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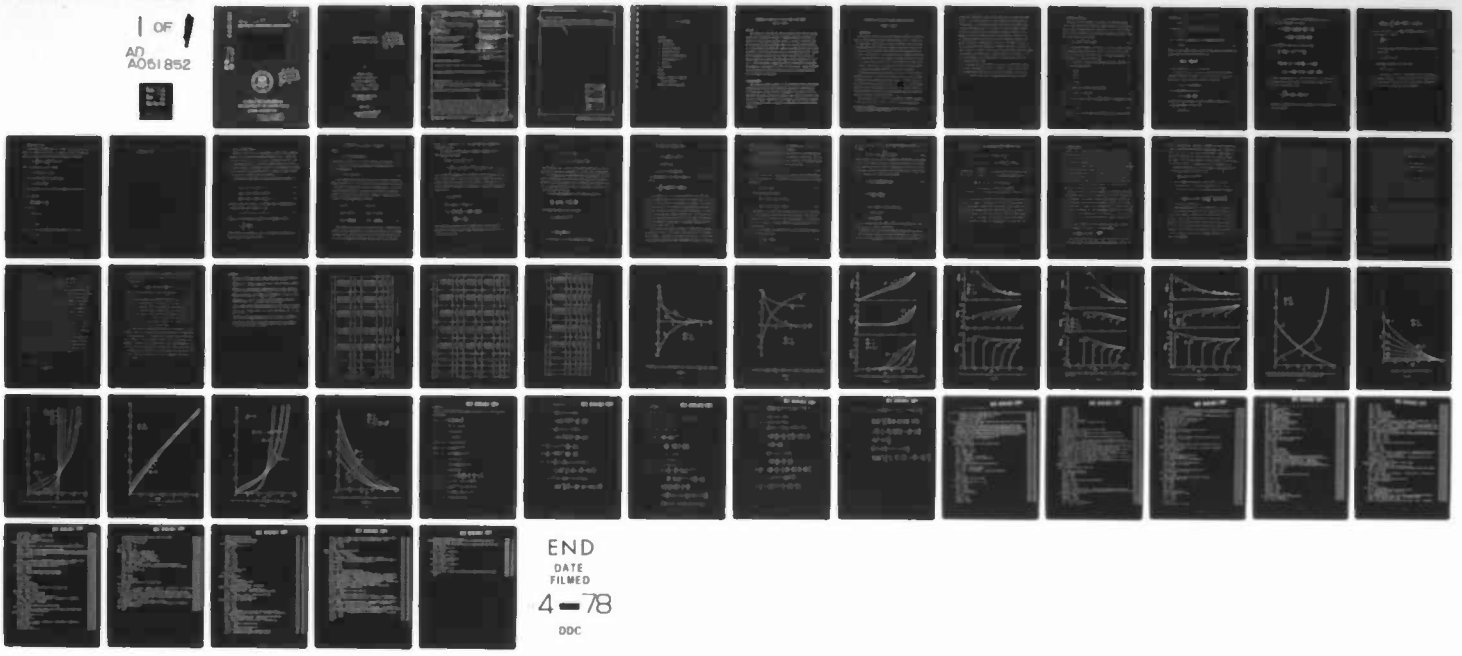

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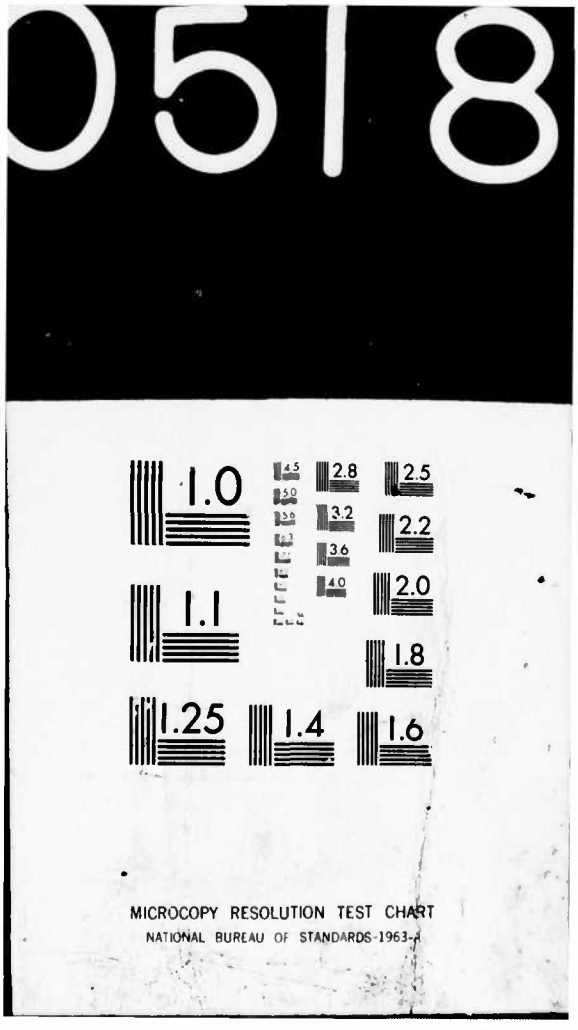
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PERTURBATION SOLUTIONS FOR VARIABLE ENERGY BLAST WAVES

by

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The trajectory of and the flow field behind blast waves with time varying energy input is determined. Freeman's Lagrangean coordinate formulation is modified to include both the geometric factor, α, for plane, cylindrical and spherical shocks and also non-integer values of β, the energy input parameter, in a single computational algorithm. Numerical problems associated with vanishing density at the fictitious piston face are then examined and solved. Second order perturbation solutions about the infinite strength shock are then obtained in Sakurai's inverse shock Mach number expansion parameter for $0 < \beta < \alpha + 1$.

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PERTURBATION SOLUTIONS FOR VARIABLE ENERGY BLAST WAVES

Edward T. Pitkin†

1. INTRODUCTION

The analytical theory of blast waves has been the subject of intensive study by many investigators since the pioneering work of G. I. Taylor[9] about thirty years ago. A large fraction of this work concerns waves of constant, instantaneously supplied energy which propagate and decay according to the dictates of inviscid isentropic flow behind the wave and Rankine-Hugoniot conditions at the wave front. This work has been well summarized by Sakurai[1] who has also developed an efficient method of expanding the general solution about the simpler limiting solution for the flow behind a very strong shock. Furthermore, he has shown that the square of the inverse shock Mach number is a most convenient and natural parameter to use for this expansion.

A lesser amount of work has been devoted to analysis of blast waves of variable energy input. In 1957 Lees and Kubota[6] examined this case briefly in connection with the hypersonic blast wave analogy and Rogers[3] presented the limiting strong shock solution in 1958. More recently in 1968 Freeman[2] examined this case in connection with cylindrical spark channel formation from exploding wires and Dabora[7] has applied variable blast wave techniques to droplet explosions in spray detonations.

The work reported here is basically an extension of Freeman's approach to cover the full spectrum of plane, cylindrical and spherical variable

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energy blast waves within a single analytic formulation and computation algorithm. It also contains corrections to some numerical errors discovered in Freeman's paper. As a consequence Freeman's notation has been adopted completely and derivations given in his paper have only been summarized here. His major contribution was to recast Sakurai's expansion procedure in Lagrangean coordinates, replacing distance from the origin, r, with mass, m, as an independent variable, but retaining the inverse shock Mach number as the expansion parameter. The advantage is two fold: (1) the number of differential equations that must ultimately be integrated numerically is reduced by nearly a factor of two, and (2) the position of the fictitious moving piston face which supplies the variable energy is uniquely defined to be at mass = 0. In Eulerian coordinates precise determination of the piston position can be a very difficult numerical task.

In sections 2 and 3, the equations of motion and the strong shock solution are reviewed. The expansion procedure is outlined in section 4 and a method of extrapolating the numerical solution to the mass origin is discussed in section 5. Numerical results are then summarized and discussed in section 6.

2. EQUATIONS OF MOTION

Let p_o and ρ_o be the pressure and density in the undisturbed medium while r is the radius from the origin to an arbitrary point in space and M is the total mass contained between the origin and r. Let r = R at the shock front so that the volume behind the shock front, when filled with gas of original density, will contain the mass

$$M_o = 2^{\alpha}\pi^{\alpha(3-\alpha)/2}R^{\alpha+1}\rho_o/(\alpha+1) \tag{2.1}$$

where α is a geometric parameter with values 0, 1, and 2 for plane, cylindrical and spherical waves respectively. Finally let u be the speed at any point and U be the speed of the shock front, C the speed of sound and γ the specific heat ratio in the undisturbed gas.

It is convenient to introduce non-dimensional variables. For mass, radius, velocity and density take

$$m \triangleq M/M_o \tag{2.2}$$

$$a \triangleq r/R \tag{2.3}$$

$$f \triangleq u/U \tag{2.4}$$

$$h \triangleq \rho/\rho_o \tag{2.5}$$

while the square of the inverse shock Mach number

$$y \triangleq C^2/U^2 = C^2/(dR/dt)^2 \tag{2.6}$$

may be combined with p_o to define a variable related to non-dimensional pressure

$$g = py/p_o \tag{2.7}$$

An arbitrary non-zero initial radius of the shock, R_o, is then used to define

a characteristic time

$$t_o \triangleq R_o/C \tag{2.8}$$

which may then be used to non-dimensionalize time through

$$\omega \triangleq t/t_o \tag{2.9}$$

while the shock front coordinate, R, is non-dimensionalized by

$$z \triangleq R/R_o \tag{2.10}$$

Finally the decay index, a ratio of the percentage changes of shock Mach number and radius, as introduced by Sakurai[1], is

$$\lambda = \frac{(dy/y)}{(dz/z)} = \frac{-2Rd^2R/dt^2}{(dR/dt)^2} , \tag{2.11}$$

the latter obtained by use of (2.6) and (2.10).

The equations of fluid flow behind the shock wave have been derived in Lagrangean coordinates by Freeman[2]. Only the results will therefore be summarized here. First the density at any point is given by

$$h = \rho/\rho_o = 1/[(\alpha+1)a^{\alpha}\partial a/\partial m] \tag{2.12}$$

and the non-dimensional velocity is

$$f = a-(\alpha+1)m\frac{\partial a}{\partial m} + \lambda y\frac{\partial a}{\partial y}. \tag{2.13}$$

Beginning with the assumption of a purely isentropic process behind the shock for each mass element it follows that

$$\partial(p/\rho^{\gamma})/\partial y = 0 \tag{2.14}$$

which upon using the non-dimensional variables above can be shown to lead to the following in Lagrangean coordinates,

$$(\alpha+1)m\left[\left(\frac{\alpha}{a}\frac{\partial a}{\partial m}+\frac{1}{\gamma g}\frac{\partial g}{\partial m}\right)\frac{\partial a}{\partial m}+\frac{\partial^2 a}{\partial m^2}\right]+\frac{\lambda}{\gamma}\frac{\partial a}{\partial m}$$

$$-\lambda y\left[\left(\frac{\alpha}{a}\frac{\partial a}{\partial y}+\frac{1}{\gamma g}\frac{\partial g}{\partial y}\right)\frac{\partial a}{\partial m}+\frac{\partial^2 a}{\partial m\partial y}\right]=0\ . \tag{2.15}$$

Similarly Newton's second law for the mass element

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial y}\frac{dy}{dt}=-(2r)^{\alpha}\pi^{\alpha(3-\alpha)/2}\frac{\partial p}{\partial m} \tag{2.16}$$

leads to

$$\frac{a^{\alpha}(\alpha+1)}{\gamma}\frac{\partial g}{\partial m}-\frac{\lambda a}{2}+(\alpha+1)\frac{2\alpha+\lambda}{2}m\frac{\partial a}{\partial m}+(\alpha+1)^2m^2\frac{\partial^2 a}{\partial m^2}$$

$$+\lambda y\left[-2(\alpha+1)m\frac{\partial^2 a}{\partial m\partial y}+\frac{2+\lambda}{2}\frac{\partial a}{\partial y}+y\frac{\partial \alpha}{\partial y}\frac{\partial \lambda}{\partial y}+\lambda y\frac{\partial^2 a}{\partial y^2}\right]=0\ . \tag{2.17}$$

The energy contained between the origin and the shock is the sum of the kinetic energy and the internal energy. The increment of energy over that of the same volume of undisturbed gas is

$$E=\int_o^{M_o}\left(\frac{u^2}{2}+\frac{p}{(\gamma-1)\rho}-\frac{p_o}{(\gamma-1)\rho_o}\right)dM\ . \tag{2.18}$$

Converting to non-dimensional form, using Equation (2.12) and letting a prime designate $\partial/\partial m$ gives

$$\frac{E\rho_o}{(\alpha+1)p_oM_o} = \frac{1}{y}\int_0^1 \left(\frac{\gamma f^2}{2(\alpha+1)} + \frac{ga^\alpha a'}{\gamma-1}\right)dm - \frac{1}{(\gamma-1)(\alpha+1)} \quad . \tag{2.19}$$

Letting $b = 1/[(\gamma-1)(\alpha+1)]$, $E_\alpha = E/\left|2\pi^{(3-\alpha)/2}\right|^\alpha$ and designating the integral J after Sakurai[1] this becomes

$$\frac{E_\alpha}{p_oR^{\alpha+1}} = \frac{J}{y} - b \quad . \tag{2.20}$$

If the total energy behind the shock varies as a power of time according to the following rule

$$E_\alpha = R_o{}^{\alpha+1}p_o(t/t_o)^\beta, \tag{2.21}$$

the energy equation reduces to the simple form

$$\omega^\beta/z^{\alpha+1} = J/y - b. \tag{2.22}$$

When $\beta = 0$ the energy is constant, a case examined by many investigators and well documented by Sakurai. Letting β take non-zero values allows one to model a fairly wide range of energy inputs which can be further extended with some modification of (2.21) as has been shown by Freeman[2].

3. STRONG SHOCK LIMIT

At very large shock Mach numbers $y \to 0$ and $J/y >> b$ so the energy equation reduces to a simple form consistent with Rogers'[3] similarity solution for the equation of motion for which the shock front advances according to

$$R = \left[R_o^{\alpha+1} C^2/n^2 J_o t_o^{\beta}\right]^{1/(\alpha+3)} t^n = Kt^n \tag{3.1}$$

where n is related to β, α and λ through

$$n = (2+\beta)/(\alpha+3) = 2/(\lambda_o+2) \tag{3.2}$$

from which the decay index for strong shocks is

$$\lambda_o = 2(\alpha+1-\beta)/(2+\beta) \tag{3.3}$$

The zero subscripts on λ and J are used to designate the strong shock case here.

Note also that

$$\frac{dz}{d\omega} = \frac{t_o}{R_o}\frac{dR}{dt} = \frac{U}{C} = \frac{1}{y^{\frac{1}{2}}} \tag{3.4}$$

while from (3.1)

$$dz/d\omega = nz/\omega \tag{3.5}$$

so that

$$z = \omega/ny^{\frac{1}{2}} \tag{3.6}$$

Combining this with the strong shock energy equation (b = 0) yields

$$z = z_o y^{1/\lambda_o} \tag{3.7}$$

where

$$z_o = \left(n^{\beta}/J_o\right)^{1/(\alpha+1-\beta)} \tag{3.8}$$

4. OUTLINE OF SERIES SOLUTION

Equations (2.15) (2.17) and (2.22) comprise a system of three dependent variables; λ, a, and g and two independent variables; m and y. Sakurai has shown that λ is a function of y only and that the pressure, density etc. are slowly varying functions of y. He therefore suggests expansion in powers of y to reduce the number of independent variables. In the Lagrangean coordinates used here this has the very desirable effect of reducing the partial differential equations to ordinary differential equations.

