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COMPRESSION OF A MAGNETIC FIELD BY A SHELL OF CONSTANT CONDUCTIVITY

by

A. Ye. Kulago

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By: A. Ye. Kulago

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О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
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GREEK ALPHABET

Alpha	Α	α	α		Nu	Ν	ν	
Beta	Β	β			Xi	Ξ	ξ	
Gamma	Γ	γ			Omicron	Ο	ο	
Delta	Δ	δ			Pi	Π	π	
Epsilon	Ε	ε	ϵ		Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ			Sigma	Σ	σ	ς
Eta	Η	η			Tau	Τ	τ	
Theta	Θ	θ	ϑ		Upsilon	Υ	υ	
Iota	Ι	ι			Phi	Φ	φ	ϕ
Kappa	Κ	ϰ	κ	ϰ	Chi	Χ	χ	
Lambda	Λ	λ			Psi	Ψ	ψ	
Mu	Μ	μ			Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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COMPRESSION OF A MAGNETIC FIELD BY A SHELL OF CONSTANT CONDUCTIVITY

A. Ye. Kulago

Equations are obtained for a magnetic field compressed by a cylindrical shell of constant conductivity. Solutions are given in some particular cases.

§1. Formulation of the problem and its solution by the integral transformation method. Ya. P. Terletskiy was the first to point out the possibility of obtaining superstrong magnetic fields through the compression of the field [1]. Fields on the order of 10^7 G were obtained in experiments [2, 3] based on this method. The plane problem on compressing a magnetic field without consideration of biased currents was solved in articles [4-6]. The authors of work [7] examined diffusion of a magnetic field in a plate and shell with the motion of the metal's melting zone. The plane and axisymmetrical problem for an ideally conducting boundary with consideration of the bias currents was solved by I. M. Rutkevich [8]. The problem of compressing a magnetic field by a cylindrical shell of finite conductivity without consideration of bias currents is presented in this work.

Let us examine a cylindrical cavity with a radius R_0 which is compressed with a rate of $\vec{V}$. We will consider region D_2 infinite. At the initial point in time $t = 0$ the magnetic field was homogeneous $H = H_0$ in the cavity (region D_1) and $H = 0$ in the conductor (region D_2). The electrical field E was absent with t = 0 and in the center of the cavity

$$E(0,t)=0. \tag{1}$$

In this case the equations for H and E are the following

$$\operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t},\quad \operatorname{div}\vec{H}=0,\quad \operatorname{div}\vec{E}=0,\qquad r\in D_1+D_2. \tag{2}$$

$$\operatorname{rot}\vec{H} = \frac{1}{c}\cdot\frac{\partial\vec{E}}{\partial t},\qquad r\in D_1, \tag{3}$$

$$\operatorname{rot}\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\cdot\frac{\partial\vec{E}}{\partial t}, \tag{4}$$

$$r\in D_2.$$

$$\vec{j} = \sigma\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{H}\right]. \tag{5}$$

Let us use the cylindrical coordinate system r, ϕ, z. The z-axis is the axis of symmetry of this problem. The vector $\vec{H}$ will have the component $H_z(r,t)$, $\vec{E} = E_\phi(r,t)$ and $\vec{v} - v_r = v$. Ignoring the bias currents and keeping in mind that div $\vec{H} = 0$, we can consider the field H_z homogeneous in D_1, that is $H_z = H(t)$.

The conditions for the uniting of the solutions on the boundary of regions D_1 and D_2 will be

$$E|_{\gamma_-} = E|_{\gamma_+},\quad H|_{\gamma_-} = H|_{\gamma_+},$$

that is the magnetic and electrical field on the boundary is continuous. Integrating the first equation of system (2) from r = 0 to r = R, where $R = R_0 - \int_0^t v(u)\,du$, and considering (1) we obtain

$$\frac{d(HR^2)}{dt} = -2cE - vH, \tag{6}$$

where H - the field in cavity D_1 and E - the electrical field on the boundary γ. Since the rate of diffusion of the magnetic field is considerably greater than the rate of motion of the boundary, we will solve the problem of diffusion of the magnetic field in a motionless conductor considering that at this stage R = R(t') is constant. Recalculation of E and H for a moving conductor is accomplished in accordance with the well-known formulas

$$\vec{H}' = \vec{H},\quad \vec{E}' = \vec{E} + \frac{1}{c}[\vec{v}\times\vec{H}].$$

where the prime sign signifies the moving coordinate system.

