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DESIGNING A SUPPORT SYSTEM FOR REPAIRABLE ITEMS

by

Donald Gross
Charles E. Pinkus

Serial T-367
10 February 1978



The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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A model is developed which yields the best system design (number and location of repair depots, number of repair channels at each depot, and size of the spares inventory at each depot) to service a widely dispersed population of repairable units, subject to random failure. Considered in the design model is a service level constraint in terms of spares availability, as well as purchase costs of spares and repair channels, fixed costs of operating repair depots, and unit repair and transportation costs. Failure and repair times are assumed to be exponential and finite source queueing theory coupled with a branch-and-bound algorithm are utilized in developing the model.

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1. Introduction

A population of units (aircraft engine components, computer modules, lathes, etc.) that randomly fail but are completely repairable requires both spares and repair facilities to provide for certain desired levels of service. For example, it may be desired to have the system perform so that the probability that at least β percent of the population operating at any given time is $1-\alpha$. Another measure of system performance often used is the percentage of requests for spares met immediately from on-hand inventory. This latter criterion is referred to as spares availability and is the criterion used in this model; that is, the complement of the percentage of requests back ordered.

The physical system under consideration is of a cyclical nature in that there is a fixed number of components, say N . If we desire M of those to be operating at a given time, then there are $N - M \equiv y$ spares in the system.¹ Any component can be in one of three states: (1) operating,

¹We assume a component to be "basic," that is, upon failure, the entire component (not parts of it) is replaced by an identical spare unit. No indenture is considered in this study, as opposed to the MOD-METRIC model [see Muckstadt (1973)].

(2) spare, or (3) in the repair facility (either being worked on or in a repair queue).

In order to determine the appropriate number of spares (we assume M is given as input and it is desired to find y) and the repair facility capacity necessary to achieve a specified service level, a stochastic model is required since the failure and repair processes are random. The real stochastic process is extremely complex; many simplifying assumptions must be made to obtain analytical results. Therefore, the stochastic process modeled here is an approximation to the true underlying stochastic process, and the simplifying assumptions are duly noted. We believe, however, that the approximation is an adequate one and that the simplifying assumptions are justified, as discussed below.

2. Mathematical Model

Letting y equal the number of spares in the system and c equal the number of simultaneous repair channels in the repair depot, we desire to find the values for c and y such that

$$\underset{c,y}{\text{Minimize}} \quad Z(c,y) = C_I(c,y) + C_{II}(c,y) \quad (1)$$

$$\text{Subject to} \quad \sum_{n=0}^{y-1} q_n \geq 1-\alpha, \quad (2)$$

where

$C_I(c,y)$ = annually prorated one-time costs such as purchase costs, salvage values, set-up costs, etc.

$C_{II}(c,y)$ = annually recurring costs such as repairmen salaries, holding costs of spares, unit repair costs, etc.

q_n = probability that n units are in the repair queue or in repair when a request for a spare occurs (an item fails).

The problem then is one of finding the combination of c and y that minimizes annual equivalent costs over the life of the system subject to providing a service level of $1-\alpha$ (percentage of requests for spares filled immediately from on-hand inventory).

Treating Equation (2) first, $\sum_{n=0}^{y-1} q_n$ is the probability that when a request occurs, at least one spare is available; that is, fewer than y units are in or awaiting repair. Considering the repair process, q_n are simply the conditional (upon a failure) probabilities that the system state, which is measured by the number in the repair system either in queue or being worked on, is n . These q_n are functions of the general time system-state probabilities (call them p_n) which, with some assumptions, can be determined from classical queueing theory. This will be elaborated on below; however, first we treat Equation (1) in detail.

Suppose the system is expected to be used for k years, and that r is the annual interest charge on capital. Then a one-time expenditure at the beginning of the first year can be converted to an annual equivalent expenditure by applying the capital recovery factor, $r(1+r)^k / [(1+r)^k - 1]$ [see, for example, Maynard (1956), pp. 7-66 to 7-77]. Salvage value received at the end of k years can be converted to an annual equivalent return (negative cost) by applying the sinking fund factor, $r / [(1+r)^k - 1]$. Using the following additional notation,

- $C_{P,y}$ = purchase cost of a spare (\$ per unit)
- $C_{P,c}$ = purchase cost of a repair channel (\$ per channel)
- $C_{S,y}$ = salvage value of a spare after k years (\$ per unit)
- $C_{S,c}$ = salvage value of a repair channel after k years (\$ per channel)
- C_O = operating cost of a repair channel (\$ per year per channel)
- C_I = carrying and handling costs of a spare (\$ per year per spare in system)
- C_R = unit repair cost (\$ per unit repaired)
- F_{CR} = capital recovery factor
- F_{SF} = sinking fund factor
- $\bar{R}(c,y)$ = average number of units repaired per year,

