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Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a mapping continuous together with derivatives up through some order r. A bifurcation surface for the equation (1) $M(\lambda, x) = 0$ is a surface in parameter space Λ for which the number of solutions x of (1) changes as λ crosses this surface. Under certain generic hypotheses on M, ~~the author and his colleagues have~~ it has shown that one can systematically determine the bifurcation surfaces by elementary

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20. Abstract

scaling techniques and the implicit function theorem. This ~~talk~~ document gives a summary of these results for the case of bifurcation near an isolated solution or families of solutions of the equation $M(\lambda_0,x) = 0$. The results have applications to the buckling theory of plates and shells under the effect of external forces, imperfections, curvature and variations in shape. The results on bifurcation near families has applications in nonlinear oscillations and the theory of homoclinic orbits.

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TOPICS IN LOCAL BIFURCATION THEORY+

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+ Talk presented at New York Academy of Sciences Conference on Bifurcation Theory and Applications October 31 - November 4, 1977.

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ACCOMPANYING STATEMENT

TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

This paper summarizes much of our work over the last few years on bifurcation in families of functions which contain several independent parameters. Applications arise in almost every discipline. We have emphasized problems of buckling in plates and shells and have discussed the effect of lateral forces, imperfections, curvature and variations in the shape. Applications have also been made to nonlinear oscillations in ordinary differential equations - the parameters being damping, amplitude and frequency of forcing.

TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

Abstract

Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a mapping continuous together with derivatives up through some order r. A bifurcation surface for the equation (1) $M(\lambda, x) = 0$ is a surface in parameter space Λ for which the number of solutions x of (1) changes as λ crosses this surface. Under certain generic hypotheses on M, the author and his colleagues have shown that one can systematically determine the bifurcation surfaces by elementary scaling techniques and the implicit function theorem. This talk gives a summary of these results for the case of bifurcation near an isolated solution or families of solutions of the equation $M(\lambda_0, x) = 0$. The results have applications to the buckling theory of plates and shells under the effect of external forces, imperfections, curvature and variations in shape. The results on bifurcation near families has applications in nonlinear oscillations and the theory of homoclinic orbits.

TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a smooth function of $(\lambda, x) \in \Lambda \times X$ and consider the equation

$$M(\lambda, x) = 0 \tag{1}$$

for $\lambda \in \Lambda$, $x \in X$. A pair (λ, x) satisfying (1) is called a solution, the set of solutions is denoted by S and

$$S_\lambda = \{x \in X: (\lambda, x) \in S\}$$

is the section of solution set at λ. The basic problem in bifurcation theory is to determine how the set S_λ varies with the parameter λ. Any point λ for which the structure of the set S_λ changes is called a <u>bifurcation point</u>.

If λ is a scalar parameter, very general results on the existence of bifurcation points can be obtained without imposing too many specific properties about the manner in which the function M depends on (λ, x). On the other hand, if λ is a vector parameter, one generally must assume more complete knowledge is available.

In the past few years, I and some of my colleagues have been attacking this latter problem under the following premises. Firstly, we assume the parameter is a vector parameter of dimension generally greater than one. On the other hand, we do not take the dimension too large because we wish to discuss the interaction of a few physical parameters at a time. It is well known that many parameters are needed to discuss a complicated bifurcation point. However, it is also known that some parameters have a more drastic effect on the qualitative nature of the bifurcation than others. It is, therefore, of interest to understand well the bifurcations in low dimensional parameter space Λ.

Secondly, we wish to devise methods which are applicable to equations which may not be the gradient of some function. Such methods will be applicable tc nonconservative physical systems.

Thirdly, we want the methods to be extremely elementary and require only calculus, the implicit function theorem and a small amount of geometric intuition.

The purpose of this talk was to survey some of our efforts in this direction. Three types of problems were discussed. First, suppose the equation

$$M(\lambda_0, x) = 0 \tag{2}$$

for a particular value of λ_0 has the isolated solution $x = 0$ and the linear operator $A = \partial M(\lambda_0,0)/\partial x$ does not have a bounded inverse. Assuming $\dim N(A)$ is one or two and assuming some generic conditions on the nonlinearities, one can give a complete description of the bifurcation set near the point $(\lambda_0,0) \in \Lambda \times X$. We give the theory and applications especially to the von Kármán equations in the papers [1,2,3]. External forces, imperfections, small curvatures and variations in shape are considered. The effect of symmetry is contained in [4,5]. Bifurcation of the nodal lines of a rectangular plate is contained in [6]. Paper [7] contains general lecture notes on bifurcation.

