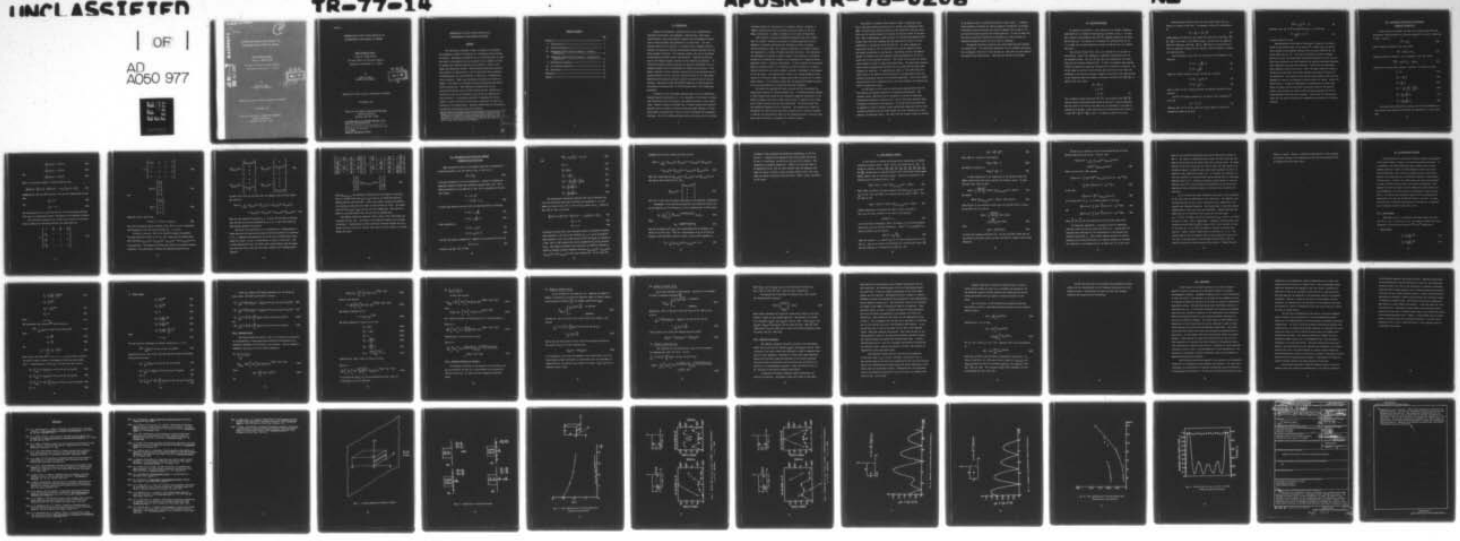
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DETERMINATION OF CAVITY FIELDS EXCITED BY AN ELECTROMAGNETIC PULSE THROUGH AN APERTURE

Final Technical Report

Grant No. AFOSR-76-3002

Air Force Office of Scientific Research

Bolling Air Force Base, D. C. 20332

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by

David K. Cheng
Principal Investigator

(Approved for public release; distribution unlimited)

31 December 1977

Electrical and Computer Engineering Department
Syracuse University
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DETERMINATION OF CAVITY FIELDS EXCITED BY AN ELECTROMAGNETIC PULSE THROUGH AN APERTURE

ABSTRACT

The state-space technique is used to formulate the transient electromagnetic problem of excitation of a rectangular cavity through an aperture. Depending on whether the E- or the H-formulation is used, the effect of the aperture can be accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order state equations by defining three new state variables in addition to an appropriate vector potential. These state equations are solved by the method of moments. Two cases are considered: parallel polarization and perpendicular polarization. Some significant singularities for the parallel-polarization case are found and the electric intensities as functions of time at two locations in the cavity are computed for a step excitation. An alternative approach of expanding the cavity fields in terms of TM and TE cavity modes is developed to reduce the dimension of the system matrix. Numerical difficulties are encountered and possible contributing causes are discussed.

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TABLE OF CONTENTS

	Page
Abstract	1
I. Introduction	3
II. Solution Procedure	7
III. Rectangular Cavity with Slot Aperture - Parallel Polarization	10
IV. Rectangular Cavity with Slot Aperture - Perpendicular Polarization	15
V. Some Numerical Results	19
VI. An Alternative Approach	24
VII. Conclusion	36
References	39
Figures	42

I. INTRODUCTION

Transient electromagnetic problems arise in the consideration of shielding effectiveness, electromagnetic compatibility, radar target identification, remote environmental sensing, and electromagnetic effects due to X rays and gamma rays generated by a nuclear blast. The time-harmonic behavior of an antenna or a scatterer can be obtained first as a function of frequency by solving the governing integral equations with the given boundary conditions. The transient response can then be determined by performing an inverse Fourier transformation. Numerical methods are used in the frequency-domain solution for many different frequencies. The brute-force numerical Fourier inversion is generally inefficient and convergence problems arise in superposing the steady-state solutions. A few simple situations, such as transient scattering from wire antennas and conducting cylinders [1]-[3], have been analyzed directly in the time domain. Basis-function expansions and inner products over both space and time are required. Space-time integro-differential equations are encountered and the numerical representations of the derivatives lead to very complicated procedures.

A variation of the time-domain approach makes use of a a Hallén-type integral-equation formulation [4]. As a consequence of the absence of space-time derivatives under the integrals, the numerical process is less complicated. However, because of retarded time, a tedious step-by-step iterative procedure has to be used in conjunction with the proper families of trajectories known as characteristic curves in order to determine the homogeneous solutions. All of the methods mentioned above are tedious and do not afford

a physical insight in the solution of a transient problem. Moreover, a complete recalculation would be necessary under any change in the wave shape, polarization, or angle of incidence of the source of excitation.

More recently a singularity-expansion method has been used to determine the transient response of scatterers of simple geometries [5]-[11]. Responses to transient excitations are expressed in terms of exterior natural frequencies, modes, and coupling coefficients, and induced currents are represented by a series of damped sinusoidal functions. This method has the advantages of providing a physical sight to the radiation or scattering problem and of allowing the response to be determined for a change in source parameters without a complete recalculation. In the evaluation of the natural-mode and coupling vectors, it is necessary to know the nature of the singularities involved in the inverse Laplace transform. Knowledge in this respect is not yet secure. The transcendental nature of its system impedance matrix results in an infinite number of complex poles whose locations must be numerically searched. It is generally agreed that the singularity-expansion method is not very satisfactory for evaluating early-time responses.

The method of characteristic modes has been used for determining the steady-state response of conducting bodies [12]. A striking similarity appears to exist between this method and the singularity-expansion formulation if the method of moments [13] using a common spatial basis is applied to both cases; but no formal relations have yet been established. The natural modes from the singularity expansion are not orthogonal. The characteristic modes are orthogonal, but they vary with the source frequency and it would be necessary to compute the characteristic modes for all frequencies before a Fourier inversion could be effected to determine the transient behavior.

The problem of transient field behavior inside a conducting cavity due to excitation through an aperture by an incident electromagnetic pulse (EMP) is particularly difficult because of the reflections from the cavity walls and the coupling between the interior and exterior fields at the aperture. Past investigations on EMP excitation of cavity-backed apertures have largely dealt with small openings and have neglected the effect of cavity reflections on the aperture field distribution. For small openings the quasi-static method is used to determine the fictitious magnetic current and charge distributions in the aperture. Equivalent electric and magnetic dipoles are defined, and their radiated fields determined with the aid of scalar and vector potentials [14]-[19]. The fields in the cavity are customarily expanded in terms of unperturbed normal modes. The quasi-static approximation cannot be applied when the aperture is not small and when early-time responses are important. In neglecting the effect of the reflections from cavity walls on the aperture field distribution, one essentially treats the external and internal portions of the problem separately. Since cavity dimensions obviously play an important part in the total problem, this approach may result in significant errors.

In this report we will avoid the quasi-static approximation and solve the internal and external portions of the problem simultaneously. New variables (state variables) will be introduced to convert the governing second-order differential equation into a set of first-order equations which correspond to normalized state equations. The field within the cavity will be expanded in terms of suitably chosen subsectional expansion functions with variable coefficients and the field outside the cavity expressed as a superposition of plane-wave fields. The cavity and the external fields are matched

at the aperture where a fictitious equivalent current exists. A combined field expression containing the unknown expansion coefficients is obtained. To determine these coefficients the method of moments [13] is used to convert the first-order equations into matrix equations. It will be shown that the typical coefficient matrix can be expressed in a form for which the singularity-expansion method [5] can be used to advantage.

The general procedure of solution for cavity-backed aperture problems is outlined first. The theoretical formulation for the transient excitation of a rectangular cavity with a slot aperture is then given for both parallel and perpendicular polarizations. Some numerical results are included.

II. SOLUTION PROCEDURE

We consider the problem of a slot aperture in an infinite conducting plane backed by a rectangular conducting box, as shown in Fig. 1. An incident transient electromagnetic wave ($\bar{E}^i$, $\bar{H}^i$) impinges normally on the plane and the aperture. The problem is to determine the scattered field in the $y > 0$ region and the field penetrated through the aperture into the conducting cavity.

The induction theorem [20], [21] can be invoked for the solution of this problem. Figure 2(a) represents a simplified 2-dimensional view of the original problem. ($\bar{E}_c$, $\bar{H}_c$) and ($\bar{E}_s$, $\bar{H}_s$) are, respectively, the cavity field and the external scattered field. In order to determine these unknown fields, we consider the case when the aperture is covered by a conductor. The entire region to the left of the infinite plane will have a null field and, according to the induction theorem, a magnetic current $\bar{M}_o$ on the right surface of the conducting plane will support a different scattered field ($\bar{E}_s^o$, $\bar{H}_s^o$), as shown in Fig. 2(b), where

$$\begin{aligned} \bar{M}_o &= \bar{E}_s^o \times \hat{n} \\ &= \hat{n} \times \bar{E}^i \\ &= \hat{y} \times \bar{E}^i \ . \end{aligned} \tag{1}$$

For a normally incident plane wave ($\bar{E}^i$, $\bar{H}^i$), the scattered field ($\bar{E}_s^o$, $\bar{H}_s^o$) from an infinite conducting plane without an aperture is easily determined. The null field to the left of the plane will be maintained if the plane is removed and a magnetic current $2\bar{M}_o$ exists in its place which will result in a field ($\bar{E}^i + \bar{E}_s^o$, $\bar{H}^i + \bar{H}_s^o$) in the $y > 0$ region, as shown in Fig. 2(c).

Subtracting the fields in Fig. 2(c) from those in Fig. 2(a), we obtain the problem in Fig. 2(d). The magnetic current $\bar{M}$ in the aperture is

$$\bar{M} = -2\bar{M}_o = -2\hat{y} \times \bar{E}^i , \tag{2}$$

which supports the field $(\bar{E}_c, \bar{H}_c)$ inside the cavity and a field $(\bar{E}_s - \bar{E}_s^o, \bar{H}_s - \bar{H}_s^o)$ to the right of the infinite plane. We note that the region in which the difference field $(\bar{E}_s - \bar{E}_s^o, \bar{H}_s - \bar{H}_s^o)$ exists is source-free and that the tangential component of the electric field is required to vanish on conducting walls.