Equation (1) expands to

$$\begin{aligned} \text{Minimize}_{c,y} \quad Z(c,y) = & (C_{P,y} F_{CR} - C_{S,y} F_{SF} + C_I)y \\ & + (C_{P,c} F_{CR} - C_{S,c} F_{SF} + C_O)c \\ & + C_R \bar{R}(c,y), \end{aligned} \quad (3)$$

where $\bar{R}$ depends on c and y and the stochastic process model.

3. Stochastic Process Model

The stochastic process modeled here is a finite population of items that randomly fail. Upon failure, the failed item is removed, transported to the repair facility, repaired, and dispatched to the spares pool. Simultaneously a spare is requested and "plugged in" immediately if one is available. If no spares are available (y or more units in or awaiting repair) the request is back ordered. The service level constraint requires the percentage of requests back ordered to be $\leq \alpha$. This process is schematically shown in Figure 1.

Assuming that failures and removal, transportation, and repair times are random yields a cyclic queueing process, since we also assume complete repair capability and no addition or withdrawal of items. When failures are Poisson and removal, transportation, and repair times exponential, analytical results can be obtained [see Gross and Ince (1977)]. A further simplifying assumption in which removals and transportation times are insignificant or are included in the repair times so that the cyclic queue consists of only two stages--the ready unit stage and the repair stage--reduces the model to that of the classical machine repair problem with spares [see Gross, Kahn, and Marsh (1977)].

The key assumptions necessary, enabling use of the classical queueing theory to determine q_n and $\bar{R}$ in Equations (2) and (3), are then

1. Poisson failure process,
2. exponential repair (transport and removal) times,
3. all units identical,

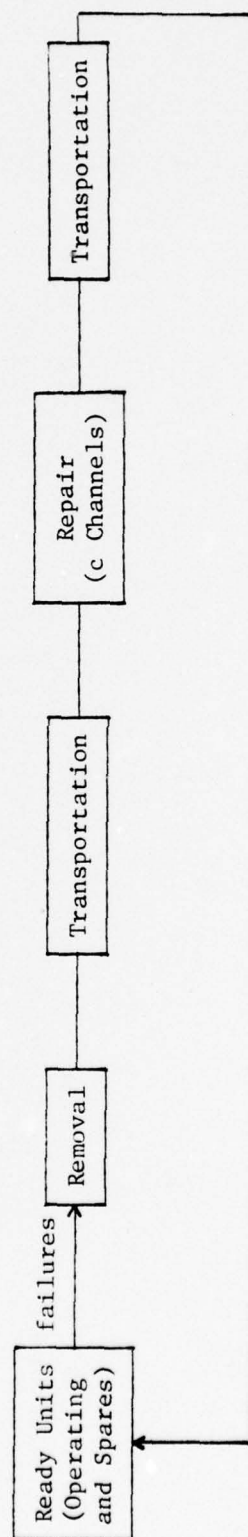


Figure 1.--Repair process.

4. continuous operation of units and repair facilities,
5. steady state.

We now discuss these in detail.

3.1 Poisson failures

A Poisson failure process, or equivalently, exponential times to failure, is commonly assumed when modeling lifetime distributions of many components, and especially those with considerable electronic hardware. It has also been used extensively when dealing with aircraft engines, particularly jet aircraft. While there is great impetus to use this assumption for mathematical convenience, its final justification must rely on how well it approximates the real world. In many cases, data have borne out the reasonableness of the Poisson assumption; however, data are not always available, for example, for a new component. In such cases, either data on similar components (justifying the assumption for gas turbine ship engines using aircraft jet engine data, for example) or characteristics of the Poisson/exponential process itself, such as the Markovian property, the long tail of the exponential distribution, and so on, must be utilized.

If one considers the shape of the exponential density function, the long tail to the right indicates that one would occasionally experience a very long time to failure, but that most of the failure times are of moderate length. The Markovian or constant failure (hazard) rate property says that the item exhibits no wear-out; that is, an item that has been operating a long time has no more chance of failing than a brand new item. These two characteristics seem to be quite reasonable for the kinds of items encountered in a repairable inventory system--jet engines, avionics equipment, etc.