The expansions, taken about the strong shock solution for which y = 0, are

$$\lambda(y) = \lambda_o + \lambda_1 y + \lambda_2 y^2 + \cdots \tag{4.1}$$

$$a(m,y) = a_o(m) + a_1(m)y + a_2(m)y^2 + \cdots \tag{4.2}$$

$$g(m,y) = g_o(m) + g_1(m)y + g_2(m)y^2 + \cdots \tag{4.3}$$

By similar reasoning the equation for the shock front is expanded about the strong shock "zero order" solution (3.7) as

$$z = z_o\, y^{1/\lambda_o} (1 + z_1\, y + z_2\, y^2 + \cdots) \quad . \tag{4.4}$$

Equation (3.4) which applies in general can be integrated to give:

$$\omega = \int y^{\frac{1}{2}} \frac{dz}{dy}\, dy \quad .$$

Substituting for z and taking z = 0 at ω = 0 then gives an expansion for the time variable as a function of y and z

$$\omega = z_o y^{(\lambda_o+2)/2\lambda_o}(\omega_o + \omega_1 z_1 y + \cdots + \omega_j z_j y^j + \cdots) \tag{4.6}$$

where

$$\omega_j = (2j\lambda_o+2)/((2j+1)\lambda_o+2) \quad , \tag{4.7}$$

Noting from (2.13) that $f = f(\lambda, a, m)$ it is clear that substitution of (4.1 - 4.3) into (2.19) will yield integrals in ascending powers of y so that J can be expressed as

$$J = J_o + J_1 y + J_2 y^2 + \cdots \tag{4.8}$$

These expansions are then substituted into the isentropic, momentum and energy equations (2.15), (2.17) and (2.22) and the coefficients of like powers of y equated to give perturbation equations of ascending order. In the isentropic and momentum equations this procedure yields ordinary differential equations for a_j and g_j with λ_j entering as a parameter. These must be solved subject to the shock jump boundary conditions.

$$a_o(1) = 1 \qquad a_1(1) = 0$$

$$g_o(1) = 2\gamma/(\gamma+1) \qquad g_1(1) = (1-\gamma)/(\gamma+1)$$

$$a_o'(1) = \frac{\gamma - 1}{(\alpha+1)(\gamma+1)} \qquad a_1'(1) = \frac{2}{(\alpha+1)(\gamma+1)} \tag{4.9}$$

The remaining coefficients are all zero at the shock. Fortunately the successive orders of approximation to the isentropic equation yield intermediate integrals for g_j. These may then be used in the momentum equation which must be solved by numerical integration from the shock front to the mass

origin. The equations and solutions for each order will be discussed in the following sections.

Now consider the energy equation. Putting the expansion (4.4), (4.6) and (4.8) into (2.22) gives

$$z_o^{\beta}\,(\omega_o + \omega_1 z_1 y + \omega_2 z_2 y^2 + \cdots)^{\beta}$$

$$= z_o^{\alpha+1}(1 + z_1 y + z_2 y^2 + \cdots)^{\alpha+1}(J_o + (J_1 - b)y + J_2 y^2 + \cdots). \quad (4.10)$$

The two series with exponents may be converted to simple power series by means of the binomial formula if the first term of each is larger than the sum of the remaining terms. This is certainly true for small y. Multiplying out on the right and equating coefficients of like powers of y then yields relations between J_j and z_j:

$$J_o = \omega_o^{\beta} z_o^{\beta-(\alpha+1)},$$

$$J_1/J_o = \left[\beta\omega_1/\omega_o - (\alpha+1)\right] z_1 + b \quad ,$$

$$J_2/J_o = \left[\beta\left(\frac{\beta-1}{2}\right)\frac{\omega_1^2}{\omega_o^2} + (\alpha+1)\left(\frac{\alpha+2}{2} - \frac{\beta\omega_1}{\omega_o}\right)\right] z_1^2$$

$$+ \left[\frac{\beta\omega_2}{\omega_o} - (\alpha+1)\right] z_2 \,. \quad (4.11)$$

These relations are equally valid for integer and non integer values of β. Freeman[2] only considered integer values. Substituting (4.4) into (2.11) and solving for the z_j gives

$$z_1 = -\lambda_1/\lambda_o^2$$

$$z_2 = -[\lambda_2 + (1+\lambda_o)z_1\lambda_1]/2\lambda_o^2$$

$$z_3 = -[\lambda_3 + (1+\lambda_o)z_1\lambda_2 + (1+2\lambda_o)z_2\lambda_1]/3\lambda_o^2 \qquad (4.12)$$

$$\vdots \qquad\qquad \vdots$$

which may then be used in (4.11) to give relations between the λ_j and J_j. These will be used later to evaluate the λ_j for each order of approximation.

The solution proceeds in ascending orders of approximation. The zeroth order, corresponding to the strong shock solution with $y \to 0$, is used in the computation of the first order solution and the first order solution is then used to compute the second and so forth. In this paper the solution will be carried only to the 2nd order.

The zeroth order approximation to the isentropic equation is

$$\frac{g_o'}{g_o} = -\left[\frac{\lambda_o}{(\alpha+1)m} + \gamma\left(\alpha\frac{a_o'}{a_o} + \frac{a_o''}{a_o'}\right)\right] \qquad (4.13)$$

which Freeman has shown to have an integral

$$g_o = K_o \Big/ (a_o{}^{\alpha} a_o')^{\gamma m \lambda_o/(\alpha+1)} \qquad (4.14)$$

where

$$K_o = \frac{2\gamma}{\gamma+1}\left[\frac{\gamma-1}{(\alpha+1)(\gamma+1)}\right]^{\gamma} \qquad (4.15)$$

The momentum equation in the zeroth approximation becomes

$$a_o'' = (B_o a_o' - C_o a_o - g_o a_o^{\alpha} \lambda_o / m)/A_o \,, \tag{4.16}$$

where

$$A_o = (\alpha+1)\,\gamma\,[a_o^{\alpha} g_o / a_o' - (\alpha+1)m^2] \,,$$

$$B_o = (\alpha+1)m\gamma(\alpha+\lambda_o/2) \,,$$

$$C_o = \gamma[\lambda_o/2 + \alpha(\alpha+1) g_o a_o^{\alpha} a_o' / a_o^2] \,. \tag{4.17}$$

The first approximation to the energy integral, J, of (4.8) will be obtained by evaluating

$$J_o = \int_o^1 \left[\frac{\gamma}{2}\,\frac{(a_o - (\alpha+1)m a_o')^2}{\alpha+1} + \frac{g_o a_o^{\alpha} a_o'}{\gamma-1}\right] dm \tag{4.18}$$

along with integration of (4.16) from the shock front to the mass origin. (Note that Freeman's forms of this and higher order integrals are only correct for $\alpha = 1$.) The first term of this expression is a measure of the kinetic energy behind the shock and the second term is proportional to the internal energy so these two factors can easily be accumulated separately. The integration of a_o is initiated with the boundary conditions (4.9). The values of $a_o(m)$, $a_o'(m)$, $g_o(m)$ and $g_o'(m)$ may be stored for use in the first order solution or, more conveniently, these integrations can be performed simultaneously with those for the higher order solutions.

In the zeroth order equation λ_o appears as a known parameter obtained from the similarity solution. In the higher order equations the λ_j's appear as unknown parameters. Fortunately, the equations are linear in λ_j and of course the perturbation equations are linear differential equations so it follows that

superposition of solutions is valid. This property will allow evaluation of the λ_j's after the equations have been integrated.

In each higher order approximation the g_j' term in the momentum equation may be eliminated with the isentropic equation and its integral, g_j. The integrals have been obtained through third order by Freeman. The momentum equation then gives higher order perturbation equations of the form

$$a_j'' = (B_j\, a_j' + C_j\, a_j + D_j + \lambda_j\, E_j)/A_o \tag{4.19}$$

in which the coefficients B_j through E_j are functions only of known quantities from lower order solutions. It follows that a_j can be expressed as the linear combination

$$a_j = a_{j1} + \lambda_j a_{j2} \tag{4.20}$$

so that (4.19) gives two equations

$$a_{j1}'' = (B_j\, a_{j1}' + C_j\, a_{j1} + D_j)/A_o \ ,$$

$$a_{j2}'' = (B_j\, a_{j2}' + C_j\, a_{j2} + E_j)/A_o \ . \tag{4.21}$$

It is convenient to let the non-zero boundary condition of (4.2) be satisfied by a_{11} so the remaining perturbations are zero at the shock. Note that the denominator, A_o, is the same for all orders of approximation. This term goes to zero at $m = 0$ when $\beta = 0$ causing computational difficulties which will be discussed later.

It will be evident upon substitution of a_j into the isentropic equation that g_j must be of the form

$$g_j = g_{j1} + \lambda_j\, g_{j2} \ . \tag{4.22}$$

Finally substituting a_j and g_j into the energy integral will show J_j to be of the form

$$J_j = J_{j1} + \lambda_j J_{j2} = \int_0^1 (J'_{j1} + \lambda_j J'_{j2})\,dm \tag{4.23}$$

where the terms J_j, and J_{j2} are to be accumulated simultaneously with integration of (4.21). When J_{j1} and J_{j2} have thus been evaluated λ_j can be determined. After using (4.12) in (4.11) to eliminate z_j in favor of λ_j, (4.23) is substituted in to the left hand side of (4.11) and the result is then solved for λ_j. The first two of these coefficients are

$$\lambda_1 = \frac{b - J_{11}}{J_{12} + J_o[\omega_1\beta/\omega_o - (\alpha+1)]/\lambda_o^2} \tag{4.24}$$

and

$$\lambda_2 = \frac{J_o\,\lambda_1^2\,G - 2\lambda_o^2\,J_{21}}{J_o\,H - 2\lambda_o^2\,J_{22}} \tag{4.25}$$

where

$$G = \beta(\beta-1)\,\omega_1^2/\omega_o^2 + (\alpha+1)(\alpha+1-\lambda_o-2\beta\omega_1/\omega_o)$$

$$+ (1+\lambda_o)\,\beta\,\omega_2/\omega_o$$

$$H = \beta(\omega_2/\omega_o) - (\alpha+1) \tag{4.26}$$

The ω_j are obtained from (4.7). Expressions for the coefficients A_j through E_j, J'_{j1} and J'_{j2} are listed in the Appendix.

Freeman has shown that given approximations through the k'th order with $k \geq 2$, it is possible to obtain an excellent approximation to the shock position R(t) as follows. Truncate the λ series at the k+2 term and evaluate the

last two λ_j in terms of those preceeding and known boundaries of the λ vs y curve at y = 1. Thus

$$\lambda(1) = \lambda_o + \lambda_1 + \cdots + \lambda_k + \lambda_{k+1} + \lambda_{k+2} = 0 \tag{4.27}$$

$$d\lambda(1)/dy = \lambda_1 + 2\lambda_2 + \cdots + k\lambda_k + (k+1)\lambda_{k+1} + (k+2)\lambda_{k+2}$$

$$= (2+\alpha)/4 \tag{4.28}$$

The higher order λ terms in (4.12) are then zero and all the z_j can be expressed in terms of λ_o through λ_{k+2}. If k = 2 the general expression for z_j when $j \geq 5$ is

$$z_j = -\Big([1+(j-4)\lambda_o]\lambda_4 z_{j-4} + [1+(j-3)\lambda_o]\lambda_3 z_{j-3}$$

$$+ [1+(j-2)\lambda_o]\lambda_2 z_{j-2} + [1+(j-1)\lambda_o]\lambda_1 z_{j-1}\Big)/j\lambda_o^2 \tag{4.29}$$

Using this and (4.12) in (4.4) and (4.6) then gives $z(\omega)$ or $R(t)$ for all values of y.

It was found that this procedure works very well for $\beta = 0$, but when β approaches $\alpha + 1$ where $\lambda \equiv 0$, satisfaction of the final slope condition (4.28) with a limited number of terms causes a false hump in the λ-y curve near y = 1. The procedure was therefore modified through multiplying λ_{k+2} as obtained by simultaneous solution of (4.27) and (4.28) by the factor $[1-\beta/(\alpha+1)]$ and solving (4.27) for λ_{k+1} once again. Thus the need to meet the final slope conditions is progressively eliminated as β approaches $\alpha + 1$ to give more realistic λ-y curves.

5. EXTRAPOLATION TO m = 0.

When $\beta = 0$ as it does in the constant energy blast wave, integration of (4.16) will show that $a_o' \to \infty$ and $a_o \to 0$ as $m \to 0$ so it follows from (4.17) that $A_o \to 0$. This latter term appears as the denominator of (4.21) so it may be anticipated that computational difficulties will be encountered near the mass origin for all orders of approximation. The problem is further compounded in evaluation of the energy integrals which contain terms proportional to a_o' and accumulate rapidly near $m = 0$. For example if $\gamma = 1.1$ and $\alpha = 1$, sixty percent of the true value of J_o is accumulated in the last one percent of mass variation. Numerical integration all the way to the origin is clearly impossible yet it is mandatory that accurate values of J_j be obtained else the next higher order approximation be seriously in error.

Freeman[2] mentioned this problem and suggested an "appropriate approximation formula" but gave no details. Similar numerical problems arising in the Eulerian formation of the problem have been noted by Sakurai[1] and Bach and Lee[4]. The procedure outlined here was developed by the author and is based upon the fact that in a region close to the mass origin the pressure term, g_o, can be taken to be constant to a very high degree of approximation. Although the procedure is only necessary when β is close to zero, it is still valid for larger values of β so it can be incorporated into a computer program valid for all values of β.

Consider the integrand of (4.18) in which the first term is related to the kinetic energy and the second to the internal energy.

$$J_o' = \frac{\gamma}{2(\alpha+1)}\left(a_o-(\alpha+1)ma_o'\right)^2 - \frac{g_o a_o^{\alpha} a_o'}{\gamma-1} \tag{5.1}$$

In the region near the origin, say $m < 10^{-3}$, the density is very low so it may be anticipated - and verified by numerical experiment - that the first

term is much smaller than the second. Furthermore it has been observed from numerical integrations that the product ma_o' approaches zero at nearly the same rate as a_o so that this first term is nearly constant. These numerical integrations also showed g_o to be constant to about four significant figures in this region. It follows that taking these factors constant over an interval of $\Delta m \leq 10^{-3}$ will introduce no significant error in the computation of J_o.

The major contributing factor to the integral is then $a_o^\alpha a_o'$ which by use of (4.14) can be expressed in terms of m as

$$a_o^\alpha a_o' = (K_o/g_o)^{1/\gamma_m - \lambda_o/\gamma(\alpha+1)} \tag{5.2}$$

which can be integrated analytically if g_o is taken to be constant.

The procedure then is to integrate numerically from m = 1 to m = δ where $0 < \delta \leq 10^{-3}$ and add a correction term obtained by analytical integration from 0 to δ, i.e.,

$$\Delta J_o = \frac{\gamma\delta}{2(\alpha+1)}(a_o - (\alpha+1)\delta a_o')^2 + \frac{g_o}{\gamma-1}\left(\frac{K_o}{g_o}\right)^{1/\gamma}\left[\frac{\delta^{1-\lambda_o/\gamma(\alpha+1)}}{1-\lambda_o/\gamma(\alpha+1)}\right] \tag{5.3}$$

where a_o and a_o' are evaluated at m = δ.

Similar considerations apply to the higher order approximations to J. In each case the kinetic energy term in the integrand can be taken as constant over the last integration interval while the internal energy term can be expressed in terms of $a_o^\alpha a_o'$, g_o and essentially constant ratios such as a_j/a_o, a_j'/a_o', etc. This method has been checked by letting δ take on values from 10^{-3} to 10^{-6} and noting that all give the same final values for each J_j. A value of $\delta = 10^{-5}$ was used in the computations leading to the results reported in the next section. Formulas for the correction term ΔJ_{11}, ΔJ_{12} etc. are given in the appendix.

6. RESULTS

The equations outlined in Sections 3, 4 and the Appendix have been numerically integrated with a fourth order Runge-Kutta algorithm in double precision and extrapolated to m = 0 as discussed in Section 3. The procedure has been carried out to the second order in the expansion variable y (some authors[1,2,4] prefer "third approximation") for plane, cylindrical and spherical blast waves, i.e., for $\alpha = 0, 1, 2$. The range $0 \leq \beta < \alpha+1$ (above which the zeroth order similarity solution ceases to exist[6]) was covered for $\gamma = 1.1$ and 1.4. The solution for the case, $\beta = \alpha + 1$, which corresponds to the well known self similar solution for which $\lambda = 0$ and $R = Kt$, is computationally unobtainable because of a zero divisor problems evident in equation 3.7. However, solutions for β very close to this value (within .01) were obtained easily. Solutions have also been obtained for other typical values of γ for integer values of β.

