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OPTIMUM QUANTIZATION PARAMETERS FOR A NORMALIZED CORRELATOR WIT--ETC(U)
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OPTIMUM QUANTIZATION PARAMETERS
FOR A
NORMALIZED CORRELATOR WITH GAUSSIAN INPUT

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by

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INTRODUCTION

True-normalized correlators which use either an analog or a quantized reference and a hard-limited-received signal have been applied to underwater acoustic measurements with a pseudorandom signal ~~(1,2)~~. This type of signal, together with a large number of quantization levels in the reference channel, yields a true-normalized correlator; that is, the average correlator output is proportional to the correlation coefficient of the inputs ~~(3)~~.

The purpose of this note is to obtain quantitative results for the deviation from a true-normalized correlator when a small number of quantization levels are used in the reference channel. Results indicate that, for a linear quantizer with a fixed number N of quantization levels, there exists an optimum-input-signal strength for a given quantization range.

METHOD

A system block diagram of the model under consideration is shown in Figure 1. The transfer characteristic of the quantizer is given in Figure 2. The total number of levels N is assumed to be an even integer.

The inputs x_1 and x_2 are assumed to be jointly Gaussian with correlation coefficient r, zero means, and unit variances; the joint probability density is

$$p(x_1,x_2) = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left\{-\frac{x_1^2+x_2^2-2rx_1x_2}{2(1-r^2)}\right\}. \qquad (1)$$

The variances of x_1 and x_2 are assumed to be one. The actual variance of x_1 will not matter since x_1 is hard limited; the actual variance of x_2 can be taken into account in the sequel by changing K to $\frac{K}{\sigma_2}$.