Another property of the exponential distribution is that the minimum of a set of exponential random variables is also exponential. Thus, if a unit is rather complex with many "pieces," each of which can fail approximately exponentially causing the unit to fail, the unit has an exponential failure distribution.

3.2 Exponential repair (transport and removal) times

In order to use classical queueing theory, repair times must be exponentially distributed unless the repair capacity has "ample servers;" that is, no queueing ever occurs [see Posner and Bernholtz (1968)]. The same holds true for removal and transport times. It is rare for repair depots to have an "ample" number of channels; that is, a potential channel for every item in the system. The ample server assumption, however, may be more realistic for the transport and removal functions.

Unless there are ample servers, the justification of exponentiality for these times must be done in a manner similar to that used to justify failure times. If there is a sizable human element in the service process, one might expect more symmetrical distributions for service times with increasing hazard rates; i.e., the longer an item has been in service, the greater the probability of its being completed in the next instant of time. However, if service is mainly diagnostic rather than routine, or if it is very heterogeneous in nature (each repair differing from the last), then the exponential assumption with its long tail and Markovian property could be quite realistic. Again, either a data analysis or knowledge of how the process is likely to work would be required for final justification.

3.3 Identical units, continuous operating, and steady state

These three assumptions are treated in some detail in Barzily, Gross, and Kahn (1977), and are shown to be adequate for repairable item systems. We discuss these briefly here and refer the interested reader to the reference.

To use classical cyclic or machine repair queueing theory, all units must have identical failure and repair characteristics, so that it is necessary only to keep track of how many are at each stage in the cycle, and not which individual units are where. In reality, this is generally not so; some repaired items may not be as good as original items and some may be better due to technological learning. If items are introduced over

time, later items may be more reliable. Newer items may also be easier to repair. To account for this while using classical queueing theory, we averaged characteristics of all items in the population and assumed a population of identical items, all operating with these "average" characteristics. An analysis in the reference cited above showed that unless item characteristics differed by an order of magnitude, the averaging assumption was adequate.

Classical queueing theory assumes that all ready items in the population (except spares) operate continuously, as do the repair facilities. Often in the real situation, while both repair facilities and operating units keep the same hours, all population units may not be operating simultaneously. For example, a fleet of aircraft with M total engines requires a support system (repair facilities and spares), but not all aircraft operate all the time. Furthermore, those not operating at any time cannot be considered to be spares, for they may be regularly or randomly scheduled for operation. This is usually accounted for by "stretching out" the mean time to failure of each unit and "pretending" all units operate at a reduced failure rate. Another adjustment method suggested in Barzily et al. (1977) is to find an average percentage of the population which is operating at any time and to adjust M by this percentage while using the true failure rate. These two methods were compared and found to be in very close agreement.

The problem of steady-state results was also analyzed in Barzily et al., where it was found that the steady state was quickly approached in most cases unless population characteristics were changing very rapidly. When designing support facilities for a population of units, interest is centered on the population once it has reached maturity, rather than during its infant years when things are changing rapidly. That is, facilities are designed to handle the estimated steady-state situation, and while the estimates of some steady-state input parameters, such as final attainable reliability, may be less certain, these can be handled by a type of sensitivity analysis that considers different possible steady-state situations.

Once the above assumptions are made and their ramifications kept in mind, a classical queueing model can be used to yield the q_n and $\bar{R}$ of Equations (2) and (3), respectively. We refer the reader to Gross, Kahn, and Marsh (1977) and to Gross and Ince (1977) for details. The best combinations of c and y that minimize the objective function $Z(c,y)$ of (3) can be found using a two-variable search or can be closely estimated using a heuristic algorithm presented in Gross, Kahn, and Marsh (1977). The model is further extended below to consider multiple repair depot locations as well as simultaneously provisioning for more than one type of item, which can share repair depots but requires dedicated repair channels within the depot.

4. Multiunit, Multilocation Design Model

The basic model(s), described above for a single population and single repair facility, can be coupled with a previously developed multi-activity, multifacility design model that utilizes a branch and bound algorithm [see Pinkus, Gross, and Soland (1973)] to determine the number of spares and the number of repair channels for each type of unit at each repair facility, as well as how many and where the repair facilities should be located among available sites. We first illustrate the concepts of the model on a small problem in which complete enumeration can be used in place of the branch and bound algorithm, and then illustrate the power of the branch and bound methodology on a large problem.

