

AD-A050 213

NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF
CMS HISTOGRAM, DENSITY ESTIMATION AND PROBABILITY PLOTTING ROUT--ETC(U)
DEC 77 6 I DANIKAS

F/6 9/2

UNCLASSIFIED

NL

1 of 3
AD
A050213



AD A 050213

2

NAVAL POSTGRADUATE SCHOOL

Monterey, California



AD No. _____
DDC FILE COPY

DDC
RECEIVED
FEB 22 1978
A D

9 Master's **THESIS**

6
CMS Histogram, Density Estimation and Probability Plotting Routines, with an Application to the Analysis of the Output of a Simulation of a Correlated Queue.
by
10 Georgios Ioannis/Danikas
11 December 1977
Thesis Advisor: P. A. W. Lewis

12 231 p.

Approved for public release; distribution unlimited.

257450

et

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CMS Histogram, Density Estimation and Probability Plotting Routines, with an Application to the Analysis of the Output of a Simulation of a Correlated Queue		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; December 1977
7. AUTHOR(s) Georgios Ioannis Danikas		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		12. REPORT DATE December 1977
		13. NUMBER OF PAGES 232
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Histogram Plotting subroutines Exponential Moving Average (EMA)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The object of this thesis has been twofold. The first object was to develop FORTRAN versions of several existing APL programs which were designed to analyze univariate data. In particular the programs were designed to test for exponentiality and normality of the data and, by sectioning or jackknifing, obtain estimates of sampling variances of sample moments. The		

second object of the thesis was to use these programs in a simulation study of first-come first-served queues in which the service times and the inter-arrival times are exponentially distributed but dependent. The dependence is introduced by using the mixed moving average autoregressive structure (EARMA (p, q)) for exponential sequences, introduced by Lewis and co-workers. Four models of correlated queues are introduced, giving autocorrelated and cross-correlated service and arrival times in various degrees. The simulation study gives a quantitative idea of the effect of correlation on the mean waiting time and the distribution of the waiting time.

Approved for public release; distribution unlimited

CMS HISTOGRAM, DENSITY ESTIMATION AND PROBABILITY PLOTTING
ROUTINES, WITH AN APPLICATION TO THE ANALYSIS OF THE OUTPUT
OF A SIMULATION OF A CORRELATED QUEUE

by

Georgios Ioannis Danikas
Major Hellenic Army
B.S., Military Academy of Greece, 1963

Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN COMPUTER SCIENCE
MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
December 1977

DDC
RECEIVED
FEB 22 1978
D

Author:

[Handwritten signature of Georgios Ioannis Danikas]

Approved by:

Peter A. W. Lewis by P. Thrapfel
Thesis Advisor

GERALD G. BROWN

Second Reader

[Handwritten signature]
Chairman, Department of Computer Science

[Handwritten signature]
Chairman, Department of Operations Research

[Handwritten signature]
Dean of Information and Policy Sciences

ACCESSION INFO	
DTIC	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist. AVAIL. and/or SPECIAL	
A	

ABSTRACT

The object of this thesis has been twofold. The first object was to develop FORTRAN versions of several existing APL programs which were designed to analyze univariate data. In particular the programs were designed to test for exponentiality and normality of the data and, by sectioning or jackknifing, obtain estimates of sampling variances of sample moments. The second object of the thesis was to use these programs in a simulation study of first-come first-served queues in which the service times and the inter-arrival times are exponentially distributed but dependent. The dependence is introduced by using the mixed moving average autoregressive structure (EARMA(p,q)) for exponential sequences introduced by Lewis and co-workers. Four models of correlated queues are introduced, giving autocorrelated and cross-correlated service and arrival times in various degrees. The simulation study gives a quantitative idea of the effect of correlation on the mean waiting time and the distribution of the waiting time.

TABLE OF CONTENTS

I.	INTRODUCTION.....	16
II.	SUBROUTINE HISTGS/HISTFS.....	18
	A. DESCRIPTION.....	18
	B. USING 'HISTGS/HISTFS' WITH TELEPHONE DATA 1 AND TELEPHONE DATA 2.....	20
III.	PLOTTING SUBROUTINES.....	29
	A. INTRODUCTION.....	29
	B. DESCRIPTION OF THE PLOTTING METHOD.....	30
	C. SUBROUTINE NORMPL.....	31
	1. Description.....	31
	2. Program Structure.....	36
	3. Interpreting The Output.....	38
	4. Using 'NORMPL' with Generated Data from Various Distributions.....	50
	5. Using 'NORMPL' With Cost Overruns Data...	54
	D. SUBROUTINE EXPLT	56
	1. Description	56
	2. Program Structure	62
	3. Interpreting The Output	63
	4. Using 'EXPLT' with Data Generated from Various Distributions	64
IV.	SUBROUTINE LIST.....	76
	A. DESCRIPTION.....	76
	B. INTERPRETING THE OUTPUT.....	77
	C. USING 'LIST' WITH TELEPHONE DATA 1 AND TELEPHONE DATA 2.....	78
V.	ASSESSMENT OF VARIABILITY ROUTINES	87
	A. INTRODUCTION.....	87
	B. SUBROUTINE SECTN	88
	1. Description.....	88

2. Program Structure	91
3. Using 'SECTN' with Telephone Data Set 1..	93
C. SUBROUTINE JACK.....	96
1. Description.....	96
2. Program Structure	99
3. Using 'JACK' with Telephone Data 1.....	101
VI. SINGLE-SERVER FIRST-COME FIRST-SERVED QUEUES	
WITH 'EARMA' STRUCTURE.....	103
A. THE EARMA(P,Q) PROCESS.....	103
B. USE OF AUTOREGRESSIVE (EAR(P)), MOVING	
AVERAGE (EMA(Q)) AND MIXED STRUCTURES	
(EARMA(P,Q)) IN MODELLING QUEUES	105
C. PROGRAM STRUCTURE.....	111
1. General.....	111
2. Main Program.....	111
3. Subroutine BETAS.....	114
4. Subroutine AUTOR.....	115
5. Subroutine EARMA	115
6. Time requirements.....	115
D. ANALYSIS OF SIMULATION RESULTS.....	116
1. Queue with Dependent Service.....	116
2. Queue with Cross-Correlated Service Time.	162
PROGRAM LISTINGS.....	192
LIST OF REFERENCES.....	229
INITIAL DISTRIBUTION LIST.....	232
LIST OF FIGURES.....	7

LIST OF FIGURES

1.	Telephone Data 1. The Entire Set of Data.....	23
2.	Telephone Data 1. 2nd Section.....	24
3.	Telephone Data 1. 3rd Section.....	25
4.	Telephone Data 2. The Entire Set of Data.....	26
5.	Telephone Data 2. 2nd Section.....	27
6.	Telephone Data 2. 3rd Section.....	28
7a.	Generated Data from an Exponential Distribution....	40
7b.	Histogram of the Data of Figure 7a.....	41
8a.	Generated Data from a Uniform (0,1) Distribution...	42
8b.	Histogram of the Data of Figure 8a.....	43
9a.	Generated Data from a Cauchy Distribution.....	44
9b.	Histogram of the Data of Figure 9a.....	45
10a.	Generated Data from a Triangular (0,2) Distribution Sample size 200.....	46
10b.	Histogram of the Data of Figure 10a.....	47
11a.	Generated Data from a Triangular (0,2) Distribution Sample size 50.....	48
11b.	Histogram of the Data of Figure 11a.....	49
12a.	Generated Data from a Normal (1000,1) Distribution.	52
12b.	Histogram of the Data of Figure 12a.....	53

13.	Cost Overruns Data.....	55
14a.	A Sample of Size 100 from a Uniform (0,1) Distribution.....	66
14b.	Histogram of the Data of Figure 14a.....	67
15a.	A Sample of Size 100 from a Triangular Symmetric (0,2) Distribution.....	68
15b.	Histogram of the Data of Figure 15a.....	69
16a.	X^2 with 10 d.f. Distributed Data.....	70
16b.	Histogram of the Data of Figure 16a.....	71
17a.	A Sample of Size 100 from a Normal (1000,1) Distribution.....	72
17b.	Histogram of the Data of Figure 17a.....	73
18a.	A Sample of size 50 from an Exponential Distribution.....	74
18b.	Histogram of the Data of Figure 18a.....	75
19a.	Using LIST with Telephone Data Set 1.....	80
19b.	Using LIST with Telephone Data Set 1 (ccnt.).....	81
19c.	Using LIST with Telephone Data Set 1 (ccnt.).....	82
20a.	Using LIST with Telephone Data Set 2.....	83
20b.	Using LIST with Telephone Data Set 2 (ccnt.).....	84
20c.	Using LIST with Telephone Data Set 2 (ccnt.).....	85
20d.	Using LIST with Telephone Data Set 2 (ccnt.).....	86
21.	Using SECTN with Telephone Data Set 1.....	95
22.	Using JACK with Telephone Data Set 1.....	102

23a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 for the Run S#50#25.....	119
23b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 for the Run S#50#25.....	120
23c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 for the Run S#50#25.....	121
23d.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#50#25.....	122
24a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 for the Run S#25#98.....	123
24b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 for the Run S#25#98.....	124
24c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 for the Run S#25#98.....	125
24d.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#25#98.....	126
24e.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 EAR for the Run S#25#98.....	127
24f.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 BAR for the Run S#25#98.....	128

24g.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 EAR for the Run S#25#98.....	129
25.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#99#25.....	130
26a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 for the Run S#99#98.....	131
26b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 for the Run S#99#98.....	132
26c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 for the Run S#99#98.....	133
26d.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#99#98.....	134
27a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 EAR for the Run S#50#90.....	137
27b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 EAR for the Run S#50#90.....	138
27c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 EAR for the Run S#50#90.....	139
28a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 EAR for the Run S#50#25.....	140

28t.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W_4 BAR for the Run S#50#25.....	141
28c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W_4 BAR for the Run S#50#25.....	142
29a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Plot of $E[W \text{ BAR}]$ versus r	145
29b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Plot of $E[W \text{ BAR}]$ versus t	146
30.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Plot of $E[W \text{ BAR}]$ versus N from the Run S#99#98.....	148
31a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W_4 EAR for the Run S#95#95.....	149
31b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W_4 BAR for the Run S#95#95.....	150
31c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W_4 BAR for the Run S#95#95.....	151
31d.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#95#95.....	152
32a.	M/M/1 Queue Histogram of W_4 BAR for the Run S#50#95.....	153
32b.	M/M/1 Queue Exponential Plot of W_4 BAR for the Run S#50#95.....	154
32c.	M/M/1 Queue Normal Plot of W_4 BAR	

	for the Run S#50#95.....	155
33a.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Histogram of W 4 EAR for the Run S#50#95.....	156
33b.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Exponential Plot of W 4 BAR for the Run S#50#95.....	157
33c.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Normal Plot of W 4 EAR for the Run S#50#95.....	158
33d.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#50#95.....	159
34.	Queue with EAR1 Autoregressive Service Times and Poisson Input. Tabulation of Sample Statistics for the Run S#50#98.....	161
35a.	Queue with Cross-Correlated Service Times and Poisson Input. Histogram of W 4 for the Run SC#50#25.....	164
35b.	Queue with Cross-Correlated Service Times and Poisson Input. Exponential Plot of W 4 for the Run SC#50#25.....	165
35c.	Queue with Cross-Correlated Service Times and Poisson Input. Normal Plot of W 4 for the Run SC#50#25.....	166
35d.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#50#25.....	167
36a.	Queue with Cross-Correlated Service Times and Poisson Input. Histogram of W 4 EAR for the Run SC#50#25.....	169

36b.	Queue with Cross-Correlated Service Times and Poisson Input. Exponential Plot of W 4 BAR for the Run SC#50#25.....	170
36c.	Queue with Cross-Correlated Service Times and Poisson Input. Normal Plot of W 4 EAR for the Run SC#50#25.....	171
37.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#99#25.....	172
38a.	Queue with Cross-Correlated Service Times and Poisson Input. Histogram of W 4 for the Run SC#50#98.....	173
38b.	Queue with Cross-Correlated Service Times and Poisson Input. Exponential Plot of W 4 for the Run SC#50#98.....	174
38c.	Queue with Cross-Correlated Service Times and Poisson Input. Normal Plot of W 4 for the Run SC#50#98.....	175
39a.	Queue with Cross-Correlated Service Times and Poisson Input. Histogram of W 4 EAR for the Run SC#50#98.....	176
39b.	Queue with Cross-Correlated Service Times and Poisson Input. Exponential Plot of W 4 BAR for the Run SC#50#98.....	177
39c.	Queue with Cross-Correlated Service Times and Poisson Input. Normal Plot of W 4 EAR for the Run SC#50#98.....	178
39d.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#50#98.....	179

40a.	Queue with Cross-Correlated Service Times and Poisson Input. Histogram of $W \bar{4}$ BAR for the Run SC#25#25.....	182
40b.	Queue with Cross-Correlated Service Times and Poisson Input. Exponential Plot of $W \bar{4}$ BAR for the Run SC#25#25.....	183
40c.	Queue with Cross-Correlated Service Times and Poisson Input. Normal Plot of $W \bar{4}$ BAR for the Run SC#25#25.....	184
40d.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#25#25.....	185
41.	Queue with Cross-Correlated Service Times and Poisson Input. Plot of $E[W \bar{4}]$ versus r	188
42.	Queue with Cross-Correlated Service Times and Poisson Input. Plot of $E[W \bar{4}]$ versus t	189
43.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#25#98.....	190
44.	Queue with Cross-Correlated Service Times and Poisson Input. Tabulation of Sample Statistics for the Run SC#99#50.....	191

ACKNOWLEDGEMENT

I would like to thank Professor F.A.W. Lewis, my thesis advisor, for guidance and direction during the writing of this thesis. I would also like to thank Professor G.G. Brown, the second reader, for advice on computing aspects of the thesis. In addition, the staff of the W.R. Church Computing Center at the Naval Postgraduate School generously and patiently made available their time and advice.

Finally, my thanks to Roula, who helped me perhaps most of all, as she does in all things, by being the wonderful wife that she is.

I. INTRODUCTION

The object of this thesis has been twofold.

The first object was to develop FCBTAN versions of several existing APL programs which were designed to analyze univariate data, in particular to test for exponentiality and normality of the data and, by sectioning or jackknifing obtain estimates of variances of sample moments.

The second object was to use these programs in a simulation study of first-come first-served queues in which the service times and the inter-arrival times are exponentially distributed but dependent. The dependence is introduced by using the mixed moving average autoregressive structure (EARMA(p,q)) for exponential sequences introduced by Gaver and Lewis (1977), Lawrence and Lewis (1977), Jacobs and Lewis (1977) and Lawrence and Lewis (1978). Four schemes are introduced which give autocorrelated and cross-correlated service and arrival times in various degrees.

Since no analytical properties of these queues can be derived, their properties are studied by simulation. Because the EARMA queues are neither regenerative nor Markovian there are several problems in the simulation study of knowing when one can assume that a steady state has been reached in the simulation and of obtaining confidence intervals for estimated parameters. This problem is handled by generating a number of independent sample paths and comparing statistics obtained from the replications at several points along the sample paths. Only the waiting

time process (W_N) is studied in this thesis, not the state space process $N(t)$.

Unfortunately the simulation study of the EARMA-type queue was hampered by the size and speed of the IBM System 360/67 system at the Naval Postgraduate School. Consequently the results were not as extensive as had been hoped for. Fairly detailed results are given for the case where the service time process is autoregressive; one case where the traffic intensity is 0.99 and the correlation parameter 0.98, was intensively investigated. This gives some idea of the length of the transient in the queue and of the inflation of the mean waiting time because of the correlation in the service process. Some investigation of the case where the service times and inter-arrival times are cross-correlated are given.

In section II we describe the basic histogram and statistics computation package 'HISTGS/F' and apply it to two sets of telephone error data. In section III the plotting method and the plotting Subroutines 'NCFMFL', 'EXPLT' are described and we test them with various generated data. In section IV we describe the assessment of variability Subroutines 'SECTN', 'JACK' and we apply them to telephone data set 1. In section V the Subroutine 'LIST' is described and we apply it to two sets of telephone error data. In section VI we describe and analyze the EARMA(p,q) model as well the program which we developed in order to simulate it. Computer program listings are provided after section VI.

II. SUBROUTINE HISTGS/HISTFS

A. DESCRIPTION

Here the Subroutine 'HISTGS/HISTFS' is presented. 'HISTGS/HISTFS' is used for estimating the probability density function from a given set of data and at the same time computing some basic statistics. Basically this Subroutine is the library FORTRAN-Subroutine HISTF/G, which was created at N.P.S. by D. W. Robinson. It has been modified by adding the new entry point 'LIMITS'.

The new entry point may be used for wild data or for data having a mixed distribution. Therefore it is a useful tool for that user involved with this kind of data. Simply, the user has to define those values in the range of data (sectioning the data into several disjoint sections) that he/she believes are useful cut points. For each section of data the user may have the probability density function as well as the basic statistics (as HISTF/G does). Also by sectioning the data over its range, the user may have a histogram of the random variable when it is conditioned to be between given limits.

The number of sections that the entry point 'LIMITS' can accept is as many as 50.

A complete description of how 'HISTGS/HISTFS' operates is given in the subroutine. On the other hand, a summary of the subroutine is given by typing on the terminal the

command DESCRIBE HISTGS under CMS (Cambridge Monitor System, Ref. 4). Typing 'DESCRIBE HISTGS' the following response will be given on the terminal:

SUBROUTINE HISTGS/HISTFS

'HISTGS/HISTFS' gives you a histogram of a set of data along with the estimate of a set of basic statistics. Also it gives you the following options:

1. Displaying a smoothed empirical probability density function over the histogram,
2. Sectioning the data into several disjoint sections (no more than 50) and then having a histogram and the estimate of basic statistics for each section,
3. Scaling the histogram,
4. Displaying just the histogram (with or without the density function) and no statistics.

The estimated basic statistics are: Mean, Median, Trimean, Midrange, Geometric Mean, Harmonic Mean, Variance, Standard Deviation, Coefficient of Variation, Mean Deviation, Range, Midspread, Third and Fourth Central Moment, Skewness, Kurtosis, Beta1, Beta2, Maximum, Minimum, and Quantiles.

CALLING SEQUENCES

```
CALL HISTGS ( X, N, NBAR )
CALL HISTFS ( X, N, NBAR )
CALL LIMITS ( X, N, XLIM, LIMS )
CALL NCSTAS
CALL STATS
CALL FIXS ( SCALE )
CALL NCFIXS
CALL PRCMAX ( PSC )
CALL NOPRMX
```

ARGUMENTS

X Array of data values
N Number of data values
NEAR Number of bars in the histogram
XLIM Array of cut points
LIMS Number of cut points
SCALE Vector of two values to scale the data
PSC A real variable (between 0. and 1.) defining
 the maximum value of the probability axis.

If the entry point NOSTAS is called before calling any of the entry points HISTFS, HISTGS, or LIMITS then no statistics are printed, otherwise the statistics are printed by default.

By calling the entry point FIXS the histogram scale may be fixed and remains set unless it is reset by another call to FIXS or allowed to float (the default) by calling NCFIXS. The reason for fixing the scales is so that when comparing more than one batch of data, we can get comparably scaled histograms.

By calling PROMAX, the maximum value of the probability axis (Y) can be set to PSC, which will remain in effect until NOPRMX is called (the default).

More information is given in the subroutine.

B. USING 'HISTGS/HISTFS' WITH TELEPHONE DATA 1 AND
TELEPHONE DATA 2

Here 'HISTGS/HISTFS' has been used to plot the set of Telephone Data 1 and Telephone Data 2. The algorithmic

procedure for both sets of data is the same. That is, the entry point 'HISTFS' is called first and then the entry point 'LIMITS'. Four cut points have been used (1, 2, 141, 86000) sectioning the data into three disjoint sections as follows:

Section 1 : Consists of data points $X_{(i)}$, such that
 $1 \leq X_{(i)} < 2$.

Section 2 : Includes the data points $X_{(i)}$, such that
 $2 \leq X_{(i)} < 141$.

Section 3 : Includes the data points $X_{(i)}$, such that
 $142 \leq X_{(i)} < 86000$.

No print-out for section 1 is given because of the constant value of the data points.

Figures 1 through 6 give the histogram and the basic statistics for both sets of data (For the entire set and for sections 2 and 3).

We observe that both of data sets are so unruly that the use of the entry point 'LIMITS' or 'HISTGS' is suggested.

Observing Figures 1 and 4 we may conclude that both data sets appear to be the same, having some geometric or exponential distribution. But as we will see later this is not true. Thus the histogram, in that case, which is obtained by using just the entry point 'HISTFS' is not helpful and leads us to a wrong conclusion.

'LIMITS' is applied now to give us a useful answer :

Observing figures 2 and 5, obtained by using the entry point 'LIMITS', we may conclude that the data sets do not have the same distribution (at least in that interval). In addition data set 2 is far from having a geometric or an

exponential distribution, as it can be seen by the histogram of the entire set of data. This is a conclusion which cannot be obtained by without use of the entry point 'LIMITS'.

Observing also Figures 3 and 6 , where the histogram of the 3rd section from each data set is presented, we may conclude that this section of both data sets may have an exponential distribution, but not the same one.

The most important facet of the data which appears is the modes at about 124. The mode is much more apparent in Telephone Data 2. In fact closer investigation reveals that this mode was due to leakage of dial pulses; this leakage caused bit errors at multiples of 124 bits.

The above is a quick and informal analysis of the Telephone Data Sets obtained by using the entry point 'LIMITS' of 'HISTGS/HISTFS', and the conclusion is that both of data sets are so unruly that any formal characterization of their distribution is not so easily obtained. In addition it is apparent that a mixed distribution governs both sets of data.

Having quickly analyzed both of the telephone data sets, the use of entry point 'LIMITS' of the Subroutine 'HISTGS/HISTFS' is obvious.

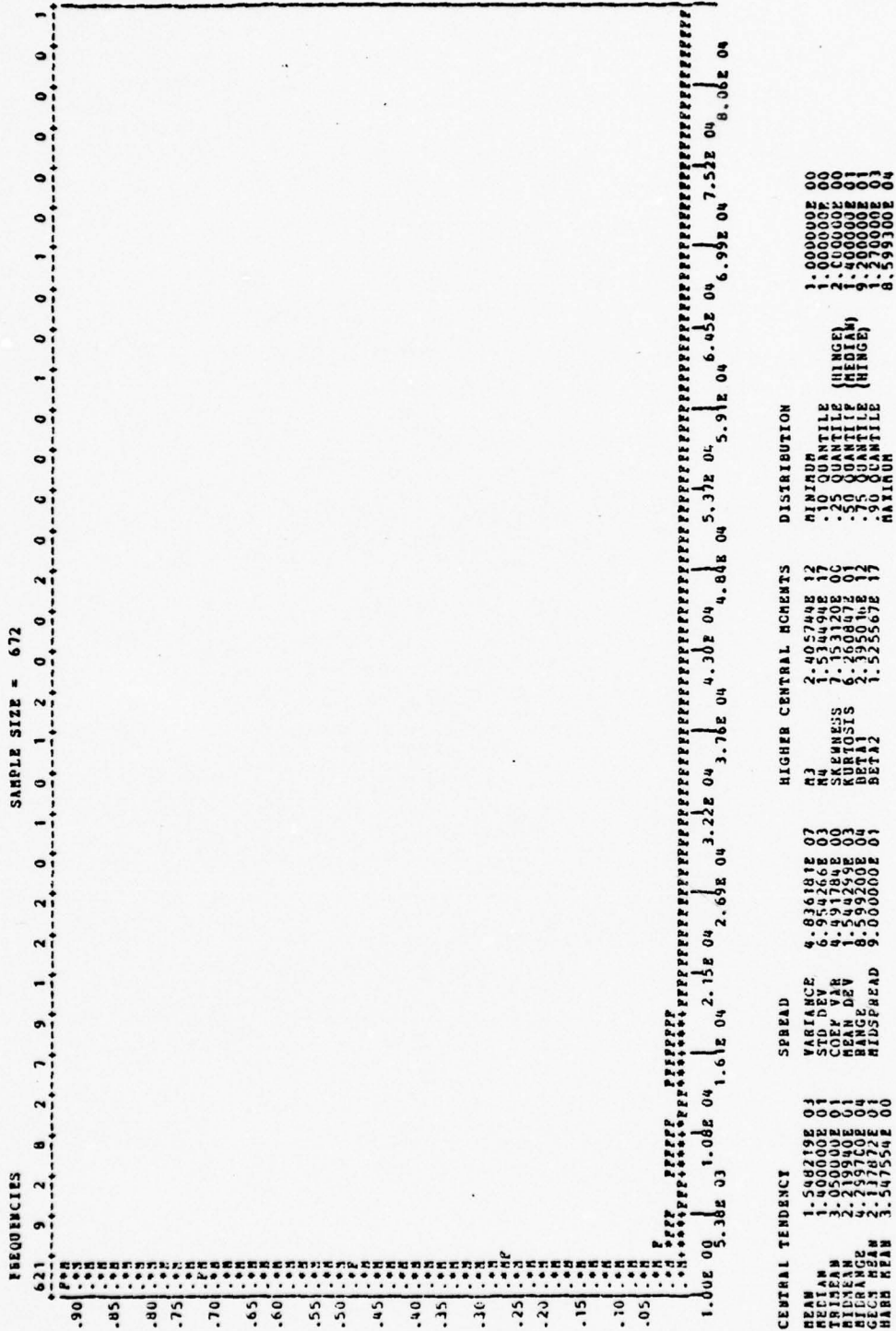


Figure 1 - TELEPHONE DATA 1. THE ENTIRE SET OF DATA.

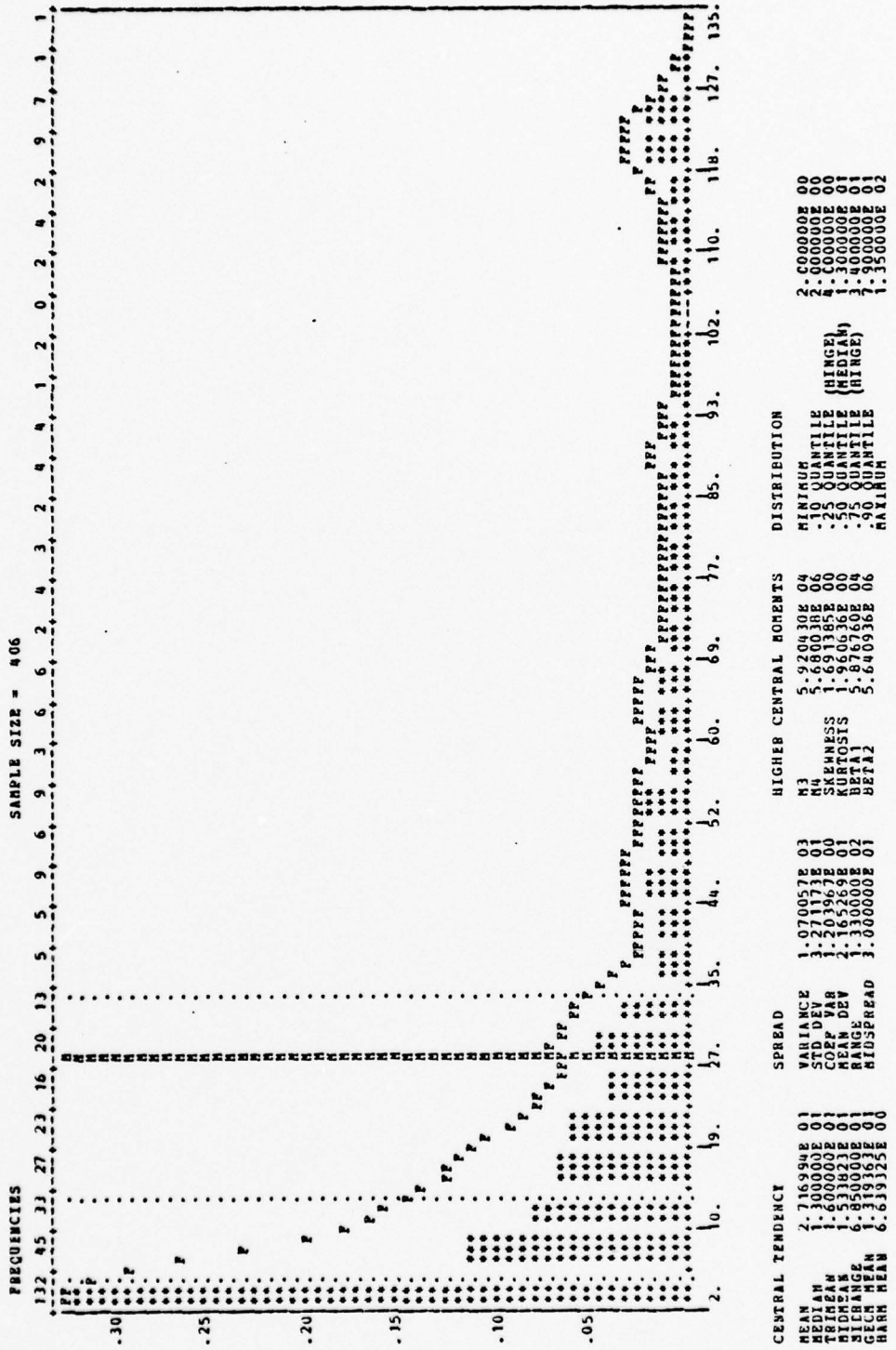


Figure 2 - TELEPHONE DATA 1. 2nd SECTION.

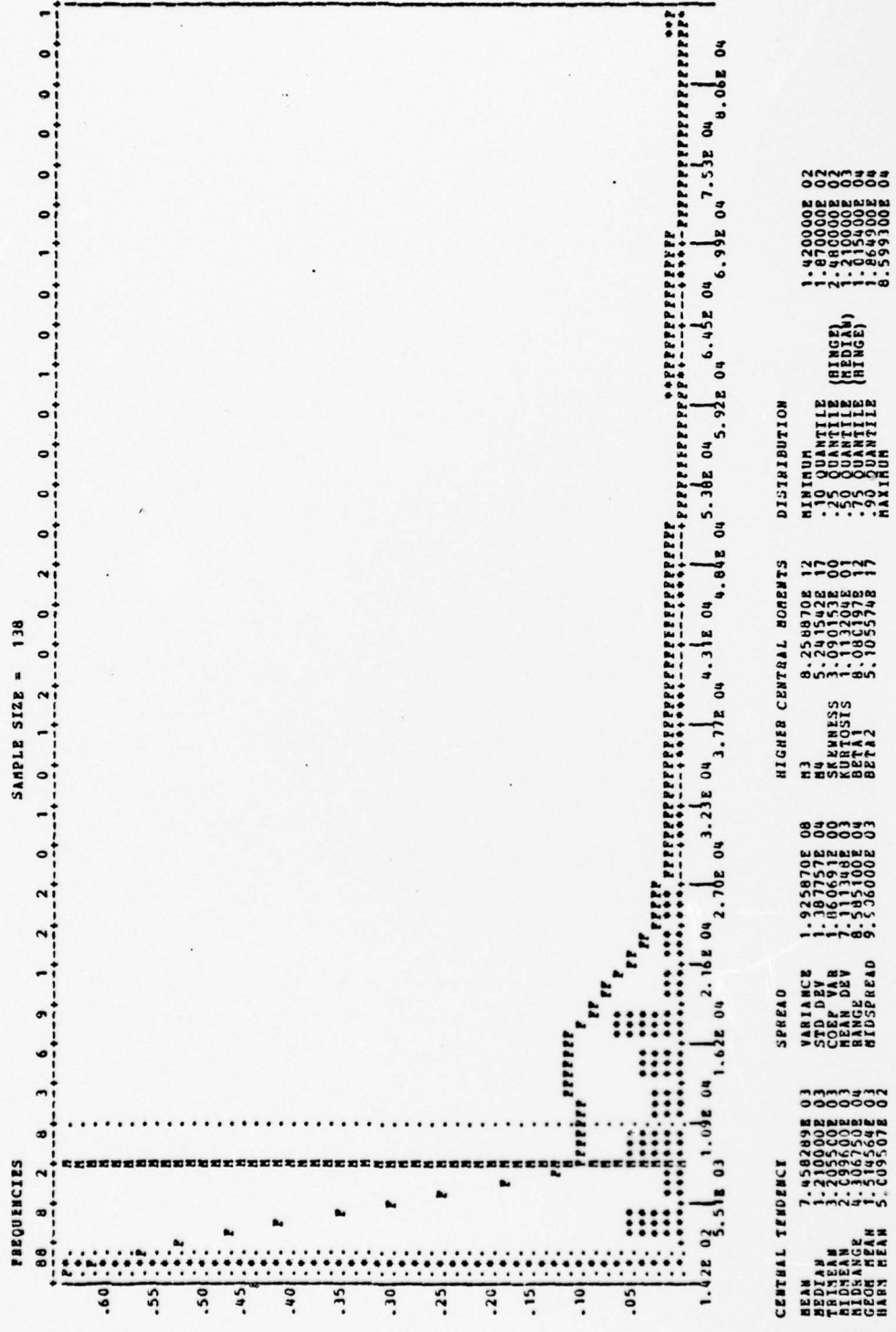


Figure 3 - TELEPHONE DATA 1. 3rd SECTION.

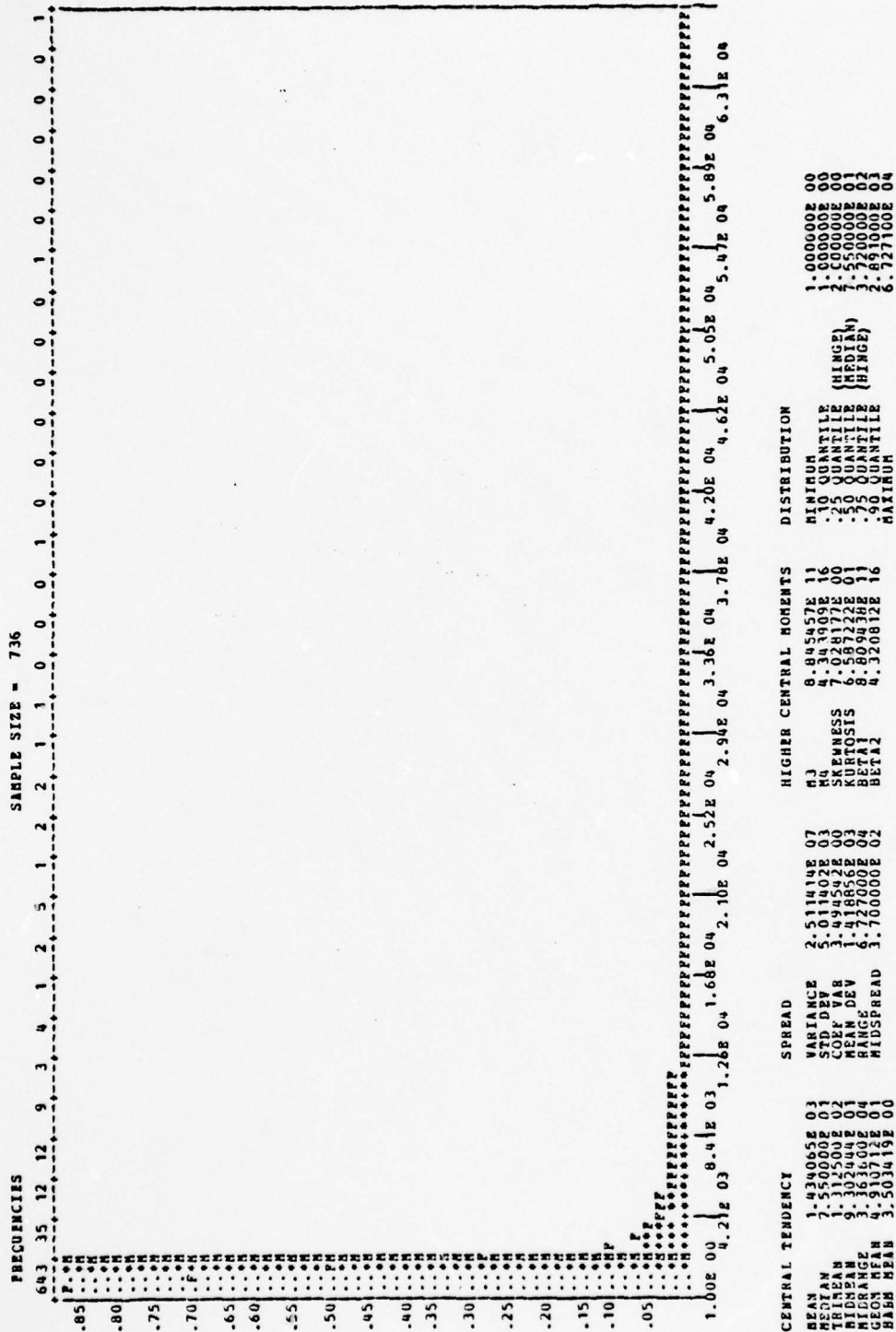


Figure 4 - TELEPHONE DATA 2. THE ENTIRE SET OF DATA.

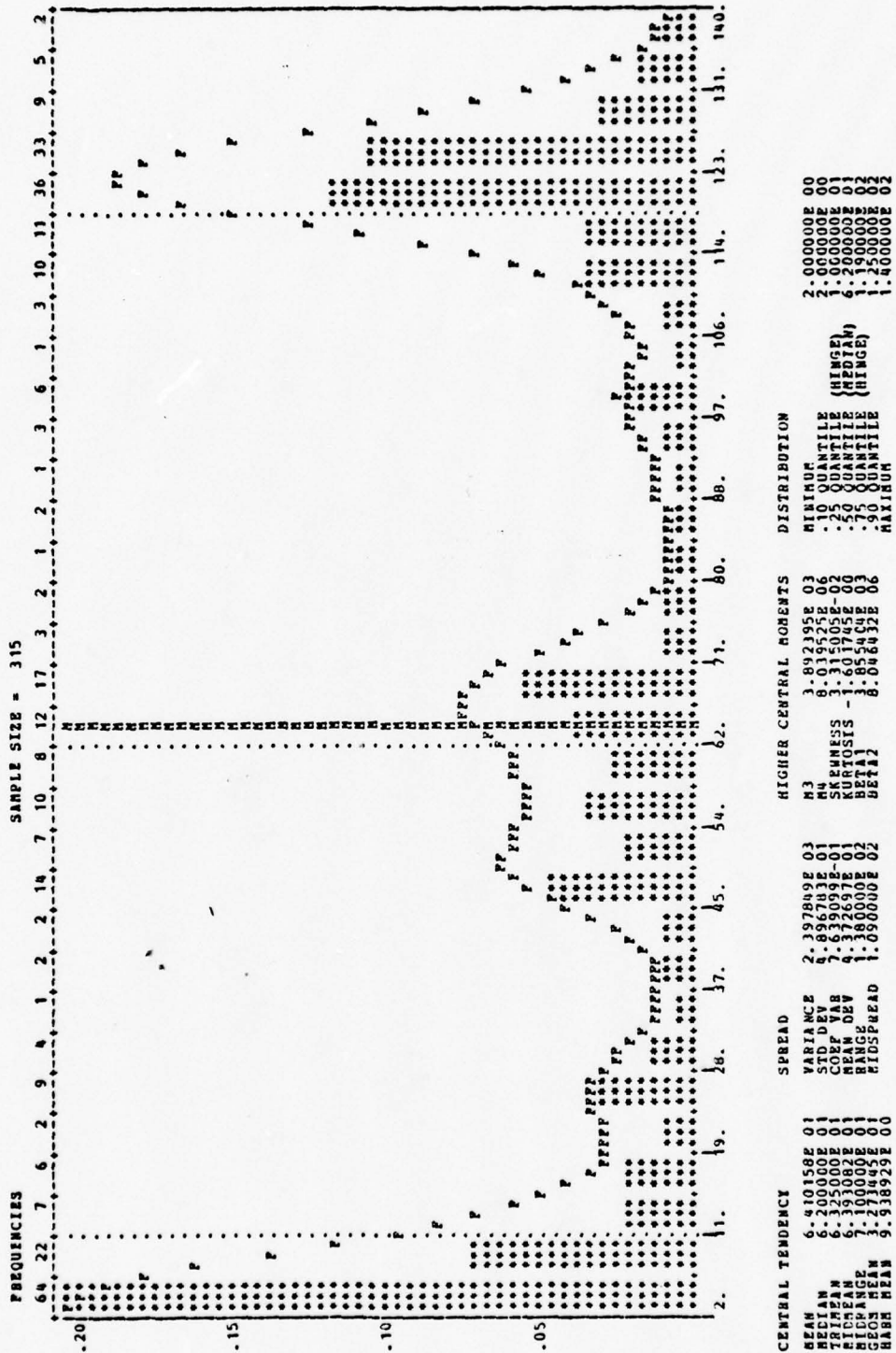


Figure 5 - TELEPHONE DATA 2. 2nd SECTION.



Figure 6 - TELEPHONE DATA 2. 3rd SECTION.

III. PLOTTING SUBROUTINES

A. INTRODUCTION

Graphical methods of assessing the validity of a probability model, and of estimating some basic parameters (especially location and scale parameters) have been widely applied over many years. A variety of different prescriptions have been advanced for the plotting positions, that is, for the set of values at which the ordered observations in a sample should be plotted.

Plotting the ordered data is an informal and quick visual method for getting an impression of how the distribution of data looks. Depending on what is known about the distribution of the data there are many probability plotting methods.

The Plotting Subroutines presented here, 'NORMPL' and 'EXPLT', use the method of plotting the order statistics ($X_{(i)}$) versus their expected value ($E[X_{(i)}]$) called 'SCORES'. This method does not require knowledge of the Cumulative Distribution Function $F(x)$ (except for its assumed continuity). Therefore it may be very attractive for those who know nothing about the data to be analyzed. In addition the indication of the graph may lead the analyst to further useful results.

Using the Plotting Subroutines 'NORMPL' and 'EXFLT' one may analyze the resulting graphs obtained to get an idea of the distribution of the data.

B. DESCRIPTION OF THE PLOTTING METHOD

The idea behind this method, as above stated, is to plot $X_{(i)}$ versus $E[X_{(i)}]$.

It is used where data X_1, X_2, \dots, X_n arise as independent observations of a continuous Random Variable X with a distribution function which is believed to have some particular form $F(x)$. Then the ordered data $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ are plotted against $E[X_{(i)}]$.

If the model for $F(x)$ used to get $E[X_{(i)}]$ is true then the resulting plot is a linear (regression) relationship and should plot, within limits of sample variations, as a straight line. Barnett [2] gives the value of $E[X_{(i)}]$ as a linear relation of the plotting position $x_{(i)}$. That is:

$$E[X_{(i)}] = m + sx_{(i)}$$

here m , and s , are the location and scale parameters. The Mean and Variance of X are not necessary. As we see the above relation uniquely defines the plotting position as :

$$x_{(i)} = \frac{E[X_{(i)} - m]}{s}$$

$$= E\left[\frac{X_{(i)} - m}{s}\right] .$$

Therefore, if the model is correct the linearity will probably be evident. In addition if the resulting plot not being linear leads us to reject the model, we may get an indication of the type of departure present and thus an indication of what action to take or what alternative model to use. It should be noted that the linearity of the graph suggests informally that the model is true, but not to accept the model. Additional formal tests must be done in order to accept the model.

C. SUBROUTINE NORMPL

1. Description

Subroutine 'NORMPL', is used to test a sample of independent observations of a continuous Random Variable X having a distribution function which is believed to have the Normal form.

'NORMPL' sorts the data into increasing order to create a set of Order Statistics. Then the Order Statistics are plotted versus either Normal Scores (if $n \leq 50$, n = number of data-points) or inverse of the Standard Normal Cumulative Distribution Function ($F^{-1}(y)$), (if $n > 50$).

Evaluation of Normal Scores is based on computing the Expected Value of the i^{th} Order Statistic ($E[X_{(i)}]$).

This evaluation follows:

Let

$g(x_{(i)})$ be the density function of the i^{th} Order Statistic of a sample X_1, X_2, \dots, X_n from a population having the Standard Normal Distribution F . Then $g(x_{(i)})$ is given by :

$$g(x_{(i)}) = \frac{n!}{(i-1)! (n-i)!} F_X(x_{(i)})^{i-1} (1-F_X(x_{(i)}))^{n-i} f_X(x_{(i)})$$

$$= \frac{n!}{(i-1)! (n-i)!} P^{i-1} Q^{n-i} Z ,$$

where:

$$Z = f_X(x_{(i)}) = \frac{1}{(2\pi)^{1/2}} e^{-x_{(i)}^2/2} ,$$

$$P = F_X(x_{(i)}) = \int_{-\infty}^{x_{(i)}} \frac{1}{(2\pi)^{1/2}} e^{-u^2/2} du ,$$

$$Q = 1-P .$$

Now

$$E[X_{(i)}] = \int_{-\infty}^{\infty} x g(x) dx$$

$$(1) \quad = \frac{n!}{(i-1)! (n-i)!} \int_{-\infty}^{\infty} x Z P^{i-1} Q^{n-i} dx .$$

The value of expression (1) has been evaluated and is given in [19]. This table gives the values of Normal Scores from 1 to $n/2$ (for $n \leq 50$). The values from $(n/2)+1$ to n are evaluated using the symmetric expression :

$$E[X_{(i)}] = - E[X_{(n-i+1)}] .$$

Also

$$E[X_{(k+1)}] = 0, \quad n = 2k+1 ,$$

'NORMPL' uses the above Normal scores if $n \leq 50$. For $n > 50$ 'NORMPL' computes its own Normal Scores using the program function 'INVNRN', which gives the value x of the inverse Standard Normal distribution ($x = F^{-1}(y)$, $0 \leq y \leq 1$). Justification of this is the fact that if the data are really normally distributed then $E[X_{(i)}] = F(i/(n+1))$. Therefore 'NORMPL' can accommodate any sample size.

In addition to the plot given by 'NORMPL', the Wilk-Shapiro test for normality can be evaluated by the same Subroutine. A detailed description of that test is given in [22]. However a summary of that test is presented here in order to help us to describe how the program works. The Wilk-Shapiro test is based on the statistic:

$$W = \frac{b^2}{s^2} ,$$

where:

$$b = \sum_{i=1}^k a_{n-i+1} (X_{(n-i+1)} - X_{(i)}) , \quad k=n/2, \quad n \text{ even}$$

$$b = a_n (X_{(n)} - X_{(1)}) + a_{n-1} (X_{(n-1)} - X_{(2)}) + \dots$$

$$+ a_{k+2} (X_{(k+2)} - X_{(k)}), \quad k = (n-1)/2, \quad n \text{ odd}$$

$$S^2 = \sum_{i=1}^n (X_i - \bar{X})^2$$

The coefficients (a_i) are defined by:

$$a_i = \frac{\sum_{j=1}^n m_j u_{ij}}{C}, \quad i = 1, 2, \dots, n$$

where:

$$m_j = E[X_{(j)}], \quad (\text{That is the 'NORMAL SCORES'})$$

$$u_{ij} = \text{Cov}[X_{(i)}, X_{(j)}], \quad i, j = 1, 2, \dots, n$$

$$C = m' V^{-1} V^{-1} m$$

where:

$$m' = (m_1, m_2, \dots, m_n)$$

$$V = (u_{i,j}), \quad \text{That is the } n \times n \text{ Covariance Matrix.}$$

There are some approximations associated with the evaluation of these coefficients. However for $n \leq 50$ the (a_i) are tabulated in the above-mentioned paper and the program uses them by FORTRAN DATA initialization statements.

For $n > 50$ the (a_i) are computed by the program using the following approximation method:

$$a_i = 2m_i/C, \quad i = 1, 2, \dots, n-1,$$

where C is given by the following Least-Square equation as a function of n :

$$C^2 = -2.722 + 4.083n.$$

Also, for $n=1$

$$a_1^2 = \frac{\Gamma(n/2 + .5)}{2^{1/2} \Gamma(n/2 + 1)}.$$

Note that $a_i = a_{n-i+1}$.

Some properties for the W Statistic can be given here, which have been taken from the above mentioned paper:

- (a). W is scale and origin invariant.
- (b). W has a distribution which depends only on the sample size n , for samples from a Normal Distribution.
- (c). W is statistically independent of S^2 and of \bar{X} for samples from a Normal Distribution.
- (d). $E[W^r] = E[b^{2r}] / E[S^{2r}]$ for any r .
- (e). The maximum value of W is 1.
- (f). The minimum value of W is $na_1^2 / (n-1)$.
- (g). The half and first moments of W are given by:

$$E[W^{1/2}] = \frac{R^2 \Gamma(n/2 - .5)}{C \Gamma(n/2) 2^{1/2}},$$

$$E[W] = \frac{R^2 (R^2 + 1)}{C^2 (n-1)}.$$

where:

$$R = \frac{2}{n} \sum_{i=1}^n V_i^{-1}$$

(h). For $n=3$ the density of W is given as:

$$f_W(w) = \frac{3}{n} (1-w)^{-1/2} w^{-1/2}, \quad .75 \leq w \leq 1.$$

2. Program Structure

'NORMPL' is a FORTRAN-callable Subroutine with each call returning (optionally) either a plot for a given set of data or a value for the W Statistic or both.

The program is divided into three parts:

The first part is the control program of 'NORMPL'. In that part the 'Normal Scores' for $n = 2(1)25$ and $n = 26(2)50$ are stored along with the Subroutine by DATA statements; on the other hand, the Normal Scores for $n > 50$ are computed by calling the program's function 'INVNRM'. In this part the user specifies just a plot or just the value of W or both. The Library-Subroutine 'PXSORT' is used by the program.

The second part of the program consists of the Subroutine 'PLOT' which accepts any data set to be tested for Normality. 'PLOT' itself scales the data according to its range and plots the scaled data along 110 equal spaced positions of the X-axis. No plot is given if the data has a constant value.

The third part of the program is the 'WILK' Subroutine. In this Subroutine the coefficients (a_i) (for $n \leq 50$) of the W Statistic are stored using FORTRAN

DATA initialization statements. For $n > 50$ the coefficients (a_i) are computed by the Subroutine itself. This Subroutine also calls the 'INVNRN' Function to get the 'Normal Scores' to be used for the computation of the (a_i) 's when $n > 50$.

A complete description of how 'NORMPL' operates is given in the Subroutine. However a summary is obtained by typing on the terminal DESCRIBE NORMPL. When the user types the command DESCRIBE NCRMPL under the CMS environment the following response is printed on the terminal:

SUBROUTINE NCRMPL

'NORMPL' takes a set of data, sorts it into ascending order and uses the created Order Statistics for:

1. Plotting $X_{(i)}$ versus either 'Normal Scores' (if $n \leq 50$, $n =$ Sample size) or Inverse of the unit Normal Cumulative Distribution (that is:

$X_{(i)}$ versus Normal scores or $F^{-1}(i/(n+1))$,)

to see if there is a linear fit.

2. Computing the value of the W test Statistic (Wilk-Shapiro test for normality).

It is called by:

```
CALL NCRMPL ( X, SCCRES, N, K )
```

ARGUMENTS

X Is the array containing the data

SCORES Is a work array of dimension N

N Is the number of data values

K User's option

For: K = 1 A plot only ,

K = 2 The W-value is given ,
K = 3 Both of the above are given.

More information is given in the subroutine.

3. Interpreting The Output

If data are really Normally distributed then a straight line is expected from the output. This is an indication that gives the user a first feeling of the distribution of data. But if the plot is not linear then the user has to reject the normality assumption. In addition the shape of the plot may lead the user to make an alternative decision of what model he may use. For example:

(a). Shape of the form of Figure 7a suggests that data are skewed to one side (specifically positive values). In that case an Exponential model could be an appropriate one. The histogram of the data is given in Figure 7b.

(b). Shape of the form of Figure 8a can be interpreted as data not as dispersed as the Normal Distribution. This set of data may have a Uniform or a Triangular Distribution, and could, for instance, arise if measuring tolerances of components hand-picked to lie within tolerances. The associated histogram is given in Figure 8b.

(c). Shape of the form of Figure 9a indicates much more dispersed data than Normal, but the data is symmetric, probably coming from a Double Exponential or Cauchy Distribution. The associated histogram is given in Figure 9b.

It should be noted here that a straight line does not always arise from Normal data. Figure 10a for example

gives an indication that data are Normal (Straight line). But these data have been generated by computer from a Symmetric Triangular Distribution in the range (0 to 2). On the other hand figure 11a plots a data set of size 50 again from a Triangular (0 to 2) distribution. But in this case we observe that the graph is not a straight line. These results demonstrate the informality of that Probability Plotting Method and in these cases the user has to continue testing the data by other formal tests. Nevertheless, since this method is primarily an initial screening device, ease of application is important. Figure 11b gives the associated histogram.

