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# THE VALUE OF INFORMATION IN COMBAT DECISION MAKING

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## ABSTRACT

The Lanchester model is a widely used abstraction of the complexities of combat. Normally, the initial friendly and enemy strengths are assumed to be deterministic. However, in reality, there may be some uncertainty associated with both variables. This paper provides a methodology for evaluating the benefit of reducing this uncertainty by information collection. This technique is derived from concepts of evaluation of perfect information developed in Decision Analysis.

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The Lanchester model is a widely used abstraction of the complexities of combat. Normally, the initial friendly and enemy strengths are assumed to be deterministic. However, in reality, there may be some uncertainty associated with both variables. This paper provides a methodology for evaluating the benefit of reducing this uncertainty by information collection. This technique is derived from concepts of evaluation of perfect information developed in Decision Analysis.


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# THE VALUE OF INFORMATION IN COMBAT DECISION MAKING

## I. Introduction

The Army's Field Manual on Combat Intelligence [1] states, "Combat intelligence is derived from the interpretation of information on the enemy (both his capabilities and his vulnerabilities) and the environment. The objective of combat intelligence is to minimize uncertainty concerning the effects of these factors on the accomplishing of the mission". * The Manual also gives as a "basic" principle, "Intelligence must increase knowledge and understanding of the particular problem under consideration in order that logical decision can be reached". **

Commanders in the past have considered intelligence as a "free" good, that is, they have considered the benefit without consideration of the cost. Recently, officers at all levels are becoming aware of the cost of information collection, analysis, and dissemination. This is particularly evident in the strategic intelligence programs where millions of dollars are spent in the development and operation of satellite collection systems. However, even the acquisition of tactical intelligence incurs a cost - a cost that may not necessarily be monetary but may be some other resource such as time, equipment, or even human lives.

Of all the many management science disciplines existing today, Decision Analysis most explicitly treats the value of information. The philosophy is quite simple. The commander, based on his present state of information, can arrive at a decision which will optimize some desired objective function. Acquisition of additional information might lead to a change in this initial decision. Any such change must provide an increased value of the objective function. Using this increase, the expected value of the additional information can be weighed against the cost of acquisition to determine if collection would be warranted.

We intend to illustrate this concept by the use of the Lanchester equations of combat.

## II. Notation

The notation used in this paper is common to Decision Analysis, particularly writings by Howard [2,3]. We let

$\{Z\}$ be the density function on a random variable Z,

and $\{Z|W\}$ be a conditional density function.

A particularly important conditional probability is

$\{Z|\varepsilon\}$, the prior distribution or the probability assigned based on the current state of information.

Additionally, we let

$$\langle Z \rangle = \text{the expected value of } Z = \int_Z Z \{Z\}\, dZ$$

$$\langle Z \rangle^V = \text{the variance of } Z$$

and

$$\langle Z|W \rangle = \text{the conditional mean} = \int_Z Z \{Z|W\}\, dZ$$

## III. Lanchester Equations

Some understanding of Lanchester's modeling of ground combat is also necessary in the development of this paper.

Frederick Lanchester postulated that combat between two forces using aimed fire (such as tank duels) is captured by the simultaneous differential equations

$$\frac{dX(t)}{dt} = -a_1 Y(t)$$

$$\frac{dY(t)}{dt} = -a_2 X(t) \tag{1}$$

where X(t) is the size of the X force at time t, Y(t) is the size of the Y force at time t, $a_1$ is the effective casualty producing rate of each Y soldier using aimed fire, and $a_2$ is the effective casualty producing rate of each X soldier using aimed fire.

Lanchester further postulated that combat between two forces using area fire (such as an artillery duel) is captured by the simultaneous differential equations:

$$\frac{dX(t)}{dt} = -b_1 X(t)\ Y(t)$$

$$\frac{dY(t)}{dt} = -b_2 X(t)\ Y(t) \tag{2}$$

where $b_1$ is the effective casualty producing rate of each Y soldier using area fire and $b_2$ is the corresponding rate for each X soldier. The solution to equation set (1) is

$$\alpha\ (X_o^{\ 2} - X_f^{\ 2}) = (Y_o^{\ 2} - Y_f^{\ 2}) \tag{3}$$

where $\alpha = a_2/a_1$ , $X_o = X(t{=}0)$ or the initial size of the X force, and $X_f = X(t{=}t_f)$ the size of the X force at some time, $t_f > 0$. Equation (3) is Lanchester's Square Law.