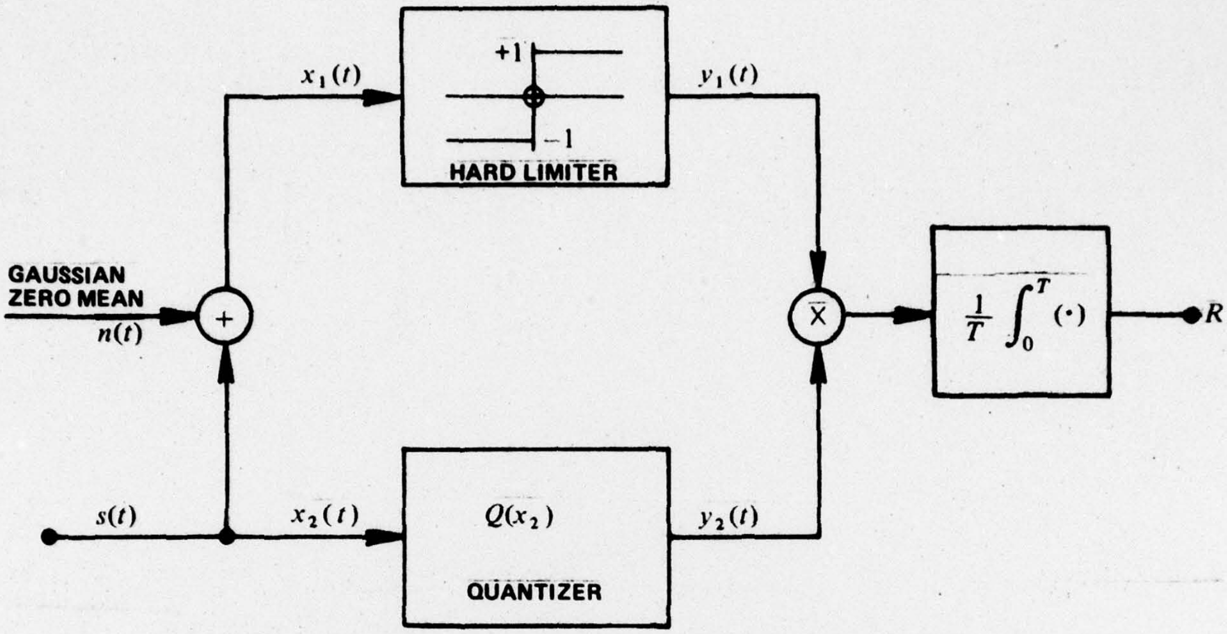

Figure 1. Block Diagram of the Model System

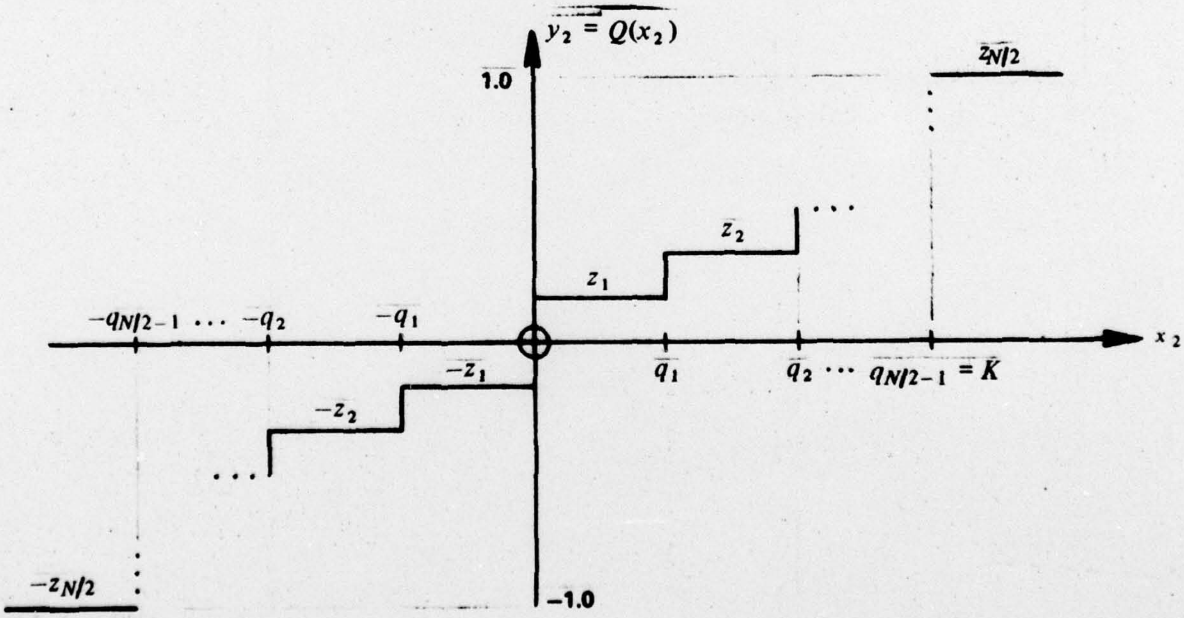

Figure 2. Transfer Characteristic of the Linear Quantizer

Under these assumptions, the average correlator output is

$$R(r,K) = \sqrt{\frac{2}{\pi}} \sum_{j=1}^{N/2-1} z_j \int_{q_{j-1}}^{q_j} \operatorname{erf}\left(\frac{ru}{\sqrt{2(1-r^2)}}\right) \exp\left(-\frac{u^2}{2}\right) du$$

$$+ \sqrt{\frac{2}{\pi}} \int_{q_{N/2-1}}^{\infty} \operatorname{erf}\left(\frac{ru}{\sqrt{2(1-r^2)}}\right) \exp\left(-\frac{u^2}{2}\right) du, \qquad (2)$$

where

$$z_j = \frac{2j-1}{N-1}$$

$$q_j = \begin{cases} \frac{2Kj}{N-2}, & N \neq 2 \\ 0, & N = 2 \end{cases}$$

$$j = 1, 2, \ldots, \left(\frac{N}{2} - 1\right)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

It can be shown from this expression that for any N and for $K = 0$ or $K >> 1$, $R(r,K)/R(r=1,K) = \frac{2}{\pi}\sin^{-1} r$; that is, the system becomes a polarity coincidence correlator because there are effectively only two quantization levels in each channel.