Tables 1 to 3 are summaries of the expansion coefficients λ_1, λ_2 and J_0, J_1, J_2 obtained in the computation. Korobienikov and Chushkin[5], Sakurai[1], Bach and Lee[4] have all published values of λ_1 computed in Eulerian coordinates for $\beta = 0$, $\gamma = 1.4$ which agree with those given here. (Note that Bach and Lee report values of $\lambda_i/2$ while the others give values of $\lambda_i/(\alpha+1)$). The values of λ_2 and λ_1 for $\beta = 0$ and $\gamma = 1.4$ are also in agreement to the full six significant figures quoted by Bach and Lee. The values of λ_1 for $\beta = 0$ and $\gamma = 1.1$ to 3.0 are in agreement with those reported by Freeman, however his values of λ_1 for $\beta = 1$ are in error because he apparently used an incorrect value of λ_0 to obtain them. His values of J_{11} and J_{12} are, however, correct. As a final check on numerical accuracy it is noted that the values of J_0 reported to 4 significant figures by Rogers for $\gamma = 1.2$ and 1.4 and various β agree with values in these tables.

It can be seen from the tables that the λ coefficients approach zero

rapidly as β is increased from zero. This is also shown in Figure 1. The maximum value of β is $\alpha+1$ where $\lambda = 0$ and the shock propagates with constant velocity. This drop off of the decay index implies much stronger convergence of the perturbation series as β increases.

Figure 2 shows typical variations of the J coefficients with β for $\alpha = 2$, and $\gamma = 1.4$. It is seen that J_2 drops off rapidly and J_o becomes fairly constant while J_1 takes on an increasing portion of the energy as β increases. Similar results obtain for other γ and α.

Radial pressure, density and velocity destributions are presented in Figures 3 to 5 for $\alpha = 2$, $\gamma = 1.4$ and $\beta = 0, 1, 2$. The hump in the density distubution for $y = .5$ and $\beta = 0$ is most likely due to truncation of the expansion at the third term. This suggests an upper limit of validity for $y = .5$, or a shock Mach number of 1.4, for the second order expression when $\beta = 0$. This limit can be raised considerably as β increases as in evidenced by the curves for $\beta = 1, 2$.

As β is increased from zero, the constant energy case, the flow region rapidly contracts to a thin shell between the shock and the fictitious piston located at the zero mass position as is evident in Figures 4 and 5. Even when $\beta = 0$, over 99% of the mass is located in the outer half radius of the sphere but as noted before, over 50% of the total energy can be confined to the inner 1% of the mass because the temperature becomes extremely large in this region. As β approaches $\alpha+1$ the solution approaches the self similar result for $\lambda = 0$ and $y = $ constant. At this limit the spherical blast wave flow field occupies a thin shell within which the velocity, pressure and density are constant. Similar results obtain for other γ and α. In particular the profiles for $\alpha = 1$, $\beta = 1$ and $\gamma = 1.4$ are given in Figure 6 as replacements for similar plots given by Freeman which, as noted before, are in error due to his use of

the incorrect value of λ.

The variation of the decay index, λ, and the shock position, z, with shock strength, y, is given in Figure 7 for a spherical shock with $\gamma = 1.4$ and $\beta = 0$. The decay index was obtained by use of accurate values of λ_0, λ_1 and λ_2 given in Table 2 and approximate values of λ_3 and λ_4 obtained by matching boundary conditions at $y = 1$ as outlined in section 4. The shock position was obtained from (4.6) with z_j from (4.12) and enough terms of (4.28) to insure that no truncation error would occur in the sixth significant figure. These results are compared with a decay index and shock position obtained from tables of shock position and over-pressure reported by Goldstine and Von Neumann[8] who solved the exact partial differential equations by a numerical technique. The comparison is most favorable over the whole range of y even though the internal structure of the flow field can only be predicted to $y \approx .5$ for $\beta = 0$. Of course when β becomes larger the internal approximations improve for larger y.

The effect of β on decay index and shock position is given in Figures 8 and 9 where λ and z are plotted against shock strength with β as a parameter. Note the smooth approach to $\lambda = 0$ as β approaches $\alpha+1$ in Figure 8.

It is evident in Figure 9 that a constant value of shock strength is approached in the limit $\beta = \alpha+1$ or $\beta = 3$ in this case. The value can easily be obtained by noting that a solution of the form $z \sim \omega^n$ is required in which $n = (2+\beta)/(\alpha+3) = 1$ to be consistent with the rest of the family of curves. Differentiating this relation and using (3.4), the definition of y, gives

$$dz/d\omega = z/\omega = y^{-1/2} \tag{6.1}$$

while the energy equation (2.22) gives

$$(\omega/z)^{\alpha+1} = Jy - 1/[(\alpha+1)(\gamma-1)] \tag{6.2}$$

Eliminating ω/z and using the second order approximation for J consistent with the other curves then yields the following equation which can be solved for the limiting value of y.

$$y^{(\alpha+3)/2} = J_o + \left(J_1 - 1/[(\alpha+1)(\gamma-1)]\right) y + J_2\, y^2 \qquad (6.3)$$

Taking $\gamma = 1.4$, the limits are $y = 0.531$, 0.528 and 0.542 for $\alpha = 2$, 1 and 0, respectively.

The shock trajectories corresponding to Figures 8 and 9 are given in Figure 10. It is clear that there is relatively little difference in trajectories as β is varied. The major effect of increasing β is the faster decay of shock strength at low z and slower decay at large z previously shown in Figure 9. In addition there appears to be only a very weak dependence of the trajectory upon γ as the curves for $\gamma = 1.1$ fall in the same narrow band indicated for $\gamma = 1.4$. The γ effect is more evident in the z-y plots of Figure 11 which also indicate the effect of α for $\beta = 0.5$ in all cases.

The apportionment of energy between kinetic and internal modes is also of interest. These two contributions can easily be separated in the computation of the energy integral, J. The fraction of the total energy associated with kinetic energy is plotted in Figure 12 versus the shock strength with β as a parameter. These results were computed with a three term approximation of J. Note that the kinetic energy fraction increases with β while in all cases most of the energy soon is transformed to internal energy as the shock decays and y increases. The values obtained for $y = 0$ are in agreement with computations made by Rogers[3].

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β	γ	J_0	λ_1	J_1	λ_2	J_2
0	1.1	2.80158	-4.17316	-0.84575	5.91593	4.14349
	1.2	1.53186	-4.08477	-0.62865	5.09973	1.95302
	1.3	1.09904	-4.01846	-0.54156	4.57118	1.25598
	1.4	0.87708	-3.96712	-0.48974	4.20319	0.92163
	5/3	0.59873	-3.87486	-0.40999	3.61729	0.54144
	2	0.44810	-3.80867	-0.35333	3.24624	0.36366
	3	0.27579	-3.72635	-0.26385	2.82464	0.19475
.25	1.4	0.70204	-2.65379	-0.02745	2.18103	0.57057
.5	1.1	0.78106	-1.79666	3.66353	1.33644	0.59187
	1.4	0.62794	-1.72133	0.22057	0.96355	0.36104
1.0	1.1	0.60523	-0.65358	4.20887	0.07644	0.15244
	1.2	0.58859	-0.65324	1.73102	0.06742	0.14316
	1.3	0.54713	-0.65407	0.91562	0.06269	0.13736
	1.4	0.56127	-0.65559	0.51408	0.06044	0.13357
	5/3	0.53222	-0.66114	0.04626	0.06044	0.12830
	2	0.50269	-0.66886	-0.17246	0.06530	0.12565
	3	0.43705	-0.68943	-0.35263	0.08495	0.12245
1.5	1.1	0.53960	-0.18017	4.45556	-0.05095	0.02116
	1.4	0.53039	-0.18812	0.69124	-0.04452	0.03804
1.95	1.4	0.51401	-0.01130	0.80246	-0.00473	0.00116
1.99	1.4	0.51286	-0.00216	0.81088	-0.00093	-0.00091

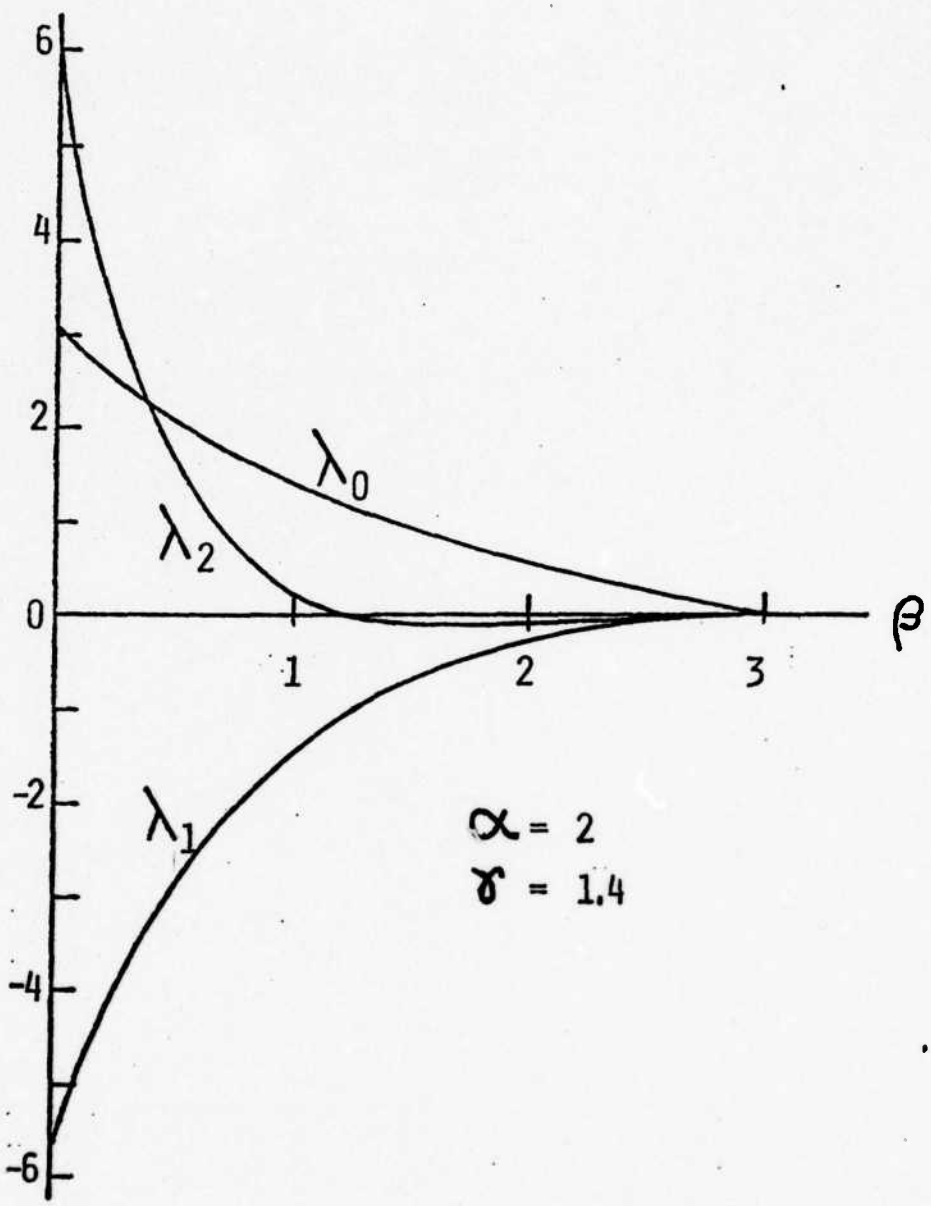
TABLE 1. EXPANSION COEFFICIENTS FOR CYLINDRICAL WAVE, $\alpha = 1$

β	γ	J_0	λ_1	J_1	λ_2	J_2
0	1.1	1.87082	-6.00300	-0.41019	7.75197	2.41710
↓	1.2	1.02644	-5.89969	-0.35189	6.80141	1.16354
	1.3	0.73942	-5.81883	-0.32307	6.15881	0.75899
	1.4	0.59261	-5.75446	-0.30338	5.69937	0.56291
	5/3	0.40915	-5.63556	-0.26860	4.94952	0.33752
	2	0.31034	-5.54866	-0.24066	4.46598	0.23100
	3	0.19774	-5.44223	-0.19204	3.91799	0.12912
.25	1.4	0.49105	-4.07792	-0.03462	3.14452	0.36095
.5	1.1	0.57496	-3.03230	2.35264	2.33198	0.43541
↓	1.4	0.44325	-2.87521	0.11646	1.57621	0.23932
1.0	1.1	0.44272	-1.43972	2.72249	0.34785	0.14331
↓	1.2	0.42440	-1.42414	1.08744	0.28346	0.12362
	1.3	0.40974	-1.41519	0.55541	0.24441	0.11135
	1.4	0.39761	-1.41027	0.29596	0.21973	0.10322
	5/3	0.37295	-1.40748	-0.00305	0.18889	0.09144
1.5	1.1	0.39238	-0.66360	2.88715	-0.07706	0.04510
↓	1.4	0.37556	-0.66763	0.40369	-0.08871	0.04054
2.0	1.1	0.36584	-0.27807	2.98455	-0.09847	0.00554
↓	1.2	0.36489	-0.28116	1.31493	-0.09717	0.00764
	1.3	0.36379	-0.28403	0.75685	-0.09595	0.00959
	1.4	0.36257	-0.28669	0.47695	-0.09485	0.01137
	5/3	0.35879	-0.29290	0.13969	-0.09243	0.01542
2.5	1.1	0.34945	-0.08812	3.05043	-0.04453	-0.01159
↓	1.4	0.35401	-0.09310	0.53054	-0.04237	-0.00205
2.95	1.4	0.34846	-0.00629	0.56801	-0.00320	-0.00752
2.99	1.4	0.34804	-0.00121	0.57097	-0.00062	-0.00782

TABLE 2. EXPANSION COEFFICIENTS FOR SPHERICAL WAVE, $\alpha = 2$

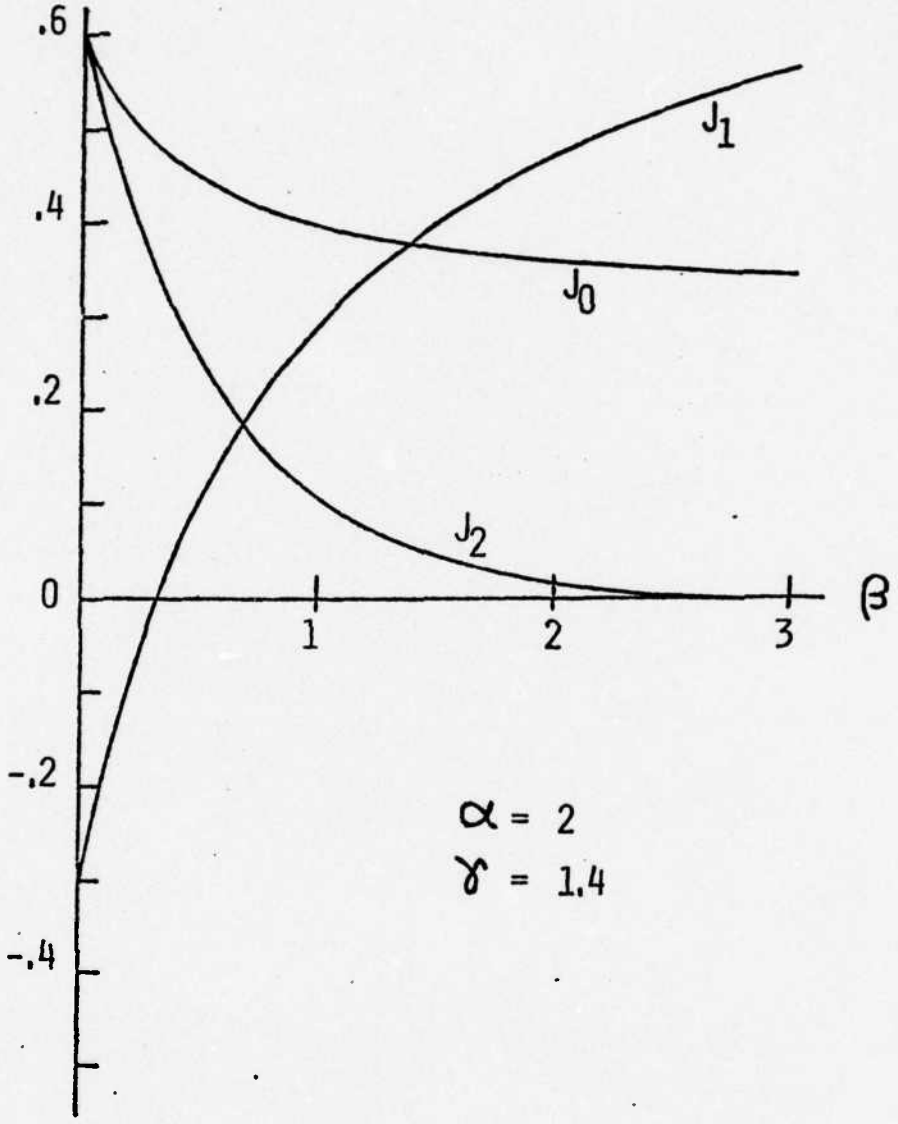
β	γ	J_0	λ_1	J_1	λ_2	J_2
0	1.1	5.57737	-2.32573	-2.97147	4.19515	11.69897
↓	1.2	3.02287	-2.24368	-1.78234	3.45473	5.22159
	1.3	2.14757	-2.18618	-1.36163	3.01386	3.23624
	1.4	1.69705	-2.14334	-1.13735	2.72112	2.30893
	5/3	1.13051	-2.06835	-0.83828	2.27262	1.28460
	2	0.82425	-2.01427	-0.66025	1.99397	0.82176
	3	0.47899	-1.94071	-0.42959	1.66803	0.39949
.25	1.4	1.23367	-1.20673	0.08085	1.19867	1.32576
.5	1.1	1.25640	-0.57074	7.87118	0.37492	0.91993
↓	1.4	1.08643	-0.56662	0.67245	0.34222	0.74140
.75	1.4	1.01463	-0.19267	1.07922	0.04087	0.36683
.95	1.4	0.97925	-0.02768	1.33264	-0.00185	0.17582
.99	1.4	0.97358	-0.00517	1.37783	-0.00061	0.14599