NUMBER OF ORDERED PAIRS = 200

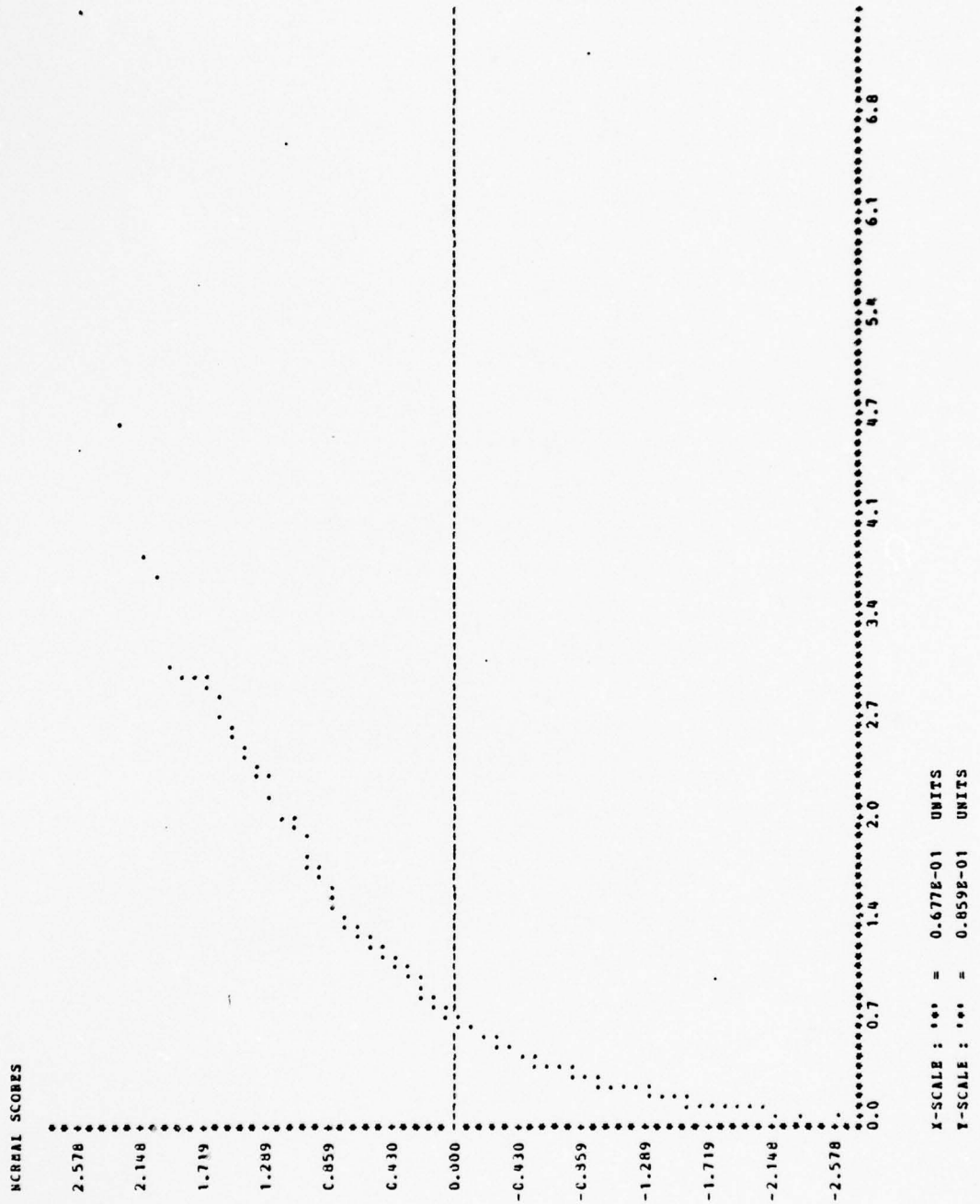


Figure 7a - GENERATED DATA FROM AN EXPONENTIAL DISTRIBUTION

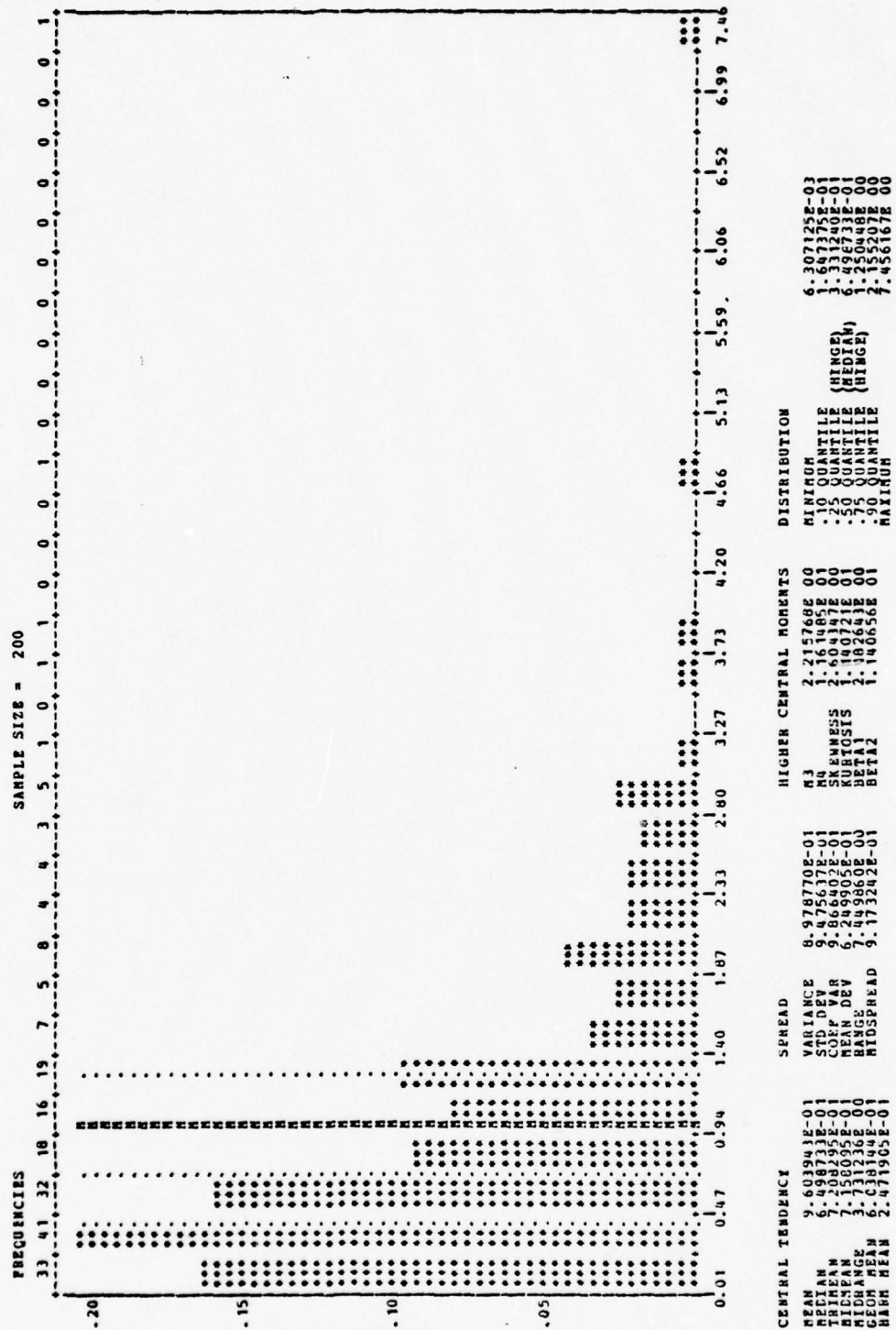


Figure 7b - HISTOGRAM OF THE DATA OF FIGURE 7a

NUMBER OF ORDERED PAIRS = 200

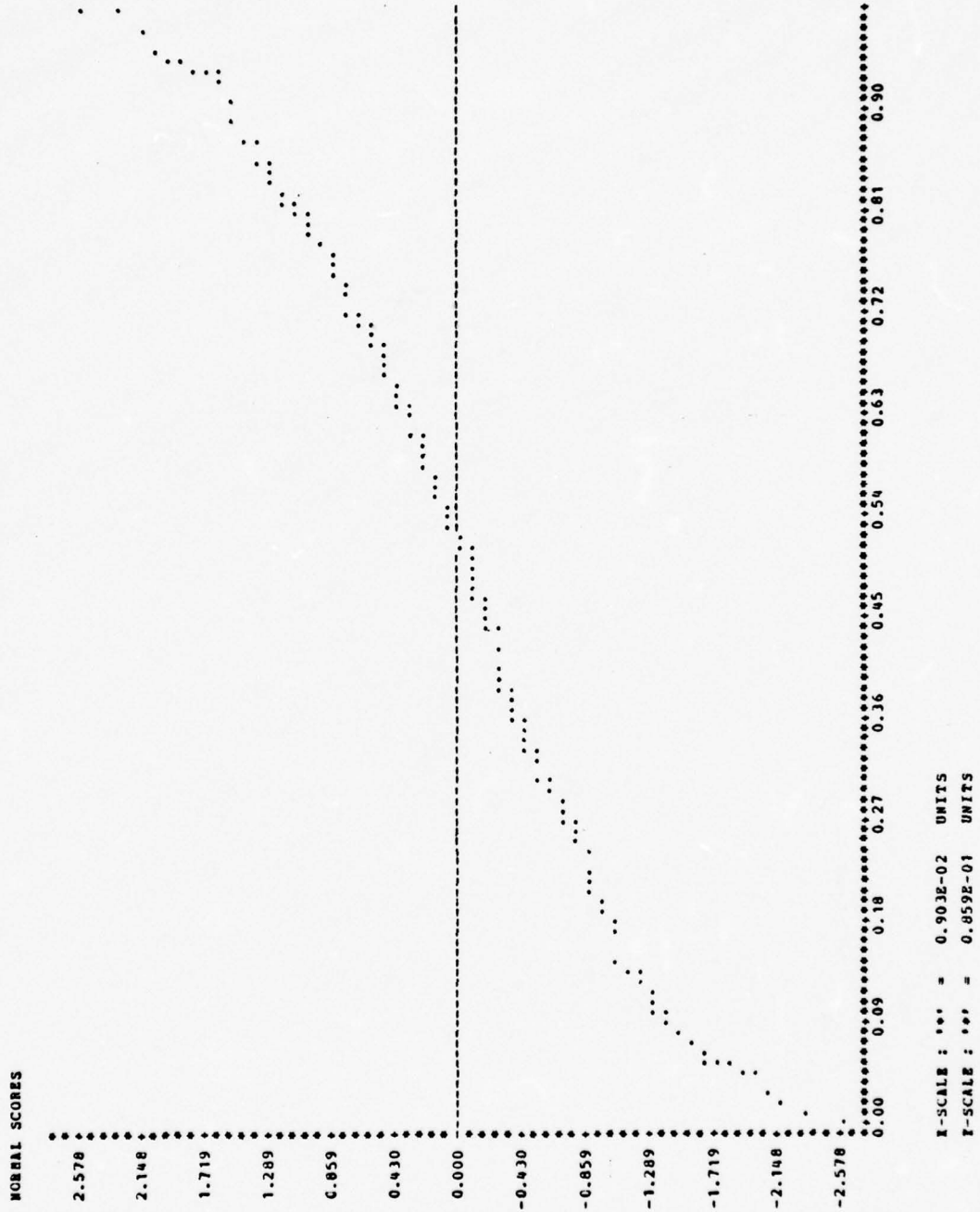


Figure 8a - GENERATED DATA FROM A UNIFORM (0, 1) DISTRIBUTION

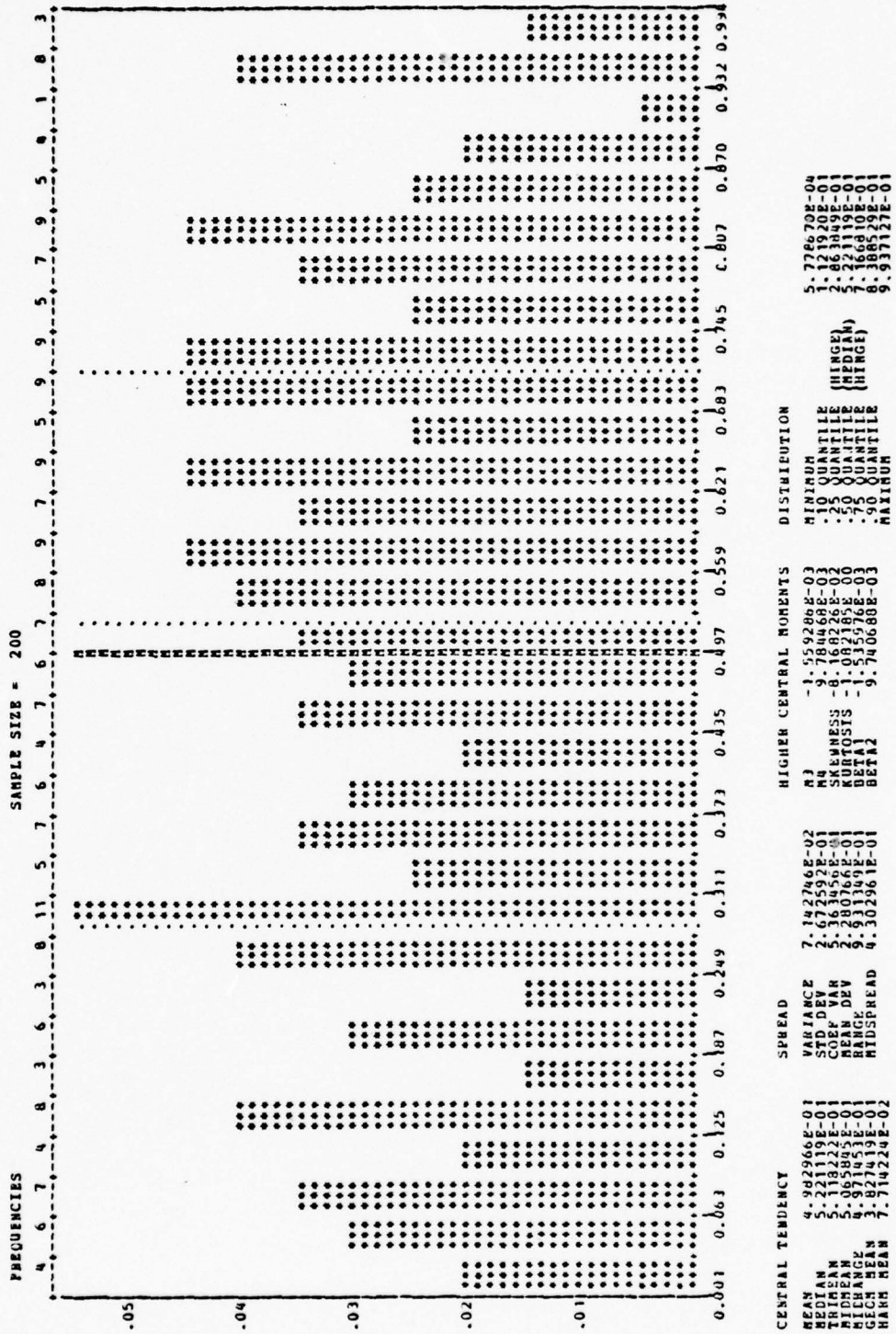


Figure 8b - HISTOGRAM OF THE DATA OF FIGURE 8a

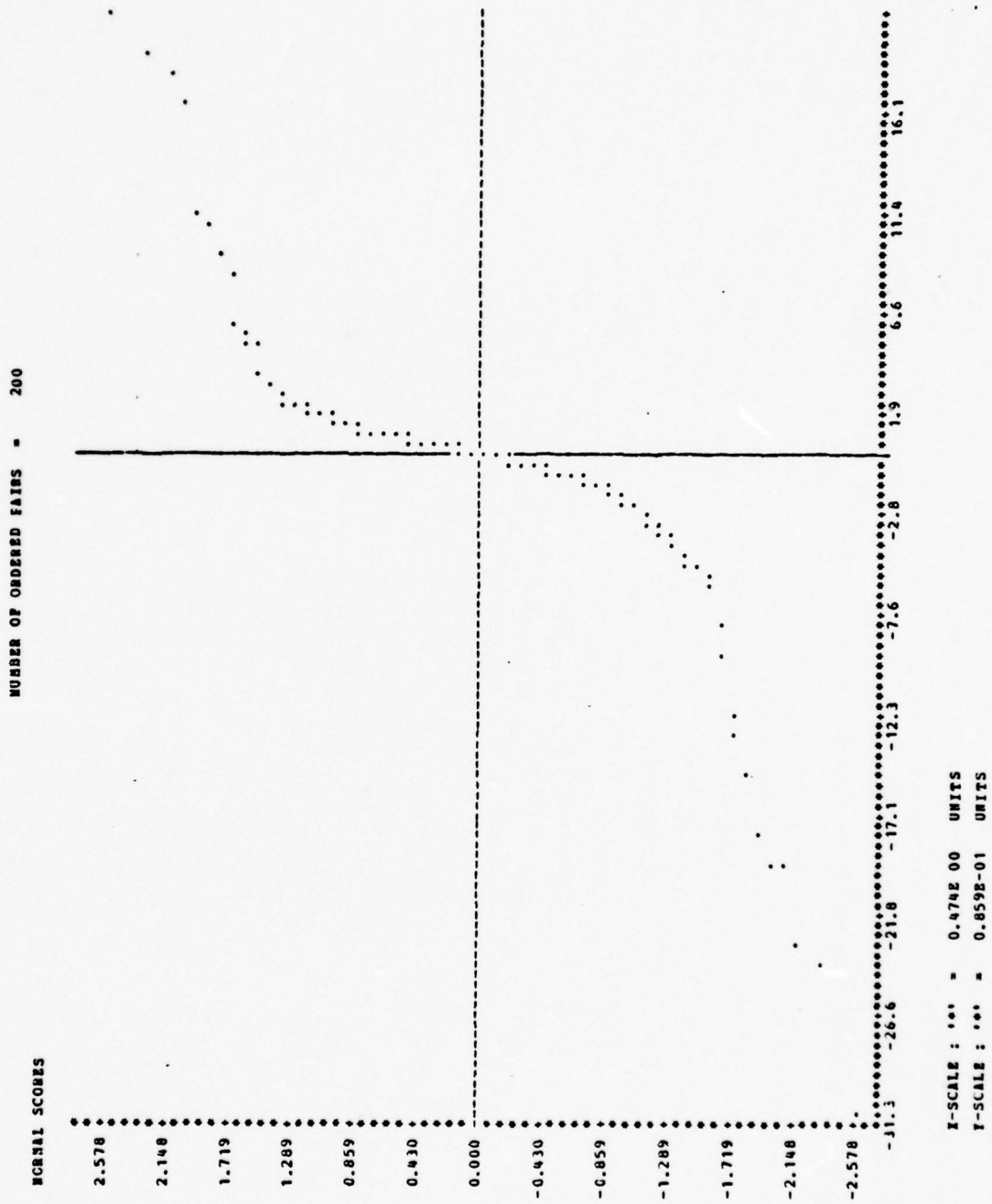


Figure 9a - GENERATED DATA FROM A CAUCHEY DISTRIBUTION

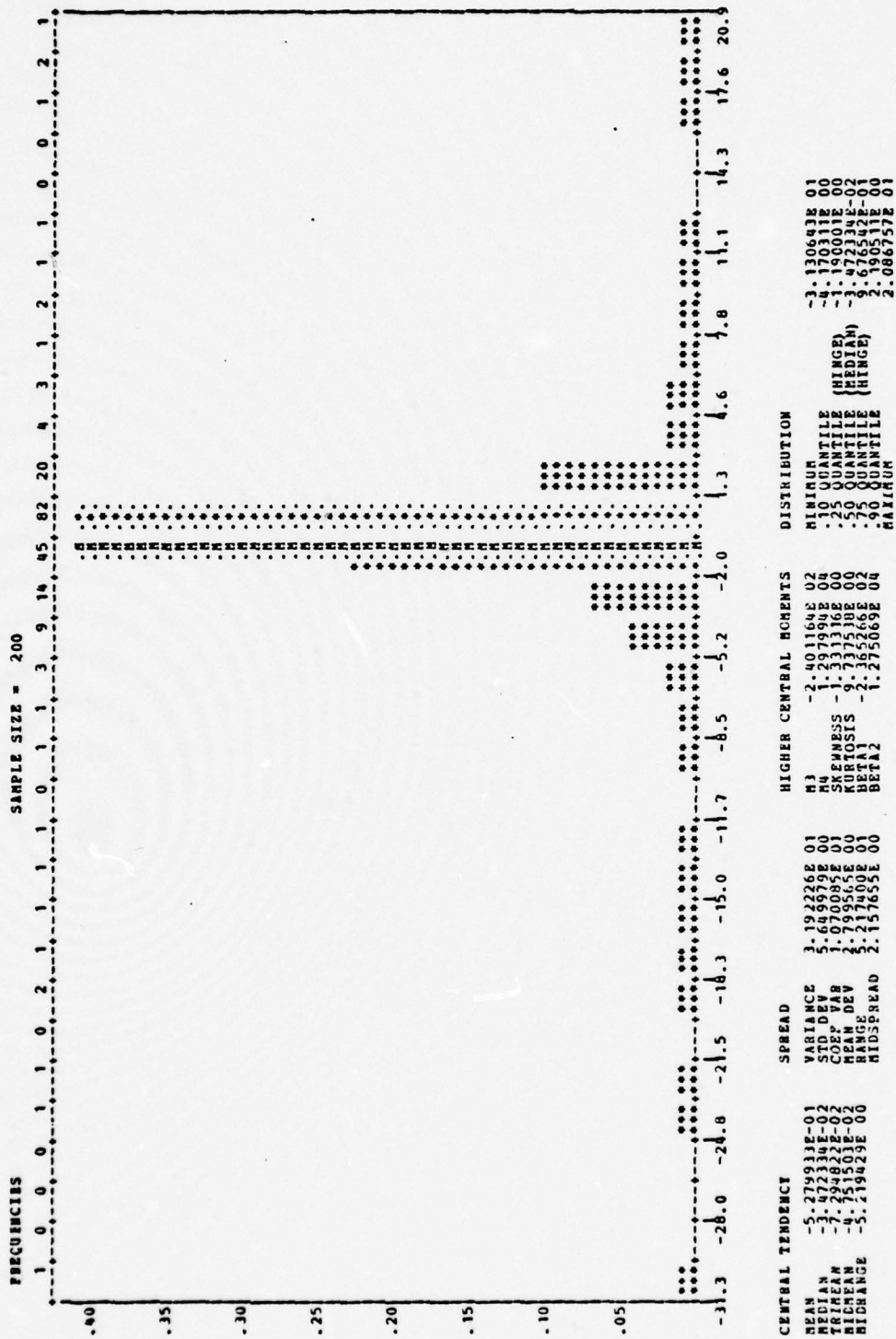


Figure 9b - HISTOGRAM OF THE DATA OF FIGURE 9a

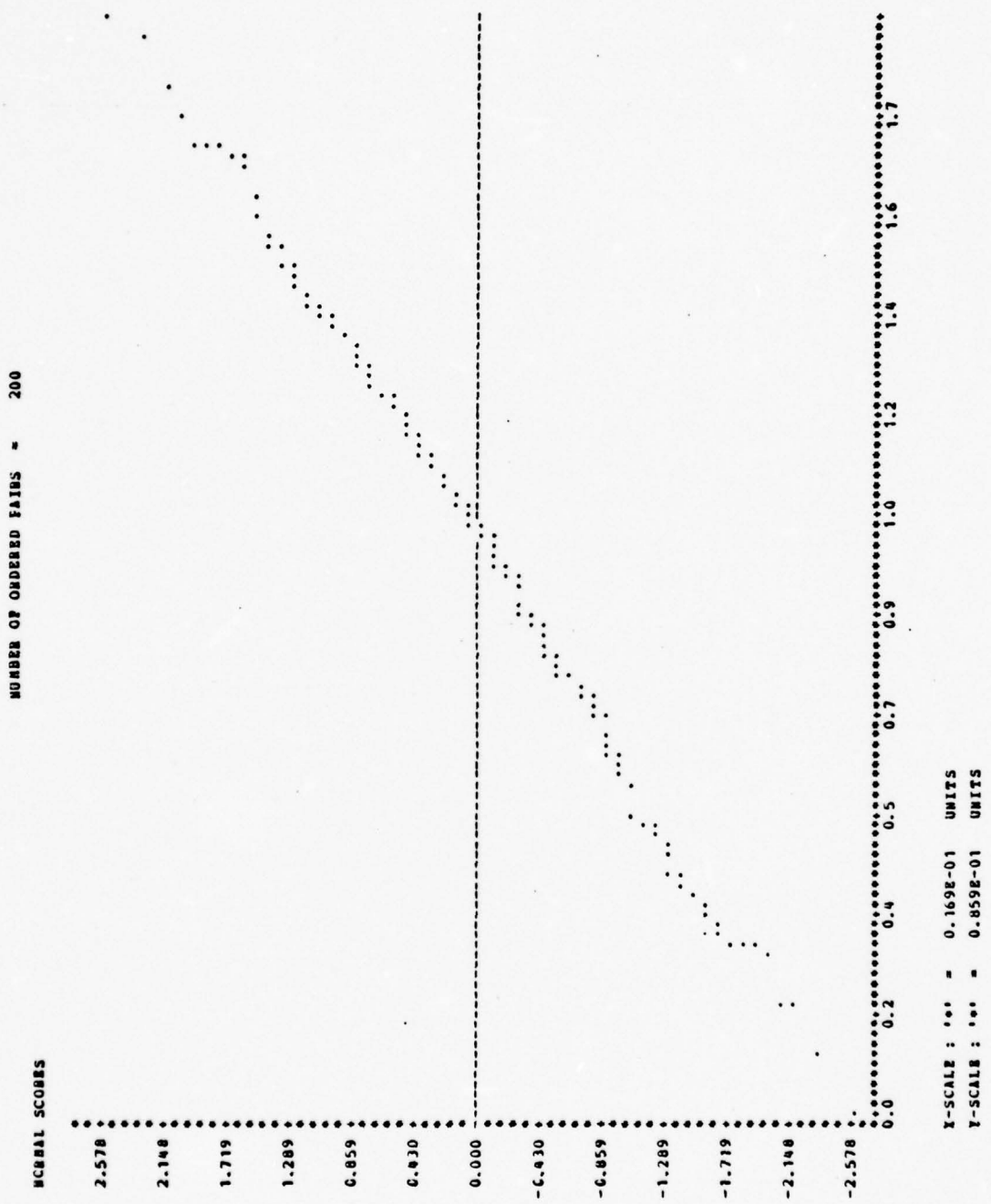


Figure 10a - GENERATED DATA FROM A TRIANGULAR (0, 1) DISTRIBUTION. SAMPLE SIZE 200

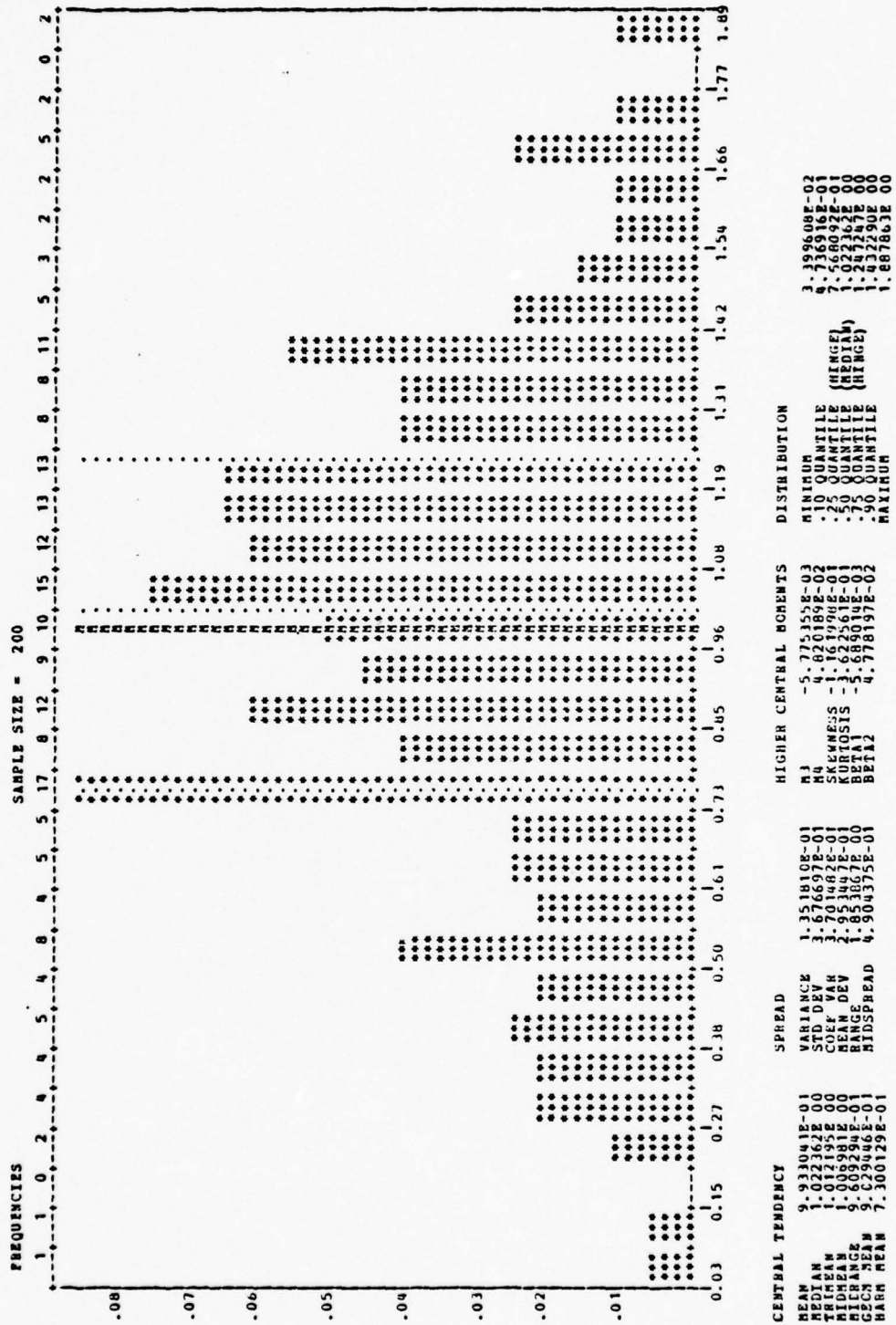


Figure 10b - HISTOGRAM OF THE DATA OF FIGURE 10a

NUMBER OF ORDERED PAIRS = 50

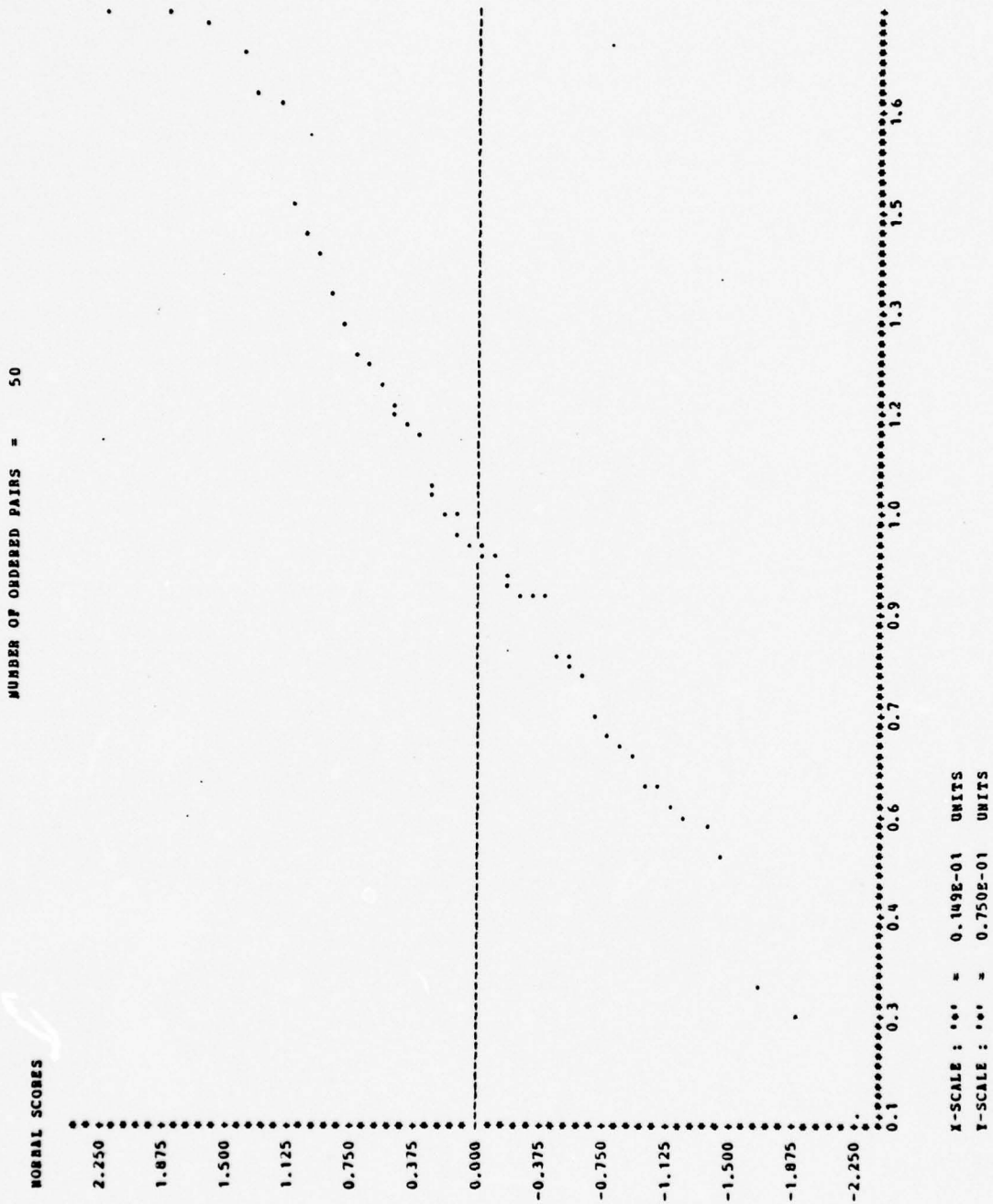


Figure 11a - GENERATED DATA FROM A TRIANGULAR (C, 1) DISTRIBUTION. SAMPLE SIZE 50

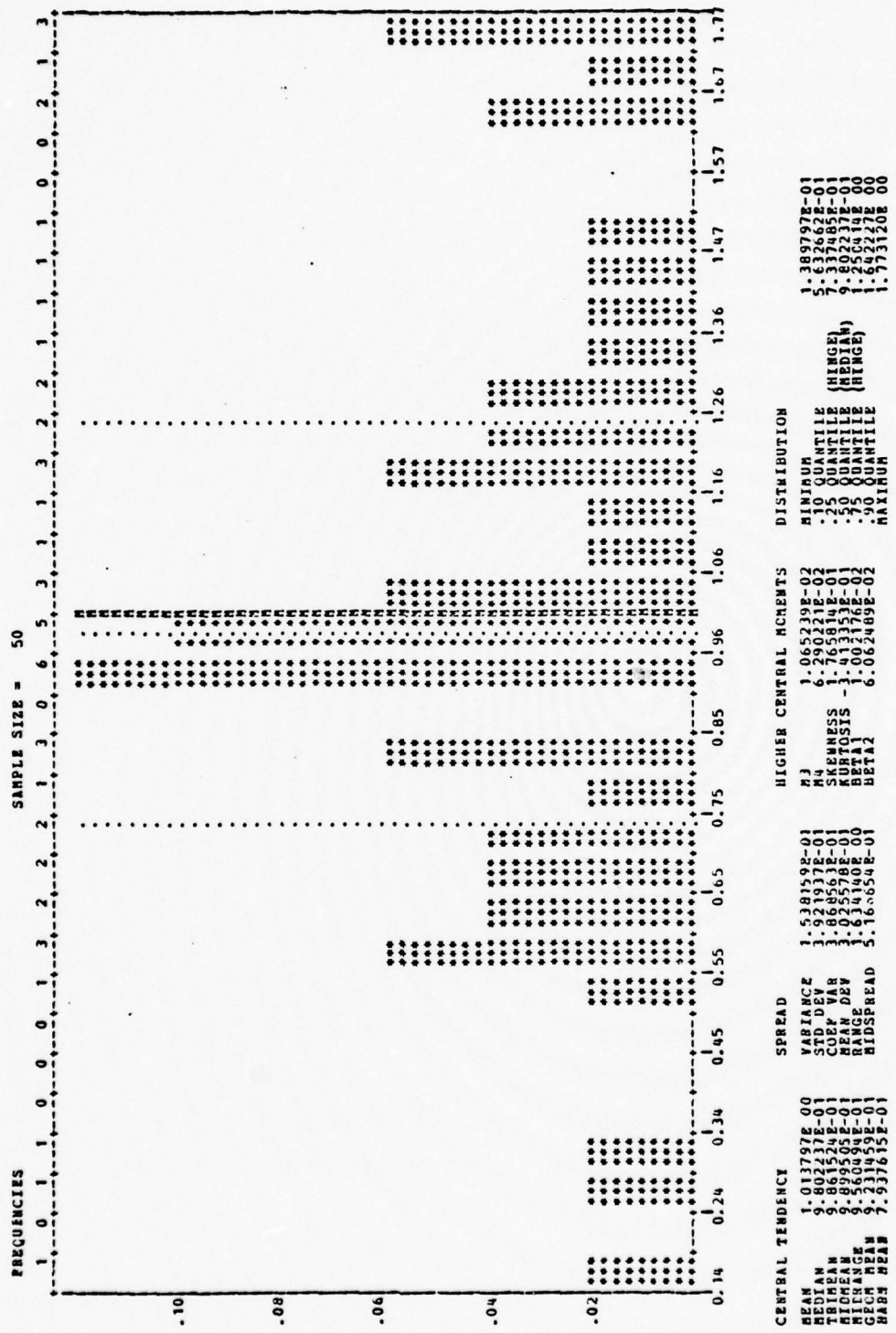


Figure 11b - HISTOGRAM OF THE DATA OF FIGURE 11a

4. Using 'NORMPL' with Generated Data from Various Distributions.

The Subroutine 'NORMEL' has been used here to test for Normality some data sets which were generated by the computer from various distributions. The plots of these data sets illustrate the advantages and disadvantages of the plotting Subroutine 'NORMPL'.

Observing Figures 7a-b, 8a-b, 9a-b and 12a-b we may note the following:

(a). Figure 12a shows a plot of a sample of 50 data points from a Normal Distribution ($N(1000,1)$). As it was expected a very straight line is fitted. In addition to that plot, the Subroutine 'NORMPL' uses the option to evaluate the 'WILK-SHAPIRO' test and the W-value is computed and printed on the same figure. Comparing this value with the percentage points of the W-test which are given in [22], for sample size $n=50$ we see that we may accept the normality assumption with a significance level = .02. Figure 12b gives the associated histogram.

(b). Figure 7a plots a sample of 200 data points of the Unit Exponential distribution. Here the nonlinear graph is obvious (as is expected) and suggests to reject the normality assumption of the data without doing any further formal test.

(c). Figure 8a gives a plot of a sample of size 200 from Uniform (0, 1) generated data. The shape of that plot indicates data not as dispersed as the normal, therefore the user is told to reject normality, getting at the same time a suggestion that data may be Uniformly

distributed (because of that particular shape of the graph).

(d). Figure 9a suggests strongly departure from a Normality assumption. Indeed this set of data have been generated from a Cauchy Distribution and its shape really indicates a symmetric distribution.

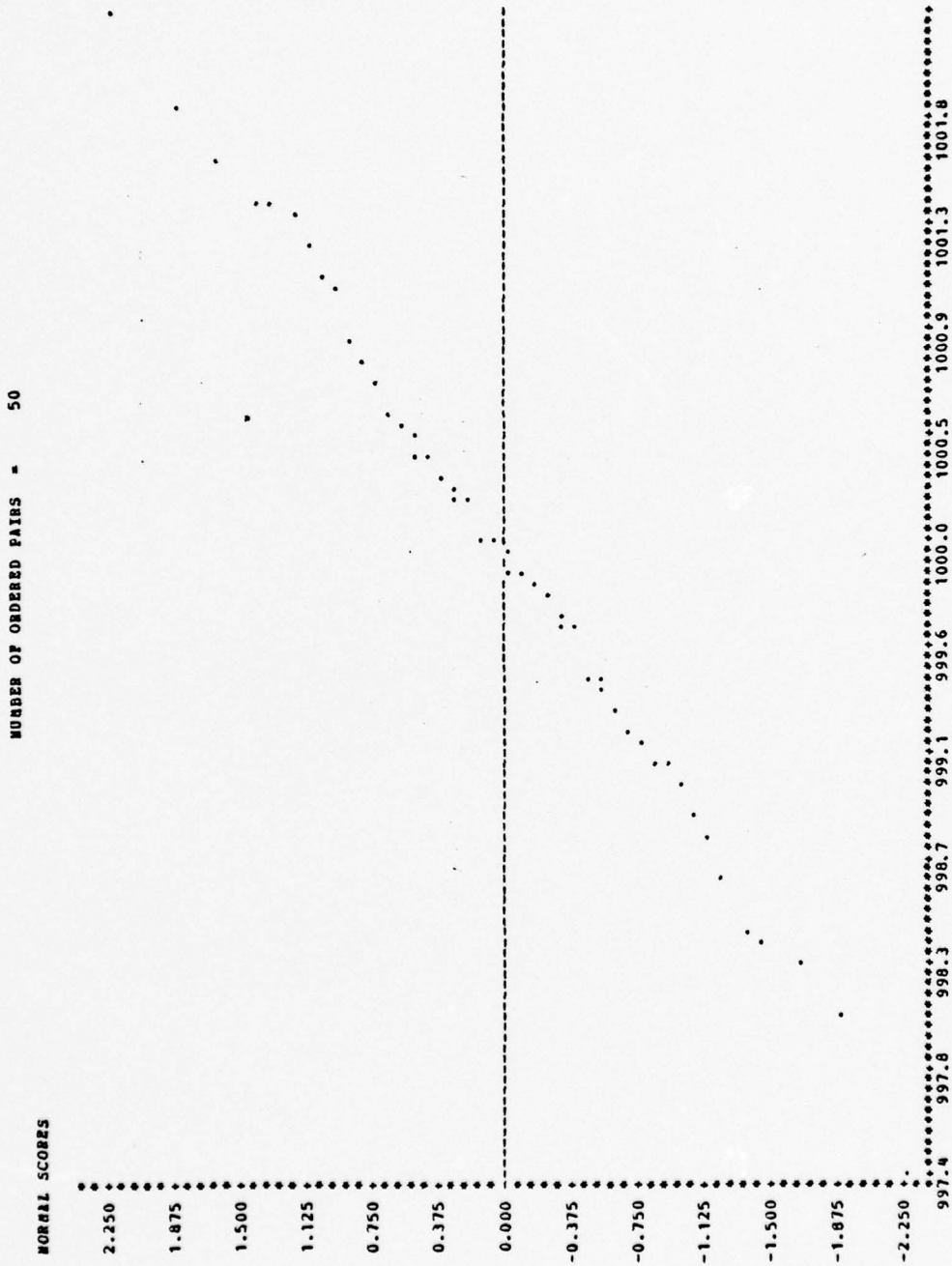


Figure 12a - GENERATED DATA FROM A NCRMAL (1000, 1) DISTRIBUTION

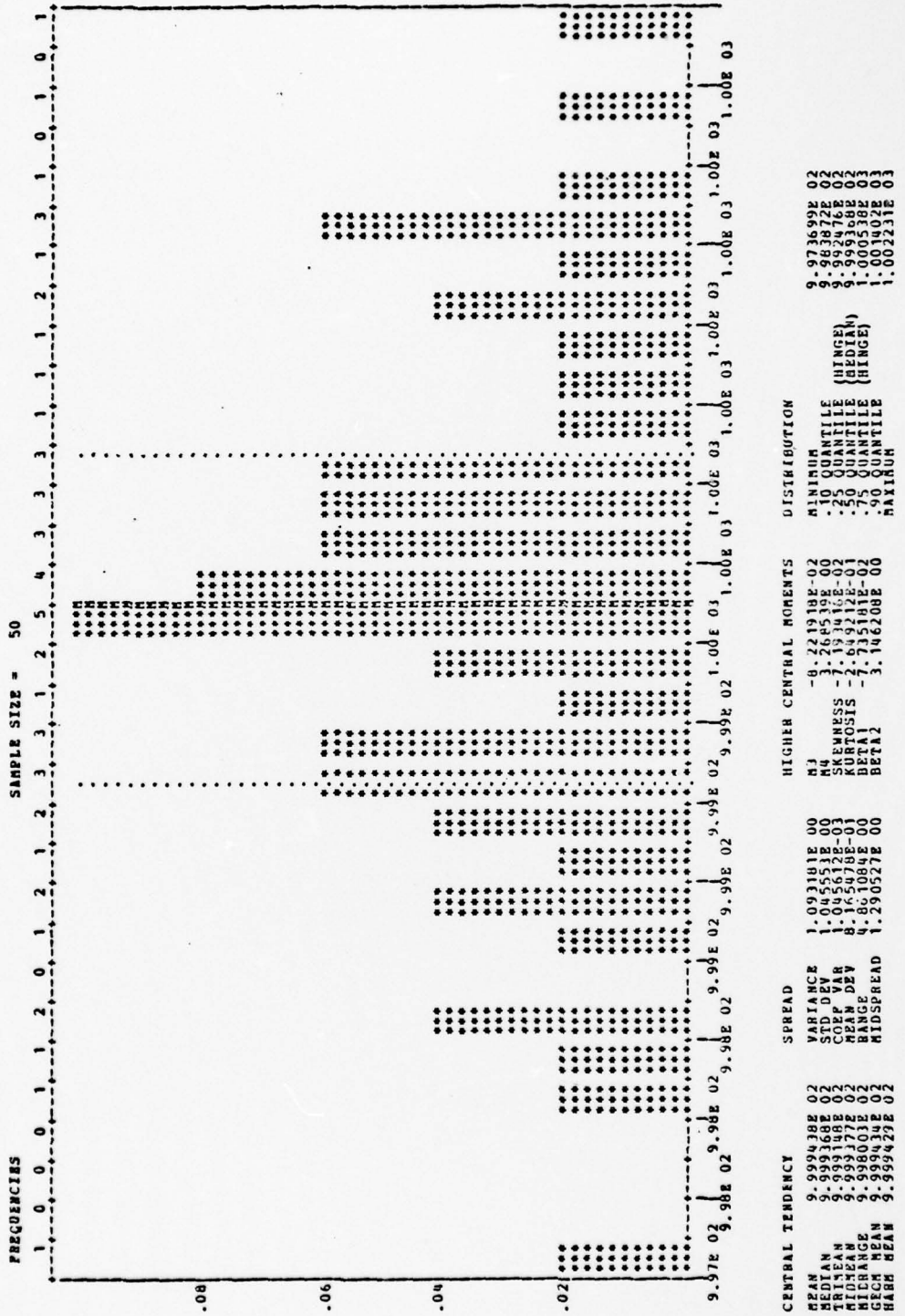


Figure 12b - HISTOGRAM OF THE DATA OF FIGURE 12a

5. Using 'NORMPL' With Cost Overruns Data

'NORMPL' was used with the cost overruns data [6], for the year 1950 to see if there is a linear fit. Looking at Figure 13 we see that data may be Normally distributed since the plot is very close to linear. Of course some data points deviate from the straight line, but these points may be considered as outliers.

NUMBER OF ORDERED PAIRS = 22

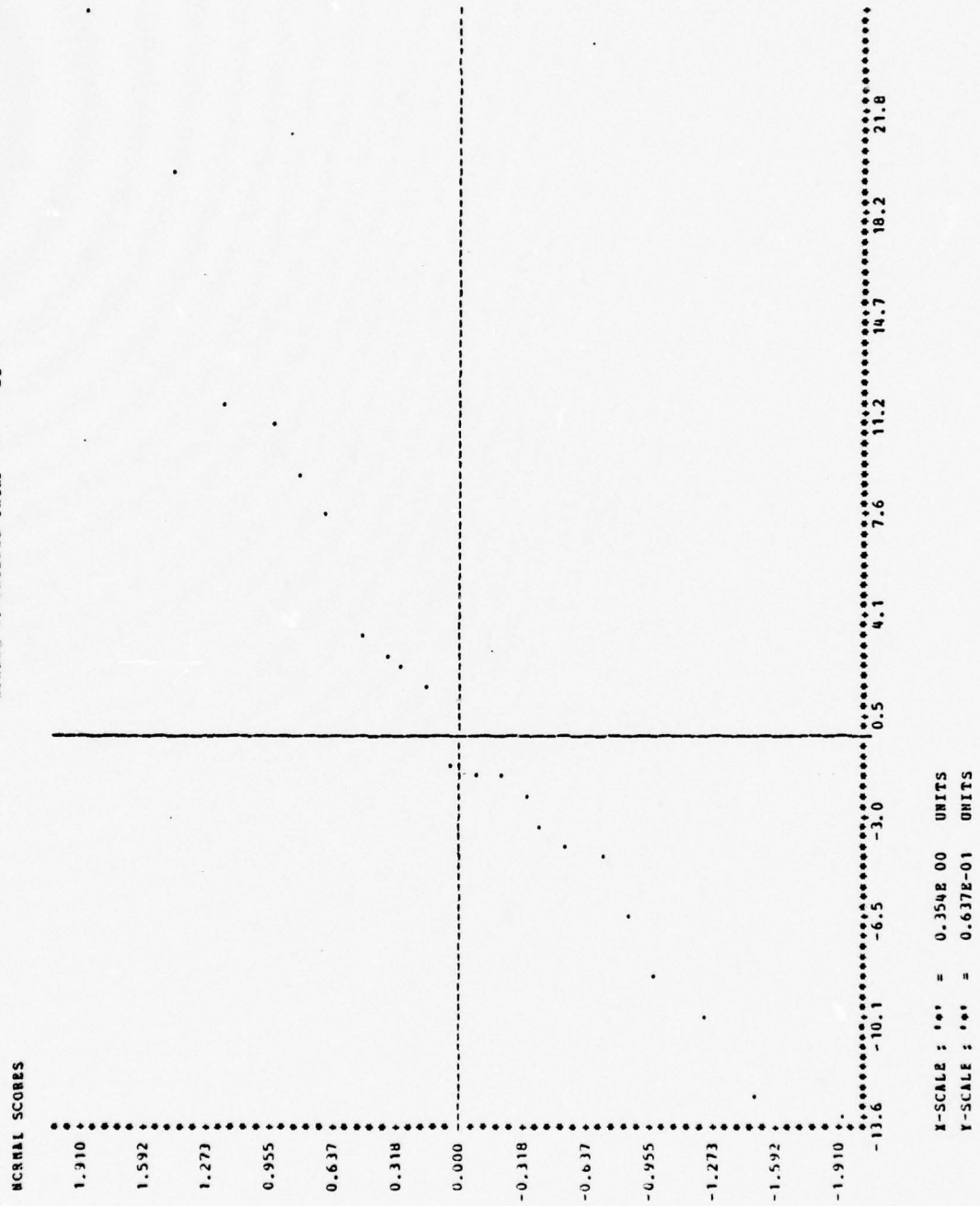


Figure 13 - COST OVERRUNS DATA

D. SUBROUTINE EXPLT

1. Description

Subroutine 'EXPLT' is intended to be used for testing a sample of independent observations of a continuous random variable X with a distribution function which is believed to be Exponential, by plotting each of the Order Statistics $X_{(i)}$ versus its Expected Value, and by estimating the parameters γ_1 and γ_2 of the observations (γ_1 and γ_2 introduced later).

This Subroutine sorts the data into increasing order, obtaining the Order Statistics. The Order Statistics are then plotted versus Exponential Scores. These scores are evaluated by the Subroutine. The Exponential Scores are the expected value of the Order Statistics ($E[X_{(i)}]$). They are more easily derived and computed than the Normal Scores. That is:

Let X be a random variable Exponentially distributed with parameter $\lambda = 1$
Then it is known:

$$f_X(x) = \lambda e^{-\lambda x}, \quad \lambda, x > 0,$$

$$F_X(x) = 1 - e^{-\lambda x}, \quad \lambda, x > 0.$$

The Density of the i^{th} Order Statistic is given by:

$$f_{X_{(i)}}(x_{(i)}) = n \binom{n-1}{i-1} F_X(x_{(i)})^{i-1} (1-F_X(x_{(i)}))^{n-i} f_X(x_{(i)})$$

Thus for the 1^{st} Order Statistic we have:

$$\begin{aligned} f_{X_{(1)}}(x_{(1)}) &= n(1 - (1 - e^{-\lambda x_{(1)}}))^{n-1} \lambda e^{-\lambda x_{(1)}} \\ &= n \lambda e^{-n \lambda x_{(1)}} \end{aligned}$$

Therefore $X_{(1)}$ is exponentially distributed with parameter $\lambda' = n \lambda$.

And for $\lambda = 1$ we have:

$$E[X_{(1)}] = \frac{1}{n \lambda} = \frac{1}{n}$$

It can be proved (see Feller [7]) that the n random variables $X_{(1)}$, $X_{(2)} - X_{(1)}$, $X_{(3)} - X_{(2)}$, ..., $X_{(n)} - X_{(n-1)}$ from an Exponential random variable X with density function:

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0,$$

are independent and the density of the random variable

$Y = X_{(k)} - X_{(k-1)}$ is given by:

$$f_Y(y) = (n-k+1) \lambda e^{-\lambda(n-k+1)y}, \quad y > 0.$$

That is, Y is an Exponential random variable with parameter $(n-k+1) \lambda$.

Therefore $E[Y]$ is given by:

$$E[X_{(k)} - X_{(k-1)}] = \frac{1}{\lambda(n-k+1)} = \frac{1}{n-k+1}, \quad \text{for } k=1.$$

This fact is used to derive $E[X_{(i)}]$, $i = 2, 3, \dots, n$.

That is:

$$\begin{aligned} E[X_{(i)}] &= E[(X_{(i)} - X_{(i-1)}) + (X_{(i-1)} - X_{(i-2)}) + \dots + \\ &\quad (X_{(2)} - X_{(1)}) + X_{(1)}] \\ &= E[X_{(1)}] + E[X_{(2)} - X_{(1)}] + \dots + \\ &\quad E[X_{(i-1)} - X_{(i-2)}] + E[X_{(i)} - X_{(i-1)}] \\ &= \frac{1}{n} + \frac{1}{n-2+1} + \frac{1}{n-3+1} + \dots + \frac{1}{n-i+1} \\ &= \sum_{k=1}^i \frac{1}{n-k+1}, \quad i = 2, \dots, n. \end{aligned}$$

'EXPLT' derives the values of $E[X_{(i)}]$ using the above formula and then it uses its Subroutine 'EPLCT' to plot the Order Statistics versus their expected value.

In addition to the plot, which is an informal and quick test for exponentiality the estimates of the statistics γ_1 and γ_2 provide another informal test since the values of γ_1 and γ_2 have a constant value for any Exponential distribution:

By definition γ_1 and γ_2 are given by:

$$\gamma_1 = E\left[-\frac{(X-\mu)^3}{\sigma^3}\right] ,$$

$$\gamma_2 = E\left[-\frac{(X-\mu)^4}{\sigma^4}\right] - 3 .$$

Thus

$$\gamma_1 = -\frac{1}{\sigma^3} E[(X-\mu)^3]$$

$$= -\frac{1}{\sigma^3} E[X^3 - 3X^2\mu + 3X\mu^2 - \mu^3]$$

$$(1) \quad = -\frac{1}{\sigma^3} (E[X^3] - 3\mu E[X^2] + 3\mu^2 E[X] - \mu^3) .$$

Now

$$\begin{aligned} E[X^3] &= \int_0^{\infty} x^3 f_X(x) dx \\ &= \int_0^{\infty} x^3 \lambda e^{-\lambda x} dx \\ &= \frac{\Gamma(4)}{\lambda^3} \int_0^{\infty} \frac{x^{4-1}}{\Gamma(4)} \lambda^4 e^{-\lambda x} dx . \end{aligned}$$

The above integral is that of the Gamma density function with parameter $\alpha = 4$, and since its value equals 1 we have:

$$E[X^3] = \frac{3!}{\lambda^3} = \frac{6}{\lambda^3} .$$

Computing $E[X^2]$ we have:

$$\begin{aligned} E[X^2] &= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx \\ &= \frac{\Gamma(3)}{\lambda^2} \int_0^{\infty} \frac{x^{3-1}}{\Gamma(3)} \lambda^3 e^{-\lambda x} dx \\ &= \frac{2}{\lambda^2} . \end{aligned}$$

Also $E[X] = \frac{1}{\lambda} = \mu$.

Substituting in (1) we have:

$$\begin{aligned} \gamma_1 &= E\left[\frac{(X-\mu)^3}{3}\right] \\ &= \frac{1}{\sigma^3} \left(\frac{6}{\lambda^3} - 3 \frac{1}{\lambda} \frac{2}{\lambda^2} + 3 \frac{1}{\lambda^2} \frac{1}{\lambda} - \frac{1}{\lambda^3} \right) \\ &= \lambda^3 \left(2/\lambda^3 \right) = 2 . \end{aligned}$$

Therefore the value of γ_1 equals 2 for any Exponential distribution.

Also γ_2 is computing as follows:

$$\begin{aligned} \gamma_2 &= E\left[\frac{(X-\mu)^4}{\sigma^4}\right] - 3 \\ &= E[X^4 - 4\mu X^3 + 6\mu^2 X^2 - 4\mu^3 X + \mu^4] / \sigma^4 - 3 \\ (2) \quad &= \{E[X^4] - 4\mu E[X^3] + 6\mu^2 E[X^2] - 4\mu^3 E[X] + \mu^4\} / \sigma^4 - 3. \end{aligned}$$

But

$$\begin{aligned} E[X^4] &= \int_0^{\infty} x^4 \lambda e^{-\lambda x} dx \\ &= \frac{\Gamma(5)}{\lambda^4} \int_0^{\infty} \frac{x^{5-1} \lambda^5}{\Gamma(5)} e^{-\lambda x} dx \\ &= 24 / \lambda^4 . \end{aligned}$$

Thus

$$\begin{aligned} \gamma_2 &= \frac{1}{\sigma^4} \left\{ \frac{24}{\lambda^4} - 4 \frac{1}{\lambda} \frac{6}{\lambda^3} + 6 \frac{1}{\lambda^2} \frac{2}{\lambda^2} - 4 \frac{1}{\lambda^3} \frac{1}{\lambda} + \frac{1}{\lambda^4} \right\} - 3 \\ &= \frac{4}{\lambda} \left(9 / \lambda \right) - 3 = 6 . \end{aligned}$$

Therefore the value of γ_2 equals 6 for any Exponential distribution.

This fact is used by the Subroutine 'EXPLT' to give the user one more informal test for exponentiality. It estimates the values of γ_1 and γ_2 and if $\gamma_1 \neq 2$ and/or $\gamma_2 \neq 6$ then the data may not have an Exponential distribution.

Estimating the values of γ_1 and γ_2 , 'EXPLT' uses unbiased estimator for σ , $E[(X-\mu)^3]$ and $E[(X-\mu)^4]$ using the formulas:

$$\sigma = \left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{(n-1)} \right)^{1/2} ,$$

$$E[(X-\mu)^3] = n \frac{\sum_{i=1}^n (X_i - \bar{X})^3}{(n-1)(n-2)} ,$$

$$E[(X-\mu)^4] = \frac{(n(n-2)+3) \left(\sum_{i=1}^n (X_i - \bar{X})^4 \right)}{(n-1)(n-2)(n-3)} - \frac{3\sigma^4(n-1)(2n-3)}{n(n-2)(n-3)} .$$

2. 'EXFIT' Structure

The Subroutine 'EXFLT' is also a FCRTRAN callable subroutine with each call producing a plot for the given set of data and at the same time calculating and printing the estimated values of the parameters γ_1 and γ_2 of the data.

Basically the program is divided into two parts:

The first part is involved with all required computations calculating the Exponential Scores and the various statistics needed to compute the γ_1 and γ_2 parameters.

The second part is the Subroutine 'EFLOT'. 'EFLOT' scales the data points according to their range and plots the scaled data along 110 equal-spaced positions of the X-axis. No plot is given if the data have constant value or if there exists a data point less than zero.

A complete description of how 'EXFLT' operates is given by the Subroutine and a summary description is given on the terminal by typing the command DESCRIBE EXFLT under the CMS environment. When the user types DESCRIBE EXFLT the following response is printed on the terminal:

```
SUBROUTINE  EXFLT
```

'EXFIT' takes a set of data, sorts it into increasing order and plots the created order statistics versus exponential scores (expected values of order statistics) computed by :

$$E[X_{(i)}] = \sum_{k=1}^i (i/(n-k+1)), \quad i = 1, 2, \dots, n.$$

Also, it computes the estimates of γ_1 and γ_2 parameters.

It is called by :

CALL EXFLT (X, SCORES, N)

ARGUMENTS

X Is the array containing the data

SCORES Is a work array of dimension N

N Is the number of data values

More information is given in the subroutine.

3. Interpreting The Output

If data have an Exponential distribution then the plot will tend to a straight line. But it should be noted that the inverse is not generally true. The linearity of the graph is an indication only and gives the user a first feeling of the distribution of data. If the plot is not linear, however, this suggests that they are not Exponential. In addition the shape of the plot may lead the user to make an alternative model selection.

On the other hand the values of the statistics γ_1 and γ_2 may be used to test informally the distribution of data. These values are an indication only for the test for exponentiality. If the estimate of γ_1 has the value about 2 and the estimate of γ_2 has the value about 6 then it suggests that data may be exponentially distributed. But a departure of these values suggests that data do not have an exponential distribution.

4. Using 'EXPLT' with Data Generated from Various Distributions

The Subroutine 'EXPLT' has been used here, plotting data generated by the computer from various distributions, in order to get a visual sense of the behavior of the plot.

Figure 14a gives the plot of 100 Uniform (0,1) variates. Obviously nonlinearity governs the plot (as we should expect) and thus it suggests departure from the Exponential distribution. Besides the nonlinearity of the plot the estimates of γ_1 and γ_2 (0.1 and -1.15 respectively) are far away from the corresponding values (2 and 6) of exponentially distributed data. Figure 14b gives the associated histogram.

Figure 15a plots variates generated from a Triangular Symmetric distribution (0,2). The nonlinearity and the values of the estimates γ_1 and γ_2 suggest that one reject the exponential assumption.

Figure 16 shows the plot of X^2 generated variates with 10 degrees of freedom. For the same reasons as above the exponential model is rejected.

Figure 17a plots Normally ($N(1000,1)$) distributed generated data and the nonlinearity of the plot is apparent. Besides the nonlinearity, the estimates of γ_1 and γ_2 (.09 and -0.63) suggest not only to reject the Exponentiality of data but in addition lead us to test the data for Normality. Figure 17b gives the associated histogram.

Figure 18 plots exponentially generated variates with parameter $\lambda = 1$. As we see the graph is linear and the user gets the idea that the data possibly are exponential. Besides the linearity here, the values of the estimates of γ_1 and γ_2 (2.06 and 5.4) are close enough to their true values to indicate an Exponential model.

NUMBER OF ORDERED PAIRS = 100

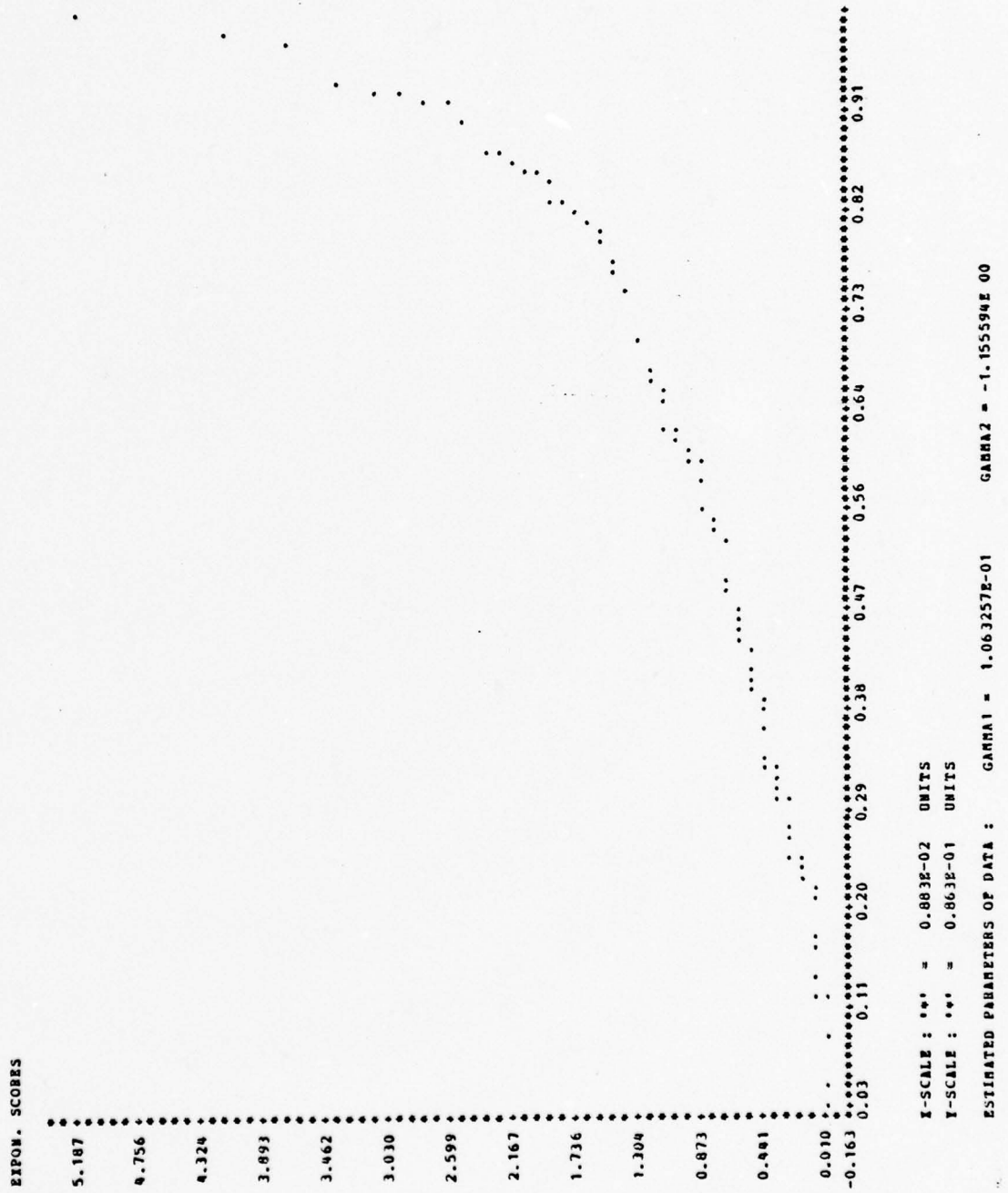


Figure 14a - A SAMPLE OF SIZE 100 FROM A UNIFORM (0,1) DISTRIBUTION

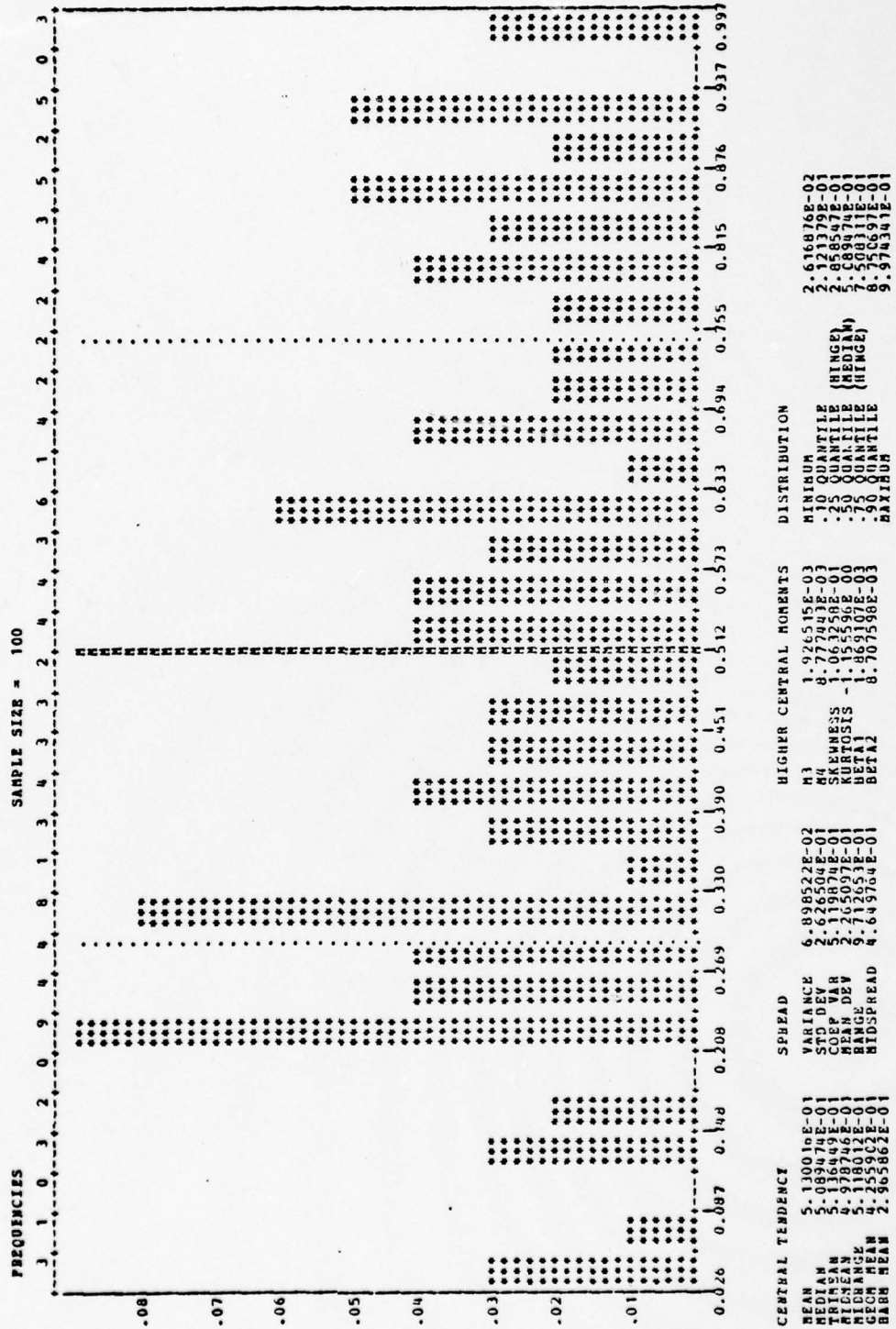
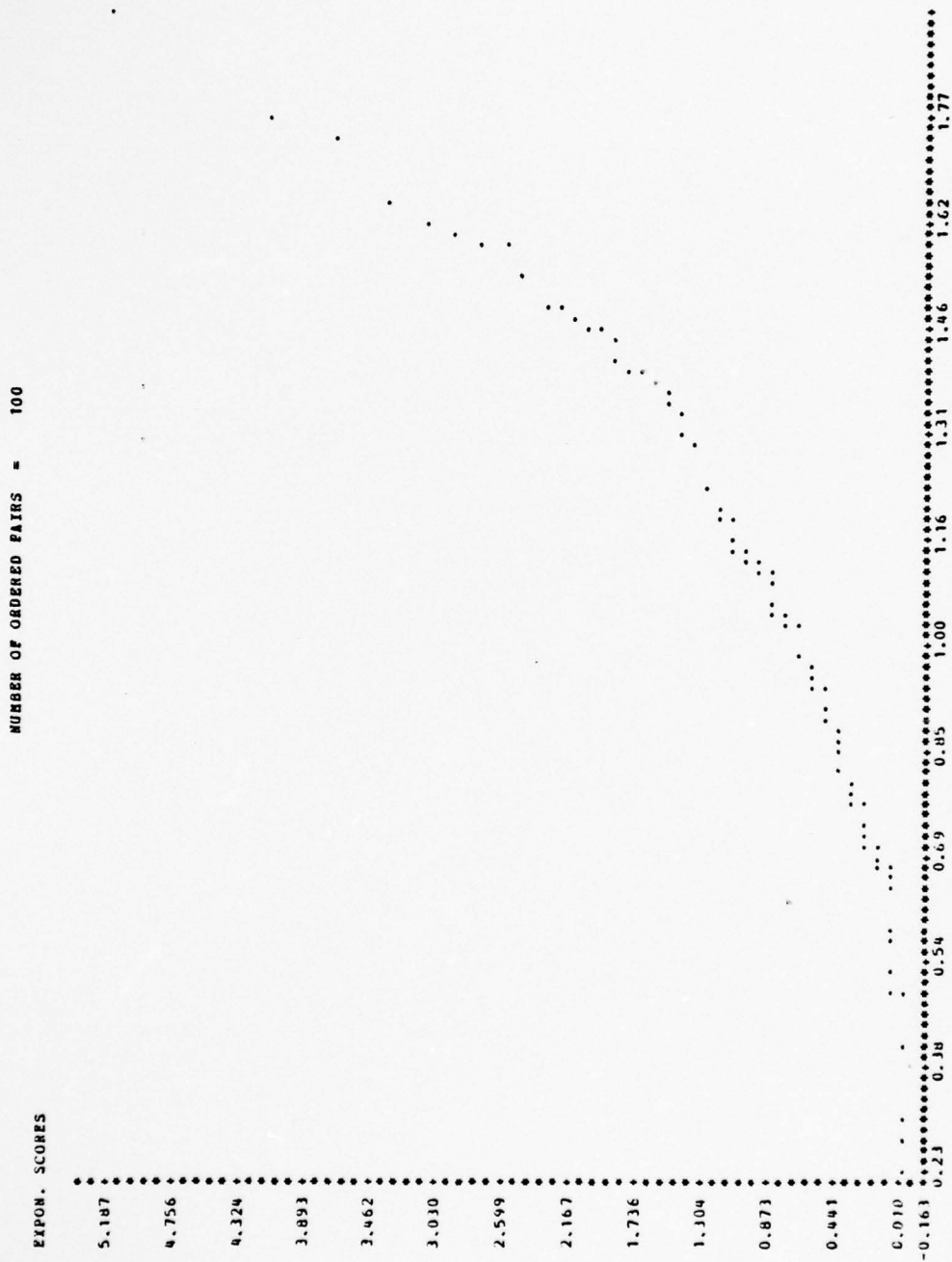