RESULTS

The expression for $R(r,K)$ was calculated directly from the above expression for a few values of N and normalized to $R_1 \equiv R(r = 1,K)$. A plot of

$$R_n(r,K) \equiv \frac{R(r,K)}{R_1} \qquad (3)$$

is shown in Figure 3 for $N = 4$ and various values of K.

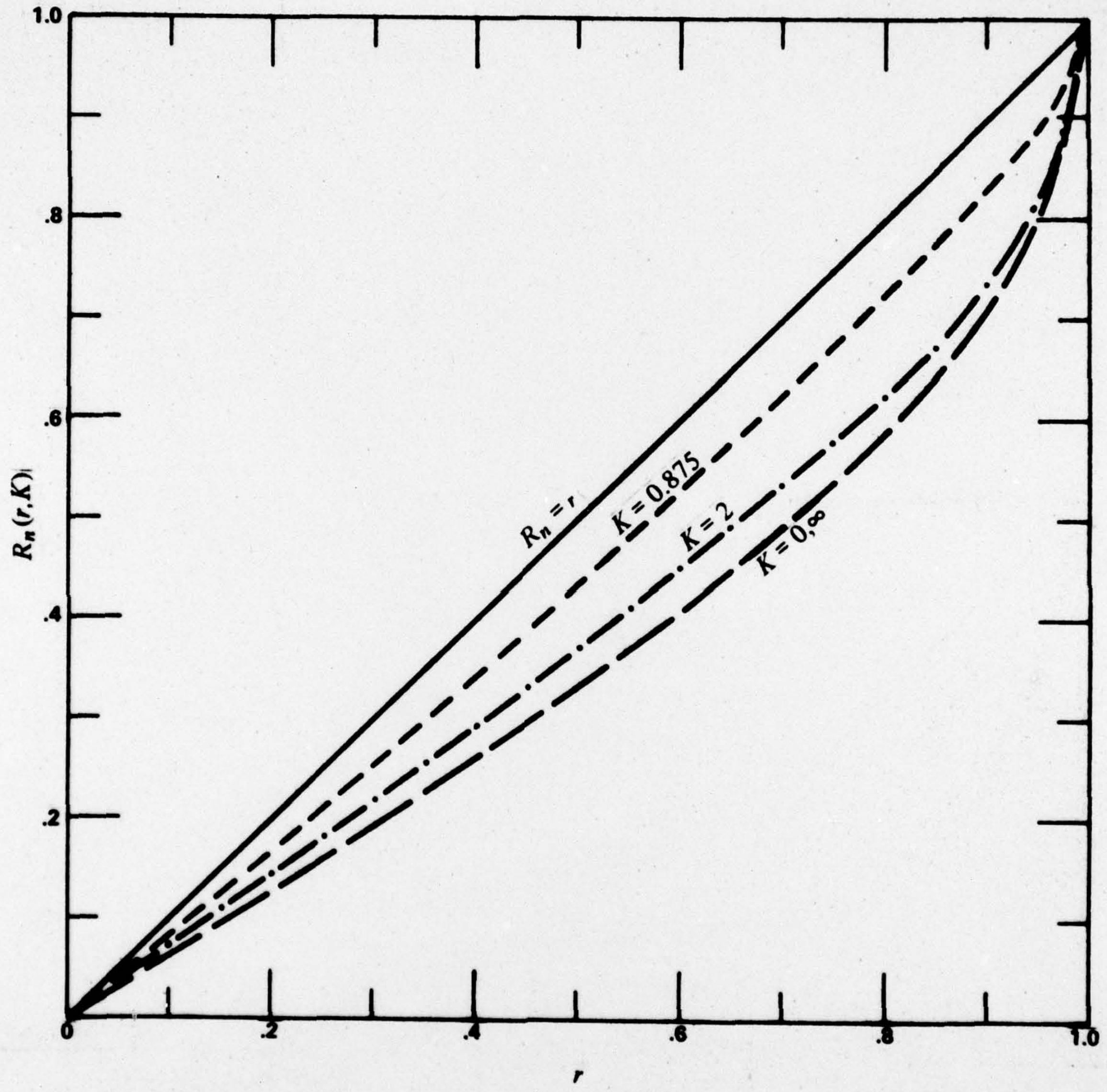

Figure 3. Normalized Average Output vs. Input Correlation Coefficient, for $N = 4$ and Various Values of K