For the problem in Fig. 2(d), we start from the two Maxwell's curl equations

$$\bar{\nabla} \times \bar{E} = -\mu_o \frac{\partial \bar{H}}{\partial t} - \bar{M} \tag{3}$$

$$\bar{\nabla} \times \bar{H} = \varepsilon_o \frac{\partial \bar{E}}{\partial t} . \tag{4}$$

Taking the Laplace transform of Eqs. (3) and (4), we obtain

$$\bar{\nabla} \times \tilde{\bar{E}} = -\mu_o s \tilde{\bar{H}} - \tilde{\bar{M}} \tag{5}$$

$$\bar{\nabla} \times \tilde{\bar{H}} = \varepsilon_o s \tilde{\bar{E}} , \tag{6}$$

where a tilde (~) over a quantity denotes the Laplace transform of that quantity.

Let $\tilde{\bar{F}}$ be the Laplace transform of an electric vector potential $\bar{F}$ such that

$$\tilde{\bar{E}} = -\bar{\nabla} \times \tilde{\bar{F}} . \tag{7}$$

Combining Eqs. (5) to (7) and using the Lorentz gauge, we have an inhomogeneous Helmholtz equation:

$$\bar{\nabla}^2 \tilde{\bar{F}} = \mu_o \varepsilon_o s^2 \tilde{\bar{F}} = - \tilde{\bar{M}} . \tag{8}$$

Solution of Eq. (8) for $\tilde{\bar{F}}$ will give $\tilde{\bar{E}}$ from Eq. (7) and $\tilde{\bar{H}}$ from

$$\tilde{\bar{H}} = \frac{1}{\mu_o s} \bar{\nabla} \times \bar{\nabla} \times \tilde{\bar{F}} \tag{9}$$

in regions where $\bar{M}$ is zero.

Specialization of these general formulas will depend on the polarization of the incident wave; but as soon as $\bar{E}^i$ is known, Eqs. (7) and (9) can be expanded into component equations and the source term $\tilde{\bar{M}}$ in Eq. (8) can be found from Eq. (2). A set of new variables can then be defined which will convert the second-order differential equation (8) into a set of first-order equations, and these equations are Laplace-transformed normalized state equations in the new state variables. Solution of the transformed state equations involves four steps. First, the space inside the cavity is divided into subsections and suitable expansion functions are chosen over the subsections. The elements of the unknown state-variable vector are then expressed in terms of the expansion functions within the cavity. Second, the field in the $y > 0$ region is expressed as a superposition of plane waves. Third, the cavity and the half-space fields are matched at the aperture. Fourth, inner products are taken so that the matrix equations for the unknown expansion coefficients are obtained. These steps are outlined separately for the cases of parallel and perpendicular excitations in following sections.

III. RECTANGULAR CAVITY WITH SLOT APERTURE - PARALLEL POLARIZATION

In this section we consider the case of an incident plane wave with the electric field polarized in a direction parallel to the slot. Referring to Fig. 1, we have

$$\bar{E}^i = \hat{z}\, E_z^i \tag{10}$$

and the Laplace transform of Eq. (2) becomes

$$\bar{\tilde{M}} = -2\hat{x}\, \tilde{E}_z^i = \hat{x}\, \tilde{M}_x \tag{11}$$

which has only an x-component. The x-component of Eq. (8) is then

$$\bar{\nabla}^2 \tilde{F}_x - \mu_o \varepsilon_o s^2\, \tilde{F}_x = -\tilde{M}_x \delta(y), \tag{12}$$

where $\delta(y)$ is a Dirac delta function. From Eqs. (7) and (9), we have

$$\tilde{E}_x = 0 \tag{13}$$

$$\tilde{E}_y = -\frac{\partial}{\partial z}\tilde{F}_x \tag{14}$$

$$\tilde{E}_z = \frac{\partial}{\partial y}\tilde{F}_x \tag{15}$$

$$\tilde{H}_x = -\frac{1}{\mu_o s}\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\tilde{F}_x \tag{16}$$

$$\tilde{H}_y = \frac{1}{\mu_o s}\frac{\partial^2}{\partial x \partial y}\tilde{F}_x \tag{17}$$

$$\tilde{H}_z = \frac{1}{\mu_o s}\frac{\partial^2}{\partial x \partial z}\tilde{F}_x . \tag{18}$$

The second-order differential equation (12) can be represented as a set of first-order equations by defining new quantities u, v, and w such that

$$-\frac{\partial}{\partial x}\tilde{F}_x(r,s) = s\,\tilde{u}(r,s) \tag{19}$$

$$-\frac{\partial}{\partial y}\tilde{F}_x(r,s) = s\,\tilde{v}(r,s) \tag{20}$$

and

$$-\frac{\partial}{\partial z}\tilde{F}_x(r,s) = s\,\tilde{w}(r,s)\ , \tag{21}$$

where r is the space variable. We have, from Eq. (12),

$$\frac{\partial}{\partial x}\tilde{u}(r,s) + \frac{\partial}{\partial y}\tilde{v}(r,s) + \frac{\partial}{\partial z}\tilde{w}(r,s) = -\mu_o\varepsilon_o s\,\tilde{F}_x(r,s) + \frac{1}{s}\tilde{M}_x\ . \tag{22}$$

Comparing Eqs. (21) and (20) with Eqs. (14) and (13) respectively, we see that

$$\tilde{E}_y = s\,\tilde{w} \tag{23}$$

and

$$\tilde{E}_z = -s\,\tilde{v}\ . \tag{24}$$

The introduction of $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ and the use of the first-order equations will result in significantly faster convergence in the numerical solution.

The first-order equations (19) to (22) can be written in a succinct form by defining the following operators and column matrices:

$$L = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{\partial}{\partial x} & 0 & 0 & 0 \\ -\frac{\partial}{\partial y} & 0 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

$$P = \begin{bmatrix} -\mu_o\varepsilon_o & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$\tilde{f}(r,s) = \begin{bmatrix} \tilde{F}_x \\ \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} \tag{27}$$

and

$$\tilde{e}_g(s) = \begin{bmatrix} \frac{1}{s}\tilde{M}_x \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{28}$$

Equations (19) to (22) become

$$L\,\tilde{f}(r,s) = s\,P\,\tilde{f}(r,s) + \tilde{e}_g(r,s)\ . \tag{29}$$

Note that the inverse Laplace transform of Eq. (29) is a set of normalized state equations in the four state variables F_x, u, v, and w.

In order to solve Eq. (29) by the method of moments we subdivide the space within the cavity in the x,y, and z directions and choose expansion functions $F_{x(i,j,k)}(r)$, $u_{(i,j,k)}(r)$, $v_{(i,j,k)}(r)$, and $w_{(i,j,k)}(r)$ over the subsections. The expansion functions must satisfy the required boundary conditions. For convenience, we define the following column vectors:

$$f^{F}_{(i,j,k)}(r) = \begin{bmatrix} F_{x(i,j,k)}(r) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad f^{u}_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ u_{(i,j,k)}(r) \\ 0 \\ 0 \end{bmatrix} \tag{30}$$

$$f^{v}_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ v_{(i,j,k)}(r) \\ 0 \end{bmatrix} \quad f^{w}_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_{(i,j,k)}(r) \end{bmatrix}.$$

In view of Eq. (27), we can then write the expanded form of $\tilde{f}(r,s)$ inside the cavity as

$$\tilde{f}(r,s) = \sum_{i,j,k} \{\tilde{\alpha}_{(i,j,k)}(s)\, f^{F}_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\, f^{u}_{(i,j,k)}(r) + \tilde{\gamma}_{(i,j,k)}(s)\, f^{v}_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\, f^{w}_{(i,j,k)}(r)\}. \tag{31}$$

Note that the expansion functions F_x, u, v, and w are functions of position only and that the inverse transformation of $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ will yield the time-varying expansion coefficients.

The field in the half-space $y \geq 0$ is expressed as a superposition of plane waves and the internal and external fields are matched at the aperture $y = 0$. A combined field expression for $\tilde{f}(r,s)$ can be obtained which holds inside the cavity, in the $y > 0$ half-space, as well as in the slot. This is then substituted in Eq. (31) which, after inner products with the expansion functions in Eq. (30) have been taken, leads to the following matrix equation:

$$\begin{bmatrix} 0 & \ell_{mn}^{Fu} & \ell_{mn}^{Fv} & \ell_{mn}^{Fw} \\ \ell_{mn}^{uF} & 0 & \ell_{mn}^{uv} & 0 \\ \ell_{mn}^{vF} & \ell_{mn}^{vu} & \ell_{mn}^{vv} & \ell_{mn}^{vw} \\ \ell_{mn}^{wf} & 0 & \ell_{mn}^{wv} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \\ \tilde{\delta} \end{bmatrix} = s \begin{bmatrix} p_{mn}^{FF} & 0 & p_{mn}^{Fv} & 0 \\ 0 & p_{mn}^{uu} & p_{mn}^{uv} & 0 \\ p_{mn}^{vF} & p_{mn}^{vu} & p_{mn}^{vv} & p_{mn}^{vw} \\ 0 & 0 & p_{mn}^{wv} & p_{mn}^{ww} \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \\ \tilde{\delta} \end{bmatrix}$$

$$+ \begin{bmatrix} q^F \\ q^u \\ q^v \\ q^w \end{bmatrix} \{\frac{1}{s} \tilde{m}_{x(i',n_y,k')}(s)\} + \begin{bmatrix} 0 \\ 0 \\ \tilde{C}(s) \\ 0 \end{bmatrix} \qquad (32)$$

where $\tilde{m}_{x(i',n_y,k')}$ are expansion coefficients for the magnetic current $\tilde{M}_x$, $\tilde{C}(s)$ is a column matrix and ℓ_{mn}'s, p_{mn}'s and q's are themselves matrices arising from the inner products. m and n are indices locating the position of a particular subsection over which an inner product is taken. The expressions for $\tilde{C}(s)$, ℓ_{mn}'s, p_{mn}'s and q's are very complicated. They have been given in a previous report [22], and will not be repeated here.

The unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$ and $\{\tilde{\delta}(s)\}$ can be solved from Eq. (32). An outline of the method of solution will be given in Section V. Determination of these coefficient matrices enables the calculation of $\tilde{f}(r,s)$ in Eq. (27) from Eq. (31), which, in turn, leads to the field inside the cavity.

