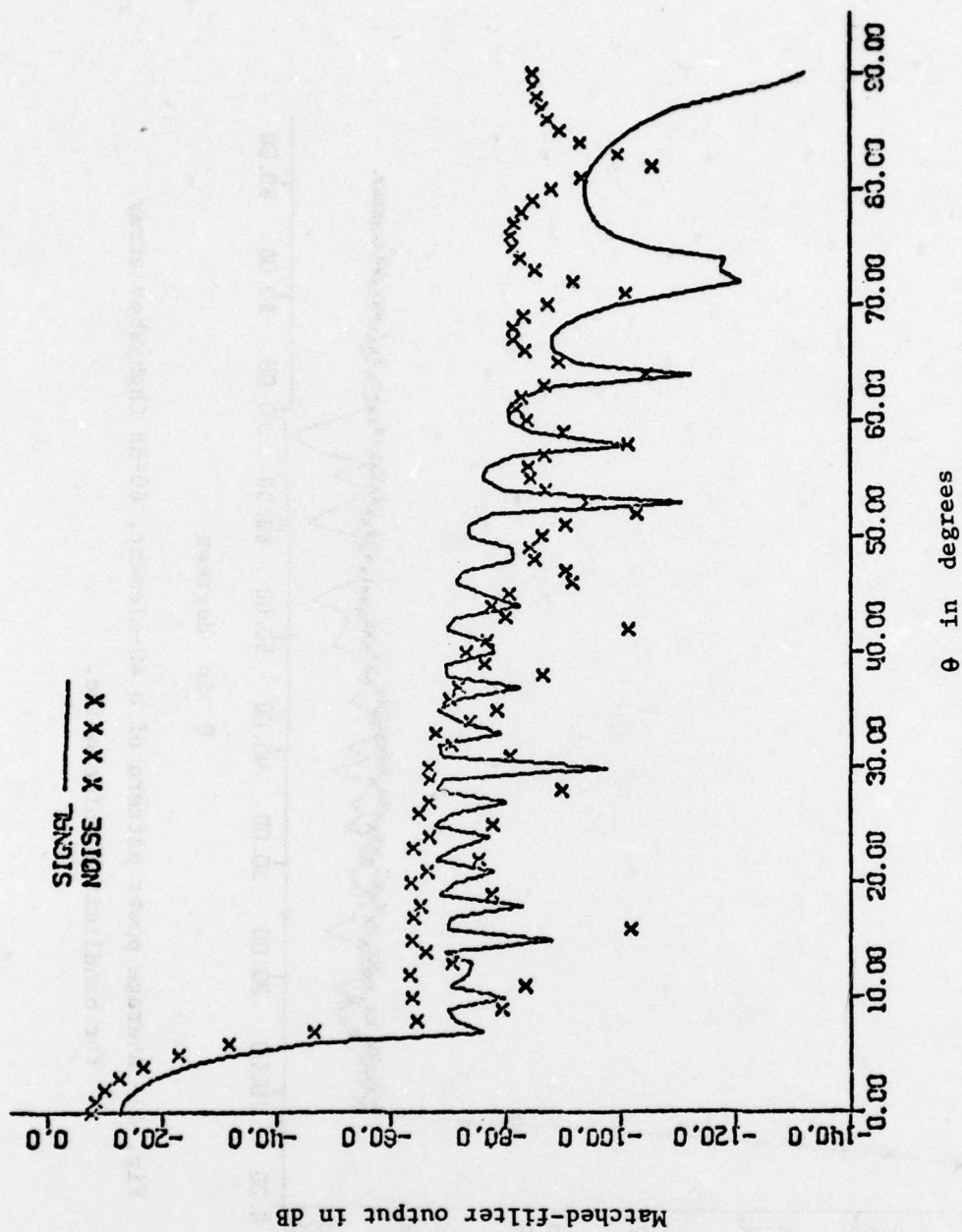


Fig. 3 - Chebyshev-array response to pulse with linear FM and white noise,
 $\omega_0 t = -25.13$ rad. ($2N = 40$, designed sidelobe level = -60 dB, $\gamma = 0.1$)