When $r - R_n(r,K)$ is differentiated with respect to r and the result set equal to zero, an equation for the $r \equiv r_{max}$ at which $r - R_n(r,K)$ is a maximum for a given K is

$$1 - \frac{\dfrac{2}{\pi\sqrt{1-r^2}}\left[1 + 2\sum_{j=1}^{N/2-1} \exp\left[-j^2\left(\dfrac{K}{N/2-1}\right)^2 (1-r^2)^{-2}\right]\right]}{N - 1 - 2\sum_{j=1}^{N/2-1} \operatorname{erf}\dfrac{jK}{\sqrt{2}\,(N/2-1)}} = 0 . \qquad (4)$$

This equation was solved for r_{max} by an iterative method.

With the values of r_{max} and K the maximum error was calculated and plotted in Figure 4. The K for which the minimum, e_{min}, occurs is labeled K_{opt}; e_{min} is the minimum maximum error.

$$e_{min} = \min_{K} \max_{r} \left[r - R_n(r,K)\right] . \qquad (5)$$

The optimum input quantizing range K_{opt} vs. the number of quantizing steps N is shown in Figure 5 and a plot of the minimum maximum error e_{min} vs. N is shown in Figure 6.

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that for a given N there exists an optimum K for which the maximum deviation of $R_n(r,K)$ from the linear relationship is minimal.

Further, inspection of Figure 4 yields two important conclusions: First, choice of the number of quantization levels N greater than 32 is unjustified, as far as system linearity is concerned; for $N = 32$ the error is less than one percent for a wide range of values of K. Second, for $N \leqslant 8$ the value chosen for K becomes quite critical; small changes in reference signal amplitude cause large changes in deviation from linearity.

Suggestions for Further Development

It is conjectured that a lower deviation from linearity can be achieved if the restriction of a linear quantizer is removed. A suitable substitute criterion might be minimum mean square quantizing error. J. Max ("On the Autocorrelation Function of Quantized Signal Plus Noise," *Information Theory*, IT-11, January 1960) has derived the optimum levels with the criterion of minimum *n*th norm of the quantization error.

Also, results similar to those derived in this paper can be derived for deterministic signals as follows:

$$R\left(\tau, \frac{S}{N}\right) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} E\left\{y_1(t) y_2(t+\tau)\right\} dt \tag{6}$$

where

$$E\left\{y_1(t) y_2(t+\tau)\right\} = Q[x_2(t+\tau)] \left\{Pr[s(t)+n(t)>0] - Pr[s(t)+n(t)<0]\right\}$$

$$= Q[s(t+\tau)] \operatorname{erf}\left(\frac{s(t)}{\sqrt{2}\,\sigma_n}\right).$$

For example, if the input signal is bi-level,

$$Q[s(t+\tau)] \operatorname{erf}\left(\frac{s(t)}{\sqrt{2}\,\sigma_n}\right) = \begin{cases} \operatorname{erf}\left(\frac{1}{\sqrt{2}\,\sigma_n}\right), & \text{if } s(t) = 1 \text{ and } s(t+\tau) = 1 \\ & \text{or } s(t) = -1 \text{ and } s(t+\tau) = -1 \\ -\operatorname{erf}\left(\frac{1}{\sqrt{2}\,\sigma_n}\right), & \text{if } s(t) = 1 \text{ and } s(t+\tau) = -1 \\ & \text{or } s(t) = -1 \text{ and } s(t+\tau) = 1 \end{cases}$$

$$= R_{ss}(\tau) \operatorname{erf}\left(\frac{1}{\sqrt{2}\,\sigma_n}\right),$$

and

$$R\left(\tau, \frac{S}{N}\right) = R_{ss}(\tau) \operatorname{erf}\left(\frac{1}{\sqrt{2}\,\sigma_n}\right), \tag{7}$$

where $R_{ss}(\tau)$ is the autocorrelation function of the bi-level signal.