Similarly, Equation set (2) reduces to

$$\beta \; (X_o - X_f) = (Y_o - Y_f) \qquad (4)$$

where $\beta = b_1/b_2$.

This is Lanchester's Linear Law.

The time, $t_f$, is frequently taken to be the termination of the battle. Battles are often assumed to be a fight to the finish, i.e., either $X_f$ or $Y_f$ is zero.

We may assure by correct choice of $X_o$, $\alpha$, and $\beta$ that the X force is always the winner, or $Y_f = 0$. Equations (3) and (4) reduce to

$$\alpha \, (X_o^{\,2} - X_f^{\,2}) = Y_o^{\,2}$$

$$X_f = (X_o^{\,2} - \frac{1}{\alpha} Y_o^{\,2})^{\frac{1}{2}} \qquad (5)$$

and

$$\beta \, (X_o - X_f) = Y_o$$

$$X_f = X_o - (1/\beta) \, Y_o \qquad (6)$$

In virtually every development of the Lanchester Equations $X_o$ and $Y_o$ are taken as deterministic. However, the friendly or X force commander will never precisely know the starting strength of the enemy (Y) force. At times, in the heat of battle, he may not even know the exact size of his own force. Therefore, we can assign probability distributions to $X_o$ and $Y_o$, viz, $\{X_o \mid \varepsilon\}$ and $\{Y_o \mid \varepsilon\}$. We also assume $X_o$ and $Y_o$ are independent random variables.

## IV. The Scenario

We assume a simple combat scenario. If the X force uses area fire, the Y force also uses area fire. (We can imagine the two forces withdraw beyond the range of small arms and other aimed fire weapons.) Similarly, if X uses aimed fire, then Y uses aimed fire. (We can imagine close combat.)

## V. Theory

### A. "No Information" Case

We now examine the commander's decision making process. He must choose whether to use aimed or area fire. A logical objective is to maximize the expected number of the surviving X force, $X_f$.

Given the use of aimed fire, the expected value of $X_f$ is

$$\langle X_f | d = \text{aim fire}, \varepsilon \rangle = \int_{X_o} \int_{Y_o} (X_o{}^2 - \frac{1}{\alpha} Y_o{}^2)^{\frac{1}{2}} \{Y_o | \varepsilon\} \{X_o | \varepsilon\} \, dY_o dX_o \qquad (7)$$

Similarly, if area fire is used, the expected value of $X_f$ is

$$\langle X_f | d = \text{area fire}, \varepsilon \rangle = \int_{X_o} \int_{Y_o} (X_o - \frac{1}{\beta} Y_o) \{X_o | \varepsilon\} \{Y_o | \varepsilon\} \, dY_o dX_o$$

$$= \langle X_o | \varepsilon \rangle - \frac{1}{\beta} \langle Y_o | \varepsilon \rangle \qquad (8)$$

Let d* be the optimal decision. Then d* = aimed fire if

$$\int_{X_o}\int_{Y_o}\left(X_o^{\,2}-\frac{1}{\alpha}Y_o^{\,2}\right)^{\frac{1}{2}}\{Y_o|\varepsilon\}\{X_o|\varepsilon\}\,dY_o dX_o \geq \langle X_o|\varepsilon\rangle-\frac{1}{\beta}\langle Y_o|\varepsilon\rangle \qquad (9)$$

and d* = area fire if inequality (9) is reversed. The value $\langle X_f \mid d = d^*, \varepsilon\rangle$ is the base case for calculation of the value of information.

B. Perfect Information on $X_o$ or $Y_o$.

The concept of perfect information is useful to establish an upper bound on the value of any information collection program as the value of actual information will always be less than the value of the perfect information.

Assume we know that $X_o$ was equal to a specific value, X.