Figure 14b - HISTOGRAM OF THE DATA OF FIGURE 14a



X-SCALE : *** = 0.155E-01 UNITS
 Y-SCALE : *** = 0.861E-01 UNITS

ESTIMATED PARAMETERS OF DATA : GARRA1 = 2.018546E-01 GARRA2 = -4.379970E-01

Figure 15a - A SAMPLE OF SIZE 100 FROM A TRIANGULAR SYMMETRIC (0,2) DISTRIBUTION

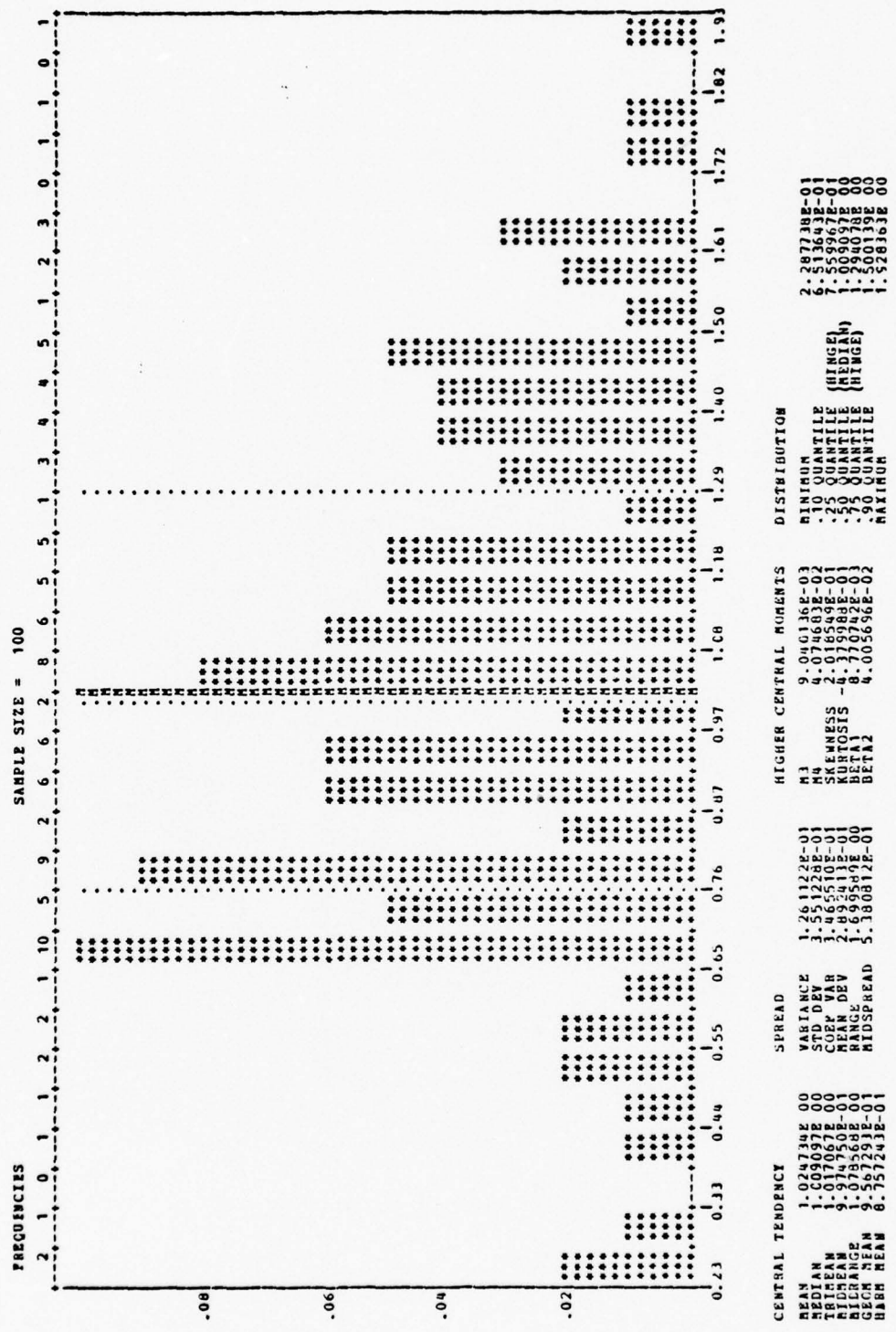


Figure 15b - HISTOGRAM OF THE DATA OF FIGURE 15a

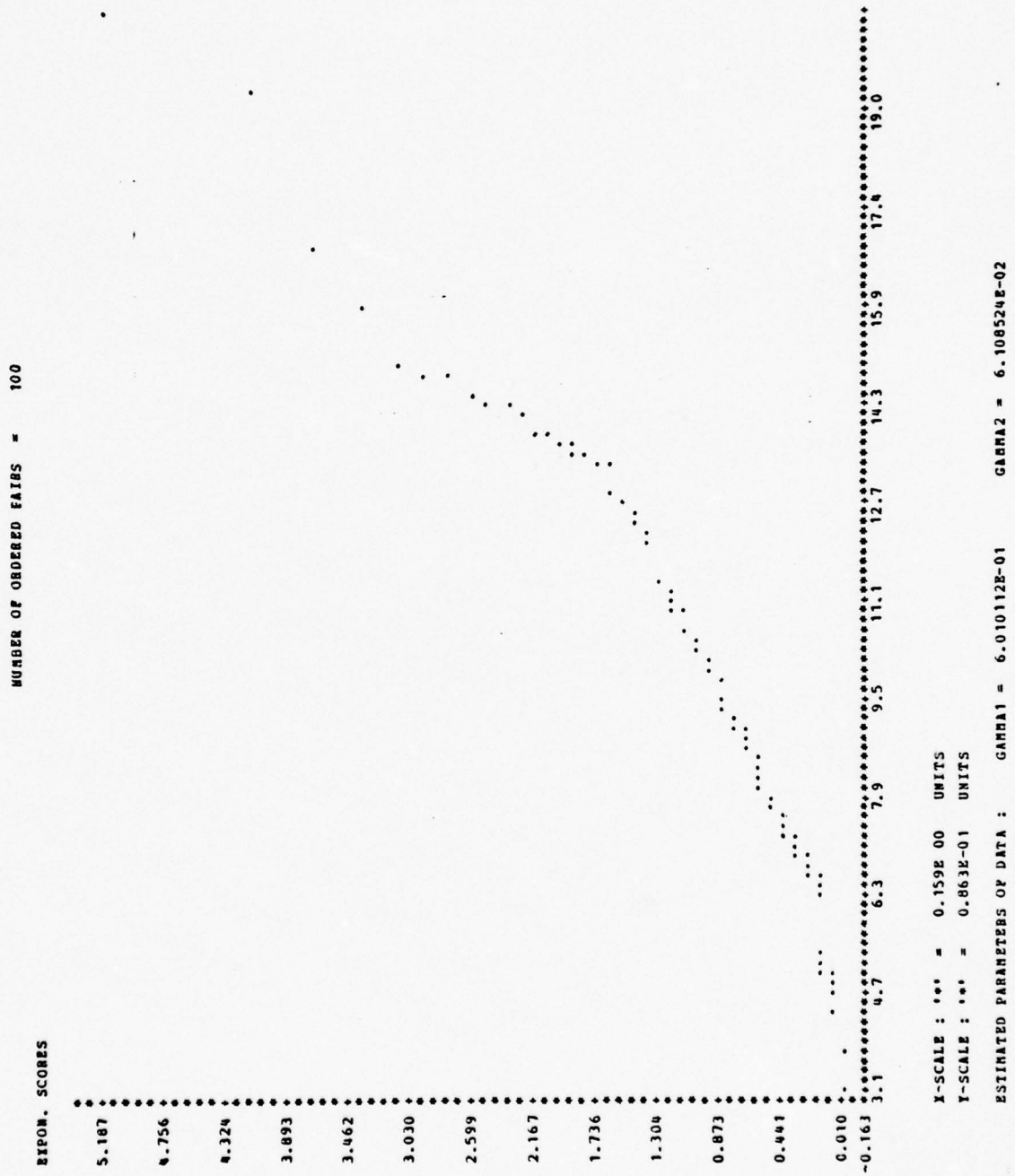


Figure 16a - X^2 (10 d.f.) DATA

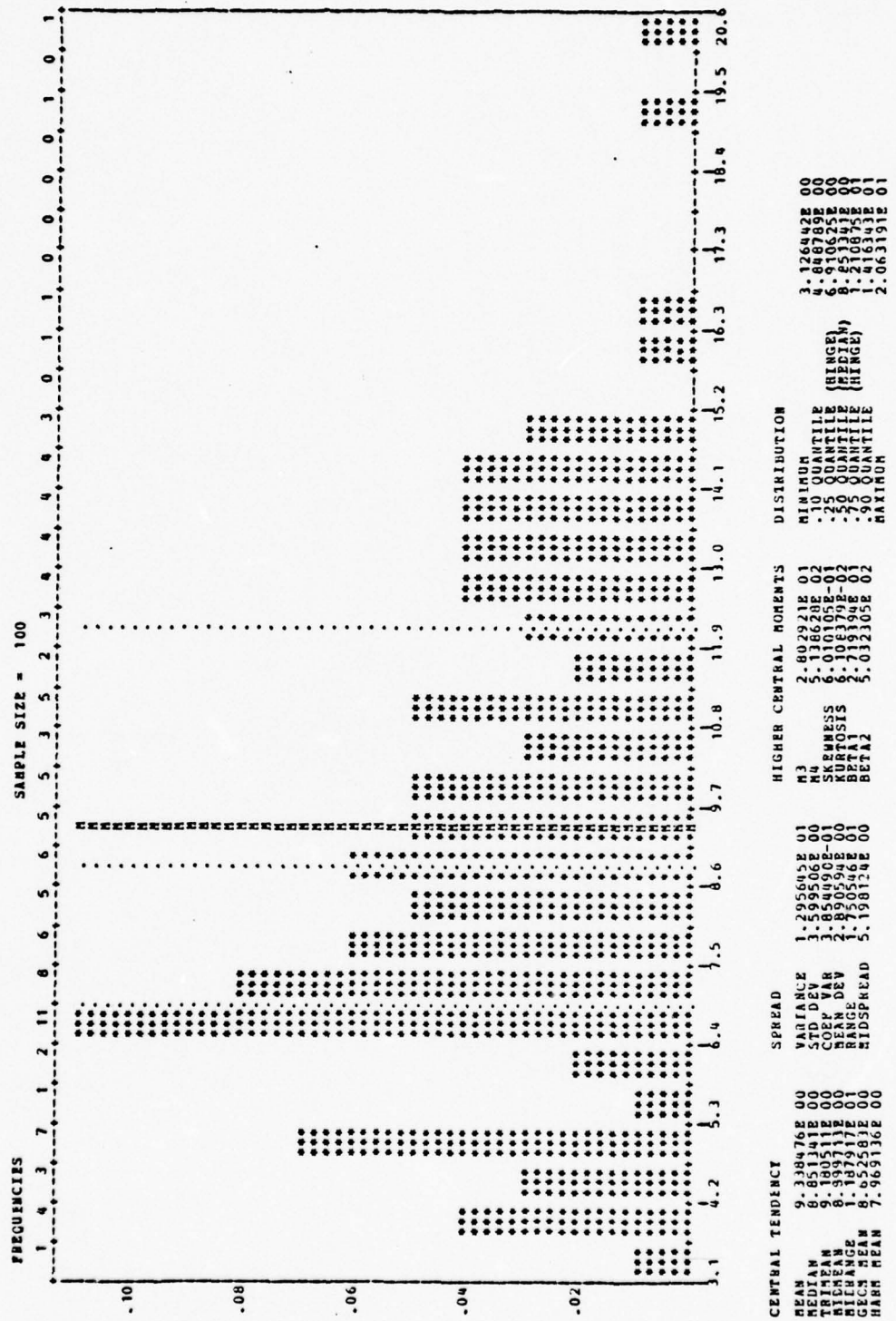


Figure 16b - HISTOGRAM OF THE DATA OF FIGURE 16a

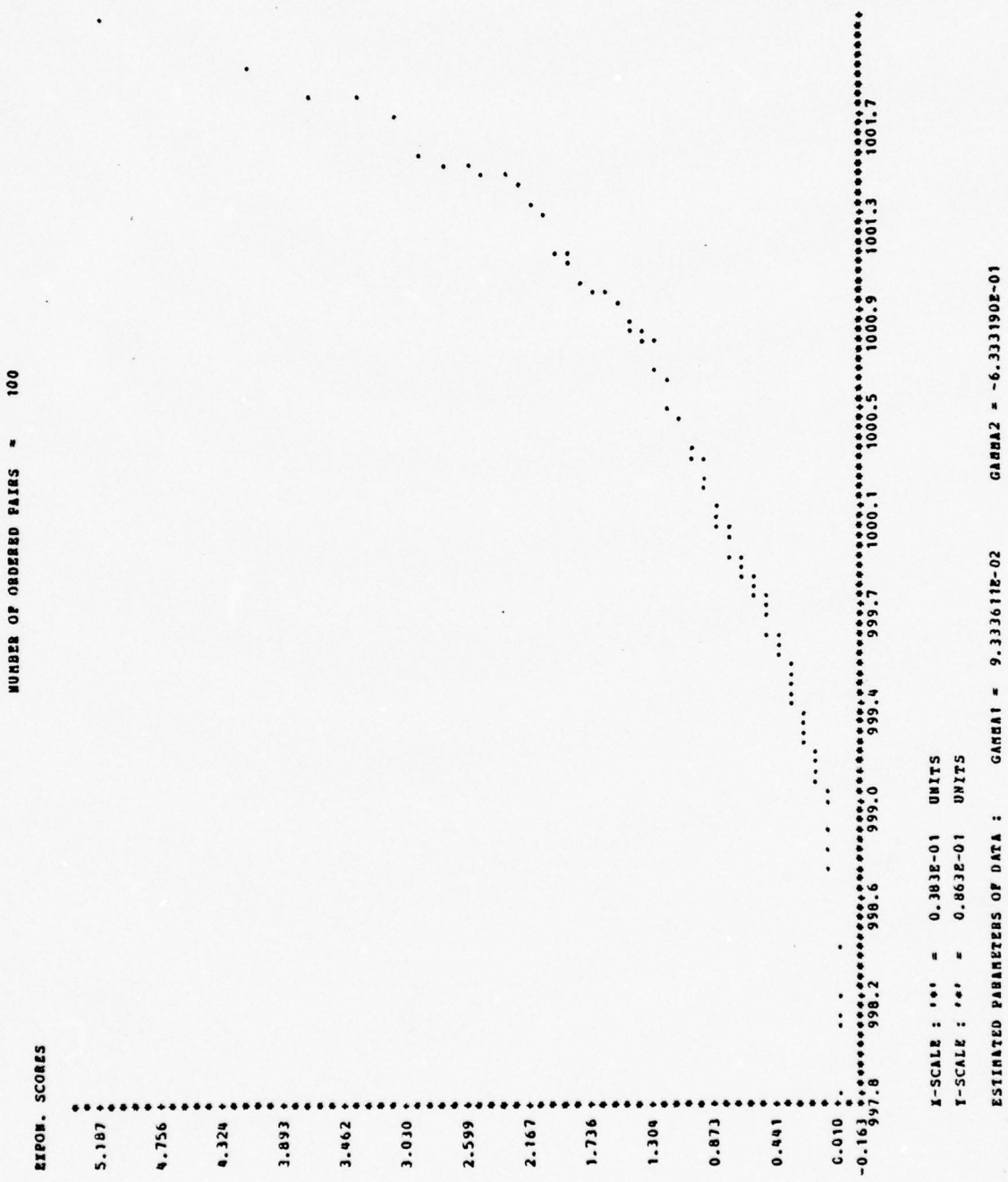


Figure 17a - A SAMPLE OF SIZE 100 FROM A NORMAL (1000,1) DISTRIBUTION

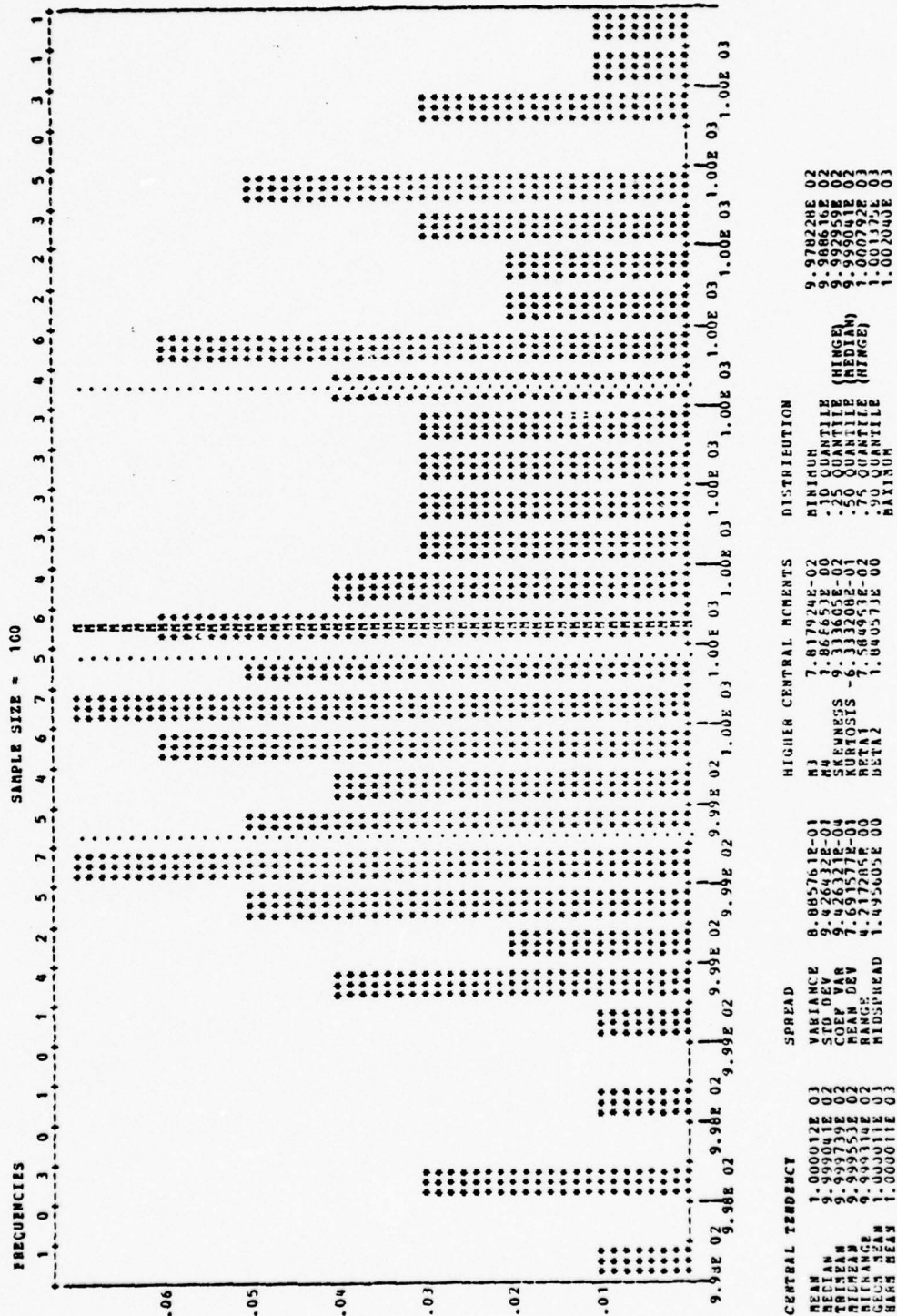


Figure 17b - HISTOGRAM OF THE DATA OF FIGURE 17a

NUMBER OF ORDERED PAIRS = 50

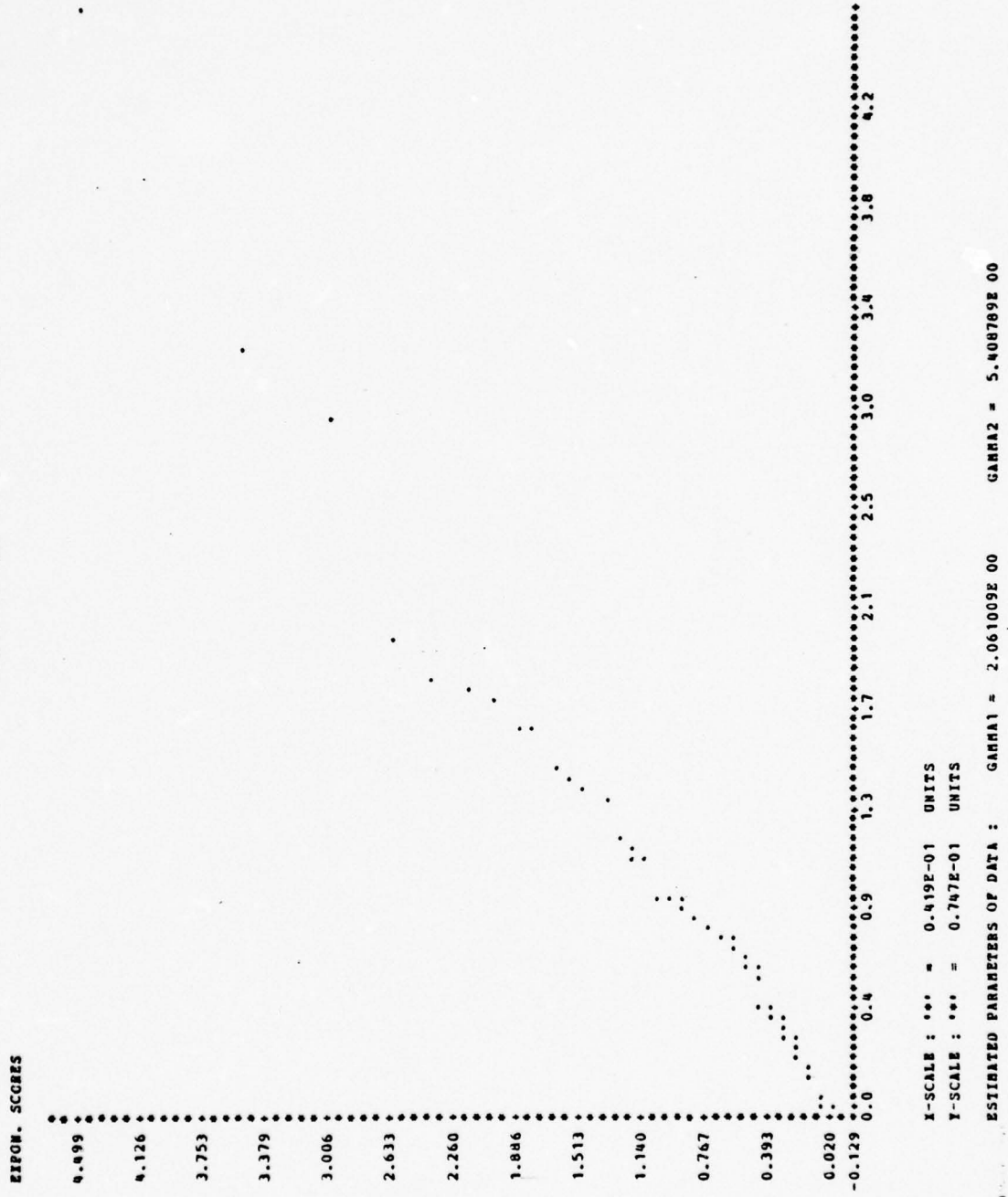


Figure 18a - A SAMPLE OF SIZE 50 FROM AN EXPONENTIAL DISTRIBUTION

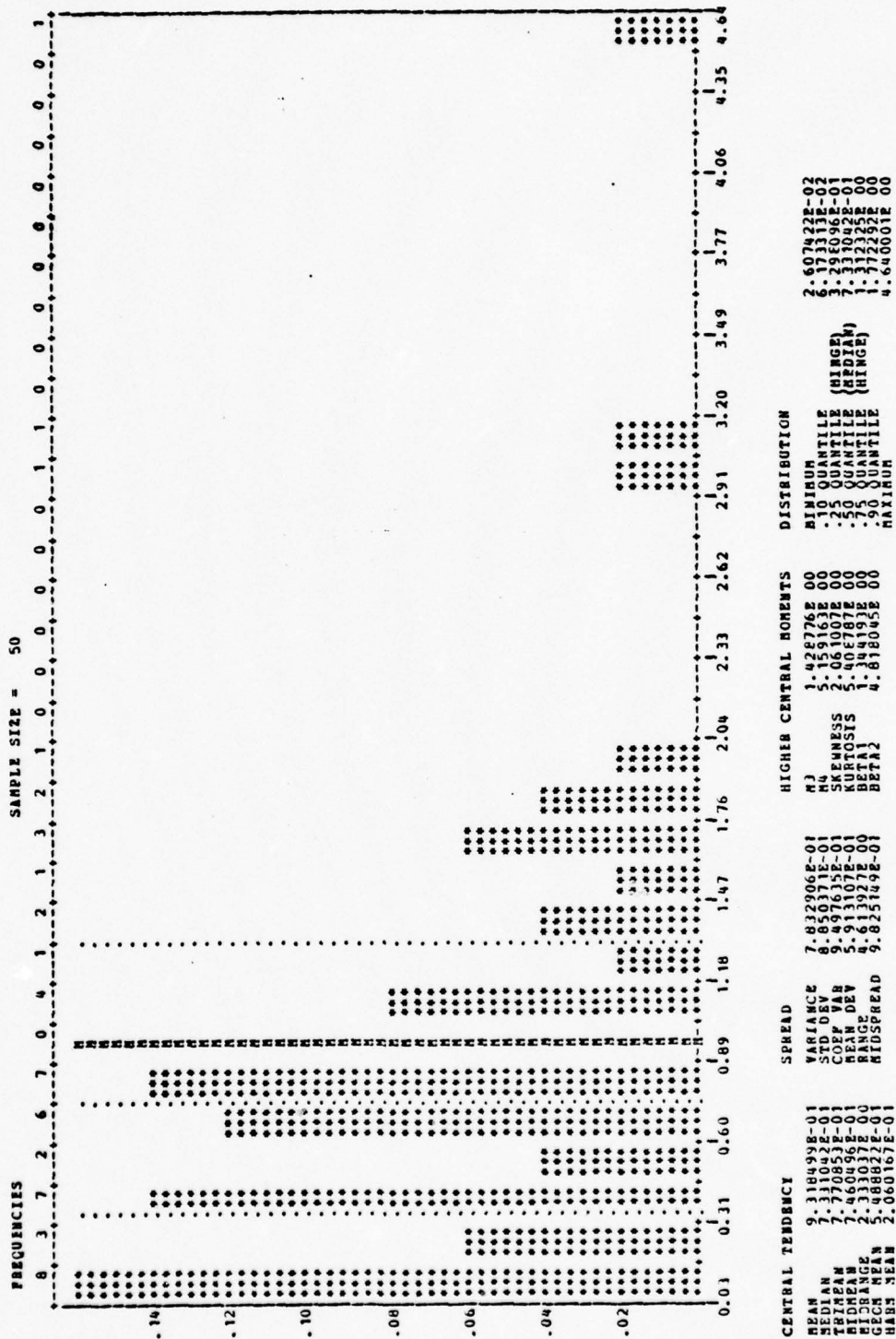


Figure 18b - HISTOGRAM OF THE DATA OF FIGURE 18a

IV. SUBROUTINE LIST

A. DESCRIPTION

Another Subroutine presented in this thesis is the Subroutine 'LIST'. Its purpose is to list a given set of data in ascending order and, taking advantage of like occurrences in the data, to print the ordered data in a compressed form. This feature becomes highly useful when listing a large number of data points that contain many repeated values. It is also a tool for finding multiplicities in supposedly continuous data, and a probability function estimating routine for data which is known to be discrete.

A complete description of how 'LIST' operates is given in the Subroutine. However a summary is given by typing DESCRIBE LIST. When the user types the command DESCRIBE LIST under the CMS environment the following response will be given on the terminal:

SUBROUTINE LIST

'LIST' sorts a set of data into increasing order and gives a 5-column print-out as follows:

1st column: Serial number of the first occurrence of this value in the ordered List

2nd column: Value of ordered data-value

3rd column: Frequency of occurrence of the value

4th column: Percent for the value
5th column: Graphical representation of the frequency
for each value.

It is called by:

CALL LIST (X, N)

where:

X Is the array of data
N Is the number of data-values.

More information is given in the subroutine.

E. INTERPRETING THE OUTPUT

The print-out is a visual representation of the data and of each data value frequency. For data points having the same value 'LIST' will print this value once with the number of occurrences. A 5-column output will be printed and its interpretation is:

1. First column gives the serial number of the first occurrence of this value in the ordered List
2. Second column gives the value of ordered data-value
3. Third column gives the frequency of occurrence of the value
4. Fourth column gives the percent for the value
5. Fifth column is a graphical representation of the frequency for each value.

Example:

Let 1, 1, 5, 3, 5, 1, 2, 2, 5, 6 be a given data set.
Then the ordered data will be: $X_{(1)} = X_{(2)} = X_{(3)} = 1,$
 $X_{(4)} = X_{(5)} = 2, X_{(6)} = 3, X_{(7)} = X_{(8)} = X_{(9)} = 5, X_{(10)} = 6.$ Then
the print-out will have the following form:

<u>SERIAL NUMBER</u>	<u>ORDERED DATA</u>	<u>FREQUENCY</u>	<u>PERCENT</u>	<u>PROB. GRAPH</u>
1	1	3	.3	***
4	2	2	.2	**
6	3	1	.1	*
7	5	3	.3	***
10	6	1	.1	*

If there are no data-points having a common value then 'LIST' gives this indication and prints only the ordered data. This happens when data have a continuous distribution.

C. USING 'LIST' WITH TELEPHONE DATA 1 AND TELEPHONE DATA 2

Subroutine 'LIST' was used with Telephone Data 1 (Figure 19) and Telephone Data 2 (Figure 20) and a brief analysis of the output follows.

Looking at Figure 19 and Figure 20 we may get some information of each data set. It can be seen that both of the data sets contain a large number of multiple occurrences of the data value one and data value two. As we can see reading the fourth column the occurrence of ones is 19% for Telephone Data 1 and 24% for Telephone Data 2.

Comparing also Figures 19a-c, 20a-d we can see that multiple occurrences happen in the range 1 to 240 for Telephone Data 1 and in the range 1 to 132 for Telephone Data 2.

Furthermore, quick visual information concerning the range where we have multiple occurrences can be obtained from the Probability Graph. Thus we can see that for Telephone Data 2 there is a region from 113 to 132 where the multiple occurrences of values is larger than in neighboring regions. Therefore 'LIST' gives the user a useful tool for analysis of data.

SERIAL NUMBER	ORDERED DATA	FREQUENCIES	PERCENT	PROBABILITY GRAPH
1	0.10000000E 01	128	0.19048	*****
129	0.20000000E 01	58	0.08036	*****
183	0.30000000E 01	28	0.04167	*****
211	0.40000000E 01	22	0.03274	*****
233	0.50000000E 01	17	0.02530	*****
250	0.60000000E 01	11	0.01637	***
261	0.70000000E 01	10	0.01488	**
271	0.80000000E 01	12	0.01786	**
283	0.90000000E 01	14	0.02083	**
297	0.10000000E 02	9	0.01339	**
306	0.11000000E 02	10	0.01488	**
316	0.12000000E 02	11	0.01637	**
327	0.13000000E 02	6	0.00893	**
333	0.14000000E 02	6	0.00893	**
339	0.15000000E 02	6	0.00893	**
345	0.16000000E 02	6	0.00893	**
353	0.17000000E 02	5	0.01190	**
361	0.18000000E 02	5	0.01190	**
366	0.19000000E 02	5	0.00744	**
378	0.20000000E 02	12	0.01786	****
379	0.20000000E 02	12	0.01786	****
384	0.21000000E 02	12	0.01786	****
389	0.22000000E 02	12	0.01786	****
392	0.23000000E 02	7	0.01042	**
399	0.24000000E 02	7	0.01042	**
402	0.25000000E 02	7	0.01042	**
405	0.26000000E 02	7	0.01042	**
407	0.27000000E 02	6	0.00446	*
410	0.28000000E 02	6	0.00446	*
415	0.29000000E 02	6	0.00446	*
421	0.30000000E 02	6	0.00446	*
425	0.31000000E 02	6	0.00446	*
429	0.32000000E 02	6	0.00446	*
431	0.33000000E 02	6	0.00446	*
435	0.34000000E 02	6	0.00446	*
438	0.35000000E 02	6	0.00446	*
440	0.36000000E 02	6	0.00446	*
442	0.37000000E 02	6	0.00446	*
443	0.38000000E 02	6	0.00446	*
444	0.39000000E 02	6	0.00446	*
445	0.40000000E 02	6	0.00446	*
447	0.41000000E 02	6	0.00446	*
448	0.42000000E 02	6	0.00446	*
449	0.43000000E 02	6	0.00446	*
453	0.44000000E 02	6	0.00446	*
456	0.45000000E 02	6	0.00446	*
457	0.46000000E 02	6	0.00446	*
459	0.47000000E 02	6	0.00446	*
460	0.48000000E 02	6	0.00446	*
461	0.49000000E 02	6	0.00446	*
463	0.50000000E 02	6	0.00446	*
464	0.51000000E 02	6	0.00446	*
465	0.52000000E 02	6	0.00446	*
468	0.53000000E 02	6	0.00446	*
470	0.54000000E 02	6	0.00446	*
472	0.55000000E 02	6	0.00446	*
474	0.56000000E 02	6	0.00446	*
475	0.57000000E 02	6	0.00446	*
477	0.58000000E 02	6	0.00446	*
480	0.59000000E 02	6	0.00446	*
481	0.60000000E 02	6	0.00446	*
483	0.61000000E 02	6	0.00446	*
484	0.62000000E 02	6	0.00446	*
487	0.63000000E 02	6	0.00446	*
488	0.64000000E 02	6	0.00446	*
489	0.65000000E 02	6	0.00446	*
490	0.66000000E 02	6	0.00446	*
492	0.67000000E 02	6	0.00446	*
493	0.68000000E 02	6	0.00446	*
496	0.69000000E 02	6	0.00446	*
497	0.70000000E 02	6	0.00446	*
498	0.71000000E 02	6	0.00446	*
499	0.72000000E 02	6	0.00446	*
501	0.73000000E 02	6	0.00446	*
502	0.74000000E 02	6	0.00446	*
504	0.75000000E 02	6	0.00446	*
505	0.76000000E 02	6	0.00446	*
506	0.77000000E 02	6	0.00446	*

Figure 19a - USING LIST WITH TELEPHONE DATA SET 1.

07	0.98000000	00	1	0.00149
08	0.99000000	00	1	0.00149
09	0.10600000	00	1	0.00149
10	0.10900000	00	1	0.00149
11	0.11100000	00	1	0.00149
12	0.11200000	00	2	0.00298
13	0.11300000	00	1	0.00149
14	0.11600000	00	1	0.00149
15	0.11700000	00	1	0.00149
16	0.11900000	00	1	0.00149
17	0.12000000	00	4	0.00595
18	0.12100000	00	2	0.00298
19	0.12200000	00	2	0.00298
20	0.12300000	00	3	0.00595
21	0.12400000	00	3	0.00446
22	0.12800000	00	1	0.00149
23	0.13500000	00	1	0.00149
24	0.14200000	00	1	0.00149
25	0.14800000	00	1	0.00149
26	0.15300000	00	1	0.00149
27	0.15600000	00	1	0.00149
28	0.15800000	00	1	0.00149
29	0.16100000	00	1	0.00149
30	0.16500000	00	1	0.00149
31	0.17500000	00	1	0.00149
32	0.17600000	00	1	0.00149
33	0.17700000	00	1	0.00149
34	0.18300000	00	1	0.00149
35	0.18600000	00	1	0.00149
36	0.18700000	00	1	0.00149
37	0.19200000	00	1	0.00149
38	0.19300000	00	1	0.00149
39	0.20200000	00	1	0.00149
40	0.21700000	00	2	0.00298
41	0.22400000	00	1	0.00149
42	0.22600000	00	1	0.00149
43	0.22800000	00	3	0.00446
44	0.23100000	00	1	0.00149
45	0.23400000	00	1	0.00149
46	0.23700000	00	1	0.00149
47	0.23900000	00	2	0.00298
48	0.24400000	00	4	0.00595
49	0.24800000	00	1	0.00149
50	0.24900000	00	1	0.00149
51	0.25100000	00	1	0.00149
52	0.25200000	00	1	0.00149
53	0.27000000	00	1	0.00149
54	0.27900000	00	1	0.00149
55	0.28600000	00	1	0.00149
56	0.29700000	00	1	0.00149
57	0.30300000	00	1	0.00149
58	0.31200000	00	1	0.00149
59	0.31800000	00	1	0.00149
60	0.33470000	00	1	0.00149
61	0.33540000	00	1	0.00149
62	0.33600000	00	1	0.00149
63	0.36400000	00	1	0.00149
64	0.37000000	00	1	0.00149
65	0.39000000	00	1	0.00149
66	0.39400000	00	1	0.00149
67	0.43500000	00	1	0.00149
68	0.44600000	00	1	0.00149
69	0.46500000	00	1	0.00149
70	0.47300000	00	1	0.00149
71	0.48000000	00	1	0.00149
72	0.54910000	00	1	0.00149
73	0.54900000	00	1	0.00149
74	0.60900000	00	2	0.00298
75	0.62100000	00	1	0.00149
76	0.71100000	00	1	0.00149
77	0.81700000	00	1	0.00149
78	0.83600000	00	1	0.00149
79	0.92700000	00	1	0.00149
80	0.11240000	04	1	0.00149
81	0.11500000	04	1	0.00149
82	0.12700000	04	1	0.00149
83	0.12890000	04	1	0.00149

*
* * * * *
*
* * * * *

Figure 19b - USING LIST WITH TELEPHONE DATA SET 1
(cont.)

606	0.12980000E04	1	0.00149
607	0.13050000E04	1	0.00149
608	0.13280000E04	1	0.00149
609	0.13480000E04	1	0.00149
610	0.13550000E04	1	0.00149
611	0.14120000E04	1	0.00149
612	0.14290000E04	1	0.00149
613	0.14890000E04	1	0.00149
614	0.14930000E04	1	0.00149
615	0.15100000E04	1	0.00149
616	0.15190000E04	1	0.00149
617	0.15470000E04	1	0.00149
618	0.16330000E04	1	0.00149
619	0.17370000E04	1	0.00149
620	0.20720000E04	1	0.00149
621	0.22483000E04	1	0.00149
622	0.22806000E04	1	0.00149
623	0.22962000E04	1	0.00149
624	0.30260000E04	1	0.00149
625	0.32010000E04	1	0.00149
626	0.35930000E04	1	0.00149
627	0.36850000E04	1	0.00149
628	0.39520000E04	1	0.00149
629	0.41570000E04	1	0.00149
630	0.44690000E04	1	0.00149
631	0.62080000E04	1	0.00149
632	0.76140000E04	1	0.00149
633	0.83220000E04	1	0.00149
634	0.90150000E04	1	0.00149
635	0.96250000E04	1	0.00149
636	0.98060000E04	1	0.00149
637	0.98180000E04	1	0.00149
638	0.10154000E05	1	0.00149
639	0.10398000E05	1	0.00149
640	0.10451000E05	1	0.00149
641	0.10939000E05	1	0.00149
642	0.11280000E05	1	0.00149
643	0.13447000E05	1	0.00149
644	0.14385000E05	1	0.00149
645	0.15155000E05	1	0.00149
646	0.15294000E05	1	0.00149
647	0.15504000E05	1	0.00149
648	0.15847000E05	1	0.00149
649	0.15868000E05	1	0.00149
650	0.16280000E05	1	0.00149
651	0.16299000E05	1	0.00149
652	0.16361000E05	1	0.00149
653	0.16408000E05	1	0.00149
654	0.16817000E05	1	0.00149
655	0.17174000E05	1	0.00149
656	0.17667000E05	1	0.00149
657	0.18218000E05	1	0.00149
658	0.18649000E05	1	0.00149
659	0.19461000E05	1	0.00149
660	0.21848000E05	1	0.00149
661	0.23499000E05	1	0.00149
662	0.24692000E05	1	0.00149
663	0.26443000E05	1	0.00149
664	0.30974000E05	1	0.00149
665	0.35644000E05	1	0.00149
666	0.38003000E05	1	0.00149
667	0.40131000E05	1	0.00149
668	0.47120000E05	1	0.00149
669	0.47592000E05	1	0.00149
670	0.61710000E05	1	0.00149
671	0.69775000E05	1	0.00149
672	0.85993000E05	1	0.00149

Figure 19c - USING LIST WITH TELEPHONE DATA SET 1
(cont.)

SERIAL NUMBER	ORDERED DATA	FREQUENCIES	PERCENT	PROBABILITY GRAPH
1	0.10000000E 01	178	0.24185	*****
179	0.20000000E 01	36	0.04891	*****
215	0.30000000E 01	11	0.01495	**
226	0.40000000E 01	6	0.00815	*
232	0.50000000E 01	6	0.00815	*
238	0.60000000E 01	6	0.00815	*
241	0.70000000E 01	6	0.00815	*
248	0.80000000E 01	6	0.00815	*
252	0.90000000E 01	6	0.00815	*
256	0.10000000E 02	9	0.01223	**
255	0.11000000E 02	9	0.01223	**
257	0.12000000E 02	3	0.00408	*
270	0.13000000E 02	1	0.00136	
271	0.14000000E 02	1	0.00136	
272	0.15000000E 02	1	0.00136	
273	0.16000000E 02	1	0.00136	
277	0.19000000E 02	1	0.00136	
278	0.21000000E 02	1	0.00136	
279	0.22000000E 02	1	0.00136	
280	0.24000000E 02	1	0.00408	*
283	0.25000000E 02	1	0.00408	*
286	0.26000000E 02	1	0.00272	*
288	0.27000000E 02	1	0.00136	*
289	0.30000000E 02	3	0.00408	*
292	0.32000000E 02	1	0.00136	*
293	0.36000000E 02	1	0.00136	*
294	0.40000000E 02	2	0.00272	*
296	0.42000000E 02	2	0.00136	*
297	0.44000000E 02	1	0.00136	*
298	0.47000000E 02	1	0.00543	*
302	0.48000000E 02	1	0.00136	*
303	0.49000000E 02	1	0.01223	**
304	0.50000000E 02	1	0.00543	**
312	0.51000000E 02	1	0.00408	*
316	0.52000000E 02	1	0.00272	*
319	0.53000000E 02	1	0.00543	*
321	0.54000000E 02	1	0.00408	*
325	0.57000000E 02	1	0.00408	*
328	0.59000000E 02	1	0.00136	*
329	0.59000000E 02	1	0.00408	*
332	0.60000000E 02	1	0.00272	*
334	0.62000000E 02	1	0.00408	*
337	0.63000000E 02	1	0.00136	*
338	0.64000000E 02	1	0.00543	*
342	0.65000000E 02	1	0.00408	*
345	0.66000000E 02	1	0.00543	*
349	0.67000000E 02	1	0.00136	*
350	0.68000000E 02	1	0.00679	*
353	0.69000000E 02	1	0.00408	*
356	0.70000000E 02	1	0.00408	*
361	0.71000000E 02	1	0.00136	*
366	0.73000000E 02	1	0.00136	*
367	0.74000000E 02	1	0.00136	*
368	0.75000000E 02	1	0.00136	*
369	0.76000000E 02	1	0.00136	*
370	0.77000000E 02	1	0.00136	*
371	0.81000000E 02	1	0.00136	*
372	0.82000000E 02	1	0.00136	*
373	0.88000000E 02	1	0.00136	*
374	0.89000000E 02	1	0.00136	*
375	0.91000000E 02	1	0.00136	*
376	0.95000000E 02	1	0.00136	*
377	0.96000000E 02	1	0.00136	*
378	0.97000000E 02	1	0.00136	*
379	0.99000000E 02	3	0.00408	*
382	0.10000000E 03	1	0.00136	*
383	0.10100000E 03	1	0.00136	*
384	0.10500000E 03	1	0.00136	*
385	0.10600000E 03	1	0.00136	*
386	0.10700000E 03	1	0.00136	*
387	0.10800000E 03	1	0.00136	*
388	0.11000000E 03	1	0.00136	*
389	0.11100000E 03	1	0.00136	*
390	0.11200000E 03	1	0.00136	*
391	0.11300000E 03	4	0.00543	*
395	0.11400000E 03	3	0.00408	*
398	0.11500000E 03	1	0.00136	*
399	0.11600000E 03	1	0.00543	*
403	0.11700000E 03	1	0.00136	*

Figure 20a - USING LIST WITH TELEPHONE DATA SET 2.

404	0.11800000E 03	5	0.00679	*
409	0.11900000E 03	8	0.01087	**
417	0.12000000E 03	9	0.01223	**
426	0.12100000E 03	7	0.00951	**
433	0.12200000E 03	12	0.01630	**
445	0.12300000E 03	8	0.01087	**
453	0.12400000E 03	7	0.00951	**
460	0.12500000E 03	9	0.01223	**
469	0.12600000E 03	6	0.00815	**
475	0.12700000E 03	3	0.00408	*
478	0.12800000E 03	2	0.00272	*
480	0.12900000E 03	4	0.00543	*
484	0.13000000E 03	3	0.00408	*
487	0.13200000E 03	3	0.00408	*
490	0.13300000E 03	2	0.00272	*
492	0.13700000E 03	1	0.00136	*
493	0.14000000E 03	1	0.00136	*
494	0.14300000E 03	1	0.00136	*
495	0.15200000E 03	1	0.00136	*
499	0.15800000E 03	1	0.00136	*
499	0.16600000E 03	1	0.00136	*
499	0.16900000E 03	2	0.00272	*
500	0.17000000E 03	1	0.00136	*
501	0.17300000E 03	2	0.00272	*
503	0.17600000E 03	1	0.00136	*
504	0.17800000E 03	1	0.00136	*
505	0.18000000E 03	2	0.00272	*
507	0.18200000E 03	1	0.00136	*
508	0.18500000E 03	1	0.00136	*
509	0.18700000E 03	1	0.00136	*
510	0.19000000E 03	1	0.00136	*
511	0.19400000E 03	1	0.00136	*
512	0.19500000E 03	1	0.00136	*
513	0.19900000E 03	1	0.00136	*
514	0.20200000E 03	1	0.00136	*
515	0.20900000E 03	1	0.00136	*
516	0.21600000E 03	1	0.00136	*
517	0.22900000E 03	1	0.00136	*
518	0.23000000E 03	1	0.00136	*
519	0.23700000E 03	1	0.00136	*
520	0.23900000E 03	3	0.00408	*
523	0.24000000E 03	2	0.00272	*
525	0.24100000E 03	1	0.00136	*
526	0.24400000E 03	2	0.00272	*
528	0.24700000E 03	2	0.00272	*
530	0.24800000E 03	2	0.00272	*
532	0.25000000E 03	1	0.00136	*
533	0.25100000E 03	1	0.00136	*
534	0.25400000E 03	2	0.00272	*
536	0.25500000E 03	2	0.00272	*
538	0.25600000E 03	1	0.00136	*
539	0.25800000E 03	1	0.00136	*
540	0.27200000E 03	1	0.00136	*
541	0.27500000E 03	1	0.00136	*
542	0.28000000E 03	1	0.00136	*
543	0.29800000E 03	1	0.00136	*
544	0.31800000E 03	1	0.00136	*
545	0.32400000E 03	1	0.00136	*
546	0.34000000E 03	1	0.00136	*
547	0.34400000E 03	1	0.00136	*
548	0.34600000E 03	1	0.00136	*
549	0.35900000E 03	1	0.00136	*
550	0.36500000E 03	2	0.00272	*
552	0.36700000E 03	1	0.00136	*
553	0.37700000E 03	1	0.00136	*
554	0.38000000E 03	1	0.00136	*
555	0.38700000E 03	1	0.00136	*
556	0.41300000E 03	1	0.00136	*
557	0.41800000E 03	1	0.00136	*
558	0.42900000E 03	1	0.00136	*
559	0.45300000E 03	1	0.00136	*
560	0.46600000E 03	1	0.00136	*
561	0.47200000E 03	1	0.00136	*
562	0.50300000E 03	2	0.00272	*
564	0.50400000E 03	1	0.00136	*
565	0.50700000E 03	1	0.00136	*
566	0.51000000E 03	1	0.00136	*
567	0.51400000E 03	1	0.00136	*
568	0.51700000E 03	1	0.00136	*
569	0.52800000E 03	1	0.00136	*

Figure 20b - USING LIST WITH TELEPHONE DATA SET 2
(cont.)

570	0.57000000	03	1	0.00136
571	0.57200000	03	1	0.00136
572	0.57300000	03	1	0.00136
573	0.57400000	03	1	0.00136
574	0.57500000	03	1	0.00136
575	0.57600000	03	1	0.00136
576	0.57700000	03	1	0.00136
577	0.57800000	03	1	0.00136
578	0.57900000	03	1	0.00136
579	0.58000000	03	1	0.00136
580	0.71600000	03	2	0.00272
582	0.74000000	03	1	0.00136
583	0.74200000	03	2	0.00272
585	0.79100000	03	1	0.00136
586	0.85300000	03	1	0.00136
587	0.86200000	03	1	0.00136
588	0.86600000	03	1	0.00136
589	0.86750000	03	1	0.00136
590	0.86850000	03	1	0.00136
591	0.86940000	03	1	0.00136
592	0.92600000	03	1	0.00136
593	0.92800000	03	1	0.00136
594	0.94100000	03	1	0.00136
595	0.97600000	03	1	0.00136
596	0.99100000	03	1	0.00136
597	0.10210000	04	1	0.00136
598	0.10230000	04	1	0.00136
599	0.10520000	04	1	0.00136
600	0.10860000	04	2	0.00272
602	0.11070000	04	1	0.00136
603	0.11190000	04	1	0.00136
604	0.11250000	04	1	0.00136
605	0.12310000	04	1	0.00136
606	0.12500000	04	1	0.00136
607	0.12610000	04	2	0.00272
609	0.12960000	04	1	0.00136
610	0.13670000	04	1	0.00136
611	0.14120000	04	1	0.00136
612	0.14130000	04	1	0.00136
613	0.14620000	04	1	0.00136
614	0.14850000	04	1	0.00136
615	0.14970000	04	1	0.00136
616	0.15040000	04	1	0.00136
617	0.15317000	04	1	0.00136
618	0.15334000	04	1	0.00136
619	0.15335000	04	2	0.00272
621	0.15620000	04	1	0.00136
622	0.15940000	04	1	0.00136
623	0.15950000	04	1	0.00136
624	0.16030000	04	1	0.00136
625	0.16520000	04	1	0.00136
626	0.16530000	04	1	0.00136
627	0.16930000	04	1	0.00136
628	0.17000000	04	1	0.00136
629	0.17150000	04	1	0.00136
630	0.17500000	04	1	0.00136
631	0.17550000	04	1	0.00136
632	0.17860000	04	1	0.00136
633	0.18220000	04	1	0.00136
634	0.18240000	04	1	0.00136
635	0.18550000	04	1	0.00136
636	0.18770000	04	1	0.00136
637	0.18930000	04	1	0.00136
638	0.18970000	04	1	0.00136
639	0.19230000	04	1	0.00136
640	0.19640000	04	1	0.00136
641	0.19850000	04	1	0.00136
642	0.20510000	04	1	0.00136
643	0.20630000	04	1	0.00136
644	0.21670000	04	1	0.00136
645	0.21850000	04	2	0.00272
647	0.22080000	04	1	0.00136
648	0.22530000	04	1	0.00136
649	0.22600000	04	1	0.00136
650	0.22710000	04	1	0.00136
651	0.22870000	04	1	0.00136
652	0.23160000	04	1	0.00136
653	0.23210000	04	1	0.00136
654	0.24290000	04	1	0.00136
655	0.24390000	04	1	0.00136

Figure 20c - USING LIST WITH TELEPHONE DATA SET 2
(cont.)

656	0.246800000	04	1	0.00136
657	0.247200000	04	1	0.00136
658	0.249000000	04	1	0.00136
659	0.268400000	04	1	0.00136
660	0.279300000	04	1	0.00136
661	0.288200000	04	1	0.00136
662	0.289100000	04	1	0.00136
663	0.293800000	04	1	0.00136
664	0.308000000	04	1	0.00136
665	0.311700000	04	1	0.00136
666	0.316500000	04	1	0.00136
667	0.328600000	04	1	0.00136
668	0.331500000	04	1	0.00136
669	0.333780000	04	1	0.00136
670	0.335010000	04	1	0.00136
671	0.336070000	04	1	0.00136
672	0.337370000	04	1	0.00136
673	0.338500000	04	1	0.00136
674	0.338580000	04	1	0.00136
675	0.338790000	04	1	0.00136
676	0.338880000	04	1	0.00136
677	0.339440000	04	1	0.00136
678	0.339780000	04	1	0.00136
679	0.437300000	04	1	0.00136
680	0.439300000	04	1	0.00136
681	0.451200000	04	1	0.00136
682	0.453700000	04	1	0.00136
683	0.475900000	04	1	0.00136
684	0.483500000	04	1	0.00136
685	0.492160000	04	1	0.00136
686	0.494920000	04	1	0.00136
687	0.506520000	04	1	0.00136
688	0.507540000	04	1	0.00136
689	0.518500000	04	1	0.00136
690	0.524100000	04	1	0.00136
691	0.537200000	04	1	0.00136
692	0.538500000	04	1	0.00136
693	0.565900000	04	1	0.00136
694	0.581600000	04	1	0.00136
695	0.582100000	04	1	0.00136
696	0.730700000	04	1	0.00136
697	0.732900000	04	1	0.00136
698	0.746000000	04	1	0.00136
699	0.792700000	04	1	0.00136
700	0.803900000	04	1	0.00136
701	0.805800000	04	1	0.00136
702	0.835300000	04	1	0.00136
703	0.884700000	04	1	0.00136
704	0.920600000	04	1	0.00136
705	0.925600000	04	1	0.00136
706	0.951700000	04	1	0.00136
707	0.954100000	04	1	0.00136
708	0.956200000	04	1	0.00136
709	0.965000000	04	1	0.00136
710	0.100200000	05	1	0.00136
711	0.103760000	05	1	0.00136
712	0.108370000	05	1	0.00136
713	0.109180000	05	1	0.00136
714	0.120420000	05	1	0.00136
715	0.127930000	05	1	0.00136
716	0.130120000	05	1	0.00136
717	0.131790000	05	1	0.00136
718	0.146920000	05	1	0.00136
719	0.158250000	05	1	0.00136
720	0.168770000	05	1	0.00136
721	0.188850000	05	1	0.00136
722	0.192730000	05	1	0.00136
723	0.196960000	05	1	0.00136
724	0.198490000	05	1	0.00136
725	0.198510000	05	1	0.00136
726	0.208460000	05	1	0.00136
727	0.214400000	05	1	0.00136
728	0.245730000	05	1	0.00136
729	0.247700000	05	1	0.00136
730	0.262780000	05	1	0.00136
731	0.272380000	05	1	0.00136
732	0.281330000	05	1	0.00136
733	0.299130000	05	1	0.00136
734	0.308670000	05	1	0.00136
735	0.331380000	05	1	0.00136
736	0.672710000	05	1	0.00136

Figure 20d - USING LIST WITH TELEPHONE DATA SET 2
(cont.)

V. ASSESSMENT OF VARIABILITY ROUTINES

A. INTRODUCTION

In the previous sections Probability Plotting Subroutines were presented for use in informal estimation of the form of the distribution of a set of independent observations. In the present section subroutines computing the variability of some basic statistics are described.

We know that \bar{X} (sample mean) is an unbiased estimator of the population mean ($= \int x f(x) dx$), where X is a population with unknown distribution ($F_X(x)$) and unknown mean, and X_1, X_2, \dots, X_n is a sample of independent observations from X . Furthermore we can compute the Variance of \bar{X} as:

$$\begin{aligned} \text{Var}[\bar{X}] &= \text{Var}\left[\sum_{i=1}^n \frac{X_i}{n}\right] \\ &= \frac{1}{n} \text{Var}\left[\sum_{i=1}^n X_i\right] \\ &= \frac{n}{n} \text{Var}[X] = \frac{\sigma^2}{n} . \end{aligned}$$

Thus we see it is possible to estimate the $\text{Var}[X]$, even if the population Variance (σ^2) is unknown, by simply using the sample variance S^2 as follows: $\text{Var}[\bar{X}] = S^2/n$, where $S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{(n-1)}$ (an unbiased estimator of σ^2). Therefore S^2/n is an unbiased estimator of $\text{Var}[\bar{X}]$ also. Furthermore, especially for a Normal population, we can obtain a confidence interval for the mean using the t-statistic.

But, although for \bar{X} there exists a direct assessment of variability, for estimates of other population parameters such as Skewness, Kurtosis, Coefficient of Variation, and so forth, this is not so simply obtained. Thus several methods have been introduced to obtain assessments of variability and for two of them (Sectioning of data and the Jackknife) the Subroutines 'SECTN' and 'JACK' will be presented.

E. SUBROUTINE SECTN

1. Description

'SECTN' Subroutine is used for assessing variability of estimates from data based on the Sectioning Method.

The basic idea of this method is:

Assume we have n independent observations X_1, X_2, \dots, X_n from a population with unknown distribution function $F(x)$. Let θ be a parameter of $F(x)$ and $\hat{\theta}(n) = \hat{\theta}(X_1, X_2, \dots, X_n)$ be a statistic which estimates θ .

Now we want to estimate the variance of $\tilde{\theta}(n)$ which is a new Random Variable and we work as follows:

1. Divide the sample into r disjoint sections, of size m (r should be such that $n=mr$. If this is not possible some of the latter data is discarded).

2. For each section form the same estimate $\tilde{\theta}_i(m)$, $i = 1, 2, \dots, r$

3. Compute the average of $\tilde{\theta}_i(m)$, that is:

$$\bar{\theta}(n) = \frac{1}{r} \sum_{i=1}^r \tilde{\theta}_i(m) ,$$

doing so, we have an estimate of θ and

$$\begin{aligned} E[\bar{\theta}(n)] &= E\left[\frac{1}{r} \sum_{i=1}^r \tilde{\theta}_i(m)\right] \\ &= \frac{1}{r} r E[\tilde{\theta}(m)] \\ &= E[\tilde{\theta}(m)] , \end{aligned}$$

also we have:

$$\begin{aligned} \text{Var}[\bar{\theta}(n)] &= \text{Var}\left[\frac{1}{r} \sum_{i=1}^r \tilde{\theta}_i(m)\right] \\ &= \frac{1}{r^2} \text{Var}\left[\sum_{i=1}^r \tilde{\theta}_i(m)\right] \\ &= \frac{1}{r} \text{Var}[\tilde{\theta}(m)] \\ &= \frac{1}{r} \frac{1}{r-1} \sum_{i=1}^r (\tilde{\theta}_i(m) - \bar{\theta}(n))^2 \\ &= \frac{1}{r} S_{\tilde{\theta}(m)}^2 . \end{aligned}$$

Thus we see that $\text{Var}[\tilde{\theta}(m)]$ and $\text{Var}[\bar{\theta}(n)]$ can be unbiasedly estimated from the sample variance of $\tilde{\theta}_i(m)$, the main advantage and the purpose of the sectioning of data method.

But the main disadvantage of this method is that we would like r to be as large as possible in order to make the variability of the variance estimate S^2 as small as possible. This however may be worse for the bias properties of the estimation procedure. Therefore the choice of r is a factor that should be considered.

It should be noted here that if (m) are approximately normal variates (This can be tested by the previously described Subroutine 'NORMPL') then confidence intervals for the unknown parameter can be obtained based on the t -Statistic, in the following way:

$$\bar{\theta}(n) \pm \frac{S \bar{\theta}(n)}{r^{1/2}} t_{(1-\alpha/2), (r-1)}$$

where:

$\bar{\theta}(n)$ is the mean of the sectioned data statistics obtained from the column named 'mean' of the second table ('estimated parameters of the sample parameters') of the program output.

$S \bar{\theta}(n) / r^{1/2}$ is the standard deviation of the sectioned data statistics divided by the square root of the number of sections, obtained from the last column of the same table of the program output.

$t_{(1-\alpha/2), (r-1)}$ is the $(1-\alpha/2)$ quantile of the t -distribution with $r-1$ degrees of freedom.

2. Program Structure

'SECTN', using the Sectioning method, estimates the following statistics: Mean, Median, Variance, Standard Deviation, Coefficient of Variation, Skewness, Kurtosis, Minimum, and Maximum. (The formulas which have been used to compute these estimates are described in the comments of the Subroutine.) The first table is then printed by the program, containing the values of these parameters. Then 'SECTN', using the computed estimates of all sections for each parameter, estimates the Mean, Median, Variance, Skewness, Kurtosis, and Standard Deviation divided by the square root of the number of sections. The second table is printed containing these values.

There are three restrictions in using 'SECTN':

1. The number of sections must be no greater than 100. If it is, a diagnostic message is printed and only estimates from unsectioned data will be given.
2. The number of data values must be greater than 3; otherwise a diagnostic message is printed without any calculation.
3. The size of each section must be greater than 3. If it is less than, or equal to 3, then the program gives estimates for the entire set of (unsectioned) data. A diagnostic is printed.

A complete description of how 'SECTN' operates is given in the Subroutine. Furthermore, a Summary is given by typing on the terminal the command DESCRIBE SECTN under the CMS environment. The following response will be given on the terminal when the user types the above command:

SUBROUTINE SECTN

'SECTN' is intended to estimate a set of basic statistics of a given set of observations using the 'Sectioning of data method'. Also for each estimated statistic, estimates of some basic statistics such as the mean, standard Deviation, and so forth, are given.

It is called by :
CALL SECTN (X, N, K)

where:

- X Is the array of data
- N Is the number of data values (must be greater than 3)
- K Is the number of desired sections (no greater than 100)

Note: k should be a number which minimizes the number of data points that will have to be discarded. 'SECTN' places the data into the equal size sections discarding any data left over.

For $k \leq 3$ or $k > 100$ or $(n/k) \leq 3$ only estimates from unsectioned data will be given and no estimates for the estimated statistics will be computed.

No output is expected if $n \leq 3$.

More information is given in the subroutine.

3. Using 'SECTIN' with Telephone Data Set 1

'SECTIN' was used on Telephone Data Set 1 to assess the variability in the Mean, Median, Variance, Standard Deviation, Coefficient of Variation, Skewness and Kurtosis.

The 672 data-points of Telephone Data Set 1 were broken down into 16 disjoint sections with 42 data-points per section. Because of this break-down no data-points were discarded. Compare the values of row 'unsectioned' (see figure 21) with the values of the corresponding statistics computed by the Subroutine 'HISTGS/HISTFS' (see figure 1). The values are the same.

Now if we want to assess the variability in any of the above parameters we proceed as follows:

1. We take the mean (\bar{y}) of the parameter, whose the variability we want to assess, from the table under 'ESTIMATED PARAMETERS OF SAMPLE PARAMETERS'.

2. We take, from the same table, the value (S) from the last column (STD. DEV./NS**.5) for the same parameter.

3. Using the t-statistic with k-1 d.f. and the formula:

$$\bar{y} \pm St_{(1-\alpha/2), (k-1)}$$

we get a (1- α)% confidence interval for the parameter.

Example

In order to find a 95% confidence interval for the Skewness of Telephone Data 1 we take:

$$\bar{y} = 4.9343, \quad S = .30312, \quad t_{.975, 15} = 2.131$$

Therefore the 95% confidence interval is:

$$4.9343 \pm .30312 * 2.131 = 4.9343 \pm .64595 \\ \implies [4.2884, 5.5802].$$

It should be noted here that the use of the Variance estimate from the sectioned data in order to get confidence intervals of the parameters is based on the normality assumption and the independence of the estimates from the sections. The normality will depend on the number of data-points in each section, which should be kept large. This requirement, however, conflicts with the need to make the number of sections large to reduce the variability in the estimate of the variance of the statistics. The skewness estimates of each of the 16 sections, can be run through the normal plotting routine to see whether the use of the t-statistic confidence interval is valid.

ESTIMATED SAMPLE PARAMETERS

SECTION	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSIS	MINIMUM	MAXIMUM
1	1.0526E 03	8.5000E 00	3.4598E 07	5.8820E 03	5.5879E 00	6.3488E 00	3.7831E 01	1.0000E 00	3.8003E 04
2	3.2135E 03	1.4500E 01	1.8494E 08	1.3599E 04	4.2320E 00	5.8106E 00	3.3017E 01	1.0000E 00	8.5993E 04
3	1.7662E 03	1.4500E 01	4.2383E 07	6.5103E 03	3.6860E 00	4.2806E 00	1.7836E 01	1.0000E 00	3.5644E 04
4	6.0669E 02	1.1000E 01	5.3412E 06	2.3111E 03	3.8094E 00	4.3148E 00	1.6486E 01	1.0000E 00	1.1280E 04
5	1.5639E 03	5.0500E 01	2.1924E 07	4.6824E 03	2.9941E 00	4.2203E 00	1.8565E 01	1.0000E 00	2.6443E 04
6	2.5443E 03	5.7000E 01	4.0337E 07	6.3511E 03	2.5061E 00	3.1573E 00	9.5654E 00	1.0000E 00	3.0974E 04
7	2.6778E 03	2.2000E 01	7.2756E 07	8.5297E 03	3.1853E 00	4.1587E 00	1.7579E 01	1.0000E 00	4.7120E 04
8	9.8881E 02	1.8500E 01	3.8282E 07	6.1873E 03	6.2573E 00	6.4801E 00	3.8995E 01	1.0000E 00	4.0111E 04
9	1.5176E 03	2.2000E 01	2.0792E 07	4.5599E 03	3.0046E 00	2.9866E 00	6.8551E 00	1.0000E 00	1.7174E 04
10	2.7682E 03	1.4000E 01	1.0906E 08	1.0443E 04	3.7726E 00	4.8932E 00	2.4134E 01	1.0000E 00	6.1710E 04
11	1.9258E 03	1.4000E 01	5.9852E 07	7.7384E 03	4.0173E 00	5.4134E 00	2.9659E 01	1.0000E 00	4.7592E 04
12	8.1955E 02	4.9500E 01	8.2895E 06	2.8791E 03	3.5131E 00	4.5999E 00	1.9785E 01	1.0000E 00	1.5868E 04
13	2.1201E 03	4.0000E 00	1.2224E 08	1.1056E 04	5.2150E 00	5.9400E 00	3.3861E 01	1.0000E 00	6.9775E 04
14	2.1062E 02	1.1500E 01	3.3035E 05	5.7476E 02	2.4923E 00	3.4695E 00	1.2010E 01	1.0000E 00	2.9620E 03
15	4.3752E 02	7.0000E 00	5.7201E 06	2.3917E 03	5.4664E 00	6.3983E 00	3.8289E 01	1.0000E 00	1.5504E 04
16	5.4838E 02	6.0000E 00	1.1340E 07	3.3675E 03	6.1408E 00	6.4765E 00	3.8964E 01	1.0000E 00	2.1848E 04
UNSECTIONED	1.5482E 03	1.4000E 01	4.8362E 07	6.9543E 03	4.4918E 00	7.1531E 00	6.2609E 01	1.0000E 00	8.5993E 04

ESTIMATED PARAMETERS OF SAMPLE PARAMETERS

PARAMETER	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF VAR	SKEWNESS	KURTOSIS	STD. DEV./MS**5
MEAN	1.5482E 03	1.5408E 03	8.6488E 05	9.2999E 02	6.0069E-01	2.7558E-01	-1.2951E 00	2.3250E 02
MEDIAN	2.0313E 01	1.4250E 01	2.8023E 02	1.6740E 01	8.2412E-01	1.4356E 00	3.0546E-01	4.1850E 00
VARIANCE	4.8637E 07	3.6440E 07	2.6217E 15	5.1203E 07	1.0529E 00	1.5503E 00	1.4675E 00	1.2801E 07
STD. DEV.	6.0664E 03	6.0346E 03	1.2625E 07	3.5532E 03	5.8572E-01	5.5874E-01	-4.7073E-01	8.8030E 02
COEF. VAR.	4.1175E 00	3.7910E 00	1.5503E 00	1.2451E 00	3.0240E-01	4.9658E-01	-1.2011E 00	3.1128E-01
SKEWNESS	4.9343E 00	4.7465E 00	1.4701E 00	1.2125E 00	2.4573E-01	-1.0259E-01	-1.4647E 00	3.0312E-01
KURTOSIS	2.4552E 01	2.1960E 01	1.2484E 02	1.1173E 01	4.5508E-01	4.0534E-03	-1.5801E 00	2.7933E 00

MS=# OF SECTS

Figure 21 - USING SECTIN WITH TELEPHCNE DATA SET 1.

AD-A050 213

NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF

F/G 9/2

CMS HISTOGRAM, DENSITY ESTIMATION AND PROBABILITY PLOTTING ROUT--ETC(U)

DEC 77 6 I DANIKAS

UNCLASSIFIED

NL

2 of 3

AD
A050213



C. SUBROUTINE JACK

1. Description

This routine is used for assessing variability of estimates of data based on the Jackknife method, and for reducing the bias in estimates also. It is particularly useful for data with small sample size.

A big picture of the method has as follows:

Let X_1, X_2, \dots, X_n be a sample of n independent, identically distributed observations from a population with unknown distribution function $F_X(x)$. Also let θ be an unknown parameter of $F_X(x)$ to be estimated. Furthermore suppose a method (biased or unbiased) is available for estimating θ . Then we proceed as follows:

1. Divide the sample size into r disjoint groups, each of size m (r should be such that $n = mr$. If this is not possible some of the last data-points will be discarded.)

2. Compute the estimator $\tilde{\theta}(n)$ of θ based on all $n = mr$ observations.

3. Compute the estimator $\tilde{\theta}_i(n-m)$ based on the $n-m$ observations, having deleted the i^{th} group.

4. Compute the so called 'PSEUDO VALUES' $\hat{\theta}_i(n)$:

$$\hat{\theta}_i(n) = r \tilde{\theta}(n) - (r-1) \tilde{\theta}_i(n-m), \quad i = 1, 2, \dots, r.$$

Then the Jackknife estimator is defined to be the average of Pseudo Values:

$$\begin{aligned}\hat{\vartheta}(n) &= \frac{1}{r} \sum_{i=1}^r \hat{\vartheta}_i(n) \\ &= \frac{1}{r} (r^2 \tilde{\vartheta}(n) - (r-1) \sum_{i=1}^r \tilde{\vartheta}_i(n-m)) \\ &= r \tilde{\vartheta}(n) - \frac{r-1}{r} \sum_{i=1}^r \tilde{\vartheta}_i(n-m) .\end{aligned}$$

Having so defined the Jackknife estimator it can be proved that this estimator is an unbiased estimator of ϑ , except for terms n^{-2} and higher order, assuming that the bias has the form of $E[\tilde{\vartheta}(n)] = \vartheta + a/n +$ terms of higher order in n . That is, the Jackknife estimator eliminates a n^{-1} bias term. Namely if

$$E[\tilde{\vartheta}(n)] = \vartheta + an^{-1} + o(n^{-2}) ,$$

then we have:

$$\begin{aligned}E[\hat{\vartheta}(n)] &= E[r \tilde{\vartheta}(n) - \frac{r-1}{r} \sum_{i=1}^r \tilde{\vartheta}_i(n-m)] \\ &= r E[\tilde{\vartheta}(n)] - \frac{r-1}{r} \sum_{i=1}^r E[\tilde{\vartheta}_i(n-m)] \\ &= r (\vartheta + an^{-1} + o(n^{-2})) - \frac{r-1}{r} (r\vartheta + \frac{ra}{n-m} + \frac{r}{n-m} o(n^{-2})) \\ &= r\vartheta + \frac{ra}{n} + \dots - (r-1) (\vartheta + \frac{a}{m(r-1)} + \dots) \\ &= r\vartheta + \frac{ra}{n} + \dots - r\vartheta - \frac{(r-1)a}{m(r-1)} + \vartheta + \dots\end{aligned}$$

$$= r\theta - r\theta + \frac{ra}{mr} - \frac{a}{m} + \theta + \dots$$

$$= \theta.$$

A special case of Jackknife estimator can be mentioned here, called the 'Complete Jackknife Estimator', where $r=n$ and $m=1$.

Some properties of the Jackknife estimator follow:

1. It is unbiased up to order n^{-2} .
2. The 'Pseudo values' can be used to obtain variance estimates for the Jackknife estimator since they can be considered (see [18]) as approximately independent and identically distributed. Thus $(r(r-1))^{-1} \sum_{i=1}^r (\hat{\theta}_i(n) - \hat{\theta}(n))^2$ should be an approximate estimate of $\text{Var}[\hat{\theta}(n)]$,

and

$$\frac{(\hat{\theta}(n) - \theta) (r(r-1))^{1/2}}{\left\{ \sum_{i=1}^r (\hat{\theta}_i(n) - \hat{\theta}(n))^2 \right\}^{1/2}}$$

should be approximately distributed as a t-statistic with $r-1$ d.f. This procedure is particularly useful if the number n of data-points is small, but it must be used with care.

3. For large n it can be shown that for the complete Jackknife estimator, under very general conditions we have:

$$\text{Var}[\hat{\theta}(n)] \rightarrow \text{Var}[\tilde{\theta}(n)].$$

2. Program Structure

Subroutine 'JACK' is a FORTRAN-callable Subroutine which takes a set of data, groups it into r disjoint groups and for each set of the r -ith group computes and prints in a table the statistics: Mean, Median, Variance, Standard Deviation, Coefficient of Variation, Skewness and Kurtosis. The same statistics are also computed for the ungrouped set of data. Then, using these statistics, the Jackknife estimator of the above parameters are computed and printed in another table along with its Variance and Standard Deviation.

The program is divided into the main program and the Subroutine JACKES. The main program groups the data and successively calls the Subroutine 'JACKES' in order to get the estimates of the above parameters for ungrouped data and for each group as well. A table is then printed containing the estimated parameters. The 'Pseudo-Values' for each parameter are also computed by the main program, and are used to compute the Jackknife estimator for each parameter and its Variance and Standard Deviation.

To avoid division by zero the number of observations must be greater than three, otherwise no output is given. For the same reason the expression $(r-1)(n/r)$ must evaluate to greater than three, otherwise the program will give only estimates of ungrouped data.

A complete description of how 'JACK' operates is given in the Subroutine. A Summary is printed on the terminal by typing DESCRIBE JACK under the CMS environment, which responds:

SUBROUTINE JACK

Jack is intended to estimate the statistics : Mean, Median, Variance, Standard Deviation, Coefficient of Variation, Skewness and Kurtosis of a given set of independent observations, using the Jackknife method. In addition to the above parameters which are given for each group, the Jackknife estimator along with its variance and standard deviation is given for each parameter.

It is called by:

```
CALL JACK ( X, XS, STAT, N, IG )
```

where:

X is the array of data of dimension N
XS is a work array of dimension N (returns ordered data)
STAT is a two-dimensional work array, dimensioned by (IG,7)
N is the number of data-values
IG is the number of groups plus one (=r+1)

Note : r must be such a number as to minimize the number of data-points that will have to be discarded. JACK places the data into the equal size groups discarding any data left over (last observations in order of original input).

No output is expected if $N \leq 3$. Furthermore if $IG \leq 2$ or $(IG-2)(N/(IG-1)) \leq 3$ only estimates for ungrouped data will be given.

More information is given in the Subroutine.

3. Using 'JACK' with Telephone Data 1

To assess the variability in the Mean, Median, Variance, Standard Deviation, Coefficient of Variation, Skewness and Kurtosis of Telephone Data 1 the Subroutine 'JACK' is now used. In order to avoid discarding any of the data-points we use 16 groups with 42 data-points per group.

Observing figure 22 we see that the values of the parameters for ungrouped data are exactly the same as the corresponding values of the same parameters of figure 1 which was produced by using the Subroutine HISTGS/HISTFS. Now in order to assess the variability in any of the above parameters we have to use the t-statistic with (r-1) d.f. along with the values printed under 'ESTIMATED JACKKNIFE PARAMETERS' using:

$$\bar{\theta} \pm St_{(1-\alpha/2), (r-1)}$$

where $\bar{\theta}$ is the Jackknife estimator and S is its Standard Deviation. For example to assess the variability of the Skewness with a confidence level $\alpha = .05$ we have:

$$\bar{\theta} = 7.37321, \quad S = .900118, \quad t_{.975, 15} = 2.131,$$

Therefore a 95% confidence interval of the Skewness is:

$$7.37321 \pm .900118 * 2.131 = 7.37321 \pm 1.91815$$

$$\implies [5.4551, 9.2914] .$$

This should be compared with the point estimate 4.9343 and the confidence interval estimate [4.2884, 5.5802] obtained from subroutine SECTN. The data is so skewed that one will inevitably have trouble here with the Jackknife procedure.