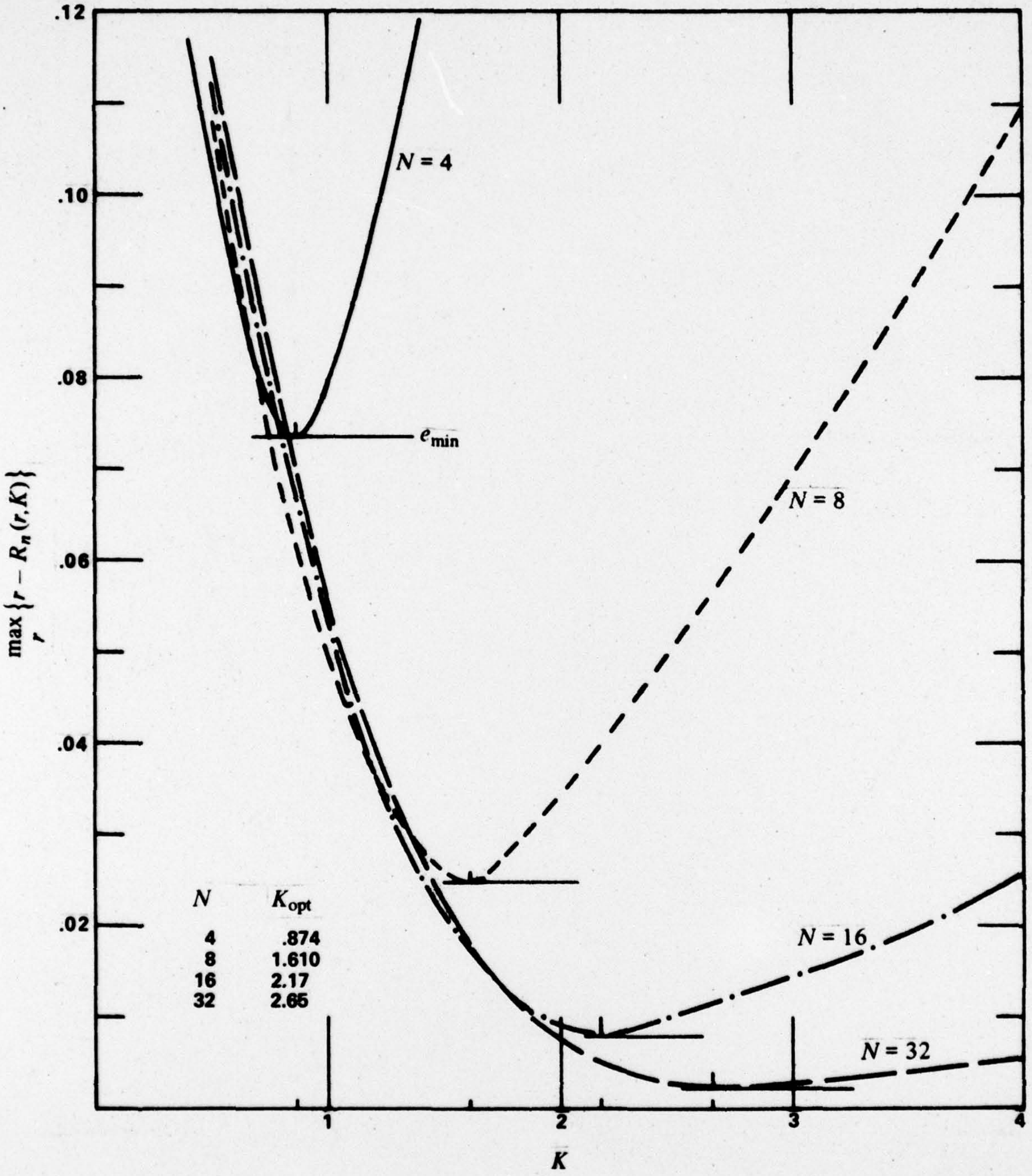

Figure 4. Maximum Deviation from Linearity vs. the Input Quantizing Range with the Number of Quantizing Steps as Parameter