IV. RECTANGULAR CAVITY WITH SLOT APERTURE

- PERPENDICULAR POLARIZATION

When the electric field of the incident plane wave is polarized in a direction normal to the slot shown in Fig. 1; that is, if

$$\bar{H}^i = \hat{z}\, H_z^i \,, \tag{33}$$

it is more convenient to use an H-formulation. Instead of invoking the induction theorem, we apply the equivalence theorem [20], [21]. The H-field discontinuity at the aperture in Fig. 2(d) is supported by an electric current

$$\begin{aligned} \bar{J} &= -2\hat{n} \times \bar{H}^i \\ &= -2\hat{x}\, H_z^i = \hat{x}\, J_x \,. \end{aligned} \tag{34}$$

To solve this problem, we start with the following Maxwell's equations

$$\bar{\nabla} \times \bar{E} = -\mu_o \frac{\partial \bar{H}}{\partial t} \tag{35}$$

$$\bar{\nabla} \times \bar{H} = \varepsilon_o \frac{\partial \bar{E}}{\partial t} + \bar{J} \,, \tag{36}$$

which transform to

$$\bar{\nabla} \times \tilde{\bar{E}} = -\mu_o s\, \tilde{\bar{H}} \tag{37}$$

$$\bar{\nabla} \times \tilde{\bar{H}} = \varepsilon_o s\, \tilde{\bar{E}} + \tilde{\bar{J}} \,. \tag{38}$$

Let $\tilde{\bar{A}}$ be the Laplace transform of a magnetic vector potential $\bar{A}$ such that

$$\tilde{\bar{H}} = \bar{\nabla} \times \tilde{\bar{A}} \,. \tag{39}$$

We have, from Eqs. (37) to (39),

$$\tilde{\nabla}^2\tilde{A}_x - \mu_o\varepsilon_o s^2\tilde{A}_x = -\tilde{J}_x\,\delta(y) \tag{40}$$

and

$$\tilde{H}_x = 0 \tag{41}$$

$$\tilde{H}_y = \frac{\partial}{\partial z}\tilde{A}_x \tag{42}$$

$$\tilde{H}_z = -\frac{\partial}{\partial y}\tilde{A}_x \tag{43}$$

$$\tilde{E}_x = -\frac{1}{\varepsilon_o s}\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\tilde{A}_x \tag{44}$$

$$\tilde{E}_y = \frac{1}{\varepsilon_o s}\frac{\partial^2}{\partial x \partial y}\tilde{A}_x \tag{45}$$

$$\tilde{E}_z = \frac{1}{\varepsilon_o s}\frac{\partial^2}{\partial x \partial z}\tilde{A}_x \,. \tag{46}$$

The second-order differential equation (40) can be converted into a set of first-order equations by defining new quantities $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ similar to those in Eqs. (19) to (21) with $\tilde{F}_x$ replaced by $\tilde{A}_x$. Instead of Eqs. (22) to (24), we now have

$$\frac{\partial}{\partial x}\tilde{u}(r,s) + \frac{\partial}{\partial y}\tilde{v}(r,s) + \frac{\partial}{\partial z}\tilde{w}(r,s) = -\mu_o\varepsilon_o s\tilde{A}_x(r,s) + \frac{1}{s}\tilde{J}_x \tag{47}$$

$$\tilde{H}_y = -s\,\tilde{w} \tag{48}$$

$$\tilde{H}_z = s\,\tilde{v} \,. \tag{49}$$

An operator equation (29) which represents Laplace transformed normalized state equations in the four state variables A_x, u, v, and w is obtained when operators L and P and column matrices $\tilde{f}(r,s)$ and $\tilde{e}_g(s)$ are defined as in Eqs. (25) to (28) respectively with $\tilde{F}_x$ replaced by $\tilde{A}_x$ and $\tilde{M}_x$ replaced by $\tilde{J}_x$. This operator equation can be solved by the method of moments as before by choosing suitable expansion functions $A_{x(i,j,k)}(r)$, $u_{(i,j,k)}(r)$, $v_{(i,j,k)}(r)$, and $w_{(i,j,k)}(r)$ over cavity subsections. We can write the

expanded form of $\tilde{f}(r,s)$ inside the cavity as [22]

$$\tilde{f}(r,s) = \sum_{i,j,k} \{\tilde{\alpha}_{(i,j,k)}(s)\ f^{A}_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\ f^{u}_{(i,j,k)}(r)$$

$$+ \tilde{\gamma}_{(i,j,k)}(s)\ f^{v}_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\ f^{w}_{(i,j,k)}(r)\}, \qquad (50)$$

where the column matrices $f^{u}_{(i,j,k)}(r)$, $f^{v}_{(i,j,k)}(r)$, and $f^{w}_{(i,j,k)}(r)$ are the same as those defined in Eq. (30),

$$f^{A}_{(i,j,k)}(r) = \begin{bmatrix} A_{x(i,j,k)}(r) \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (51)$$

and $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ are the Laplace transforms of the expansion coefficients.

Analogously to the parallel-polarization case treated in Section III, we can expand $\tilde{H}_z$ in the half-space $y \geq 0$ as a superposition of plane waves:

$$\tilde{H}_z = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}^{H}(k_x,k_z) e^{-j(k_x x + k_y y + k_z z)}\, dk_x\, dk_z\ , \qquad (52)$$

where

$$jk_y = \sqrt{(s/c)^2 + k_x^2 + k_z^2} \qquad (53)$$

and the new quantity $g^{H}(,_x,k_z)$ can be determined from the boundary condition at the slot [22]. There is a discontinuity in H_z at the slot on account of the equivalent electric current J_x which can be expanded as

$$\tilde{J}_x = \sum_{i,k} \tilde{j}_{x(i,n_y,k)}(s)\ v_{(i,n_y,k)}(x,\ 0,\ z) \qquad (54)$$

A system of three equations are obtained by matching $\tilde{A}_x$, $\tilde{u}$, and $\tilde{v}$ at the slot. A combined field expression which holds inside the cavity, in the $y > 0$ half-space, as well as in the slot can be written. This expression is extremely complicated. Suffice it to say that when it is substituted in Eq. (50) and inner products with the expansion functions are taken, we obtain a matrix equation similar to Eq. (32), from which the unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ can be solved.

V. SOME NUMERICAL RESULTS

In this section we outline the procedure for determining the unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ from Eq. (32). Although the component matrices (ℓ_{mn}^{uv}, ℓ_{mn}^{vu}, ℓ_{mn}^{vv}, ℓ_{mn}^{vw}, p_{mn}^{Fv}, p_{mn}^{uv}, p_{mn}^{vF}, p_{mn}^{vu}, p_{mn}^{vw}, and p_{mn}^{wv}) representing the coupling between cavity and external fields appear highly complex, they are relatively sparse. Typically an equation of the following form is obtained from Eq. (32):

$$[\tilde{Z}(s)]\ \{\tilde{\alpha}(s)\} = [H(s)]\ \{\frac{1}{s}\,\tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\}\ , \tag{55}$$

where $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ are square matrices containing ℓ_{mn}'s, p_{mn}'s, and q_{mn}'s in Eq. (32), and $\{\tilde{K}(s)\}$ is in general not the same as $\{\tilde{C}(s)\}$. From Eq. (55) we have

$$\{\tilde{\alpha}(s)\} = [\tilde{Z}(s)]^{-1}\ ([\tilde{H}(s)]\ \{\frac{1}{s}\,\tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\})\ . \tag{56}$$

Similar expressions are obtained for $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$.

Let s_α be the zeros of $|\tilde{Z}(s)|$ or the roots of the equation

$$\det[\tilde{Z}(s)] = 0\ . \tag{57}$$

In circuit-theory terminology, $[\tilde{Z}(s)]$ corresponds to the system impedance matrix and s_α are the natural frequencies. $[\tilde{Z}(s)]^{-1}$ can be expanded in a partial-fraction form as follows:

$$[\tilde{Z}(s)]^{-1} = \sum_\alpha \frac{[R_\alpha]}{s - s_\alpha}\ , \tag{58}$$

where the constant square matrix $[R_\alpha]$ is the system residue matrix at the pole s_α. $[R_\alpha]$ can be written as the product of a natural mode vector $\{R_\alpha^m\}$ and the transpose of a coupling vector $\{R_\alpha^c\}$ [5], [9]:

$$[R_\alpha] = \{R_\alpha^m\}\ \{R_\alpha^c\}^T\ , \tag{59}$$

where $\{R_\alpha^m\}$ is a solution of the equation

$$[Z(s_\alpha)]\ \{R_\alpha^m\} = 0 \tag{60}$$

and $\{R_\alpha^c\}$ is a solution of

$$[Z(s_\alpha)]^T\ \{R_\alpha^c\} = 0\ . \tag{61}$$

A close examination of the composition of the matrices $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ reveals that their poles coincide and therefore cancel. We have, from Eqs. (56), (58) and (59),

$$\{\tilde{a}(s)\} = \sum_\alpha \frac{\{R_\alpha^m\}\{R_\alpha^c\}^T}{s - s_\alpha}\ ([\tilde{H}(s)]\ \{\frac{1}{s}\ \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\})\ . \tag{62}$$

Now define

$$[\tilde{H}(s)]\ \{\frac{1}{s}\ \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\} = \tilde{N}(s)\ \{\tilde{V}_o(s)\}\ , \tag{63}$$

where $\{\tilde{V}_o(s)\}$ is the excitation vector when the incident wave is a pulse. We can then write Eq. (62) as

$$\begin{aligned}\{\tilde{a}(s)\} &= \sum_\alpha \frac{\{R_\alpha^m\}\{R_\alpha^c\}^T}{s - s_\alpha}\ \tilde{N}(s)\ \{\tilde{V}_o(s)\} \\ &= \sum_\alpha \frac{\{R_\alpha^m\}}{s - s_\alpha}\ \tilde{\eta}_\alpha(s)\ \tilde{N}(s)\ ,\end{aligned} \tag{64}$$

where

$$\tilde{\eta}_\alpha(s) = \{R_\alpha^c\}^T\{\tilde{V}_o(s)\} \tag{65}$$

is called the coupling coefficient [5]. We note that $\tilde{N}(s)$ itself may contain poles in the finite plane, but this fact does not result in any serious difficulty.

We are now in a position to write the expressions for the field distributions within the cavity. From Eq. (31),

$$\tilde{F}_x(x,y,z,s) = \sum_{i,j,k} \tilde{a}_{(i,j,k)}(s)\, F_{x(i,j,k)}(x,y,z)$$

$$= \{\tilde{a}(s)\}^T \{F_{x(i,j,k)}(x,y,z)\} \tag{66}$$

which, in view of Eq. (64), becomes

$$\tilde{F}_x(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\, \{R_\alpha^m\}^T \{F_{x(i,j,k)}(x,y,z)\}\, (s - s_\alpha)^{-1}\, \tilde{N}(s)$$

$$= \sum_\alpha \tilde{\eta}_\alpha(s)\, \nu_\alpha^F(x,y,z)\, (s - s_\alpha)^{-1}\, \tilde{N}(s)\ . \tag{67}$$

In Eq. (67),

$$\nu_\alpha^F(x,y,z) = \{R_\alpha^m\}^T \{F_{x(i,j,k)}(x,y,z)\} \tag{68}$$

is a natural mode for F_x. In a similar manner, we will get

$$\tilde{E}_y(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\, \nu_\alpha^{E_y}(x,y,z)\, (s - s_\alpha)^{-1}\, \tilde{N}(s) \tag{69}$$

and

$$\tilde{E}_z(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\, \nu_\alpha^{E_z}(x,y,z)\, (s - s_\alpha)^{-1}\, \tilde{N}(s), \tag{70}$$

where $\nu_\alpha^{E_y}$ and $\nu_\alpha^{E_z}$ are the natural modes for $\tilde{E}_y$ and $\tilde{E}_z$ respectively.

For numerical computation, a rectangular cavity of dimensions 4×6×2 was chosen and the slot width was a/10 or 0.4. Considerable difficulties were experienced in the determination of the singularities (natural frequencies) s_α. After having compared several root-finding procedures and checked meticulously our computer programs, we obtained the locations of the singularities in the upper part of the left half-

plane for the parallel-polarization case with E-formulation as shown in Fig. 3. Two layers of singularities were found: the first layer was very close to the jω-axis and the second layer had a positive slope. The singularities in the first layer fall on a smooth locus, and they appear to be irregularly spaced. No method exists which could ascertain the effect of numerical noise, subdivision scheme, etc. It is known that numerical results are sensitive to round-off-errors and may diverge with an increasing sampling density [23]. Problems of numerical anomaly appear in even the simplest cases [23], and precluded the determination of the natural frequencies of some well-defined geometric structures such as oblate spheroids [24].