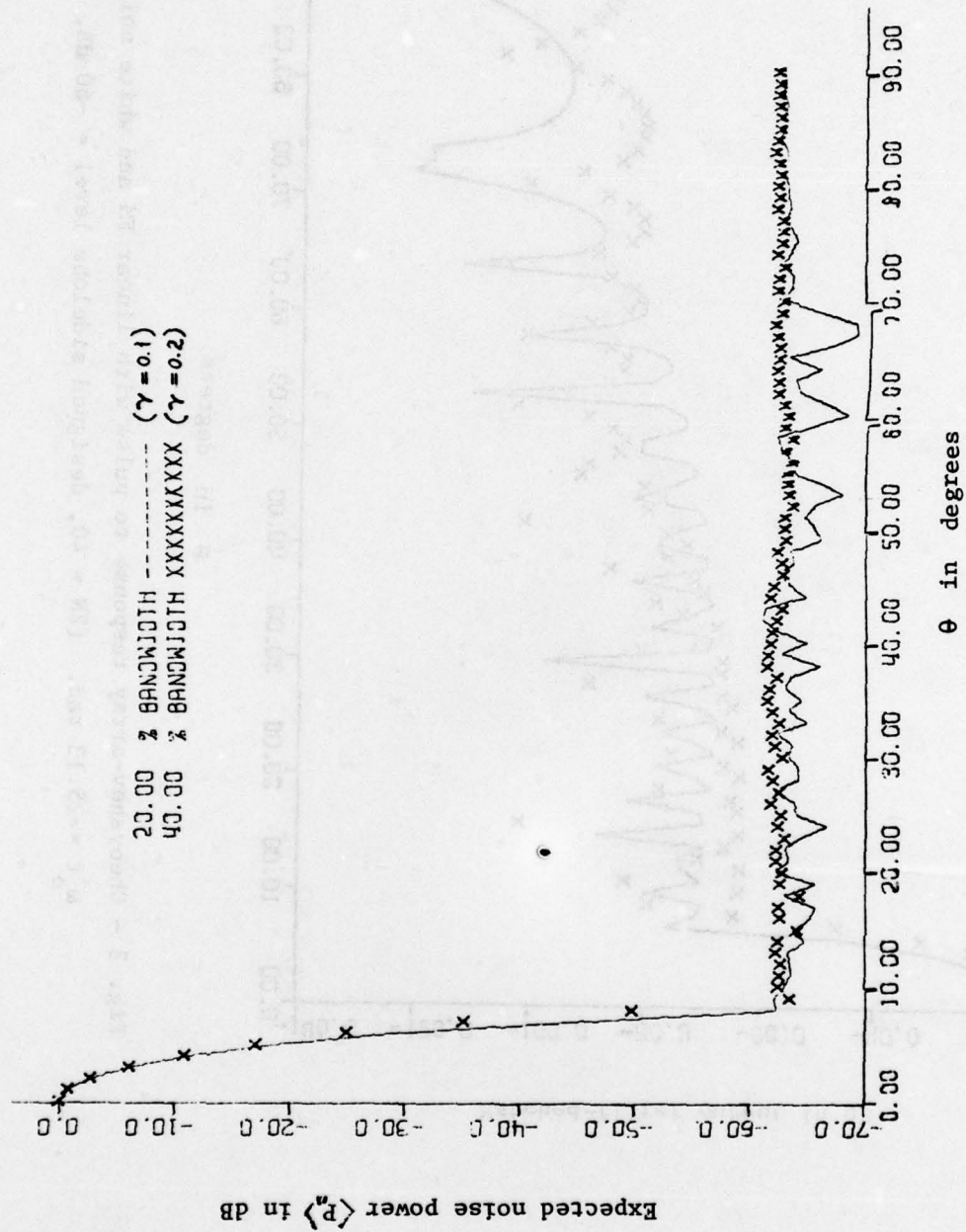


Fig. 4 - Average power pattern of a 40-element, 60-dB Chebyshev array for bandlimited white noise.