ESTIMATED PARAMETERS

GROUP	MEAN	MEDIAN	VARIANCE	STD. DEV.	COEF. VAR	SKEWNESS	KURTOSIS
1	1.58126E 03	1.50000E 01	4.93184E 07	7.02270E 03	4.44121E 00	7.17459E 00	6.30249E 01
2	1.43720E 03	1.40000E 01	3.93384E 07	6.27203E 03	4.36406E 00	6.52195E 00	5.06898E 01
3	1.53369E 03	1.40000E 01	4.88250E 07	6.98749E 03	4.55601E 00	7.30927E 00	6.47616E 01
4	1.61099E 03	1.40000E 01	5.11798E 07	7.15400E 03	4.44076E 00	6.97807E 00	5.92573E 01
5	1.54717E 03	1.35000E 01	5.01620E 07	7.08251E 03	4.57771E 00	7.14944E 00	6.18267E 01
6	1.48248E 03	1.30000E 01	4.88925E 07	6.99232E 03	4.71663E 00	7.36629E 00	6.51602E 01
7	1.47291E 03	1.30000E 01	4.67577E 07	6.83796E 03	4.64248E 00	7.50811E 00	6.88309E 01
8	1.58551E 03	1.40000E 01	4.90734E 07	7.00524E 03	4.41828E 00	7.18504E 00	6.33708E 01
9	1.55026E 03	1.35000E 01	5.02357E 07	7.08771E 03	4.57196E 00	7.15921E 00	6.17726E 01
10	1.46689E 03	1.40000E 01	4.43761E 07	6.66154E 03	4.54127E 00	7.45724E 00	6.96176E 01
11	1.52105E 03	1.40000E 01	4.76796E 07	6.90504E 03	4.53370E 00	7.32003E 00	6.59486E 01
12	1.59680E 03	1.30000E 01	5.10129E 07	7.14233E 03	4.47291E 00	7.00921E 00	5.97006E 01
13	1.51010E 03	1.60000E 01	4.35998E 07	6.60302E 03	4.37258E 00	7.14932E 00	6.50323E 01
14	1.63606E 03	1.40000E 01	5.14459E 07	7.17258E 03	4.38406E 00	6.92004E 00	5.85264E 01
15	1.62226E 03	1.50000E 01	5.11303E 07	7.15055E 03	4.40775E 00	6.97844E 00	5.93194E 01
16	1.61487E 03	1.50000E 01	5.07497E 07	7.12606E 03	4.41276E 00	7.02911E 00	6.01293E 01
UNGROUPED	1.54822E 03	1.40000E 01	4.83610E 07	6.95427E 03	4.49178E 00	7.15313E 00	6.26085E 01

ESTIMATED JACKKNIFE PARAMETERS

PARAMETER	JACKKNIFE	VARIANCE	STD. DEV. OF JACK. PARAMETER
MEAN	1.54825E 03	8.64880E 05	2.32497E 02
MEDIAN	1.30625E 01	1.56563E 02	3.12812E 00
VARIANCE	4.83451E 07	2.54534E 15	1.26128E 07
STD. DEV.	7.01556E 03	1.38786E 07	9.31350E 02
COEF. VAR.	4.50542E 00	2.42612E 00	3.89400E-01
SKEWNESS	7.37321E 00	1.29634E 01	9.00118E-01
KURTOSIS	6.70793E 01	4.68056E 03	1.71037E 01

Figure 22 - USING JACK WITH TELEPHCNE DATA SET 1.

VI. SINGLE-SERVER FIRST-COME FIRST-SERVED QUEUES WITH
'EARMA' STRUCTURE

A. THE EARMA(P,Q) PROCESS

The starting point for these processes is the definition of a first-order autoregressive model for a stationary sequence of random variables $\{X_i\}$:

$$X_i = rX_{i-1} + \varepsilon_i; \quad i = 0, \pm 1, \pm 2, \dots$$

If the marginal distribution of the X_i is fixed to be exponential with parameter λ for all i ;

$$P\{X_i \leq x\} = 1 - e^{-\lambda x}, \quad \lambda > 0, x \geq 0,$$

then ε_i is zero with probability r and exponential(λ) with probability $1-r$. Thus

$$X_i = \begin{cases} rX_{i-1} & \text{w.p. } r \\ rX_{i-1} + E_i & \text{w.p. } 1-r, \end{cases} \quad (1)$$

where $\{E_i\}$ is a sequence of i.i.d. exponential(λ) random variables. The process defined by (1) is the exponential autoregressive process of order 1, the EAR(1) process. The correlation structure is $r(j) = r^j$, $j = 0, 1, 2, \dots$

The first order moving average exponential process, EMA(1) is defined as

$$X_i = \begin{cases} b E_i & \text{w.p. } b \\ b E_i + E_{i-1} & \text{w.p. } (1-b) \end{cases} \quad (0 \leq b \leq 1; \quad i = 0, \pm 1, \pm 2, \dots)$$

and has correlations $\rho(1) = (1-b)$ and $\rho(1) = 0, j = 2, 3, \dots$

The general EMA(q) model takes the form

$$X_i = \begin{cases} b_q E_i & \text{w.p. } b_{q+1} \\ b_q E_i + b_{q-1} E_{i-1} & \text{w.p. } b_q \\ \dots & \dots \\ b_q E_i + b_{q-1} E_{i-1} + \dots + b_1 E_{i-q+1} & \text{w.p. } b_2 \\ b_q E_i + b_{q-1} E_{i-1} + \dots + b_1 E_{i-q+1} + E_{i-q} & \text{w.p. } b_1 \end{cases} \quad (2)$$

for $0 \leq b_1, b_2, \dots, b_q \leq 1; \quad i = 0, \pm 2, \dots$ where

$$b_i = \begin{cases} b_q & i = q+1, \\ (1-b_q) \dots (1-b_i) b_{i-1} & q \geq i \geq 2, \\ (1-b_q) \dots (1-b_i) & i = 1. \end{cases}$$

The serial correlations are given by the equation

$$\rho^{(q)}(j) = \text{corr}(X_i, X_{i-r}) = \begin{cases} \sum_{v=j}^{q-j+1} b_v b_{v+j} & (1 \leq j \leq q) \\ 0 & (q+1 \leq j < \infty) \end{cases}$$

To convert this EMA(q) process into a mixed autoregressive moving-average exponential process of orders 1 and q, called EMA(1,q) we replace E_{i-q} in (2) with

$$A_{i-q} = \begin{cases} rA_{i-q-1} & \text{w.p. } r \\ rA_{i-q-1} + E_{i-q} & \text{w.p. } (1-r) \end{cases}$$

For further results on these processes and their properties see Gaver and Lewis (1978), Lawrance and Lewis (1977), Jacobs and Lewis (1977), and Lawrance and Lewis (1978).

E. USE OF AUTOREGRESSIVE (EAR(P)), MOVING AVERAGE (EMA(Q)) AND MIXED STRUCTURES (EARMA(P,Q)) IN MODELLING QUEUES

Consider for simplicity a queue with a single input stream and a single server, and a first-come-first-served (FIFO) service discipline. Let S_i , $i = 0, 1, 2, \dots$, denote the service time for the i^{th} arrival, and let X_i , $i = 1, 2, \dots$, denote times between arrival of the i^{th} and $(i-1)^{\text{st}}$ customers. As is usual we assume that the first customer (with service time S_0) arrives at time zero and finds the queue empty.

If the $\{S_i\}$ and $\{X_i\}$ sequences are i.i.d. exponential random variables with parameters λ and α respectively, we have the M/M/1 queue.

Now let

E_i be exponential (λ) and independent, $i = 0, +1, \dots$;

\mathcal{E}_i be exponential (α) and independent, $i = 0, +1, \dots$.

We want to model queues with correlated (autocorrelated and/or cross-correlated) service and inter-arrival times, the service and inter-arrival times both having marginally exponential distributions. We also want the queuing model to include the M/M/1 queue as a special case.

There are five simple possibilities which we put forward here, based on the use of EARMA(p,q) processes, giving what we call an EARMA MD/MD/I queue.

In what follows let p_s and p_x be, respectively, the order of the autoregressive components of the service time sequence $\{S_i\}$ and the inter-arrival time sequence $\{X_i\}$, and let q_s, q_x be, respectively, the order of the moving average component of $\{S_i\}$ and $\{X_i\}$. These parameters can take values $0, 1, \dots$. If $p = 0, q = 0$, the sequence is independent; if $p = 0$, the process is purely moving average and if $q = 0$, then the process is purely autoregressive.

The five possibilities or cases are as follows:

- 1) Let $\{S_i\}$ be EARMA(p_s, q_s) over $\{E_i, E_{i-1}, \dots\}$;
let $\{X_i\} = \{\mathcal{E}_i\}$.

Thus the arrivals are a Poisson process and the service times are autocorrelated, i.e. a mixed autoregressive moving average exponential sequence.

- 2) Let $S_i = E_i$, $i = 0, 1, 2, \dots$;
 let X_i be EARMA(p_X, q_X) over $\{\xi_i, \xi_{i-1}, \dots\}$.

Then the service times are independent and the arrival process is non-Poisson because of the dependency between the inter-arrival times, i.e. the arrival process is a point process with EARMA(p, q) structures.

- 3) Let $\{S_i\}$ be EARMA(p_S, q_S) over $\{E_i, E_{i-1}, \dots\}$.
 Let $\{X_i\}$ be EARMA(p_X, q_X) over $\{\xi_i, \xi_{i-1}, \dots\}$.

The service and arrival processes are autocorrelated, but the two processes are independent. The marginal distributions of $\{S_i\}$ and $\{X_i\}$ are still exponential.

- 4) To couple the two processes, with resultant dependence in the arrival and service processes, the simplest procedure seems to be the following.

Let $\{S_i\}$ be EARMA(p_S, q_S) in the following sense:

$\{S_i\}$ is EARMA(p_S, q_S) over $\{E_i, \frac{\alpha}{\lambda} \xi_i, \frac{\alpha}{\lambda} \xi_{i-1}, \dots\}$.

Then if $X_i = \xi_i$, $i = 0, 1, 2, \dots$ we have that $\{S_i\}$ is an autocorrelated sequence and also cross-correlated with $\{X_i = \xi_i\}$, i.e. $\{S_i, X_i\}$ is a bivariate dependent sequence of random variables with exponential marginal distributions, although $\{X_i\}$ is still Poisson. Note that the S_i sequence is an autocorrelated sequence because of the cross-coupling, but not a pure

EARMA(p_s, q_s) sequence. It does have an exponential marginal distribution however.

As an example let $p_s = 1, q_s = 1$.

Let

$$S_0 = E_0$$

$$S_1 = b_S E_1 \quad \text{w.p. } b_S \quad A_1 = \frac{\alpha}{\lambda} \varepsilon_1$$

$$= b_S E_1 + A_1 \quad \text{w.p. } (1-b_S)$$

$$S_2 = b_S E_2 \quad \text{w.p. } b_S \quad A_2 = rA_1 \quad \text{w.p. } r$$

$$= b_S E_2 + A_2 \quad \text{w.p. } (1-b_S) = rA_1 + \frac{\alpha}{\lambda} \varepsilon_2 \quad \text{w.p. } (1-r)$$

.

.

.

$$S_i = b_S E_i \quad \text{w.p. } b_S \quad A_i = rA_{i-1} \quad \text{w.p. } r$$

$$= b_S E_i + A_i \quad \text{w.p. } (1-b_S) = rA_{i-1} + \frac{\alpha}{\lambda} \varepsilon_i \quad \text{w.p. } (1-r)$$

.

.

.

In these equations one can replace ε_i with X_i and then the cross-coupling between the sequences $\{S_i\}$ and $\{X_i\}$ becomes apparent.

Interpretation. We have positive correlation between S_i and q_s previous inter-arrival times (when $p_s = 0$). If

the ξ_j 's ($j = i, i-1, \dots, i-p_S-1$) are short, then S_i will be short if all the q_S previous inter-arrival times are short. This models the case where the server tends to speed up if the queue gets long. Of course he also slows down when the queue gets short and it is not immediately clear what the effect on an average waiting time will be.

- 4') Similar to 4 but $\{X_i\}$ is EARMA(p_X, q_X) over $\xi_i, \frac{\lambda E_i}{\alpha_i}, \frac{\lambda E_{i-1}}{\alpha_{i-1}}, \dots$ and $S_i = E_i$.

Interpretation is not clear, but it would have the same effect as balking in the input stream. If service times get long (and presumably the queue gets long) then inter-arrival times get long.

- 5) A more general possibility which allows one to model dependency in the input stream, in the service stream and dependency which couples them is as follows:

Let $\{D_i\}$ be a unit exponential independent sequence, a driving sequence.

Let $\{E_i\}$ be EARMA(p_S, q_S) over $\{E_i, E_{i-1}, \dots\}$.

Let $\{X_i\}$ be EARMA(p_X, q_X) over $\{\xi_i, \xi_{i-1}, \dots\}$.

Let $\{S_i\}$ be EARMA(p_{c_1}, q_{c_1}) over $\{E_i, \lambda D_i, \lambda D_{i-1}, \dots\}$.

Let $\{X_i\}$ be EARMA(p_{c_2}, q_{c_2}) over $\{X_i, \alpha D_i, \alpha D_{i-1}, \dots\}$.

This is a very general scheme which reduces to all of the previous schemes as special cases. For example if

$p_{c_1} = q_{c_1} = p_{c_2} = q_{c_2} = 0$, then we have case 3. If $q_{c_2} = 1$, $p_{c_2} = 0$, $q_{c_1} = 0$, $p_S = q_S = 0$; $p_X = 0$ (and p_{c_1} and q_{c_1} are not equal to zero) we have case 4.

In all cases the basic equation for M/M/1 queue waiting time W_n still holds:

$$W_0 = 0$$

$$W_{n+1} = (W_n + S_n - X_{n+1})^+ \quad n = 1, 2, \dots$$

These equations can be used to generate successive W_n 's in a simulation. We have not touched on how the correlated sequences are started. This is somewhat arbitrary as it is not known (except for the EMA(1) and MA(1) processes) how to start them so as to produce a stationary exponential sequence. The problem is aggravated, as in cases 4, 4' and 5, when we are dealing with bivariate exponential sequences. Then the marginal processes may be stationary but not the bivariate process.

C. PROGRAM STRUCTURE

1. General

To simulate the EARMA(p,q) model, a FORTRAN Program has been written to take care of all the existing cases of that model and also of the M/M/1 queue. Depending on the input values which the parameters (as described below) will take, the program can be used for creating in each run a particular case of the EARMA(p,q) queue. Because of that generality of the program it is not suggested that it be used for those EARMA cases where either the arrival times or the service times or both are not autocorrelated or cross-correlated. The reason for this suggestion is efficiency. It is suggested for use with the EARMA(p,q) cases where both arrival and service times are autocorrelated and, furthermore, cross-correlated with any order. The program is divided into the main program and into the Subroutines BETAS, AUTOR and EARMA.

2. Main Program

The main program performs all I/C operations and calculations required for the desired statistics. Specifically it is designed:

a) To read

1. All random number generator seeds required to generate the sequences of the Exponential and Uniform variates to be used during the execution.

2. The values for the variables:

N : Number of arrivals to be generated.

M : Number of replications.
RX, RS : Arrival rate, service rate.
CFX, CPS, CPG : Taking the values 1 or 0 (1 for the coupled process).
KX1, FX, KS1, KS : The order of moving average for arrival and service processes.
QX1, QX, QS1, QS : Taking the values 1 or 0 (1 for autoregressive case).
EX, BX1, BS, BS1 : Arrays with the values of β 's for moving average part.
RHOX1, RHOX, RHCS1, RHOS : Parameter values for the autoregressive parts.

b) To calculate and print

SUMX(I,J); SUMXM(I,J);
 SUMS(I,J); SUMSM(I,J);
 W(I,J); WM(I,J);
 WB(I,J); WMB(I,J);
 D(I,J); DM(I,J);
 for I = 1,2,3,4, J = 1,2,...,M,

where:

(In what follows k gets the values N/4, N/2, 3N/4, N for I=1,2,3,4 respectively and J=1,2,...,M)

$$\text{SUMX}(I,J) = \sum_{r=1}^k X_r^{(J)}, \quad X_r^{(J)} \text{ the } r^{\text{th}} \text{ arrival for}$$

the J^{th} replication in the EARMA queue;

$$\text{SUMXM}(I,J) = \sum_{r=1}^k (XM)_r^{(J)}, \quad (XM)_r^{(J)} \text{ the } r^{\text{th}} \text{ arrival}$$

for the J^{th} replication in M/M/1 Queue;

$$\text{SUMS}(I,J) = \sum_{r=1}^k S_r^{(J)}, \quad S_r^{(J)} \text{ the } r^{\text{th}} \text{ service for}$$

the J^{th} replication in the EARMA queue;

$$\text{SUMSM}(I, J) = \sum_{r=1}^k (\text{SM})_r^{(J)}, \quad (\text{SM})_r^{(J)} \text{ the } r^{\text{th}} \text{ service}$$

for the J^{th} replication in M/M/1 Queue;

$$W(I, J) = W_k^{(J)} = \max\{ (W_{k-1}^{(J)} + S_{k-1}^{(J)} - X_k^{(J)}), 0 \}$$

= waiting time of k^{th} arrival in J^{th} replication of the correlated queue;

$$WM(I, J) = (WM)_k^{(J)} \quad \text{The same as } W(I, J) \quad \text{but}$$

for M/M/1 queue;

$$WB(I, J) = \sum_{r=1}^k W_r^{(J)} / k$$

$$WME(I, J) = \sum_{r=1}^k (WM)_r^{(J)} / k$$

$$D(I, J) = \{ \text{SUMS}(I, J) - \text{SUMX}(I, J) \} / K - 1/RS + 1/RX$$

$$DM(I, J) = \{ \text{SUMSM}(I, J) - \text{SUMXM}(I, J) \} / K - 1/RS + 1/RX$$

$$\text{SUMSM}(I, J) = \sum_{r=1}^k (\text{SM})_r^{(J)},$$

Note: Between the group-variables (KX1, KX, KS1, KS), (QX1, QX, QS1, QS), (CPX, CPS, CFG), (RHOX1, RHCX, RHOS1, RHCS) and (BX1, EX, BS1, BS) there are the following relations:

1. The number of elements of each variable (array) from B-group should be the same as the value of the corresponding variable from K-group.

2. The '0' value of a variable from Q-group implies that the corresponding variable from R-group is not needed.

3. The value '1' of CPG dominates any value of CFX, or CFS.

Any valid combination in the values of the above variables specially of the Q-group, C-group and K-group (0 or not 0), generates a particular EARMA(p,q) case and is accepted by the program.

The basic approach in the main program is to generate at once and for each replication three independent exponential sequences with parameters λ (arrival rate), μ (service rate) and 1 (unit exponential) which are stored in the arrays EXP λ , EXP μ and EXP1 respectively. Then the Subroutine AUTOR which is involved with the autoregressive part of the model is called (if it is needed for the particular EARMA case). A loop follows which is executed as many times as the value of N. From the loop the EARMA Subroutine is called which is involved with the moving average part of the model. The number of calls of Subroutine EARMA depends on the particular case which is being run. The statistics and all information that will help us to analyze the model are also computed inside the loop and stored in the arrays described above. The program continues execution until the desired statistics for all M replications have been calculated and gathered. Then the output part of the program follows which gives us the values of calculated statistics on paper and on punched cards for analysis.

3. Subroutine BETAS

This is a simple Subroutine and its purpose is to return an array SUMBX1/X/S1/S with element-values as follows:

$$\text{SUMBX}(I) = \sum_{r=1}^I \text{EX}(r), \quad I = 1, 2, \dots, \text{value of K-group}$$

to be used for the choice of the order in the moving average. It is called at most 4 times during the execution and only if the K-group variables have value greater than '0'.

4. Subroutine AUTCR

'AUTCR' accepts in each call an exponential sequence (generated previously) of variates and transforms it into an autocorrelated sequence. It is called only if any of the Q-group variables equals 1. For each Q-group variable having value 1 it is called M times.

5. Subroutine EARMA

This is the main Subroutine of the program and it has been created so that it can be used for all EARMA(p,q) cases. Because of the generality quite a few parameters are transferred and in each call just one value is returned. This value is the I^{th} arrival or the I^{th} service time as it has been modified because of the moving average. It is called only if the particular EARMA(p,q) case requires moving average; that is only if any of the K-group variables is greater than 0.

6. Time requirements

Some CPU-times have been gathered in the course of the simulation of some EARMA(p,q) cases, using the IBM 360/67, FORTRAN H compiler as follows:

N	M	K-GROUP	QS1	C-GROUP	TIME
2000	500	0	1	0	9 Min. 51 Sec.
5000	500	0	1	0	22 Min. 25 Sec.
10000	500	0	1	0	43 Min. 30 Sec.
10000	500	0	1	CPS=1	55 Min. 20 Sec.

From these results we see that the value of N is the main factor that affects the CPU-time (linearly). We see also that one of the C-group variables may increase the CPU-time by 25%. We ignore the factor of M (it is another main factor as N is) because we may stop the program at any replication, getting the last random number generator seeds and then continuing another time using these seeds.

D. ANALYSIS OF SIMULATION RESULTS

Since no analytical properties of the EARMA(p,q) models can be derived, a simulation has been done to study their properties. But as we have noticed, this program is a general program accomodating all EARMA(p,q) cases. Since our study is limited to only the two cases below, in order to save CPU-time a modified program has been used that simulates these particular two cases.

1. Queue with Dependent Service

The model:

$$\begin{aligned} \{X_i\} &= \xi_i \\ \{S_i\} &\text{ is EARMA}(1,0) \quad \text{over} \quad \{E_i, E_{i-1}, \dots\} \end{aligned}$$

That is:

$$S_i = \begin{cases} rA_{i-1} & \text{w.p. } r \\ rA_{i-1} + E_i & \text{w.p. } (1-r) \end{cases} \quad \text{where } A_0 = E_0$$

To simulate this model we chose the values .25, .50, .95, .99 for traffic intensity t and the values .25, .50,

.90, .95, .98 for the correlation r , in order to cover a representative range. In what follows the string $S\#tt\#rr$ stands for the run with $t=tt$ and $r=rr$. That is the run $S\#25\#50$ stands for the run with $t=.25$ and $r=.50$. For each t and r we ran the program and the required statistics were gathered, analyzed and plotted using the Subroutines HISTGS/FS, NCRMPL and EXPLT. The sample statistics from HISTGS/FS were also tabulated separately from the figures so that one could obtain an overall picture of whether or not the waiting times had converged in mean and in distribution to the limiting distribution.

From the analysis of all runs and plots we have the following results:

1. Simulating the type $S\#25\#rr$ and $S\#50\#rr$ model (with $r = .25, .50$ and $.90$) it was possible for the \bar{W} and \bar{W} to reach the steady state for $N=2000$. On the other hand for the type $S\#25\#rr$ and $S\#50\#rr$ (with $rr = .95, .98$) it was necessary to go up to $N=10000$, in order for the \bar{W} and \bar{W} to reach the steady state. Thus we see that the high correlation affects the choice of the value of N , requiring N to increase as the correlation is increasing. But not only the correlation affects the choice of N . The traffic intensity affects it much more, since for the type $S\#95\#rr$ and $S\#99\#rr$ the \bar{W} and \bar{W} do not reach the steady state for $N=10000$ even if r is low. Because of the CPU-time requirements we restricted ourselves to a detailed study of the particular case $S\#99\#98$ increasing successively the value of N up to 320,000, where the \bar{W} and \bar{W} appear to be close to the steady state. Thus we may conclude that high traffic intensity and/or high correlation require a large value of N in order to achieve steady state. Figures 23a through 23d, 24a-24c, 25 and 26a-26d justify the above conclusions. In figures 23a-23d, where we are dealing with

the waiting time of the 2000th (W_{2000}) arrival for the case $t=0.50$, $r=0.25$, we can see that W_{2000} has converged to a value of about 6.3, and furthermore a straight line appears in the plot under EXPLT. We also see approximate convergence for the correlated queue for the \bar{W}_{2000} , which is presented in figures 28a-28c, and its value has converged to a value of about 3.03 (figure 23d). Note that the introduction of correlation into the service time has increased the average waiting time from 2.5 to about 3.03, or by approximately 20%.

We cannot say the same for figures 24a-24c where, because of the high correlation ($r=0.98$) we do not have convergence of W and \bar{W} , although $N=10,000$, and for figure 25 where, because of the high traffic intensity ($t=0.99$) we do not have convergence of W and \bar{W} . Figures 26a-26d also show us the non-convergence of W , \bar{W} , because of the high correlation and the high traffic intensity ($t=0.99$, $r=0.98$). This is the case for which we eventually had to carry the simulation out to about $N=500,000$ to observe a stationary queue.

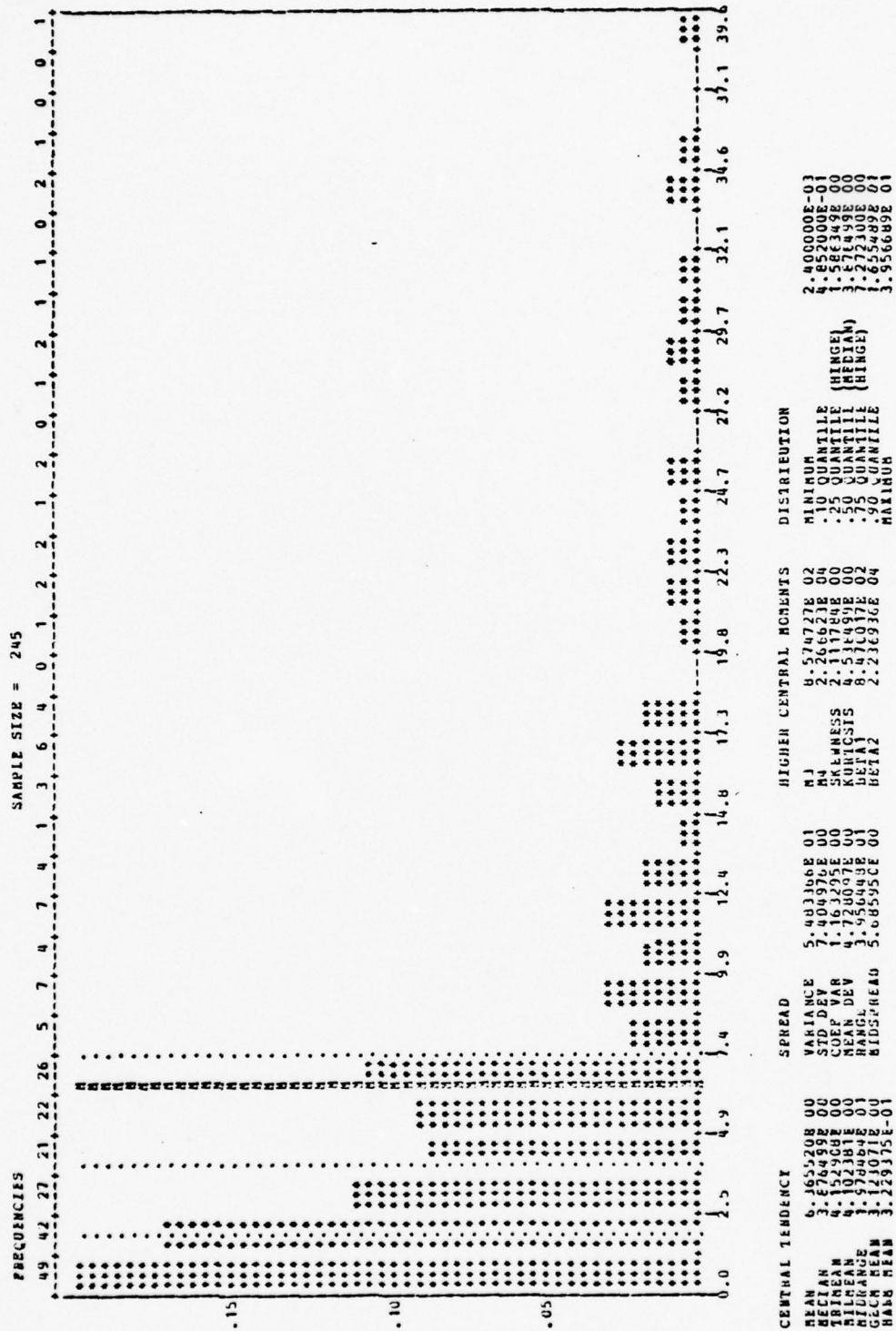


Figure 23a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE WAITING TIMES WITHOUT ZEROS FROM THE RUN S#50#25; m=500 REPLICATIONS, RX=.2, RS=.4. No OF ZEROS=255.

NUMBER OF ORDERED FAIRS = 245

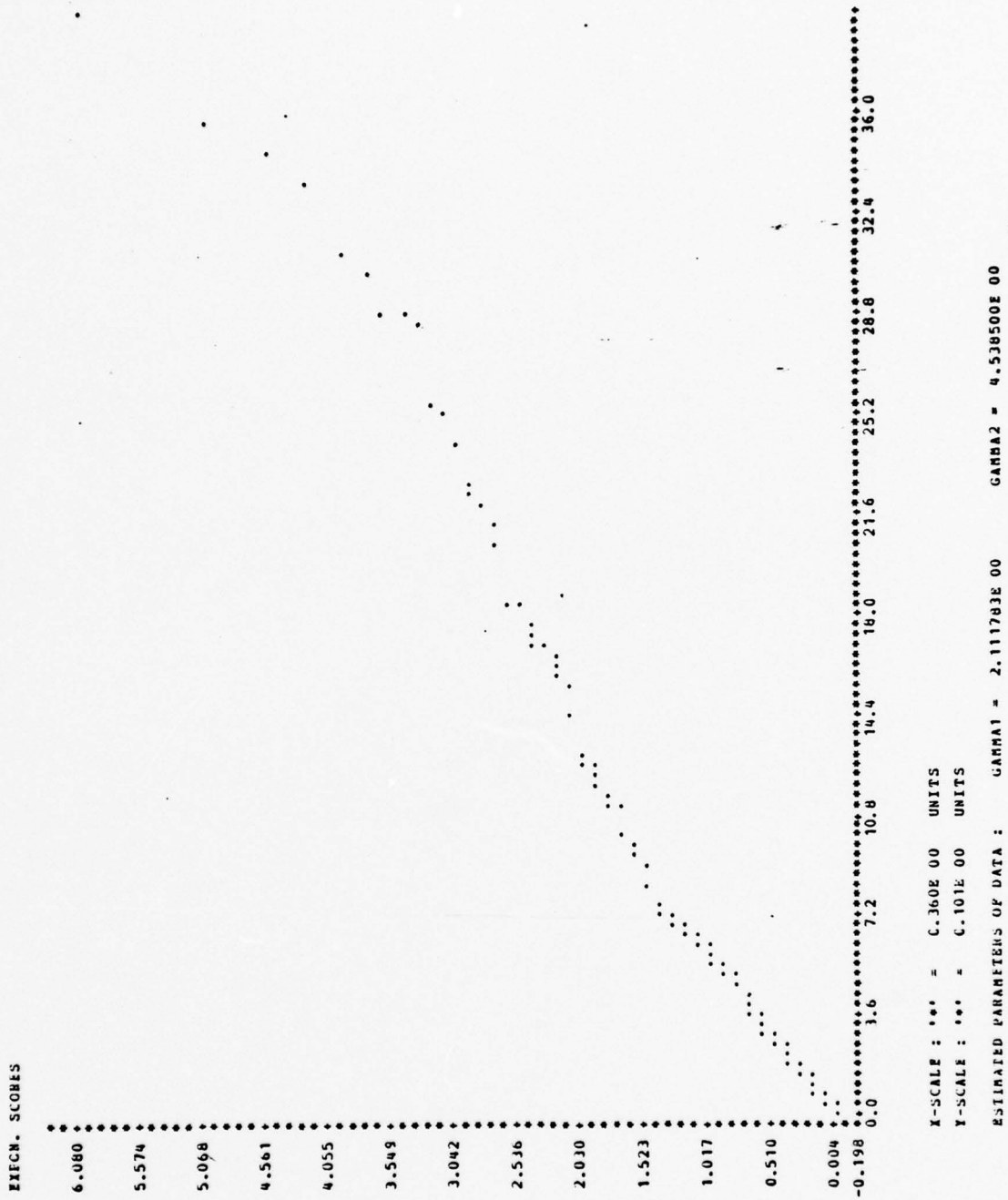


Figure 23t - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PICT (EXPLT) OF THE WAITING TIMES W_{2000} WITHOUT ZEROS FROM THE RUN S#50#25; $m=500$ REPLICATIONS, $R_X=.2$, $R_S=.4$. No OF ZERCS=255.

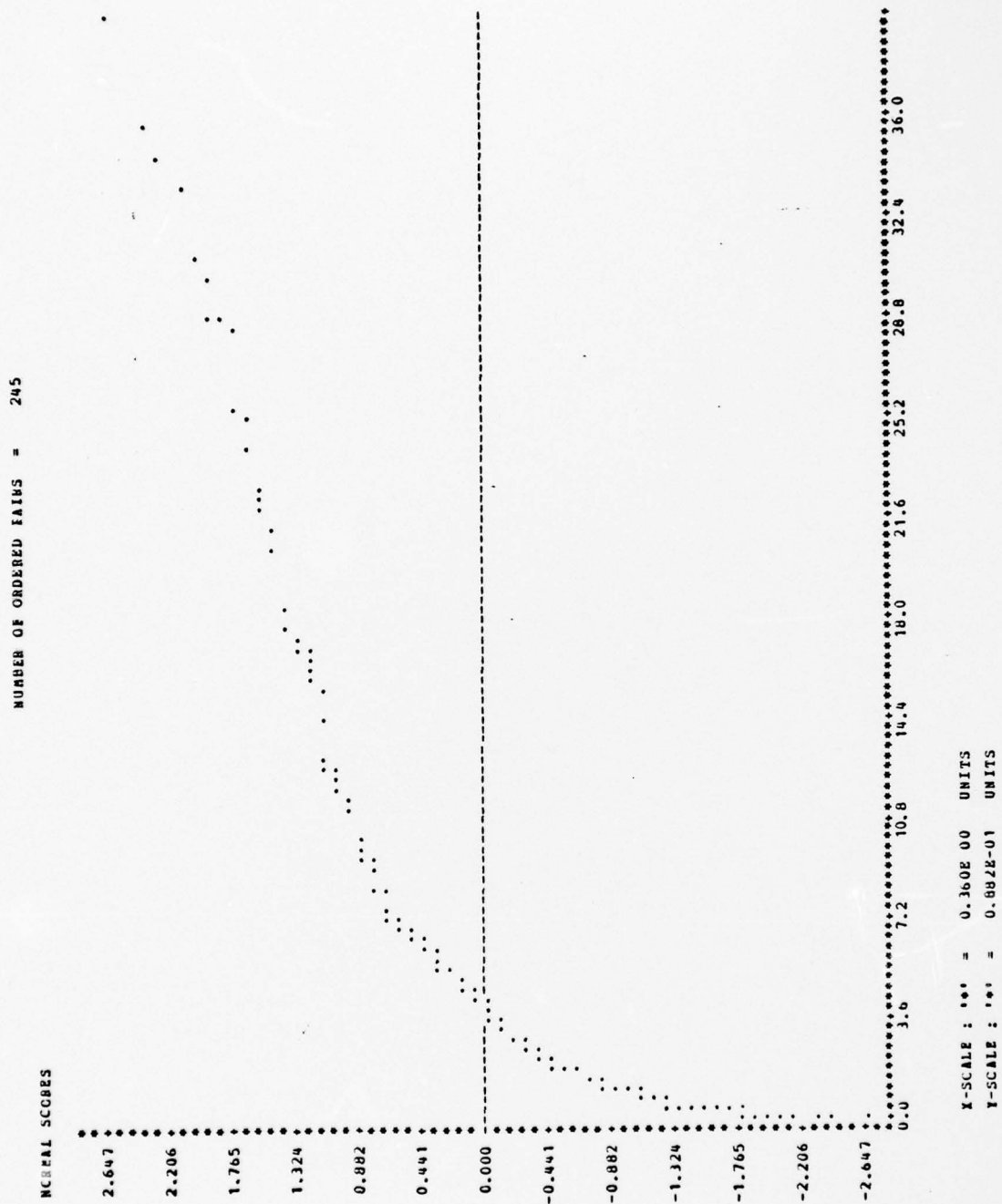


Figure 23c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE WAITING TIMES W_{2000} WITHOUT ZEROS FROM THE RUN S#50#25; $m=500$ REPLICATIONS, $R_X=.2$, $R_S=.4$. NO OF ZEROS=255.

CCORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	5001.965	9598.750	2498.716	4992.211	3.07756	6.08216	3.06554	6.01085	3.12886	6.28286	3.11910	6.36552
S (MEAN)	7.0361	5.0354	4.5052	6.5871	0.24952	0.41368	0.26895	0.45710	0.24779	0.41009	0.27188	0.47309
ST. DEV.	157.3321	202.0377	100.7388	147.2915	5.57942	6.57991	6.01397	7.29935	5.54076	6.47113	6.07936	7.40498
SKEWNESS	-0.2306	-0.2395	0.0667	0.2586	4.06781	3.55424	3.42195	2.55988	2.43921	1.61747	3.03096	2.11178
KURTOS.	0.1442	0.1382	-0.0233	0.1448	31.44217	24.58147	17.09064	10.08711	6.44455	2.54303	10.37912	4.53850
SHEL SIZE	500.00000	0.0	500.00000	500.00000	500.00000	253.00000	500.00000	255.00000	500.00000	249.00000	500.00000	245.00000

PARAMETER	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	3.0040035	3.0257206	3.0361691	3.0264702	-0.0032524	-0.0003933
S (MEAN)	0.0384256	0.0288684	0.0229700	0.0199248	0.0082827	0.0060741
ST. DEV.	0.8592227	0.8455169	0.5136244	0.4456213	0.1852060	0.1358213
SKEWNESS	1.2956314	1.0151809	0.8071278	0.7198124	0.1208596	0.0878487
KURTOS.	2.9877872	1.3304682	0.8637352	0.7924871	-0.1567421	0.3080139
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

UNCORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	5001.965	9598.750	2499.844	4999.156	2.50055	5.00109	2.57197	5.14395	2.57820	5.13586	2.72569	5.14280
S (MEAN)	7.0361	5.0354	3.5683	5.1405	0.19253	0.31361	0.21395	0.36101	0.19706	0.31915	0.20248	0.31484
ST. DEV.	157.3321	202.0377	79.7889	114.9458	4.30517	4.95860	4.78403	5.70811	4.40636	5.05628	4.52758	5.12527
SKEWNESS	-0.2306	-0.2395	0.0893	0.1489	2.72664	2.06557	3.80018	3.13685	2.40065	1.65638	2.46332	1.82443
KURTOS.	0.1442	0.1382	-0.0001	-0.0751	10.26487	6.32769	24.89131	17.73781	6.42046	2.84681	6.98296	3.81722
SHEL SIZE	500.00000	0.0	500.00000	500.00000	500.00000	250.00000	500.00000	250.00000	500.00000	251.00000	500.00000	265.00000

PARAMETER	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	2.4863586	2.4965744	2.5005102	2.4961290	-0.0021236	0.0030810
S (MEAN)	0.0275175	0.0194881	0.0155533	0.0144737	0.0078315	0.0056511
ST. DEV.	0.6153103	0.457680	0.3477832	0.3012817	0.1751172	0.1263634
SKEWNESS	1.1364145	0.8768911	0.6178330	0.4541365	0.2281407	0.1533930
KURTOS.	1.8650570	0.9444056	0.3653927	0.1611725	-0.0018263	0.5103149
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

Figure 23D - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND ECISCON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE LISTIFICATIONS OF CORRELATED INTERARRIVAL TIMES AT N=1000 AND N=2000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZEROS (M 1 + OS, M 1, I FOR CASE N=500, 1000, 1500, 2000), CUMULATED WAITING TIMES (M 1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S2 AND X2 (D2); OBTAINED FROM n=500 REPLICATIONS OF THE RUN S450#25; EX=-2; RS=.4 .

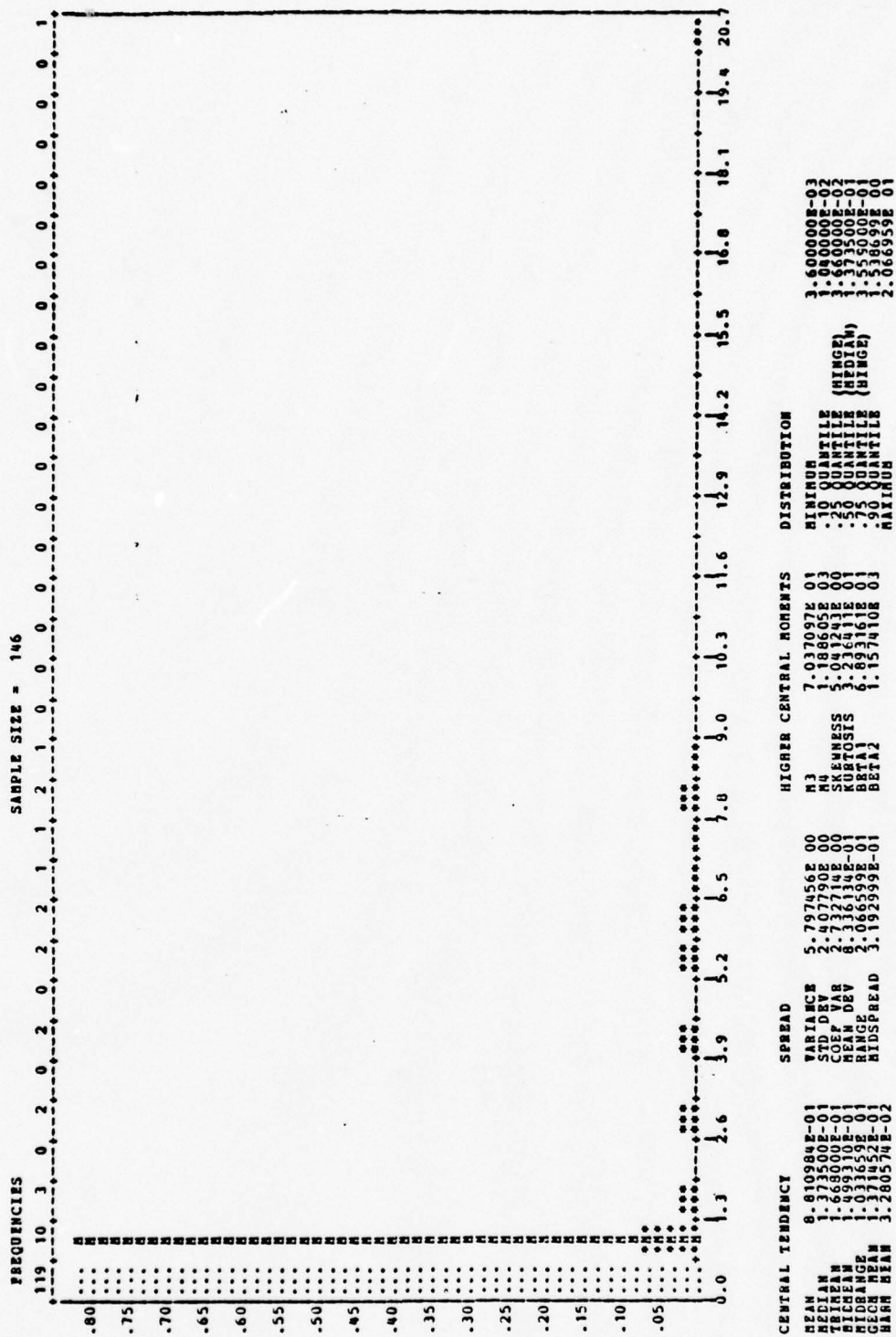


Figure 24a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE WAITING TIMES WITHOUT ZEROS FROM THE RUN S#25#98; m=500 REPLICATIONS, RX=2.5, RS=10; NO OF ZEROS=354.

MURUM OF ORDERED FAIRS = 146

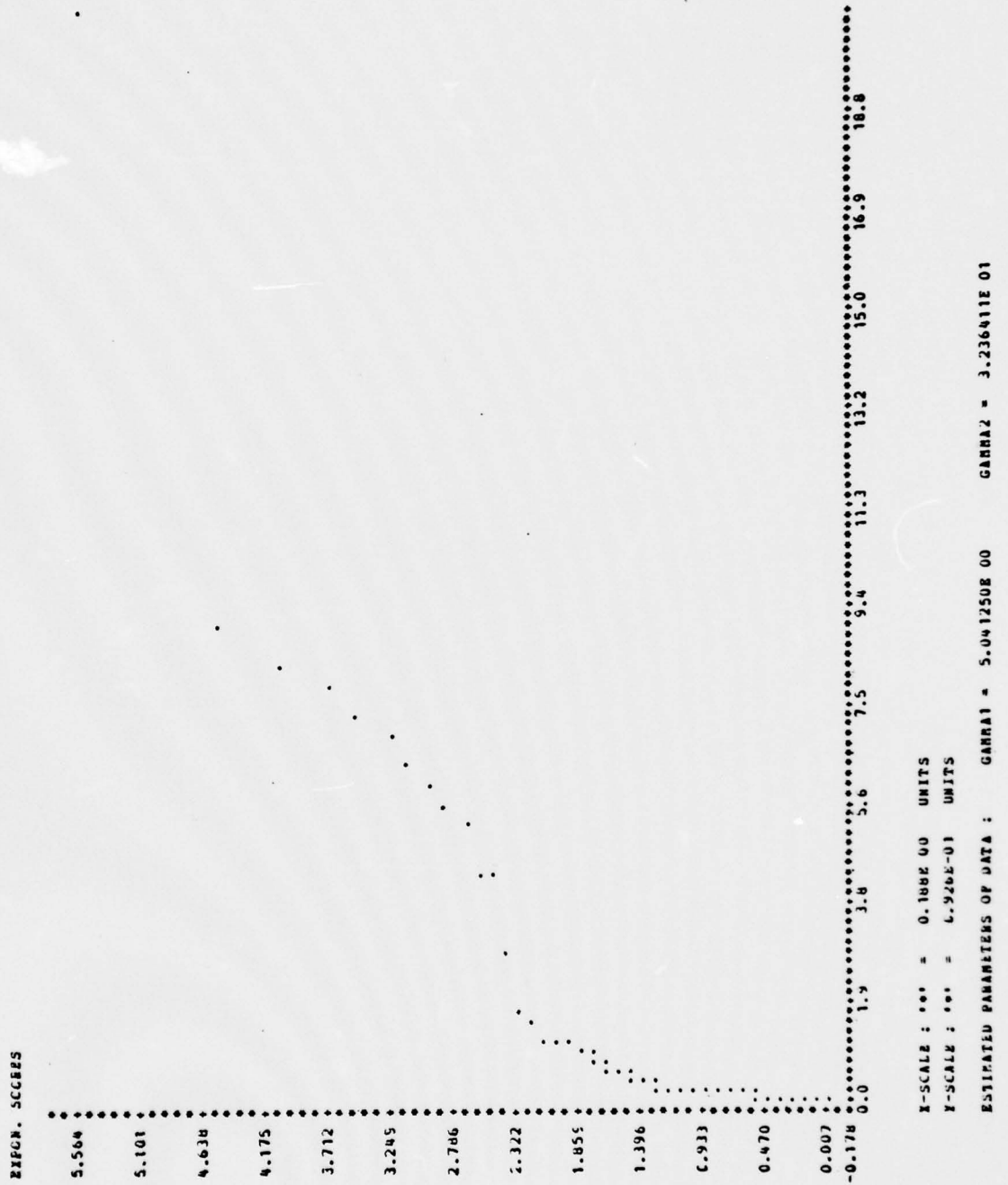


Figure 24t - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PICT (EXPLT) OF THE WAITING TIMES M_{10000} WITHOUT ZEROS FROM THE RUN S#25#98; $m=500$ REPLICATIONS, $R_X=2.5$, $R_S=10$; No OF ZEROS=354.

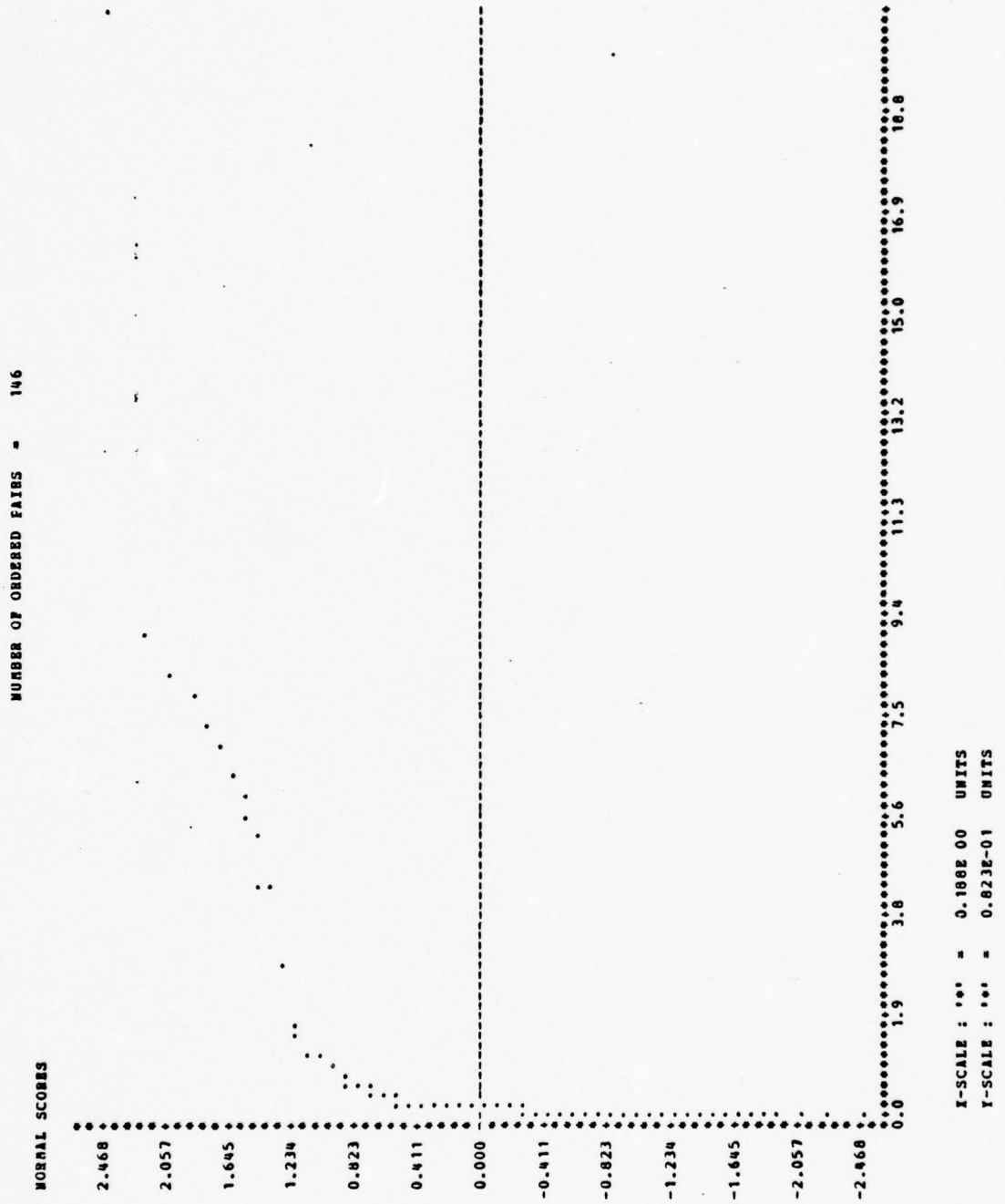


Figure 24c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE WAITING TIMES W_{10000} WITHOUT ZEROS FROM THE RUN S#25#98; $n=500$ REPLICATIONS, $RX=2.5$, $RS=10$; No OF ZEROS=354.

CORRELATED QUEUE

PARTS	X 2	X 4	S 2	S 4	M 1 + 05	M 1	M 2 + 05	M 2	M 3 + 05	M 3	M 4 + 05	M 4
MEAN	1959.456	1559.234	495.182	993.617	0.18153	0.78246	0.10872	0.42469	0.11847	0.59834	0.25728	0.88110
S (MEAN)	1.2384	1.7382	2.5624	4.3755	0.05074	0.20991	0.02280	0.08739	0.03447	0.16622	0.06075	0.19927
ST. DEV.	27.0907	38.8671	60.2412	57.8387	1.14456	2.26080	0.53219	0.96873	0.77081	1.65365	1.35647	2.40779
SKEWNESS	-0.0261	-0.2054	0.2000	0.2402	11.70774	5.67824	9.35595	4.79691	11.72616	5.23263	9.36337	5.04124
KURTOS.	0.1781	0.1013	-0.0917	0.1235	159.47882	36.05627	101.97438	25.38643	155.19870	29.24665	113.06558	32.36411
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	116.00000	500.00000	128.00000	500.00000	99.00000	500.00000	146.00000

PARTS	D 1	D 2	D 3	D 4
MEAN	0.106366E	0.1690013	0.1701906	0.1707705
S (MEAN)	0.008341E	0.0068714	0.0056008	0.0047813
ST. DEV.	0.1865285	0.1536453	0.1252386	0.1069135
SKEWNESS	3.7013730	3.3361349	2.6342487	2.0241365
KURTOS.	23.4013167	17.2875519	10.3973818	5.6284195
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000

UNCORRELATED QUEUE

PARTS	X 2	X 4	S 2	S 4	M 1 + 05	M 1	M 2 + 05	M 2	M 3 + 05	M 3	M 4 + 05	M 4
MEAN	1959.496	1559.234	495.456	998.720	0.02767	0.11245	0.02637	0.11986	0.04066	0.16530	0.04013	0.14645
S (MEAN)	1.2384	1.7382	0.3155	0.4621	0.00305	0.00873	0.00310	0.01112	0.00513	0.01742	0.00432	0.01165
ST. DEV.	27.6907	38.8671	7.0557	10.3337	0.06815	0.09685	0.07377	0.11664	0.11920	0.15324	0.09664	0.13631
SKEWNESS	-0.0261	-0.2054	0.2372	0.0994	3.11780	1.21369	4.00196	1.76561	4.78121	2.43433	3.59277	1.95713
KURTOS.	0.1781	0.1013	0.0277	0.0293	10.78555	1.41914	19.01723	3.32073	29.88892	7.72525	17.81947	6.29365
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	123.00000	500.00000	110.00000	500.00000	123.00000	500.00000	137.00000

PARTS	D 1	D 2	D 3	D 4
MEAN	0.0312546	0.0332368	0.0312770	0.0312694
S (MEAN)	0.0001375	0.0000957	0.0000833	0.0000733
ST. DEV.	0.003083E	0.0021191	0.0018641	0.0016391
SKEWNESS	0.4119023	0.3522976	0.2679055	0.3216345
KURTOS.	0.4359894	0.0770941	-0.0173473	-0.0419693
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000

Figure 24d - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE LISTED STATISTICS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZERCS (M 1 + 0, M 1, I FOR CASE N=2500, 5000, 7500, 10000), CUMULATED WAITING TIMES (M 1 BAR ETC.) AND THE AVERAGED DIFFERENCES BETWEEN S2 AND X2 (D2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN S42549E; BX=2.5; RS=10.

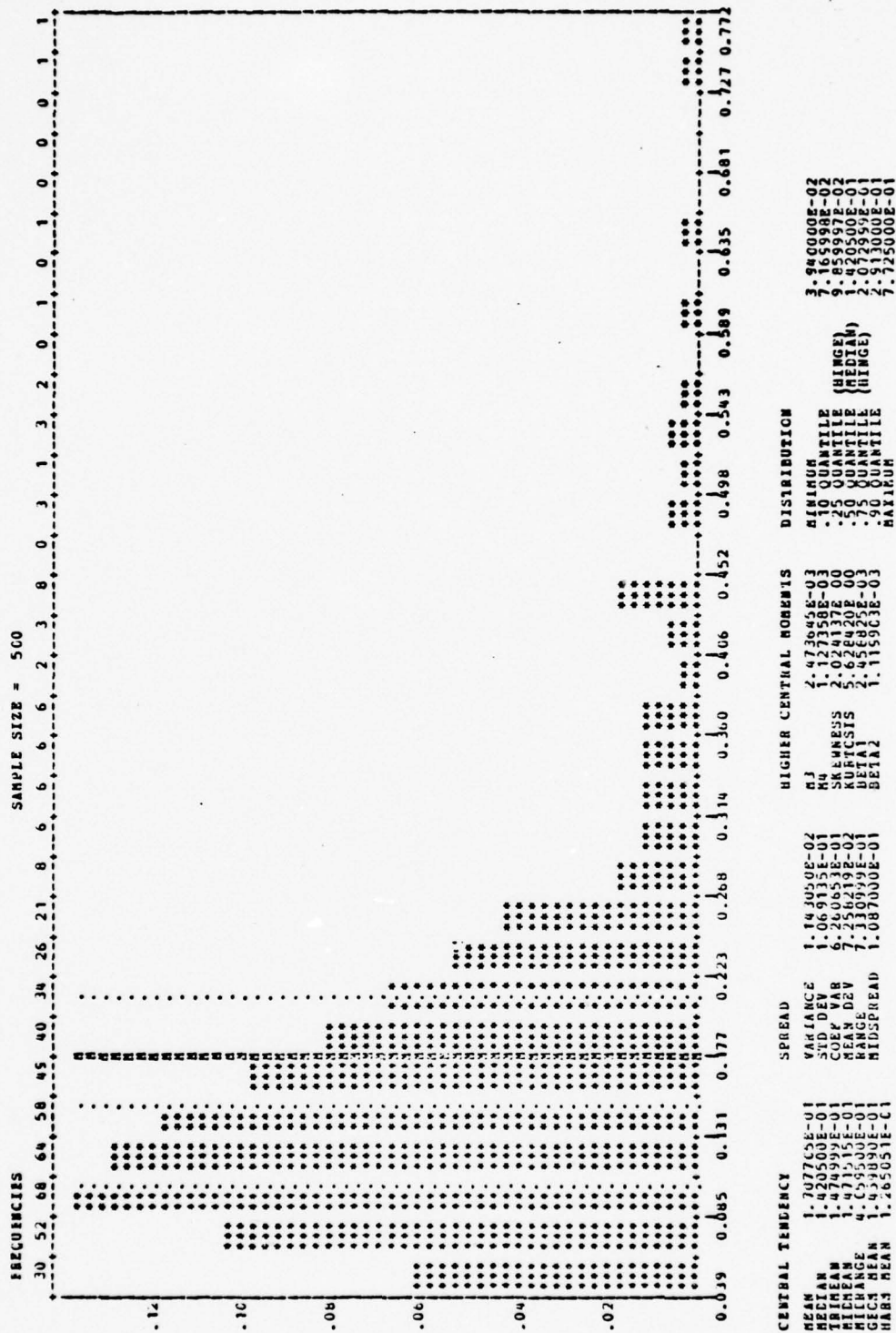
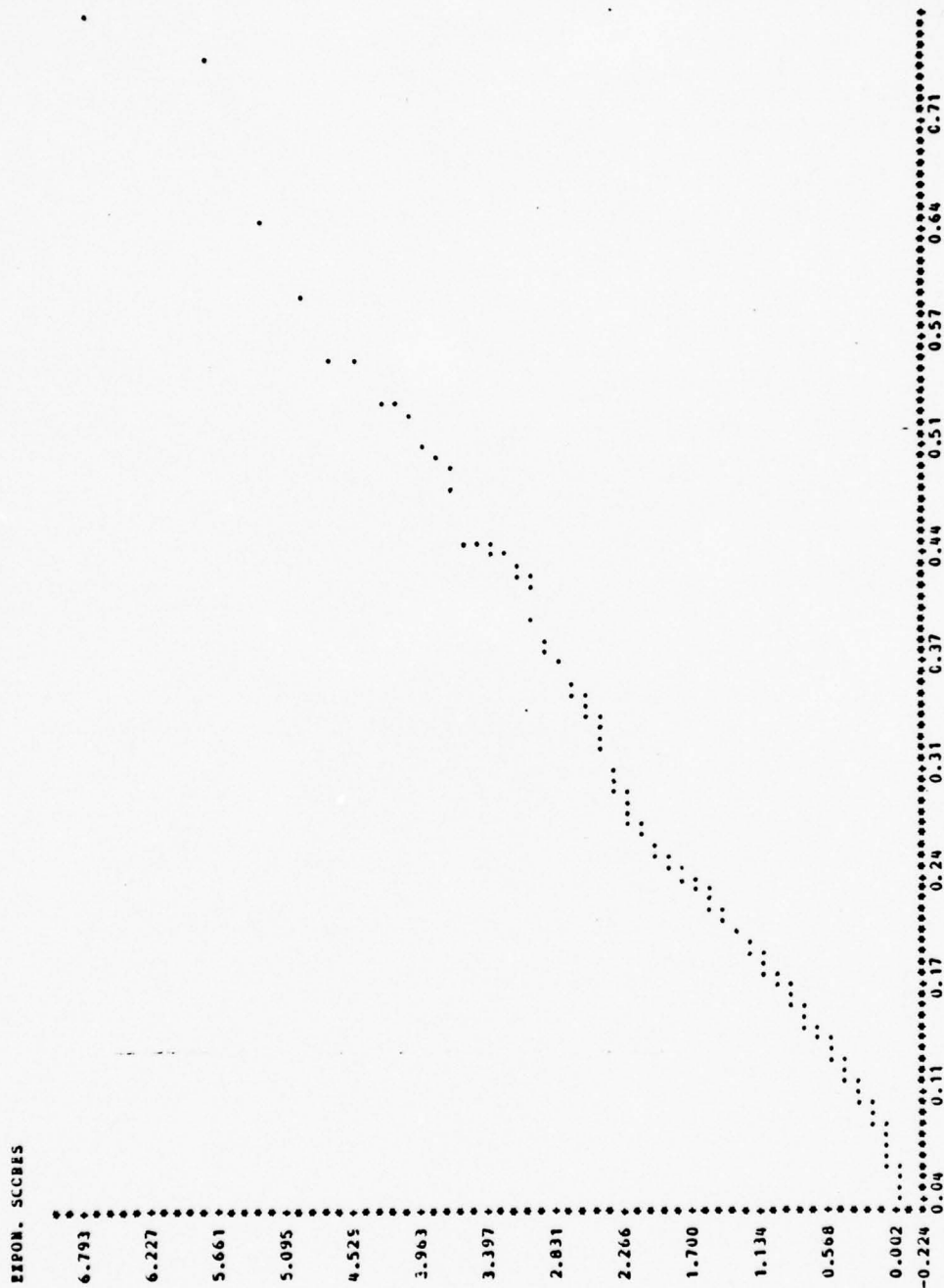


Figure 24e - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#25#96; $n=500$ REPLICATIONS, $RX=2.5$, $RS=10$.

NUMBER OF ORDERED PAIRS = 500



X-SCALE : ** = 0.666E-02 UNITS
 Y-SCALE : ** = 0.113E 00 UNITS

ESTIMATED PARAMETERS OF DATA : GARHA1 = 2.024137E 00 GARHA2 = 5.626021E 00

Figure 24f - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLOT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE RUN S#25#98; $m=500$ REPLICATIONS, $RX=2.5$, $RS=10$.

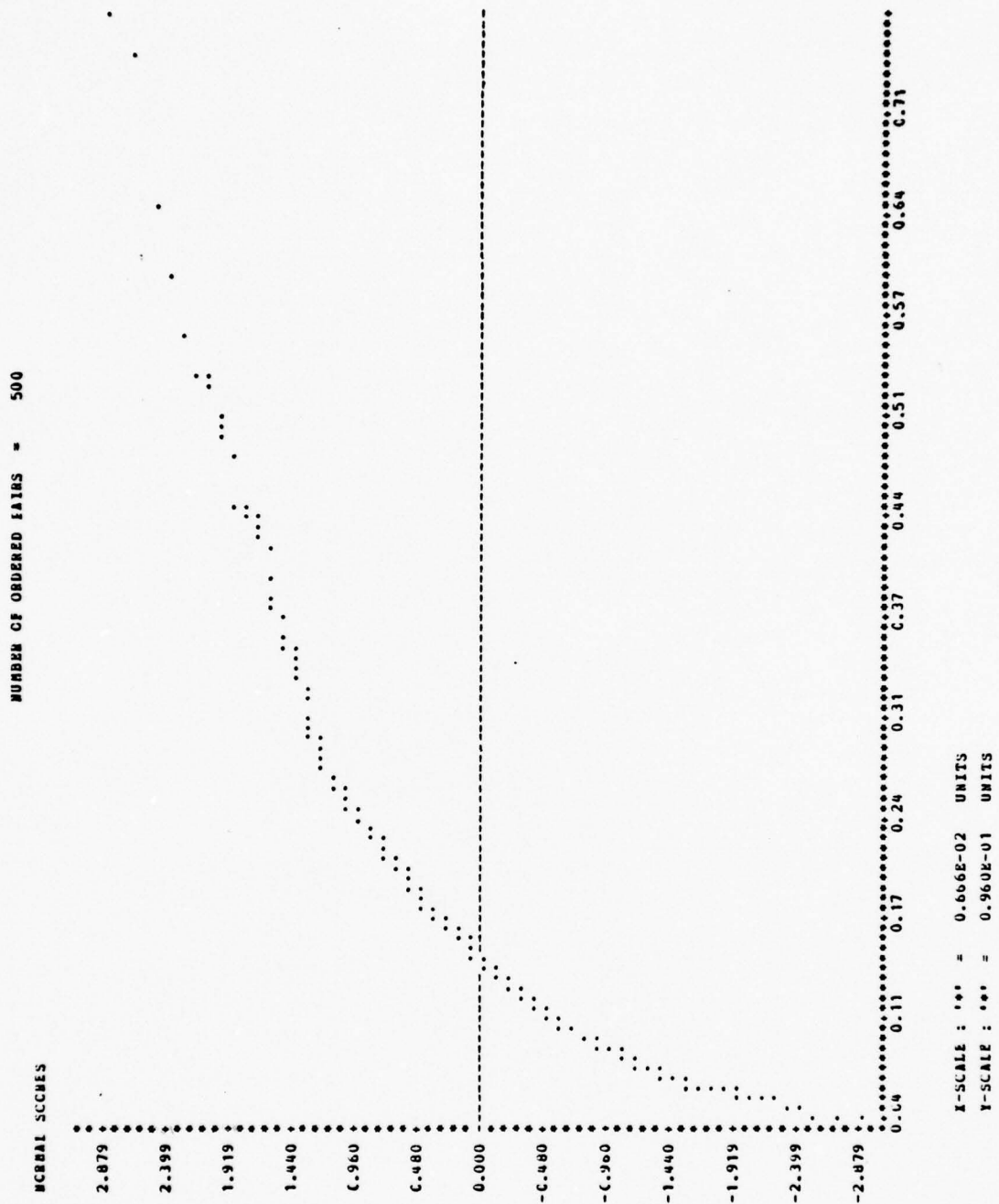


Figure 24g - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE RUN S#25#98; $m=500$ REPLICATIONS, $R_X=2.5$, $R_S=10$.

CORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	W 1 + CS	W 1	W 2 + OS	W 2	W 3 + OS	W 3	W 4 + OS	W 4
MEAN	1683.007	3366.192	1664.411	3330.068	16.96955	17.24553	22.27824	22.59456	25.56830	25.96404	28.15018	28.78342
STDEV	1.0424	1.4631	1.2989	1.8697	0.63766	0.64054	0.82874	0.82978	1.02681	1.03253	1.14832	1.15816
ST. DEV.	23.3062	32.7162	29.0446	41.8077	14.25854	14.20781	18.48635	18.42409	22.96030	22.91158	25.67712	25.61078
SKEWNESS	-0.0261	-0.2053	-0.0554	-0.0217	1.09584	1.09515	1.18767	1.19065	1.42630	1.42657	1.75002	1.75614
KURTOS.	0.1783	0.1013	-0.1659	-0.1877	0.82240	0.82057	1.30003	1.30472	2.64276	2.65312	4.63546	4.65158
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

PARAM	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	11.8439808	15.920297	16.7871857	20.7761688	-0.0003522	-0.0002452
STDEV	0.3253745	0.4183092	0.4843423	0.5419208	0.0003232	0.0002345
ST. DEV.	6.9170371	9.3536816	10.8239188	12.1177225	0.0072230	0.0052435
SKEWNESS	1.4605024	1.3524457	1.4713373	1.7270740	0.0722044	0.0792252
KURTOS.	2.2101507	1.5983324	2.2263470	3.4483662	-0.0658259	0.0503588
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

UNCORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	W 1 + OS	W 1	W 2 + OS	W 2	W 3 + OS	W 3	W 4 + OS	W 4
MEAN	1683.007	3366.192	1665.384	3331.023	14.82400	15.03446	18.62451	15.12897	21.96606	22.27795	23.66727	23.90634
STDEV	1.0424	1.4631	1.0524	1.5408	0.56681	0.56525	0.75166	0.76221	0.91403	0.91537	1.02615	1.03093
ST. DEV.	23.3062	32.7162	23.5216	34.4538	12.07428	12.03930	16.49151	16.50674	20.43625	20.41335	22.56553	22.93684
SKEWNESS	-0.0261	-0.2053	0.2369	0.0945	1.14055	1.13940	1.44110	1.44187	1.34763	1.34367	1.93359	1.90408
KURTOS.	0.1783	0.1013	0.0275	0.0294	0.79820	0.79072	2.37382	2.37591	1.81030	1.80023	5.34098	5.34051
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

PARAM	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	10.3716206	13.8082275	16.1548462	17.7791555	-0.0001574	-0.0001499
STDEV	0.2197136	0.3830756	0.4490776	0.5010585	0.0002948	0.0002138
ST. DEV.	6.2555255	8.5658340	10.0416851	11.2049074	0.0065921	0.0047798
SKEWNESS	1.4325445	1.4735689	1.6260328	1.7394295	0.2896550	0.2765061
KURTOS.	1.5141417	2.0772343	2.9112806	3.5485595	-0.1195488	0.1767348
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

Figure 25 - QUEUE WITH EARLY AUTOREGRESSIVE SERVICE TIMES AND ECISSON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE DISTRIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZEROS (W 1 + OS, W 1, W 2 + OS, W 2, W 3 + OS, W 3, W 4 + OS, W 4) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND X 2 (D 2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN S89825; RM=2.97; RS=3.