In our ccalculations eight subsections were used in the y-direction of the cavity and six subsections in the x-direction. The expansion (and testing) functions in the y- and x- directions were, respectively, rectangular and triangular functions. The natural modes of the first several singularities of the first layer were determined. Typical amplitude and phase plots at two locations in the cavity for the first pole, $s_{11} = (-0.110 + j1.258)c$, and for the third pole, $s_{13} = (-0.141 + j2.296)c$, are given in Figs. 4 and 5 respectively. Assuming a step excitation, the normalized cavity field $|E_z|/E_z^i$ is shown in Fig. 6 as a function of time at the point (2, -5, 1), which is (5/6)th of the way in from the slot aperture. Figure 7 shows a similar plot at the point (2, -3, 1). The figures indicate that the maximum value of $|E_z|$ inside the cavity is in the order of one-thousandth of the incident field intensity, E_z^i, for parallel polarization (good shielding effectiveness). Maxima and nulls

exist as t varies. However, a physical interpretation of their spacings is difficult because of the complicated nature that the penetrated field is reflected from the cavity walls.

VI. AN ALTERNATIVE APPROACH

The formulations in the previous sections required the expansion of the cavity fields in terms of the selected expansion functions. As a consequence, the three-dimensional moment method leads to system impedance matrices, $[\tilde{Z}(s)]$, of large dimensions which often tend to be ill-conditioned. With a view to avoiding numerical difficulties inherent with large matrices, an alternative approach is investigated in this section. Briefly, the fields inside the cavity are expressed in terms of TM and TE cavity modes with unknown coefficients. This approach effectively changes the three-dimensional moment method into a two-dimensional one, thus reducing the dimensions of the resulting matrix. We develop this alternative approach for the perpendicular-polarization case with the incident $\bar{E}^i$ normal to the slot. As shown in Section IV, the $\bar{H}$-field discontinuity at the aperture is supported by an equivalent current $\bar{J}$ given in Eq. (34).

VI-1. Cavity Modes

Referring to Fig. 1, we separate the fields inside the cavity into TM to z ($H_z = 0$) and TE to z ($E_z = 0$) modes and express the field components in terms of wave functions ϕ^{TM} and ϕ^{TE} respectively.

I. TM to z mode

$$\tilde{E}_x = \frac{1}{\tilde{y}} \frac{\partial^2}{\partial x \partial z} \tilde{\phi}^{TM} \tag{71}$$

$$\tilde{E}_y = \frac{1}{\tilde{y}} \frac{\partial^2}{\partial y \partial z} \tilde{\phi}^{TM} \tag{72}$$

$$\tilde{E}_z = \frac{1}{\tilde{y}} \left(\frac{\partial^2}{\partial z^2} - \frac{s^2}{c^2}\right) \tilde{\phi}^{TM} \tag{73}$$

$$\tilde{H}_x = \frac{\partial}{\partial y} \tilde{\phi}^{TM}$$

$$\tilde{H}_y = - \frac{\partial}{\partial x} \tilde{\phi}^{TM} \tag{75}$$

$$\tilde{H}_z = 0 \tag{76}$$

where

$$\tilde{y} = s\varepsilon_o \tag{77}$$

The transformed wave function $\tilde{\phi}^{TM}$ can be written as

$$\tilde{\phi}^{TM} = \sum_{m,n} A_{mn}(s) \sin k_x x \cos k_z z \sin k_y (y+b) \tag{78}$$

where

$$k_x = \frac{m\pi}{a} \tag{79}$$

$$k_z = \frac{n\pi}{c} \tag{80}$$

and

$$k_y = \left(- \frac{s^2}{c_o^2} - k_x^2 - k_z^2\right)^{1/2} \tag{81}$$

Note from Eq. (78) that $\tilde{\phi}^{TM} = 0$ at $y = - b$. c_o in Eq. (81) is used for the speed of light in order to distinguish it from the cavity dimension c. Substituting Eq. (78) in Eqs. (71)-(73) and (76), we have

$$\tilde{E}_x = \frac{1}{\tilde{y}} \sum_{m,n} (-1) k_x k_z \tilde{A}_{mn}(s) \cos k_x x \sin k_z z \sin k_y (y+b) \tag{82}$$

$$\tilde{E}_y = \frac{1}{\tilde{y}} \sum_{m,n} (-1) k_y k_z \tilde{A}_{mn}(s) \sin k_x x \sin k_z z \cos k_y (y+b) \tag{83}$$

$$\tilde{E}_z = \frac{1}{\tilde{y}} \sum_{m,n} (-1) \left(k_z^2 + \frac{s^2}{c_o^2}\right) \tilde{A}_{mn}(s) \sin k_x x \cos k_z z \sin k_y (y+b) \tag{84}$$

$$\tilde{H}_z = 0 \tag{85}$$

II. TE to z mode

$$\tilde{E}_x = -\frac{\partial}{\partial y}\tilde{\phi}^{TE} \tag{86}$$

$$\tilde{E}_y = \frac{\partial}{\partial x}\tilde{\phi}^{TE} \tag{87}$$

$$\tilde{E}_z = 0 \tag{88}$$

$$\tilde{H}_x = \frac{1}{\tilde{z}}\frac{\partial^2}{\partial x \partial z}\tilde{\phi}^{TE} \tag{89}$$

$$\tilde{H}_y = \frac{1}{\tilde{z}}\frac{\partial^2}{\partial y \partial z}\tilde{\phi}^{TE} \tag{90}$$

$$\tilde{H}_z = \frac{1}{\tilde{z}}\left(\frac{\partial^2}{\partial z^2} - \frac{s^2}{c_o^2}\right)\tilde{\phi}^{TE} \tag{91}$$

where

$$\tilde{z} = s\varepsilon_o \tag{92}$$

The wave function satisfying the boundary condition at $y = -b$ is

$$\tilde{\phi}^{TE} = \sum_{m,n} \tilde{B}_{mn}(s) \cos k_x x \sin k_z z \cos k_y(y+b) \tag{93}$$

Substitution of Eq. (93) in Eqs. (86)-(88) and (91) yields the $\tilde{E}$-fields and $\tilde{H}_z$ for the TE mode.

$$\tilde{E}_x = \sum_{m,n} k_y \tilde{B}_{mn}(s) \cos k_x x \sin k_z z \sin k_y(y+b) \tag{94}$$

$$\tilde{E}_y = \sum_{m,n} (-1)\, k_x \tilde{B}_{mn}(s) \sin k_x x \sin k_z z \cos k_y(y+b) \tag{95}$$

$$\tilde{E}_z = 0 \tag{96}$$

$$\tilde{H}_z = \frac{1}{\tilde{z}} \sum_{m,n} (-1)\left(k_z^2 + \frac{s^2}{c_o^2}\right) \tilde{B}_{mn}(s) \cos k_x x \sin k_z z \cos k_y(y+b) \tag{97}$$

Finally we combine the $\tilde{\bar{E}}$ and $\tilde{H}_z$ expressions for the TM and TE modes in Eqs. (82)-(85) and (94)-(97) to obtain

$$\tilde{E}_x = \sum_{m,n} (-1)\left[\frac{1}{\tilde{y}} k_x k_z \tilde{A}_{mn}(s) - k_y \tilde{B}_{mn}(s)\right] \cos k_x x \sin k_z z \sin k_y(y+b) \quad (98)$$

$$\tilde{E}_y = \sum_{m,n} (-1)\left[\frac{1}{\tilde{y}} k_y k_z \tilde{A}_{mn}(s) + k_x \tilde{B}_{mn}(s)\right] \sin k_x x \sin k_z z \cos k_y(y+b) \quad (99)$$

$$\tilde{E}_z = \sum_{m,n} (-1)\,\frac{1}{\tilde{y}}\left(k_z^2 + \frac{s^2}{c_o^2}\right) \tilde{A}_{mn}(s) \sin k_x x \cos k_z z \sin k_y(y+b) \quad (100)$$

and

$$\tilde{H}_z = \sum_{m,n} (-1)\,\frac{1}{\tilde{z}}\left(k_z^2 + \frac{s^2}{c_o^2}\right) \tilde{B}_{mn}(s) \cos k_x x \sin k_z z \cos k_y(y+b) \quad (101)$$

VI-2. External Fields

The fields outside the cavity in the $y \geq 0$ region can be expressed as a superposition of plane waves with coefficients determined by the tangential components of the $\bar{E}$-field at the aperture. The two tangential components, $\tilde{E}_x$ and $\tilde{E}_z$, are to be treated separately.

(a) $\tilde{E}_x = 0$, $\tilde{E}_x \neq 0$

Let

$$\tilde{E}_x\Big|_{y\geq 0} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z'\, \tilde{E}_x(k_x', k_z') e^{-j(k_x' x + k_y' y + k_z' z)} \quad (102)$$

where

$$jk_y' = \left(\frac{s^2}{c_o^2} + k_x'^2 + k_z'^2\right)^{1/2} \quad (103)$$

and

$$\tilde{E}_x(k_x', k_z') = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz\, \tilde{E}_x(x,o,z) e^{j(k_x'x + k_z'z)} \tag{104}$$

Choose a wave function

$$\psi = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z'\, f(k_x', k_z') e^{-j(k_x'x + k_y'y + k_z'z)} \tag{105}$$

The Fourier transform of ψ is

$$\tilde{\psi} = f(k_x', k_z') e^{-jk_y'y} \tag{106}$$

The field components in terms of $\tilde{\psi}$ are [21]

$$\tilde{E}_x = jk_y'\tilde{\psi} \tag{107}$$

$$\tilde{E}_y = -jk_x'\tilde{\psi} \tag{108}$$

$$\tilde{E}_z = 0 \tag{109}$$

$$\tilde{H}_x = -\frac{k_x'k_z'}{s\mu_o} \tilde{\psi} \tag{110}$$

$$\tilde{H}_y = -\frac{k_y'k_z'}{s\mu_o} \tilde{\psi} \tag{111}$$

$$\tilde{H}_z = -\frac{(s/c_o)^2 + k_z'^2}{s\mu_o} \tilde{\psi} \tag{112}$$

Combining Eqs. (102), (107) and (112), we obtain

$$\tilde{H}_z(x,y,z) =$$

$$\frac{1}{4\pi^2} \int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z' \frac{j(s^2/c_o^2 + k_z'^2)}{s\mu_o k_y'} \tilde{E}_x(k_x', k_z') e^{-j(k_x'x + k_y'y + k_z'z)} \tag{113}$$

It is noted that $\tilde{E}_x(k_x', k_z')$ can be determined from Eq. (104) for a known $\tilde{E}_x(x,o,z)$ at the aperture.

(b) $\tilde{E}_x = 0,\ \tilde{E}_z \neq 0.$

In this case we write

$$\tilde{E}_z\Big|_{y\geq 0} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z'\, \tilde{E}_z(k_x', k_z')e^{-j(k_x'x + k_y'y + k_z'z)} \tag{114}$$

where

$$\tilde{E}_z(k_x', k_z') = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz\, \tilde{E}_z(x,o,z)e^{j(k_x'x + k_z'z)} \tag{115}$$

By a similar procedure, we obtain $\tilde{H}_z(x,y,z)$ in the $y\geq 0$ region as

$$\tilde{H}_z(x,y,z) =$$

$$\frac{1}{4\pi^2}\int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z' \left(\frac{-jk_x' k_z'}{s\mu_o k_y'}\right)\tilde{E}_z(k_x', k_z')e^{-j(k_x'x + k_y'y + k_z'z)} \tag{116}$$

Combining Eqs. (113) and (116), we have

$$\tilde{H}_z(x,y,z) =$$

$$\frac{-j}{4\pi^2}\int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z' \left\{\frac{k_x'k_z'E_z(k_x',k_z') - (s^2/c_o^2+k_z'^2)E_x(k_x',k_z')}{s\mu_o k_y'}\right\}$$

$$\cdot e^{-j(k_x'x + k_y'y + k_z'z)} \tag{117}$$

VI-3. Boundary Conditions at Aperture

The boundary conditions at the aperture are such that $\tilde{E}_z$ and $\tilde{E}_x$ are continuous and that $\tilde{H}_z$ is discontinuous by the equivalent electric current $\tilde{J}_x$. We treat the three components separately below.