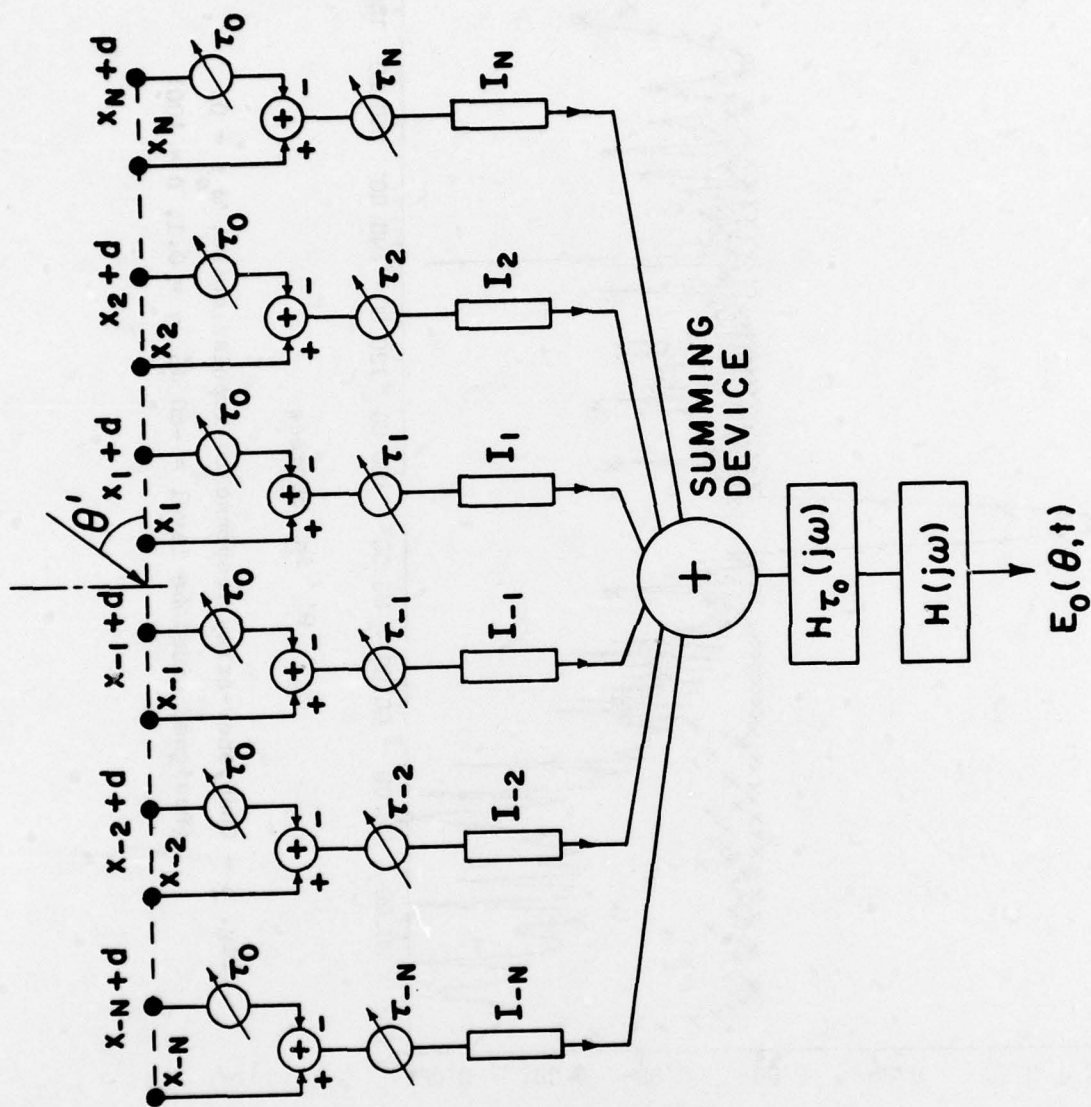


Fig. 5 - Linear array with doublets for null-steering.