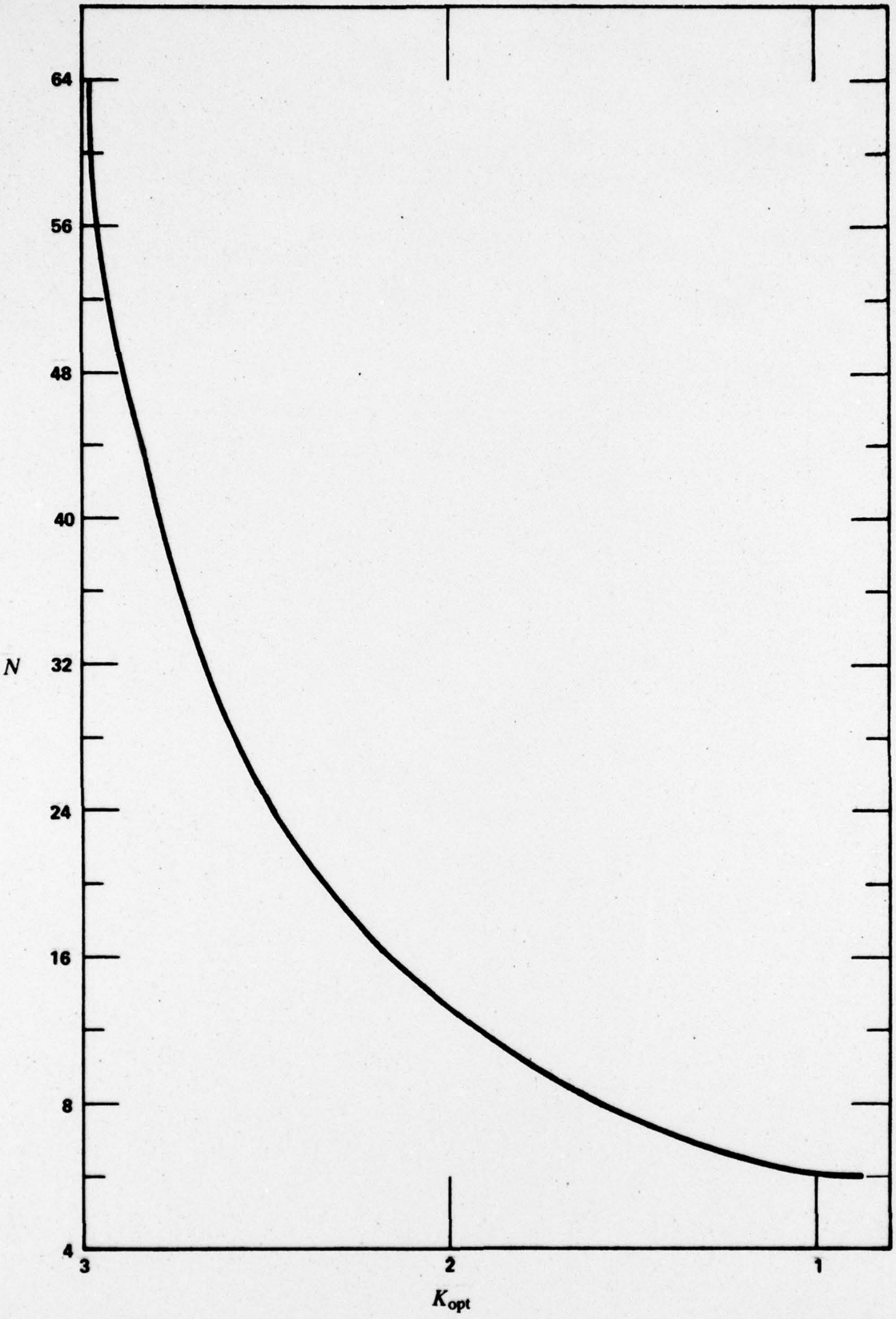

Figure 5. The Optimum Input Quantizing Range, K_{opt}, vs. the Number of Quantizing Steps, N