(a) Boundary condition for $\tilde{E}_z$.

$\tilde{E}_z$ is continuous at the aperture, y=0. Applying the method of moments, we expand $\tilde{E}_z$ at y=0 over the aperture region in terms of appropriate expansion functions $f_{pq}^{E_z}$ with unknown coefficients $\tilde{\beta}_{pq}(s)$.

$$\tilde{E}_z\Big|_{y=0} = \begin{cases} \sum_{p,q} \tilde{\beta}_{pq}(s)\, f_{pq}^{E_z}, & \text{at aperture} \\ 0, & \text{elsewhere} \end{cases} \tag{118}$$

Equating Eq. (118) to the $\tilde{E}_z$ of the cavity field in Eq. (100) at y=0, we have

$$\sum_{m,n} (-1)\frac{1}{\tilde{y}}\left(k_z^2 + \frac{s^2}{c_o^2}\right)\tilde{A}_{mn}(s) \sin k_x x \cos k_z z \sin k_y b$$

$$= \sum_{p,q} \tilde{\beta}_{pq}(s) f_{pq}^{E_z} \tag{119}$$

Now we take the inner product of Eq. (119) with testing functions and the result can be put in the following form.

$$\{\tilde{A}_{mn}(s)\} = [\tilde{M}]\{\tilde{\beta}_{pq}(s)\} \tag{120}$$

It is important to note that the dimension of the matrix $[\tilde{M}]$ is not very large because neither the number of cavity modes (m,n) nor the number of subsections over the aperture (p,q) needs to be large. $\tilde{E}_z(k_x', k_z')$ can be obtained from Eq. (115).

(b) Boundary condition for $\tilde{E}_x$.

$\tilde{E}_x$ is also continuous at the aperture. $\tilde{E}_x$ at y=0 can be expanded in terms of expansion functions $f_{pq}^{E_x}$:

$$\tilde{E}_x\Big|_{y=0} = \begin{cases} \sum_{p,q} \tilde{\alpha}_{pq}(s) f_{pq}^{E_x}, & \text{at aperture} \\ 0, & \text{elsewhere} \end{cases} \tag{121}$$

Equating Eq. (121) to the $\tilde{E}_x$ of the cavity field in Eq. (98) at y-0, we have

$$\sum_{m,n} (-1)\left[\frac{1}{\tilde{y}} k_x k_z \tilde{A}_{mn}(s) - k_y \tilde{B}_{mn}(s)\right] \cos k_x x \sin k_z z \sin k_y b$$

$$= \sum_{m,n} \tilde{\alpha}_{pq}(s)\, f_{pq}^{E_x} . \tag{122}$$

Inner product of Eq. (122) with testing functions yields

$$\{\tilde{B}_{mn}(s)\} = [\tilde{N}_1]\{\tilde{\alpha}_{pq}(s)\} + [\tilde{N}_2]\{\tilde{\beta}_{pq}(s)\} \tag{123}$$

(c) Boundary condition for $\tilde{H}_z$.

The condition for the discontinuity of $\tilde{H}_z$ at y=0 is obtained by combining Eqs. (101) and (117). We have

$$\sum_{m,n} (-1)\frac{1}{\tilde{z}}\left(k_z^2 + \frac{s^2}{c_o^2}\right) \tilde{B}_{mn}(s) \cos k_x x \sin k_z z \cos k_y b$$

$$= \tilde{J}_x(s) - \frac{j}{4\pi^2} \int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dk_z' \left\{\frac{k_x' k_z' \tilde{E}_z(k_x', k_z') - (s^2/c_o^2 + k_z'^2)\tilde{E}_x(k_x', k_z')}{s\mu_o k_y'}\right\} \cdot e^{-j(k_x' x + k_z' z)} \tag{124}$$

where $\tilde{E}_z(k_x', k_z')$ and $\tilde{E}_x(k_x', k_z')$ can be determined by substituting Eqs. (118) and (121) into Eqs. (115) and (104) respectively.

The application of the method of moments to Eq. (124) yields the following matrix equation:

$$[\tilde{Z}'(s)] \begin{Bmatrix} \tilde{\alpha}_{pq}(s) \\ \\ \tilde{\beta}_{pq}(s) \end{Bmatrix} = \{\tilde{v}(s)\} \tag{125}$$

where $\{\tilde{v}(s)\}$ represents the excitation column matrix which is the inner product of $\tilde{J}_x(s)$ and the testing functions. The procedure of solution is to determine $\{\tilde{\alpha}_{pq}(s)\}$ and $\{\tilde{\beta}_{pq}(s)\}$ from Eq. (125). With $\{\tilde{\alpha}_{pq}(s)\}$ and $\{\tilde{\beta}_{pq}(s)\}$, $\{\tilde{A}_{mn}(s)\}$ and $\{\tilde{B}_{mn}(s)\}$ can be found from Eqs. (120) and (123) respectively, which are then used to obtain the field distribution inside the cavity from Eqs. (98)-(101).

VI-4. Numerical Techniques

The numerical technique outlined in Section V for determining $\{\tilde{a}(s)\}$ can be used here for finding $\{\tilde{\alpha}_{pq}(s)\}$ and $\{\tilde{\beta}_{pq}(s)\}$ from Eq. (125). In Section V the cavity fields were expressed in terms of expansion functions in three dimensions, resulting in a $[\tilde{Z}(s)]$ with large dimensions, whereas the alternative approach developed in this Section expresses the cavity fields in relatively few cavity modes and the field at the aperture is a two-dimensional expansion. Hence the matrix $[\tilde{Z}'(s)]$ in Eq. (125) has a much smaller dimension than $[\tilde{Z}(s)]$.

In applying the method of moments, twenty subsections were used for the aperture. Rectangular pulses were chosen as the expan-

sion function in the x-direction and triangular functions as that in the z-direction. The starting points of the root-searching procedure for det[Z'(s)] = 0 were the resonant frequencies of the cavity modes without the slot aperture. Unfortunately [Z'(s)], though of a smaller dimension, was also ill-conditioned and convergence to pole singularities was not apparent in many cases. This difficulty could not be alleviated by changes in the number of subsections taken at the aperture. (This was tried without success.) An effort was made to reduce computation errors by increasing the magnitude of each element of [Z'(s)] by a hundredfold. This yielded the two layers of pole singularities shown in Fig. 8. The convergence was very slow and a time limit of 30 minutes was set for locating each pole on the Honeywell 6000 computer. As can be seen from Fig. 8, some of the points do not fall on the smoothed layers, indicating computing inaccuracies. Much time was spent on isolating possible causes for the inaccuracies without noticeable success. The computer program was checked and rechecked many times. Possible contributing factors to the slow convergence and numerical inaccuracies are round-off errors, internal resonances of [Z'(s)], and existence of singularities other than simple poles.

The normalized natural mode for the first pole singularity of the first layer, $s_{11} = (-0.121 + j2.039)c$, is plotted in Fig. 9. The natural modes for the other pole singularities have also been plotted. They are not reproduced here because, except for slight differences in the phase, they are surprisingly similar. Explanations for this phenomenon and for the different curvatures of the loci in Fig. 8 as compared with those in Fig. 3 are lacking.

Attempts were made to obtain the transient electric field at typical points inside the cavity for an incident step excitation, but the resulting response violated causality and showed peculiar fluctuations and therefore does not appear to warrant inclusion in this report.

As a last resort, an FFT (fast-Fourier-transform) algorithm was applied to obtain a time-domain response directly from the frequency-domain response.

$$E(t) = \frac{1}{2\pi j} \int_C \tilde{E}(s) e^{st} ds \tag{126}$$

Substituting $s = j\omega$, we have

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(j\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \lim_{\Omega \to \infty} \int_{-\Omega}^{\Omega} \tilde{E}(j\omega) e^{j\omega t} d\omega \tag{127}$$

Let $t_k = k\Delta t = kT/N$, $f_n = n\Delta f = n/T$. Equation (127) can be approximated as

$$E(k) = (\Delta f) \sum_{n=0}^{N-1} \tilde{E}(n) e^{j2\pi nk/N} \tag{128}$$

where $E(k)$ and $\tilde{E}(n)$ stand for $\tilde{E}(k\Delta t)$ and $\tilde{E}(j2\pi n\Delta f)$ respectively. In order to find $\tilde{E}(n)$, Eq. (125) must first be solved for $\{\alpha_{pq}(j\omega_n)\}$ and $\{\beta_{pq}(j\omega_n)\}$ which are used for obtaining $\{A_{mn}(j\omega_n)\}$ and $\{B_{mn}(j\omega_n)\}$ from Eqs. (120) and (123). The frequency-domain field components can then be determined from Eqs. (98)-(101).

Two FFT runs were made on the problem with perpendicular polarization: one for evaluating $\tilde{E}(n)$ at 257 frequency points and one at 513 frequency points. Unfortunately the plots for E(k) also violated causality and showed peculiar fluctuations.

VII. CONCLUSION

In this report the transient excitation of a cavity through an aperture is used to illustrate the state-space solution of transient electromagnetic problems. Depending on whether the E- or the H-formulation is used, the effect of the aperture is accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order normalized state equations by defining three new state variables in addition to the appropriate vector potential. These state equations subject to the associated boundary conditions are solved by the method of moments. The cavity region is first divided into subsections and the fields within the cavity expressed in terms of appropriate expansion functions with time-dependent coefficients. The fields in the half-space outside the cavity are represented as superpositions of plane waves. At the aperture, the cavity and external fields are properly matched. Inner products are taken with testing functions and the first-order equations are converted into matrix equations containing the expansion coefficients as unknown column vectors. Evaluation of these expansion-coefficient vectors leads to the determination of field distributions. The procedure for evaluating a typical coefficient vector by the singularity-expansion method has been outlined.

This solution procedure was applied to the problem of electromagnetic excitation of a rectangular cavity through a slot aperture. Two cases were considered: an E-formulation for parallel polarization and an H-formulation for perpendicular polarization. For the case of parallel polarization, some

significant pole singularities (natural frequencies) in the upper left complex plane were found for a sample cavity. The corresponding natural modes were determined and combined to give the electric intensities as functions of time at two locations in the cavity for a step-excitation. Both the shape and the magnitude of the penetrated electric field appear reasonable. However, it would not be wise to claim absolute accuracy because of the inherent noise and instability in the numerical procedure and because of a lack of a theoretical proof that there would be no multiple poles or branch points.

For the case of perpendicular polarization, the system impedance matrix resulted from the application of the three-dimensional moment method was ill-conditioned and it was not possible to determine the system singularities. In order to avoid the numerical difficulties inherent with large matrices, an alternative analytical approach for expanding the fields inside the cavity in terms of TM and TE cavity modes with unknown coefficients was developed. This approach effectively changes the three-dimensional moment method into a two-dimensional one, thus reducing the dimensions of the resulting matrix. Some pole singularities were found. However, the system matrix was also unstable and convergence to the singularities very slow. The matrix appears to contain internal resonances. An FFT algorithm was written to determine the transient time-domain response directly from the frequency-domain response. Unfortunately the numerical results violated causality and showed peculiar fluctuations.