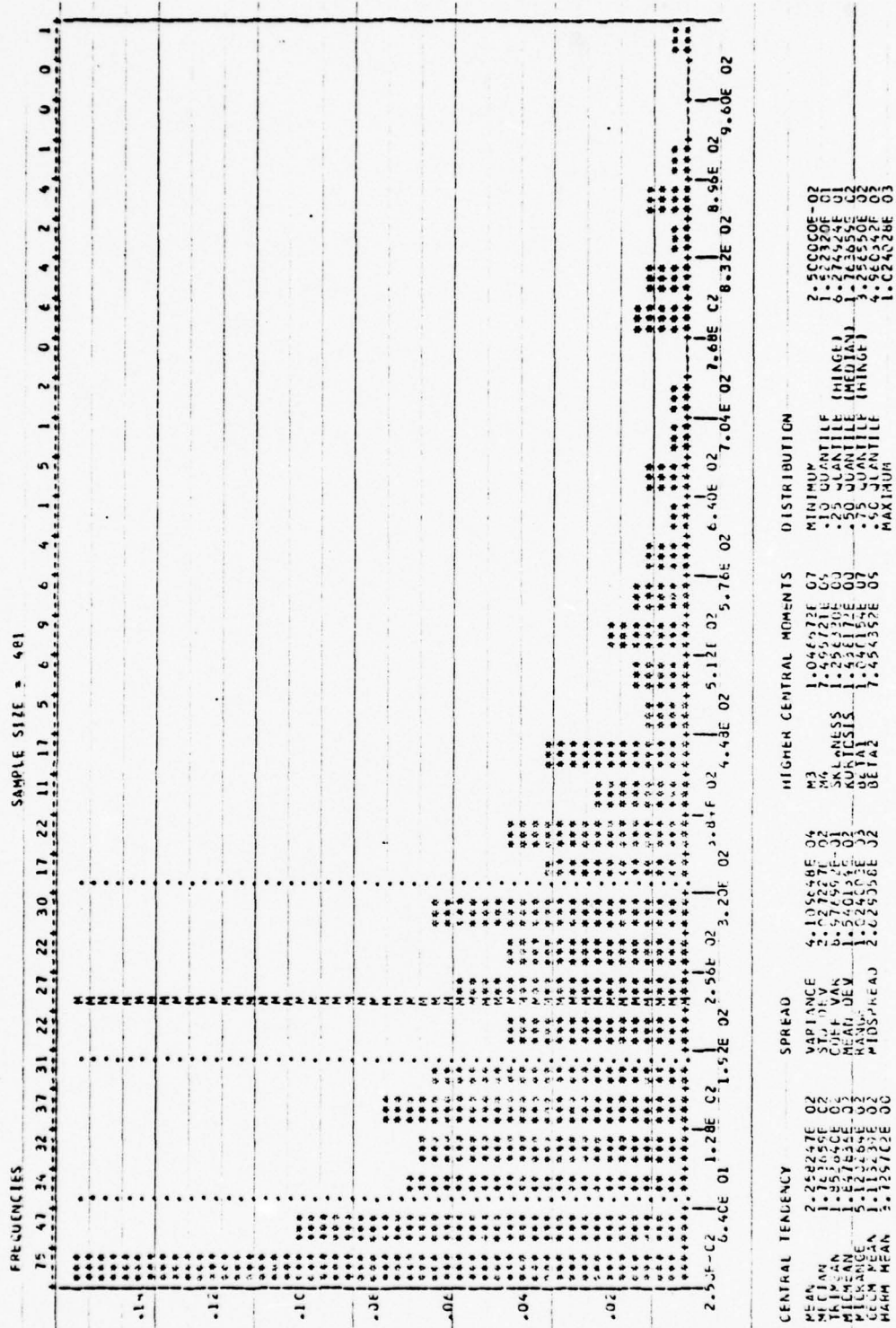
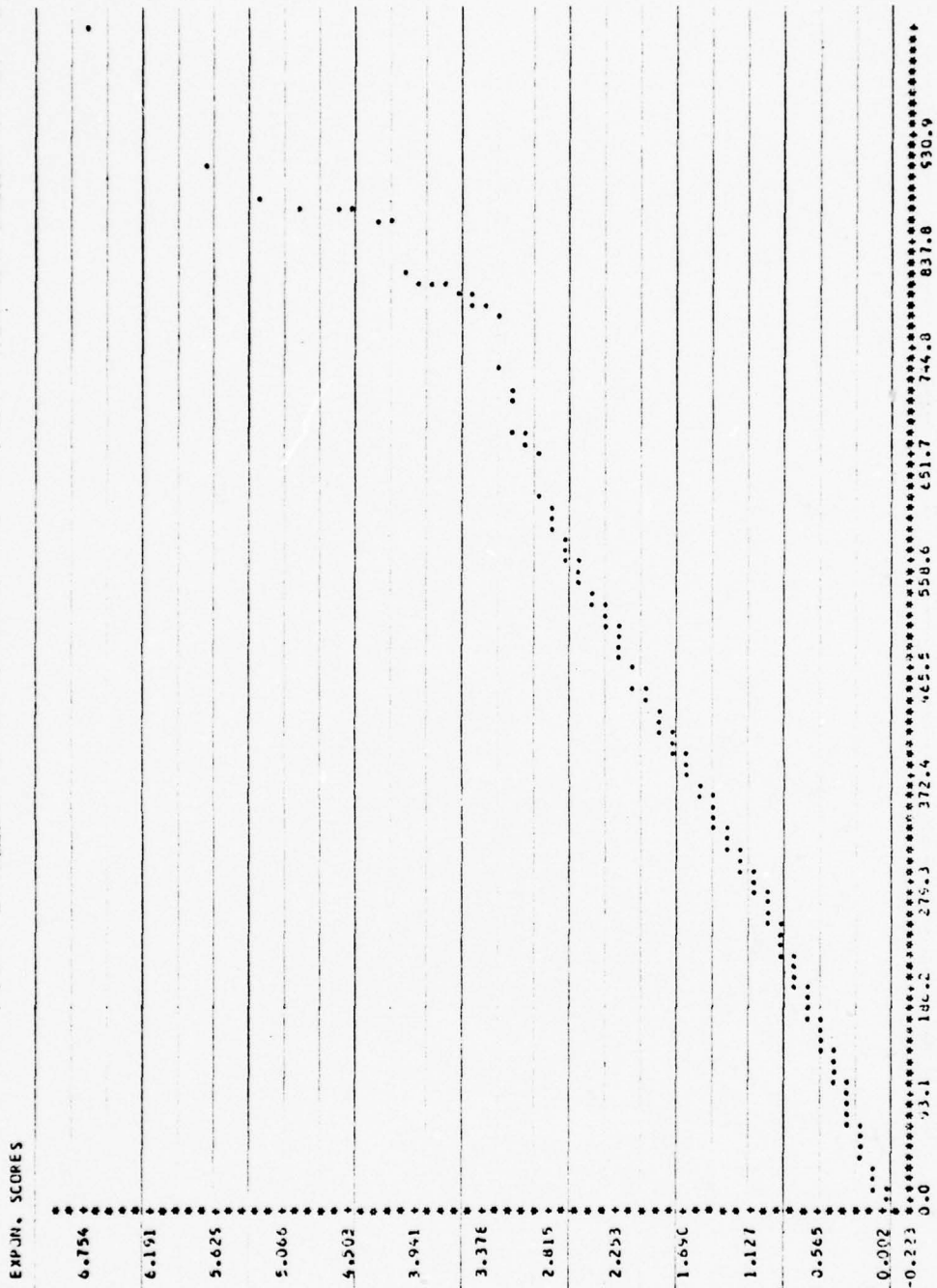


Figure 26a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE WAITING TIMES W WITHOUT ZEROS FROM THE S#99#98; m=500 REPLICATIONS, RX=2.97; RS=3 ; No OF ZEROS=19.

NUMBER OF ORDERED FAIRNS = 481



X-SCALE : *** = C.931E-01 UNITS

Y-SCALE : *** = C.113E 00 UNITS

ESTIMATED PARAMETERS OF DATA : GAMMA1 = 1.25623CE 00 GAMMA2 = 1.438177E 00

Figure 26b - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLOT (EXPLII) OF THE WAITING TIMES W_{10000} WITHOUT ZEROS FROM THE RUN S#99#98; $m=500$ REPLICATIONS, $RX=2.97$; $RS=3$; No OF ZEROS=19.

NUMBER OF ORDERED PAIRS = 481

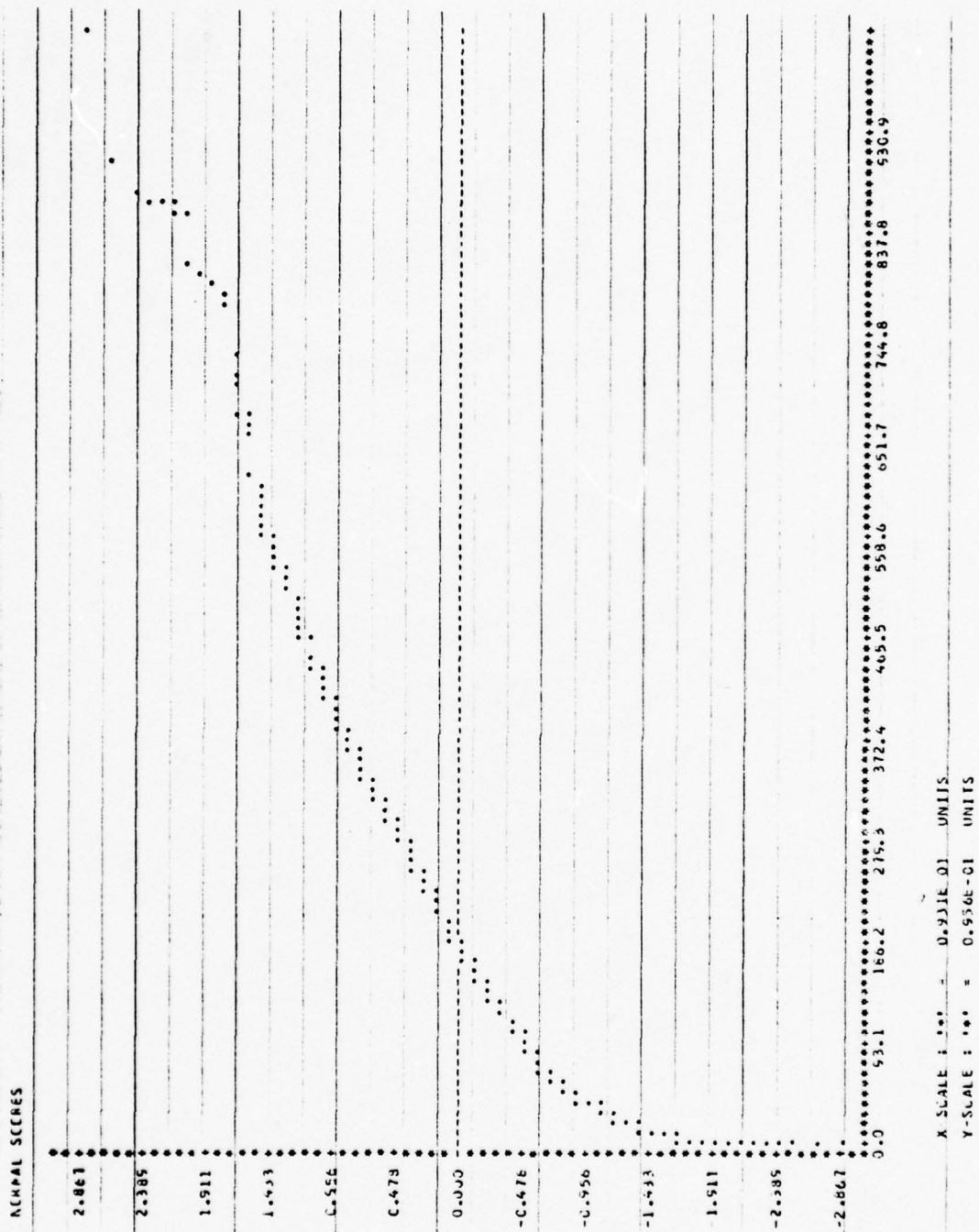


Figure 26c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE WAITING TIMES W_{10000} WITHOUT ZEROS FROM THE RUN S#99#98; $m=500$ REPLICATIONS, $RX=2.97$; $RS=3$; No OF ZEROS=19.

CORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	W 1 + S	W 1	W 2 + S	W 2	W 3 + S	W 3	W 4 + S	W 4
MEAN	1683.007	3366.192	1651.803	3316.000	111.15244	120.55688	144.63333	153.54070	183.04311	155.14820	217.24336	225.82469
STDEV	1.924	1.4331	9.48803	14.5871	4.63631	4.77766	6.02042	6.17073	7.62512	7.76455	9.09591	9.24335
ST. DEV.	23.3082	32.7162	220.9243	326.1775	103.67105	102.58667	134.84435	133.92052	170.50285	165.03450	203.44504	232.72266
SKEWNESS	-0.3261	-0.2053	0.1594	0.2394	1.13324	1.10072	1.26589	1.25510	1.26027	1.25310	1.26550	1.25433
KURTOS.	0.1783	0.1018	0.0521	0.1221	1.30831	0.96592	1.51514	1.48726	1.80714	1.82455	1.6775	1.43817
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000

PARAM	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	65.8666272	57.8470517	121.5777417	142.2246246	-0.0028737	-0.0018523
STDEV	2.5285578	3.1955700	3.7718716	4.4213005	0.0019880	0.0014556
ST. DEV.	58.5411747	71.4551344	84.5410443	58.8633423	0.0444540	0.0325485
SKEWNESS	1.5251767	1.3165245	1.3310013	1.4051094	0.1860167	0.2517110
KURTOS.	2.5143533	1.7875567	1.6947069	2.0297522	-0.1270666	0.1280117
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000

UNCORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	W 1 + S	W 1	W 2 + S	W 2	W 3 + S	W 3	W 4 + S	W 4
MEAN	1683.007	3366.192	1665.384	3331.023	14.82400	15.03446	18.62291	19.12697	21.96606	22.27755	23.66727	23.90634
STDEV	1.6424	1.4631	1.0524	1.5408	0.56681	0.56925	0.75766	0.76221	0.91403	0.91937	1.02615	1.03043
ST. DEV.	23.3362	32.7162	23.5316	34.4538	12.67428	12.63930	16.54151	16.50674	20.43825	20.41335	22.54553	22.93685
SKEWNESS	-0.0261	-0.0053	0.2369	0.0945	1.14055	1.13940	1.44110	1.44187	1.34763	1.34267	1.90339	1.90468
KURTOS.	0.1783	0.1018	0.0275	0.0294	0.79820	0.79072	2.37382	2.37581	1.81030	1.80023	5.34058	5.34651
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000

PARAM	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	10.3710206	13.9082275	16.1548462	17.7791595	-0.0001574	-0.0001499
STDEV	2.2757726	0.3520756	0.4450776	0.5010585	0.0002548	0.0002138
ST. DEV.	6.2559255	8.5628340	10.0416851	11.2049074	0.0065921	0.0047758
SKEWNESS	1.4325485	1.4739689	1.6200328	1.7354295	0.2894500	0.2765061
KURTOS.	1.5141417	2.0772343	2.9112905	3.5989599	-0.1195488	0.1787348
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000

Figure 26d - QUEUE WITH EARLY AUTOREGRESSIVE SERVICE TIMES AND EXPONENTIAL SERVICE TIMES. TABLE OF STATISTICS FOR THE DISTRIBUTIONS OF CALCULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (W 1 + S, W 1, W 2 + S, W 2, W 3 + S, W 3, W 4 + S, W 4), CUMULATED WAITING TIMES (W 1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND S 4 (D 2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN S89598; RS=2.97; RS=3.

2. As for the steady state of the value of $E[W]$ and $E[\bar{W}]$ a large N was required for high t and/or r , also for the steady state of the distribution of W and \bar{W} a large value of N was required. Informally testing their distributions, by using the Plotting Subroutines and the values of their parameters skewness and kurtosis, we may conclude that:

(i). The distribution of W , given that $W > 0$, has an exponential form, if the plot results in a straight line under EXPLT and the γ_1 and γ_2 parameters have the values of about 2 and 6 respectively (see figures 23a-23d). This is true only if the steady state has been reached while it is not evident what their distribution is if the steady state has not been reached. Thus observing the figures 24a-24d, where the W_{10000} has not reached the steady state yet, we can obtain nothing about the distribution of W .

Note that for the M/M/1 queue the steady state distribution of W , given that $W > 0$, is exponential. The distribution is not known for the correlated queue and this was one of the objectives of this study.

(ii). The distribution of \bar{W} has the normal form if we get a straight line under NORMPL and if the values of skewness and kurtosis are around '0'. The normality assumption holds for the steady state only, while the non-steady state distribution looks like an exponential. Figures 27a-27c give us the feeling that \bar{W}_{10000} has an exponential form, since a straight line appears under EXPLT (see figure 27b) and the parameters γ_1, γ_2 have the values 2.3 and 7.3 respectively. These are close to the actual values 2 and 6 of the exponential distribution. We can see

from the plots that as the \bar{W} gets close to the steady state, it leaves the exponential form and comes closer and closer to the normal form. Figures 28a-28c, 23d show us that \bar{W}_{10000} has already reached the steady state and furthermore its distribution has eventually taken the normal form, since a straight line appears under NORMPL (figure 28c) and the skewness and kurtosis have the value .7 and .8 respectively. Compare its distribution with the distribution of \bar{W}_{10000} of figure 27b where \bar{W} is not in the steady state. Again we may state here that convergence to the steady state of the distribution of \bar{W} and \bar{W} requires much larger N than convergence to the steady state of their mean values for the same t and r . And also the higher t and/or r the larger N should be. As an example we can see that even though the steady state (in value) of \bar{W} for S#25#98 type has been reached for $N=10000$, the steady state in distribution has not been reached yet. See figures 24e-24g where we can observe that the distribution of \bar{W}_{10000} starts to leave the exponential form and to go to the normal form.

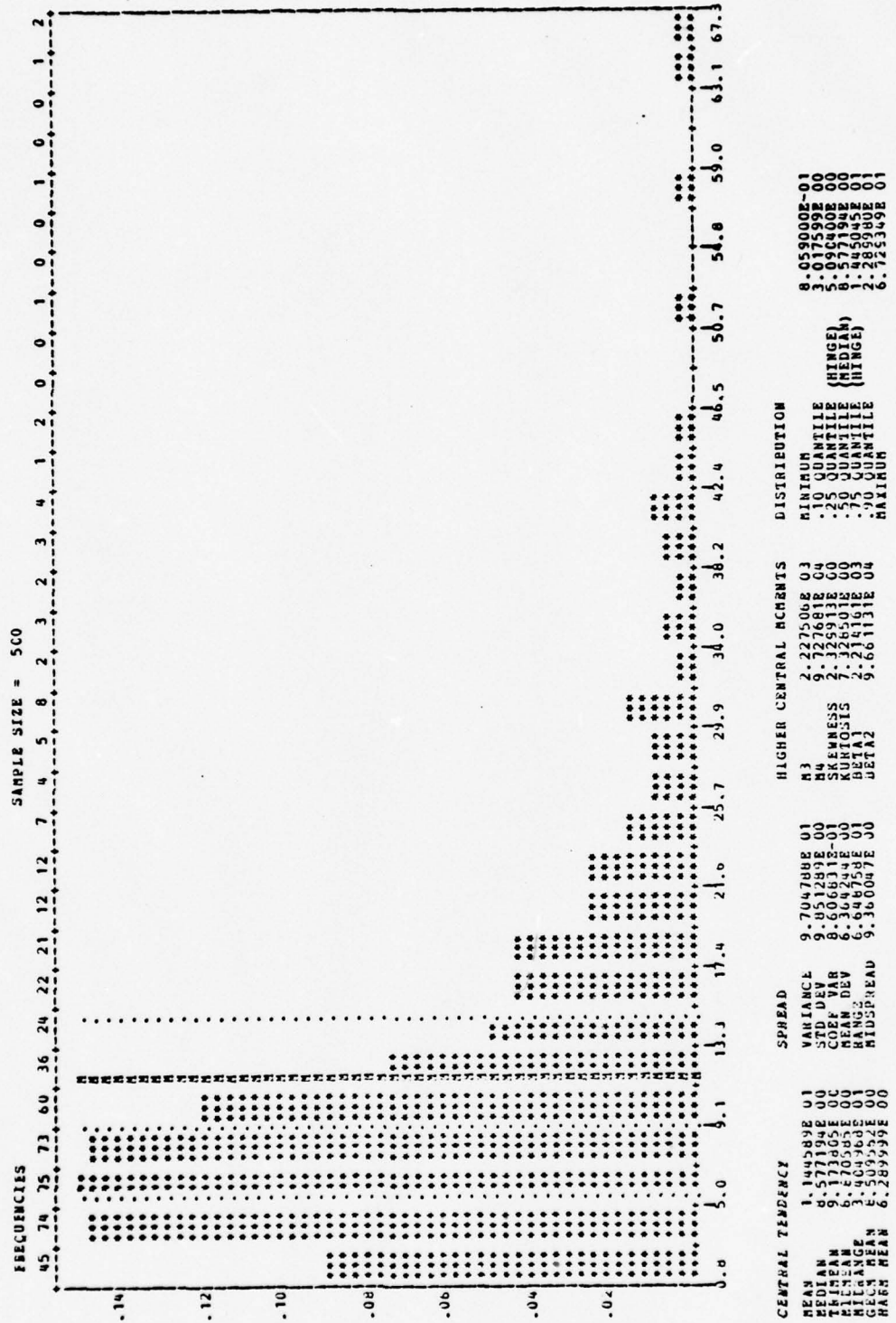


Figure 27a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{w}_{2000} FROM THE RUN S#50#90; M=500 REPLICATIONS, RX=.2; RS=.4 .

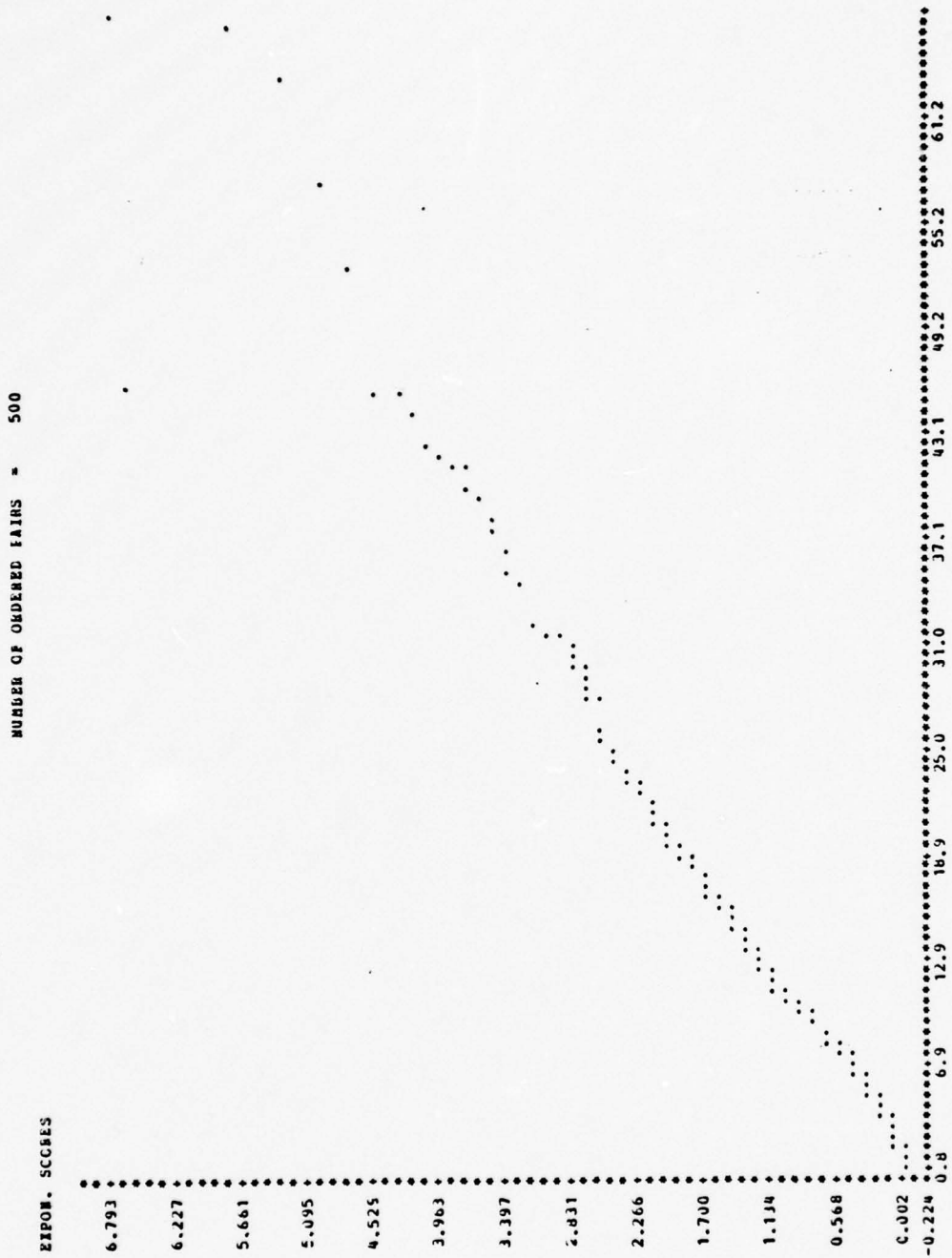


Figure 27b - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLCT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{2000} FROM THE RUN S#50#90; m=500 REPLICATIONS, RX=.2; RS=.4 .

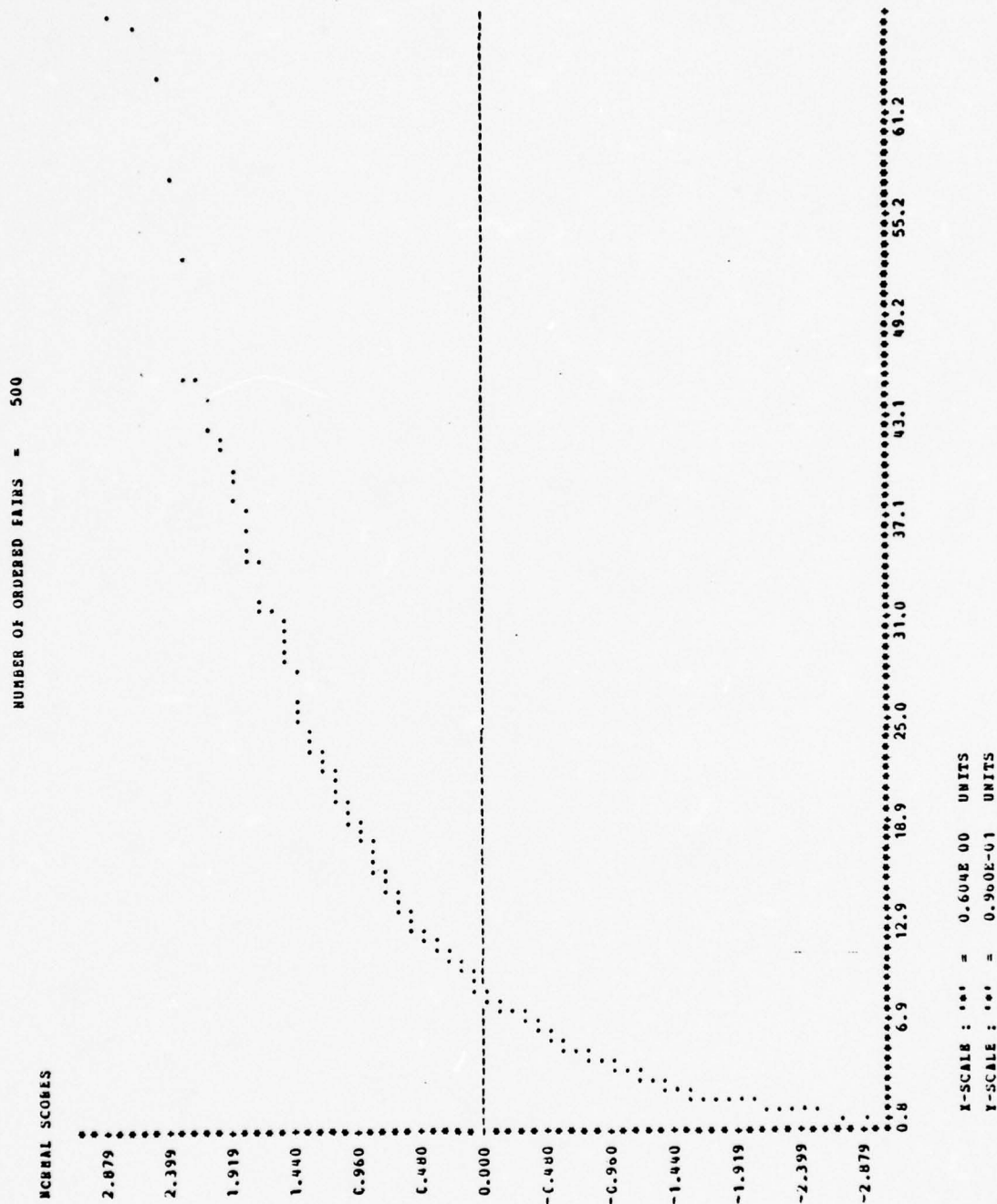


Figure 27c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{2000} FROM THE RUN S#50#90; $m=500$ REPLICATIONS, $RX=.2$; $RS=.4$.

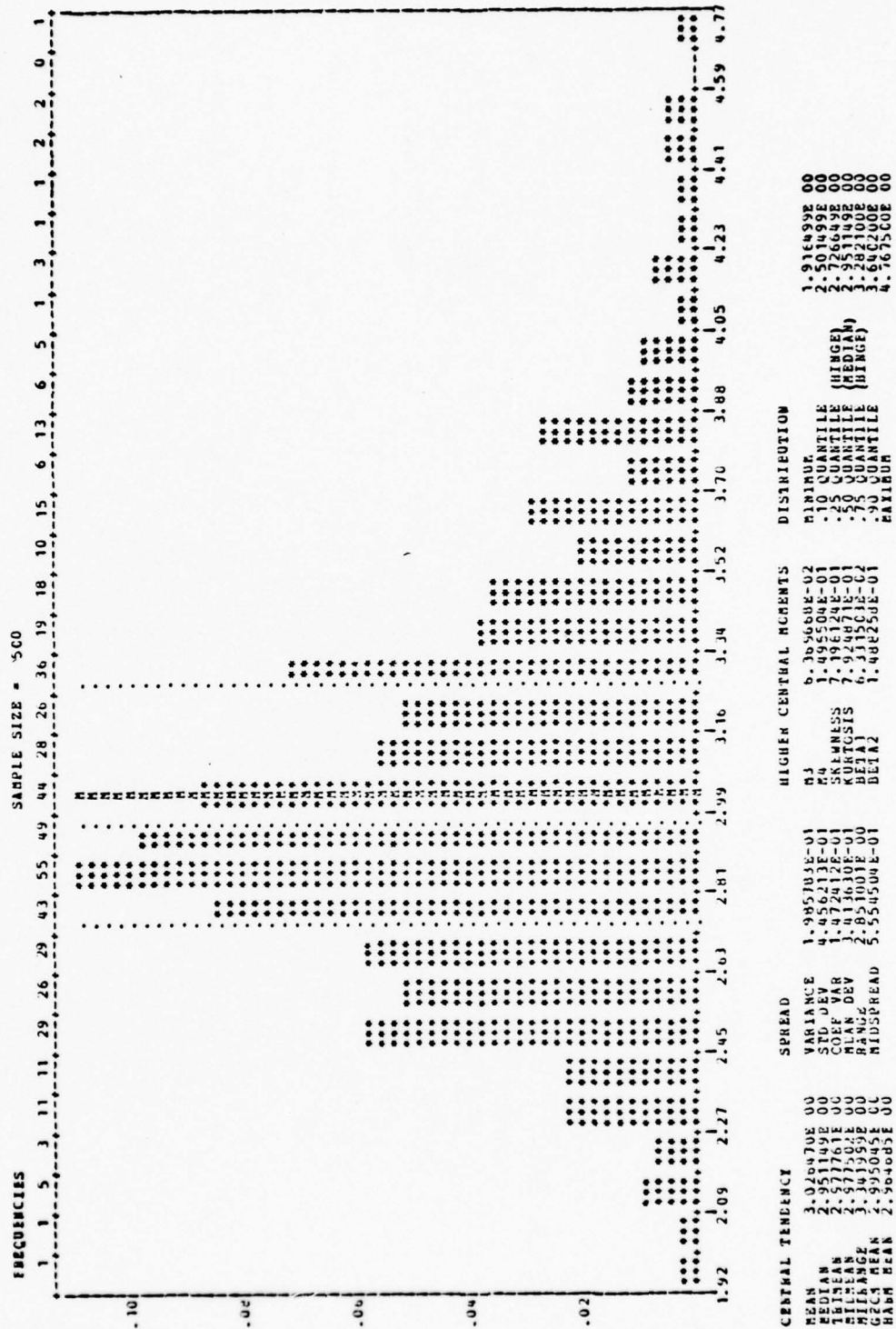


Figure 28a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{2000} FROM THE RUN S#50#25; m=500 REPLICATIONS, RX=.2; RS=.4 .

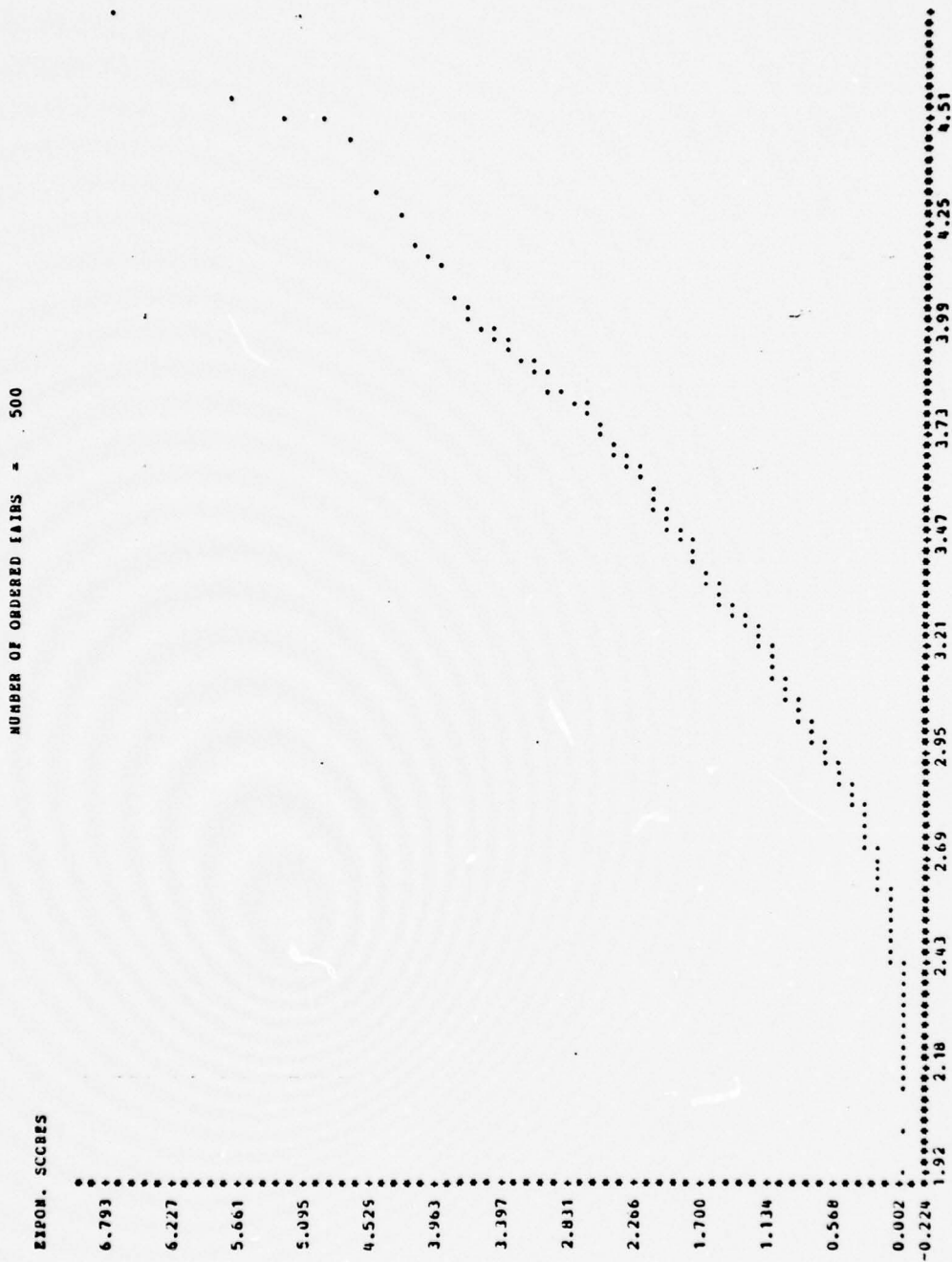


Figure 28t - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLCT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{2000} FROM THE RUN S#50#25; m=500 REPLICATIONS, RX=.2; RS=.4 .

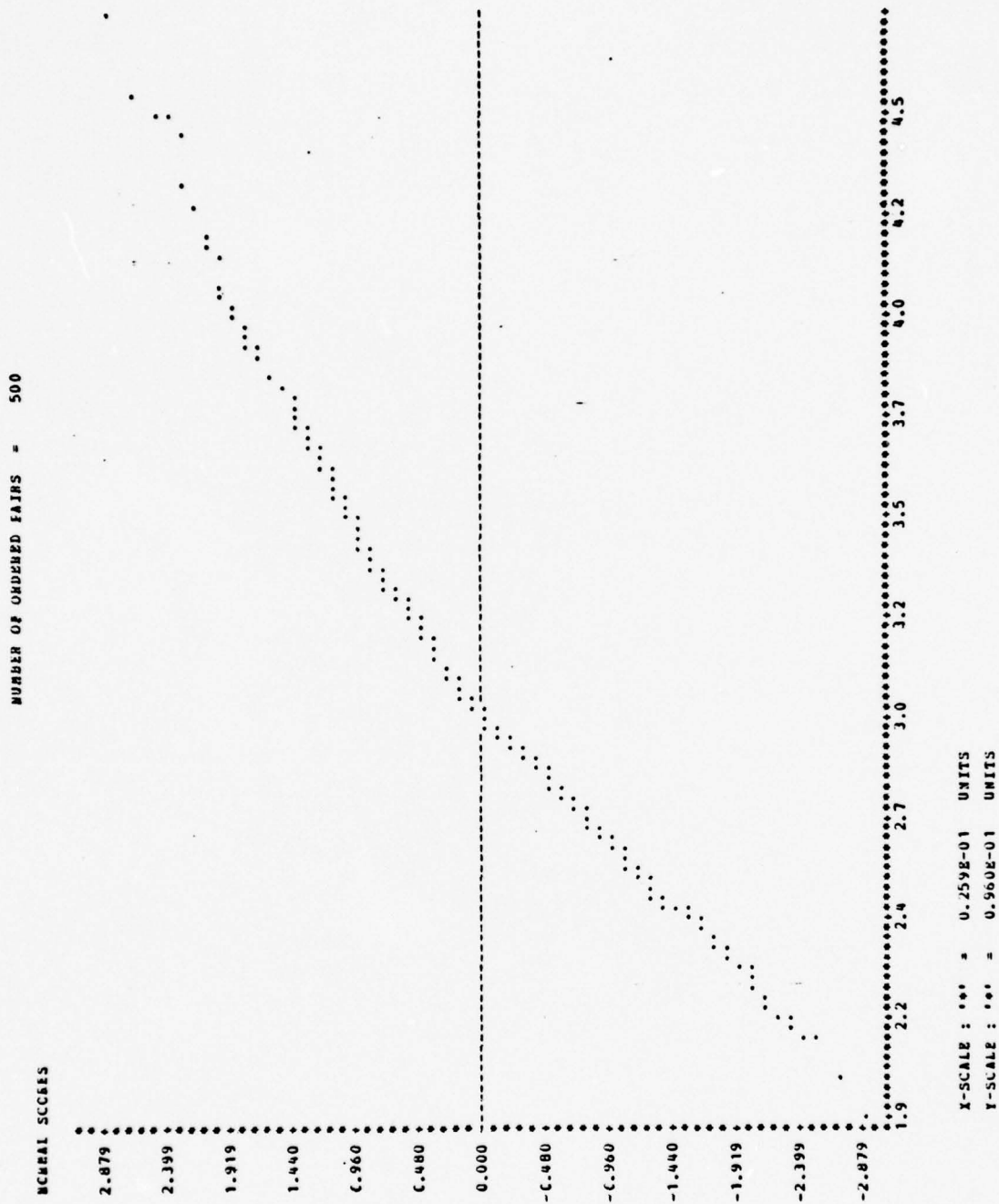


Figure 28c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND PCISSCN INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{2000} FROM THE RUN S#50#25; $n=500$ REPLICATIONS, $R_X=.2$; $R_S=.4$.

Continuing the analysis of the results we present below the results we got from all runs, concerning the values of $E[W]$ for various values of t , r , N .

N	t	r	$E[W]$ (correlated)	$E[W]$ (M/M/1)
2000	.25	.25	.3720	.3332
2000	.25	.50	.4326	.3332
2000	.25	.90	.7578	.3332
10000	.25	.95	1.0576	.3332
10000	.25	.98	1.7077	.3332
2000	.50	.25	1.2106	.9985
2000	.50	.50	1.5811	.9985
2000	.50	.90	5.0785	.9985
10000	.50	.95	9.3196	.9985
10000	.50	.98	21.0542*	.9985
10000	.95	.25	23.5651	18.3528
10000	.95	.50	33.3367	18.3528
10000	.95	.90	123.9350*	18.3528
10000	.95	.95	196.2460*	18.3528
10000	.95	.98	342.9240*	18.3528
10000	.99	.25	62.3285*	53.3371
10000	.99	.50	77.9370*	53.3371
10000	.99	.90	193.9113*	53.3371
10000	.99	.95	271.7713*	53.3371
10000	.99	.98	426.6750*	53.3371

Note that these values have been scaled from the values obtained in the simulations by multiplying by RS . Thus for $t=0.50$ and $r=0.25$ the table gives $E[W]$ (correlated) as 1.2106; the value in Figure 23d is $3.026 \times 0.4 = 1.2106$. The values marked * have not converged.

A study on these results gives us the following observations:

1. For given traffic intensity t the value of $E[\bar{W}]$ is not constant, as it is for $M/M/1$ queues, but it depends on the value of the correlation parameter r . Thus we see that the value of $E[\bar{W}]$ increase, as the value of r becomes larger and larger. Furthermore from figure 29a, where $E[\bar{W}]$ is plotted versus r we can see that the rate of increment is not constant (therefore is not a linear function of r) but it increases with r and eventually goes up infinitely as r goes to 1.

2. For given correlation r , we see that the value of $E[\bar{W}]$ increases with t , the same as happens in the $M/M/1$ queue (see figure 29b). But regardless of the value of r , $E[\bar{W}]$ for $M/M/1$ queue is always less than for the EARMA model. Thus we may state that the waiting time of the autocorrelated service time model is greater than the corresponding waiting time of the $M/M/1$ model.

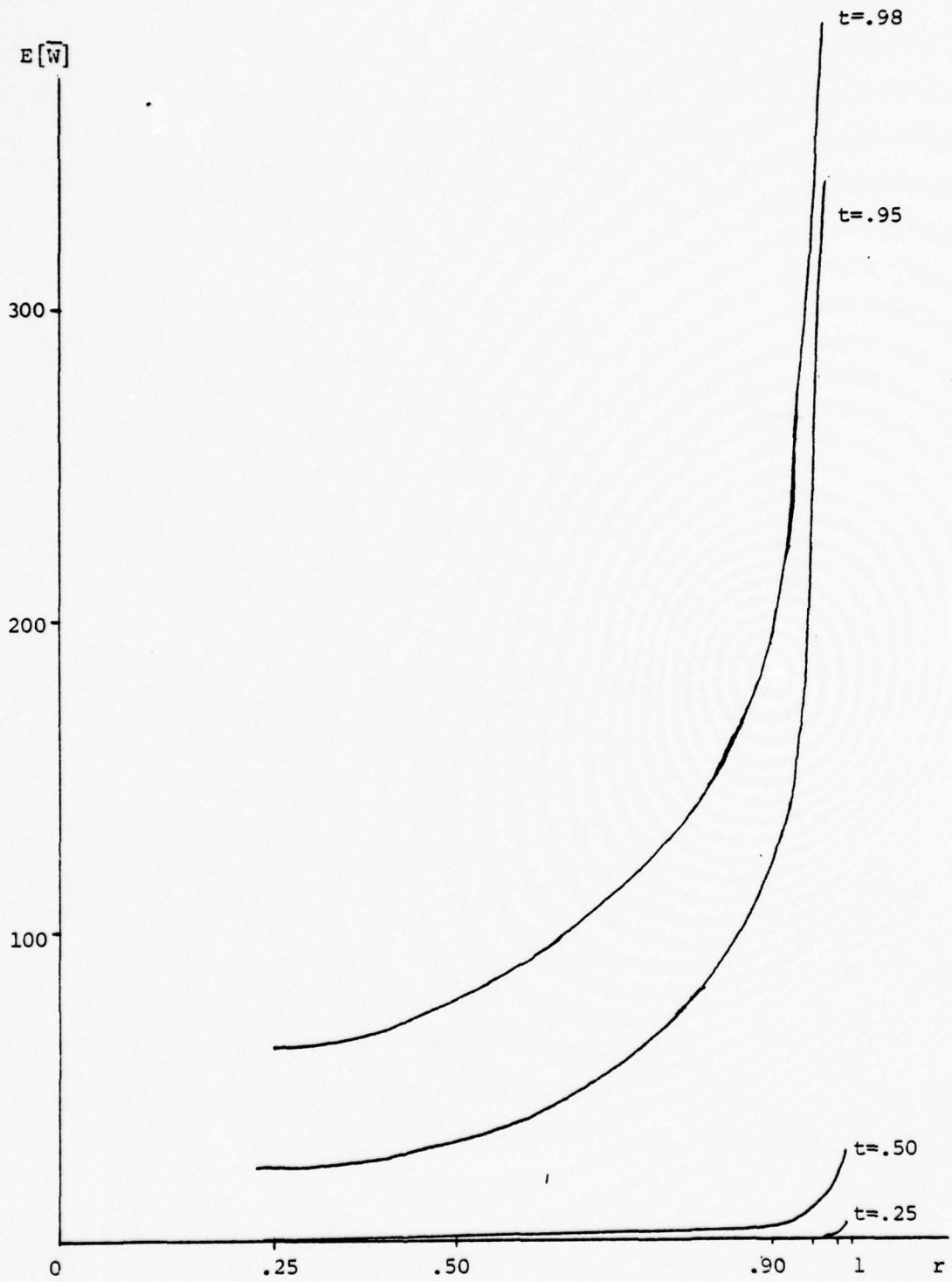


Figure 29a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. PLCT OF $E[\bar{W}]$ VERSUS r .

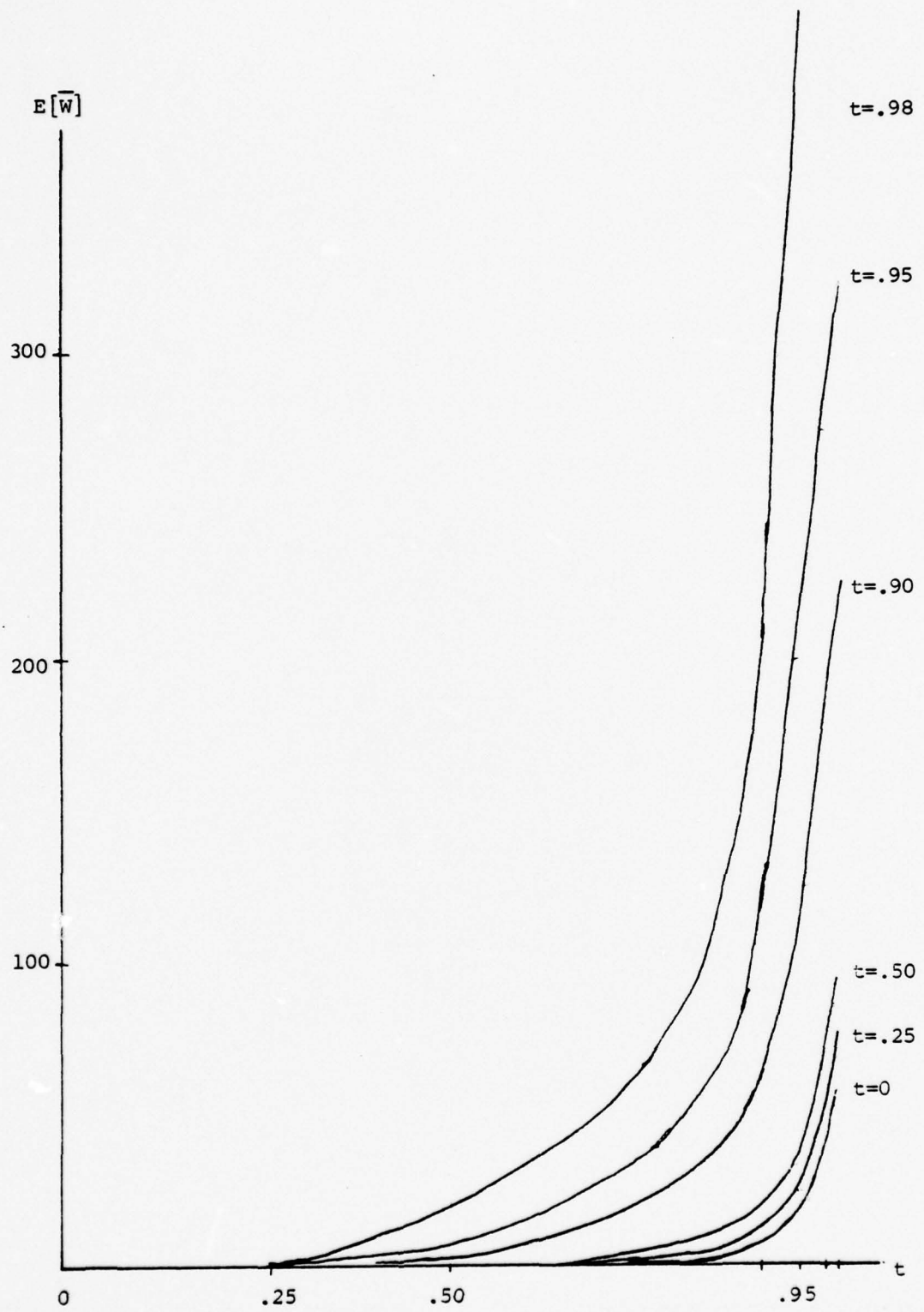


Figure 29b - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. PLOT OF $E[W]$ VERSUS t .

3. The values of $E[W]$ for the type S#99#98 model increases with N following the curve in figure 30 a fact that is known to happen in the M/M/1 queue also. We can see from that figure that the rate of increment comes down as N increases and hopefully should come to zero as W reaches the steady state. Comparing then the M/M/1 queue with the correlated queue we can see that the M/M/1 queue converges faster than the correlated especially when r is high. Thus we can see from figures 32a-32c that both the value and distribution of the \bar{W}_{10000} for M/M/1 queue have reached the steady state, but the corresponding \bar{W}_{10000} of the correlated queue (figures 33a-d) has not (at least in its distribution). (See the skewness and kurtosis values for W1 EAR, W2 EAR, W3 BAR and W4 BAR in Figure 33d. These appear to be decreasing to zero, but have certainly not reached zero by $N=10,000$.) Looking at figure 31d, we can obtain the result that neither \bar{W}_{10000} for the correlated queue, or \bar{W}_{10000} for the uncorrelated queue has reached equilibrium. This is because the skewness and kurtosis values are definitely not close to zero.

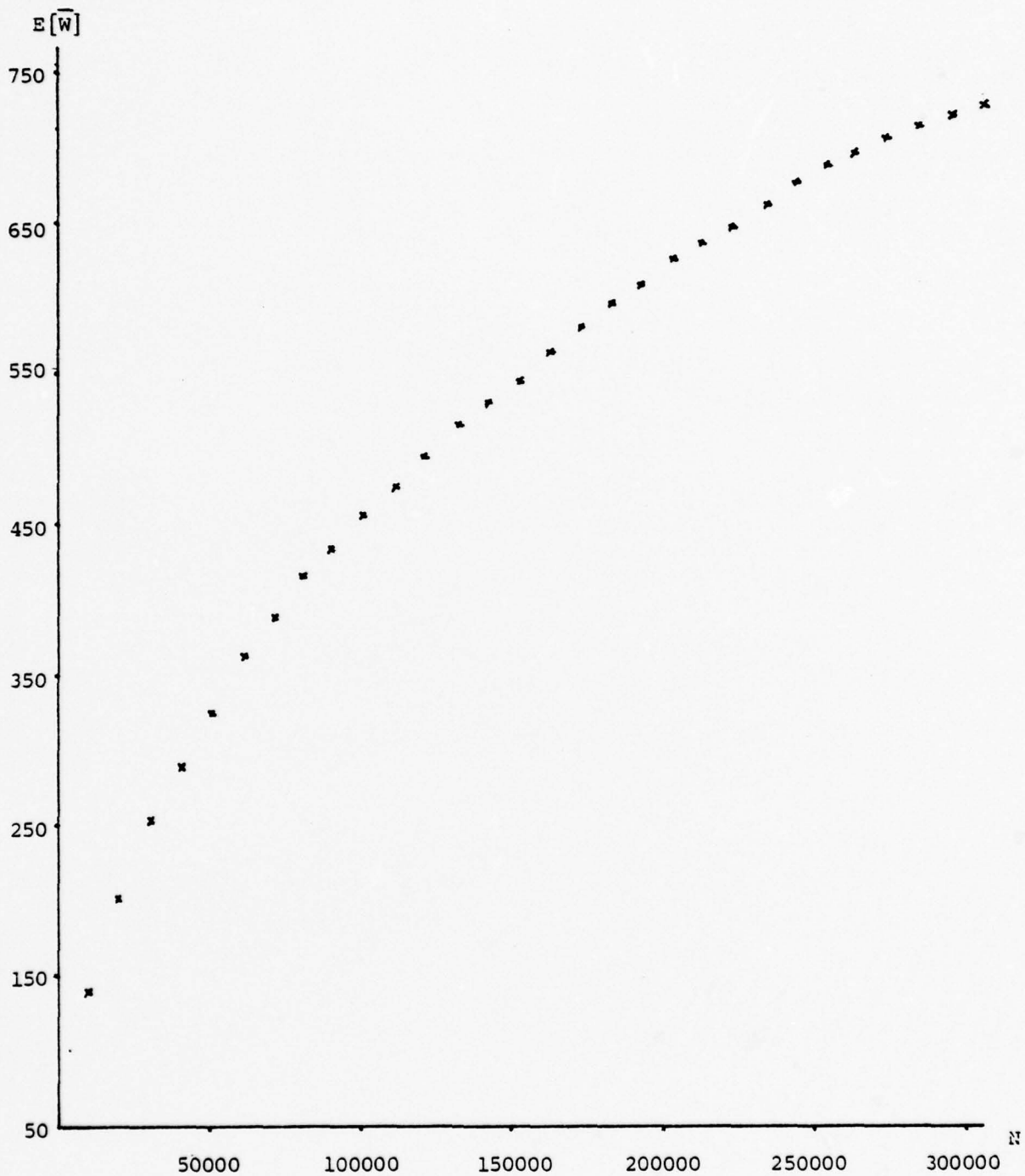


Figure 30 - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. PLOT OF $E[W]$ FROM THE RUN S#99#98, vs N.

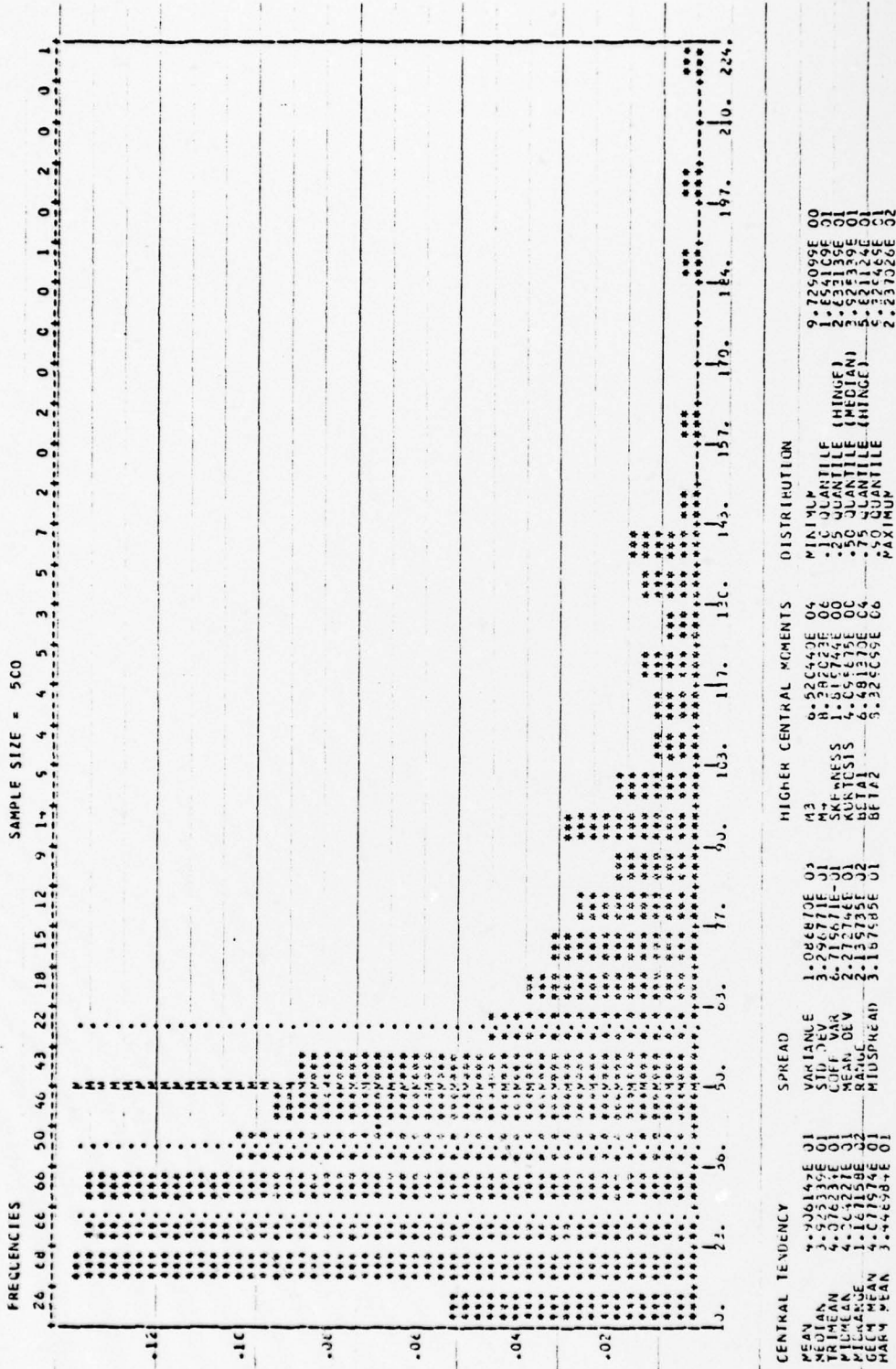
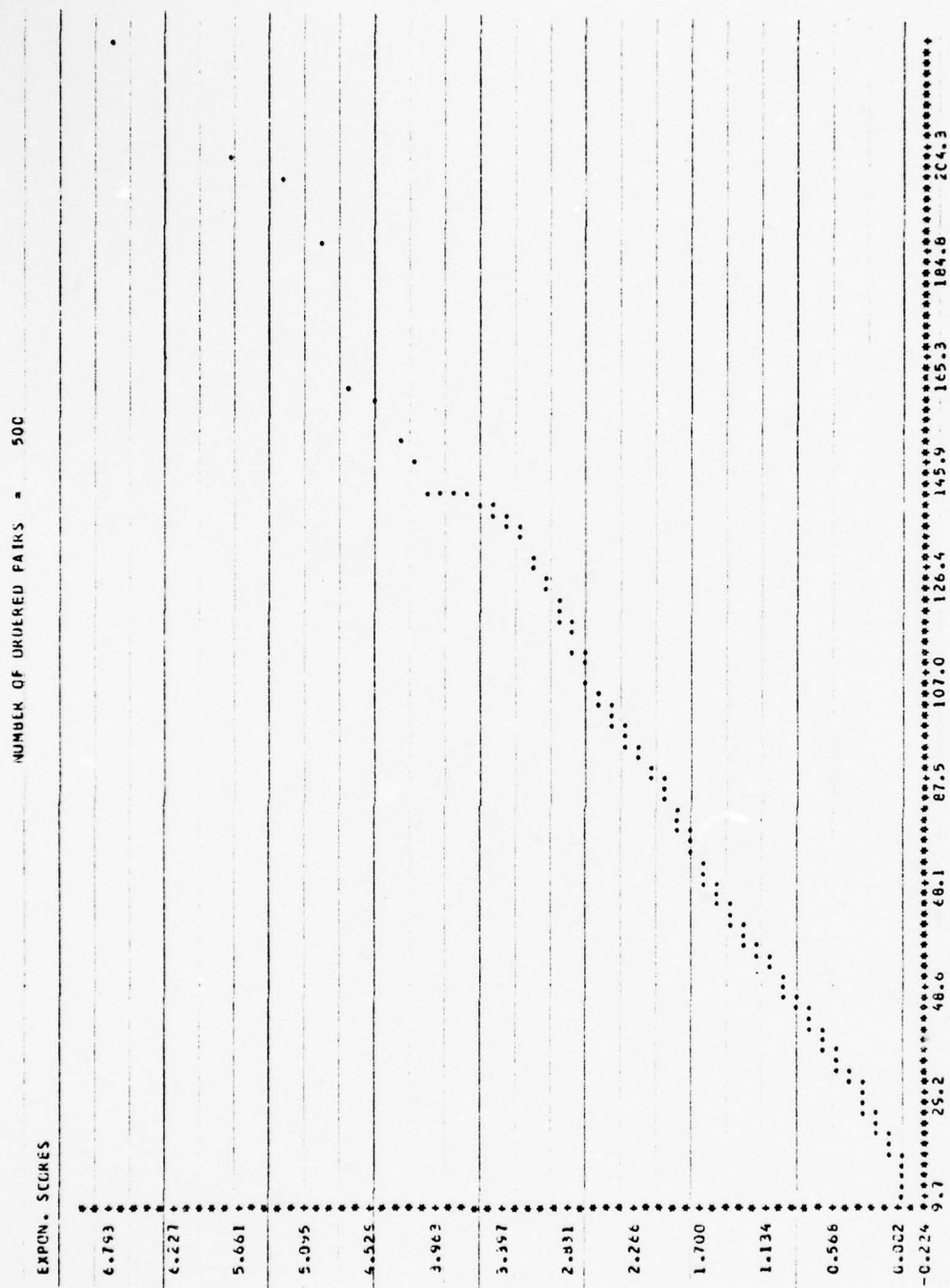


Figure 31a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#95#95; $n=500$ REPLICATIONS, $RX=3.8$; $RS=4$.



X-SCALE : ** = C.155E 01 UNITS
 Y-SCALE : ** = C.115E 00 UNITS
 ESTIMATED PARAMETERS OF DATA : GAMMA1 = 1.01574E 00 GAMMA2 = 4.095074E 00

Figure 31b - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLOT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE RUN S#95#95; m=500 REPLICATIONS, RX=3.8; RS=4 .

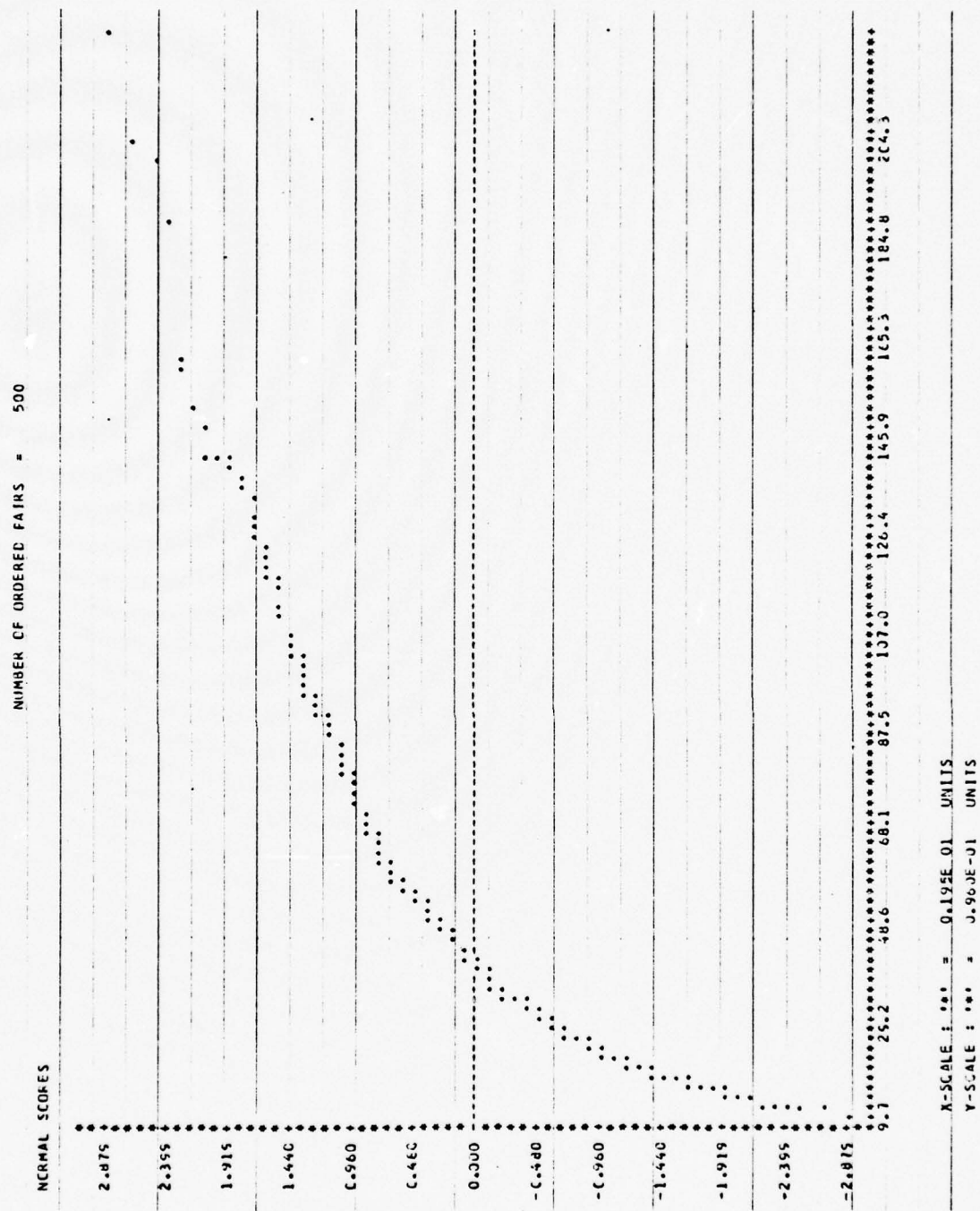


Figure 31c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE RUN S#95#95; $m=500$ REPLICATIONS, $RX=3.8$; $RS=4$.