We find E and H in D_2. For this purpose we use the integral Laplace transforms

$$H(r, p) = \int_0^\infty e^{-pt} H(r, t)\,dt, \quad E(r, p) = \int_0^\infty e^{-pt} E(r, t)\,dt.$$

Using equations (2) and (4) and disregarding the bias currents, we obtain

$$\frac{d^2H}{dr^2} + \frac{1}{r}\cdot\frac{dH}{dr} - \frac{4\pi\sigma}{c^2}\,pH = 0. \tag{7}$$

The solution of equation (7) will be

$$H(r, p) = A(p)\,Y_0(k, r),$$

where Y_0 - the Weber function of the zero index which tends to zero at infinity,

$$k^2 = -\frac{4\pi\sigma}{c^2}\,p.$$

We determine the function A(p) from the boundary condition in the following manner. Let

$$H(r, p) \doteqdot \tilde{H}(r, t), \quad A(p) \doteqdot \tilde{A}(t).$$

Then [9]

$$\tilde{H}(r, t) = \int_0^t \tilde{A}(t-u)\,\tilde{Y}_0(r, u)\,du,$$

and on the boundary with r = R

$$\tilde{H}(R, t) = \int_0^t \tilde{A}(t-u)\,\tilde{Y}_0(R, u)\,du.$$

Moving on again to the representations, we obtain

$$H(R, p) = A(p)\,Y_0(k, R).$$

where

$$H(R, p) \doteqdot \tilde{H}(t).$$

Then

$$H(r, p) = H(R, p)\,\frac{Y_0(kr)}{Y_0(kR)}.$$

Using the well-known theorems for operations analysis [9]:

$$\frac{A(p)}{pB(p)} \doteq \frac{A(0)}{B(0)} + \sum_{k=1}^{\infty} \frac{A(p_k)}{p_k B'(p_k)} e^{p_k t},$$

$$p\,\Phi_1(p)\,\Phi_2(p) \doteq \frac{d}{dt} \int_0^t f_1(t-u)\, f_2(u)\, du,$$

where A(p), B(p) - polynomials for p, we find

$$H(r, t) = \frac{d}{dt} \int_0^t H(u) \left[1 + 2 \sum_{k=1}^{\infty} \frac{Y_0\left(q_k \frac{r}{R}\right)}{Y_1(q_k)\, q_k^2} e^{-\frac{q_k^2}{\alpha(t)}(t-u)} \right] du,$$

where q_k - the roots of $Y_0(q) = 0$, $kR = q = \sqrt{\frac{4\pi\sigma}{c^2} R^2}\,\sqrt{p} = \sqrt{\alpha}\,\sqrt{p}$.

$Y_1(q) = \frac{dY_0}{dq}$. Here H(r, t) - the field in D_2 and H(t) - the field in D_1.

If we disregard the bias currents, the magnetic field H for moving and motionless conductors is the same. The electrical field for the motionless conductor equals

$$E = \frac{c}{4\pi\sigma} \cdot \frac{\partial H}{\partial r} = -\frac{c}{2\pi\sigma} \cdot \frac{d}{dt} \int_0^t H(u) \sum_{k=1}^{\infty} \frac{Y_1\left(q_k \frac{r}{R}\right)}{q_k R(t)\, Y_1(q_k)} e^{-\frac{q_k^2}{\alpha(t)}(t-u)} du.$$

Then

$$E|_r = \frac{c^2}{2\pi\sigma} \cdot \frac{d}{dt} \int_0^t H(u) \sum_{k=1}^{\infty} \frac{1}{R(t)\, q_k} e^{-\frac{q_k^2}{\alpha(t)}(t-u)} du. \tag{8}$$

Using equations (5), (6), (8) we obtain the equation for the intensity of the magnetic field in the cavity

$$\frac{d(RH)}{dt} = \frac{c^2}{\pi\sigma} \cdot \frac{d}{dt} \int_0^t \frac{H(u)}{R(t)} \sum_{k=1}^{\infty} \frac{1}{q_k} e^{-\frac{q_k^2}{\alpha}(t-u)} du + vH. \tag{9}$$