As mentioned previously, numerical anomalies appear in even the simplest cases and preclude the determination of the natural frequencies

of well-defined geometric structures [23],[24]. Numerical difficulties in cavity-backed slot aperture problems are really not very surprising. Previous studies [25], [26] on the penetration of transient electromagnetic excitation through apertures in an infinite ground screen (without a cavity) have not yielded numerical results. Various difficulties were encountered in attempts to obtain self-consistent pole locations because of numerical instability [26]. The problem is vastly more complicated when there is a cavity behind the aperture. It has been shown [27] that the structure of a conducting cylinder within a parallel-plate region gives rise to two types of singularities; namely, poles and branch points, in the complex-frequency plane. Whether a cavity-backed slot has branch points or multiple poles is an unanswered question. It is felt that a concerted effort to find a definitive answer to this question would be a worthwhile undertaking.

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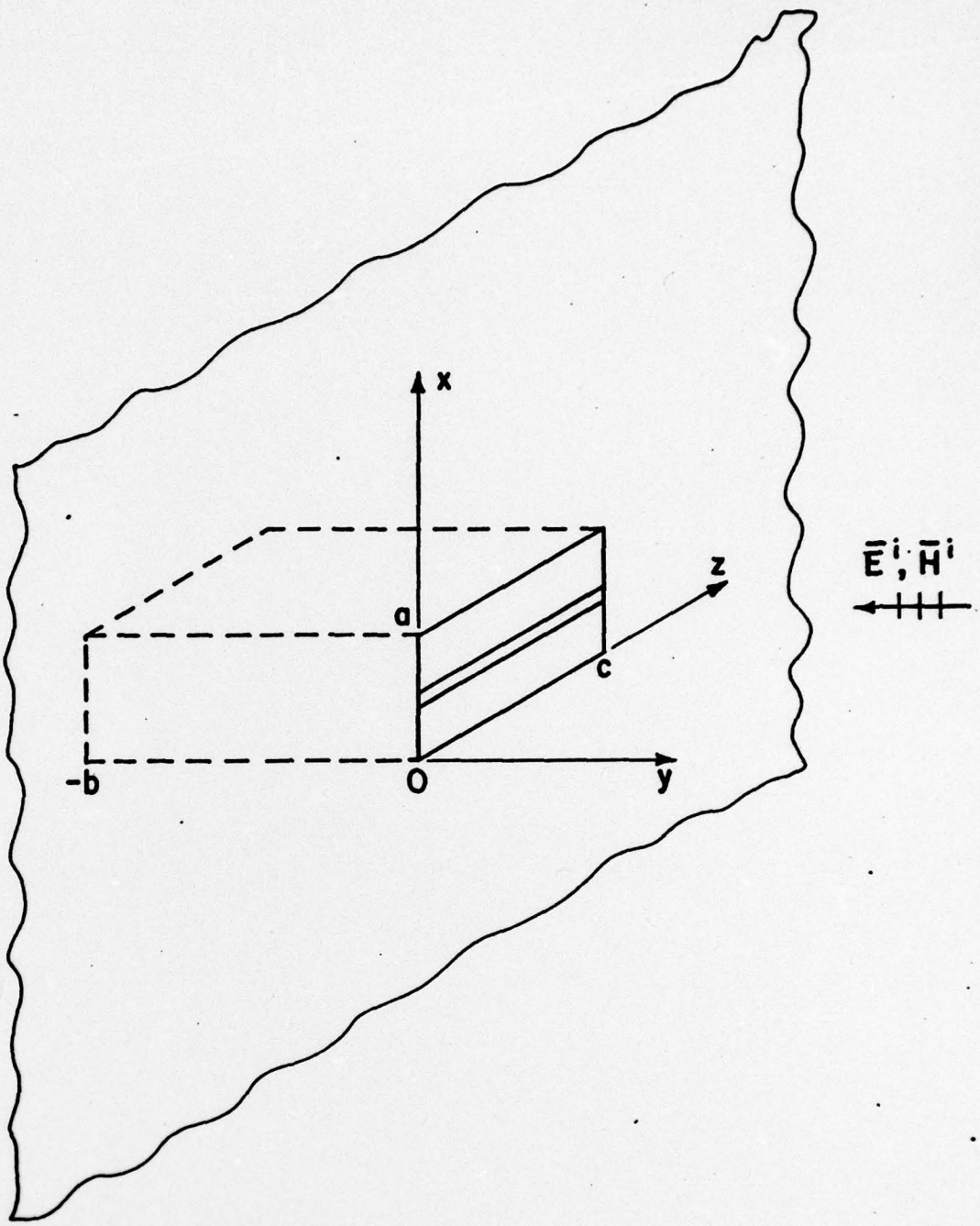


Fig. 1. A Cavity-Backed Slot Aperture Problem

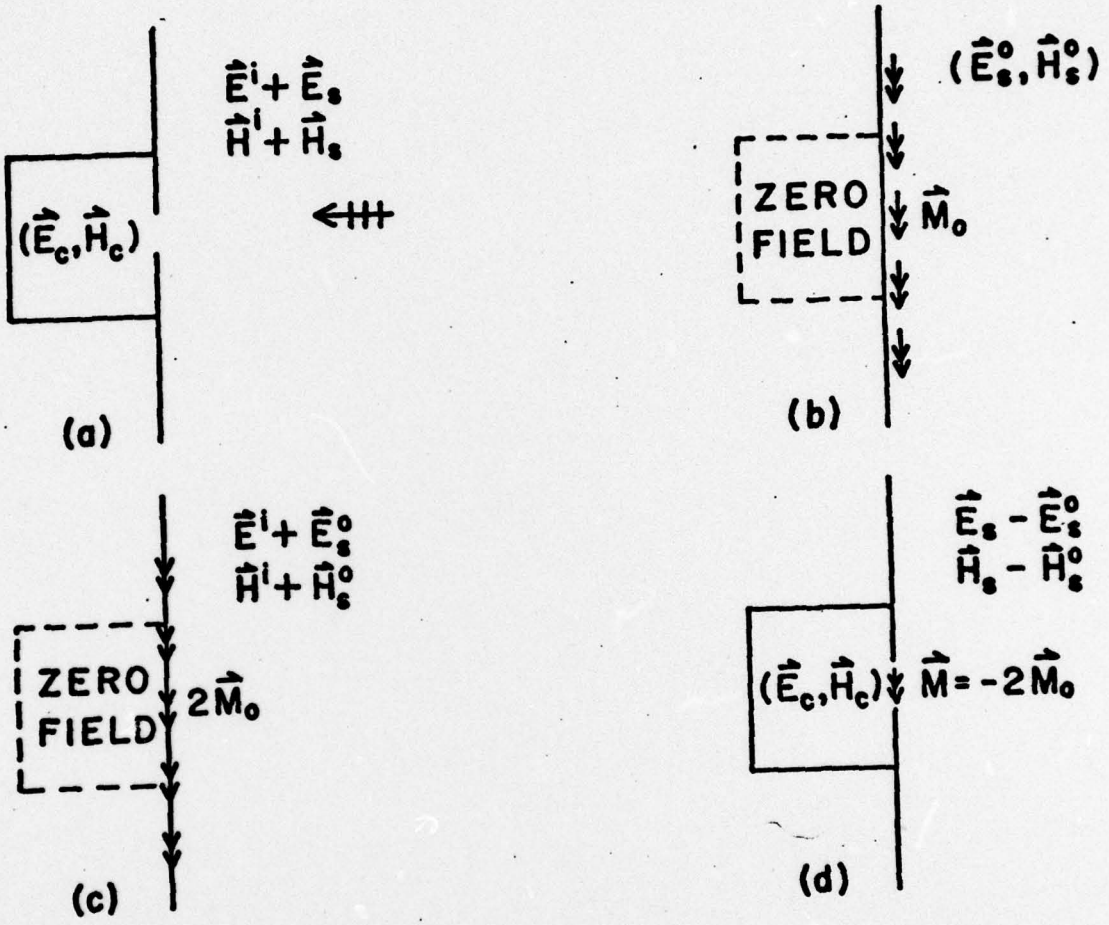
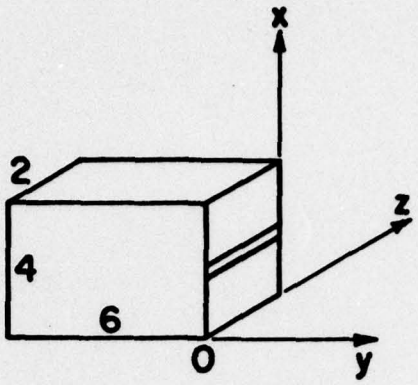


Fig. 2. Application of induction theorem



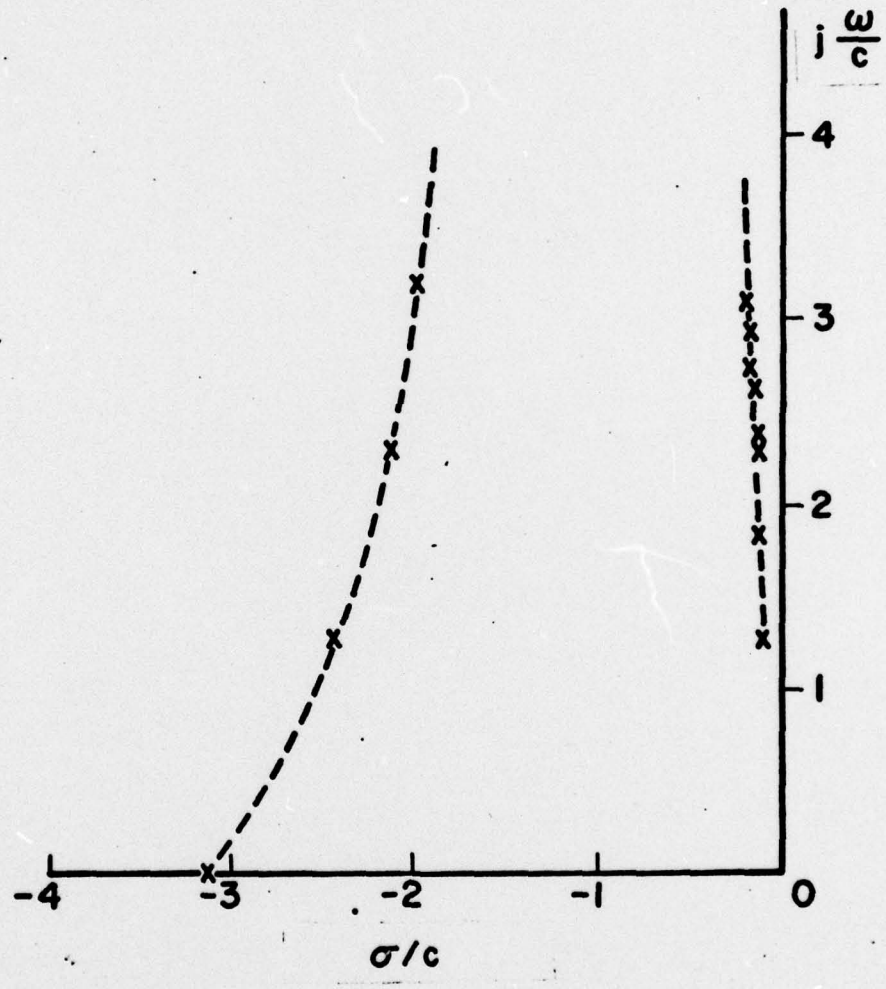


Fig. 3. Pole Singularities of Cavity-Backed Slot (Parallel Polarization)