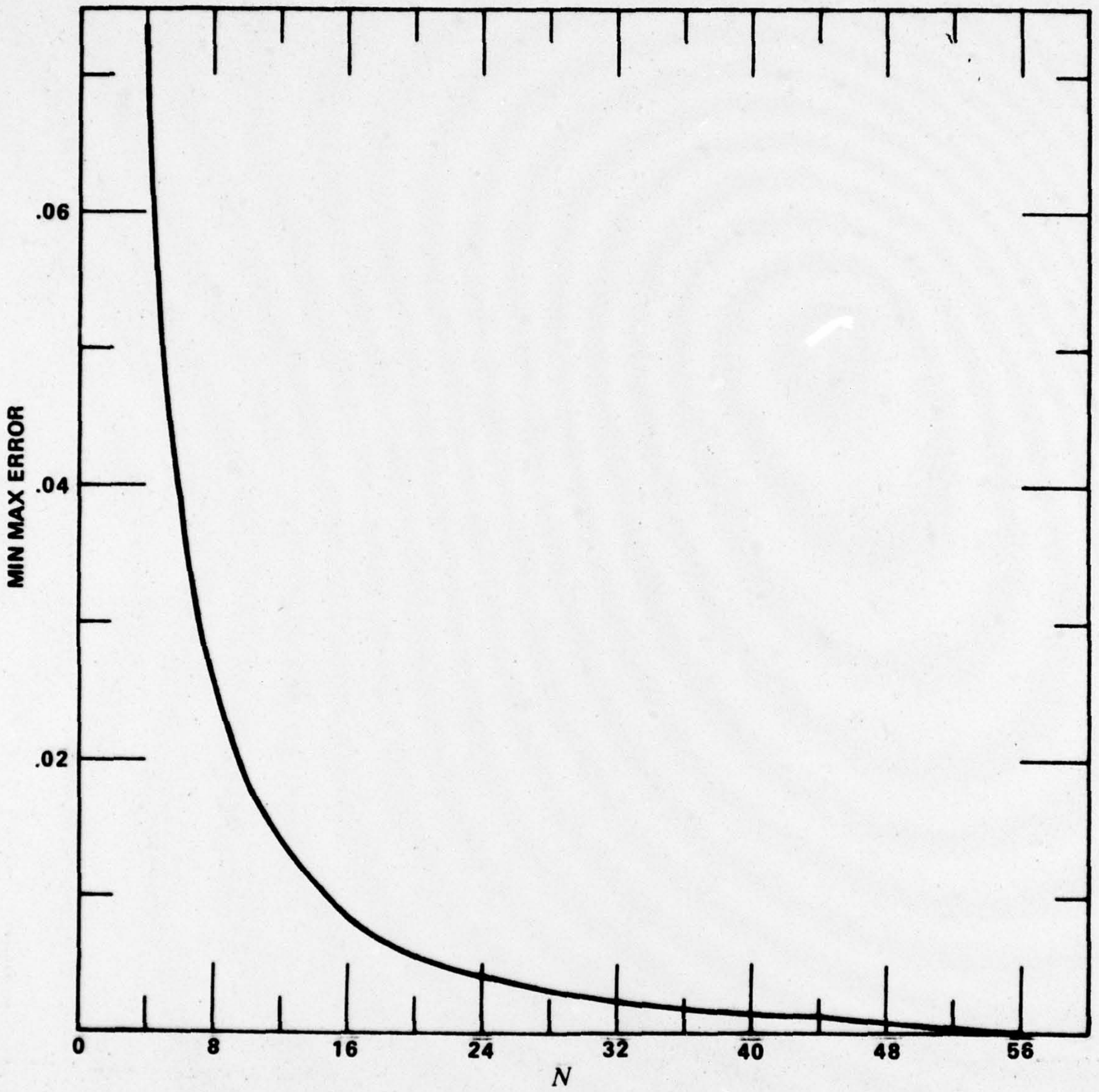

Figure 6. The Minimum Maximum Deviation from Linearity vs. the Number of Quantizing Steps