Integrating equation (9) we obtain

$$H(t) = \frac{c^2}{\pi\sigma} \cdot \frac{d}{dt} \int_0^t \frac{H(u)}{R^2(t)} \sum_{k=1}^{\infty} \frac{1}{q_k} e^{-\frac{q_k^2}{\alpha(t)}(t-u)} du + \int_0^t \frac{v(u)}{R(t)} H(u)(du) + \frac{H_0 R_0}{R(t)}. \tag{10}$$

Equation (10) is the Volterra integral equation.

§2. Uniform movement of the boundary. The limiting case for great conductivity. The following asymptotics for the roots are known [10]:

$$q_k \cong q_1 + (k-1)\pi.$$

Let us limit ourselves to one term of the series in equaiton (10) and let us consider the great significance of σ. We will consider that $R = R_0 - v_0 t$. Here v_0 - const. The equation (10) takes the form

$$H(t) = -\frac{c^2}{\pi q_1 \sigma}\int_0^t \frac{H(u)}{R^2(t)}\,du + \int_0^t \frac{v_0}{R(t)} H(u)\,du + \frac{H_0 R_0}{R(t)}. \qquad (11)$$

Integrating equation (11) for t and substituting in the obtained equation $\int_0^t H(u)\,du$ from equation (11), we obtain a first-order differential equation for H(t):

$$\frac{dH}{dt} = \frac{(c^2 + \pi q_1 \sigma v_0 R)^2 + \pi q_1 \sigma R (2 v_0 c^2 + \pi q_1 \sigma R v_0^2)}{\pi q_1 \sigma R^2 (c^2 + \pi q_1 \sigma v_0 R)} H - \frac{H_0 R_0 v_0 c^2}{R^2 (c^2 + \pi q_1 \sigma v_0 R)}. \qquad (12)$$

The solution for equation (12) will be

$$H = H_0 \frac{(c^2 + \pi q_1 \sigma v_0 R) R_0^3}{(c^2 + \pi q_1 \sigma v_0 R_0) R^3}\left[2 - \frac{R_0}{R} + \frac{\pi q_1 \sigma v_0 R_0}{c^2} \ln \frac{R_0 (c^2 + \pi q_1 \sigma v_0 R)}{R (c^2 + \pi q_1 \sigma v_0 R_0)}\right] e^{\frac{c^2 (R_0 - R)}{\pi q_1 \sigma v_0 R R_0}}. \qquad (13)$$

If, in equation (13), we direct σ toward infinity we obtain

$$H = H_0 \frac{R_0^2}{R^2}. \qquad (14)$$

Here the following relationship is used

$$\lim_{\sigma \to \infty} \frac{\pi q_1 \sigma v_0 R_0}{c^2} \ln \frac{R_0 (c^2 + \pi q_1 \sigma v_0 R)}{R (c^2 + \pi q_1 \sigma v_0 R_0)} = \frac{R_0 - R}{R}.$$

Solution (14) is the solution for an ideal conductor [1]. A comparison of solution (14) with solution (13) is shown in the drawing. The value of σ_1 is taken for copper at room temperature.

The curve for σ_3 has the maximum. Thus, the solution is valid up to certain values of compression of the cavity, from which it follows that the approximate equation (11) is valid only for sufficiently large values of σR.

In conclusion, I would like to acknowledge my graditude to V. V. Lokhin for help in the work.

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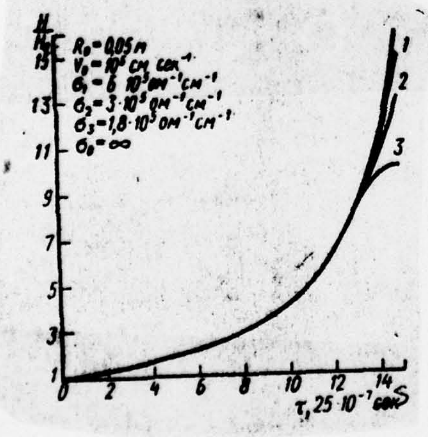

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