TABLE 3. EXPANSION COEFFICIENTS FOR PLANE WAVE, $\alpha = 0$



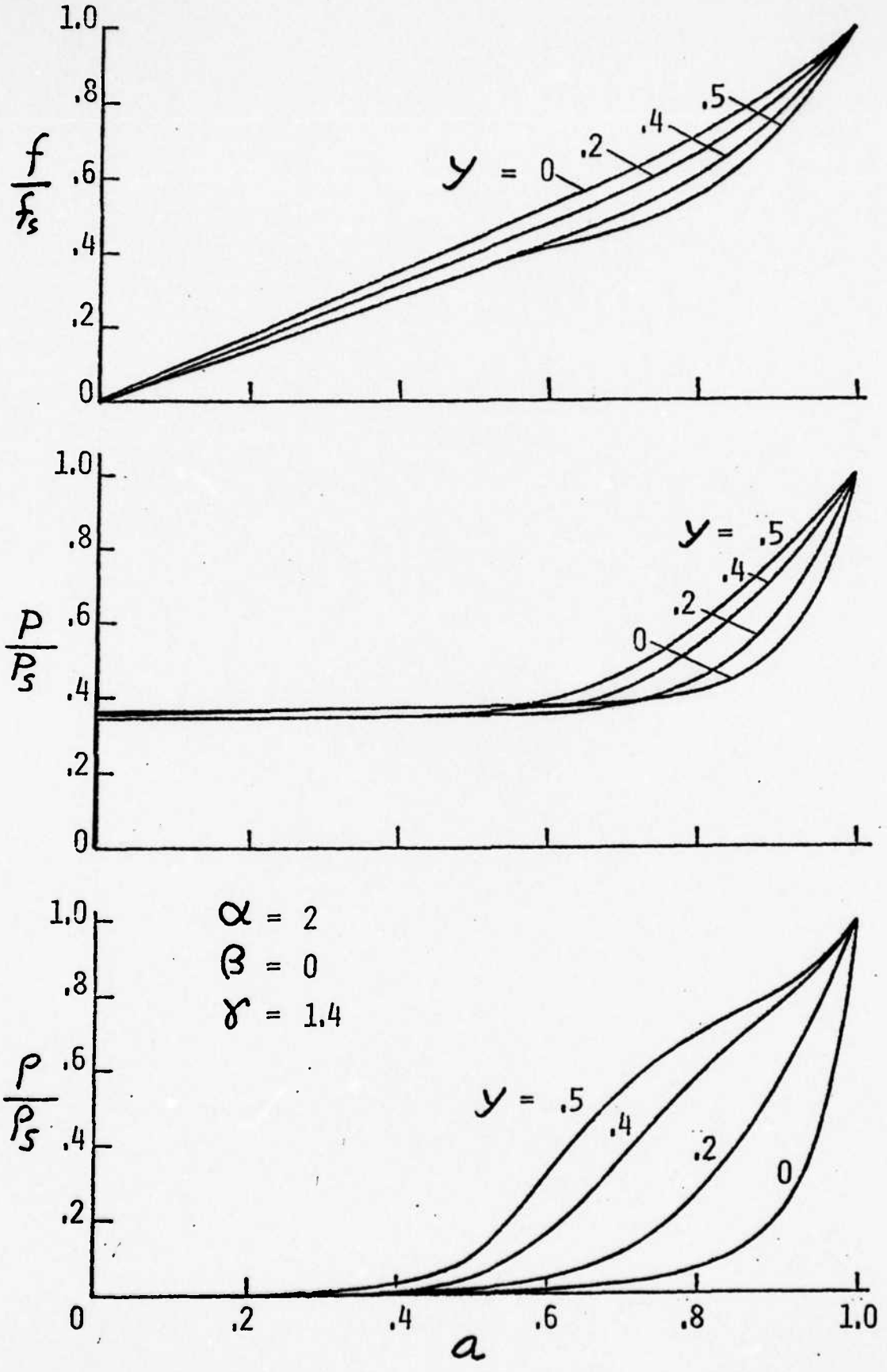
Coefficients for Decay Index Expansion as a Function of Energy Input Parameter.

Figure 1



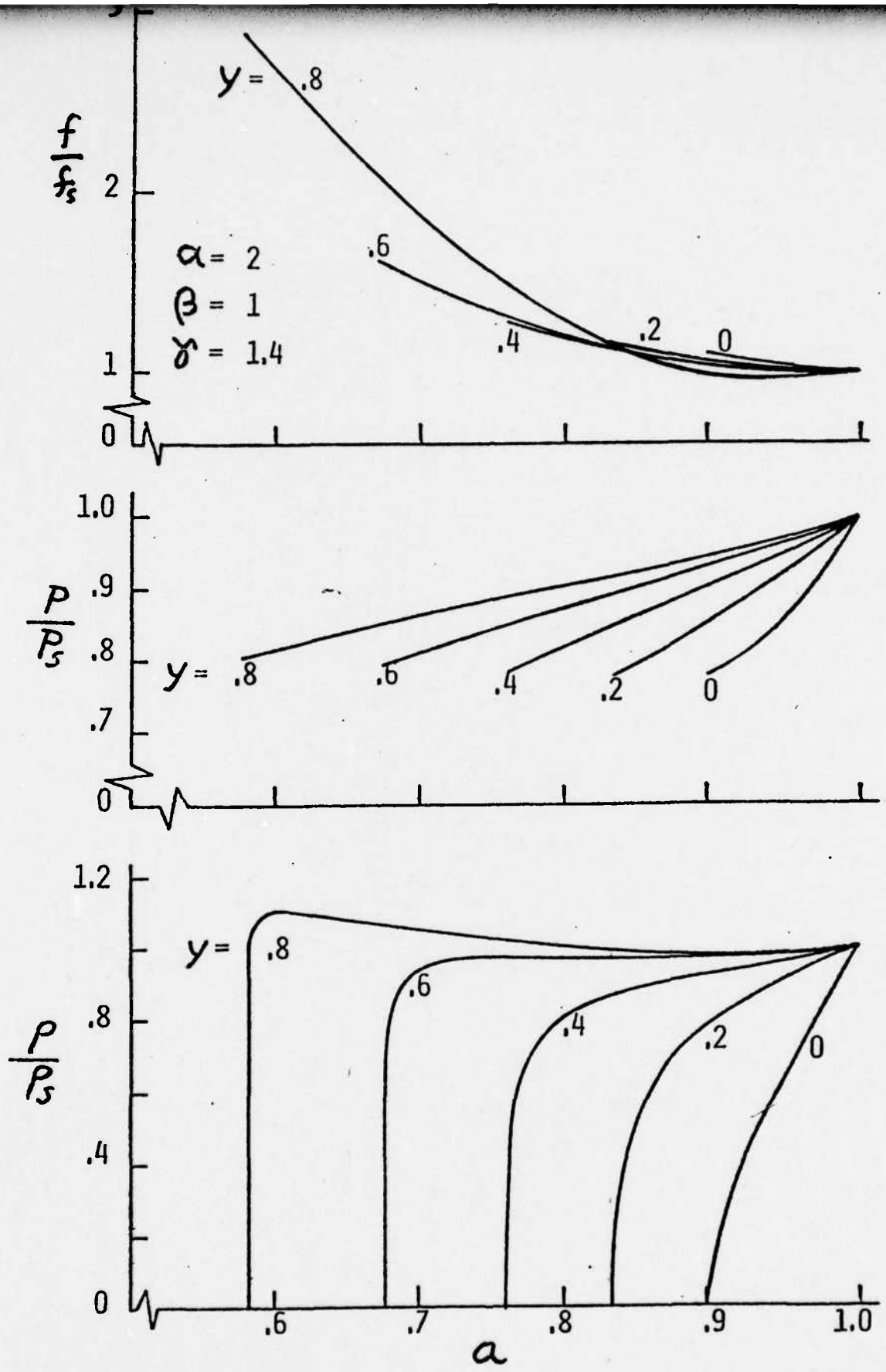
Coefficients of Energy Integral Expansion as a Function of Energy Input Parameter.

Figure 2



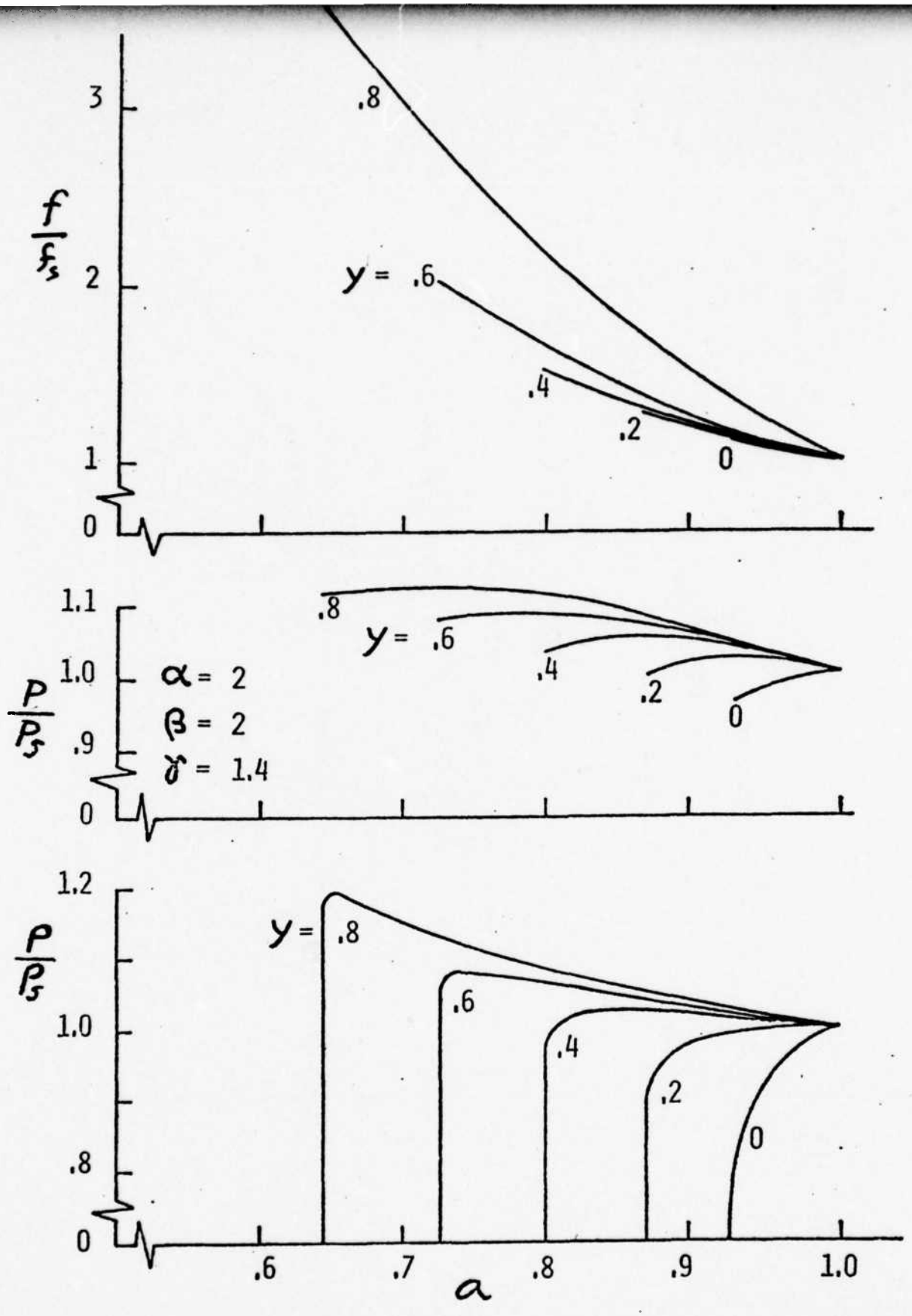
Flow Field Structure Behind Spherical Blast Wave for Instantaneous Energy Input.

Figure 3



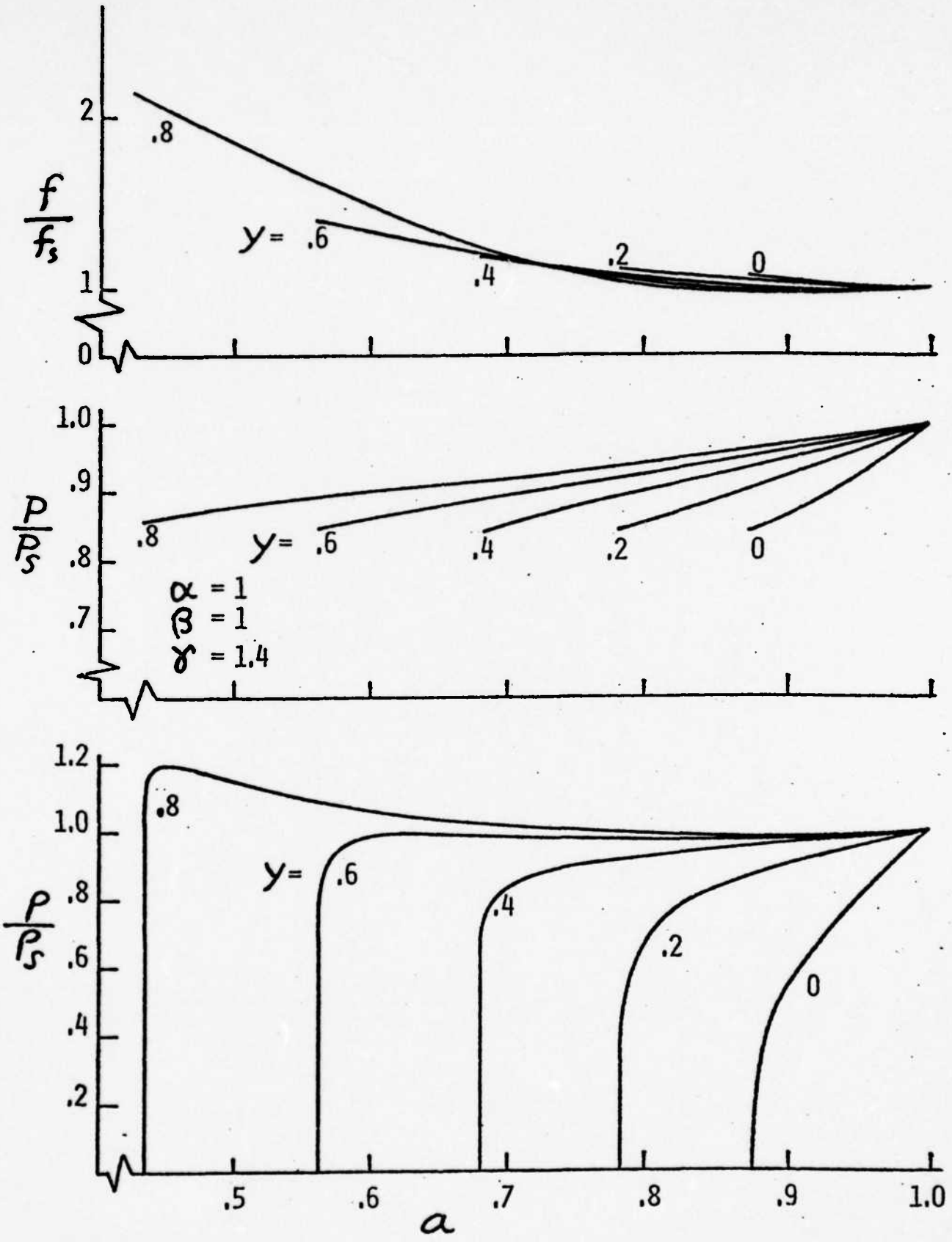
Flow Field Structure Behind Spherical Blast Wave for Constant Energy Input Rate.