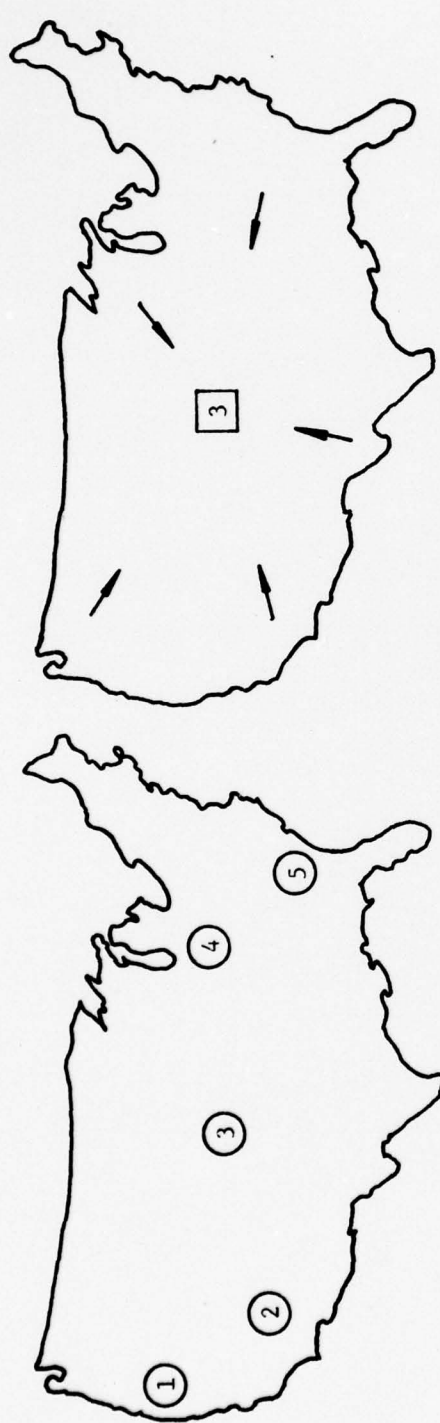
Consider a hypothetical fleet of 200 domestic aircraft with four types of repairable units that can be accommodated in the same repair depot as long as separate repair channels are used for each of the four unit types. Each aircraft requires three each of unit types 1 and 2 and two each of unit types 3 and 4. For example, one unit type might be a major engine component, another a landing gear assembly, and so on. Since on the average only half the planes operate at any one time, our effective fleet size is 100. Letting M_i denote the operating population size of unit type i , we have $M_1 = M_2 = 300$ and $M_3 = M_4 = 200$. We have three possible

repair depot designs to consider. Design 1 consists of one central repair depot in the midwest that supports the entire fleet. Design 2 consists of two additional sites on, say, the east and west coasts, so that each depot serves one-third of the fleet. Design 3 has two sites in addition to that, or five depot sites in all, which are distributed around the country so that each services one-fifth of the fleet. No transshipping is allowed; that is, each depot services a specific portion of the fleet and it is assumed that each site has the potential to support a depot of any desired capacity. Figure 2 shows the possible designs schematically.

Given any design, it is known what portion of the fleet is served by each depot, so that the previous model [the model giving the solution to Equations (2) and (3)] can be used separately for each unit type population at each depot for each design. We use the acronym SASPRO (Spare and Server Provisioning) to refer to this model. For example, consider Design 2 (three depot locations) for unit type 3. Each of the three depots serves a population of size $67 \left(\frac{1}{3} [100 \times 2] \right)$. With the appropriate costs, failure rate, and turnaround (removal + transport + repair) time,¹ the number of repair channels and spares for each depot can be determined from SASPRO, along with total costs, from Equation (3). To compare different designs for a given unit type, we must account for a reduction in repair time due to closer depots, and must also introduce transportation costs, which were ignored in the basic model since it had always been assumed that all units went to a central depot. Further, the fixed cost of a depot must be considered when the possibility of various design configurations exists.