40 doublets, $d = 0.4\lambda$, $\theta'_j = 40^\circ$ -----
 40 elements, $d = 0.8\lambda$, no θ'_j XXXX

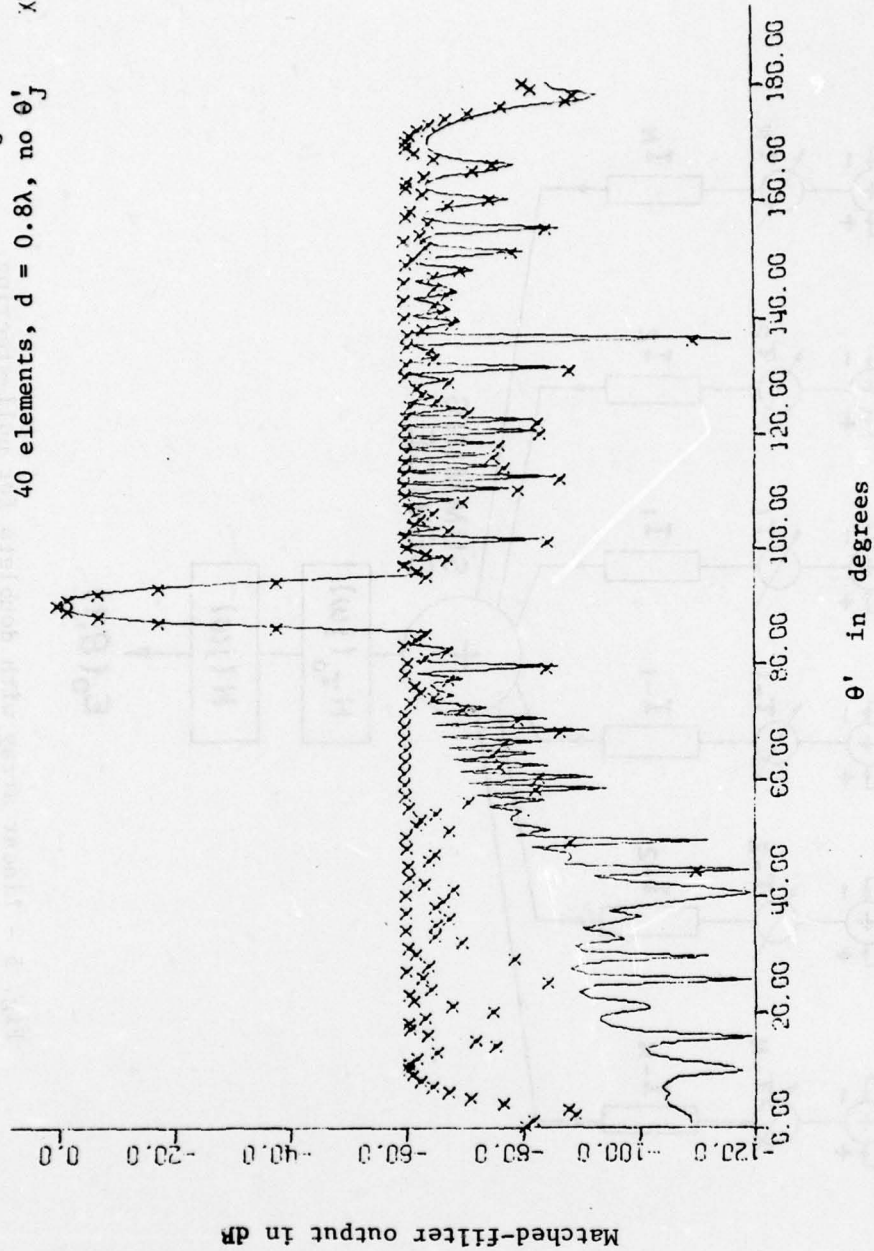


Fig. 6 - Chebyshev-array response to impulse noise, $\omega_0 t = 0$.
 (Designed sidelobe level = -60 dB, $\gamma = 0.1$, $D = 100$)