The expected value for the area and aimed fire cases are

$$\langle X_f \mid d = \text{aimed fire},\ X_o = X,\ \varepsilon\rangle = \int_{Y_o}(X^{\,2}-\frac{1}{\alpha}Y_o^{\,2})^{\frac{1}{2}}\{Y_o|\varepsilon\}\,dY_o$$

$$\langle X_f \mid d = \text{area fire},\ X_o = X,\ \varepsilon\rangle = \int_{Y_o}(X-\frac{1}{\beta}Y_o)\,\{Y_o|\varepsilon\}\,dY_o \qquad (10)$$

$$= X - \frac{1}{\beta}\langle Y_o \mid \varepsilon\rangle$$

We can define a breakeven value of $X_o$, $X_b$, such that

$$\int_{Y_o}(X_b^2-\frac{1}{\alpha}Y_o^{\,2})^{\frac{1}{2}}\ \{Y_o|\varepsilon\}\ dY_o = X_b - \frac{1}{\beta}\langle Y_o|\varepsilon\rangle \qquad (11)$$

The range of $X_o$ is taken from $X_\ell$ to $X_u$. If $X_\ell < X_b < X_u$, then d* switches at $X_b$. For illustrative purposes, we assume d* = aimed fire for $X_\ell < X_o < X_b$ and d* = area fire for $X_b < X_o < X_u$.

The expected value of $X_f$ conditioned on receipt of perfect information on X is

$$\langle X_f | d=d^*,\ PI(X_o),\ \varepsilon \rangle = \int_{Y_o} \int_{X_\ell}^{X_b} \left( X_o^2 - \frac{1}{\alpha} Y_o^2 \right)^{\frac{1}{2}} \{X_o|\varepsilon\}\{Y_o|\varepsilon\}\ dX_o dY_o$$

$$+ \int_{Y_o} \int_{X_b}^{X_u} (X_o - \frac{1}{\beta} Y_o) \ \{X_o|\varepsilon\}\{Y_o|\varepsilon\}\ dX_o dY_o \qquad (12)$$

where $PI(X_o)$ is used to denote perfect information on $X_o$.

The expected value of perfect information on $X_o$, EVPI ($X_o$), is

$$EVPI\ (X_o) = \langle X_f | d = d^*,\ PI(X_o),\ \varepsilon \rangle - \langle X_f | d = d^*,\ \varepsilon \rangle \quad (13)$$

We can similarly define

$$EVPI\ (Y_o) = \langle X_f | d = d^*,\ PI(Y_o),\ \varepsilon \rangle - \langle X_f | d = d^*,\ \varepsilon \rangle \quad (14)$$

C. Perfect Information on Both $X_o$ and $Y_o$.

The expected value of perfect information on both $X_o$ and $Y_o$ does not necessarily equal the sum of the value of perfect information on each separate random variable.

We first establish

$$\langle X_f | d = \text{aimed fire},\ X_o = X,\ Y_o = Y,\ \varepsilon \rangle = (X^2 - \frac{1}{\alpha} Y^2)^{\frac{1}{2}} \qquad (15)$$

and

$$\langle X_f | d = \text{area fire},\ X_o = X,\ Y_o = Y,\ \varepsilon \rangle = X - \frac{1}{\beta} Y \qquad (16)$$

Equating (16) and (17) yields

$$X - \frac{1}{\beta} Y = (X^2 - \frac{1}{\alpha} Y^2)^{\frac{1}{2}} \tag{17}$$

Equation (17) implies

$$d^* = \text{aimed fire for } Y < \frac{2\alpha\beta}{\alpha + \beta^2} X$$

$$d^* = \text{area fire for } Y > \frac{2\alpha\beta}{\alpha + \beta^2} X \tag{18}$$

Let $\frac{2\alpha\beta}{\alpha + \beta^2} = k$

We may now calculate

$$\langle X_f | d = d^*, \text{PI } (X_o, Y_o), \ \varepsilon \rangle =$$

$$\int_{X_o} \int_{Y_\ell}^{kX_o} (X_o^{\ 2} - \frac{1}{\alpha} Y_o^{\ 2})^{\frac{1}{2}} \ \{X_o | \varepsilon\}\{Y_o | \varepsilon\} \ dY_o dX_o \tag{19}$$

$$+ \int_{X_o} \int_{kX_o}^{Y_u} (X_o - \frac{1}{\beta} Y_o) \ \{X_o | \varepsilon\}\{Y_o | \varepsilon\} \ dY_o dX_o$$