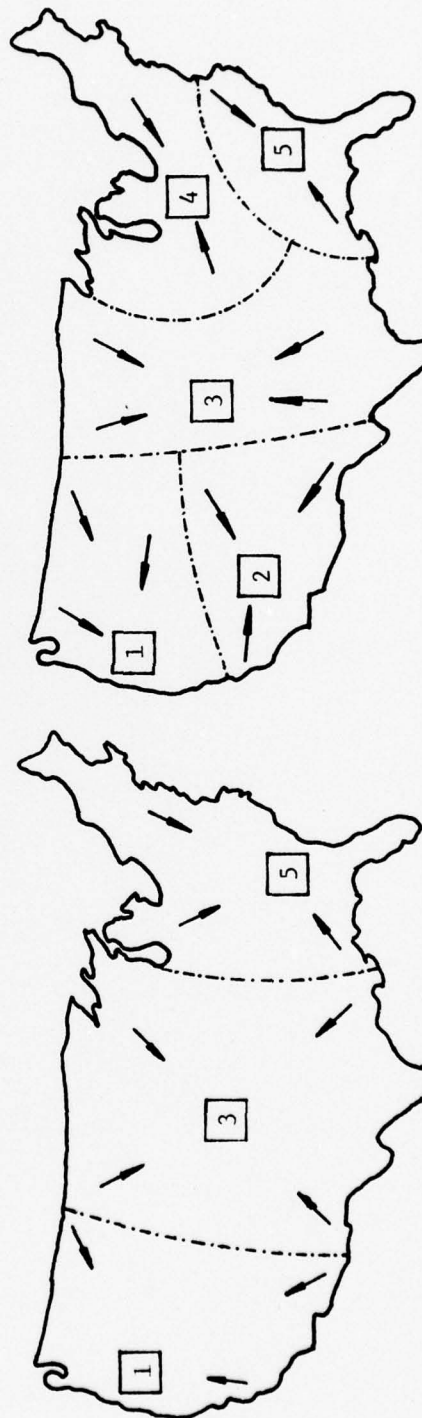
Basically, the design model considers four additional factors that must be traded off: (i) the decrease in transportation costs when more depots are involved, (ii) the reduction in turnaround time due to the proximity of depots, (iii) the increase in fixed facility costs when depots are added, and (iv) the possible economy of scale (fewer total system spares

¹We use in these examples the machine repair (two-stage cyclic queue) model, so that the service operation is considered to be one operation that includes removal, transport, and repair.



(a) Potential sites

(b) Design 1--one depot, Site 3



(c) Design 2--three depots, Sites 1, 3, 5

(d) Design 3--five depots, all sites

Figure 2.--Potential system designs

and repair channels may result from greater centralization of depots; that is, a few large depots rather than many small ones).

First, we consider only the costs shown in Equation (3) for all possible designs for each unit type. We assume a 20-year system life and an annual interest charge of 10%. The desired spares availability $(1-\alpha)$ is 90%. The failure rate, λ , is in units of failures per day, while the mean turnaround time is in units of days. All costs are in thousands of dollars. Table I shows the results of applying SASPRO to all possible designs for each type of unit. Note that the mean repair (turnaround) time decreases as more depots are added because of the smaller transport distance. It was assumed that transport time was 20% of the total turnaround time, and it was cut proportionately when additional depots were introduced. The costs shown in the next to last column are only the costs given by (3) that were incurred at each depot. Thus, for a design with three depots the costs must be tripled, as must c and y when compared with the single depot case. The costs for the total population are shown in the last column. The economy of scale is rather evident here when the total system spares and repair channels required for a single depot are compared to those required for the multidepot case. The last column clearly reflects this.

If transportation time were a significantly large portion of total turnaround time, the economy of scale of a centralized depot could be lost. However, even without this, when transportation costs are considered, a multilocation depot design could be preferable. Table II introduces transportation costs.

While transport time was cut in proportion to the number of depots (that is, n depots cut the transport time portion of the turnaround time to $1/n$), the costs would not be reduced by the same magnitude. We assume here that for three depots, transportation costs are cut in half, and for five depots, the transit costs are cut by two-thirds. Furthermore, we assume that the transport cost is a certain percentage of the purchase cost of an item, but that this percentage varies depending on the particular

TABLE I
SPARES, REPAIR CAPACITY, AND COSTS (10^3 DOLLARS) PER DEPOT AREA

Unit Type	Design No.	No. Depots	M	λ	$1/\mu$	$C_{p,c}$	$C_{s,c}$	C_o	$C_{p,y}$	$C_{s,y}$	C_I	C_R	c	y	Cost/Depot Area	Cost/Total Population
1	1	1	300	.00062	55.00	132	13	30	1,400	150	250	15	17	15	7,965	7,965
1	2	3	100	—	47.67	—	—	—	—	—	—	—	6	6	3,081	9,243
1	3	5	60	—	46.20	—	—	—	—	—	—	—	5	4	2,077	10,385
2	1	1	300	.00056	45.00	60	6	15	850	80	150	8.5	12	12	3,765	3,765
2	2	3	100	—	39.00	—	—	—	—	—	—	—	5	5	1,525	4,575
2	3	5	60	—	37.80	—	—	—	—	—	—	—	3	4	1,157	5,785
3	1	1	200	.001	60.00	15	5	7	210	20	45	35	24	17	3,940	3,940
3	2	3	67	—	52.00	—	—	—	—	—	—	—	6	7	1,389	4,167
3	3	5	40	—	50.40	—	—	—	—	—	—	—	4	5	888	4,440
4	1	1	200	.0005	120.00	350	20	80	350	40	65	67	16	19	6,377	6,377
4	2	3	67	—	104.00	—	—	—	—	—	—	—	6	7	2,279	6,837
4	3	5	40	—	100.80	—	—	—	—	—	—	—	4	5	1,497	7,485