Figure 4



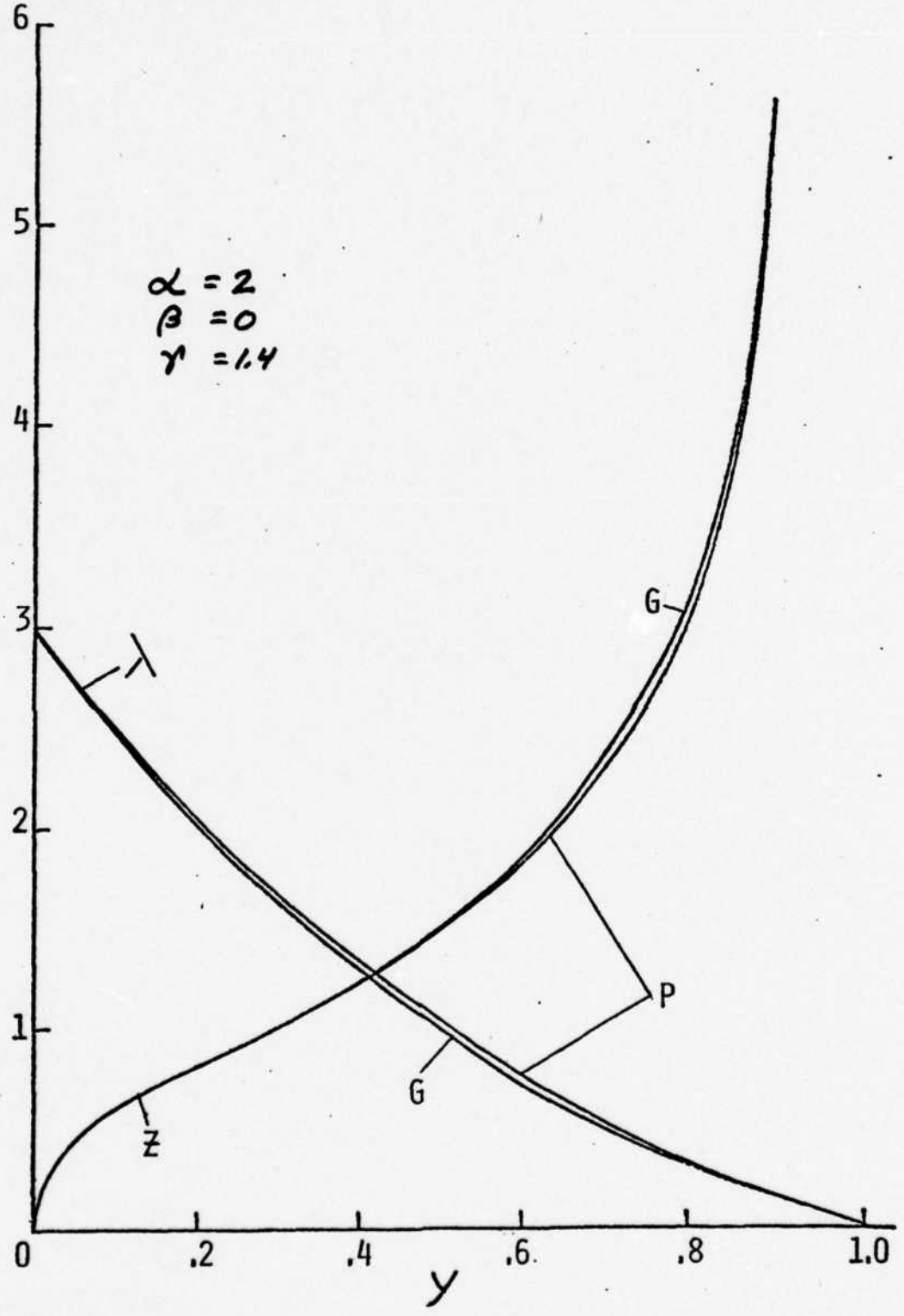
Flow Field Structure Behind Spherical Blast Wave for Linearly Increasing Energy Input Rate.

Figure 5



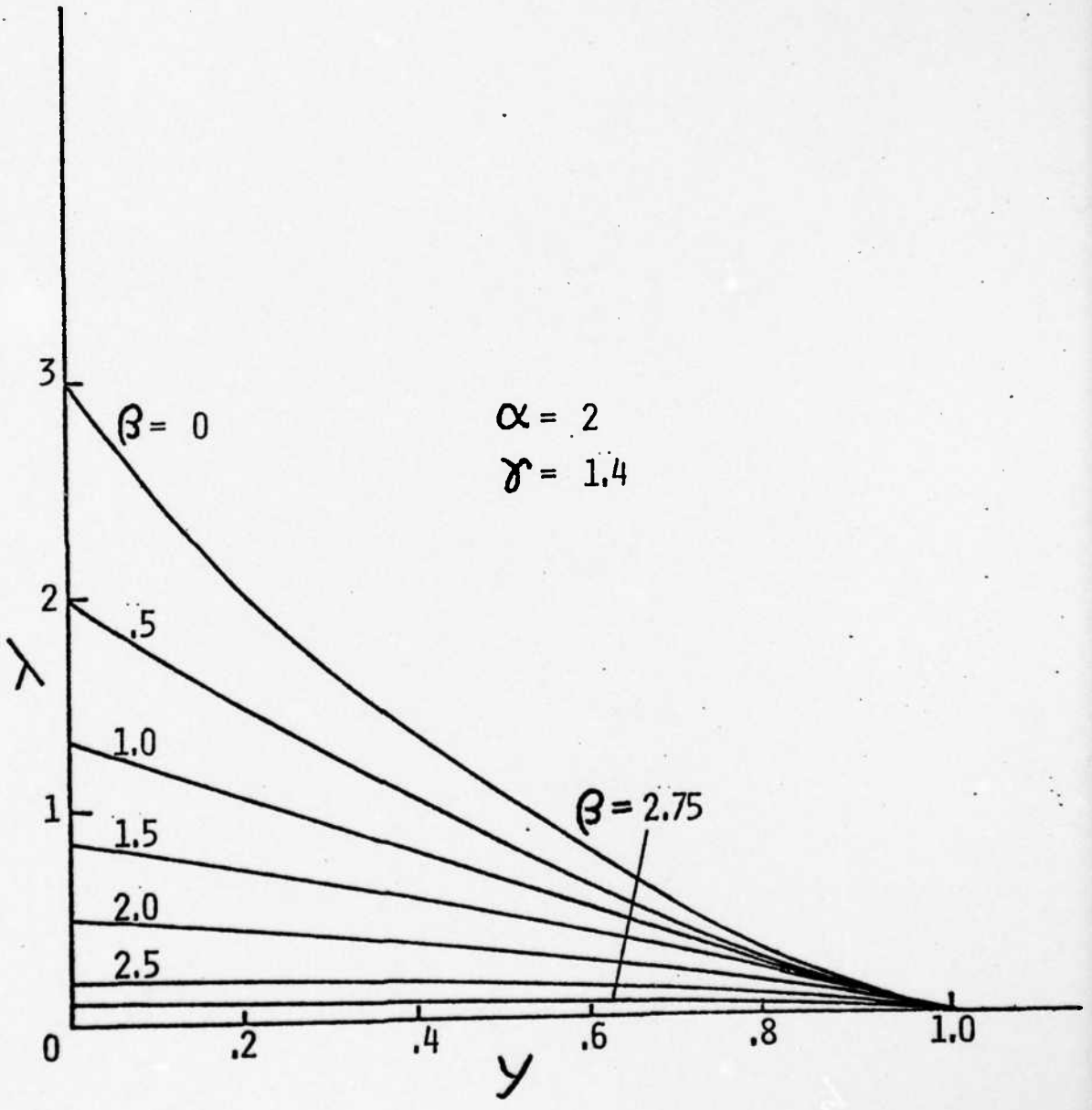
Flow Field Structure Behind Cylindrical Blast Wave for Constant Energy Input Rate.

Figure 6



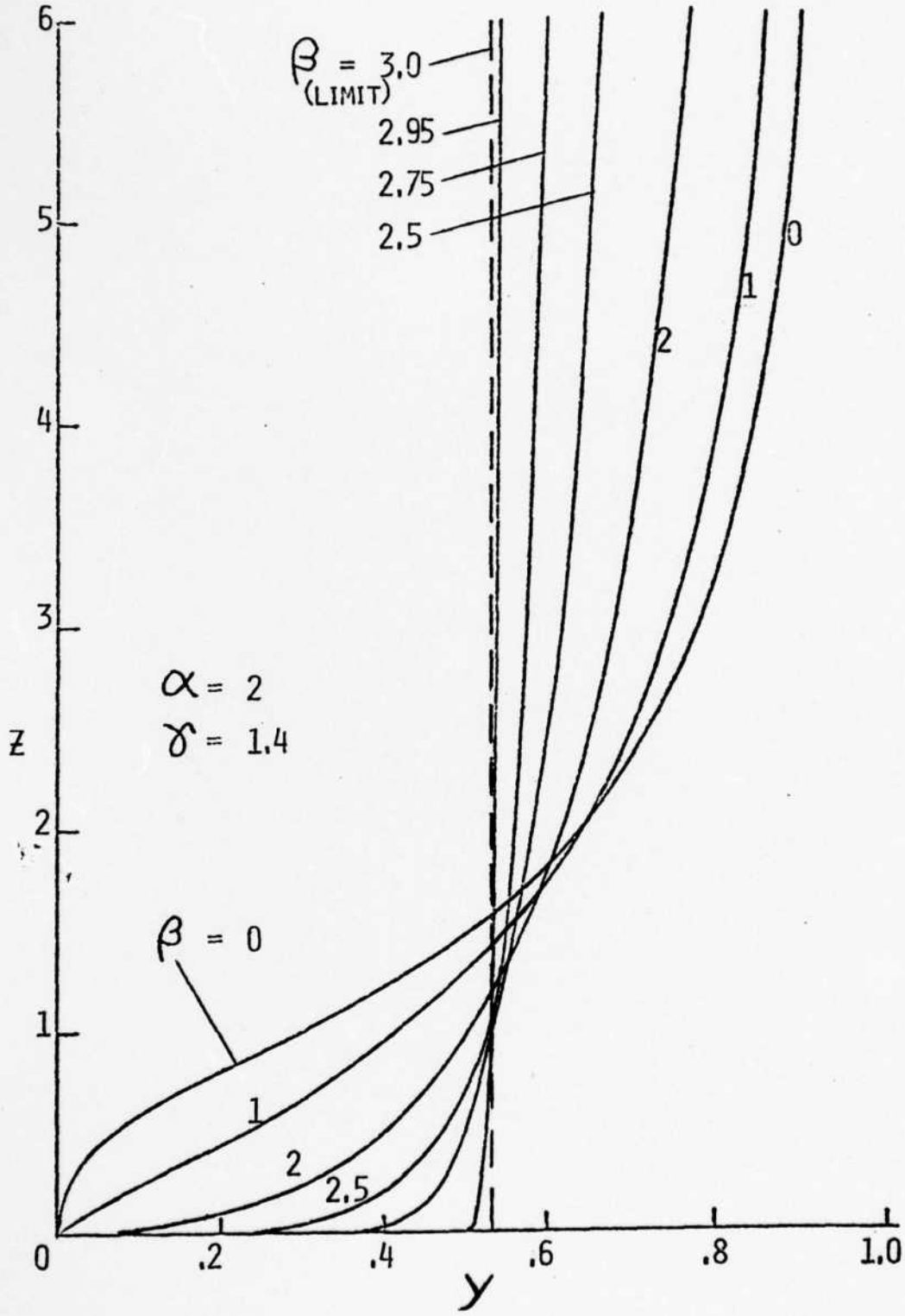
Decay Index and Shock Front Position from Second Order Perturbation Solution (P) Compared with "Exact Solution" of Goldstine and Von Neumann (G)

Figure 7



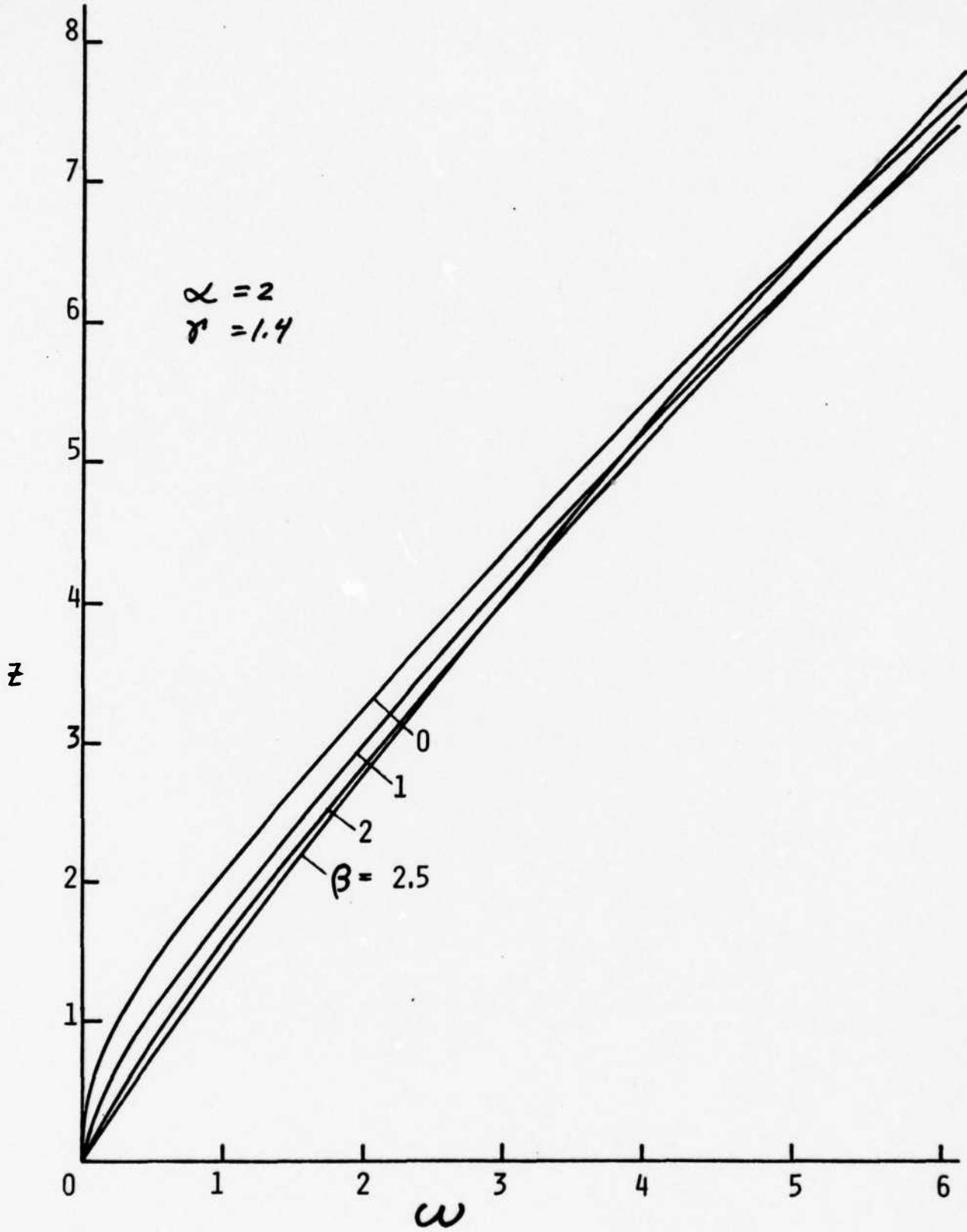
Variation of Decay Index with Shock Strength.