The second problem discussed concerns the case in which the equation (2) has a compact family of solutions. More specifically, suppose there is a C^2 function $p(t) = p(t+1)$, $t \in \mathbb{R}$, such that

$$M(\lambda_0,p(t)) = 0, \qquad t \in \mathbb{R}, \tag{3}$$

and, for each $t \in \mathbb{R}$, the operator $A(t) = \partial M(\lambda_0,p(t))/\partial x$ does not have a bounded inverse. One is interested in the bifurcation of solutions near the "circle" $\Gamma = \{p(t),\ 0 \le t \le 1\} \subset M$ for λ near λ_0. The complete structure of the bifurcation is given in [8,9] with applications to nonlinear oscillations under the assumption that $\dim N(A(t)) = 1$ for all t.

Implications in classical perturbation theory are also given. The case where $\dim N(A(t)) = 2$ is discussed in [10].

The third problem concerns bifurcation from a noncompact family of solutions of (3). This can arise in several different ways in the applications. If the function M has the form

$$M(\lambda,x) = Ax + N(\lambda,x)$$
$$N(0,x) = 0 \quad \text{for all} \quad x \in X;$$

this is the classical problem of a small perturbation of a linear operator. If $\dim N(A) \geq 1$, then for $\lambda = 0$ the equation (3) has a linear subspace of solutions; that is, a noncompact set of solutions. These problems are not well understood and are extremely difficult. In [11], we give a complete description of the bifurcation sets for the classical Duffing equation with or without damping with all parameters being treated as independent.

Another way in which a noncompact family can arise is when there is a family of solutions $p(t)$, $t \in \mathbb{R}$, of Equation (3) with the set $\Gamma = \{p(t),\ t \in \mathbb{R}\} \subset X$ bounded but not compact. For example, in a second order autonomous ordinary differential equation, Γ could be an orbit whose α- and ω-limit sets are the same critical point. When this system is subjected to a small periodic forcing, it has been known for a

long time that homoclinic points may occur near Γ. This problem is discussed in more detail in [12] when the equation is subjected to both damping and forcing which is not necessarily periodic.

REFERENCES

[1] Chow, S., Hale, J.K. and J. Mallet-Paret, Applications of generic bifurcation. I, Arch. Rat. Mech. Anal. 59(1975), 159-188; II, Ibid. 62(1976), 209-235.

[2] List, S., Generic bifurcation with applications to the von Kármán equations. Ph.D. Thesis, Brown University, June, 1976; Preprint, Univ. of Minnesota; to appear in Journal of Differential Equations.

[3] Mallet-Paret, J., Buckling of cylindrical plates with small curvature. Quart. Appl. Math. To appear.

[4] Vanderbauwhede, A.L., Generic and nongeneric bifurcation for the von Kármán equations. J. Math. Ana. Appl., to appear.

[5] Rodrigues, H.M. and Vanderbauwhede, A.L., Symmetric perturbations of nonlinear equations: symmetry of solutions. J.Nonlinear Analysis:Theory, Methods, Applications, to appear.

[6] Mallet-Paret, J., Bifurcation in nodal lines. Preprint.

[7] Hale, J.K., Generic bifurcation with applications. Heriot-Watt Lectures, Vol. 1, Pitman Publishers, 1977.

[8] Hale, J.K., Bifurcation near families of solutions. Proc. International Conf. on Differential Eqns. Uppsala (1977). Lecture Notes in Math. Springer. To appear.

[9] Hale, J.K. and Táboas, P.Z., Interaction of damping and forcing in a second order equation. J. Nonlinear Analysis: Theory, Methods, Applications. 2(1977).

[10] Hale, J.K. and Táboas, P.Z., Bifurcation near degenerate families. Submitted to SIAM J. Applied Math.

[11] Hale, J.K. and Rodrigues, H.M., Bifurcation in the Duffing equation with several parameters, I, Proc. Royal Soc. Edinburgh, Ser. A 78(1977); II, Ibid. 79(1977).

[12] Hale, J.K. and Mallet-Paret, J., Homoclinic orbits with general forcing and damping. To appear.