TABLE II
TOTAL VARIABLE SYSTEM COSTS (10^3 DOLLARS)

Unit Type	Purchase Cost of Spares, $C_{p,y}$	Transport Cost as a % of $C_{p,y}$	Design Number	No. of Depots	Transport Cost per Repaired Item	Expected Number of Repaired Items per Depot per Year (R)	Total Transport Costs at All Depots per Year	Total Yearly Costs of Spares Provisioning (Table I)	Total Variable Yearly Costs
1	1,400	2	1	1	28	67.8	1,882	7,965	9,847
		↓	2	3	14	22.6	949	9,243	10,192
			3	5	9.33	13.6	634	10,385	11,019
2	850	23	1	1	195	61.3	11,953	3,765	15,718
		↓	2	3	97.5	20.4	5,967	4,575	10,542
			3	5	65	12.2	3,965	5,785	9,750
3	210	3.33	1	1	7	72.9	510	3,940	4,450
		↓	2	3	3.5	24.3	255	4,167	4,422
			3	5	2.33	14.5	169	4,440	4,609
4	350	20	1	1	70	36.4	2,548	6,377	8,925
		↓	2	3	35	12.2	1,281	6,837	8,118
			3	5	23.33	7.3	852	7,485	8,337

unit type. The percentages used are shown in Table II. Note, for example, that unit type 1 may be transported quite inexpensively with respect to its value (purchase cost); namely, its transport cost is only 2%, while the transport cost of unit type 2 is relatively expensive, or 23%.

The expected number repaired at each depot per year, $\bar{R}$, comes out of the SASPRO model. This figure, multiplied by transportation costs and number of depots, yields the total transportation cost per year per population of each type of unit for each possible repair depot design. Adding this to the costs given in Table I yields what we refer to as the total variable cost associated with each unit type population and depot design, and is given in the last column in Table II.

If the fixed costs of operating a repair depot are ignored, by searching the final column of Table II we see that the best design for each unit type is as follows:

Unit Type	Design Number	(Number of depots)
1	1	1
2	3	5
3	2	3
4	2	3

When introducing the fixed costs of operating a depot, however, it may be advisable to select a design other than that which appears best when only the variable costs are considered. Table III displays both variable and fixed costs. The circled elements indicate the best designs when only the variable costs are considered. Table IV, however, shows by complete enumeration the total costs; that is, the variable plus the fixed costs for all designs. Note that Design 2 includes Design 1 and that Design 3 includes Designs 1 and 2. In other words, if we decide upon Design 2, then if the variable costs of a particular unit type are less for Design 1, we would use only one depot for that particular component. Thus, when considering which is the best design, when examining Design 2 we can choose the lowest variable cost between Designs 1 and 2, and when considering Design 3, we can select the lowest variable cost of all three. Table IV indicates the best solution

TABLE III
VARIABLE AND FIXED COSTS

Design	Number of Depots	Variable Costs for Unit Type (\$/yr)				Fixed Costs for Depot Number (\$/yr)					Fixed Cost for Design
		1	2	3	4	1	2	3	4	5	
1	1	(9,847)	15,718	4,450	8,925	0	0	1,750	0	0	1,750
2	3	10,192	10,542	(4,422)	(8,118)	1,750	0	1,750	0	1,000	4,500
3	5	11,019	(9,750)	4,609	8,337	1,750	2,000	1,750	1,350	1,000	6,500

TABLE IV
SOLUTION BY ENUMERATION

Unit Type	Design 1	No. of Depots to Use	Design 2 (or 1)	No. of Depots to Use	Design 3 (or 1 or 2)	No. of Depots to Use
1	9,847	1	9,847	1	9,847	1
2	15,718	1	10,542	3	9,750	5
3	4,450	1	4,422	3	4,422	3
4	8,295	1	8,118	3	8,118	3
Total Variable Costs	38,310		32,929		32,137	
Fixed Depot Costs	1,750		4,500		6,500	
Total System Cost	40,060		37,429		38,637	

to be Design 2 (three depots) where unit types 2, 3, and 4 are to use three depots (one-third of the population serviced by each), and unit type 1 to use only the central depot; that is, the entire population is to be sent to one central depot. Here the economy of scale for number of spares and channels outweighs the transportation savings.