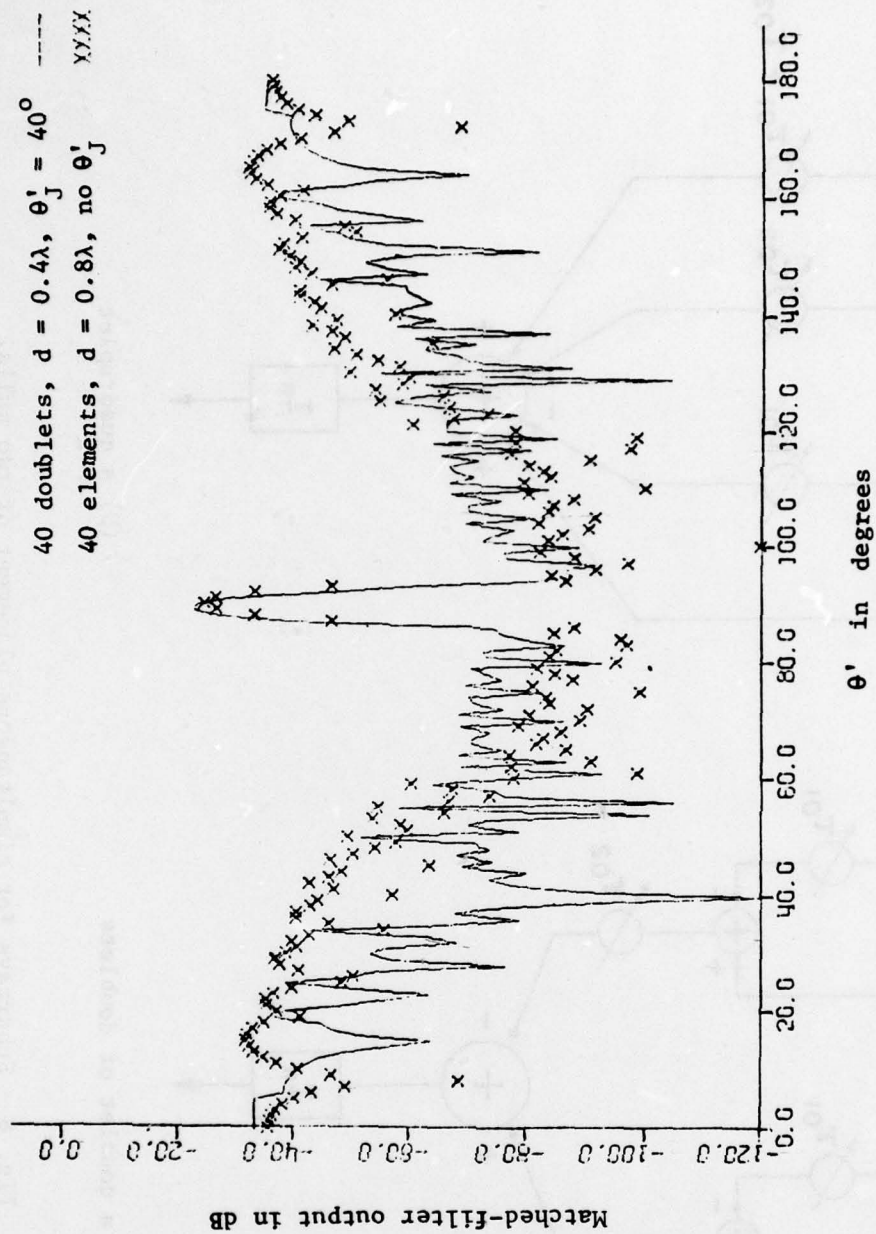


Fig. 7 - Chebyshev-array response to impulse noise, $\omega_0 t = -1520.791$ rad.
 (Designed sidelobe level = -60 dB, $\gamma = 0.1$, $D = 100$)