CCORRELATED QUEUE

PARAMTR	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	1315.513	2630.721	1237.182	2486.526	41.83807	45.27930	51.31895	55.50302	60.93990	65.10672	65.76263	71.77728
S(MEAN)	C.8147	1.1435	4.8361	6.6572	1.89864	1.97078	2.32053	2.44396	2.70564	2.74321	3.03678	3.11920
ST. DEV.	19.2169	25.2657	108.1386	148.8606	42.45010	42.36031	52.06149	51.93159	60.59399	60.42641	68.35172	68.24694
SKEWNESS	-0.2260	-0.2054	0.0080	0.0779	1.34562	1.30511	1.35181	1.34699	1.50149	1.46071	1.53079	1.51625
KURTOS.	C.1782	C.1021	-0.1010	-0.1893	1.61421	1.49580	1.63750	1.45824	2.60345	2.54736	2.86478	2.82892
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	479.00000

PARAMTR	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	24.4075623	37.4663427	44.2106934	49.0614975	-0.0024681	-0.0012617
S(MEAN)	C.5553651	1.1918144	1.3660173	1.4743595	0.0009737	0.0006720
ST. DEV.	22.2570486	26.6497555	30.5451050	32.9677124	0.0217730	0.0150274
SKEWNESS	1.5593653	1.4049540	1.6803305	1.8197441	0.0476807	0.0951524
KURTOS.	2.4249315	1.5788355	3.2507801	4.0956755	-0.1309185	-0.1001110
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

UNCORRELATED QUEUE

PARAMTR	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	1315.313	2630.721	1248.937	2493.015	4.67354	4.89885	4.62055	4.80307	4.68507	5.03770	4.81275	5.00286
S(MEAN)	0.3147	1.1435	C.7653	1.1556	0.21404	0.21914	0.22754	C.23264	C.22734	0.23652	0.24120	0.24675
ST. DEV.	18.2168	25.2697	17.6482	25.8401	4.78606	4.78612	5.08190	5.10225	5.08358	5.10018	5.39336	5.41172
SKEWNESS	-0.0260	-0.2054	0.2370	0.0945	1.70475	1.69067	2.40052	2.35665	2.00690	1.98759	2.61904	2.61761
KURTOS.	0.1782	C.1021	0.0275	0.0293	3.06231	2.99011	8.73606	8.68852	5.76017	5.66502	11.39535	11.34556
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	481.00000

PARAMTR	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	4.0176134	4.3716478	4.5287819	4.5802139	-0.0001173	-0.0001126
S(MEAN)	0.0934711	0.0992408	0.0949966	0.0903672	C.0002257	0.0001636
ST. DEV.	2.1913041	2.2195669	2.1152468	2.0206738	0.0050478	0.0036572
SKEWNESS	1.9176502	2.6313343	3.0940571	3.5639057	0.2856022	0.2776073
KURTOS.	4.5888758	10.0241823	16.1781958	22.3170471	-0.1193161	0.1654472
SPL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000

Figure 31d - QUEUE WITH EARLY AUTOREGRESSIVE SERVICE TIMES AND FCISSEN INPUT. TABULATION OF SAMPLE STATISTICS FOR THE LISTED STATISTICS OF CUMULATED INTERARRIVAL TIMES AT M=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (M 1 + OS, M 1, M 2 + OS, M 2, M 3 + OS, M 3, M 4 + OS, M 4), CUMULATED WAITING TIMES (M 1 BAR ETC.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND Y 2 (D 2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN S895895; RX=238, RS=4.

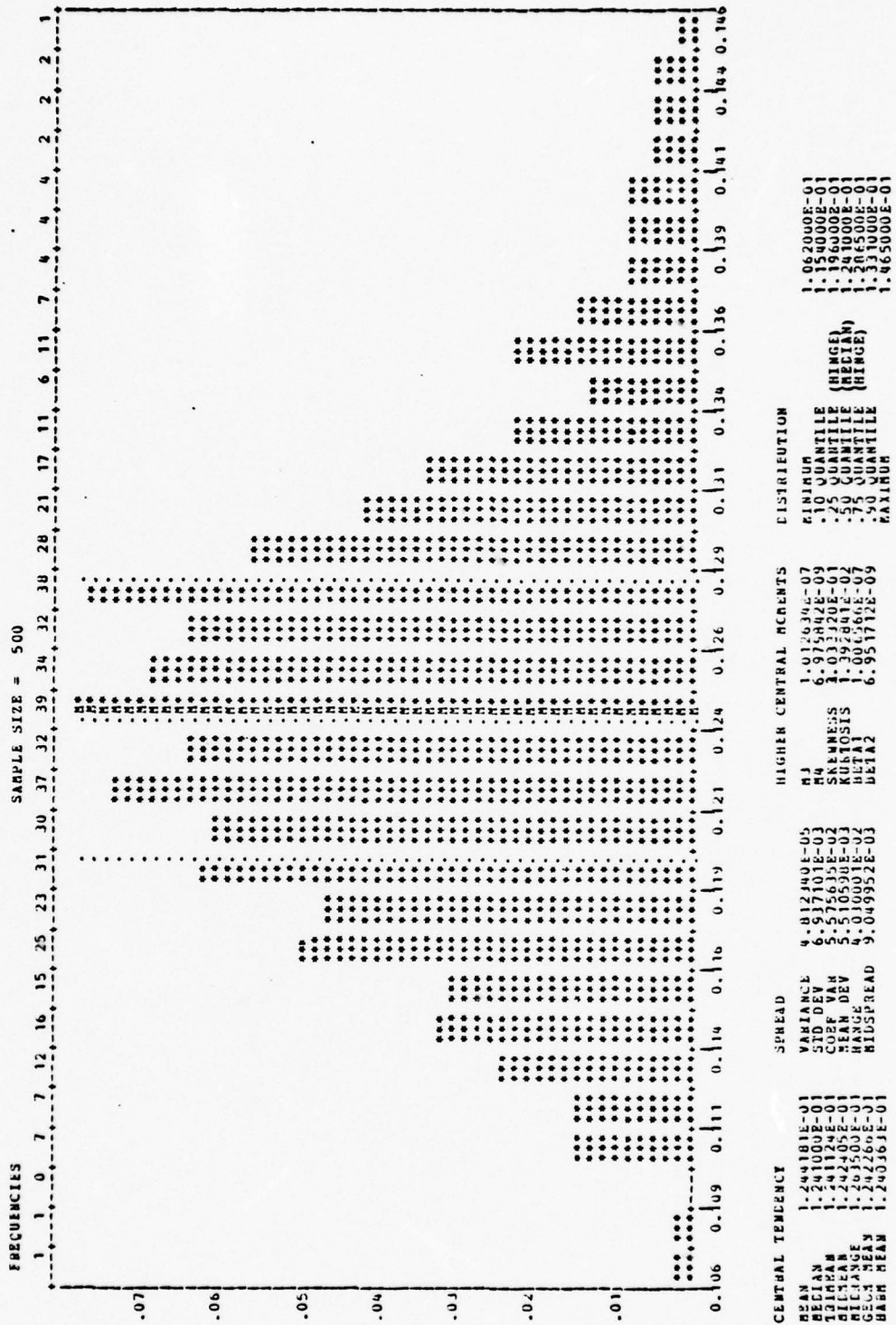
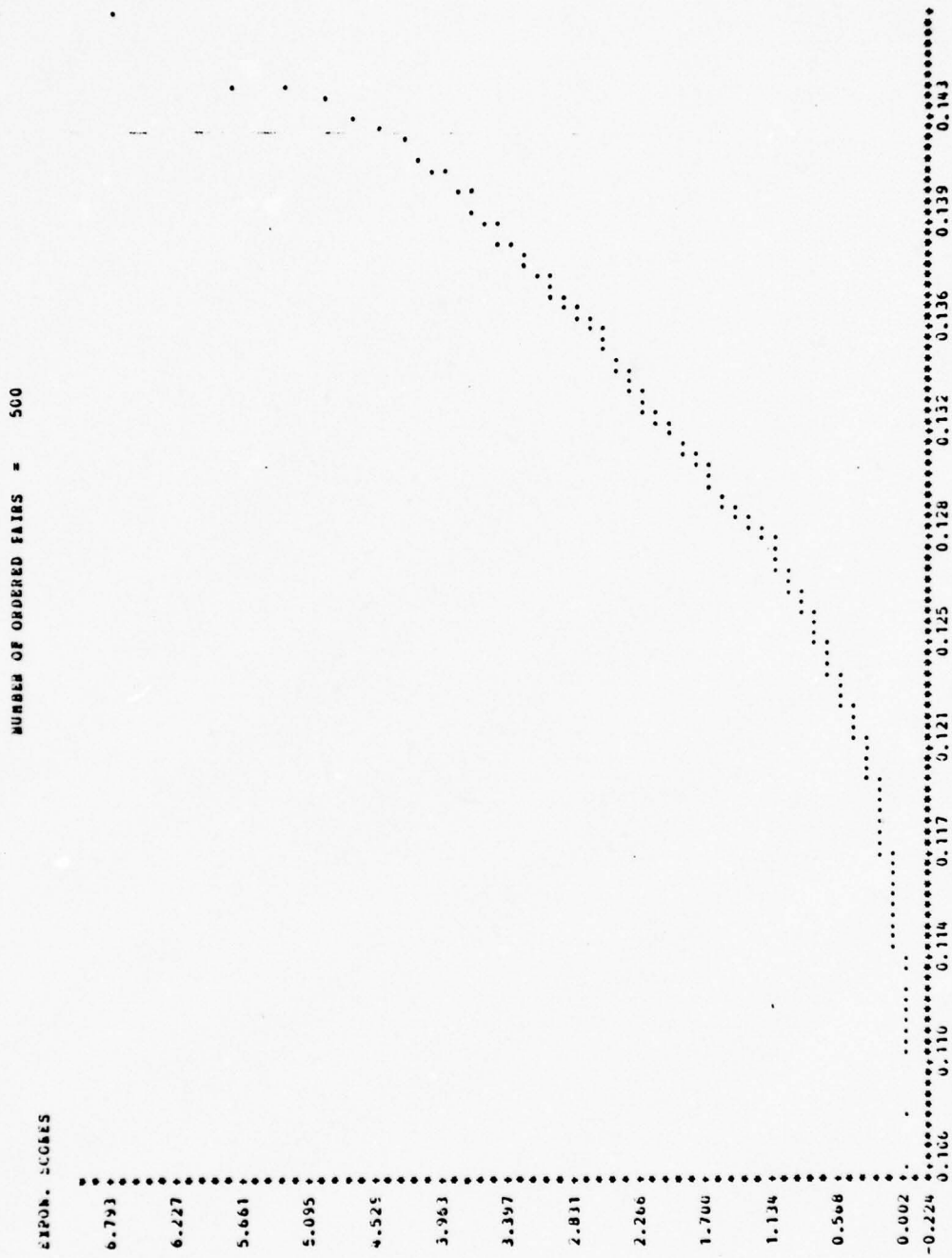


Figure 32a - M/M/1 QUEUE. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#50#95; $m=500$ REPLICATIONS, $RX=4$; $RS=8$.



X-SCALE : ** = 0.266E-03 UNITS
 Y-SCALE : ** = 0.113E 00 UNITS
 ESTIMATED PARAMETERS OF DATA : GAMMA1 = 3.033320E-01 GAMMA2 = 1.39368E-02

Figure 32b - M/M/1 QUEUE. EXPONENTIAL FLOT (EXPLI) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#50#95; m=500 REPLICATIONS, RX=4; RS=8 .

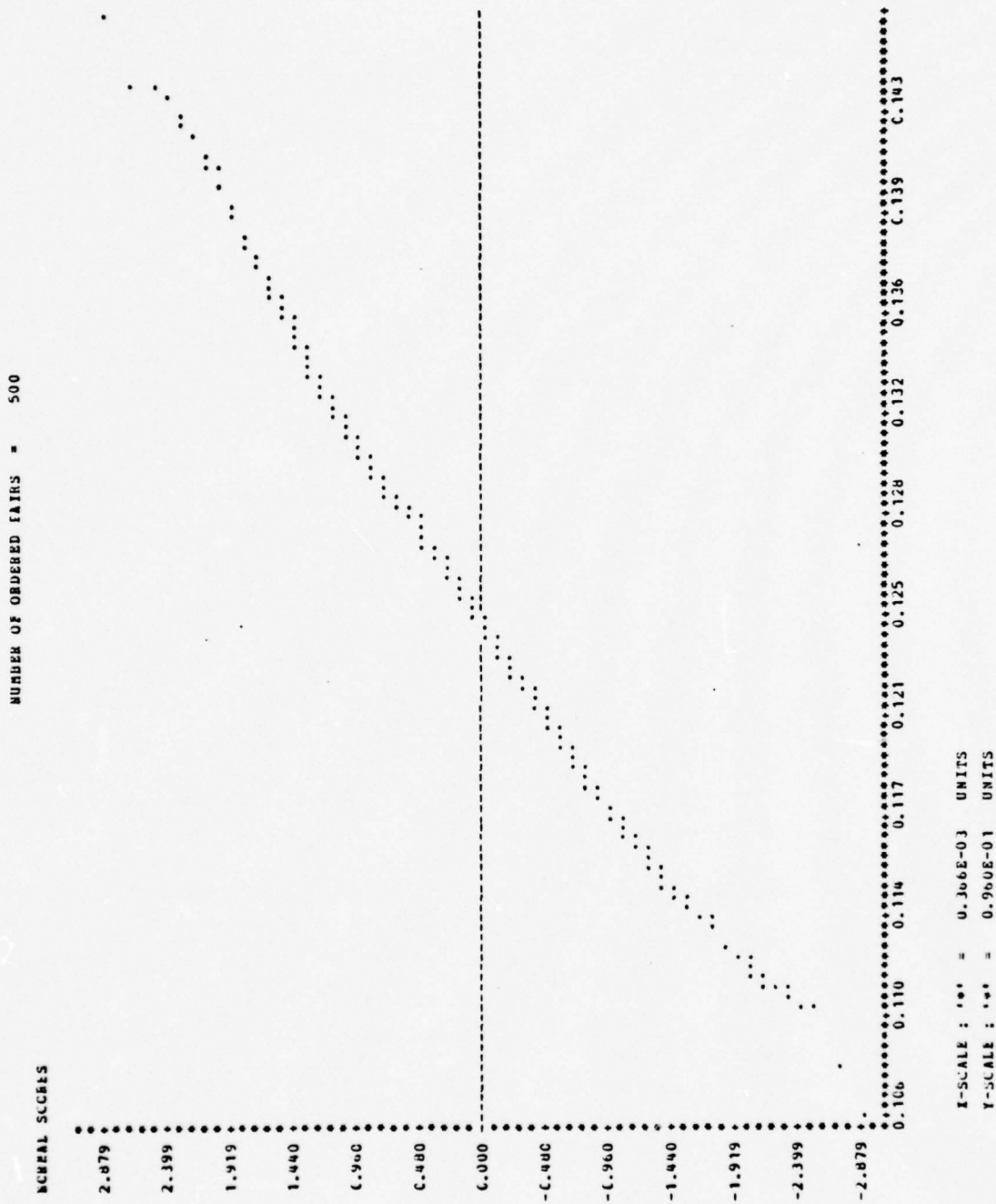


Figure 32c - M/M/1 QUEUE. NORMAL PLOT (NCRMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#50#95; m=500 REPLICATIONS, RX=4; RS=8 .

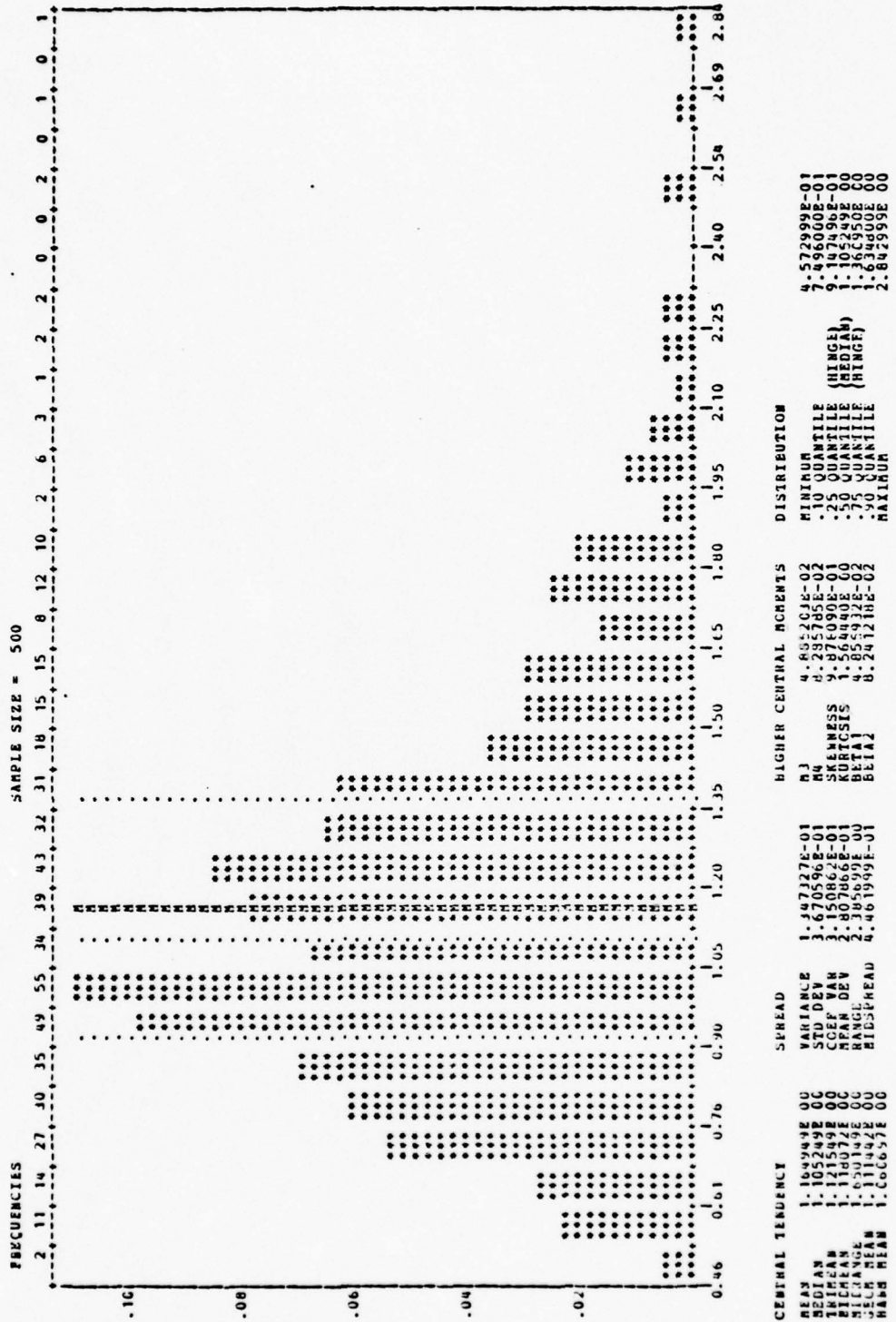
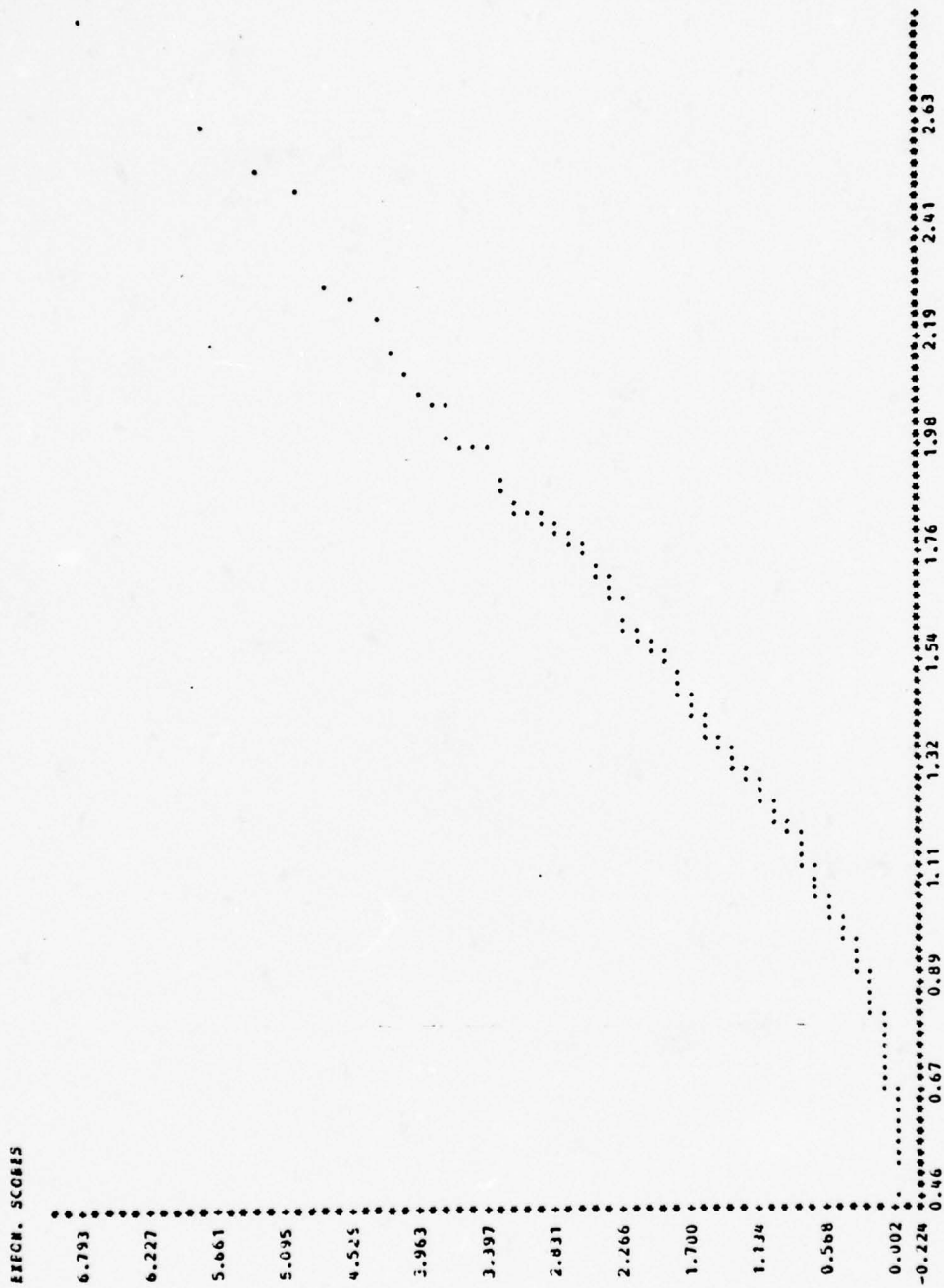


Figure 33a - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN S#50#95; $n=500$ REPLICATIONS, $RX=4$; $RS=8$.

NUMBER OF ORDERED PAIRS = 500



X-SCALE : '00' = 0.2172-01 UNITS
 Y-SCALE : '00' = 0.1132 00 UNITS

ESTIMATED PARAMETERS OF DATA : GAMMA1 = 9.878092E-01 GAMMA2 = 1.564445E 00

Figure 33b - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. EXPONENTIAL PLCT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE RUN S#50#95; m=500 REPLICATIONS, RX=4; RS=8 .

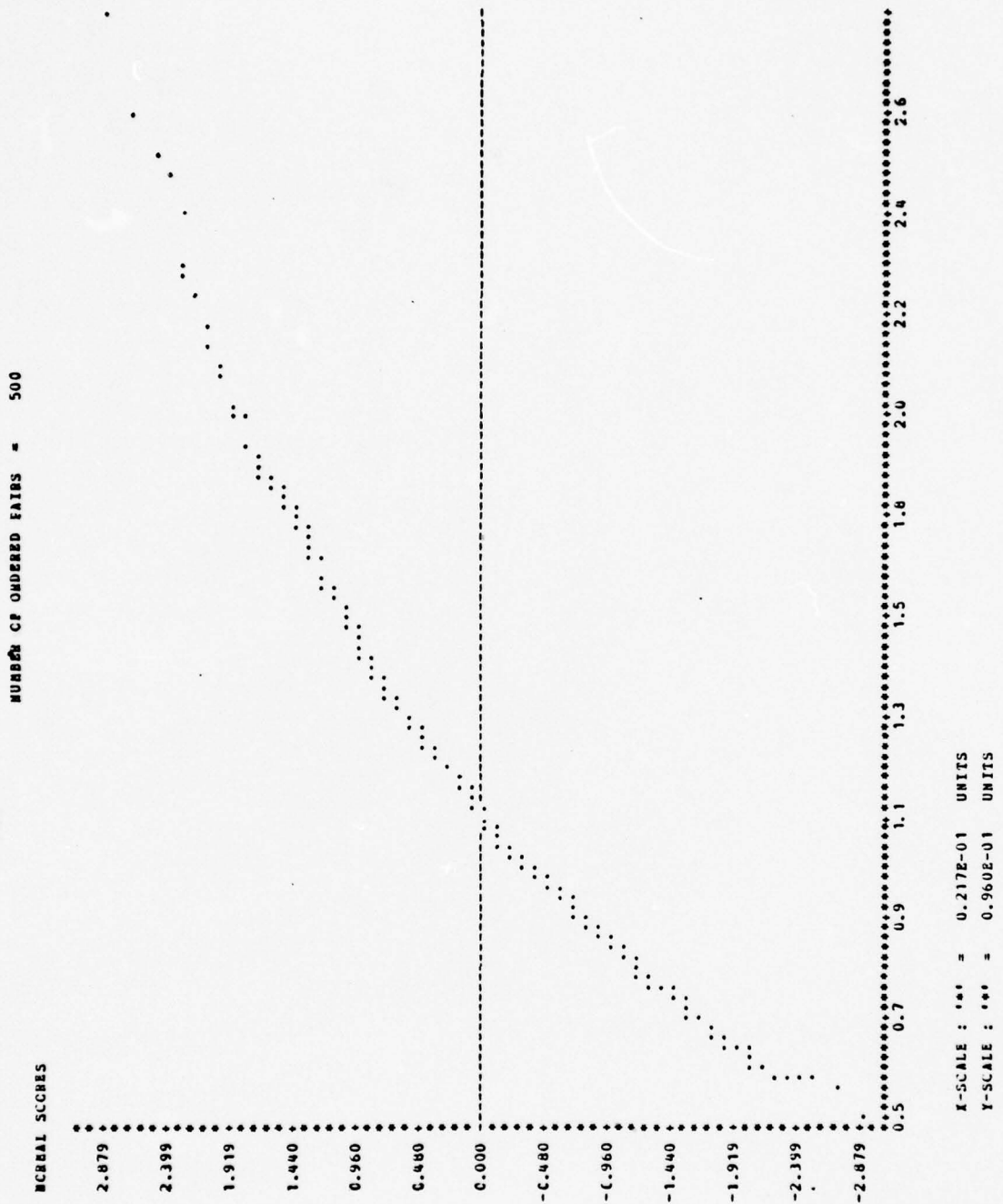


Figure 33c - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGE WAITING TIMES W_{10000} FROM THE RUN S#50#95; $m=500$ REPLICATIONS, $RX=4$; $RS=8$.

CORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	W 1 + 05	W 1	W 2 + 05	W 2	W 3 + 05	W 3	W 4 + 05	W 4
MEAN	1249.528	2499.137	618.464	1242.831	1.10977	2.19321	1.11538	2.24676	1.06352	2.42813	1.22590	2.26181
S (MEAN)	0.7739	1.0863	2.4174	3.3282	0.12115	0.21912	0.11521	0.20915	0.11936	0.24341	0.13227	0.22578
ST. DEV.	17.3060	24.2914	54.0547	74.4219	2.70909	3.48526	2.57611	3.29364	2.66889	3.60218	2.95759	3.71680
SKEWNESS	-0.0260	-0.2054	0.0081	0.0779	4.06847	2.85987	3.71708	2.56583	3.46160	2.07075	3.87350	2.78568
KURTOSIS	0.1782	0.1119	-0.1109	-0.1893	22.30673	11.21000	17.27135	8.11302	12.78089	3.69474	19.32036	10.00003
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	253.00000	500.00000	248.00000	500.00000	219.00000	500.00000	271.00000

PARAMETER	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	1.1084175	1.1266356	1.1483183	1.1649494	-0.0012129	-0.0006308
S (MEAN)	0.0312151	0.0227753	0.0187032	0.0164154	0.0005011	0.0003470
ST. DEV.	0.6979922	0.5092710	0.4482159	0.3670596	0.0112045	0.0077584
SKEWNESS	2.1959677	1.4561481	1.0985785	0.9878090	0.0689812	0.0926246
KURTOSIS	9.9568624	3.4234600	1.6901579	1.5644398	-0.1391373	-0.0192480
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

UNCORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	W 1 + 05	W 1	W 2 + 05	W 2	W 3 + 05	W 3	W 4 + 05	W 4
MEAN	1249.528	2499.137	624.340	1248.573	0.12396	0.23127	0.11235	0.22292	0.13562	0.22855	0.13832	0.25426
S (MEAN)	0.7739	1.0863	0.3945	0.5777	0.00913	0.01406	0.00817	0.01284	0.01183	0.02112	0.01018	0.01556
ST. DEV.	17.3060	24.2914	8.8215	12.5186	0.20414	0.23016	0.18258	0.20378	0.26447	0.32379	0.22771	0.25868
SKEWNESS	-0.0260	-0.2054	0.2370	0.0946	2.27932	1.60773	2.11332	1.39834	3.08997	2.15515	2.87303	2.36012
KURTOSIS	0.1782	0.1119	0.0273	0.0296	5.69143	2.64637	4.49536	1.63658	12.53375	6.30021	13.30220	10.00946
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	268.00000	500.00000	252.00000	500.00000	235.00000	500.00000	272.00000

PARAMETER	W 1 BAR	W 2 BAR	W 3 BAR	W 4 BAR	D 2	D 4
MEAN	0.1242058	0.1242967	0.1244361	0.1244181	-0.0000377	-0.0000564
S (MEAN)	0.0005801	0.0004132	0.0003521	0.0003102	0.0001731	0.0001237
ST. DEV.	0.0129725	0.0092394	0.0078728	0.0069371	0.0038699	0.0027650
SKEWNESS	0.4769654	0.3978707	0.1912445	0.3033320	0.1976776	0.2695435
KURTOSIS	0.0190020	0.0134497	0.1096792	0.0139284	-0.0576553	-0.0083609
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

Figure 33 - QUEUE WITH EARLY AUTOREGRESSIVE SERVICE TIMES AND ECISSEN INPUT. TABULATION OF SAMPLE STATISTICS FOR THE LISTINGS OF CULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (W 1 + 0, W 1, I FOR CASE N=2500, 5000, 7500, 10000), CULATED WAITING TIMES (W 1 BAR ETC.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND X 2 (D 2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN S#50095; R#4; RS=8.

Another result obtained from the simulation concerns the comparison of convergence of \bar{W} and \bar{W} . From the programs we have run we observe that \bar{W} converges slightly faster than \bar{W} (at least for low t and r). Also the standard deviation of \bar{W} is much less than of \bar{W} . In figure 24d we can see that \bar{W}_{10000} has already reached the steady state but not \bar{W}_{10000} . Compare also the values of their standard deviations (.005 versus .2). Figure 34 gives the results of the type S#50#98 model for $N=10000$. We can see that while \bar{W} is already close to the steady state, the fact that \bar{W} has not reached it yet forces us to continue the simulation by increasing the value of N .

CCORRELATED QUEUE

	X 2	X 4	S 2	S 4	W 1 + C5	W 1	W 2 + C5	W 2	W 3 + C5	W 3	W 4 + C5	W 4
PARMTR												
MEAN	1249.528	2459.137	619.247	1242.193	2.45903	4.95772	2.51849	4.51853	2.15700	4.81473	3.53149	6.28379
STDEV	17.3060	24.2514	82.8198	122.3699	6.91220	9.16586	6.84229	8.5367	6.12388	8.42550	8.12391	10.02027
SKWNESS	-0.0260	-0.2054	0.1558	0.2401	5.75748	4.17755	4.30115	2.58666	4.78043	3.15508	3.76556	2.30045
KURTOS.	0.1782	0.1019	-0.0518	0.1234	50.79430	27.25491	22.45005	10.56665	27.63375	11.59648	12.12130	5.70293
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000
PARMTR												
MEAN	2.6210012	2.5524268	0.0016494	2.6378973	2.6317711	-0.0010562	0.0007576	0.0005500	0.0006543	0.0006543	0.0006543	0.0006543
STDEV	0.1389667	1.8257370	1.5561271	1.5561271	1.3808832	0.0165393	0.0122991	0.0122991	0.0122991	0.0122991	0.0122991	0.0122991
SKWNESS	2.1862211	2.1714894	1.9596882	1.9596882	1.9567585	0.1692543	0.2547509	0.2547509	0.2547509	0.2547509	0.2547509	0.2547509
KURTOS.	5.9681206	7.2361326	6.4361839	6.4361839	3.3526345	-0.1520501	0.1401863	0.1401863	0.1401863	0.1401863	0.1401863	0.1401863
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000

UNCORRELATED QUEUE

	X 2	X 4	S 2	S 4	W 1 + C5	W 1	W 2 + C5	W 2	W 3 + C5	W 3	W 4 + C5	W 4
PARMTR												
MEAN	1249.528	2459.137	624.340	1248.573	0.12356	0.23121	0.11235	0.22292	0.13562	0.28855	0.13832	0.25426
STDEV	17.3060	24.2514	82.8198	122.3699	6.91220	9.16586	6.84229	8.5367	6.12388	8.42550	8.12391	10.02027
SKWNESS	-0.0260	-0.2054	0.1558	0.2401	5.75748	4.17755	4.30115	2.58666	4.78043	3.15508	3.76556	2.30045
KURTOS.	0.1782	0.1019	-0.0518	0.1234	50.79430	27.25491	22.45005	10.56665	27.63375	11.59648	12.12130	5.70293
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000
PARMTR												
MEAN	0.1242058	0.1242967	0.1244361	0.1244181	0.0003377	-0.0000564	0.0001237	0.0001237	0.0001237	0.0001237	0.0001237	0.0001237
STDEV	0.0125125	0.0092394	0.0078728	0.0069371	0.0058695	0.0027650	0.0027650	0.0027650	0.0027650	0.0027650	0.0027650	0.0027650
SKWNESS	0.4769654	0.3578707	0.1512445	0.3033220	0.1576776	0.2695435	0.2695435	0.2695435	0.2695435	0.2695435	0.2695435	0.2695435
KURTOS.	0.0150020	0.0134497	0.1090792	0.0135264	-0.0517653	-0.0083609	-0.0083609	-0.0083609	-0.0083609	-0.0083609	-0.0083609	-0.0083609
SPL SIZE	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000	500.000000

Figure 34 - QUEUE WITH EAR1 AUTOREGRESSIVE SERVICE TIMES AND ECISSEN INPUT. TABULATION OF SAMPLE STATISTICS FOR THE DISTRIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZEROS (W1 + 0, W1, I FOR CASE N=2500, 5000, 7500, 10000), CUMULATED WAITING TIMES (W1 BAR ETC.) AND THE AVERAGED DIFFERENCES BETWEEN S2 AND X2 (D2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN S45C498; BX=N; RS=8.

2. Queue with Cross-Correlated Service Time

The model for the cross-correlated queue is:

$$\{X_i\} = \varepsilon_i$$

$\{S_i\}$ is EABMA(1,1) over $\{E_i, \quad i \quad RX/RS, \quad i-1 \quad RX/RS, \quad \dots\}$

That is:

$$S_i = \begin{cases} \beta E_i & \text{w.p. } \beta \\ \beta E_i + A_i & \text{w.p. } (1-\beta) \end{cases}$$

where

$$A_i = \begin{cases} rA_{i-1} & \text{w.p. } r \\ rA_{i-1} + \varepsilon_i \quad RX/RS & \text{w.p. } (1-r) \end{cases}$$

Again we chose the values .25, .50, .95, .99, for t and .25, .50, .90, .95, .98 for r and we run a total of 20 runs to cover all combinations of t and r , naming each run as SC#tt#rr.

Analyzing the results obtained from the simulation we observe the following:

1. The value of the traffic intensity t and the correlation r is the main factor for convergence of W and \bar{W} . That is, high traffic intensity and/or high correlation require N to be large in order for W and \bar{W} to reach the steady state. Thus simulating the models with $t \leq .50$ and $r \leq .95$ for $N=10000$ it was possible for W and \bar{W} to reach the steady state. In figures 35a-35d, 36a-36c we can see the justification of that conclusion. The values .1078 and .0957 of W_{10000} , and \bar{W}_{10000} represent the steady state of

the SC#50#25 model. On the other hand, regardless of the value of t the models with $r \geq .98$ did not reach the steady state for $N=10000$ and regardless of the value of r the models with $t \geq .95$ did not reach the steady state for $N=10,000$. Figure 37 gives us the non-convergence of the model SC#99#25. Because of the high traffic intensity and in order to get the steady state of that model we have to continue simulating by increasing the value of N .

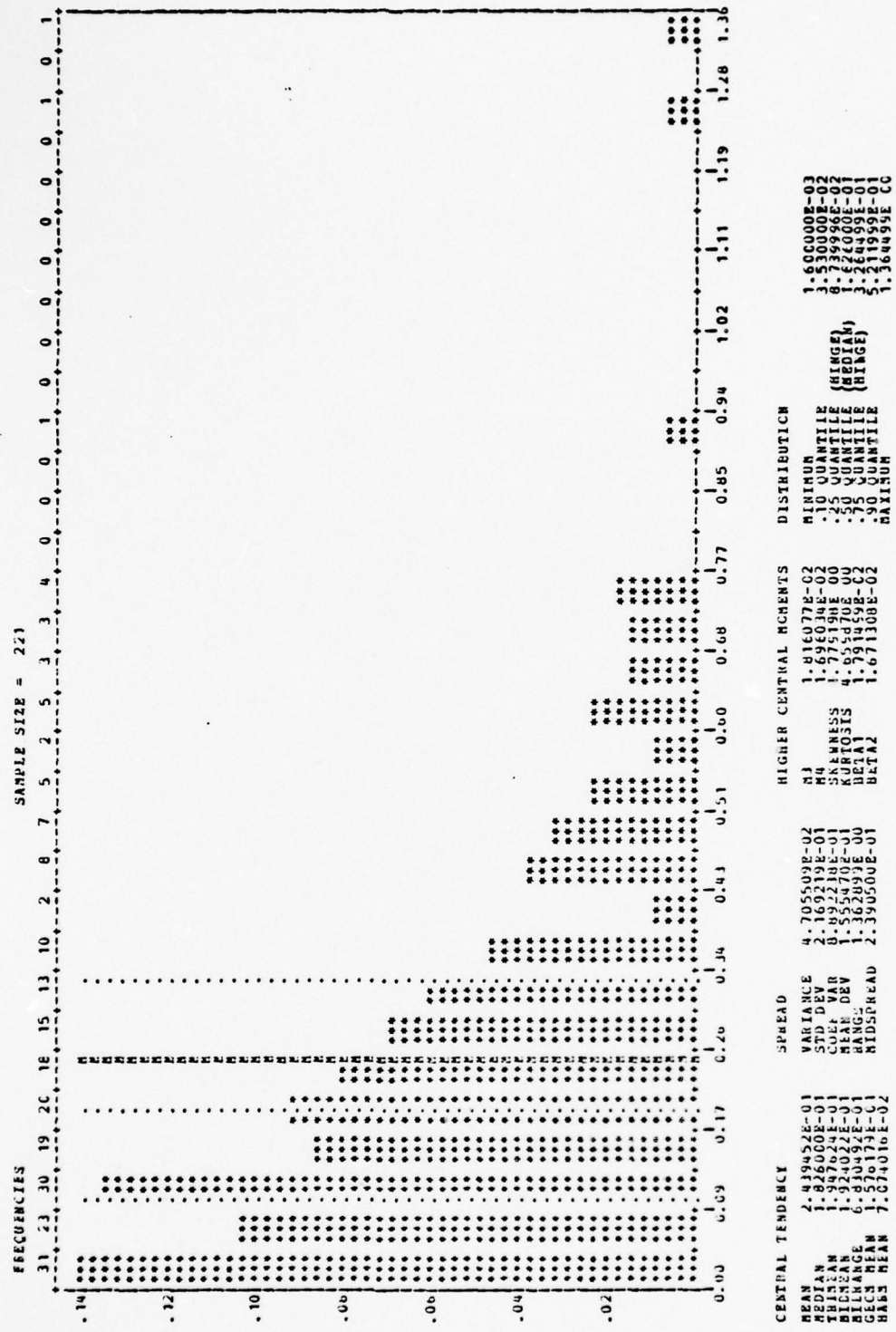


Figure 35a - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE WAITING TIMES W_{10000} WITHOUT ZERCS FROM THE RUN SC#50#25; $m=500$ REPLICATIONS, $RX=2$; $RS=4$. No OF ZEROS=279.

NUMBER OF ORDERED PAIRS = 221

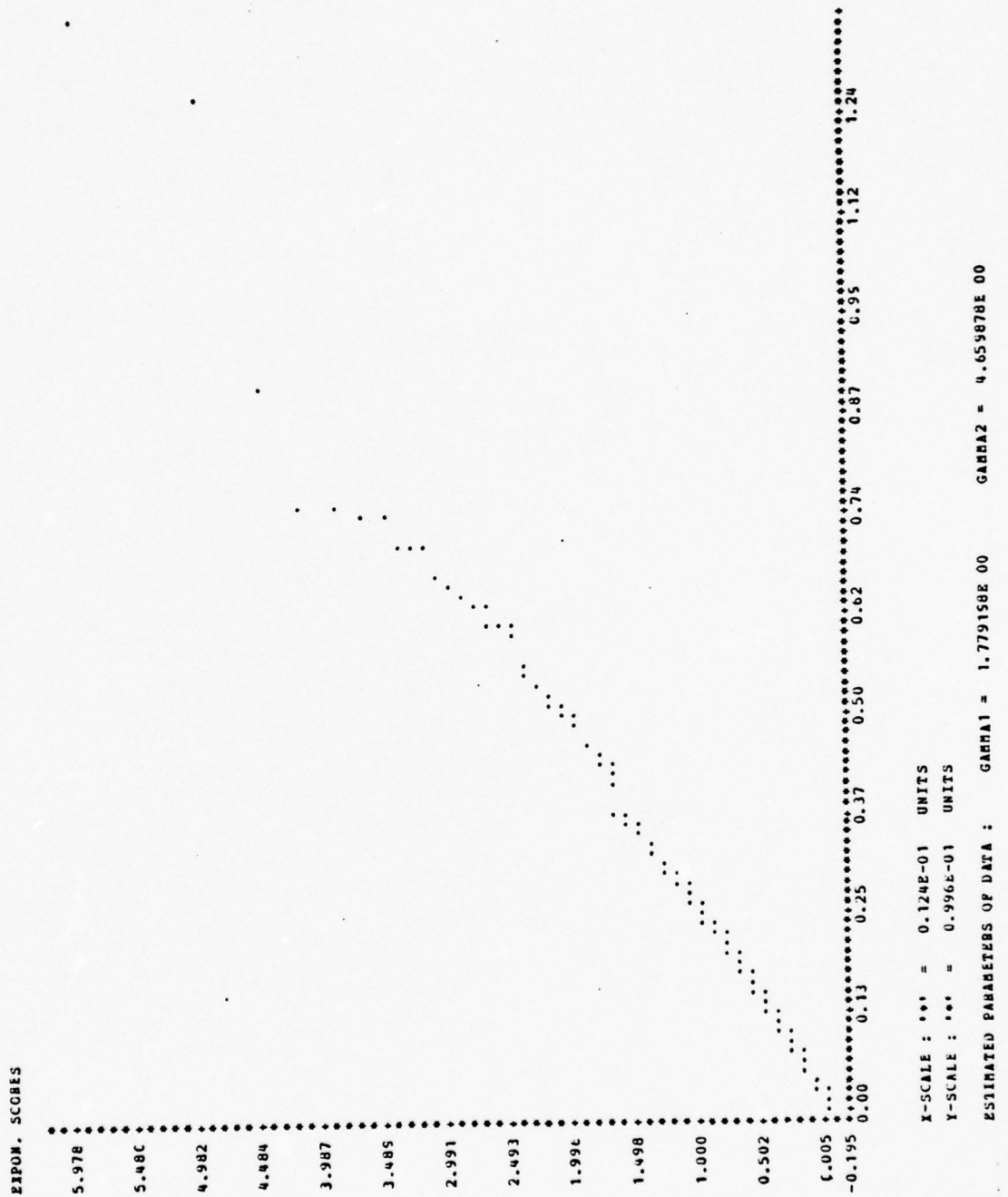


Figure 35k - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PLCT (EXPLT) OF THE WAITING TIMES W_{10000} WITHOUT ZEROS FROM THE RUN SC#50#25; $m=500$ REPLICATIONS, $R_X=2$; $R_S=4$. NO OF ZERCS=279.

NUMBER OF ORDERED PAIRS = 221

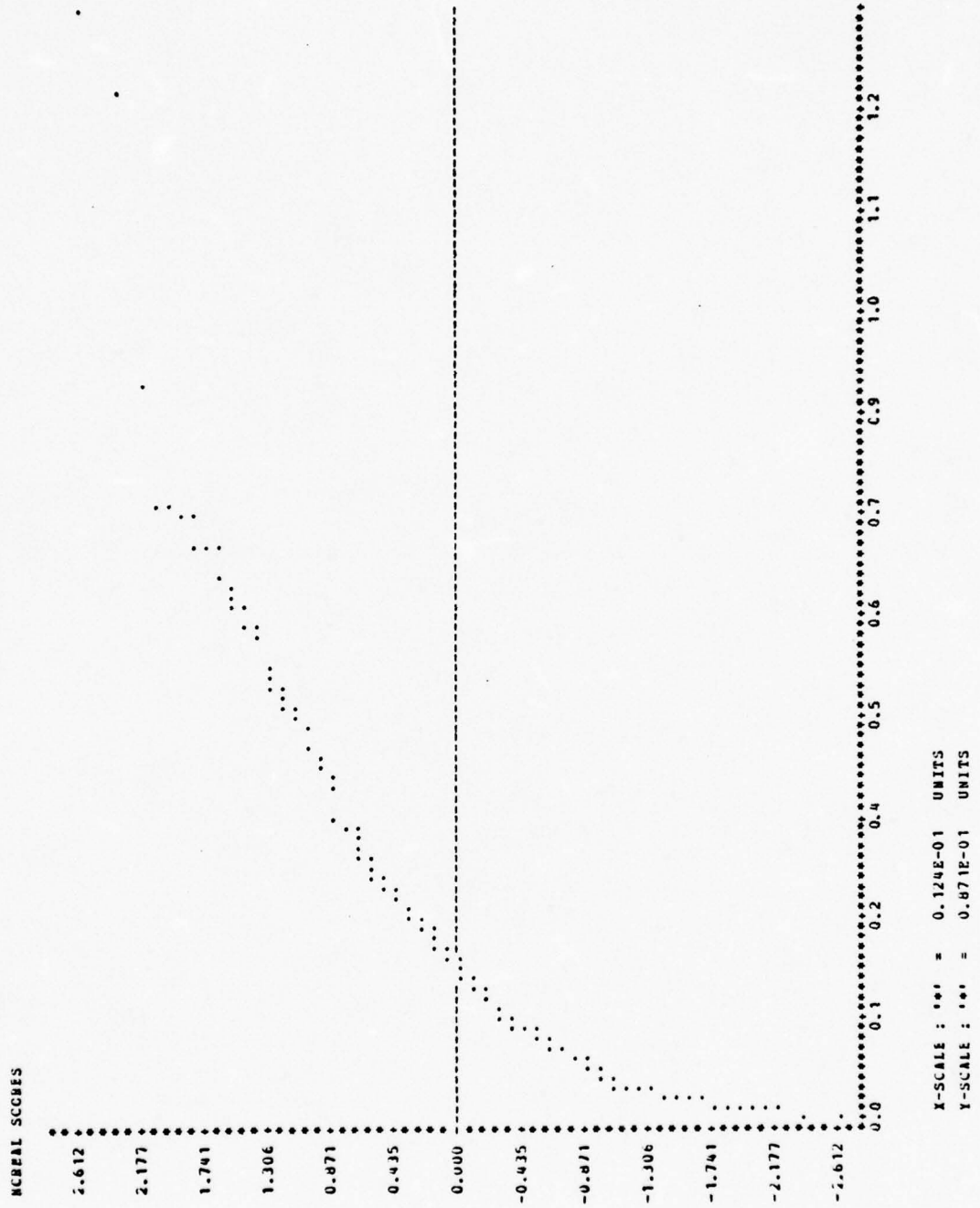


Figure 35c - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE WAITING TIMES W_{10000} WITHOUT ZEROS FROM THE RUN SC#50#25; $m=500$ REPLICATIONS, $RX=2$; $RS=4$. No OF ZERCS=279.

CORRELATED QUEUE

PARENTS	X 2	X 4	S 2	S 4	M 1 + 05	M 1	M 2 + 05	M 2	M 3 + 05	M 3	M 4 + 05	M 4
MEAN	2459.491	4596.012	1248.523	2498.391	0.090003	0.22174	0.08832	0.21130	0.10627	0.25670	0.10782	0.24395
S (MEAN)	1.5480	2.1660	0.8761	1.1685	0.00725	0.01336	0.00683	0.01195	0.00940	0.01814	0.00842	0.01459
ST. DEV.	34.6141	48.4334	18.4721	26.1284	0.16256	0.19040	0.15270	0.17272	0.21009	0.26098	0.18829	0.21692
SKEWNESS	-0.0260	-0.2050	0.0727	-0.1522	2.16911	1.10451	2.14244	1.23810	2.86387	1.79610	2.53022	1.77920
KURTOS.	0.1793	0.1020	0.1292	-0.1513	4.50274	0.72695	4.59767	1.23196	9.57336	3.54677	8.39917	4.65987
SMBL SIZE	500.00000	500.00000	500.00000	500.00000	203.00000	500.00000	203.00000	500.00000	207.00000	500.00000	221.00000	500.00000

PARENTS	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.0953327	0.0953571	0.0957121	0.0957863	-0.0001937	0.0002375
S (MEAN)	0.0003439	0.0002440	0.0002007	0.0001769	0.0002094	0.0001920
ST. DEV.	0.0076895	0.0054555	0.0044877	0.0039563	0.0060232	0.0042922
SKEWNESS	0.0066125	0.3031107	0.1048896	0.1783530	0.1998498	0.0720455
KURTOS.	0.7583609	0.2003317	-0.1696115	-0.1895771	0.4037561	0.0656462
SMBL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

UNCORRELATED QUEUE

PARENTS	X 2	X 4	S 2	S 4	M 1 + 05	M 1	M 2 + 05	M 2	M 3 + 05	M 3	M 4 + 05	M 4
MEAN	2459.491	4596.012	1248.937	2498.015	0.24793	0.46255	0.22471	0.44585	0.27123	0.57709	0.27664	0.50852
S (MEAN)	1.5480	2.1660	0.7893	1.1556	0.01826	0.02812	0.01633	0.02567	0.02366	0.04224	0.02037	0.03113
ST. DEV.	34.6141	48.4334	17.6482	25.8401	0.40829	0.46032	0.36516	0.40795	0.52894	0.64755	0.45542	0.51337
SKEWNESS	-0.0260	-0.2050	0.2370	0.0945	2.27934	1.60777	2.11334	1.35835	3.09001	2.15518	2.87300	2.38006
KURTOS.	0.1783	0.1020	0.0275	0.0293	5.69149	2.64647	4.49550	1.63665	12.53415	6.30046	13.30202	10.00915
SMBL SIZE	500.00000	500.00000	500.00000	500.00000	268.00000	500.00000	252.00000	500.00000	235.00000	500.00000	272.00000	500.00000

PARENTS	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.2484031	0.2486218	0.2489119	0.2488776	-0.0001108	0.0002000
S (MEAN)	0.0011609	0.0008263	0.0007043	0.0006207	0.0003462	0.0002467
ST. DEV.	0.0259509	0.0184774	0.0151478	0.0138790	0.0077402	0.0055169
SKEWNESS	0.4764770	0.3773002	0.1904184	0.3035822	0.1977163	0.2697002
KURTOS.	0.0193195	0.0144081	0.1101370	0.0134916	-0.0576601	-0.0075665
SMBL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

Figure 35d - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE LISTED STATISTICS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (M 1 + 0, M 1, I FOR CASE N=2500, 5000, 7500, 10000), CUMULATED WAITING TIMES (M 1 BAR ETC.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND X 2 (D 2); OBTAINED FROM M=500 REPLICATIONS OF THE RUN SC#50825; BX=2; BS=4.

2. High traffic intensity and correlation affects the convergence of the distribution of W and \bar{W} also. Furthermore the steady state of their distributions, requires larger N than is required for the steady state of their expected values. In examining that conclusion, look for example at the results of the simulated model SC#50#98, where although W and \bar{W} have already reached the steady state for $N=10000$, their distribution has not converged yet. The above results have been obtained from Figures 38a-38c, 39a-39d where W_{10000} and \bar{W}_{10000} are presented. We can see from these figures (38a-38c) that although the value of W_{10000} has reached the steady state (see Figure 39d), its distribution cannot be characterized since the plot under EXFIT (figure 38b) is not a straight line and the parameters $\chi_L=3.34$ and $\gamma_L=11.0$ are far away from the desired values 2 and 6. The same results can be obtained from the figures 39a-39c which deal with \bar{W}_{10000} .

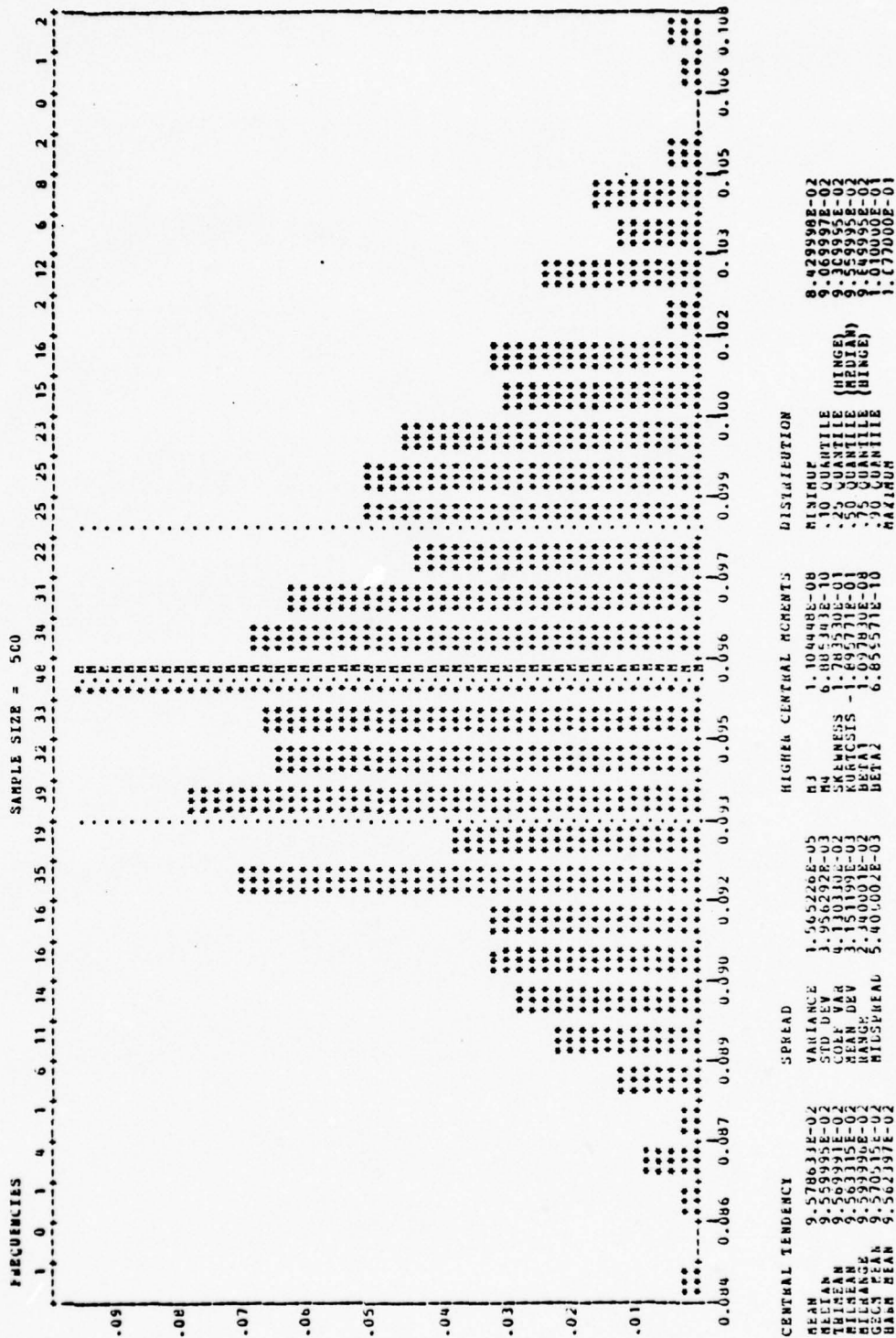
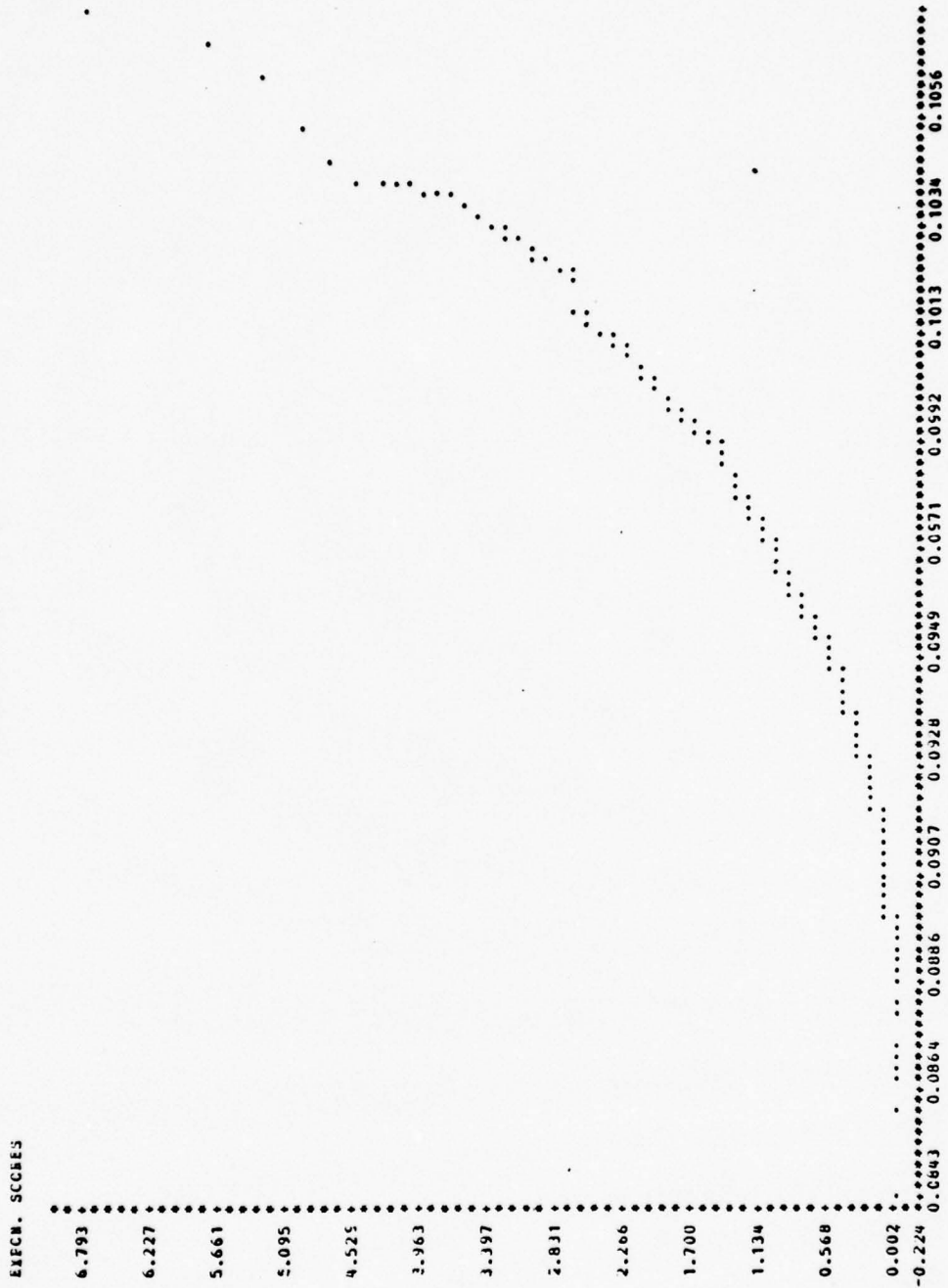


Figure 36a - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN SC#50#25; $n=1500$ REPLICATIONS, $RX=2$; $RS=4$.

NUMBER OF ORDERED FAIRS = 500



X-SCALE : '0' = 0.211E-03 UNITS
 Y-SCALE : '0' = 0.111E 00 UNITS

ESTIMATED PARAMETERS OF DATA : GANNA1 = 1.783515E-01 GANNA2 = -1.895754E-01

Figure 36t - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL FICT (EXPLT) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN SC#50#25; m=500 REPLICATIONS, RX=2; RS=4 .

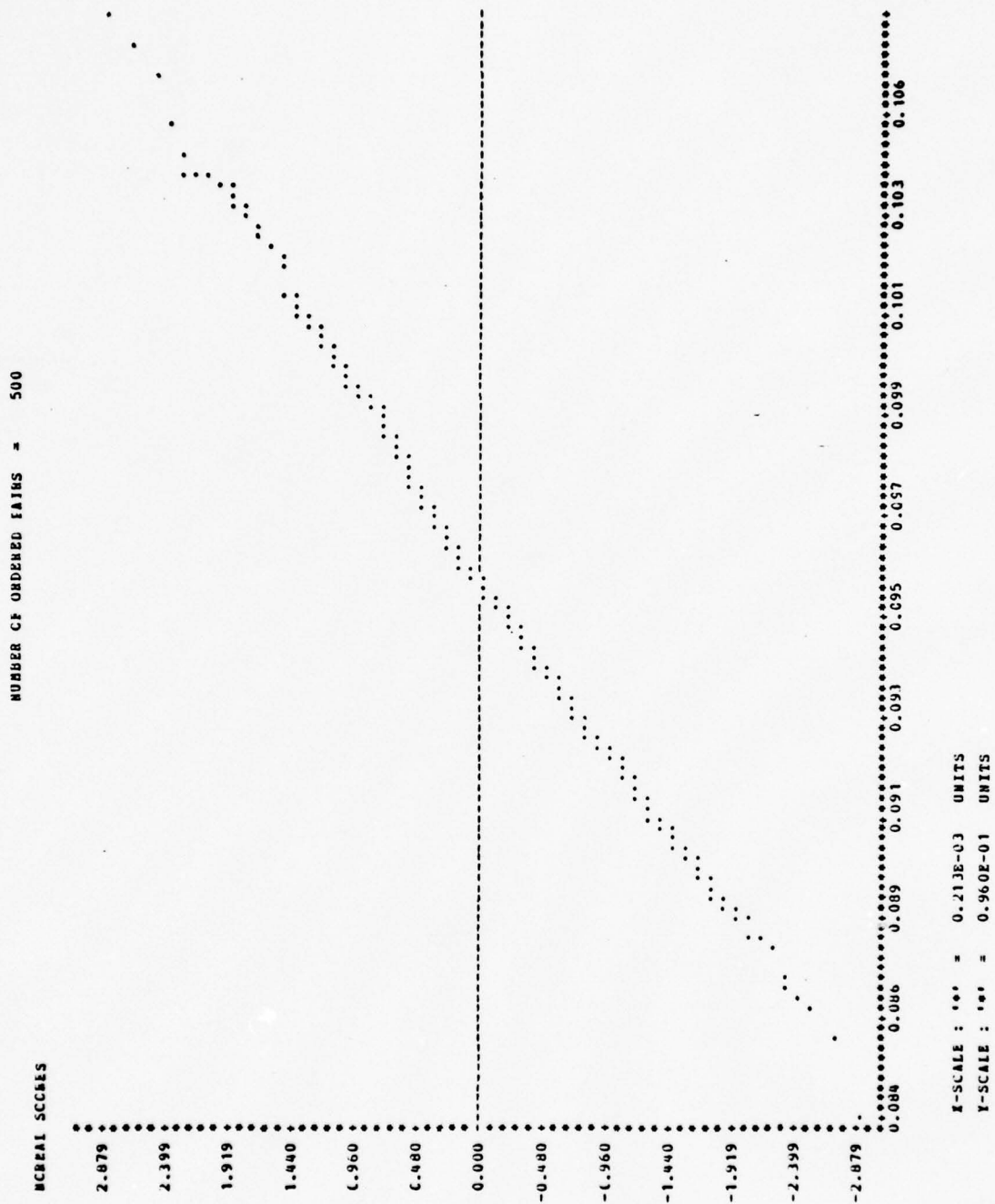


Figure 36c - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE RUN SC#50#25; $m=500$ REPLICATIONS, $RX=2$; $RS=4$.

CORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + CS	M 1	M 2 + CS	M 2	M 3 + CS	M 3	M 4 + CS	M 4
MEAN	1683.007	3366.192	1683.091	3366.192	10.19420	10.25717	12.70306	13.06703	14.64458	14.82286	15.51440	15.60805
STDEV	1.5424	1.5431	1.5580	1.5580	0.38230	0.38231	0.50462	0.50462	0.57637	0.57637	0.65565	0.65565
ST. DEV.	22.3082	22.3102	24.6300	24.6300	0.54841	0.54841	11.27030	11.27030	12.85521	12.85521	13.63212	13.63212
SKWNESS	-0.0051	-0.0053	0.0128	0.0128	1.30525	1.30525	1.55115	1.55115	1.50307	1.50307	1.43627	1.43627
KURTOS.	0.1783	0.1818	0.1253	0.1253	2.40125	2.40125	3.25545	3.25545	1.57342	1.57342	2.85161	2.85161
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000
PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR
MEAN	1.234251	9.393286	10.913062	11.860242	-0.002682	-0.002682	-0.002682	-0.002682	-0.002682	-0.002682	-0.002682	-0.002682
STDEV	0.187230	0.247376	0.275785	0.297485	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158
ST. DEV.	4.160114	5.590428	6.167046	6.652048	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828
SKWNESS	1.705758	2.945274	1.930573	1.824581	0.137454	0.137454	0.137454	0.137454	0.137454	0.137454	0.137454	0.137454
KURTOS.	4.571205	6.823605	5.644484	5.201917	0.123612	0.123612	0.123612	0.123612	0.123612	0.123612	0.123612	0.123612
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000

UNCORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + CS	M 1	M 2 + CS	M 2	M 3 + CS	M 3	M 4 + CS	M 4
MEAN	1683.007	3366.192	1683.284	3321.025	14.82400	15.03440	16.82291	15.12897	21.06930	22.27745	23.66727	23.30634
STDEV	1.0424	1.4031	1.3524	1.5400	0.56681	0.56525	0.75766	0.76221	0.51463	0.51463	1.02815	1.03093
ST. DEV.	22.3082	22.7162	23.5316	34.4530	12.07428	12.63530	16.54151	16.50074	20.43825	20.43825	22.54552	22.93684
SKWNESS	-0.0051	-0.0053	0.0205	0.0294	1.49055	1.13940	1.44110	1.44187	1.24762	1.24762	1.50335	1.50335
KURTOS.	0.1783	0.1818	0.0275	0.0294	0.79072	0.79072	2.37242	2.37242	1.81030	1.81030	5.34098	5.34098
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000
PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR	M 4 BAR
MEAN	10.391620	12.892275	16.154846	17.773155	-0.0001574	-0.0001574	-0.0001574	-0.0001574	-0.0001574	-0.0001574	-0.0001574	-0.0001574
STDEV	0.275785	0.297485	0.440376	0.501058	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158	0.002158
ST. DEV.	6.255255	6.559340	10.041089	11.2049074	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828	0.004828
SKWNESS	1.432545	1.473699	1.626329	1.734255	0.289450	0.289450	0.289450	0.289450	0.289450	0.289450	0.289450	0.289450
KURTOS.	1.914141	2.072253	2.911200	3.598955	-0.115468	-0.115468	-0.115468	-0.115468	-0.115468	-0.115468	-0.115468	-0.115468
SPL SIZE	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000

Figure 37 - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND ECSSON INPUT. TABLETION OF SAMPLE STATISTICS FOR THE DISTIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (M 1 + 0, M 1, I FOR CASE N=2500, 5000, 7500, 10000), CUMULATED WAITING TIMES (M 1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND S 2 (D 2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN SC499825; BX=2.97; BS=3.