APPENDIX I

DERIVATION OF THE AVERAGE CORRELATOR OUTPUT

The average output of the hybrid correlator is

$$R(r,K) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{sgn} x_1 \; Q(x_2)\, p(x_1,x_2)\, dx_1\, dx_2,$$

where $\operatorname{sgn} x_1 = \begin{cases} 1, x_1 > 0 \\ -1, x_1 < 0 \end{cases}$,

and $Q(x_2)$ is the transfer characteristic of the linear quantizer (see Fig. 2).

$$R(r,K) = 2 \int_0^{\infty} \int_{q_{N/2-1}}^{\infty} [p(x_1,x_2;r) - p(x_1,x_2;-r)]\; dx_1\, dx_2$$

$$+2 \sum_{j=1}^{N/2-1} z_j \int_0^{\infty} \int_{q_{j-1}}^{q_j} [p(x_1,x_2;r) - p(x_1,x_2;-r)]\; dx_1\, dx_2 .$$

Use of

$$2 \int_0^{\infty} p(x_1,x_2;r)\, dx_1 = \frac{1}{\pi\sqrt{1-r^2}} \int_0^{\infty} \exp\left[\frac{x_1^2 + x_2^2 - 2rx_1x_2}{2(1-r^2)}\right] dx_1$$

$$= \frac{1}{\pi\sqrt{1-r^2}} \exp\left(-\frac{x_2^2}{2}\right) \int_0^{\infty} \exp\left\{-\left[\frac{x_1 - rx_2}{\sqrt{2(1-r^2)}}\right]^2\right\} dx_1$$

$$= \frac{1}{\pi\sqrt{1-r^2}} \exp\left(-\frac{x_2^2}{2}\right) \int_{\frac{-rx_2}{\sqrt{2(1-r^2)}}}^{\infty} \sqrt{2(1-r^2)}\; e^{-v^2}\, dv$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_2^2}{2}\right)\left[1 - \mathrm{erf}\left(\frac{-rx_2}{\sqrt{2(1-r^2)}}\right)\right],$$

yields

$$R(r,K) = \sum_{j=1}^{N/2-1} z_j \int_{q_{j-1}}^{q_j} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)\left[1 - \mathrm{erf}\left(\frac{-ru}{\sqrt{2(1-r^2)}}\right) - 1 + \mathrm{erf}\left(\frac{ru}{\sqrt{2(1-r^2)}}\right)\right] du$$

$$+ \int_{q_{N/2-1}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)\left[1 - \mathrm{erf}\left(\frac{-ru}{\sqrt{2(1-r^2)}}\right) - 1 + \mathrm{erf}\left(\frac{ru}{\sqrt{2(1-r^2)}}\right)\right] du.$$

Then, with the fact that the error function is odd the result is the average correlator output (see eq. 2).

APPENDIX II

APPROXIMATION USED FOR THE ERROR FUNCTION

The integrals required for calculation of $R(r,K)$ used Simpson's integration rule and the following approximation for the error function (4):

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \approx 1 - [1 + a_1 x + a_2 x_2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6]^{-16}$$

where $a_1 = 7.05230784 \times 10^{-2}$

$a_2 = 4.22820123 \times 10^{-2}$

$a_3 = 9.2705272 \times 10^{-3}$

$a_4 = 1.520143 \times 10^{-4}$

$a_5 = 2.765672 \times 10^{-4}$

$a_6 = 4.30638 \times 10^{-5}$

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