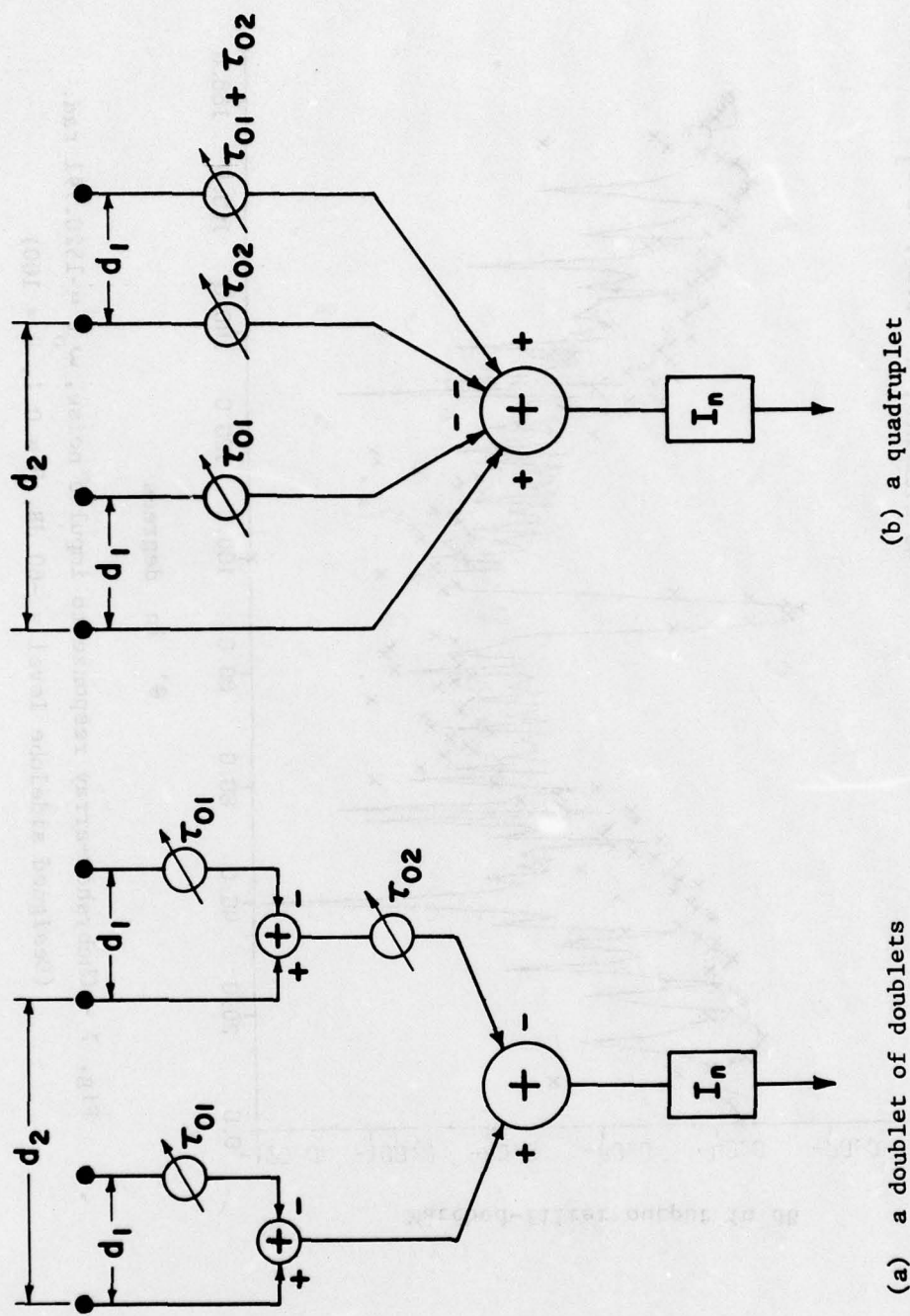


Fig. 8 - Subarrays for simultaneous placement of two nulls.