Figure 8



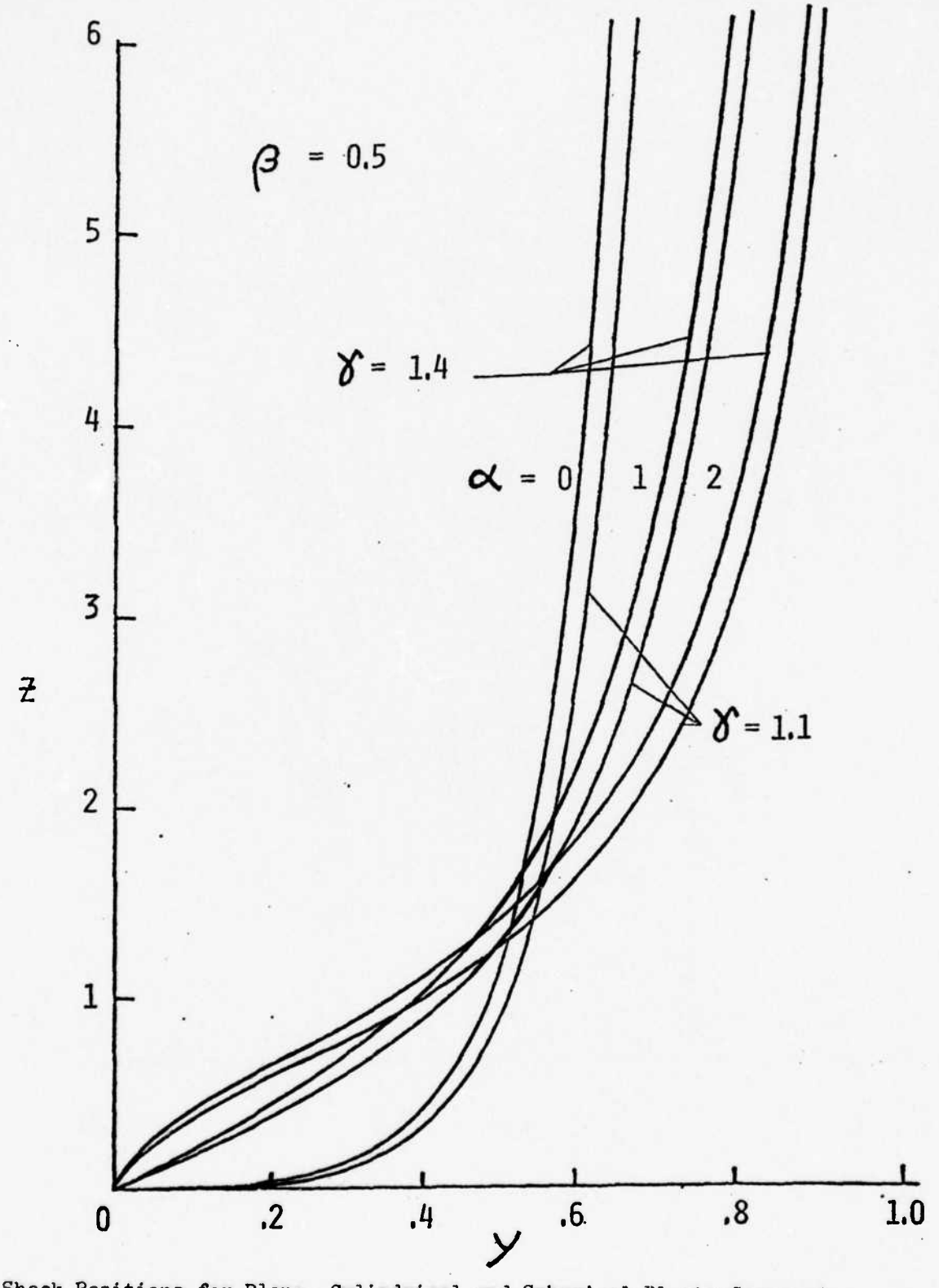
Shock Position as a Function of Shock Strength and Energy Input Parameter.

Figure 9



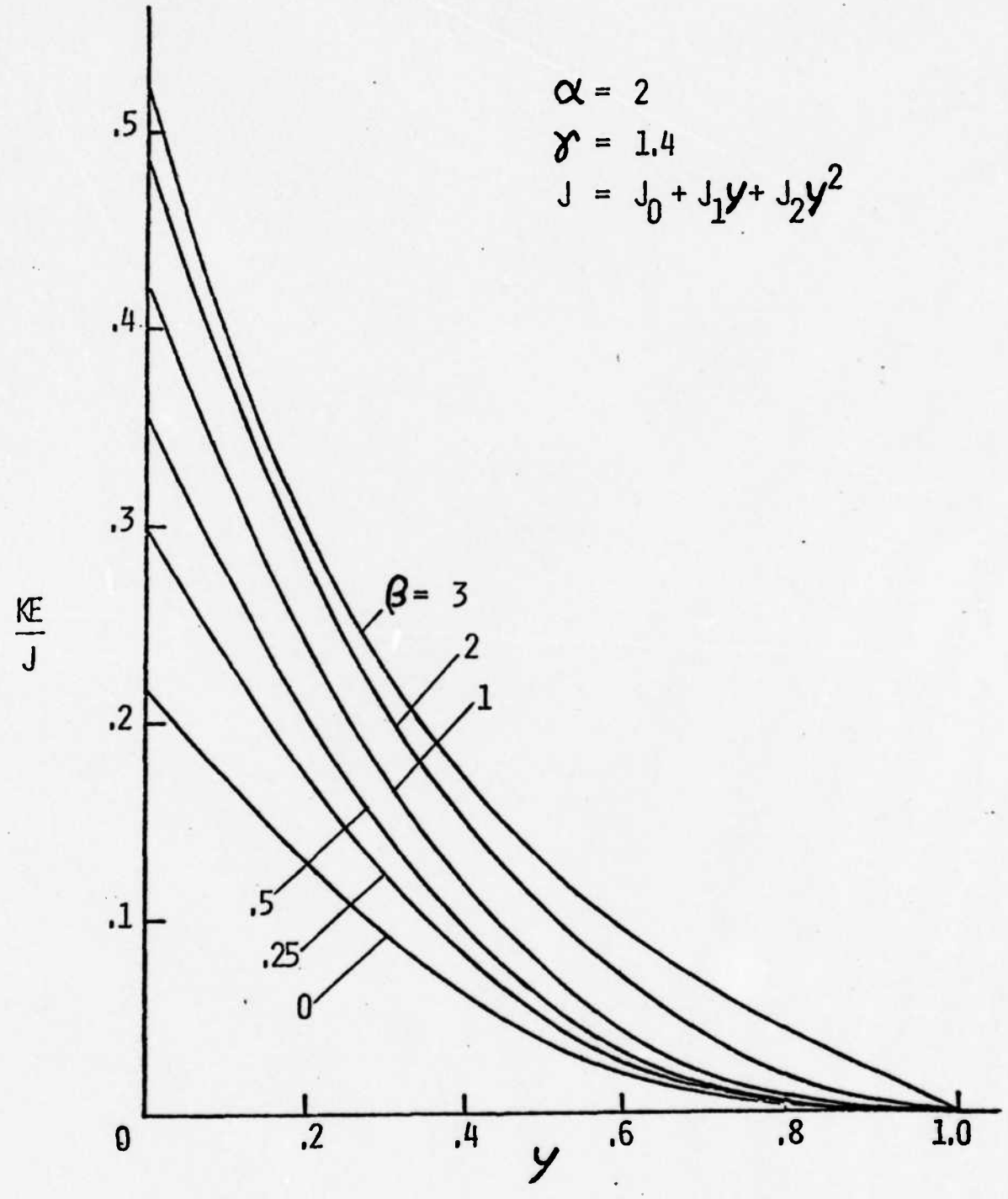
Shock Position as a Function of Time.

Figure 10



Shock Positions for Plane, Cylindrical and Spherical Blasts Compared.

Figure 11



Kinetic Energy Fraction Variation with Shock Strength and Energy Input Parameter.

Figure 12.

APPENDIX A

Summary of first and second order equations required for blast wave computation

FIRST ORDER:

(A-1) $$K_o = \frac{2\gamma}{\gamma+1}\left[\frac{\gamma-1}{(\alpha+1)(\gamma+1)}\right]^{\gamma}$$

(A-2) $$K_{11} = 2\gamma/(\gamma-1) - (\gamma-1)/2\gamma$$

(A-3) $$e_2 = 1+(\gamma-1)\lambda_o/\gamma(\alpha+1)$$

(A-4) $$e_1 = 1 - \lambda_o/\gamma(\alpha+1)$$

(A-5) $$e_3 = 1 + (2\gamma-1)\lambda_o/\gamma(\alpha+1)$$

(A-6) $$g_o = K_o/(a_o^{\alpha}a_o')^{\gamma}m^{\lambda_o/(\alpha+1)}$$

(A-7) $$A_o = \gamma(\alpha+1)[a_o^{\alpha}g_o/a_o' - (\alpha+1)m^2]$$

(A-8) $$B_o = \gamma(\alpha+1)(2\alpha+\lambda_o)m/2$$

(A-9) $$C_o = \gamma[\alpha(\alpha+1)g_o a_o^{\alpha}a_o'/a_o^2 + \lambda_o/2]$$

(A-10) $$F = \gamma(a_o - (\alpha+1)ma_o')$$

(A-11) $$B_1 = B_o + \gamma(\alpha+1)\left[a_o^{\alpha}\frac{g_o a_o''}{(a_o')^2} - \frac{g_o'}{a_o'} - \alpha\frac{g_o}{a_o} - 2m\lambda_o\right]$$

(A-12) $$C_1 = C_o + \gamma\lambda_o^2/2 - \alpha(\alpha+1)(\gamma-1)a_o^{\alpha}g_o'/a_o$$

(A-13) $$D_1 = K_{11}a_o^{\alpha}m^{\lambda_o/(\alpha+1)}[(\alpha+1)g_o' + g_o\lambda_o/m]$$

(A-14) $$E_1 = [(\alpha+1)a_o^{\alpha}g_o' - D_1/K_{11}]/\lambda_o - F/2$$

(A-15) $$a_{11}'' = (B_1a_{11}' + C_1a_{11} + D_1)/A_o$$

$$(A\text{-}16)\quad a''_{12} = (B_1 a'_{12} + C_1 a_{12} + E_1)/A_o$$

$$(A\text{-}17)\quad J'_{11} = \frac{F}{(\alpha+1)}[(1+\lambda_o)a_{11}-(\alpha+1)ma'_{11}] + g_o a_o^{\alpha} a'_o\left[K_{11}\frac{m^{\lambda_o/(\alpha+1)}}{(\gamma-1)} - \left(\frac{a'_{11}}{a'_o} + \alpha\frac{a_{11}}{a_o}\right)\right]$$

$$(A\text{-}18)\quad J'_{12} = \frac{F}{(\alpha+1)}[(1+\lambda_o)a_{12} - (\alpha+1)ma'_{12}] + g_o a_o^{\alpha} a'_o\left[\frac{1-m^{\lambda_o/(\alpha+1)}}{(\gamma-1)\lambda_o} - \left(\frac{a'_{12}}{a'_o} + \alpha\frac{a_{12}}{a_o}\right)\right]$$

$$(A\text{-}19)\quad \frac{g_{11}}{g_o} = K_{11}m^{\lambda_o/(\alpha+1)} - \gamma\left(\frac{a'_{11}}{a'_o} + \alpha\frac{a_{11}}{a_o}\right)$$

$$(A\text{-}20)\quad \frac{g_{12}}{g_o} = \frac{1-m^{\lambda_o/(\alpha+1)}}{\lambda_o} - \gamma\left(\frac{a'_{12}}{a'_o} + \alpha\frac{a_{12}}{a_o}\right)$$

$$(A\text{-}21)\quad \Delta J_{11} = \int_o^{\delta} J'_{11}\,dm = \frac{F\delta}{(\alpha+1)}[(1+\lambda_o)a_{11}-(\alpha+1)\delta a'_{11}]_{\delta} + \left[g_o\left(\frac{K_o}{g_o}\right)^{1/\gamma}\right]_{\delta}\left[K_{11}\frac{\delta^{e_2}}{e_2(\gamma-1)} - \left(\frac{a'_{11}}{a'_o} + \alpha\frac{a_{11}}{a_o}\right)_{\delta}\frac{\delta^{e_1}}{e_1}\right]$$

$$(A\text{-}22)\quad \Delta J_{12} = \int_o^{\delta} J'_{12}\,dm = \frac{F\delta}{(\alpha+1)}[(1+\lambda_o)a_{12}-(\alpha+1)\delta a'_{12}]_{\delta} - \left[g_o\left(\frac{K_o}{g_o}\right)^{1/\gamma}\right]_{\delta}\left(\frac{\delta^{e_2}}{e_2\lambda_o(\gamma-1)} + \left[\frac{a'_{12}}{a'_o} + \alpha\frac{a_{12}}{a_o} - \frac{1}{\lambda_o(\gamma-1)}\right]_{\delta}\frac{\delta^{e_1}}{e_1}\right)$$

SECOND ORDER:

(A-23). $K_1 = K_{11} - \lambda_1/\lambda_o$

(A-24) $K_{21} = K_{11}\,\lambda_1/\lambda_o - 2\gamma/(\gamma-1)^2 - (\gamma-1)^2/8\gamma^2 - \lambda_1^2/2\lambda_o^2$

(A-25) $a_1 = a_{11} + \lambda_1 a_{12}$

(A-26) $g_1/g_o = g_{11}/g_o + \lambda_1 g_{12}/g_o$

(A-27) $$\frac{g_1'}{g_o} = \frac{g_1 g_o'}{g_o^2} + \frac{K_1 \lambda_o m^{\lambda_o/(\alpha+1)}}{(\alpha+1)m} - \gamma\left[\alpha\left(\frac{a_1'}{a_o} - \frac{a_1 a_o'}{a_o^2}\right) + \frac{a_1''}{a_o'} - \frac{a_1' a_o''}{(a_o')^2}\right]$$