For larger problems, complete enumeration is not possible, but the branch and bound algorithm referred to previously and described in Pinkus *et al.* (1973) is quite efficient. We present an illustration in the following section.

4.1 Use of the Pinkus *et al.* branch and bound algorithm

In general, if there are m allowable designs, the total number of cases that must be tried to completely enumerate a solution is $\sum_{x=1}^m \binom{m}{x} = 2^m - 1$, since all combinations of designs taken one at a time, two at a time, ..., m at a time must be looked at. If there are n possible locations, the maximum number of allowable designs is $2^n - 1$ (any location can be in or out of the design) so that the total number of cases to consider for complete enumeration is

$$\sum_{x=1}^{2^n-1} \binom{2^n-1}{x} = 2^{2^n-1} - 1. \quad (4)$$

In the example to follow, there are seven allowable designs which would require $2^7 - 1 = 127$ cases to evaluate.

In the preceding example, which was solved by complete enumeration, there were three possible designs yielding a total of $2^3 - 1 = 7$ cases. Although it appears in Table IV that only three cases are being evaluated, Column 2 is really two cases (Design 2 and Design 2 or 1) and Column 3 is really four cases (Design 3, Design 3 or 1, Design 3 or 2, and Design 3 or 2 or 1). In certain cases, when one design is embedded within another,

the larger always dominates, making it unnecessary to actually evaluate the smaller. For example, since Design 1 is embedded within Design 2, Design 2 or 1 would always be less expensive than Design 2 by itself since the fixed costs are the same for both, but the variable costs might be less as some unit types might prefer Design 1 while others might prefer Design 2. Thus, in reality there may be fewer than $2^m - 1$ cases to consider, and when all combinations of designs involving n locations are allowable there are considerably fewer cases required to be searched than the upper bound given by (4). Because of the embedding of designs, it turns out that the number of cases to be searched is never more than $2^n - 1$. Hence a better upper bound to the number of cases required to be searched when there are m ($\leq 2^n - 1$) allowable designs is $\min\{2^m - 1, 2^n - 1\}$.

The branch and bound algorithm structure, under the worst case, would require searching $2^{n+1} - 1$ possible nodes [see Pinkus *et al.* (1973), for details]. However, the algorithm in all problems tried so far has needed to search only a small portion of these prior to finding the optimal design. The use of this algorithm is illustrated by the following example.

Consider a support system for repairing and storing the spares of 16 unit types. Seven different designs for the system have been proposed, with at most nine depots being considered for the entire system. These seven designs range from a completely centralized system (Design 1), which includes only one depot, to a decentralized system (Design 7) with nine depots. The depots included in each design are given in Table V.

TABLE V
SYSTEM DESIGNS FOR PROBLEM CONSISTING
OF 16 UNIT TYPES

Design Number	Depot Numbers
1	5
2	3,7
3	1,5,9
4	2,4,6,8
5	2,4,5,6,8
6	1,3,4,6,7,9
7	1,2,3,4,5,6,7,8,9

The 16 unit types to be repaired under this system have varying population sizes, failure rates, average repair times, and costs associated with their repair, the purchase and storage of spares, the purchase of repair channels, and travel to depots for repair. Although too numerous to include here in their entirety, these parameters, along with the required number of servers and repair channels given by SASPRO, are presented in Table VI for each unit type if repaired under Design 1. The entire set of parameters were considered for all seven designs in turn, for all unit types, in order to determine which of the seven designs minimized the total variable cost of the repair support system for each unit type. The results, that is, the "best" design for each unit type, are given in Table VII. However, using these designs requires a system that consists of all nine depots, resulting in an additional cost (fixed cost) of \$49 million (see Table VIII). This brings the total cost of the system, that is, the sum of the fixed and variable costs, to \$246.6 million.

Applying the branch and bound algorithm to minimize the sum of the variable and fixed costs of this repair support system produces the system design and variable costs given in Table IX. With the exception of unit types 1, 4, and 9, the variable costs are higher than those shown in Table VII. Thus, 13 of the 16 unit types are not being repaired under the designs that minimize their individual variable unit costs. However, the overall system design found by the branch and bound algorithm has reduced the fixed cost of operating depots to \$23.5 million (only five of the nine depots are required), resulting in a total system cost of \$242.6 million--a saving of \$4 million.