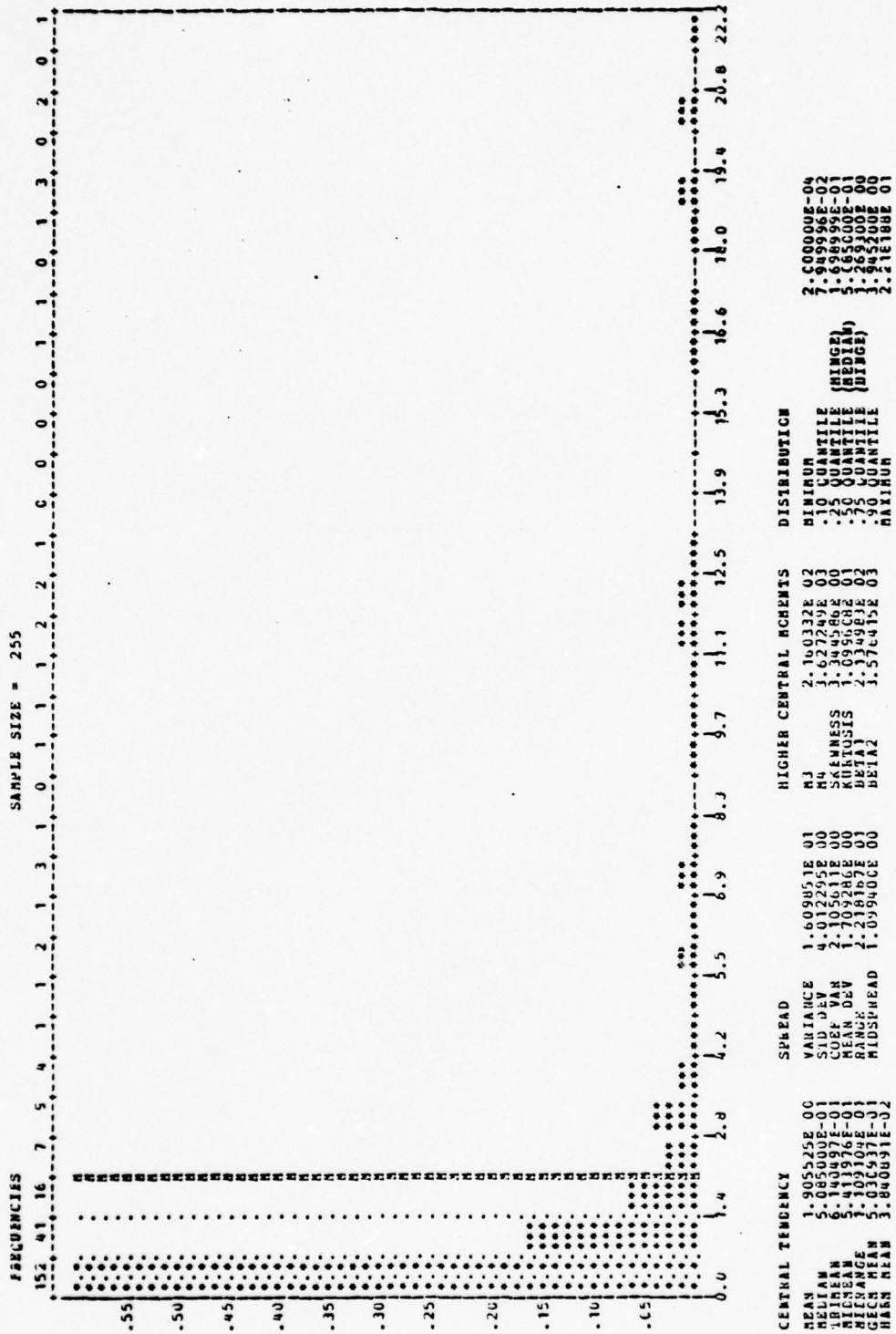
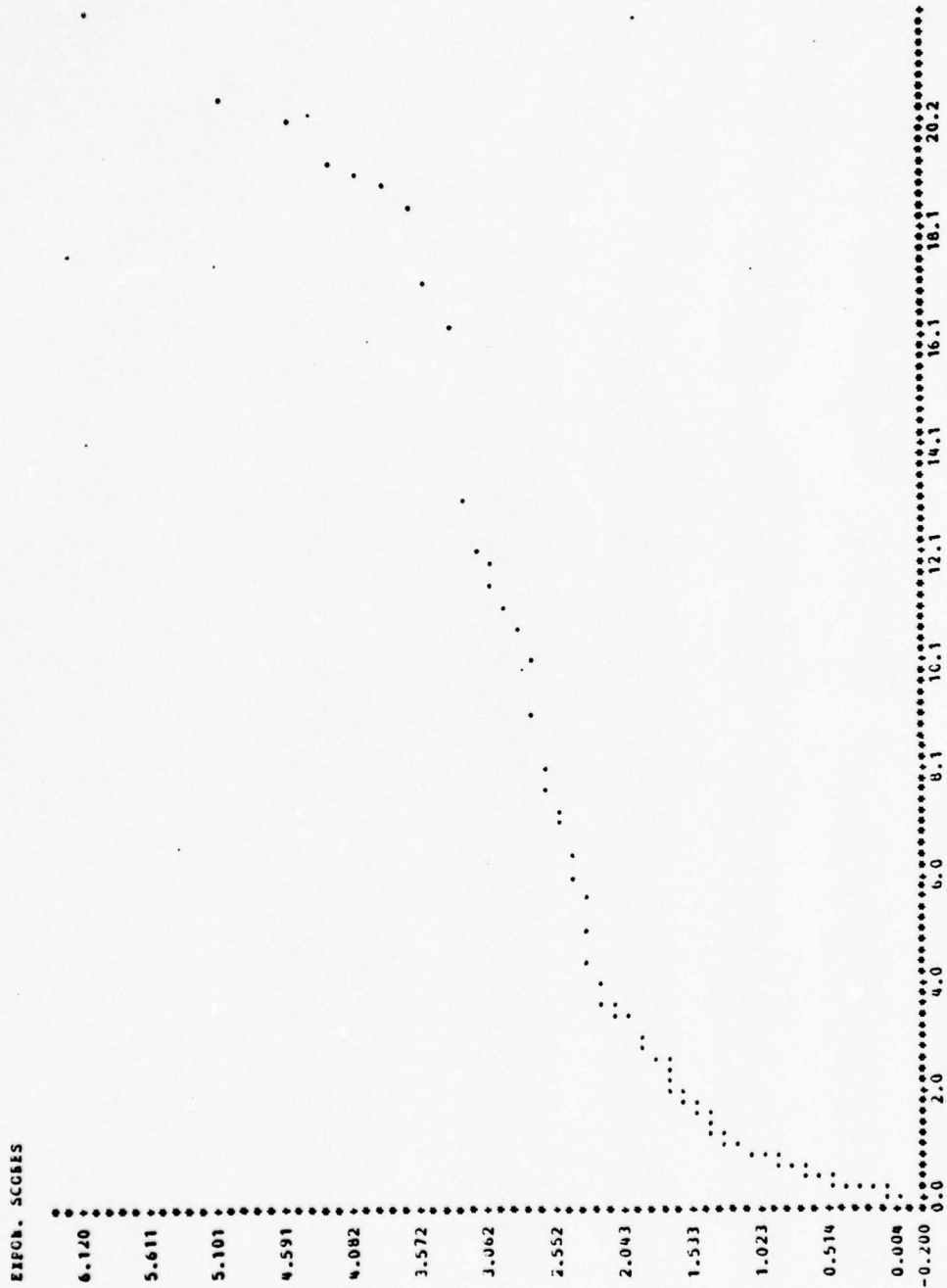


Figure 38a - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE WAITING TIMES W₁₀₀₀₀ WITHOUT ZEROS FROM THE RUN SC#50#98; m=500 REPLICATIONS, RX=2; RS=4. No OF ZEROS=245.

NUMBER OF ORDERED PAIRS = 255



X-SCALE : * * * = 0.202E 00 UNITS
Y-SCALE : * * * = C.102E 00 UNITS

ESTIMATED PARAMETERS OF DATA : GAMMA1 = 3.344565E 00 GAMMA2 = 1.099609E 01

Figure 38b - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PLOT (EXPLT) OF THE WAITING TIMES W_{10000} WITHOUT ZERCS FROM THE RUN SC#50#98; $m=500$ REPLICATIONS, $RX=2$; $RS=4$. No OF ZERCS=245.

NUMBER OF ORDERED PAIRS = 255

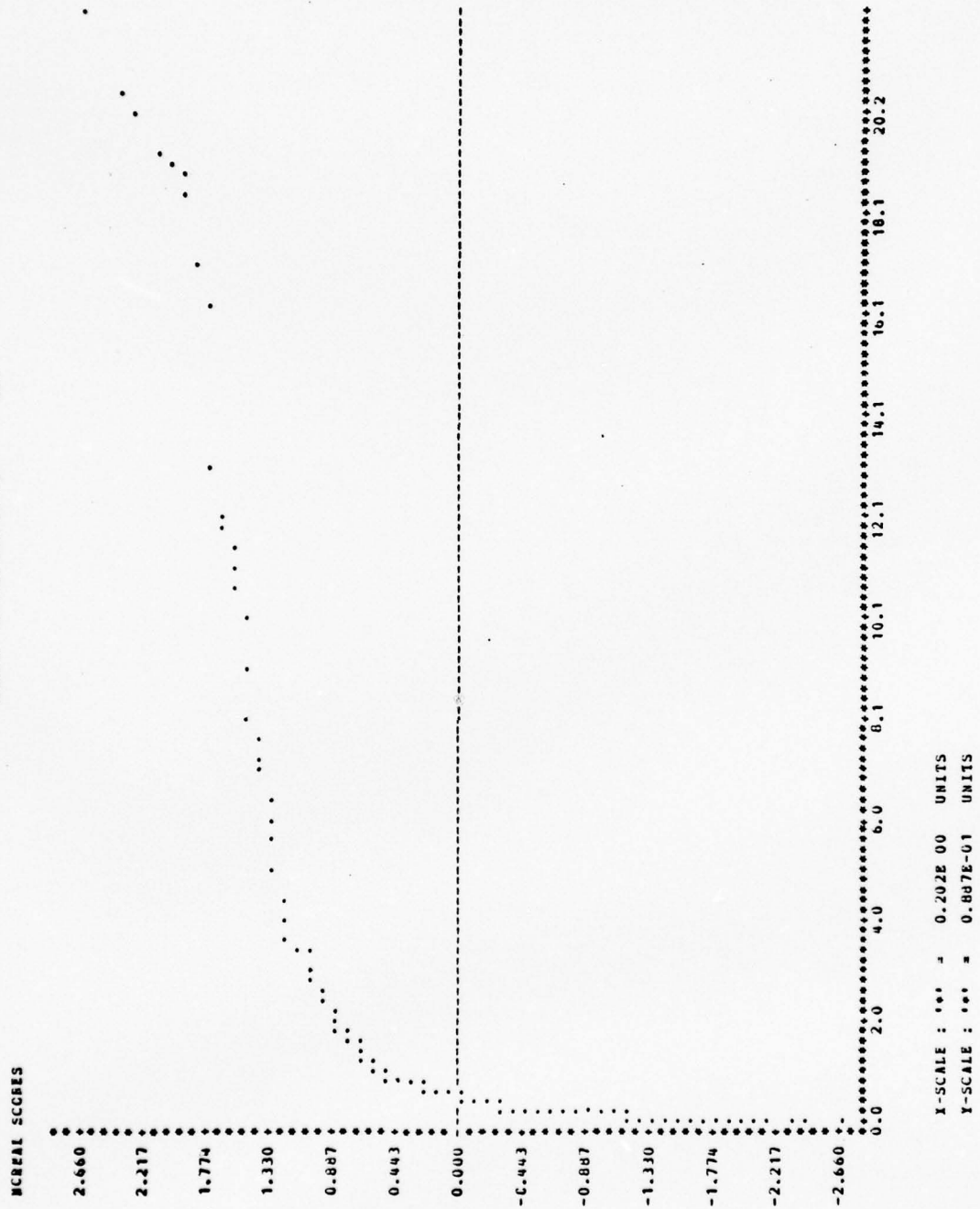


Figure 38c - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLCT (NORMPL) OF THE WAITING TIMES W_{10000} WITHOUT ZERCS FROM THE RUN SC#50#98; $m=500$ REPLICATIONS, $RX=2$; $RS=4$. No OF ZERCS=245.

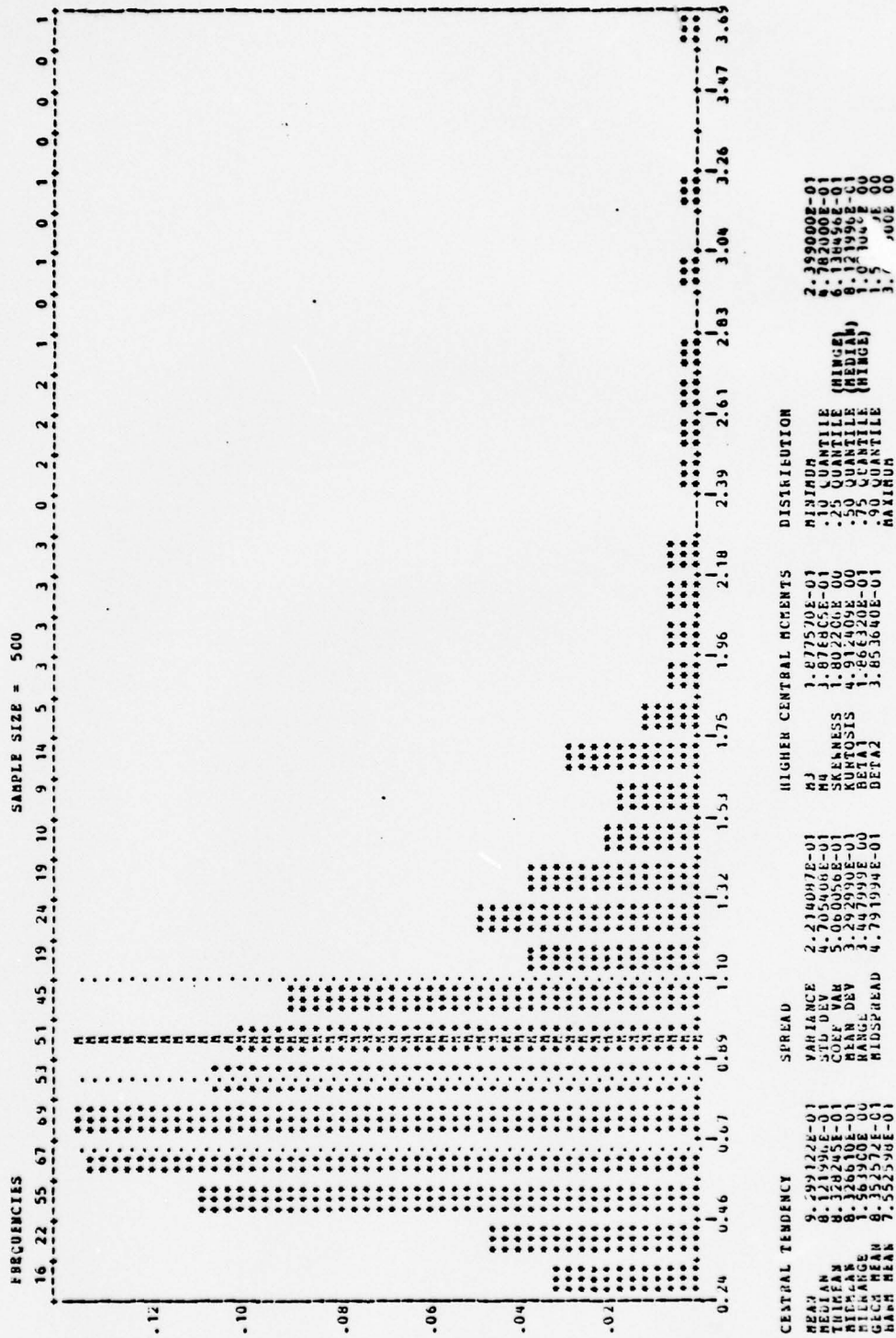
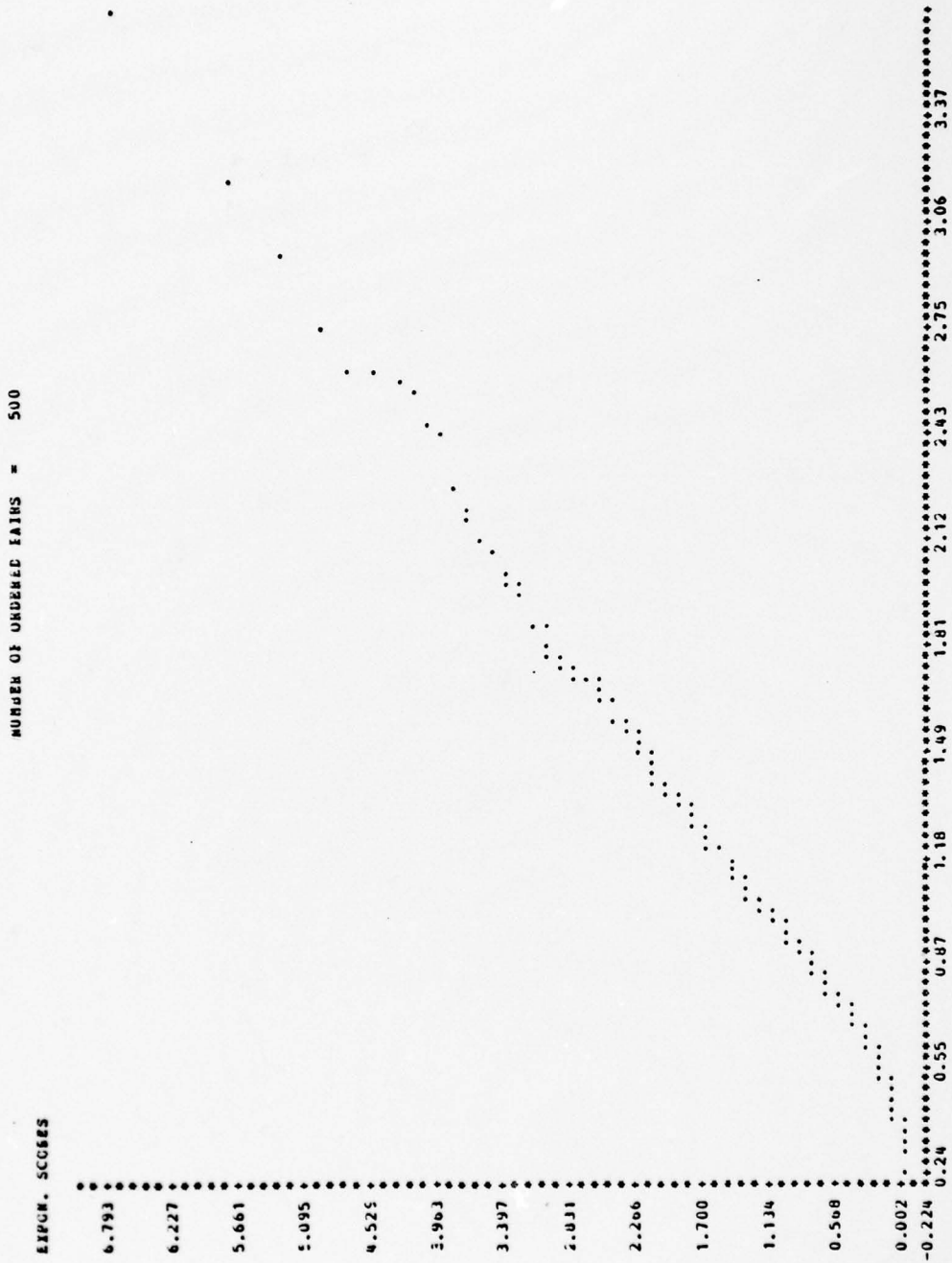


Figure 39a - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE SC#50#98; $m=500$ REPLICATIONS, $RX=2$; $RS=4$.



X-SCALE : ** = 0.313E-01 UNITS
Y-SCALE : ** = 0.113E 00 UNITS

ESTIMATED PARAMETERS OF DATA : GAMMA1 = 1.802204E 00 GAMMA2 = 4.912409E 00

Figure 39b - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PLOT (EXPFIT) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE SC#50#98; $n=500$ REPLICATIONS, $R_X=2$; $R_S=4$.

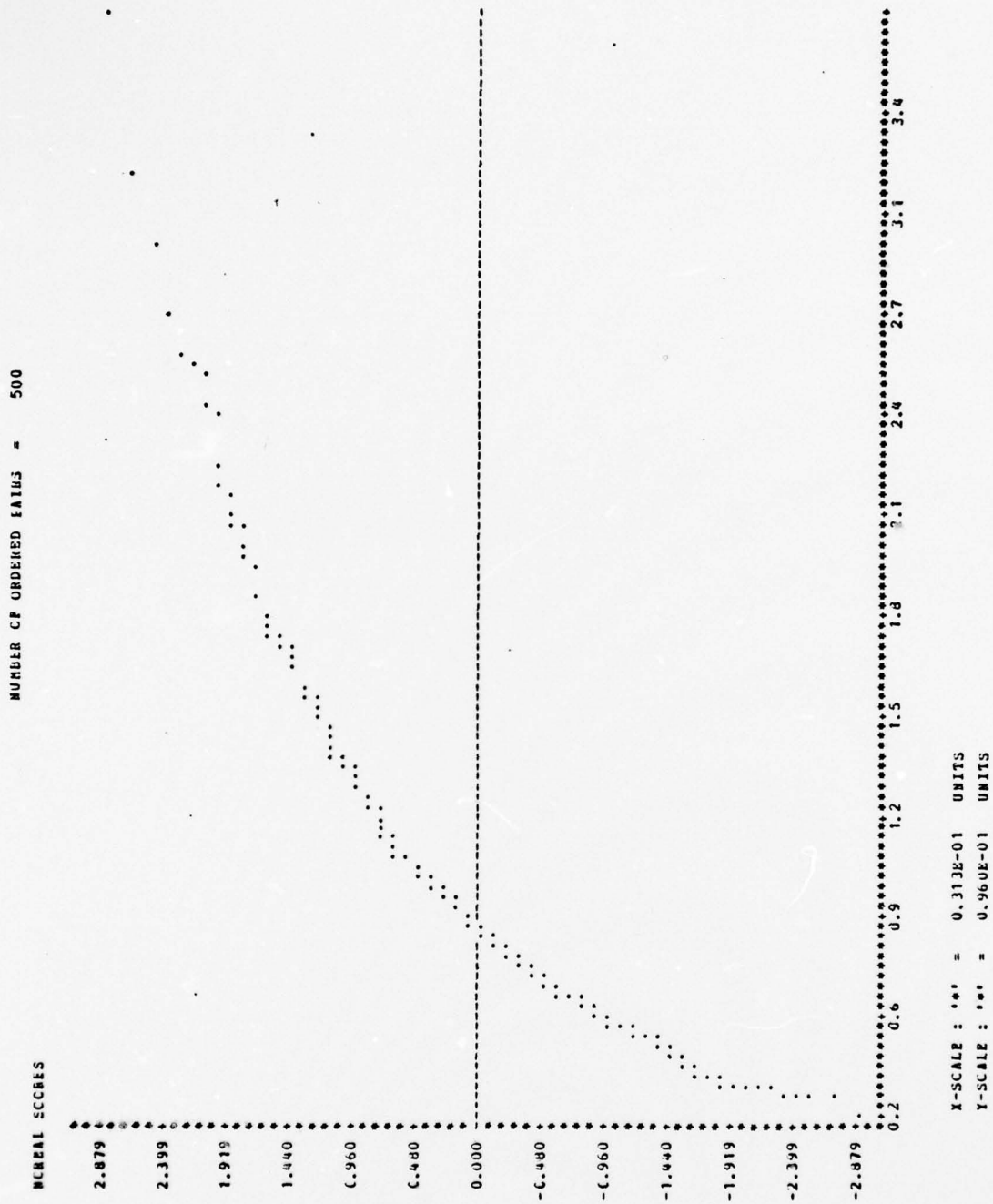


Figure 39c - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE SC#50#98; n=500 REPLICATIONS, RX=2; RS=4 .

CORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + S	M 1	M 2 + S	M 2	M 3 + S	M 3	M 4 + S	M 4
MEAN	2459.491	4996.012	1253.389	2501.876	0.82910	1.77159	0.78178	1.53894	1.01661	2.15383	0.97182	1.90553
S (MEAN)	1.5480	2.1660	3.8788	5.7193	0.15756	0.32625	0.13082	0.24868	0.15299	0.30862	0.13493	0.25126
ST. DEV.	34.6141	48.4334	86.7337	127.8872	3.52316	4.90663	2.92525	3.96327	3.42097	4.73185	3.01722	4.01229
SKEWNESS	-0.0260	-0.2050	0.2924	0.2197	9.29953	6.46620	10.00820	7.43863	-6.11774	4.20099	4.82571	3.34459
KURTOS.	0.1783	0.1020	0.1695	0.0154	108.90057	51.79210	141.44316	76.10095	46.89284	21.70930	24.65054	10.99608
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	500.00000	234.00000	500.00000	254.00000	500.00000	236.00000	500.00000	255.00000

D 2 D 4

PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.9309662	0.9259205	0.9456646	0.9299122	0.0007794	0.0005059
S (MEAN)	0.0460532	0.0299628	0.0245545	0.0210432	0.0008164	0.0005802
ST. DEV.	1.0297813	0.6704358	0.5490551	0.4705406	0.0182555	0.0129748
SKEWNESS	4.7568645	3.0633430	2.0192347	1.8022060	0.1497339	0.1467554
KURTOS.	32.0446930	15.3456116	6.8067226	4.9124088	-0.0639658	-0.0135975
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

UNCORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + S	M 1	M 2 + S	M 2	M 3 + S	M 3	M 4 + S	M 4
MEAN	2459.491	4996.012	1248.937	2498.015	0.24793	0.46255	0.22471	0.44585	0.27123	0.57769	0.27664	0.50852
S (MEAN)	1.5480	2.1660	0.7893	1.1556	0.01826	0.02812	0.01633	0.02567	0.02366	0.04224	0.02037	0.03113
ST. DEV.	34.6141	48.4334	17.6482	25.8401	0.40829	0.46032	0.36516	0.40755	0.52894	0.64759	0.45542	0.51337
SKEWNESS	-0.0260	-0.2050	0.2370	0.0945	2.27934	1.60777	2.11334	1.39835	3.09001	2.15518	2.87300	2.38006
KURTOS.	0.1783	0.1020	0.0275	0.0293	5.69149	2.64647	4.49550	1.63665	12.53415	6.30046	13.30202	10.00915
SHEL SIZE	500.00000	500.00000	500.00000	500.00000	268.00000	500.00000	252.00000	500.00000	235.00000	500.00000	272.00000	500.00000

D 2 D 4

PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.2484031	0.2486218	0.2489119	0.2480778	-0.0001100	0.0002000
S (MEAN)	0.0011608	0.0008263	0.0007043	0.0003207	0.0003462	0.0002467
ST. DEV.	0.0259565	0.0184774	0.0157478	0.0138790	0.0077402	0.0055169
SKEWNESS	0.4764770	0.3973002	0.3035622	0.3035622	0.1977163	0.2697002
KURTOS.	0.0193195	0.0144081	0.1103370	0.0134916	-0.0576601	-0.0075665
SHEL SIZE	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000	500.0000000

Figure 39d - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE DISTRIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZEROS (M 1 + S, M 1, M 2, M 3, M 4), CUMULATED WAITING TIMES (M 1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S2 AND X2 (D2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN SC#50858; EX=1.5; RS=6 .

3. To test the distribution of W and \bar{W} we also used the plotting subroutines and the values of their skewness and kurtosis parameters as we did with the queue with dependent service.

The results obtained from the analysis are as follows:

(i). Given that $W > 0$, we may say that W has an exponential form, since the plots under EXPLT Subroutine result in a straight line and the values of γ_1 and γ_2 are around their actual values 2 and 6 respectively. For example the SC#50#25 model (which is represented by the figures 35a-35d), where the steady state has been reached, gave us a straight line and the values 1.8 and 4.7 for γ_1 and γ_2 respectively. We have to state here that the exponential distribution comes only if W has reached the steady state, and we cannot have the same result if the W is still in a transient state.

(ii) For the distribution of \bar{W} we can say that the normality assumption is an appropriate one, since the graphs obtained under NORMFL result in a straight line and the value of skewness and kurtosis is around '0'. The results, for example, obtained from the SC#25#25 model gave us a straight line under NORMFL and the values .17 and -.05 for the skewness and kurtosis respectively (see figures 40a-40c). The normality assumption holds if the \bar{W} has reached the steady state. At the beginning of the transient state it is not possible to characterize its distribution, while when \bar{W} is in a transient state we can see that its distribution takes an exponential or a X^2 form. This is left little by little as \bar{W} comes close to the steady state

and then eventually it takes the normal form. The model SC#25#95 is a representative example of that result. In that model, although \bar{W} has not reached the steady state yet ($N=10000$) we can observe the values of skewness having as 3.5, 1.8, 1.6, 1.5 and the values of kurtosis as 24.7, 7.2, 5.7, 4.3 for $N=2500, 5000, 7500, 10000$ respectively. That is the values start from a high level, then come down, they pass from the actual values of γ_1, γ_2 for an exponential model and then continue decreasing and hopefully in the steady state they should reach a value around '0' where the value of skewness and kurtosis for the normal distribution. The same assumptions are obtained analyzing the plots of that model also. That is for $N=2500$ the plots are not linear under both EXPLT and NORMPL, for $N=5000$ a straight line appears under EXPLT, for $N=7500$ and 10000 the plot leaves gradually the straight line under EXPLT and begins to give us a straight line under NORMPL.

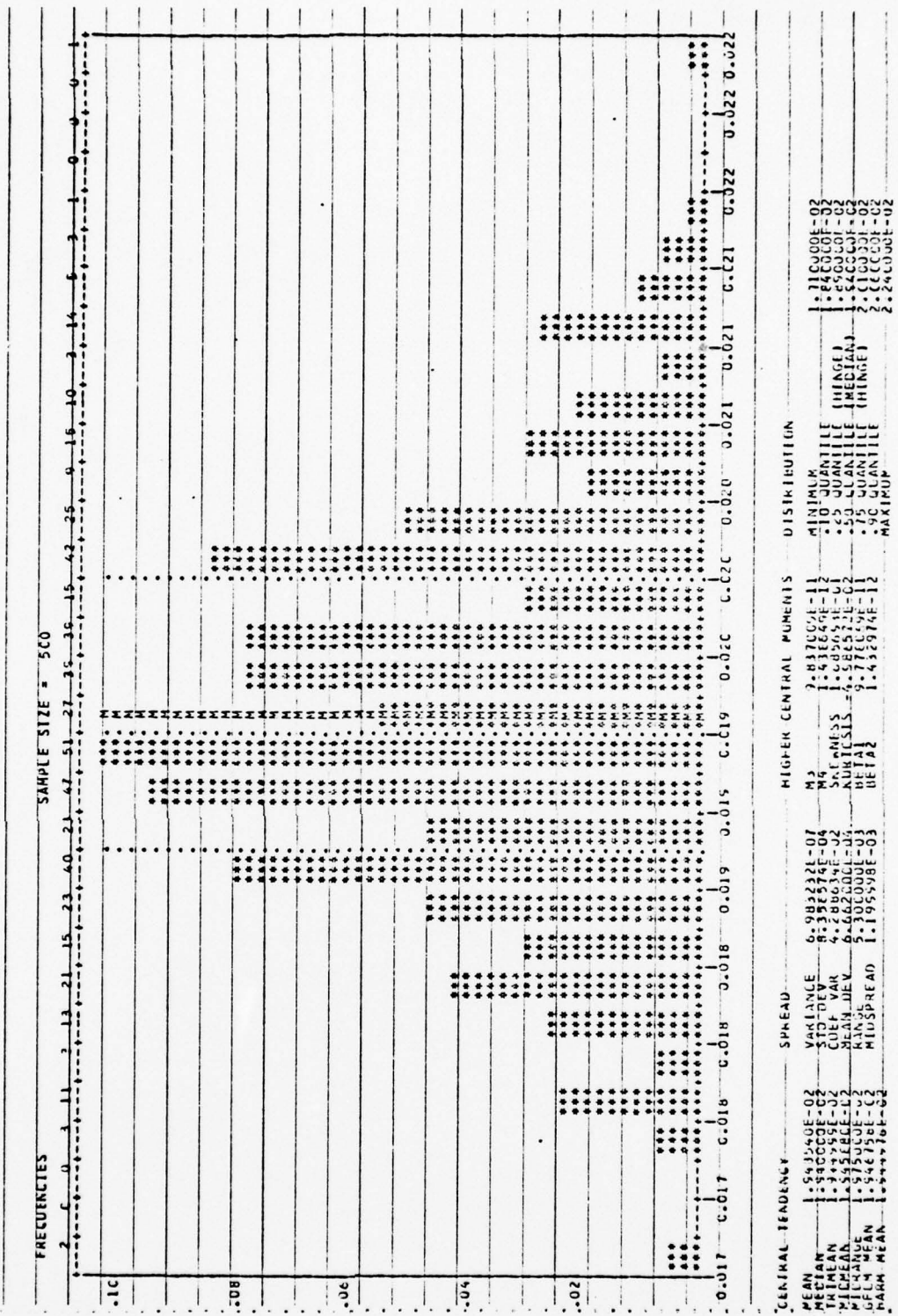


Figure 40a - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. HISTOGRAM OF THE CUMULATED AND AVERAGED WAITING TIMES \bar{W}_{10000} FROM THE SC#25#25; $n=500$ REPLICATIONS, $R_X=1.5$; $R_S=6$.

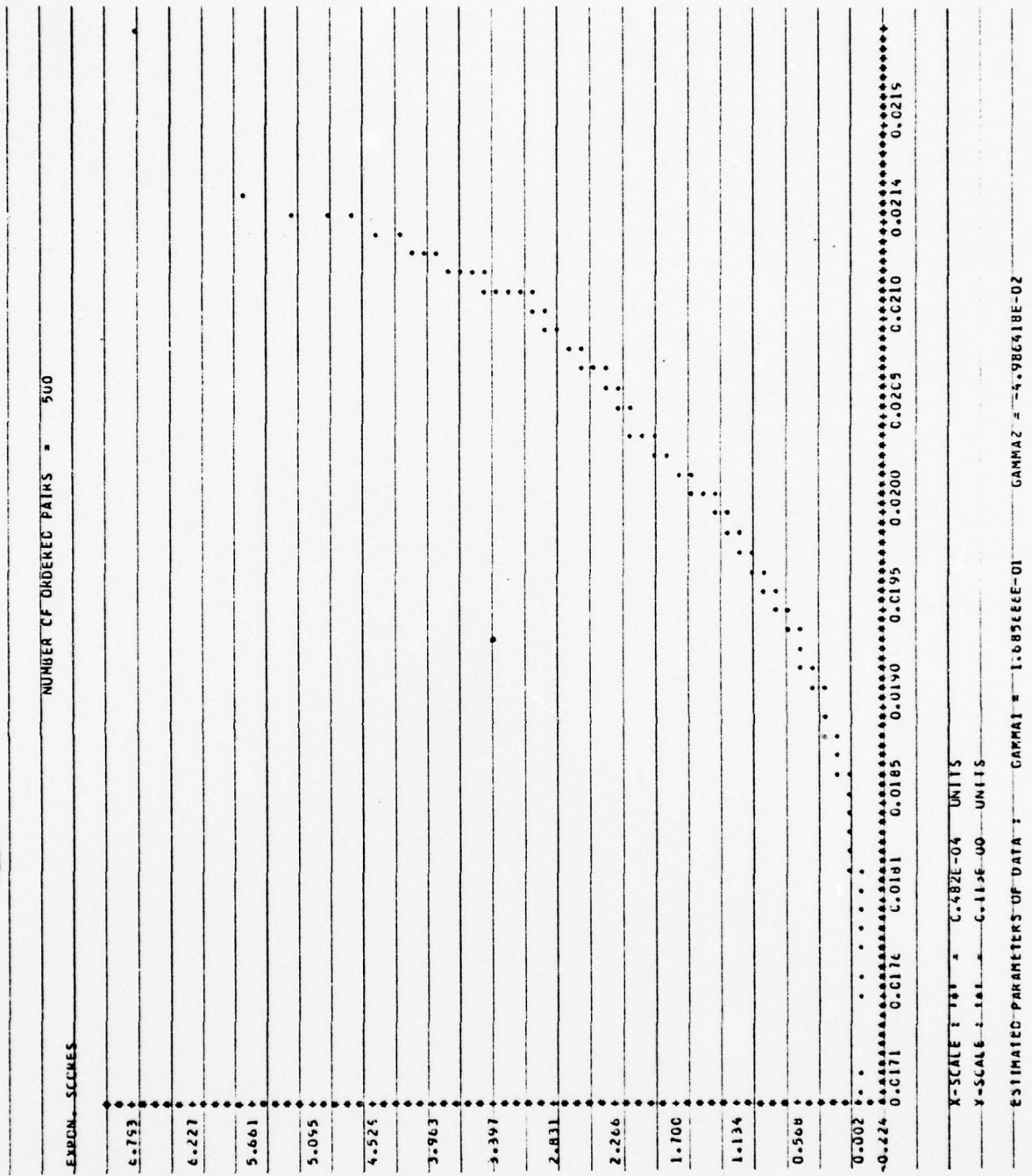


Figure 40b - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. EXPONENTIAL PLOT (EXPIT) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE SC#25#25; $m=500$ REPLICATIONS, $RX=1.5$; $RS=6$.

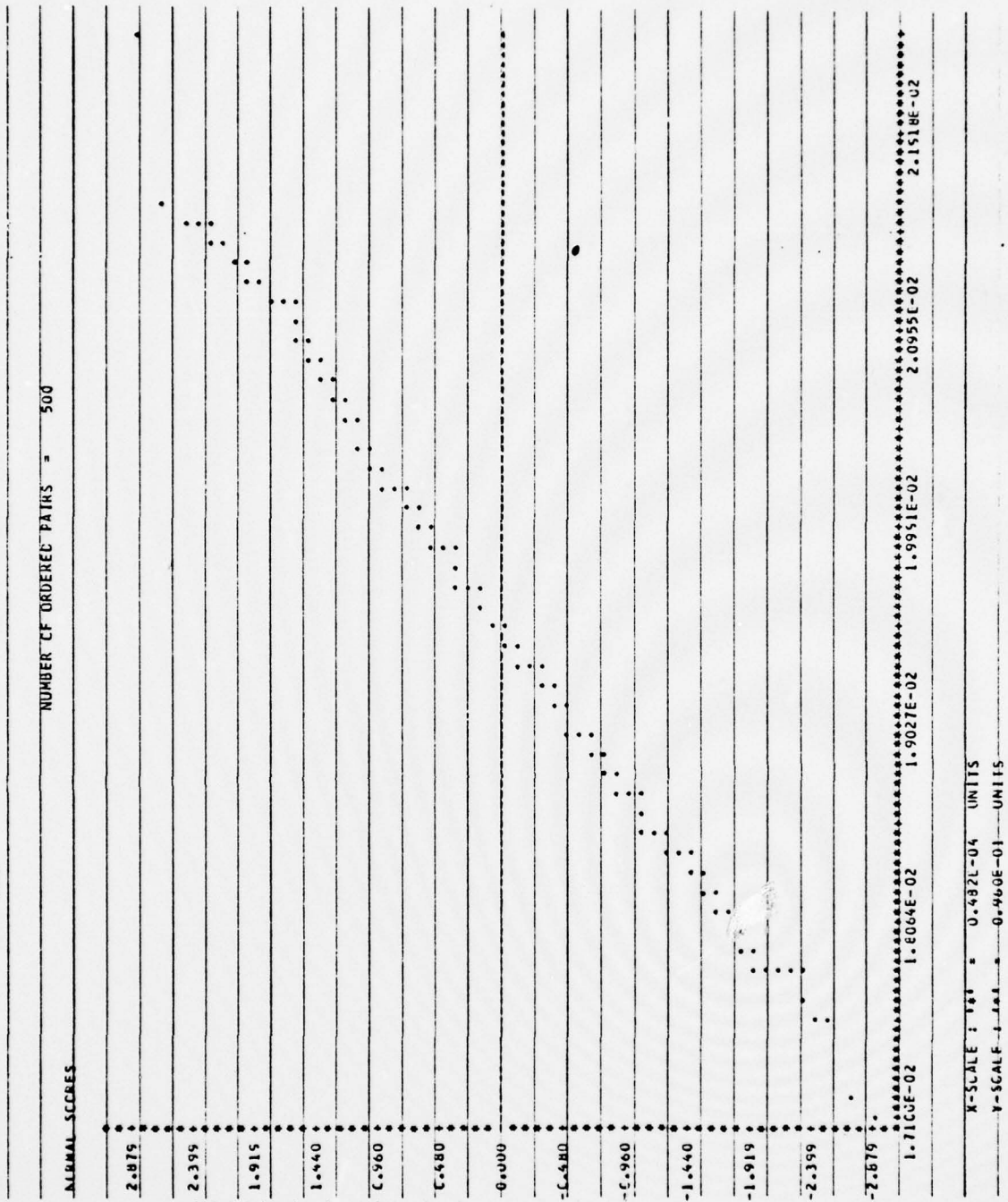


Figure 40c - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. NORMAL PLOT (NORMPL) OF THE CUMULATED AND AVERAGED WAITING TIMES W_{10000} FROM THE SC#25#25; $m=500$ REPLICATIONS, $RX=1.5$; $RS=6$.

UNCORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	M 1 + G 5	M 1	M 2 + G 5	M 2	M 3 + G 5	M 3	M 4 + G 5	M 4
MEAN	3332.823	6659.674	832.243	1665.285	0.01722	0.10012	0.01023	0.05329	0.2002	0.12210	0.02410	0.11930
STDEV	2.6640	2.8722	0.5507	0.7760	0.00225	0.00865	0.00230	0.00567	0.00309	0.01435	0.00324	0.01209
SKWENESS	0.0260	-0.2024	0.0729	-0.1921	0.00035	0.00054	0.00153	0.00019	0.00020	0.00054	0.00020	0.00020
KURTOSIS	0.1783	0.1020	0.1244	-0.1511	0.00055	0.00015	0.00015	0.00027	0.00088	0.00054	0.00047	0.00054
ZEROS #	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

PARAMETER	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	M 4 BAR	U 2	U 4
MEAN	0.0194382	0.0194240	0.0194238	0.0194238	0.0194238	-0.0001157	0.0006211
STDEV	0.0000745	0.0000511	0.0000225	0.0000374	0.0000374	0.0000374	0.0002615
SKWENESS	0.0016660	0.0011417	0.0009492	0.0008357	0.0008357	0.0008357	0.0008566
KURTOSIS	0.000052	0.0000527	0.0000527	0.0000527	0.0000527	0.0000527	0.0000527
ZEROS #	0.0	0.0	0.0	0.0	0.0	0.0	0.0

UNCORRELATED QUEUE

PARAMETER	X 2	X 4	S 2	S 4	M 1 + G 5	M 1	M 2 + G 5	M 2	M 3 + G 5	M 3	M 4 + G 5	M 4
MEAN	3332.823	6659.674	832.243	1665.033	0.04612	0.18748	0.04395	0.15576	0.06777	0.27550	0.06886	0.24408
STDEV	2.8650	2.8522	0.5461	0.7164	0.00508	0.01455	0.00450	0.01454	0.00888	0.02904	0.00720	0.01941
SKWENESS	0.1521	0.0718	0.0944	0.0944	0.11355	0.16141	0.12295	0.15440	0.19866	0.32205	0.15106	0.22719
KURTOSIS	0.1783	0.1020	0.0274	0.0292	0.07894	0.14154	0.08183	0.13216	0.25888	0.72531	0.17819	0.24408
ZEROS #	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

PARAMETER	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	M 4 BAR	D 2	D 4
MEAN	0.0257256	0.0257560	0.0257562	0.0257256	0.0257256	-0.0030607	0.0005961
STDEV	0.0002257	0.0001594	0.0001590	0.0001222	0.0001222	0.0004250	0.0003002
SKWENESS	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357
KURTOSIS	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357	0.0001357
ZEROS #	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Figure 4cd - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. TABULATION OF QUEUE STATISTICS FOR THE LISTING OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (M 1 + G 5, M 1, M 2, M 3, M 4, M 4 BAR, M 4 BAR), CUMULATED WAITING TIMES (M 1 BAR ETC.) AND THE AVERAGED DIFFERENCES BETWEEN S 2 AND X 2 (U 2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN SC#25825; BX=1.5; BS=6.

4. The following values for \bar{W} have been obtained from the runs with various values of t and r and for $N=10000$

N	t	r	E[\bar{W}]	E[\bar{W}](M/M/1)
10000	.25	.25	.1169	.3332
10000	.25	.50	.1724	.3332
10000	.25	.90	.3538	.3332
10000	.25	.95	.4075	.3332
10000	.25	.98	.4679	.3332
10000	.50	.25	.3831	.9985
10000	.50	.50	.5648	.9985
10000	.50	.90	1.4589	.9985
10000	.50	.95	2.1109	.9985
10000	.50	.98	3.7196*	.9985
10000	.95	.25	10.4672	18.3528
10000	.95	.50	13.4784*	18.3528
10000	.95	.90	29.1312*	18.3528
10000	.95	.95	75.1180*	18.3528
10000	.95	.98	145.3484*	18.3528
10000	.99	.25	35.5986*	53.3371
10000	.99	.50	43.1616*	53.3371
10000	.99	.90	98.9556*	53.3371
10000	.99	.95	139.0167*	53.3371
10000	.99	.98	219.7920*	53.3371

Analyzing the above results we may state conclusions as follows:

(i). For a given traffic intensity t the value of \bar{W} is not constant, but depends on the value of correlation r , as happens in the queue with dependent service. We observe also that \bar{W} increases as r increases, but the rate of increment is larger as r goes to 1, and very small for low

values of r ; therefore \bar{W} is not a linear function of r . Figure 41 gives the plots of \bar{W} versus r for $t = .25, .50, .95, .98$ and also for the M/M/1 queue ($r=0$). Furthermore we can observe that the value of \bar{W} for the correlated queue, comparing it with the result for the M/M/1 queue, is not always greater as in the case of dependent service, but depends on the value of r . It is less for $r \leq .50$ and greater for $r \geq .90$.

(ii). For a given correlation the value of \bar{W} increases as t increases. Furthermore high values of t cause high rate of increment, which becomes larger as the correlation increases. Figure 42 gives the plots of \bar{W} versus t for $r = 0, .25, .50, .90, .95, .98$. Observing that figure we can see also that for a given t the order of increment of \bar{W} is for $r = .25, .50, 0, .90, .95, .98$ respectively. Thus we can conclude that the waiting time for the cross-correlated queue is generally greater than the waiting time for M/M/1 queue when the correlation is high.

5. The last result obtained from the simulation of that model was the convergence of W and \bar{W} . (These statistics also converge in value and in distribution slower than the same statistics in M/M/1 queue). The model SC#25#98 reveals that \bar{W} converges faster than W (see figure 43). But the model SC#99#50 gave us the conclusion that W converges faster than \bar{W} (see figure 44). Thus we can state here that for low values of t \bar{W} converges faster than W and for high values of t the opposite is true.

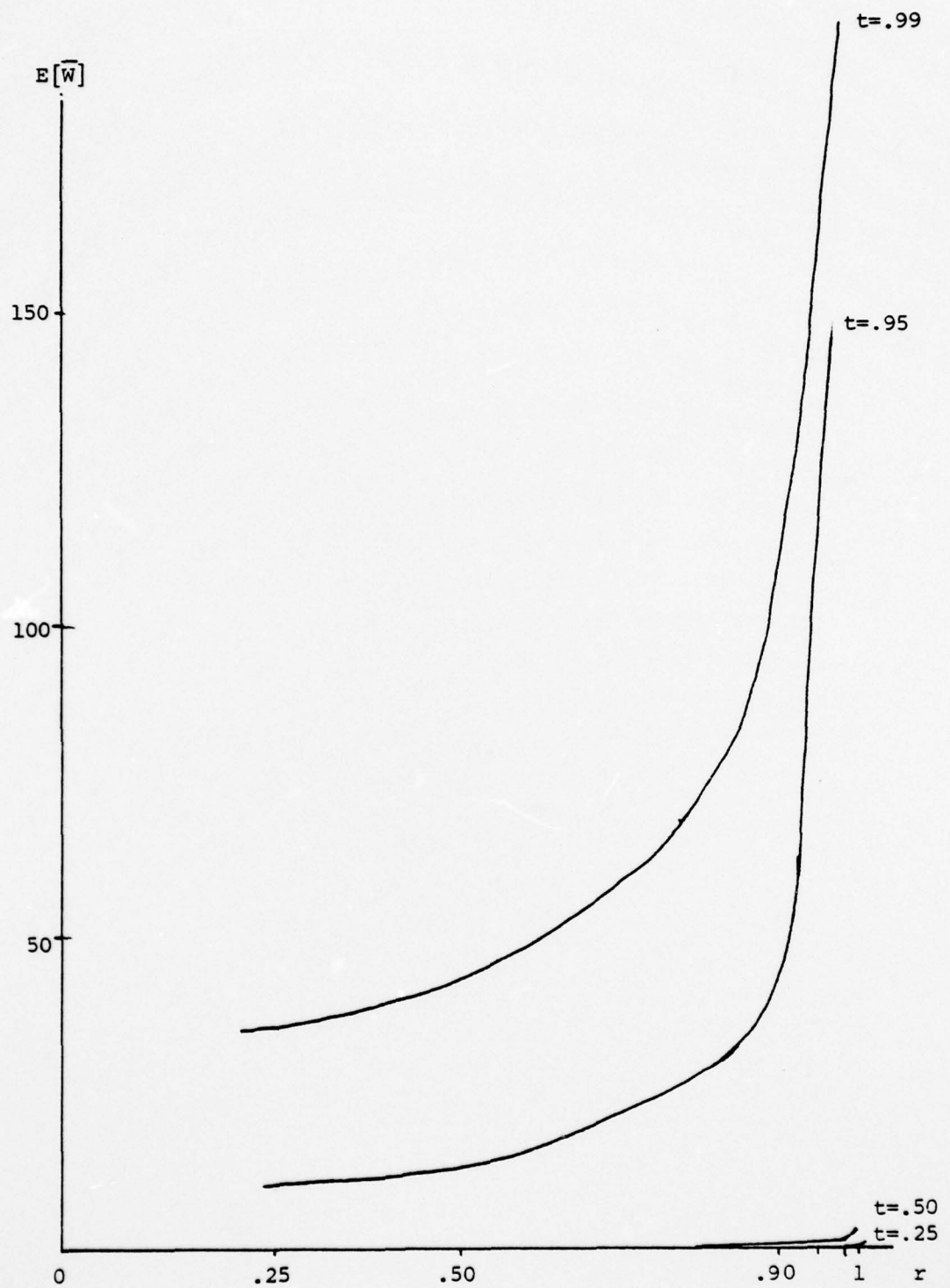


Figure 41 - QUEUE WITH CROSS-CORRELATED SERVICE TIMES AND FCISSON INPUT. PLOT OF $E[W]$ VERSUS r .

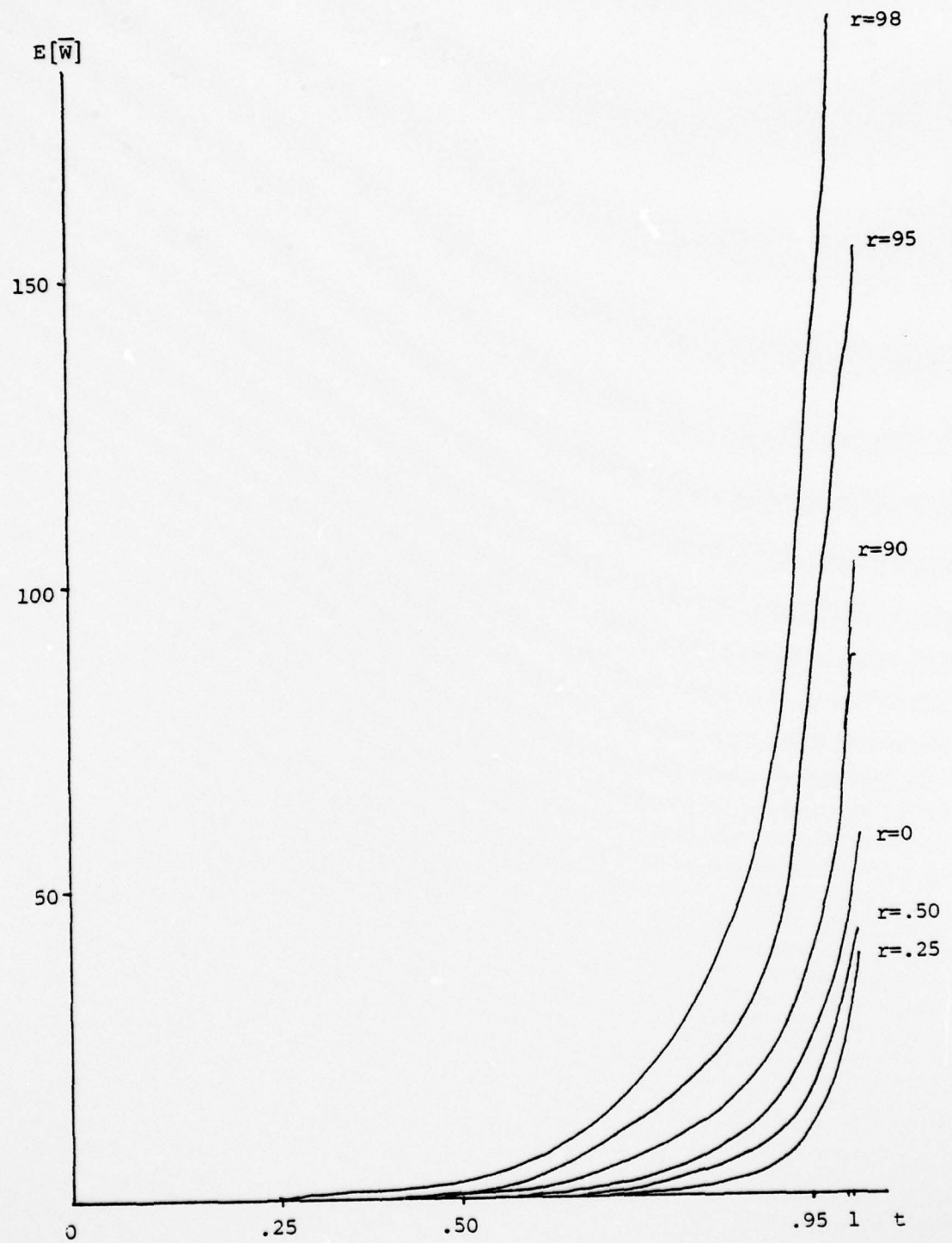


Figure 42. - QUEUE WITH CROSS-CORRELATED SERVICE TIMES AND POISSON INPUT. PLOT OF $E[\bar{W}]$ VERSUS t .

CORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	332.823	659.074	835.486	1667.608	0.07436	0.29745	0.06703	0.27473	0.05545	0.25671	0.07050	0.31422
STDEV	2.8650	2.8522	2.5856	3.8127	0.01383	0.05042	0.01071	0.03831	0.00833	0.03164	0.01259	0.03742
SKWNESS	46.1521	64.6718	57.8163	65.2540	0.30920	0.56374	0.23952	0.42316	0.16617	0.33068	0.28163	0.45364
KURTOS	-0.0260	-0.2054	0.2524	0.2197	9.71833	5.26761	9.66360	5.25638	5.2440	2.58360	6.16225	3.00450
ZEROS	0.1783	0.1020	0.1694	0.0155	118.93422	32.78246	118.62845	37.47824	39.10585	7.81845	51.61862	16.94318
	0.0	0.0	0.0	0.0	374.00000	0.0	374.00000	0.0	352.00000	0.0	356.00000	0.0

PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.0812984	0.0783889	0.0789859	0.0779790	0.0005325	0.0008533
STDEV	0.0042510	0.0022484	0.0016570	0.0013266	0.0006404	0.0004425
SKWNESS	0.950555	0.0502757	0.0303524	0.0296675	0.0145202	0.0098544
KURTOS	14.1116304	11.5986555	8.8927116	7.3945675	0.0089558	0.0501304
ZEROS	245.4133148	187.1348114	129.2479706	96.4026337	-0.1295948	-0.1657343
	0.0	0.0	0.0	0.0	0.0	0.0

UNCORRELATED QUEUE

PARAM	X 2	X 4	S 2	S 4	M 1 + OS	M 1	M 2 + OS	M 2	M 3 + OS	M 3	M 4 + OS	M 4
MEAN	332.823	659.074	835.486	1667.608	0.04612	0.18748	0.04595	0.19576	0.06777	0.27550	0.06688	0.24408
STDEV	2.8650	2.8522	2.5856	3.8127	0.00508	0.01455	0.00550	0.01854	0.00848	0.02904	0.00720	0.01941
SKWNESS	46.1521	64.6718	57.8163	65.2540	0.16141	0.12259	0.19440	0.19866	0.32205	0.13106	0.22719	0.22719
KURTOS	-0.1783	0.1020	0.0274	0.0292	10.78564	1.41545	19.01831	3.32118	29.88885	7.72531	17.92196	6.24490
ZEROS	0.0	0.0	0.0	0.0	377.00000	0.0	377.00000	0.0	377.00000	0.0	363.00000	0.0

PARAM	M 1 BAR	M 2 BAR	M 3 BAR	M 4 BAR	D 2	D 4
MEAN	0.0594256	0.0594960	0.0594962	0.0594928	-0.0000607	0.0005961
STDEV	0.0002227	0.0001594	0.0001394	0.0001222	0.0004250	0.0003002
SKWNESS	0.0051357	0.0035648	0.0031081	0.0027328	0.0095036	0.0067129
KURTOS	0.4125256	0.3556815	0.2726265	0.3175142	0.1141228	0.2426832
ZEROS	0.4375257	0.0793972	-0.0206966	-0.0520191	0.0510197	-0.0220528
	0.0	0.0	0.0	0.0	0.0	0.0

Figure 43 - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND ECISSEN INPUT. TABULATION OF QUEUE STATISTICS FOR THE DISTRIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH OR WITHOUT ZEROS (M 1 + OS, M 1, M 2 + OS, M 2, M 3 + OS, M 3, M 4 + OS, M 4), CUMULATED WAITING TIMES (M 1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S 2 AND X 2 (D 2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN SC425458; BX=1.5; RS=6.

CORRELATED CURVE

PARAMTR M 2 M 4 S 2 S 4 M 1 + 05 M 1 M 2 + 05 M 3 M 4 + 05 M 4
 MEAN 1683.007 3366.192 1684.470 3330.441 11.92533 12.14755 15.76974 16.02815 18.10533 18.31572 18.65758 18.82250
 S (MEAN) 1.6424 1.6431 1.2327 1.7161 0.46255 0.46564 0.56151 0.55415 0.72168 0.72464 0.72730 0.74070
 ST. DEV. 22.2082 32.7162 27.5631 38.3276 10.35185 10.31835 13.22781 13.17584 16.11274 16.11026 16.26302 16.22423
 SKEWNESS -0.0261 -0.2053 0.0879 -0.0557 1.17517 1.17321 1.25462 1.25536 1.20156 1.19602 1.26234 1.26211
 KURTOSIS 0.1783 0.1018 0.1017 0.0342 1.32534 1.33176 1.84084 1.85598 1.10207 1.05401 1.43117 1.43475
 SMPLE SIZE 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000

PARAMTR W 1 BAR W 2 BAR W 3 BAR W 4 BAR W 2 0 2 D 4
 MEAN 6.6141525 11.2737417 13.2417622 14.3086736 -0.0003402 -0.0002082
 S (MEAN) 0.2322564 0.2975569 0.3347952 0.3670427 0.0002464 0.0001764
 ST. DEV. 4.2837544 6.6535778 7.4863409 8.2073265 0.0055166 0.0039436
 SKEWNESS 1.6180201 1.6068602 1.6253405 1.5430333 0.0038786 -0.1288727
 KURTOSIS 3.3314552 3.0002041 3.1190050 2.6351555 0.5324717 -0.0878372
 SMPLE SIZE 500.000000 500.000000 500.000000 500.000000 500.000000 500.000000

UNCORRELATED CURVE

PARAMTR X 2 X 4 S 2 S 4 M 1 + 05 M 1 M 2 + 05 M 3 M 4 + 05 M 4
 MEAN 1683.007 3366.192 1665.284 3351.023 14.82400 15.03446 18.82291 19.12597 21.96606 22.27155 23.66727 23.90634
 S (MEAN) 1.0424 1.4631 1.0524 1.5408 0.56661 0.56525 0.75766 0.76221 0.91463 0.91537 1.02615 1.03093
 ST. DEV. 23.3082 32.7162 23.5316 34.4538 12.67428 12.63530 16.54151 16.43829 20.41335 20.44553 22.93604
 SKEWNESS -0.0261 -0.2053 0.0879 0.0342 1.17517 1.17321 1.25462 1.25536 1.20156 1.19602 1.26234 1.26211
 KURTOSIS 0.1783 0.1018 0.1017 0.0342 1.32534 1.33176 1.84084 1.85598 1.10207 1.05401 1.43117 1.43475
 SMPLE SIZE 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000 500.0000

PARAMTR W 1 BAR W 2 BAR W 3 BAR W 4 BAR W 2 0 2 D 4
 MEAN 10.591626 13.8082275 16.1598462 17.7791595 -0.0001499 -0.0001499
 S (MEAN) 0.2151716 0.2833726 0.3597176 0.5010982 0.0002548 0.0002138
 ST. DEV. 6.2559255 8.5658340 10.0416851 11.2049074 0.0065521 0.0047798
 SKEWNESS 1.4325465 1.4735689 1.626328 1.7394255 0.2092550 0.2765061
 KURTOSIS 1.9141477 2.0172393 2.9112906 3.5989555 -0.1199466 0.1787948
 SMPLE SIZE 500.000000 500.000000 500.000000 500.000000 500.000000 500.000000

Figure 44 - QUEUE WITH CROSS-CORRELATED SERVICE TIME SEQUENCE AND POISSON INPUT. TABULATION OF SAMPLE STATISTICS FOR THE DISTRIBUTIONS OF CUMULATED INTERARRIVAL TIMES AT N=5000 AND N=10000 (X 2 AND X 4), CUMULATED SERVICE TIMES (S 2 AND S 4), WAITING TIMES, WITH CB WITHOUT ZEROS (W1 + 0, W1, I FOR CASE N=2500, 5000, 7500, 10000), CUMULATED WAITING TIMES (W1 BAR etc.) AND THE AVERAGE DIFFERENCES BETWEEN S2 AND X2 (D2); OBTAINED FROM N=500 REPLICATIONS OF THE RUN SC199150; EX=2.97; ES=3.

AD-A050 213

NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF
CMS HISTOGRAM, DENSITY ESTIMATION AND PROBABILITY PLOTTING ROUT--ETC(U)
DEC 77. 6 I DANIKAS

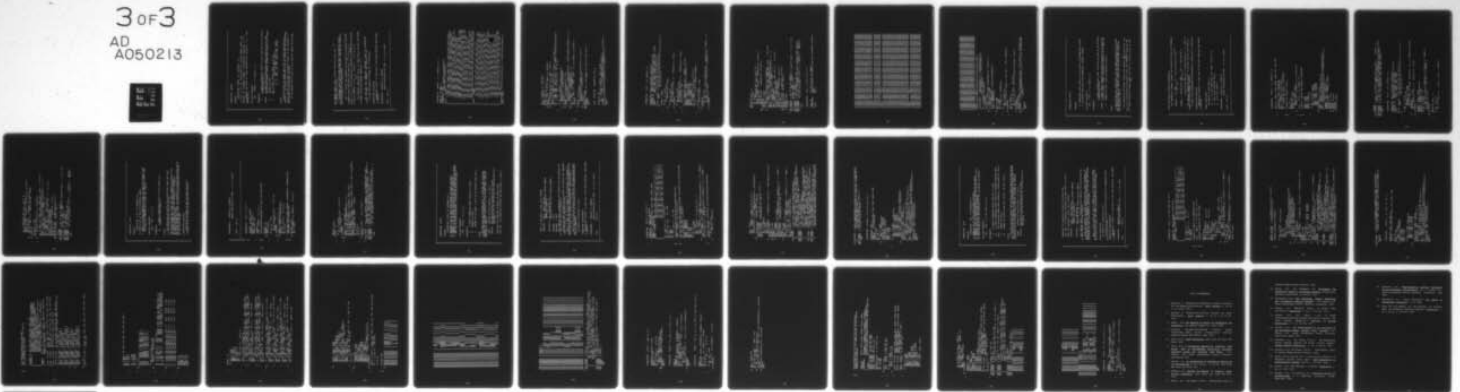
F/G 9/2

UNCLASSIFIED

NL

3 of 3

AD
A050213



END

DATE

FILMED

4 - 78

DDC

```

-----
SUBROUTINE NORMPL
PURPOSE
    SUBROUTINE NCRMPL IS INTENDED TO :
1. PLOT A SET OF DATA-POINTS VS EITHER NORMAL SCORES OR
   INVERSE OF STANDARD NORMAL DISTRIBUTION FUNCTION ( F-1 ).
2. COMPUTE THE VALUE OF WILK-SHAPIRO TEST STATISTIC ( W ) FOR
   NORMALITY.
CALLING SEQUENCES
    CALL NORMPL ( X, SCORES, N ).
ARGUMENTS
    X      A SINGLE DIMENSIONAL ARRAY OF DATA (REAL*4) TO BE PLOTTED
           DIMENSIONED BY N .
    SCORES A CUMMY SINGLE DIMENSIONAL ARRAY (REAL*4) TO BE USED
           FOR STORING NORMAL SCORES. DIMENSIONED EXACTLY AS X ARRAY
    N      NUMBER OF DATA
    K      AN INTEGER VALUE FROM 1 TO 3
           FOR :  K = 1  A PLOT IS ONLY GIVEN
                  K = 2  THE 'W'-VALUE IS ONLY GIVEN
                  K = 3  BOTH OF THE ABOVE ARE GIVEN.
USAGE
    THE DATA WILL BE SORTED AND A SET OF ORDER STATISTICS IS
    PRODUCED. THE ORDER STATISTICS THEN ARE SCALED ON THE HORIZONTAL
    X-AXIS. ON THE OTHER HAND THE NORMAL SCORES OR THE COMPUTED F-1
    ARE SCALED ON THE VERTICAL Y-AXIS.
    IF THE NUMBER (N) OF DATA IS LESS THAN CR EQUALS TO 50 THEN

```

CC

THE NORMAL SCORES (FOR Y-AXIS) ARE USED, O.W. THE F^{-1} IS COMPUTED AND IS USED.

THE NORMAL SCORES HAVE BEEN TAKEN FROM 'BICMETRIKA TABLES FOR STATISTICIANS' VOL. I, THIRD EDITION P.190 AND ARE GIVEN BY DATA STATEMENT IN THE SUBROUTINE.

THE F^{-1} VALUES ARE COMPUTED BY CALLING THE PROGRAM FUNCTION 'INVNRM' WHICH EVALUATES THE VALUE OF $F^{-1}(F(X_{(I)}))$ WHERE $F(X_{(I)})$ IS THE EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTION OF DATA GIVEN BY: $F(X_{(I)}) = I/(N+1)$, $I = 1, 2, \dots, N$

THE PROGRAM SUBROUTINE 'PLOT' IS THEN CALLED TO PLOT THE PAIRS OF ORDER STATISTICS-NORMAL SCORES/ F^{-1} .

IF $K=2$ OR $K=3$ THEN THE PROGRAM CALLS ITS SUBROUTINE 'WILKH' IN ORDER TO COMPUTE THE VALUE OF 'W'.

IF DATA ARE NORMAL A LINEAR FIT IS EXPECTED. SUBROUTINES REQUIRED.

SUBROUTINE PXSORT IS USED AND IT HAS BEEN COMPILED AND ADDED TO MP SLIB.