(A-28) $G = a_1(1+\lambda_o) - (\alpha+1)ma_1'$

(A-29) $B_2 = B_1 - 2(\alpha+1)\gamma m\lambda_o$

(A-30) $C_2 = C_1 + (1+5\lambda_o/2)\gamma\lambda_o$

(A-31) $$D_2 = (\alpha+1)a_o^{\alpha}\left\{K_{21}\lambda_o\left[\frac{g_o'}{\lambda_o} + \frac{2g_o}{(\alpha+1)m}\right] m^{2\lambda_o/(\alpha+1)} - K_1\lambda_1\left[\frac{g_o'}{\lambda_o} + \frac{g_o}{(\alpha+1)m}\right] m^{\lambda_o/(\alpha+1)} + g_1'\left[\frac{g_1}{g_o} + \alpha\frac{a_1}{a_o}\right] + \frac{g_o'}{2}\left(\gamma\left[\alpha\frac{a_1^2}{a_o^2} + \left(\frac{a_1'}{a_o'}\right)^2\right] - \frac{g_1^2}{g_o^2} - \frac{\lambda_1^2}{\lambda_o^2}\right) + \gamma g_o\left(\frac{\alpha a_1}{a_o^3}[a_o a_1' - a_1 a_o'] + \frac{1}{(a_o')^3}[a_o' a_1' a_1'' - (a_1')^2 a_o'']\right)\right\} + \frac{\gamma\lambda_1}{2}[a_1(1+4\lambda_o) - 3(\alpha+1)ma_1'] + \frac{1}{2}(\alpha-1)\alpha(\alpha+1)a_1^2 g_o'$$

$$(A\text{-}32)\quad E_2 = a_o^{\alpha}\left(\frac{(\alpha+1)g_o'}{2\lambda_o}\left[1-m^{2\lambda_o/(\alpha+1)}\right] - \frac{g_o}{m}m^{2\lambda_o/(\alpha+1)}\right) - \frac{F}{2}$$

$$(A\text{-}33)\quad a_{21}'' = (B_2 a_{21}' + C_2 a_{21} + D_2)/A_o$$

$$(A\text{-}34)\quad a_{22}'' = (B_2 a_{22}' + C_2 a_{22} + E_2)/A_o$$

$$(A\text{-}35)\quad J_{21}' = \frac{1}{\alpha+1}[F(\lambda_1 a_1 + (1+2\lambda_o)a_{21} - (\alpha+1)ma_{21}') + \frac{\gamma}{2}G^2]$$

$$+ \frac{g_o a_o^{\alpha} a_o'}{\gamma-1}\left[\frac{a_{21}'}{a_o'} + \frac{\alpha a_{21}}{a_o} + \frac{\alpha a_1}{a_o}\left(\frac{a_1'}{a_o'} + \frac{(\alpha-1)a_1}{2a_o}\right) + \frac{g_{21}}{g_o}\right.$$

$$\left. + \frac{g_1}{g_o}\left(\frac{a_1'}{a_o'} + \frac{\alpha a_1}{a_o}\right)\right]$$

$$(A\text{-}36)\quad J_{22}' = \frac{F}{\alpha+1}[(1+2\lambda_o)a_{22} - (\alpha+1)ma_{22}']$$

$$+ \frac{g_o a_o^{\alpha} a_o'}{\gamma-1}\left[\frac{a_{22}'}{a_o'} + \frac{\alpha a_{22}}{a_o} + \frac{g_{22}}{g_o}\right]$$

$$(A\text{-}37)\quad \frac{g_{21}}{g_o} = \frac{1}{2}\left(\frac{g_1^2}{g_o^2} - \frac{\lambda_1^2}{\lambda_o^2}\right) - \gamma\left[\alpha\left(\frac{a_{21}}{a_o} - \frac{a_1^2}{2a_o^2}\right) + \frac{a_{21}'}{a_o'} - \frac{1}{2}\left(\frac{a_1'}{a_o'}\right)^2\right]$$

$$+ K_{21}m^{2\lambda_o/(\alpha+1)} - \frac{K_1\lambda_1 m^{\lambda_o/(\alpha+1)}}{\lambda_o}$$

$$(A\text{-}38)\quad \frac{g_{22}}{g_o} = \frac{1}{2\lambda_o}\left(1-m^{2\lambda_o/(\alpha+1)}\right) - \gamma\left(\frac{\alpha a_{22}}{a_o} + \frac{a_{22}'}{a_o'}\right)$$

$$(A\text{-}39)\quad \Delta J_{21} = \int_0^{\delta} J'_{21}\,dm = \frac{\delta}{\alpha+1}\left[F(\lambda_1 a_1 + (1+2\lambda_o)a_{21} - (\alpha+1)\delta a'_{21} + \frac{\gamma}{2}G^2\right]_{\delta}$$

$$+\left[\frac{g_o}{\gamma-1}\left(\frac{K_o}{g_o}\right)^{1/\gamma}\right]_{\delta}\left\{\frac{\delta^{e_1}}{e_1}\left[\frac{g_1}{g_o}\left(\frac{a'_1}{a'_o}+\frac{\alpha a_1}{a_o}\right)+\frac{\alpha a_1}{a_o}\left(\frac{a'_1}{a'_o}+\frac{(\alpha-1)a_1}{2a_o}\right)\right.\right.$$

$$\left.+\frac{1}{2}\left[\frac{g_1^2}{g_o^2}-\frac{\lambda_1^2}{\lambda_o^2}+\gamma\left(\frac{\alpha a_1^2}{a_o^2}+\left(\frac{a'_1}{a'_o}\right)^2\right)\right]-(\gamma-1)\left(\frac{\alpha a_{21}}{a_o}+\frac{a'_{21}}{a'_o}\right)\right]_{\delta}$$

$$\left.+\frac{K_{21}\delta^{e_3}}{e_3}-\frac{K_1\lambda_1}{\lambda_o}\frac{\delta^{e_2}}{e_2}\right\}$$

$$(A\text{-}40)\quad \Delta J_{22} = \int_0^{\delta} J'_{22}\,dm = \frac{\delta}{\alpha+1}\left[F(a_{22}(1+2\lambda_o) - (\alpha+1)\delta a'_{22}\right]_{\delta}$$

$$+\left[\frac{g_o}{\gamma-1}\left(\frac{K_o}{g_o}\right)^{1/\gamma}\right]_{\delta}\left\{\frac{1}{2\lambda_o}\left(\frac{\delta^{e_1}}{e_1}-\frac{\delta^{e_3}}{e_3}\right)-(\gamma-1)\left(\frac{\alpha a_{22}}{a_o}+\frac{a'_{22}}{a'_o}\right)_{\delta}\frac{\delta^{e_1}}{e_1}\right\}$$