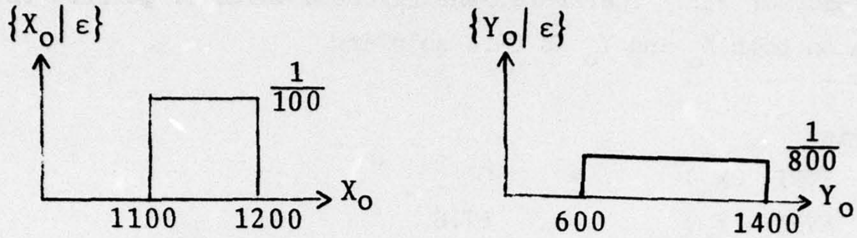
The expected value of perfect information on both variables is

$$\text{EVPI } (X_o, Y_o) = \langle X_f | d = d^*, \text{PI } (X_o, Y_o), \ \varepsilon \rangle - \langle XP | d = d^*, \varepsilon \rangle \tag{20}$$

We will illustrate this theory by consideration of a specific example.

## VI. AN EXAMPLE

Let $\{X_o \mid \varepsilon\}$ and $\{Y_o \mid \varepsilon\}$ be described as shown by the uniform distribution in Figure 1.




$\{X_o \mid \varepsilon\}$ and $\{Y_o \mid \varepsilon\}$

Figure 1.

Also assume $\alpha = 2/3$ (the enemy's aimed fire is more effective than the friendly force's), and $\beta = \frac{10}{9}$ (the friendly area fire is superior). Using equations (7) and (8) we may calculate

$$\langle X_f \mid d = \text{aimed fire}, \varepsilon \rangle = 261.6, \text{ and}$$

$$\langle X_f \mid d = \text{area fire}, \varepsilon \rangle = 250.0.$$

Thus, with only prior information the commander of the X force should choose aimed fire and should expect 261.6 men remaining following a fight to the finish with the Y force.

We now consider perfect information on $X_o$. Calculation of $X_b$, using equation (11), reveals that $X_b$ is greater than the upper limit on $X_o$. Thus PI $(X_o) = 0$.

Calculation of $Y_b$ using equation (11) reveals that $Y_b = 891$. The expected remaining friendly force, conditioned on receipt of perfect information on $Y_o$, is 329.4 soldiers. The value of perfect information of $Y_o$ is 67.8 soldiers.

This example illustrates that the value of simultaneous information on two variables does not necessarily equal the sum of the value each variable taken individually. Equation (18) indicates that k = .7792. This value in conjunction with equation (19) yields an expected remaining force of 331. Therefore, the expected value of perfect information on both $X_o$ and $Y_o$ is 69.4 soldiers.

To summarize:

$$EVPI\ (X_o) = 0$$

$$EVPI\ (Y_o) = 67.8$$

$$EVPI\ (X_o, Y_o) = 69.4 \neq EVPI\ (X_o) + EVPI\ (Y_o).$$

## VII. EXTENSIONS AND CONCLUSIONS

There are several promising extensions to this basic theory. These include:

a. Examination of the sensitivity of the results to force ratios $X_o/Y_o$, the variance of $\{X_o \mid \varepsilon\}$ and $\{Y_o \mid \varepsilon\}$, as well as changes in force effectiveness $\alpha$ and $\beta$.

b. Incorporation of Lanchester Theory that includes battles that end prior to the total destruction of the enemy force.

c. Implicit detailing of intelligence resource allocation based on this theory.

The theory and example of this paper are based on a simple combat situation. However, the philosophy and methodology are valid in more complex situations and lead to a more rational evaluation of the value of information - an evaluation that will become increasingly important on the battlefield of the future.

## ENDNOTES

*U.S. Department of the Army, Combat Intelligence, Field Manual 30-5 (Washington, D.C.: U.S. Government Printing Office, 1963), p. 2-1.

**Ibid., p. 2-13.

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