PROGRAMMER: GEORGIOS J. DANIKAS
UNDER DIRECTIONS OF PROF. P. A. W. LEWIS

DATE: APR 77

SLBROUTINE NORMPL (X, SCORES, N, K)

REAL *8 INVNRM, W

LOGICAL *1 ODD

DIMENSION X(N), SCORES(N), BIOMET(403),

* DATA SCORE1(200), SCORE2(203)

1.564, .846, 1.029, .297, 1.165, .495, 1.267, .642, .202, .572,
1.352, .757, 1.424, .852, .473, .153, .485, .932, .729,
.275, 1.253, 1.656, .376, .537, .312, .586, 1.062, .668,
.462, .225, 1.001, 1.116, .793, .537, 1.208, .501, .662, .456,
1.164, .850, 1.603, .388, .19, 1.703, .715, .335, .165,
1.766, 1.088, 1.736, 1.248, .574, .396, .446, .077, .82, .794,
1.319, 1.029, .807, .763, .451, .295, .146, 1.335, 1.099,
1.066, .848, .665, .502, .351, .208, .065, 1.844, 1.338,
.745, .707, .548, .402, .187, .062, .131, 1.867, 1.6, .781,
.63, .59, .448, .315, .12, 1.01, .585, .676, .75, .67,
.41, .29, .17, .06, 1.193, 1.24, 1.21, .88, .73, .91, .93, .98,
.33, .22, .11, 1.05, 1.15, 1.42, 1.07, 1.09, 1.14, .62, 1.36,
.37, .26, .16, 1.97, 1.52, 1.54, 1.29, 1.33, .04, 1.62, 1.65,
.64, .52, .41, .3, 2.14, .05, 2.01, 1.58, .04, 2.04, 2.07, 1.65,
.55, .44, .34, .24, .14, .05, 2.01, 1.3, .21, .12, .04, 2.07, 1.65,
.85, .73, .61, .51, .41, .32, .22, .13, .29, .29, .21, .12, .04, 2.07, 1.65,
1.18, 1.03, .89, .78, .67, .57, .47, .38, .29, .21, .12, .04, 2.07, 1.65,
SCORE2

DATA

1.4, 1.22, 1.07, .94, .82, .72, .62, .53, .44, .34, .25, .11, .98,
36, .28, .12, .04, .2, .09, 1.68, 1.43, 1.18, .84, .54, .21, .98,
.87, .76, .67, .58, .41, .24, .81, .26, 1.18, .63, .54, .47, .47,
1.7, 1.46, 1.24, 1.14, .91, .73, .71, .49, 1.32, 1.17, .17,
1.05, .94, .85, .75, .67, .52, .51, .44, .37, .37, .23, .16, .179,
.71, .63, .55, .49, .42, .35, 1.22, 1.08, .98, .88, .83, .67, .6,
2.18, 1.78, 1.54, 1.37, 1.21, 1.11, .91, .83, .75, .67, .57,
1.53, 1.46, 1.4, 1.3, 1.21, 1.09, .93, .83, .75, .67, .57,
1.44, 1.38, 1.14, .95, .86, .78, .71, .64, .55, .55, .54,
1.42, 1.28, 1.32, 1.17, .98, .89, .81, .74, .67, .54, .54,
1.48, 1.42, .36, 1.31, 1.25, .98, .84, .77, .71, .64, .64,
1.61, 1.44, 1.19, 1.19, .95, 1.24, .84, .77, .71, .64, .64,
1.58, 1.52, .46, 1.34, 1.25, 1.24, .13, .95, .87, .8, .74,
1.85, 1.63, 1.46, 1.33, 1.22, 1.12, .28, .23, .18, .13,
.07, .03, .03, .03, .03, .03, .03, .03, .03, .03, .03, .03,
BIOMET(1), SCORE1(1), (BIOMET(201), SCORE2(1))
EQUIVALENCÉ (BIOMET(1), SCORE1(1), (BIOMET(201), SCORE2(1))
ODD = .TRUE

```

IF ( N.LE.1 ) GO TO 35
CALL PXSORT ( X, 1, N )
NHALF = N / 2
GC TO ( 15, 25, 15), K
25 CALL WILK ( X, N, NHALF, W, SCCRES )
RETURN
15 IF ( MOD(N,2) .EQ. 0 ) ODD = .FALSE.
IF ( N.GT.50 ) GO TO 40
INDEX = ( (N-1)**2 + 3 ) / 4 - 1
IF ( ODD .AND. N.LE.25 ) INDEX = (N-1)**2 / 4
IF ( .NOT. ODD .AND. N.GE. 26 ) INDEX = (N-26)*(N+24)/8+156
IF ( ODD .AND. N.GE. 26 ) GC TO 20

C
DC 11 I = 1, NHALF
SCORES(N-I+1) = BICMET(INDEX+I)
SCORES(I) = -BIOMET(INDEX+I)
11 GC TO 30
20 INDEX = (N-27)*(N+23)/8 + 156
DC 22 I = 1, NHALF
SCORES(N-I+1) = ( BIOMET(INDEX+I) + BIOMET(INDEX+I+NHALF) ) / 2
22 SCORES(I) = -SCORES(N-I+1)
30 IF ( ODD ) SCORES(NHALF+1) = 0.
CALL PLOT ( X, SCORES, N )
IF ( K.EQ.3 ) CALL WILK ( X, N, NHALF, W, SCCRES )
RETURN

C
40 NHALF = N / 2
DC 44 I = 1, NHALF
SCORES(N-I+1) = INVNRM ( DFLOAT(N-I+1) / (N+1) )
44 SCORES(I) = -SCORES(N-I+1)
GC TO 30
35 WRITE (6, 200)
200 FORMAT ( '1', 5X, '*** NUMBER OF DATA TO SMALL ***' )
RETURN
END

REAL FUNCTION INVNRM *8 ( PR )
REAL *8 PR
IF ( PR.LT. 0.500 ) PR = 1.-PR
PR = DSQRT ( DLOG(1.000/(1.000-PR)**2) )
INVNRM = PR - ( (.010328*PR + .802852)*PR + 2.515517 ) /
* ( (.001308*PR+.189269)*PR + 1.432788)*PR + 1. )
RETURN
END

```

```

SUBROUTINE PLOT ( X, SCORES, N )

DIMENSION X(N), SCORES(N), XAXIS(12)
LOGICAL #1 GRAPH(65,112), FMTF(16), DIGITS(7)
LOGICAL #1 VERT, BAR, STAR, BLANK, CROSS, DOT, SAMEX
DATA VERT, BAR, STAR, BLANK, CROSS, DOT, SAMEX
* DATA DIGITS / 1,2,3,4,5,6,7 /
* FMTF / 1,2,3,4,5,6,7 /
* SAMEX = .TRUE.

C DC 11 I = 1,65
DO 11 J = 1,112
  GRAPH(I,J) = BLANK
C 11

C DG 22 I = 1,65
  GRAPH(I,1) = STAR
  IF ( MOD(I,5) .EQ. 3 ) GRAPH(I,1) = CROSS
C 22 CONTINUE

C DC 33 J = 1,112
  GRAPH(65,J) = STAR
  IF ( MOD(J,10) .EQ. 2 ) GRAPH(65,J) = CROSS
C 33

C IF ( X(1)*X(N) .LT. 0 ) SAMEX = .FALSE.
  XMAX = AMAX1 ( ABS(X(1)), ABS(X(N)) )
  X RANGE = X(N) - X(1)
  Y RANGE = 2*SCORES(N)
  X DELTA = X RANGE / 110
  Y DELTA = Y RANGE / 60
  IF ( X RANGE .LE. 0 ) GO TO 45
  IF ( SAMEX ) GO TO 10
  JZERO = 2 + INT( .5-110*X(1)/X RANGE )

C DO 44 I = 1,65
  GRAPH(I, JZERO) = VERT
C 44

C 1C CONTINUE
DC 66 K = 1,N
  I = INT( .5 + 60 * (SCORES(N-K+1) - SCORES(1)) / Y RANGE ) + 3
  J = INT( .5 + 110 * ( X(K) - X(1) ) / X RANGE ) + 2
  GRAPH(I,J) = DOT
C 66 CONTINUE

```

```

11 = -3
WRITE (6,201) N
DC 77 I = 1,65
11 = 11+1
IF (MOD(I,5) .NE. 3) GO TO 15
YAXIS = SCORES(N) - YDELTA*I
WRITE (6,200) YAXIS, ( GRAPH(I,J), J=1,112 )
GO TO 77
WRITE (6,400) ( GRAPH(I,J), J=1,112 )
15 CCNTINUE
77 XAXIS(1) = X(I)
XAXIS(J) = 2,12
DO 88 XAXIS(J) = XAXIS(J-1) + 10*XDELTA
IF ( XMAX .LT. .001 .OR. XMAX .GE. 1.E04 ) GO TO 25
NDIG = INT ( 2. + ABS (ALOG10(XMAX) ) )
NFRCT = INT ( 1. + ABS (ALOG10 ( X RANGE/11. ) ) )
IF ( X RANGE .GE. 11. ) NFRCT = 1
NDNF = NDIG+NFRCT 4 .OR. NDNF .GE. 7 ) GC TO 25
IF ( NFRCT .GT. NFRCT+3 )
FMTF(2) = DIGITS(NFRCT)
FMTF(11) = DIGITS(NFRCT)
WRITE (6,600) ( XAXIS(J), J=1,11 )
GO TO 40
25 WRITE (6,600) ( XAXIS(J), J=1,11,2 )
40 RETURN (6,604) XDELTA, YDELTA
45 RETURN (6,601)

C
200 FORMAT ( 1X, F6.3, 1X, 112A1 )
201 FORMAT ( 1, 50X, NUMBER OF ORDERED PAIRS = , I6 // 2X,
* NORMAL SCORES , // )
400 FCRMAT ( 8X, 112A1 )
600 FORMAT ( 4X, 6(1PE11.4, 9X ) )
601 FORMAT ( // 5X, *** GIVEN DATA HAVE CONSTANT VALUE *** )
604 FORMAT ( // //, 8X, X-SCALE : //, E10.3, ,
//, 8X, Y-SCALE : //, E10.3, ,
* UNITS, )
END

SUBROUTINE WILK ( X, N, K, W, A )
REAL *8 W, SUM, S2, B, C, INVNRM
DIMENSION A(N), WD(825), WD1(159), WD2(209), WD3(209), WD4(8),
* DATA WD1 /

```

* 7071	7071	6872	1677	6646	2413	6431	2806	0875	6233	3031
* 1401	6052	3169	1743	0561	4888	3244	1576	0947	5739	3291
* 1586	0922	0303	5359	3315	2260	2429	0699	0547	5325	3347
* 2460	1802	1240	0727	0240	5149	3306	1099	0539	5251	3318
* 2433	5056	3290	0221	1939	1447	3005	2455	1878	4968	3873
* 2540	1988	1524	1109	0725	1359	4886	0553	0196	2027	3273
* 1197	0837	0496	0163	4808	0232	1086	2059	0155	1711	1587
* 0612	4643	4734	3278	2565	2085	2561	1334	1641	1711	0932
* 0140	3185	3185	2578	2119	1336	1686	1052	0804	0530	0426
* 4590	3156	2571	2131	1764	1443	1350	0878	0618	0368	0122
* 4542	3126	2563	2139	1787	1480	1201	0941	0696	0459	0228
* 4493	3098	2554	2145	1807	1512	1245	0957	0764	0459	0321
* 0107										
* 4450	3069	2543	2148	1822	1539	1283	1046	0823	0610	0403
* 0200	4407	3043	2533	2151	1836	1563	1316	1089	0876	0672
* 0476	0284	0094	4366	3018	2522	2152	1848	1584	1346	1128
* 0923	0728	0540	0358	0178	4328	2992	2510	2151	1857	1601
* 1372	1162	0955	0778	0598	0424	0253	0084	4291	2568	2499
* DATA										
* 2150	1864	MD2	1395	1192	1002	0822	0650	0482	0320	0159
* 4254	2944	2487	2148	1870	1630	1475	1219	1036	0862	0697
* 0537	0381	0899	0739	0585	0921	0289	2144	1874	0642	0633
* 1243	1066	0899	1449	1265	1093	0931	0777	0629	2098	1463
* 2141	1878	1651	1456	2876	1095	2262	1880	1660	0485	0344
* 2206	0068	3894	0669	0530	0395	0268	0131	0127	1465	1284
* 1118	0961	0812	0675	1301	0340	0988	0134	0706	2854	2439
* 2134	1882	1662	4096	2834	1427	0987	0883	1673	1487	0441
* 1160	1013	0873	0610	0484	0361	2127	1119	0770	2813	1317
* 2121	1883	1678	1496	1331	1175	0239	0900	0688	2645	2415
* 0404	0287	0172	0057	4040	2794	1036	2116	0770	1683	0523
* 1344	1196	2391	0924	0798	0677	0559	2116	1883	0645	1505
* 4015	2774	2391	2110	1881	1686	1513	1356	1231	1075	0947
* 0824	0706	0592	0481	0372	0264	0158	0053	3985	2755	2380
* 0515	1800	1689	1420	1366	1225	1092	0968	0848	0622	0622
* 0515	0409	0305	0203	0101	0964	0737	2368	2058	1878	1691
* 1526	1376	1237	1108	0986	0870	0759	0651	0546	0444	0343
* 0244	0146	0049	3940	2719	2357	2091	1876	1693	1531	1384
* DATA										
* 1249	1123	MD3	0891	0782	0677	0575	0476	0379	0283	0188
* 0094	3917	2701	2345	2085	1874	1694	1535	1392	1259	1136
* 3894	0909	0804	0701	0602	0506	0411	0318	0229	0149	0045
* 0927	2684	2334	2078	1871	1695	1539	1398	1269	1087	1035
* 2667	0824	0722	0628	0534	0442	0352	0278	0175	0087	0872
* 0842	2323	2072	1868	1695	1542	1405	1278	1160	1049	0943
* 2651	0745	0651	0560	0471	0383	0296	0211	0126	0042	0850
* 2651	2313	2065	1865	1695	1545	1410	1286	1170	1062	0959

```

* 0860, .0765, .0673, .0584, .0497, .0412, .0328, .0245, .0163, .0081, .3830,
* 2635, .2302, .2058, .1862, .1695, .1548, .1415, .1293, .1180, .1073, .0972,
* 0876, .0783, .0694, .0607, .0522, .0439, .0357, .0277, .0197, .0118, .0039,
* 3808, .2620, .2291, .2052, .1859, .1695, .1550, .1420, .1300, .1189, .1085,
* 0986, .0892, .0801, .0713, .0628, .0546, .0465, .0385, .0307, .0229, .0153,
* 0076, .3789, .2604, .2281, .2045, .1855, .1693, .1551, .1423, .1306, .1197,
* 1095, .0998, .0906, .0817, .0731, .0648, .0568, .0489, .0411, .0335, .0259,
* 0185, .0111, .0037, .3770, .2589, .2271, .2038, .1851, .1692, .1553, .1427,
* 1312, .1205, .1105, .1010, .0919, .0832, .0748, .0667, .0588, .0511, .0436,
* 0361, .0288, .0215, .0143, .0071, .3751, .2574, .2260, .2032, .1847, .1691,
* 1554, .1430, .1317, .1212, .1113, .1020, .0932, .0846, .0764, .0685, .0608,
DATA
WD4 /
* 0532, .0459, .0386, .0314, .0244, .0174, .0104, .0035 /
C
EQUIVALENCE ( WD(1),WDI(1) ), ( WD(200),WD2(1) ),
( WD(409),WD3(1) ), ( WD(618),WD4(1) )
*
IF ( N .LE. 50 ) GO TO 15
C = -2.722D0 + N*.083D0
C = DSQRT(C)
DO 11 I = 2,K
A(I) = 2*INVRM (DFLOAT(N-I+1) / (N+1)) / C
11 B = (N+1.D0) / 2.D0
C = N/2.D0 + 1.D0
A(I) = DGAMMA(B) / ( SQRT(2.)*DGAMMA(C) )
A(1) = SQRT(A(1))
C
15 SUM = 0.D0
S2 = 0.D0
B = 0.D0
IA = K*(K-1) + K*MOD(N,2)
C
DC 22 I = 1,N
SUM = SUM + X(I)
IF ( I .LE. K .AND. N .LE. 50 ) A(I) = WD(IA+I)
22 CONTINUE
C
SUM = SUM/N
DO 33 I = 1,N
S2 = S2 + ( X(I) - SUM )**2
IF ( I .LE. K ) B = B + A(I)*( X(N-I+1) - X(I) )
33 CONTINUE
W = B**2/S2
WRITE (6,200) W
FORMAT (1//8X, 'WILK-SHAPIRO TEST: TEST VALUE = ', IPE10.3, 4X,
: SEE BIOMETRIKA (1965), V. 52, P. 591, FCR CRITICAL',
: VALUES' )
*
RETURN
END

```

```

-----
SUBROUTINE EXPLT

PURPOSE
1. SUBROUTINE EXPLT PLOTS A SET OF DATA VS EXPONENTIAL SCORES
2. ESTIMATES AND GIVES THE PARAMETERS GAMMA1, GAMMA2.

CALLING SEQUENCES
CALL EXPLT ( X, SCORES, N )

ARGUMENTS
X      A SINGLE DIMENSIONAL ARRAY (REAL*4) OF DATA TO BE PLOTTED
      DIMENSIONED BY N.
SCORES A DUMMY SINGLE DIMENSIONAL ARRAY (REAL*4) TO BE USED FOR
      STORING EXPON. SCORES. DIMENSIONED EXACTLY AS X ARRAY IS.
N      NUMBER OF DATA (MUST BE GREATER THAN 3)

USAGE

THE DATA WILL BE SORTED AND A SET OF ORDER STATISTICS IS GIVEN.
THE ORDER STATISTICS THEN ARE SCALED ON THE HORIZONTAL X-AXIS.
ON THE OTHER HAND THE EXPON. SCORES ARE EVALUATED AND SCALED ON
THE VERTICAL Y-AXIS.

FOR EVALUATION OF EXPON. SCORES THE FORMULA  $\sum_{J=1}^I 1/(N-J+1)$ , I=1,
2, ..., N IS USED BASED ON THE FACT THAT THE EXPECTED VALUE OF THE
I TH ORDER STATISTIC EQUALS  $\sum_{J=1}^I 1/(N-J+1)$ , J=1,2, ..., I, IF THE

```

DATA ARE REALLY EXPON. DISTRIBUTED.

THE PARAMETERS GAMMA1, GAMMA2 ARE EVALUATED USING THE FORMULAS:

$$\text{GAMMA1} = \left(N * \sum_{J=1}^I (X_{(I)} - \bar{X}) / ((N-1)*(N-2)) \right) / \text{VAR}^{1.5}$$

$$\text{GAMMA2} = \left((N*(N-2)+3) \sum_{J=1}^I (X_{(I)} - \bar{X}) / ((N-1)*(N-2)) - \right. \\ \left. - 3 * \text{VAR}^2 (N-1)(2*N-3) / (N*(N-2)*(N-3)) \right) / \text{VAR}^2 - 3$$

THE PROGRAM SUBROUTINE 'EPILOT' IS CALLED TO PLOT THE PAIRS OF ORDER STATISTICS-EXPON. SCORES.

IF DATA ARE EXPCNENTIALLY DISTRIBUTED A LINEAR FIT IS EXPECTED.

SUBROUTINES REQUIRED

SUBROUTINE 'PXSORT' IS USED AND IT HAS BEEN COMPILED AND ADDED TO MPSSLIB.

ERROR CONDITIONS

NO OUTPUT IS EXPECTED IF :

1. X IS LESS THAN ZERO FOR EVERY I=1,2,...,N.
2. DATA HAVE CONSTANT VALUE.
3. N IS LESS THAN 4 .

PROGRAMMER: GEORGIOS J. DANIKAS

UNDER DIRECTIONS OF PROF. P. A. W. LEWIS

DATE: APR 77

CC

```

SUBROUTINE EXPLT ( X, SCORES, N )

REAL *8 SUM1, SUM2, SUM3, SUM4, DIF, XM3, XM4
DIMENSION X(N), SCORES(N)
IF ( N .LE. 3 ) GO TO 35
CALL PXSORT ( X, 1, N )
IF ( X(1) .LT. 0. ) GO TO 25
IF ( X(1) .EQ. X(N) ) GO TO 30
CCOMPUTE THE EXPON. SCORES.
C
C
XN = N
SUM1 = X(1)
SCORES(1) = 1/XN
DO 11 I = 2, N
SUM1 = SUM1 + X(I)
SCORES(I) = SCORES(I-1) + 1./(XN-I+1)
11
C
C
CALL EPLOT ( X, SCORES, N )
C
C
COMPUTE GAMMA1 AND GAMMA2 PARAMETERS
SUM2 = 0.00
SUM3 = 0.00
SUM4 = 0.00
XMEAN = SUM1/XN
C
DC 22 I = 1, N
DIF = X(I) - XMEAN
SUM2 = SUM2 + DIF**2
SUM3 = SUM3 + DIF**3
SUM4 = SUM4 + DIF**4
22 CONTINUE
XVAR = SUM2 / (XN-1)
XM3 = XN*SUM3 / ((XN-1)*(XN-2))
XM4 = SUM4*(3+(XN-2)*XN) / ((XN-1)*(XN-2)*(XN-3))
* GAMMA1 = XVAR**1.5
GAMMA2 = XM3 / XVAR**2 - 3.
WRITE (6,200) GAMMA1, GAMMA2
RETURN (6,400)
25
RETURN (6,401)
30
RETURN (6,402)
35

```



```

IF ( MOD(I,5) .NE. 3 ) GO TC 15
YAXIS = SCORES(N) - YDELTA#11
WRITE (6,200) YAXIS, ( GRAPH(I,J), J=1,112 )
GO TO 33
15 WRITE (6,400) ( GRAPH(I,J), J=1,112 )
   IF ( I .EQ. 65 ) WRITE (6,202) YAXIS
33 CONTINUE
C
XAXIS(1) = X(1)
DO 44 J = 2,12
44 XAXIS(J) = XAXIS(J-1) + 10*XDELTA
C
IF ( X(N) .LT. .001 .OR. X(N) .GE. 1.E04 ) GO TC 25
NDIG = INT ( 1. + ABS ( ALOG10(X(N)) ) )
NFRCT = INT ( 1.5 + ABS ( ALOG10 ( X RANGE / 11. ) ) )
IF ( X RANGE .GE. 11. ) NFRCT = 1
NDNF = NDIG + NFRCT
IF ( X(N) .LE. 1 ) NDNF = NFRCT
IF ( NFRCT .GT. 4 .OR. NDNF .GE. 7 ) GO TC 25
FMIF(2) = DIGITS(NFRCT+3)
FMIF(11) = DIGITS(NFRCT)
WRITE (6,FMIF) ( XAXIS(J), J=1,11 )
GO TO 40
25 WRITE (6,600) ( XAXIS(J), J=1,11,2 )
40 WRITE (6,604) XDELTA, YDELTA
C
RETURN
200 FFORMAT ( 1X, F6.3, 1X, 112A1 )
201 FFORMAT ( 1, 50X, NUMBER OF ORDERED PAIRS = , I6 // 2X,
*EXPCN. SCORES, // )
202 FFORMAT ( 1, 4X, 112A1 )
400 FFORMAT ( 4X, 6(11PE11.4, 9X) )
600 FFORMAT ( // // 8X, X-SCALE : , * , = , E10.3, :
604 FFORMAT ( // // 8X, Y-SCALE : , * , = , E10.3, :
* UNITS. )
END

```

SUBROUTINE LIST

PURPOSE

SUBROUTINE LIST IS INDENDED TO :

1. ORDER A SET OF DATA IN ASCENDING ORDER.
2. PRINT THE SERIAL NUMBER OF DATA, THE VALUE OF ORDERED DATA POINTS, THE FREQUENCY AND PERCENT OF EACH DATA POINT, AND THE GRAPH OF PROBABILITY OF EACH DATA POINT.

CALLING SEQUENCES

CALL LIST (X, N).

ARGUMENTS

X A SINGLE DIMENSIONAL ARRAY OF DATA (REAL *4).
N NUMBER OF DATA POINTS.

USAGE

THE DATA WILL BE SORTED INTO INCREASING ORDER AND THEN A FIVE-COLUMN OUTPUT WILL BE GIVEN.

THE FIRST COLUMN WILL GIVE THE SERIAL NUMBER OF ORDERED DATA
THE SECOND COLUMN WILL GIVE THE DATA POINT VALUES.
THE THIRD COLUMN WILL GIVE THE FREQUENCY OF OCCURENCE
OF EACH DATA POINT VALUE.
THE FORTH COLUMN WILL GIVE THE PERCENTAGE OF EACH DATA POINT VALUE
THE FIFTH COLUMN WILL GIVE A GRAPHICAL REPRESENTATION OF VALUES
OF THE FORTH COLUMN.

SUBROUTINES REQUIRED

SUBROUTINE 'PXSORT' IS USED TO ORDER THE DATA.
'PXSORT' HAS BEEN COMPILED AND ADDED TO MPSSLIB.

PROGRAMMER : GEORGIOS J. DANIKAS
UNDER DIRECTIONS OF PROF. P. A. W. LEWIS

DATE : APR 77

SUBROUTINE LIST (X, N)

INTEGER COUNT, SERNUM
LOGICAL *I STAR
DATA STAR / :*
DIMENSION X(N)

SERNUM = 1
CALL PXSORT (X, 1, N)

FIND MAXIMUM FREQUENCY TO SCALE THE GRAPH PART

I1 = 1
PROMAX = 0.
COUNT = 0
TCP = X(I1)

DC 22 I = I1, N
K = I
IF (X(I) .GT. TOP) GO TO 10
COUNT = COUNT + 1

22 CONTINUE

10 PROB = FLOAT(COUNT) / N
PROMAX = AMAX1 (PROMAX, PROB)
I1 = K
IF (K .EQ. N) GO TO 25
GO TO 20

FIND AND PRINT SERIAL NUMBER, FREQUENSIES, PROBABILITIES
AND GRAPH THE PROBABILITY

25 PROMIN = 1. / N
IF (PROMAX .LE. PROMIN) GO TO 35
TOP = X(I1)

```

I1 = 1
WRITE (6,200)
5 COUNT = 0
C
DC 11 I = I1,N
IF ( X(I) .NE. TOP ) GO TO 15
COUNT = COUNT + 1
11 CONTINUE
C
15 PROB = FLOAT (COUNT) / N
NSTAR = INT ( 40*PROB/PROMAX + .5 )
IF ( NSTAR .EQ. 0 )
*WRITE (6,401) SERNUM, X(SERNUM), COUNT, PRCB
IF ( NSTAR .GT. 0 )
*WRITE (6,400) SERNUM, X(SERNUM), COUNT, PRCB, ( STAR, J=1,NSTAR)
SERNUM = SERNUM + COUNT
IF ( SERNUM .EQ. (N+1) ) RETURN
I1 = I + 1
TOP = X(I)
GO TO 5
C
35 WRITE (6,600) ( X(I), I=1,N )
RETURN
C
200 *
400 *
401 *
600 *
FORMAT ( '1', 5X, 'SERIAL NUMBER', 6X, 'ORDERED DATA', 6X,
'FREQUENCIES', 6X, 'PERCENT', 18X, 'PROBABILITY GRAPH', /// )
FORMAT ( '7X,16', 9X, 'E16.8,5X,16,11X, F7.5,8X, 45A1 / )
FORMAT ( '7X,16', 5X, 'E16.8,5X,16,11X, F7.5 )
FORMAT ( '1', 5X, 'EACH OF DATA POINTS HAS ITS OWN VALUE',
//, 5X, 'THE ORDERED DATA ARE', //, 5( 5X, E15.7 ) )
END

```

SUBROUTINE SECTN

PURPOSE

SUBROUTINE SECTN IS INTENDED TO :

1. SECTION A SET OF DATA INTO SEVERAL DISJOINT SECTIONS.
2. COMPUTE A SET OF BASIC STATISTICS FOR EACH SECTION.
3. COMPUTE A SET OF BASIC STATISTICS FOR THE ENTIRE SET OF DATA.
4. COMPUTE AN ESTIMATE OF EACH OF THE BASIC STATISTICS BY AVERAGE THE SET OF BASIC STATISTICS OBTAINED FROM THE SECTIONS.

CALLING SEQUENCES

CALL SECTN (X, N, K)

ARGUMENTS

- X A SINGLE DIMENSION ARRAY (REAL*4) CF DATA.
N NUMBER OF DATA.
K NUMBER OF SECTIONS. (MUST BE NO GREATER THAN 100).

USAGE

THE DATA WILL BE SECTIONED INTO K SECTIONS AND FOR EACH SECTION THE FOLLOWING STATISTICS WILL BE COMPUTED, USING THE FORMULAS AS THEY ARE DESCRIBED IN 'SCR STANC. MATH. TABLES, EDITION 22 (1974).

- MEAN AVERAGE OF EACH SECTION (P. 570)
MEDIAN MID-VALUE OF EACH SECTION IF N/K IS ODD
AVERAGE OF THE TWO MID-VALUES OF EACH SECTION O.W.
(P. 571)
VARIANCE THE UNBIASED ESTIMATE HAS BEEN USED.
STD. DEV. THE UNBIASED ESTIMATE HAS BEEN USED. (P.573)

COEF. OF VARIANCE = STAD. DEV. / (MEAN)

SKENNESS = $M_3 / \text{STAD. DEV.}^{**3}$
WHERE M_3 IS THE THIRD CENTRAL MOMENT
UNBIASLSSY COMPUTED

KURTOSSIS = $M_4 / \text{VARIANCE}^{**2}$ - 3
WHERE M_4 IS THE FORTH CENTRAL MOMENT.

MINIMUM = X(1)

MAXIMUM = X(N)

THE SAME AS ABOVE STATISTICS ARE COMPUTED FOR THE ENTIRE SET
OF DATA (USING THE SAME FORMULAS)

EVENTUALLY SOME ESTIMATES OF THE ABOVE STATISTICS ARE COMPUTED
USING THE RESULTS FROM EACH SECTION

NOTE : K MUST BE SUCH A NUMBER AS TO MINIMIZE THE NUMBER OF
DATA POINTS THAT WILL HAVE TO BE DISCARDED. SECTN PLACES THE DATA
INTO THE EQUAL SIZE SECTION DISCARDING ANY DATA LEFT OVER.

FOR $K \leq 3$ OR $K > 100$ OR $(N/K) \leq 3$ ESTIMATES FRCH UNSECTICNED DATA
WILL BE ONLY GIVEN AND NO ESTIMATES FOR THE ESTIMATED STATISTICS
WILL BE COMPUTED.

NO OUTPUT IS EXPECTED IF $NK \leq 3$.

SUBROUTINE REQUIREMENTS

SUBROUTINE 'PXSORT' IS USED ANC IT HAS BEEN COMPILED AND ADDED
TO MPSSLIB

PROGRAMMER : GEORGIOS J. DANIKAS

UNDER DIRECTIONS CF PROF. P. A. W. LEWIS

DATE : MAY 77

CC

```

SUBROUTINE SECTN ( X, ND, K )
DIMENSION X(N), SORT(100)
LCGICAL #1 PARMTR(70)
REAL #8 DIF, XM3, XM4, SUM1, SUM2, SUM3, SUM4
COMMON STAT(108,9)
DATA PARMTR /
*E, *A, *I, *N, *
*E, *D, *I, *A, *N, *
*V, *A, *R, *I, *A, *N, *
*S, *T, *D, *I, *D, *E, *
*C, *O, *E, *F, *W, *A, *R, *
*S, *K, *E, *F, *W, *N, *E, *S, *
*U, *R, *T, *O, *S, *I, *S, *
IF ( ND .LE. 3 ) GO TO 10
K1 = K+1
IF ( K .GT. 100 ) K1 = 101
XK = K
M = ND/K
N = M*K
IF ( M .LE. 3 ) N = ND
COMPUTE DESIRED STATISTICS OF UNSECTIONED DATA
CALL ESTIMA ( X, N, O, N, K1 )
SECTION THE DATA AND ESTIMATE DESIRED STATISTICS FOR EACH SECTION.
IF ( K .GT. 100 ) GO TO 15
IF ( M .LE. 3 ) GO TO 15
MODM2 = MOD(M,2)
MCCN2 = MOD(N,2)
MCK2 = M/2
MOC2 = N/2
NOC2 = K/2
KOC2 = J = 1, K
L1 = (J-1)*M
L2 = J*M
CALL ESTIMA ( X, N, L1, L2, J )
LPI = L1+1
CALL PXSORT ( X, LPI, L2 )
STAT ( J, 2 ) = ( (1 - MODM2)*X(L1+MOC2) + X(LPI+MOC2) ) / ( 2 - MODM2 )
CALL PXSORT ( X, 1, N )
STAT ( K1, 2 ) = ( (1 - MCCN2)*X(NC2) + X(NOC2+1) ) / ( 2 - MCCN2 )
IF ( K .LE. 3 ) GO TO 30
COMPUTE DESIRED STATISTICS FOR EACH ESTIMATED STATISTIC
DO 22 J = 1, 7
DO 33 I = 1, K
DO 33 SORT(I) = STAT(I, J)

```

C

C

C

C

```

KIJ = K1+J
CALL ESTIMA ( SORT, K, O, K, KIJ )
CALL PXSORT ( SORT, I, K, )
STAT (KIJ,2) = ( (1 - MODK2)*SQRT(KO2) + SQRT(KC2+1) )
/ ( (2 - MODK2) )
*
22 STAT(KIJ,8) = STAT(KIJ,4) / SQRT(XK)
WRITE (6,200)
WRITE (6,400)
WRITE (6,600)
DC 44 J = 1,7
JP = (J-1)*10+1
J10 = J*10
KIJ = K1+J
WRITE (6,800)
44 ( PARMTR(I), I=JP, J10 ), ( STAT(KIJ,I), I=1,8)
RETURN
10 WRITE (6,201)
30 WRITE (6,200)
CALL PXSORT ( X, I, N, )
STAT(KI,2) = ( (1 - MCDN2)*X(NO2) + X(NO2+1) ) / ( 2 - MCDN2 )
WRITE (6,400)
WRITE (6,604)
RETURN
15 WRITE (6,200)
CALL PXSORT ( X, I, N, )
STAT(KI,2) = ( (1 - MCDN2)*X(NO2) + X(NO2+1) ) / ( 2 - MODN2 )
IF ( M.LE.3 ) WRITE (6,601)
IF ( K.GT.100 ) WRITE (6,603)
WRITE (6,602) ( STAT(KI,J), J=1,9 )
RETURN
200 * ( '1'// 48X, 'ESTIMATED SAMPLE PARAMETERS'
// 3X, 'SECTION', 7X, 'MEAN', 7X, 'MEDIAN', 5X,
'VARIANCE', 3X, 'STD. DEV.', 4X, 'CCCEF VAR.', 4X, 'SKWNESS',
3X, 'KURTOSISS', 4X, 'MINIMUM', 5X, 'MAXIMUM', /
5X, '*** SAMPLE SIZE TCC SMALL ***' )
( / 4X, 13, 5X, 1P9E12.4 )
( // 1X, 'UNSECTIONED', 1P5E12.4 )
( // 1X, 'PARAMETER', 7X, 'MEAN', 8X, 'MEDIAN', 6X,
'VARIANCE', 4X, 'STD. DEV.', 5X, 'CCCEF VAR.', 5X,
'SKWNESS', 4X, 'KURTOSISS', 2X, 'STD. DEV.', /
/ 4X, '*** SECTION SIZE TCC SMALL ***'// 35X )
( 'PARAMETERS FROM UNSECTIONED DATA HAVE BEEN ESTIMATED' )
601 * FCRMAT ( / 1X, 'UNSECTIONED', 1P5E12.4 )
602 * FCRMAT ( / 4X, '*** NUMBER OF SECTIONS GREATER THAN 100'
603 * FCRMAT ( 'NO ESTIMATES BY SECTIONING CAN BE COMPUTED **'// 35X,
'PARAMETERS FROM UNSECTIONED DATA HAVE BEEN ESTIMATED' )
*
201 * FCRMAT ( / 1X, 'ESTIMATED SAMPLE PARAMETERS'
400 * FCRMAT ( / 3X, 'SECTION', 7X, 'MEAN', 7X, 'MEDIAN', 5X,
600 * FCRMAT ( / 3X, 'VARIANCE', 3X, 'STD. DEV.', 4X, 'CCCEF VAR.', 4X, 'SKWNESS',
3X, 'KURTOSISS', 4X, 'MINIMUM', 5X, 'MAXIMUM', /
5X, '*** SAMPLE SIZE TCC SMALL ***' )
( / 4X, 13, 5X, 1P9E12.4 )
( // 1X, 'UNSECTIONED', 1P5E12.4 )
( // 1X, 'PARAMETER', 7X, 'MEAN', 8X, 'MEDIAN', 6X,
'VARIANCE', 4X, 'STD. DEV.', 5X, 'CCCEF VAR.', 5X,
'SKWNESS', 4X, 'KURTOSISS', 2X, 'STD. DEV.', /
/ 4X, '*** SECTION SIZE TCC SMALL ***'// 35X )
( 'PARAMETERS FROM UNSECTIONED DATA HAVE BEEN ESTIMATED' )
601 * FCRMAT ( / 1X, 'UNSECTIONED', 1P5E12.4 )
602 * FCRMAT ( / 4X, '*** NUMBER OF SECTIONS GREATER THAN 100'
603 * FCRMAT ( 'NO ESTIMATES BY SECTIONING CAN BE COMPUTED **'// 35X,
'PARAMETERS FROM UNSECTIONED DATA HAVE BEEN ESTIMATED' )
*

```

```

604 * ( //IX, 'UNSECTIONED', 1P5E12.4, //IX, '*** NO ESTIMATES',
      * OF ESTIMATED STATISTICS CAN BE COMPUTED. NUMBER OF
      * SECTIONS TOO SMALL *** )
800 FORMAT ( //IX, 10A1, 1P7E13.4, 2X, 1PE12.4 )
801 FORMAT ( //104X, 'NS=# OF SECTS', )

```

```

SUBROUTINE ESTIMA ( X, N, L1, L2, IS )

```

```

DIMENSION X(N)
REAL #8 DIF, XM3, XM4, SUM1, SUM2, SUM3, SUM4
COMMON STAT(108,9)

```

```

SUM1 = 0.00
M = L2-L1

```

```

11 DC 11 I = 1, M
    SUM1 = SUM1 + X(L1+I)
    STAT(IS,1) = SUM1/XN
    STAT(IS,8) = X(L1+I)
    STAT(IS,9) = X(L2)
    SUM2 = 0.00
    SUM3 = 0.00
    SUM4 = 0.00

```

```

C DC 22 I = 1, M
    DIF = X(L1+I) - STAT(IS,1)
    SUM2 = SUM2 + DIF**2
    SUM3 = SUM3 + DIF**3
    SUM4 = SUM4 + DIF**4

```

```

22 STAT(IS,8) = AMINI ( X(L1+I), STAT(IS,8) )
    STAT(IS,9) = AMAXI ( X(L1+I), STAT(IS,9) )
    STAT(IS,3) = SUM2 / (XN-1)
    STAT(IS,4) = SQR ( STAT(IS,3) )

```

```

IF ( ABS ( STAT(IS,5) ) .GT. 1.E-30 )
* XM3 = XN*SUM3 / ( XN-1)* (XN-2)
* XM4 = - 3*STAT(IS,3)**2*(XN-1)* (2*XN-3) / ( XN*(XN-2)*(XN-3) )
    STAT(IS,6) = XM3 / STAT(IS,4)**3
    STAT(IS,7) = XM4 / STAT(IS,3)**2-3.
RETURN
END

```

SUBROUTINE JACK

PURPOSE

SUBROUTINE JACK IS INTENDED TO :

1. GROUP A SET OF DATA INTO SEVERAL GROUPS
2. COMPUTE A SET OF PSEUDO-VALUES FOR JACKKNIFE ESTIMATES
3. COMPUTE A SET OF BASIC STATISTICS FOR THE ENTIRE SET OF DATA.
4. COMPUTE THE BASIC JACKKNIFE ESTIMATES
5. GIVE ESTIMATES OF THE VARIANCES OF THE JACKKNIFE ESTIMATES.

CALLING SEQUENCES

CALL JACK (X, XS, STAT, N, IG)

ARGUMENTS

- X A SINGLE DIMENSION ARRAY (REAL*4) CF DATA.
- XS A SINGLE DIMENSION ARRAY (REAL*4). RETURNS ORDERED DATA.
- STAT TWO DIMENSION DUMMY ARRAY (REAL*4). DIMENSICNED BY (IG,7)
- N NUMBER OF DATA.
- IG NUMBER OF GROUPS PLUS ONE (=R+1). NO LESS THAN 3.

USAGE

THE DATA WILL BE GROUPED INTO R=IG-1 GROUPS AND FOR EACH SET OF THE (R-ITH) GROUP, I=1,2,...,R THE PSEUDO-VALUES FOR THE FOLLOWING STATISTICS WILL BE COMPUTED USING THE FORMULAS AS THEY ARE DESCRIBED IN 'SCR STAND. MATH. TABLES', EDITION 22 (1974).

- MEAN AVERAGE OF EACH SECTION (P. 570)
- MEDIAN MID-VALUE OF EACH SECTION IF N/K IS ODD
AVERAGE OF THE TWO MID-VALUES OF EACH SECTION O.W.
(P. 571)

CC

VARIANCE THE UNBIASED ESTIMATE HAS BEEN USED.
STD. DEV. THE UNBIASED ESTIMATE HAS BEEN USED. (P.573)
COEF. OF VARIANCE = STAD. DEV./ |MEAN|

SKENNESS = M3 / STAD. DEV.**3
WHERE M3 IS THE THIRD CENTRAL MOMENT
UNBIASLSSLY COMPUTED

KURTOSSIS = M4 / VARIANCE** -2
WHERE M4 IS THE FORTH CENTRAL MOMENT.

THE SAME AS ABOVE STATISTICS ARE COMPUTED FOR THE ENTIRE SET
OF DATA (USING THE SAME FORMULAS)

USING THE COMPUTED AS ABOVE PSEUDO-VALUES, JACK ESTIMATES THE
BASIC (MEAN, MEDIAN, VARIANCE, STAD. DEV., SKENNESS, KURTOSSIS)
STATISTICS OF DATA BY JACKKNIFE METHOD AND THE VARIANCE AND STAND.
DEVIATION FOR EACH STATISTIC IS ALSO GIVEN.

NOTE : R=IG-1 MUST BE SUCH A NUMBER AS TO MINIMIZE THE NUMBER
OF DATA POINTS THAT WILL HAVE TO BE DISCARDED. JACK PLACES THE
DATA INTO THE EQUAL SIZE GROUP DISCARDING ANY DATA LEFT OVER.

ERROR CONDITIONS

NO OUTPUT IS EXPECTED IF NK=3

IF IG<=2 OR (IG-2)*(N/(IG-1))<=3 ESTIMATES OF UNGROUPED DATA
WILL BE ONLY GIVEN.

SUBROUTINE REQUIREMENTS

SUBROUTINE 'PXSORT' IS USED AND IT HAS BEEN COMPILED AND ADDED
TO MPSSLIB

PROGRAMMER : GEORIOS J. DANIKAS
UNDER DIRECTIONS OF PROF. P. A. W. LEWIS

DATE : MAY 77

CC


```

C
C
C
WRITE (6,400) ( I, (STAT(I,J), J=1,7), I=1,IR )
WRITE (6,401) ( STAT(IG,J), J=1,7 )

COMPUTE JACKKNIFE ESTIMATES.

DC 44 J = 1,7
   STAT(IG,J) = STAT(IG,J)*IR
   T1 = 0.
   PS1 = 0.
   PS2 = 0.
DO 66 I = 1,IR
   T1 = T1+STAT(I,J)
   PS2 = PS2+(STAT(IG,J)-(IR-1)*STAT(I,J))**2
   PS1 = PS1+STAT(IG,J)-(IR-1)*STAT(I,J)**2
66   STAT(1,J) = STAT(IG,J)-(IR-1)*T1/IR
   STAT(2,J) = (PS2-PS1**2/IR) / (IR-1)
   STAT(3,J) = SQRT (STAT(2,J)/IR)
44   DC 77 J = 1,7
   JP = (J-1)*10+1
   J10 = J*10
   WRITE (6,600) ( PARMTR(I), I=JP,J10 ), (STAT(I,J), I=1,3 )
15   RETURN (6,800)
25   RETURN (6,200)
   IF ( L2 .LE. 3 ) WRITE (6,801)
   IF ( IR .LT. 2 ) WRITE (6,802)
DC 88 I = 1,N
   XS(I) = X(I)
CALL XS(I) PXSORT ( XS, I, N )
STAT(IG,2) = ( (1-MODN2)*XS(N02) + XS(N02+1) ) / ( 2-MODN2 )
WRITE (6,802) ( STAT(IG,J), J=1,7 )
RETURN
FORMAT ( '1'// 46X, 'ESTIMATED PARAMETERS', 'MEDIAN', 'COEF.', 'VAR', '6X,
// 3X, 'GROUP', '10X, 'MEAN', '10X, 'DEV.', '6X, 'COEF.', 'VAR', '6X,
8X, 'VARIANCE', '7X, 'STD. DEV.', '6X, 'KURTOSIS', '//' )
// 3X, 'SKEWNESS', '7X, 'KURTOSIS', '//' )
401 FCRMAT ( 3X, '13', 'UNGROUPED', '1X, IP7E15.5 )
401 FCRMAT ( // 1X, 'UNGROUPED', '1X, IP7E15.5 )
// 1X, 'PARAMETER', '7X, 'JACKKNIFE', '7X, 'VARIANCE', '7X,
// 1X, 'STD. DEV.', '7X, 'JACKKNIFE', '7X, 'PARAMETER', '7X, 'JACKKNIFE', '7X, 'VARIANCE', '7X,
// 1X, 'UNGROUPED', '1X, IP3E16.5 )
600 FCRMAT ( // 1X, '10A1', 'SAMPLE SIZE TOO SMALL ***' )
800 FCRMAT ( // 4X, '***', 'INVALID GROUP SIZE ***', '//' )
801 FCRMAT ( // 4X, '***', 'INVALID GROUP SIZE ***', '//' )
802 FCRMAT ( // 3X, 'PARAMETERS FROM UNGROUPED DATA HAVE BEEN ESTIMATED', '//' )
// 1X, 'UNGROUPED', '1X, IP7E15.5,

```

```

803 * FORMAT ( // 4X, 'JACKKNIFE PARAMETERS CANNOT BE ESTIMATED, ' )
      * RETURN
      * END
      * PARAMETERS FROM UNGROUPED DATA HAVE BEEN ESTIMATED, // )

```

```

SUBROUTINE JACKES ( X, STAT, IS, IG, N )

```

```

DIMENSION X(N), STAT(IG,7)
REAL #8 DIF, XM3, XM4, SUM1, SLM2, SUM3, SUM4

```

```

C
XN = N
SUM1 = 0.00
DC I 1 = 1, N
  SUM1 = SUM1 + X(I)
  STAT(IS,1) = SUM1/XN
SUM2 = 0.00
SUM3 = 0.00
SUM4 = 0.00

```

```

C
DO 22 I = 1, N
  DIF = X(I) - STAT(IS,1)
  SUM2 = SUM2 + DIF**2
  SUM3 = SUM3 + DIF**3
  SUM4 = SUM4 + DIF**4

```

```

C
STAT(IS,3) = SUM2 / (XN-1)
STAT(IS,4) = SQRT ( STAT(IS,3) )
IF ( ABS ( STAT(IS,1) ) .GT. 1.E-30 )
  * XM3 = STAT(IS,5) = STAT(IS,4) / ABS ( STAT(IS,1) )
  * XM4 = SUM3 / ( (XN-1)*(XN-2) )
  * STAT(IS,6) = XM3 / ( (XN-1)*(XN-2)*(XN-3) )
  * STAT(IS,7) = XM4 / ( (XN-1)*(XN-2)*(XN-3) )
  * RETURN
  * END

```

```

C THE GENERAL EARMA(P,Q) PROGRAM.
C THIS PROGRAM CAN BE USED TO SIMULATE
C ALL THE EARMA(P,Q) CASES.

INTEGER G, QS, QSI, QX, QX1, CPS, CPX, UX(10000),
DIMENSION US(10000), FS(10000), FSI(10000), FX(10000),
* * * * * SUMB(31), SUMBS(31), SUMBS1(31), SUMBX(31),
* * * * * EXPL(10030), EXPSL(10030), BX(30), BX1(30),
* * * * * BS(30), BSI(30), SUMXM(4,500), D(4,500), DM(4,500),
* * * * * SUMS(4,500), SUMSM(4,500), WM(4,500), WMB(4,500)
COMMON N, BS(1), BSI(1), BX(1), EX1(1), SUMES(1), SUMBS(1),
DATA SUMBX(1), SUMBS1(1) / 8*1.0 /

C READ (5,100) IUS1, IUS, IUX1, IUX, IE1, IS1, IX1, IFS, IFS1,
IFX1, IFX
* READ (5,30C) N, M, KS, KSI, KX, KX1, QS, QSI, CX, QX1, CPS,
CPX, RS, RHOS, RHCSI, RHOX, RHOXI
* READ (5,301) RX, RS, KS, KSI, KX, KX1, QS, QSI, QX, QX1, CPS, CFX, CPG

C WRITE (6,200) N, M, KS, KSI, KX, KX1, QS, QSI, QX, QX1, CPS, CFX, CPG
IF ( KS .LT. 1 ) GO TO 5
WRITE (5,301) ( BS(I), I=1,KS )
WRITE (6,202) ( BS(I), I=1,KS )
CALL BETAS ( SUMBS, KS, BS )
5 IF ( KSI .LT. 1 ) GO TO 25
WRITE (5,301) ( BSI(I), I=1,KSI )
WRITE (6,202) ( BSI(I), I=1,KSI )
CALL BETAS ( SUMBS1, KSI, BSI )
25 IF ( KX .LT. 1 ) GO TO 35
WRITE (5,301) ( BX(I), I=1,KX )
WRITE (6,202) ( BX(I), I=1,KX )
CALL BETAS ( SUMBX, KX, BX )
35 IF ( KX1 .LT. 1 ) GO TO 45
WRITE (5,301) ( BX1(I), I=1,KX1 )
WRITE (6,202) ( BX1(I), I=1,KX1 )
CALL BETAS ( SUMBX1, KX1, BX1 )
45 J = 0
* WRITE (6,201) J, IUS1, IUS, IUX1, IUX, IE1, IS1, IX1, IFS, IFS1,
IFX1, IFX

```

```

K = MAXO ( KS, KSI, KX, KXI, QS, CSI, QX, QXI )
N1 = N/4
N2 = N/2
N3 = 3*N/4
NK = N+K

SL = 1./RS
XL = 1./RX
CS = RX/RS
CX = 1./CS

C 10 J = J+1
CALL EXPON ( IEL, EXP1, NK )
CALL EXPON ( IXI, EXPXL, NK )
CALL EXPON ( ISI, EXPSL, NK )
CALL RANDOM ( IUX, UX, N )
CALL RANDOM ( IUXI, UXI, N )
CALL RANDDCM ( IUS, US, N )
CALL RANDDCM ( IUSI, USI, N )

C DO 11 I = 1, NK
EXP1(I) = EXP1(I)*SL
EXPXL(I) = EXPXL(I)*XL

C 11 IF ( CPG .EQ. 0 ) GO TO 65
IF ( QS .EQ. 1 ) CALL AUTOR ( FS, EXP1, SL, RHOS, IFS, 1 )
IF ( QX .EQ. 1 ) CALL AUTOR ( FX, EXPXL, XL, RHOC, IFX, 1 )
IF ( QSI .EQ. 1 ) CALL AUTOR ( FSI, EXPSL, 1., RHOSI, FSI, 0 )
IF ( QXI .EQ. 1 ) CALL AUTOR ( FXI, EXPXL, 1., RHCXI, IFXI, 0 )
GO TO 50

C 65 IF ( QSI .EQ. 1 .AND. CPS .EQ. 0 ) CALL AUTOR ( FSI,
EXP1, 1., RHOSI, FSI, 1 )
IF ( QSI .EQ. 1 .AND. CPS .EQ. 1 ) CALL AUTOR ( FSI,
EXPXL, CS, RHOSI, FSI, 1 )
IF ( QXI .EQ. 1 .AND. CPX .EQ. 0 ) CALL AUTOR ( FXI,
EXPXL, 1., RHCXI, IFXI, 1 )
IF ( QXI .EQ. 1 .AND. CPX .EQ. 1 ) CALL AUTOR ( FXI,
EXPXL, CX, RHCXI, IFXI, 1 )

C 50 ARIV = 0.
SERV = 0.
ARIVM = 0.
SERVM = 0.
WAIT = 0.
SNEWM = 0.
WAITM = 0.
WEAR = 0.

```

```

C      WMBAR = 0.
DC 22  I = I,N
      SOLD = SNEW
      WOLD = WAIT
      WMOLD = WAITM
      IES = KSI + I
      IEX = KXI + I
      VAL = 0
      IF ( CPG .EQ. 1 ) GO TO 75

C      SNEW = BSI(1) * EXPSL(IES)
      IF ( CPS .EQ. 0 .AND. KSI .GT. 0 ) CALL EARMA ( QSI,
*      KSI, I, I, BSI, EXPSL, FSI, SUMBS1, VAL, 0, US1(I) )
*      IF ( CPS .EQ. 1 .AND. KSI .GT. 0 ) CALL EARMA ( QSI,
      KSI, I, CS, BSI, EXPXL, FSI, SUMBS1, VAL, 1, US1(I) )
      SNEW = SNEW + VAL
      IF ( QSI .EQ. 1 .AND. KSI .EQ. 0 ) SNEW = FSI(I)
      XNEW = BX1(I) * EXPXL(IEX)
      VAL = 0
      IF ( CPX .EQ. 0 .AND. KXI .GT. 0 ) CALL EARMA ( QX1,
*      KXI, I, I, BX1, EXPXL, FX1, SUMBX1, VAL, 0, UX1(I) )
*      IF ( CPX .EQ. 1 .AND. KXI .GT. 0 ) CALL EARMA ( QX1,
      KXI, I, CX, BX1, EXPXL, FX1, SUMBX1, VAL, 1, UX1(I) )
      XNEW = XNEW + VAL
      IF ( QX1 .EQ. 1 .AND. KXI .EQ. 0 ) XNEW = FX1(I)
      GO TO 70

C      IF ( KSI .GT. 0 ) CALL EARMA ( QSI, KSI, I, 1., BSI,
      EXPXL, FSI, SUMBS1, VAL, 0, US1(I) )
      SNEW = BSI(1) * EXPSL(IES) + VAL
      IF ( QSI .EQ. 1 .AND. KSI .EQ. 0 ) SNEW = FSI(I)
      VAL = 0
      IF ( KS .GT. 0 ) CALL EARMA ( QS, KS, I, SL, BS, EXPL,
      FS, SUMBS, VAL, 1, US(I) )
      SNEW = SNEW * BS(1) + VAL
      IF ( QS .EQ. 1 .AND. KS .EQ. 0 ) SNEW = FS(I)
      VAL = 0
      IF ( KXI .GT. 0 ) CALL EARMA ( QX1, KXI, I, 1., BX1,
      EXPXL, FX1, SUMBX1, VAL, 0, UX1(I) )
      XNEW = BX1(1) * EXPXL(IEX) + VAL
      IF ( QX1 .EQ. 1 .AND. KXI .EQ. 0 ) XNEW = FX1(I)
      VAL = 0
      IF ( KX .GT. 0 ) CALL EARMA ( QX, KX, I, XL, BX, EXPL,
      FX, SUMBX, VAL, 1, UX(I) )
      XNEW = XNEW * BX(1) + VAL
      IF ( QX .EQ. 1 .AND. KX .EQ. 0 ) XNEW = FX(I)

```

```

7C      ARIV = ARIV + XNEW
        SERV = ARIVM + SNEW
        ARIVM = ARIVM + EXPXL(I)
        SERV = SERV + EXPSL(I)
        IF ( I .EQ. 1 ) GO TO 22
        WAIT = WOLD + SCLD - XNEW
        IF ( WAIT .LE. 0 ) WAIT = 0
        WAITM = WMOLD + EXPSL(I-1) - EXPXL(I)
        IF ( WAITM .LE. 0 ) WAITM = 0
        WBAR = WBAR + WAIT
        WMBAR = WMBAR + WAITM
        IF ( I .EQ. N1 ) .OR. I .EQ. N2 .OR. I .EQ. N3 .OR.
*      GO TO 22
        GO TO 22

C      85      G = I/N1
        SUMX(G,J) = ARIV
        SUMS(G,J) = SERV
        W(G,J) = WAIT
        WB(G,J) = WBAR/I
        SUMXM(G,J) = ARIVM
        SUMSM(G,J) = SERV
        WM(G,J) = WAITM
        WMB(G,J) = WMBAR/I
        D(G,J) = (SERV - ARIV)/I - SL + XL
        DM(G,J) = (SERV - ARIVM)/I - SL + XL
22 CONTINUE

C      WRITE ( 6,201 ) J, IUS1, IUS, IUX1, IUX, IE1, IS1, IX1, IFS, IFS1,
*      IF ( J .EQ. M ) GO TO 15
        GO TO 10

C      OUTPUT PART
C      15      WRITE ( 6,211 )
        WRITE ( 6,212 )
        WRITE ( 6,214 )
        WRITE ( 6,222 )
        WRITE ( 6,224 )
        WRITE ( 6,231 )
        WRITE ( 6,232 )
        WRITE ( 6,233 )
        WRITE ( 6,234 )
        WRITE ( 6,241 )
        ( SUMX ( 1,1 ) , I = 1,J )
        ( SUMX ( 2,1 ) , I = 1,J )
        ( SUMX ( 4,1 ) , I = 1,J )
        ( SUMS ( 2,1 ) , I = 1,J )
        ( SUMS ( 4,1 ) , I = 1,J )
        ( W ( 2,1 ) , I = 1,J )
        ( W ( 3,1 ) , I = 1,J )
        ( W ( 4,1 ) , I = 1,J )
        ( WB ( 1,1 ) , I = 1,J )

```



```
SUBROUTINE BFTAS ( SUM, K, B )
```

```
  DIMENSION SUM(31), B(30)
  SUM(1) = B(1)
  PROD = 1
  IF ( K.EQ. 1 ) GO TO 15
  DO 11 I = 2, K
    PROD = PROD*(1.-B(I-1))
    SUM(I) = SUM(I-1) + PROD*B(I)
  11 SUM(K+1) = SUM(K) + PROD*( 1.-B(K) )
  RETURN
  END
```

```
SUBROUTINE AUTOR ( F, EXP, L, R, ISD, I1 )
```

```
  REAL L, N
  COMMON F(10000), EXP(10030), U(10000)
  DIMENSION F(1) = L*( I1+I-1), *EXP(1)
  CALL RANDOM ( ISD, U, N )
  IF ( I1*U(1) .GT. R ) F(I) = F(1) + L*EXP(2)
  DO 11 I = 2, N
    F(I) = R*F(I-1)
  11 IF ( U(I) .GT. R ) F(I) = F(I) + L*EXP(I+11)
  CONTINUE
  RETURN
  END
```

```
SUBROUTINE EARMA ( IQ, K, IS, L, B, EXP, F, SUM, VAL, I1, U )
```

```
  REAL L
  DIMENSION B(31), EXP(10030), F(10000), SUM(31)
  VAL = 0.
  IF ( U .LE. SUM(1) ) RETURN
  KPI = K+1
  IF ( K.EQ. 1 ) GO TO 15
  DO 11 ID = 2, KPI
    IR = ID
    IF ( U .LE. SUM(IR) ) GC TO 10
  11 CONTINUE
```

C

```
1C IS1 = IS+I1+KPI
   IRM1 = IR
   IF ( IR .EQ. KPI )   IRM1 = IR-1
   DO 22 I = 2,IRM1
   22 VAL = VAL + B(I)*L*EXP(IS1-I)
      IF ( IR .EQ. KPI ) VAL = VAL + L*(1-IQ)*EXP(IS+I1) + F(IS)*IQ
   RETURN
15 VAL = L*(1-IQ)*EXP(IS+1) + F(IS)*IQ
   RETURN
   END
```

```

C THIS PROGRAM CAN BE USED ONLY FOR THE PARTICULAR
C EARMA(1,0) CASE. IT IS A MODIFICATION OF THE GENERAL
C EARMA(P,Q) PROGRAM.

REAL *8 ARIV, SERV, WBAR, ARIVM, SERV, WMBAR
INTEGER G
DIMENSION U(10000), F(10000), EX(10000), ES(10000),
SUMX(500), SUMS(500), SUMXM(500), SUMSM(500),
D(500), DM(500), W(500), WB(500), WM(500), WMB(500)
* *
COMMON N, RX, RS, RHOS
READ (5,300) N, RX, RS, RHOS
WRITE (6,200) N, RX, RS, RHOS
READ (5,100) G, K
READ (5,100) IF, IEX, IES
J = 0
WRITE (6,201) J, IF, IEX, IES
NK = N + K
SL = 1./RS
XL = 1./RX

C 10 J = J + 1
CALL EXPON ( IEX, EX, N )
CALL EXPON ( IES, ES, NK )

C DC 11 I = 1, N
ES(I) = ES(I) * SL
EX(I) = EX(I) * XL

C READ (5,304) ARIV, SERV, SNEW, WAIT, WBAR, ARIVM, SERV,
SMNEW, WAITM, WMBAR
* WRITE (6,304) ARIV, SERV, SNEW, WAIT, WBAR, ARIVM, SERV,
SMNEW, WAITM, WMBAR
CALL AUTOR ( F, ES, RHOS, IF, K, SNEW )
WRITE (6,201) J, IF, IEX, IES

C DC 22 I = 1, N
SOLD = SNEW
WOLD = WAIT
WMOLD = WAITM
SMOLD = SMNEW
ARIV = ARIV + EX(I)
SERV = SERV + F(I)
ARIVM = ARIVM + ES(I)
SERVM = SERV + ES(I)
SMNEW = F(I)
SNEW = F(I)
WAIT = WOLD + SOLD - EX(I)

```

```

IF ( WAIT .LE. 0. ) WAIT = 0.
WAITM = WMOLD + SMOLD - EX(I)
IF ( WAITM .LE. 0. ) WAITM = 0.
WMBAR = WBAR + WAIT
WMBAR = WMBAR + WAITM
22 CONTINUE

SUMX(J) = ARIV
SUMS(J) = SERV
W(J) = WAIT
WB(J) = WBAR / N/G
SLXM(J) = ARIVM
SUMSM(J) = SERV
WM(J) = WAITM
WMB(J) = WMBAR / N/G
D(J) = (SERV - ARIV) / N/G - SL + XL
DM(J) = (SERV - ARIV, SNEW, WBAR, ARIVM, SERV,
SMNEW, WAITM, WMBAR,
J, ARIV, SERV, SNEW, WAIT, WEAR, ARIVM, SERV,
* WRITE (7,303)
* IF ( J.EQ. 500) SMNEW, WAITM, WMBAR
GO TO 10
15 WRITE (7,203) IF, IEX, IES
100 FFORMAT ( /, 3I20 )
200 FFORMAT ( /, 1, 57X, 'N = ', I6, '///', 5X, 'SERV = ', F10.3, 5X, 'WBAR = ', F14.3,
F6.3, / )
* F6.3, 10X, 'RS = ', F6.3, //, 5X, 'RHOS = ',
F6.3, / )
* J = ', I4, 5X, 'SEEDS : ', 5I20 )
201 FFORMAT ( /, 120 )
203 FFORMAT ( /, 26X, 'ARIV = ', F10.3, 5X, 'SERV = ', F10.3, 5X, 'F10.3, 5X, 'F14.3
/ , 26X, 'ARVM = ', F10.3, 5X, 'SRVM = ', F10.3, 5X, 'F10.3, 5X,
* SMNW = ', F8.3, 5X, 'WAITM = ', F9.4, 5X, 'WMBR = ',
F14.3, // )
* F10, 3, 3F10.3 )
300 FFORMAT ( /, 10, 5 )
301 FFORMAT ( /, 13, 2X, 5F15.4, /, 5X, 5F15.4 )
303 FFORMAT ( /, 5X, 5F15.4, /, 5X, 5F15.4 )
WRITE (6,212) ( SUMX(I), I = 1, J )
WRITE (6,222) ( W(I), I = 1, J )
WRITE (6,232) ( WB(I), I = 1, J )
WRITE (6,242) ( D(I), I = 1, J )
WRITE (6,256) ( SUMXM(I), I = 1, J )
WRITE (6,412) ( SUMSM(I), I = 1, J )
WRITE (6,422) ( WM(I), I = 1, J )
WRITE (6,432) ( WM(I), I = 1, J )

```


LIST OF REFERENCES

1. Barnett, V. "Convenient Probability Plotting Positions for the Normal Distribution." Appl. Statist. v. 25, No 1, p. 47-50, 1976.
2. Barnett, V. "Probability Plotting Methods and Order Statistics." Appl. Statist., v. 24, No 1, p. 95-107, 1975.
3. Beyer, W.H. CRC Handbook of Tables for Probability and Statistics, The Chemical Rubber Co., Ohio, 1966.
4. "Control Program-67/Cambridge Monitor System (CP-67/CMS) Users Guide", International Business Machines Manual GH20-0859-2, May, 1972.
5. David, H.A., Order Statistics, John Wiley and Sons, New York, 1970.
6. Dixon, M.W., A Statistical Analysis of Deviation from Target Cost in NAVAIRSCOMHQ Fixed-Price Incentive Contracts during the 1949-1965 Time Frame. M.S. Thesis, U.S. Naval Postgraduate School, Monterey, California, 1973.
7. Feller, W., An Introduction to Probability Theory and its Applications, v. II, 2nd ed., p. 19-21, John Wiley and Sons, New York, 1971.
8. Fishman, G.S. Concepts and Methods of Discrete Event Digital Simulation. John Wiley and Sons, New York, 1973.
9. Gaver, D.P. and Lewis, P.A.W., (forthcoming paper on

- further EARMA process results), 1978.
10. Gaver, D.P. and Thompson, G.L. Programming and Probability Models in Operations Research, Brooks-Cole, Monterey, California, p. 452-527, 1973.
 11. Hutchinson, D.R., "APL Histogram, Density Estimation and Probability Plotting Routines". M.S. Thesis, U.S. Naval Postgraduate School, Monterey, California, 1976.
 12. Harter, H.L. "Expected Values of Normal Order Statistics." Biometrika, v. 52, p. 151-157, 1961.
 13. Jacobs, P.A. and Lewis, P.A.W. "A Mixed Autoregressive-Moving Average Exponential Sequence and Point Process (EARMA1,1)." Advances in Applied Probability, v. 9, p. 87-104, 1977.
 14. Jathro, T.L., "An Investigation of the Properties of the Exponential Moving Average Point Process". M.S. Thesis U.S. Naval Postgraduate School, Monterey, California, March 1976.
 15. Lawrence, A.J. and Lewis, P.A.W., "An Exponential Moving Average Sequence and Point Process (EMA1)". Journal Applied Probability, v. 14, p. 98-113, 1977.
 16. Lawrence, A.J. and Lewis, P.A.W., (forthcoming paper on further EARMA process results), 1978.
 17. Lewis, P.A.W. and Cox D.R. "A Statistical Analysis of Telephone Circuit Error Data." IEEE Transactions on Communications Systems, Com-14, p. 382-389, 1966.
 18. Miller, G.R. "The Jackknife - A Review." Biometrika, v. 61, p. 1-16, 1974.
 19. Pearson, E.S. and Hartley, H.W. Biometrika Tables for Statisticians, v. I, Cambridge University Press, Cambridge, 1966.

20. Robinson, D.W., "Non-Parametric Quantile Estimation Through Stochastic Approximation". Ph. D. Thesis, U.S. Naval Postgraduate School, Monterey, California, June 1975.
21. Rosenblatt, M., "Curve Estimates." The Annals of Mathematical Statistics, v. 27, 1956.
22. Wilk, M.B. and Shapiro, S.S. "An Analysis of Variance Test for Normality (Complete Samples)." Biometrika, v. 52, 3 and 4, p. 591-610, 1965.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3. Department Chairman, Code 52 Department of Computer Science Naval Postgraduate School Monterey, California 93940	1
4. Department Chairman, Code 55 Department of Operation Research Naval Postgraduate School Monterey, California 93940	1
5. Professor P. A. W. Lewis Code, 55Lw Department of Operation Research Naval Postgraduate School Monterey, California 93940	3
6. Asscc. Professor G.G. Brown, code 52Bw Department of Computer Science Naval Postgraduate School Monterey, California 93940	1
7. Major Georgios I. Danikas, Hellenic Army Thrakis kai thermcylon 4 Agia Paraskevi, Athens, GREECE	3