Thirty-three iterations of the branch and bound algorithm were required to solve this problem. This took 11.52 seconds on an IBM 370/148 computer.

4.2 Further work

Two areas of further work are being considered. First, we would like to incorporate space constraints into the branch and bound algorithm. These constraints would enable limitations to be placed on the total amount

TABLE VI
PARAMETERS AND RESULTING SASPRO SOLUTIONS AND VARIABLE COSTS FOR EACH
OF THE 16 UNIT TYPES REPAIRED UNDER THE SUPPORT SYSTEM OF DESIGN 1*

Unit Type	M	λ	1/u	$C_{p,c}$	$C_{s,c}$	C_o	$C_{p,y}$	$C_{s,y}$	C_I	C_R	c	y	Transportation Cost per Repaired Unit	Variable Cost
1	360	.00062	80.0	132	13	30	1,400	150	250	15	27	24	42.0	15,747.2
2	180	.001	60.0	15	5	7	210	20	45	35	16	16	21.0	4,921.4
3	540	.0005	200.0	350	20	80	350	40	65	57	62	67	52.5	25,328.0
4	720	.0005	45.0	60	6	15	850	80	150	10	26	22	8.5	8,466.5
5	360	.001	55.0	150	15	30	1,000	100	200	10	26	27	100.0	24,193.2
6	180	.002	40.0	30	5	10	500	50	100	20	22	20	6.0	6,865.1
7	540	.001	10.0	200	10	100	200	40	100	60	9	10	20.0	18,100.7
8	720	.0003	60.0	300	30	50	400	50	75	60	18	19	120.0	18,009.7
9	450	.001	30.0	350	50	20	700	200	100	50	18	20	10.5	14,585.3
10	400	.0005	60.0	100	20	20	500	200	50	50	17	18	150.0	17,016.3
11	300	.003	50.0	200	100	15	400	200	50	30	55	55	4.0	18,315.2
12	270	.003	15.0	200	100	15	400	200	50	30	17	18	4.0	12,346.3
13	360	.001	55.0	150	15	30	1,000	100	200	10	26	27	100.0	24,193.2
14	180	.002	60.0	30	5	10	500	50	100	20	28	29	6.0	8,363.0
15	540	.001	50.0	200	10	100	200	40	100	60	35	35	20.0	24,370.2
16	720	.0003	40.0	300	30	50	400	50	75	60	13	14	120.0	16,982.6

*All costs in \$ x 1,000.

TABLE VII
INDIVIDUAL "BEST" DESIGN
FOR EACH UNIT TYPE

Unit Type	Design Number	Variable Cost (\$ × 1,000)
1	1	15,747.2
2	6	4,478.4
3	7	18,706.8
4	1	8,466.5
5	6	18,732.2
6	6	6,731.0
7	7	15,908.1
8	7	10,310.7
9	1	14,585.3
10	7	9,451.7
11	7	13,490.4
12	6	12,335.1
13	7	11,925.9
14	7	7,170.5
15	7	19,214.3
16	7	10,310.7
Total Variable Cost:		\$197,564.8

TABLE VIII
FIXED DEPOT COSTS

Depot	Fixed Cost (10 ³ dollars)
1	7,000
2	3,000
3	4,500
4	8,000
5	4,500
6	5,000
7	7,000
8	3,000
9	7,000

TABLE IX
BRANCH AND BOUND SOLUTION

Unit Type	Design Number	Variable Cost (\$ × 1,000)
1	1	15,747.2
2	4	4,743.2
3	5	21,214.3
4	1	8,466.5
5	5	19,389.6
6	1	6,865.1
7	4	16,808.7
8	4	13,758.8
9	1	14,585.3
10	5	11,358.4
11	5	16,033.4
12	1	12,346.3
13	5	16,473.6
14	4	7,823.1
15	4	21,225.5
16	5	12,287.1
Total Variable Cost:		\$219,126.1

of space available at each depot for repair channels and the storage of spares. At present the design model assumes that a depot can handle, from a space point of view, anywhere from one to k unit types, where k in the above problem was 16.

Second, we are investigating the possibility of performing a sensitivity analysis of designs for unit types, in terms of their various possible parameters. Hopefully, this would lead to a narrowing of the system designs that would need to be considered for large-scale problems.

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