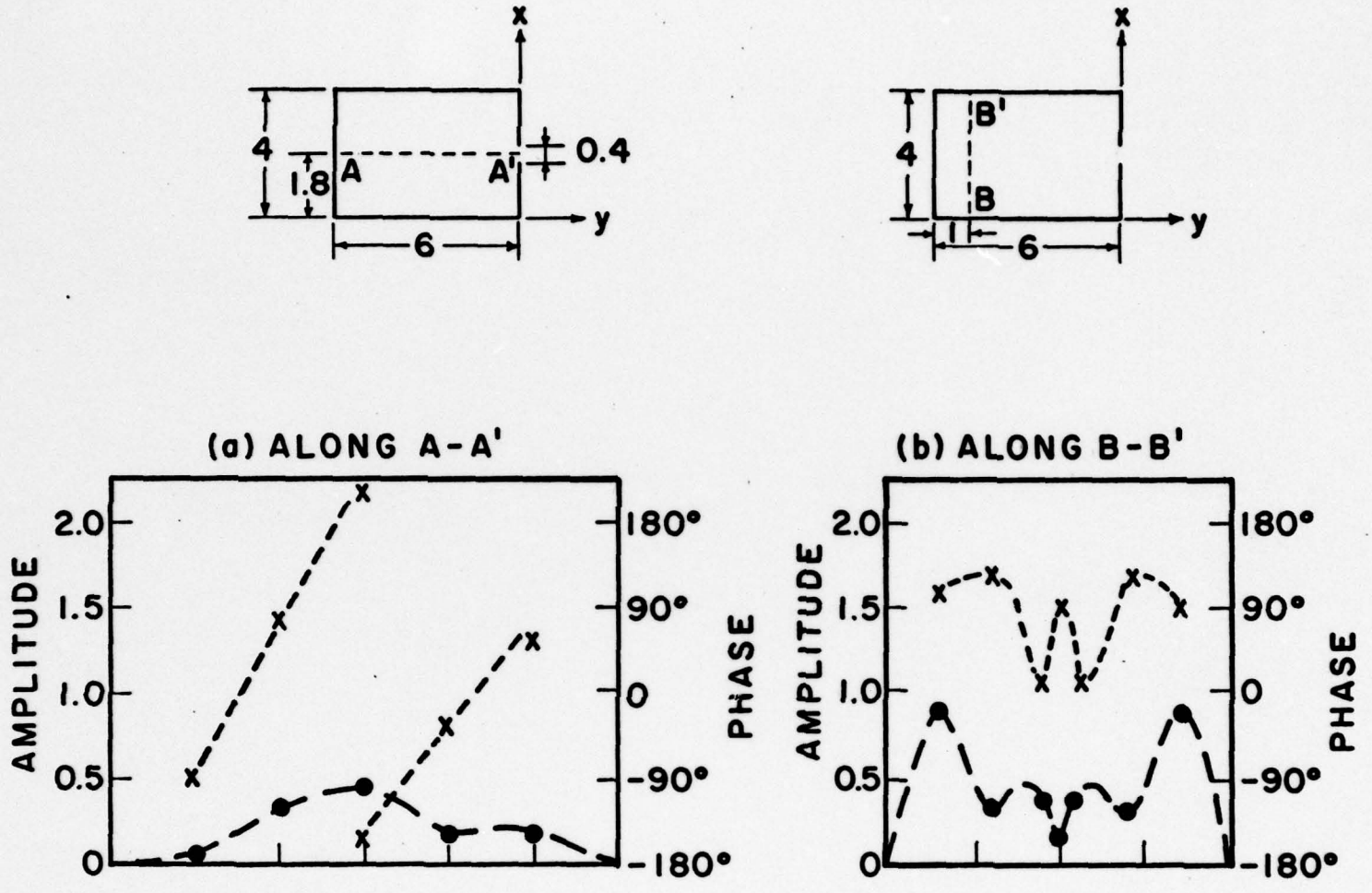


Fig. 4. Natural Mode for s_{11} = (-0.110 + j1.258)c -- Parallel Polarization.

— — — Amplitude, - - - - - Phase

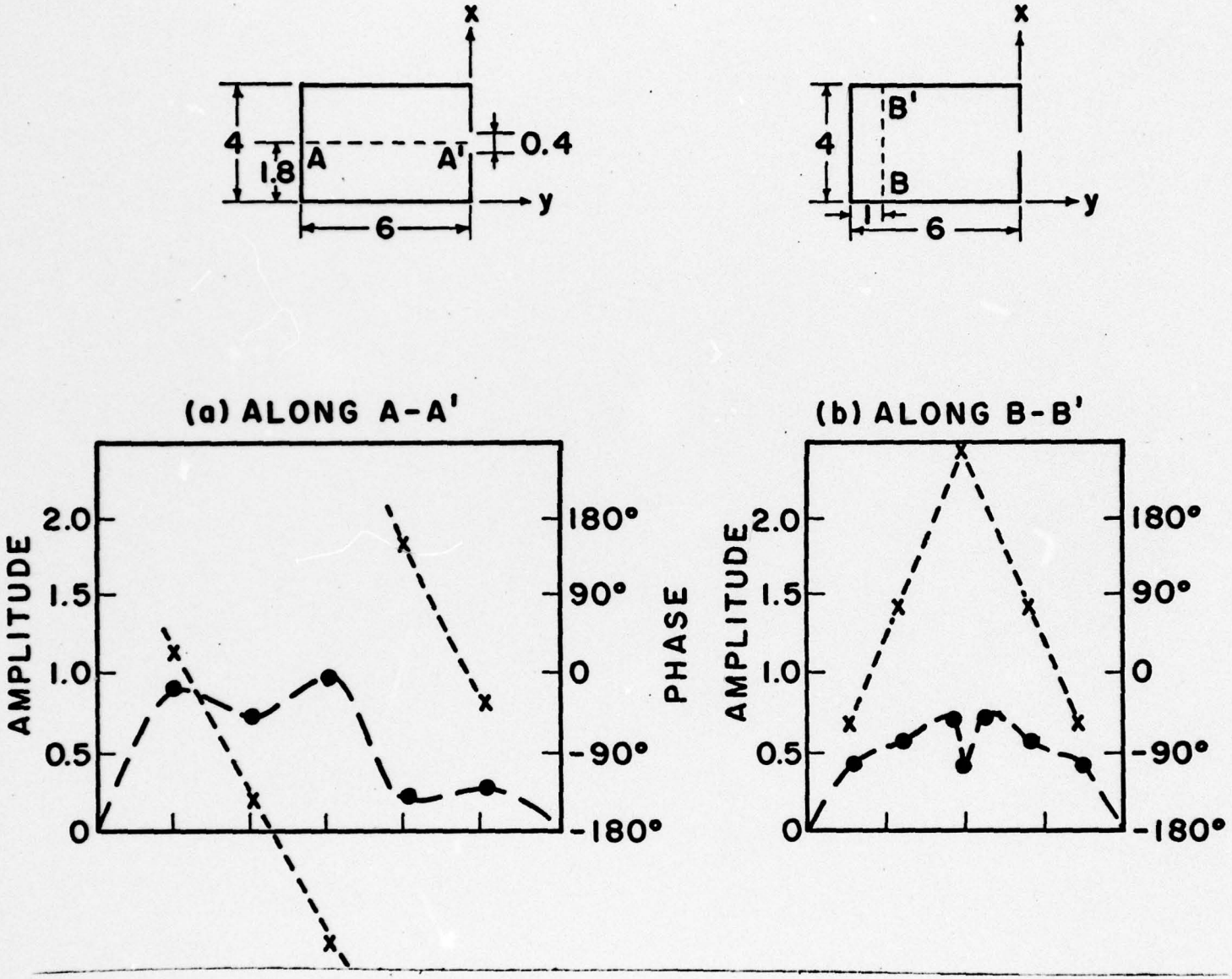


Fig. 5. Natural Mode for s_{13}= (-0.141 + j2.296)c -- Perpendicular Polarization.
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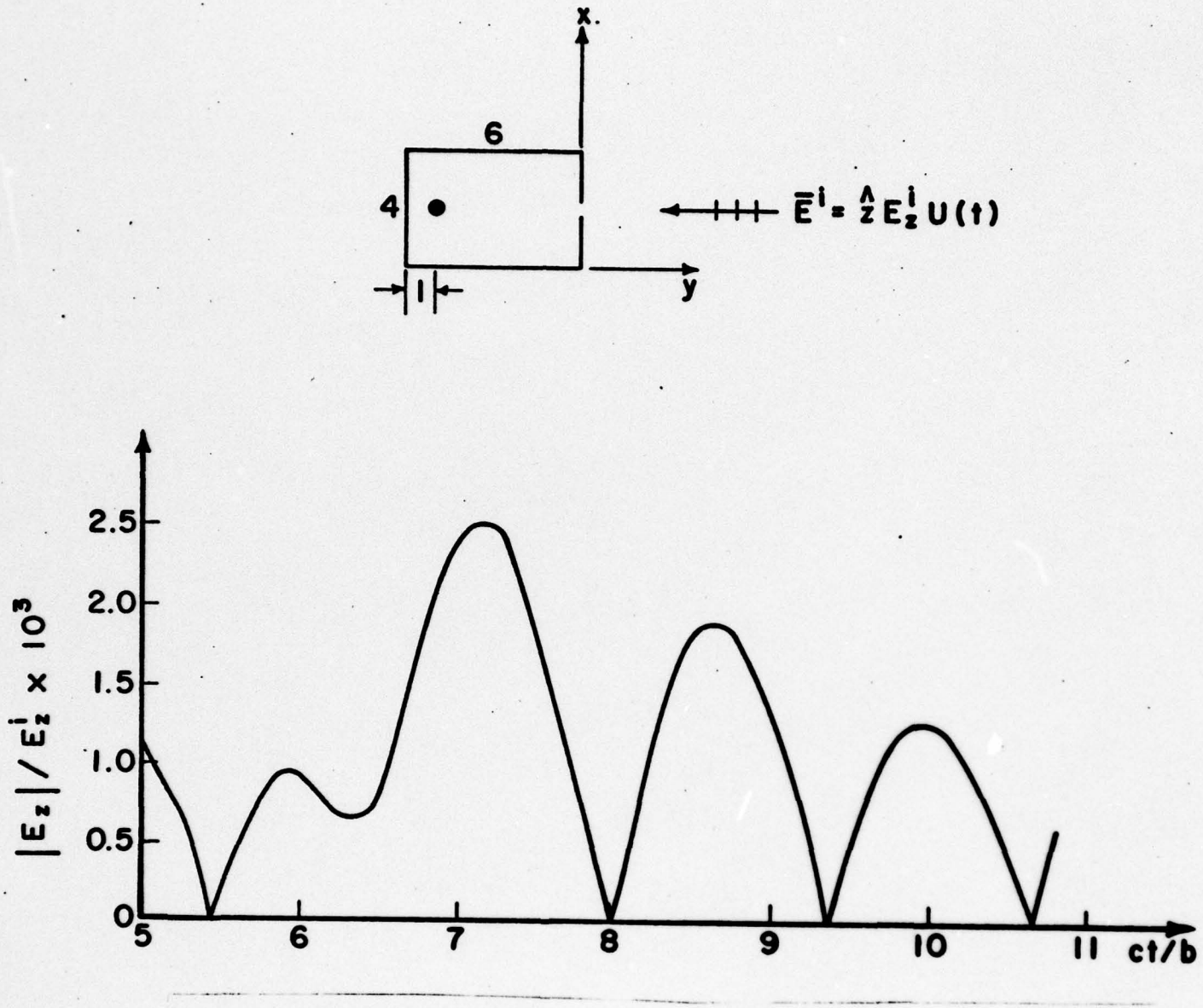


Fig. 6. Response to Step Excitation (Parallel Polarization).

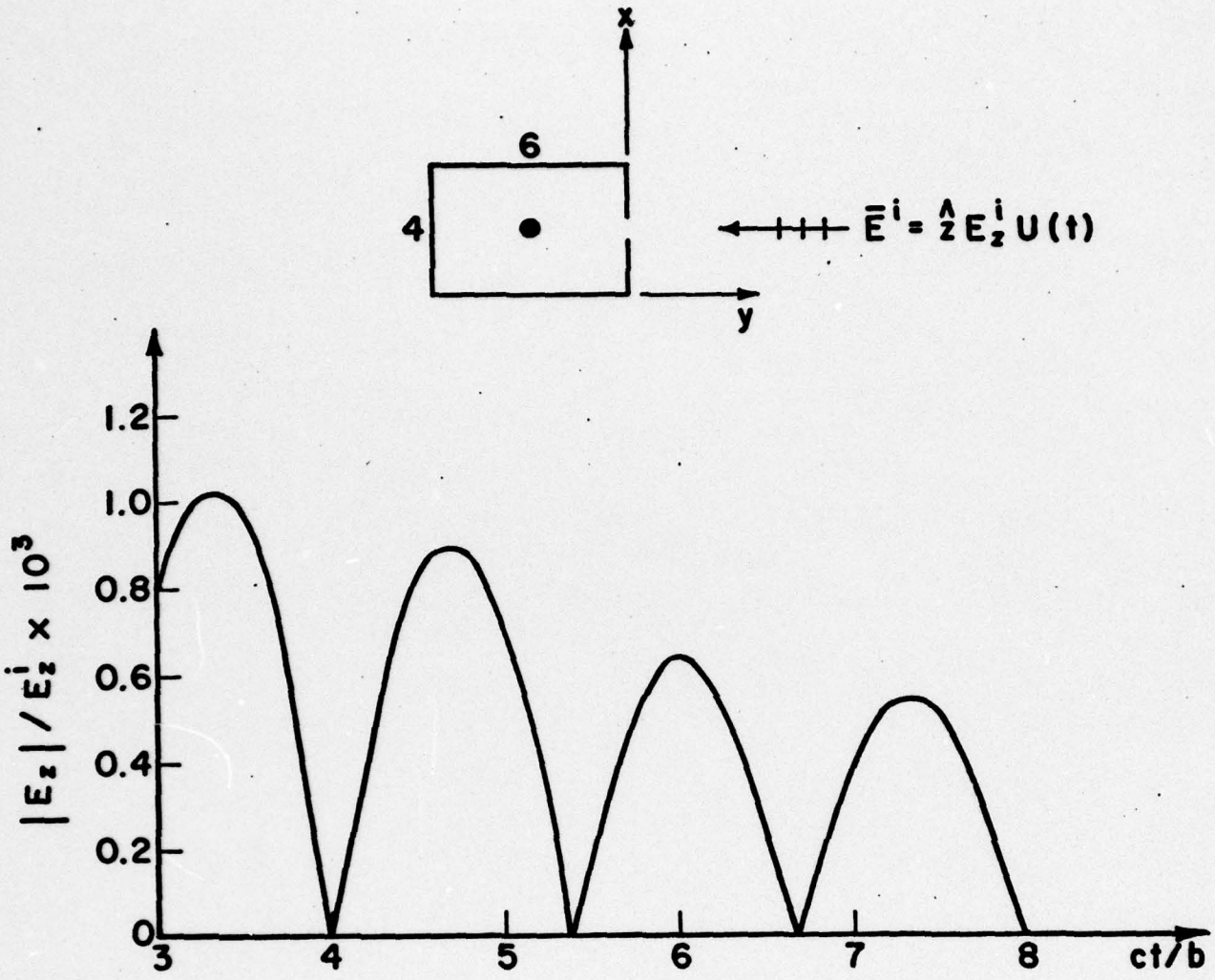


Fig. 7. Response to Step Excitation (Parallel Polarization).

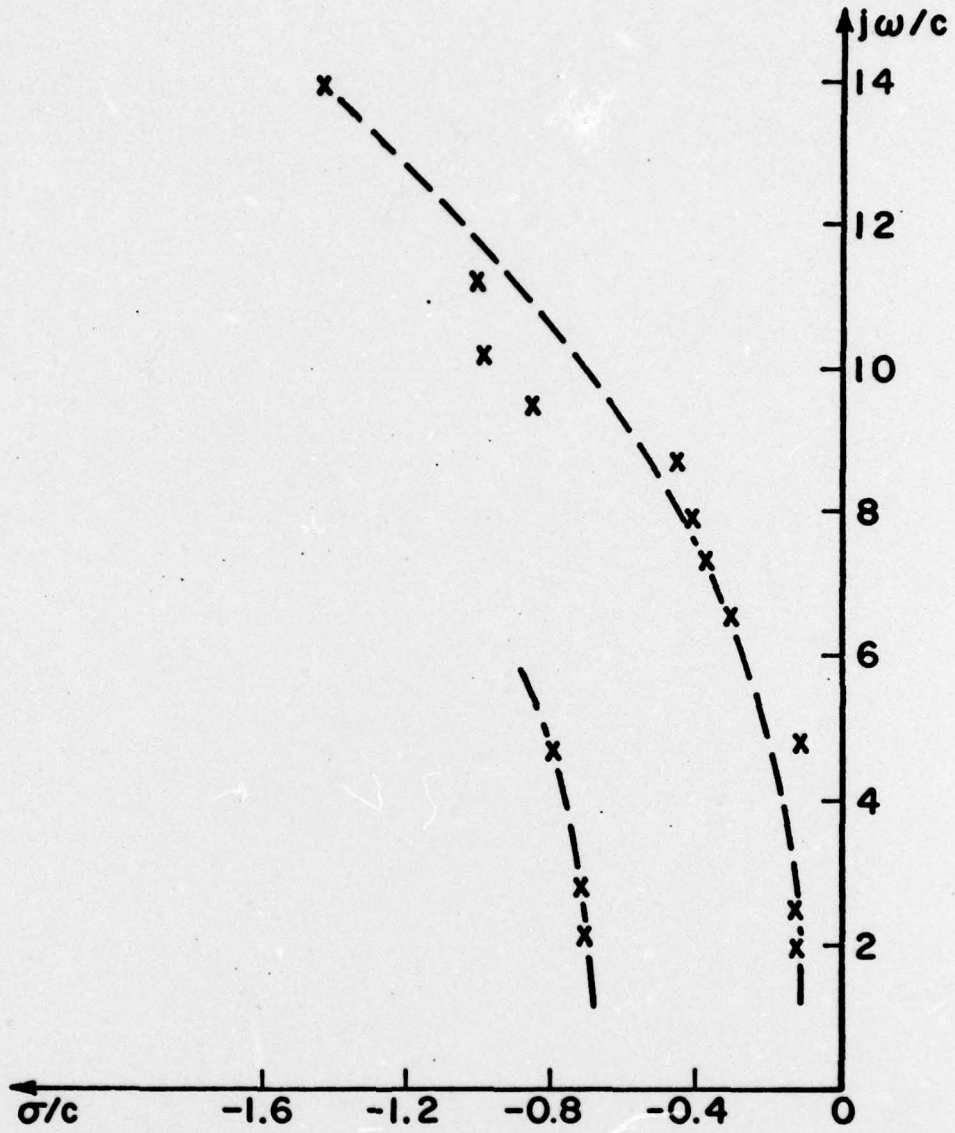


Fig. 8. Pole Singularities of Cavity-Backed Slot (Perpendicular Polarization)

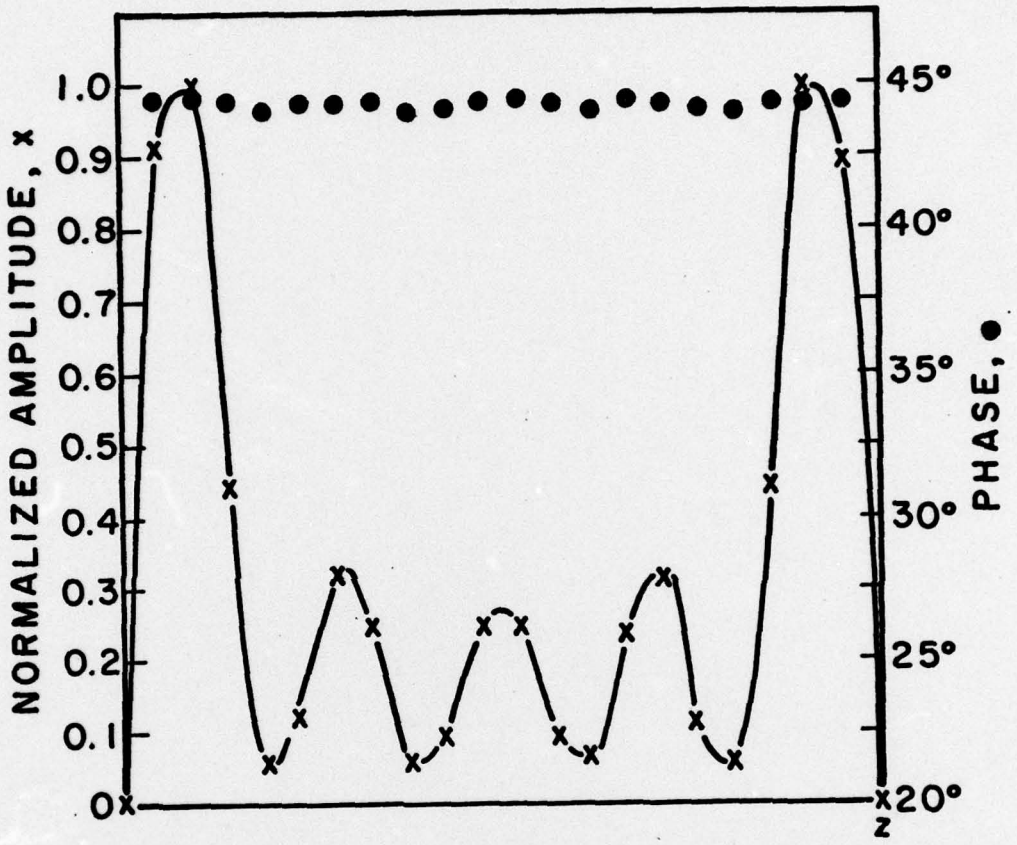


Fig. 9. Natural Mode for s_{11}= (-0.121 + j2.039)c
(Perpendicular Polarization)

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The state-space technique is used to formulate the transient electromagnetic problem of excitation of a rectangular cavity through an aperture. Depending on whether the E- or the H-formulation is used, the effect of the aperture can be accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order state equations by defining three new state variables in addition to an

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appropriate vector potential. These state equations are solved by the method of moments. Two cases are considered: parallel polarization and perpendicular polarization. Some significant singularities for the parallel-polarization case are found and the electric intensities as functions of time at two locations in the cavity are computed for a step excitation. An alternative approach of expanding the cavity fields in terms of TM and TE cavity modes is developed to reduce the dimension of the system matrix. Numerical difficulties are encountered and possible contributing causes are discussed.