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C<<<<<<<<<<<<<<<<<<< BLAST WAVE,VARIABLE ENERGY >>>>>>>>>>>>>>>>>>>>>>>>>>CBLST  10
C         PROGRAM CODED BY PROFESSOR E.T. PITKIN                          CBLST  20
C         MECHANICAL ENGINEERING DEPARTMENT                               CBLST  30
C         UNIVERSITY OF CONNECTICUT                                       CBLST  40
C         STORRS, CONN., 06268                                            CBLST  50
C<<<<<<<<<<<<<<<<<<< SECOND ORDER COMPUTATION >>>>>>>>>>>>>>>>>>>>>>>>>>>CBLST  60
      IMPLICIT REAL*8 (A-H,O-Z)                                           BLST  70
      REAL*8 MASS,LAMN,LAM1,LAM2,LAM3,LAM4,KN,K1,LAMP1,LAMDA,LAMN2        BLST  80
      DIMENSION ZA11(200),ZAP11(200),ZA12(200),ZAP12(200),ZG11(200),      BLST  90
     1ZG12(200),ZA21(200),ZA22(200),ZAP21(200),ZAP22(200),ZG21(200),      BLST 100
     2ZG22(200),C(16),ZA(200),ZAP(200),ZM(200),ZG(200),RZ(100),OM(100)    BLST 110
      EQUIVALENCE (C(1),LAMN),(C(2),KN),(C(3),K1),(C(4),ALP),(C(5),GM),   BLST 120
     1 (C(6),GMOG),(C(7),AK11),(C(8),AK21),(C(9),GN),(C(10),RLAM),        BLST 130
     2 (C(11),LAM1),(C(12),ALFA),(C(13),GAMMA)                            BLST 140
CONSTRUCT INITIAL VALUES FOR INTEGRATION                                  BLST 150
      SAVE = -.005D0                                                      BLST 160
    4 FORMAT(3F10.4,I5)                                                   BLST 170
   10 READ(5,4,END=999) ALFA,BETA,GAMMA,KFIELD                            BLST 180
      L = ALFA + .001D0                                                   BLST 190
      LP = L+1                                                            BLST 200
      ALP = ALFA + 1.D0                                                   BLST 210
      GPL = GAMMA + 1.D0                                                  BLST 220
      GM = GAMMA - 1.D0                                                   BLST 230
      GMOG = GM/GAMMA                                                     BLST 240
      AK11 = 2.D0/GMOG - GMOG/2.D0                                        BLST 250
      LAMN = 2.D0*(ALP-BETA)/(2.D0+BETA)                                  BLST 260
      LAMN2 = LAMN**2                                                     BLST 270
      LAMP1 = LAMN + 1.D0                                                 BLST 280
      KN = 2.D0*GAMMA*(GM/GPL/ALP)**GAMMA/GPL                             BLST 290
      K1 = 0.D0                                                           BLST 300
      F1 = 1.D0 - LAMN/ALP/GAMMA                                          BLST 310
      F2 = 1.D0 + GMOG*LAMN/ALP                                           BLST 320
      F3 = 1.D0 + (2.D0*GAMMA - 1.D0)*LAMN/GAMMA/ALP                      BLST 330
      KPASS = 1                                                           BLST 340
      NV = 12                                                             BLST 350
      WRITE(6,40)                                                         BLST 360
      WRITE(6,41) ALFA,BETA,GAMMA                                         BLST 370
   20 STEP = SAVE                                                         BLST 380
      KK = DABS(.01D0/STEP) + .1D0                                        BLST 390
      DO 21 I=1,15                                                        BLST 400
   21 Z(I) = 0.D0                                                         BLST 410
      MASS = 1.D0                                                         BLST 420
      Z(1) = 1.D0                                                         BLST 430
      Z(2) = GM/GPL/ALP                                                   BLST 440
      Z(5) = 2.D0/ALP/GPL                                                 BLST 450
      J = 191                                                             BLST 460
      ZM(J) = 1.D0                                                        BLST 470
      ZA(J) = 1.D0                                                        BLST 480
      ZAP(J) = Z(2)                                                       BLST 490
      ZA11(J) = 0.D0                                                      BLST 500
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      ZA12(J) = 0.D0                                                    BLST 510
      ZA21(J) = 0.D0                                                    BLST 520
      ZA22(J) = 0.D0                                                    BLST 530
      ZAP11(J) = Z(5)                                                   BLST 540
      ZAP12(J) = 0.D0                                                   BLST 550
      ZAP21(J) = 0.D0                                                   BLST 560
      ZAP22(J) = 0.D0                                                   BLST 570
      ZG(J) = KN/Z(2)**GAMMA                                            BLST 580
      ZG11(J) = ZG(J)*(AK11 - GAMMA*Z(5)/Z(2))                          BLST 590
      ZG12(J) = 0.D0                                                    BLST 600
      ZG21(J) = 0.D0                                                    BLST 610
      ZG22(J) = 0.D0                                                    BLST 620
   25 WRITE(6,42) ZA(J),ZAP(J),Z(3),ZG(J),ZM(J)                         BLST 630
      IF(KPASS.EQ.2) GO TO 30                                           BLST 640
      WRITE(6,43) ZA11(J),ZAP11(J),ZA12(J),ZAP12(J),Z(8),Z(9)           BLST 650
      IF(KPASS.EQ.1) GO TO 50                                           BLST 660
   30 WRITE(6,44) ZA21(J),ZAP21(J),ZA22(J),ZAP22(J),Z(14),Z(15)         BLST 670
   40 FORMAT(////)                                                      BLST 680
   41 FORMAT(' ALFA =',F5.3,3X,'BETA =',F5.3,3X,'GAMMA =',F6.4,//)      BLST 690
   42 FORMAT(/,3X,'A0 =',E10.4,3X,'A0PRIME =',E10.4,3X,'J0 =',E11.5,3X  BLST 700
     1,'G0 =',E11.5,3X,'MASS =',E10.3)                                  BLST 710
   43 FORMAT(/,3X,'A11=',E10.4,3X,'A11PRIME=',E10.4,3X,'A12=',E10.4,3X  BLST 720
     1,'A12PRIME=',E10.4,3X,'J11=',E11.5,3X,'J12=',E11.5,/)             BLST 730
   44 FORMAT(/,3X,'A21=',E10.4,3X,'A21PRIME=',E10.4,3X,'A22=',E10.4,3X  BLST 740
     1,'A22PRIME=',E10.4,3X,'J21=',E11.5,3X,'J22=',E11.5,/)             BLST 750
   45 FORMAT(/,' DJ0 =',F10.5,2X,'EK0 =',F10.5,2X,'EFRAC0 =',F10.5)     BLST 760
   46 FORMAT(/,' RLAM =',F10.5,2X,'K21 =',F10.5)                        BLST 770
   47 FORMAT(/,' DJ11 =',F10.5,2X,'DJ12 =',F10.5,2X,'LAM1 =',F10.5,2X,  BLST 780
     1'J1 =',F10.5,2X,'EK1 =',F10.5,2X,'K1 =',F10.5)                    BLST 790
   48 FORMAT(/,' DJ21 =',F10.5,2X,'DJ22 =',F10.5,2X,'LAM2 =',F10.5,2X,  BLST 800
     1'J2 =',F10.5,2X,'EK2 =',F10.5)                                    BLST 810
COMMENCE INTEGRATION                                                    BLST 820
   50 DO 100 I=1,KK                                                     BLST 830
  100 CALL RUNKUT(Z,MASS,C,STEP,NV)                                     BLST 840
      J = J-1                                                           BLST 850
      FM1 = MASS**(LAMN/ALP)                                            BLST 860
      IF(KPASS.EQ.2) GO TO 98                                           BLST 870
      ZM(J) = MASS                                                      BLST 880
      ZA(J) = Z(1)                                                      BLST 890
      ZAP(J) = Z(2)                                                     BLST 900
      ZG(J) = KN/((Z(1)**L*Z(2))**GAMMA*FM1)                            BLST 910
      ZA11(J) = Z(4)                                                    BLST 920
      ZA12(J) = Z(6)                                                    BLST 930
      ZAP11(J) = Z(5)                                                   BLST 940
      ZAP12(J) = Z(7)                                                   BLST 950
      ZG11(J) = ZG(J)*(AK11*FM1 - GAMMA*(ALFA*Z(4)/Z(1)+Z(5)/Z(2)))     BLST 960
      ZG12(J) = ZG(J)*((1.D0-FM1)/LAMN-GAMMA*(ALFA*Z(6)/Z(1)+Z(7)/Z(2)))BLST 970
   98 IF(KPASS.EQ.1) GO TO 99                                           BLST 980
      ZA21(J) = Z(10)                                                   BLST 990
      ZA22(J) = Z(12)                                                   BLST1000
      ZAP21(J) = Z(11)                                                  BLST1010
      ZAP22(J) = Z(13)                                                  BLST1020
      ZG1 = ZG11(J) + LAM1*ZG12(J)                                      BLST1030
      ZA1 = Z(4) + LAM1*Z(6)                                            BLST1040
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      ZAP1 = Z(5) + LAM1*Z(7)                                          BLST1050
      ZG21(J) =(((ZG1/ZG(J))**2-RLAM**2)/2.DO - GAMMA*(ALFA*Z(10)/Z(1) BLST1060
     1 + Z(11)/Z(2) - (ALFA*(ZA1/Z(1))**2 + (ZAP1/Z(2))**2)/2.DO)      BLST1070
     2 + AK21*FM1**2 - K1*RLAM*FM1)*ZG(J)                             BLST1080
      ZG22(J) = ((1.DO-FM1**2)/2.DO/LAMN - GAMMA*(ALFA*Z(12)/Z(1) +    BLST1090
     1 Z(13)/Z(2)))*ZG(J)                                             BLST1100
CUT DOWN STEP SIZE AS MASS APPROACHES ZERO                            BLST1110
   99 IF(J.EQ.101) STEP = STEP/10.DO                                  BLST1120
      IF(J.EQ.101) GO TO 101                                          BLST1130
      IF(J.EQ.11) GO TO 101                                           BLST1140
      IF(J.GT.2) GO TO 50                                             BLST1150
  101 Z3 = -Z(3)                                                      BLST1160
      WRITE(6,42) ZA(J),ZAP(J),Z3,ZG(J),ZM(J)                         BLST1170
      IF(KPASS.EQ.2) GO TO 104                                        BLST1180
      Z8 = -Z(8)                                                      BLST1190
      Z9 = -Z(9)                                                      BLST1200
      WRITE(6,43)  ZA11(J),ZAP11(J),ZA12(J),ZAP12(J),Z8,Z9            BLST1210
      IF(KPASS.EQ.1) GO TO 105                                        BLST1220
  104 Z14 = -Z(14)                                                    BLST1230
      Z15 = -Z(15)                                                    BLST1240
      WRITE(6,44) ZA21(J),ZAP21(J),ZA22(J),ZAP22(J),Z14,Z15           BLST1250
  105 IF(J.GT.2) GO TO 50                                             BLST1260
COMPUTE EXTRAPOLATED VALUES FOR MASS = 0 WITH GO = CONSTANT           BLST1270
      ENSTP = -.0000100                                               BLST1280
      DO 202 JK=1,2                                                   BLST1290
      DO 200 I=1,90                                                   BLST1300
  200 CALL RUNKUT(Z,MASS,C,ENSTP,NV)                                  BLST1310
      FM1 = MASS**(LAMN/ALP)                                          BLST1320
      GN = KN/((Z(1)**L*Z(2))**GAMMA*FM1)                             BLST1330
      Z3 = -Z(3)                                                      BLST1340
      WRITE (6,42) Z(1),Z(2),Z3,GN,MASS                               BLST1350
      IF(KPASS.EQ.2) GO TO 199                                        BLST1360
      Z8 = -Z(8)                                                      BLST1370
      Z9 = -Z(9)                                                      BLST1380
      WRITE(6,43) Z(4),Z(5),Z(6),Z(7),Z8,Z9                           BLST1390
      IF(KPASS.EQ.1) GO TO 201                                        BLST1400
  199 Z14 = -Z(14)                                                    BLST1410
      Z15 = -Z(15)                                                    BLST1420
      WRITE(6,44) Z(10),Z(11),Z(12),Z(13),Z14,Z15                     BLST1430
  201 IF(JK.EQ.2) GO TO 202                                           BLST1440
      Z2S = Z(2)                                                      BLST1450
      Z4S = Z(4)                                                      BLST1460
      Z5S = Z(5)                                                      BLST1470
      Z6S = Z(6)                                                      BLST1480
      Z7S = Z(7)                                                      BLST1490
      GNS = GN                                                        BLST1500
      ENSTP = ENSTP/10.DO                                             BLST1510
      IF(KPASS.EQ.1) GO TO 202                                        BLST1520
      Z10S = Z(10)                                                    BLST1530
      Z11S = Z(11)                                                    BLST1540
      Z12S = Z(12)                                                    BLST1550
      Z13S = Z(13)                                                    BLST1560
  202 CONTINUE                                                        BLST1570
      IF(KPASS.EQ.2) GO TO 206                                        BLST1580
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      AN = Z(1)                                                          BLST1590
      ANL = AN**L                                                        BLST1600
      ANP = Z(2)                                                         BLST1610
      FAJ = AN -ALP*ANP*MASS                                             BLST1620
      GNO = (GN/GNS)*GN                                                  BLST1630
      GN = .7D0*GN + .3D0*GNO                                            BLST1640
      GNKF = GN*(KN/GN)**(1.D0/GAMMA)/GM                                 BLST1650
      FJ1 = GNKF*MASS**E1/E1                                             BLST1660
      FJ2 = GNKF*MASS**E2/E2                                             BLST1670
      FJ3 = GNKF*MASS**E3/E3                                             BLST1680
      ANO = AN**LP - ALP*(GM/GN)*FJ1                                     BLST1690
      IF(ANO.LE.1.D-13) ANO=1.D-13                                       BLST1700
      ANO = ANO**(1.D0/ALP)                                              BLST1710
      ANPO = (Z(2)/Z2S)*Z(2)                                             BLST1720
      DJO = GAMMA*MASS*FAJ**2/ALP/2.D0 + FJ1                             BLST1730
      AJO = DJO - Z(3)                                                   BLST1740
      EKO = DJO - FJ1 - Z(10)                                            BLST1750
      EFRACO = EKO/AJO                                                   BLST1760
      AMASS = 0.D0                                                       BLST1770
      WRITE(6,42) ANO,ANPO,AJO,GNO,AMASS                                 BLST1780
      WRITE(6,45) DJO,EKO,EFRACO                                         BLST1790
      A11 = Z(4)                                                         BLST1800
      AP11 = Z(5)                                                        BLST1810
      A12 = Z(6)                                                         BLST1820
      AP12 = Z(7)                                                        BLST1830
      A110 = (A11/Z4S)*A11                                               BLST1840
      A120 = (A12/Z6S)*A12                                               BLST1850
      AP110 = (AP11/Z5S)*AP11                                            BLST1860
      AP120 = (AP12/Z7S)*AP12                                            BLST1870
      G11 = GN*(AK11*FM1 - GAMMA*(ALFA*Z(4)/Z(1)+Z(5)/Z(2)))             BLST1880
      G12 = GN*((1.D0-FM1)/LAMN-GAMMA*(ALFA*Z(6)/Z(1)+Z(7)/Z(2)))        BLST1890
      EK11 = GAMMA*MASS*FAJ*(LAMP1*A11/ALP - MASS*AP11)                  BLST1900
      EK12 = GAMMA*MASS*FAJ*(LAMP1*A12/ALP - MASS*AP12)                  BLST1910
      DJ11 = EK11 + AK11*FJ2 -GM*(ALFA*A11/AN+AP11/ANP)*FJ1              BLST1920
      DJ12 = EK12 + (FJ1-FJ2)/LAMN - GM*(ALFA*A12/AN+AP12/ANP)*FJ1       BLST1930
      EK11 = EK11 - Z(11)                                                BLST1940
      EK12 = EK12 - Z(12)                                                BLST1950
      AZ8 = DJ11 - Z(8)                                                  BLST1960
      AZ9 = DJ12 - Z(9)                                                  BLST1970
      WRITE(6,43) A110,AP110,A120,AP120,AZ8,AZ9                          BLST1980
      ZM(1) = MASS                                                       BLST1990
      ZA(1) = AN                                                         BLST2000
      ZAP(1) = ANP                                                       BLST2010
      ZG(1) = GN                                                         BLST2020
      ZA11(1) = A11                                                      BLST2030
      ZA12(1) = A12                                                      BLST2040
      ZAP11(1) = AP11                                                    BLST2050
      ZAP12(1) = AP12                                                    BLST2060
      ZG11(1) = G11                                                      BLST2070
      ZG12(1) = G12                                                      BLST2080
      IF(KPASS.EQ.1) GO TO 210                                           BLST2090
  206 WRITE(6,42) ANO,ANPO,AJO,GNO,AMASS                                 BLST2100
      A1 = A11 + LAM1*A12                                                BLST2110
      AP1 = AP11 + LAM1*AP12                                             BLST2120
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```
      A21 = Z(10)                                                          BLST2130
      A22 = Z(12)                                                          BLST2140
      AP21 = Z(11)                                                         BLST2150
      AP22 = Z(13)                                                         BLST2160
      A210 = (A21/Z10S)*A21                                                BLST2170
      A220 = (A22/Z12S)*A22                                                BLST2180
      AP210 = (AP21/Z11S)*AP21                                             BLST2190
      AP220 = (AP22/Z13S)*AP22                                             BLST2200
      G1 = G11 + LAM1*G12                                                  BLST2210
      EK21 = GAMMA*MASS*(FAJ*(A21*(LAMP1+LAMN) - ALP*MASS*AP21 + LAM1      BLST2220
     1*A1) + (LAMP1*A1 - ALP*MASS*AP1)**2/2.D0)/ALP                        BLST2230
      DJ21 = EK21 + AK21*FJ3 - K1*RLAM*FJ2 + FJ1*((G1/GN)*(ALFA*A1/AN +    BLST2240
     1 AP1/ANP) + ((G1/GN)**2 - RLAM**2 + GAMMA*(ALFA*(A1/AN)**2 + (AP1    BLST2250
     2/ANP)**2))/2.D0 - GM*(ALFA*A21/AN+AP21/ANP) + ALFA*(A1/AN)*((ALFA    BLST2260
     3 -1.D0)*A1/AN/2.D0+AP1/ANP))                                         BLST2270
      EK22 = GAMMA*MASS*FAJ*(A22* (LAMP1+LAMN) - ALP*MASS*AP22)/ALP        BLST2280
      DJ22 = EK22 + (FJ1-FJ3)/LAMN/2.D0 - GM*(ALFA*A22/AN+AP22/ANP)*FJ1    BLST2290
      EK21 = EK21 - Z(8)                                                   BLST2300
      EK22 = EK22 - Z(9)                                                   BLST2310
      AZ14 = DJ21 - Z(14)                                                  BLST2320
      AZ15 = DJ22 - Z(15)                                                  BLST2330
      WRITE(6,44) A210,AP210,A220,AP220,AZ14,AZ15                          BLST2340
      ZA21(1) = A21                                                        BLST2350
      ZA22(1) = A22                                                        BLST2360
      ZAP21(1) = AP21                                                      BLST2370
      ZAP22(1) = AP22                                                      BLST2380
      ZG21(1) = (((G1/GN)**2 - RLAM**2)/2.D0 - GAMMA*(ALFA*Z(10)/Z(1)      BLST2390
     1 + Z(11)/Z(2) - (ALFA*( A1/Z(1))**2 + ( AP1/Z(2))**2)/2.D0)          BLST2400
     2 + AK21*FM1**2 - K1*RLAM*FM1)*GN                                     BLST2410
      ZG22(1) = ((1.D0-FM1**2)/2.D0/LAMN - GAMMA*(ALFA*Z(12)/Z(1) +        BLST2420
     1 Z(13)/Z(2)))*GN                                                     BLST2430
      IF(KPASS.EQ.2) GO TO 220                                             BLST2440
C COMPUTE LAMEDAS AND ENERGY INTEGRAL CONTIBUTIONS                         BLST2450
  210 OMEGO = 2.D0/(LAMN+2.D0)                                             BLST2460
      OMEG1 = (2.D0*LAMN+2.D0)/(3.D0*LAMN+2.D0)                            BLST2470
      LAM1 = (1.D0/GM/ALP-AZ8)/(AZ9+AJ0*(BETA*OMEG1/OMEGO-ALP)/LAMN2)      BLST2480
      RLAM = LAM1/LAMN                                                     BLST2490
      K1 = AK11 - RLAM                                                     BLST2500
      A21 = AK11*RLAM - 2.D0/GMOG/GM - GMOG**2/8.D0 - RLAM**2/2.D0         BLST2510
      AJ1 = AZ8 + LAM1*AZ9                                                 BLST2520
      EK1 = EK11 + LAM1*EK12                                               BLST2530
      WRITE(6,47) DJ11,DJ12,LAM1,AJ1,EK1,K1                                BLST2540
      WRITE(6,46) RLAM,AK21                                                BLST2550
      NV = 15                                                              BLST2560
      KPASS = 2                                                            BLST2570
      GO TO 20                                                             BLST2580
  220 BZ1 = -LAM1/LAMN2                                                    BLST2590
      OMEG2 = (4.D0*LAMN+2.D0)/(5.D0*LAMN+2.D0)                            BLST2600
      SA = (BETA*OMEG2/OMEGO - ALP)/2.D0                                   BLST2610
      SB = -AJ0*SA/LAMN2                                                   BLST2620
      SA =AJ0*BZ1**2*(SA*LAMP1 + BETA*(BETA-1.D0)*(OMEG1/OMEGO)**2         BLST2630
     1/2.D0 + ALP*((ALFA+2.D0)/2.D0 - BETA*OMEG1/OMEGO))                   BLST2640
      LAM2 = (SA - AZ14)/(AZ15 - SB)                                       BLST2650
      LAM4 =-(1.D0-BETA/ALP) *((ALFA+2.D0)/4.D0 - LAM2 - 2.D0*LAM1 -       BLST2660
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     1 3.D0*LAMN)                                                          BLST2670
       LAM3 = -LAMN - LAM1 - LAM2 - LAM4                                   BLST2680
       AJ2 = AZ14 + LAM2*AZ15                                              BLST2690
       EK2 = EK21 + LAM2*EK22                                              BLST2700
       WRITE(6,48) DJ21,DJ22,LAM2,AJ2,EK2                                  BLST2710
C COMPUTE SHOCK TRAJECTORY                                                 BLST2720
       WRITE(6,40)                                                         BLST2730
       BZ0 = (((2.D0+BETA)/(ALP+3.D0))**BETA/AJ0)**(1.D0/(ALP-BETA))       BLST2740
       BZ(1) = BZ1                                                         BLST2750
       BZ(2) = -(LAM2 + LAMP1*LAM1*BZ1)/2.D0/LAMN2                         BLST2760
       BZ(3) = -(LAM3+(LAMP1+LAMN)*LAM1*BZ(2)+LAMP1*LAM2*BZ1)/3.D0/LAMN2   BLST2770
       BZ(4) = -(LAM4+(1.D0+3.D0*LAMN)*LAM1*BZ(3)+(LAMP1+LAMN)*LAM2*BZ(2)  BLST2780
      1+LAMP1*LAM3*BZ1)/4.D0/LAMN2                                         BLST2790
       DO 221 K=5,100                                                      BLST2800
       AK = K                                                              BLST2810
       BZ(K) = -((1.D0+(AK-4.D0)*LAMN)*BZ(K-4)*LAM4 + (1.D0+(AK-3.D0)      BLST2820
      1*LAMN)*BZ(K-3)*LAM3+(1.D0+(AK-2.D0)*LAMN)*BZ(K-2)*LAM2              BLST2830
      2 + (1.D0+(AK-1.D0)*LAMN)*BZ(K-1)*LAM1)/LAMN2/AK                     BLST2840
   221 OM(K) = 1.D0/(1.D0+.5D0/(AK+1.D0/LAMN))                             BLST2850
       OM(1) = OMEG1                                                       BLST2860
       OM(2) = OMEG2                                                       BLST2870
       OM(3) = 1.D0/(1.D0+.5D0/(3.D0+1.D0/LAMN))                           BLST2880
       OM(4) = 1.D0/(1.D0+.5D0/(4.D0+1.D0/LAMN))                           BLST2890
       Y = 0.D0                                                            BLST2900
       DO 225 I=1,20                                                       BLST2910
       SUMZ = 0.D0                                                         BLST2920
       SUMO = 0.D0                                                         BLST2930
       DO 222 K=1,100                                                      BLST2940
       TERMZ = BZ(K)*Y**K                                                  BLST2950
       TERMO = OM(K)*TERMZ                                                 BLST2960
       IF(TERMO.LT.1.D-5.AND.TERMZ.LT.1.D-5) GO TO 223                     BLST2970
       SUMZ = SUMZ + TERMZ                                                 BLST2980
   222 SUMO = SUMO + TERMO                                                 BLST2990
   223 FIRST = BZ0*Y**(1.D0/LAMN)                                          BLST3000
       BZS = (1.D0 + SUMZ)*FIRST                                           BLST3010
       OMEGA = (OMEG0 + SUMO)*FIRST*DSQRT(Y)                               BLST3020
       LAMDA = LAMN + LAM1*Y + LAM2*Y**2 + LAM3*Y**3 + LAM4*Y**4           BLST3030
       AJ = AJ0 + AJ1*Y + AJ2*Y**2                                         BLST3040
       EK = EK0 + EK1*Y + EK2*Y**2                                         BLST3050
       EFRAC = EK/AJ                                                       BLST3060
       WRITE(6,440) Y,OMEGA,BZS,AJ,EFRAC,LAMDA                             BLST3070
   225 Y = Y + .05D0                                                       BLST3080
       WRITE(6,442) ALFA,BETA,AJ0,AJ1,AJ2,LAMN,LAM1,LAM2,LAM3,LAM4,        BLST3090
      1 AZ8,AZ9,AZ14,AZ15,GAMMA,SAVE                                       BLST3100
C COMPUTE FLOW FIELD                                                       BLST3110
       IF(KFIELD.EQ.0) GO TO 10                                            BLST3120
       DO 350 II=1,9                                                       BLST3130
       Y = 0.1D0*(II-1)                                                    BLST3140
       WRITE(6,40)                                                         BLST3150
       LAMDA = LAMN + LAM1*Y + LAM2*Y**2 + LAM3*Y**3 + LAM4*Y**4           BLST3160
       WRITE(6,445) Y,LAMDA                                                BLST3170
       WRITE(6,446)                                                        BLST3180
       DO 350 KK=1,110                                                     BLST3190
       IF(KK.EQ.1) GO TO 302                                               BLST3200
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      IF(MOD((KK-1),5).NE.0.AND.KK.LT.100) GO TO 350                    BLST3210
      IF (KK.LT.92) GO TO 302                                           BLST3220
      IF(KK.GT.100) GO TO 301                                           BLST3230
      JJ = 101 - (KK-91)*10                                             BLST3240
      GO TO 303                                                         BLST3250
  301 JJ = 111 - KK                                                     BLST3260
      GO TO 303                                                         BLST3270
  302 JJ = 192 - KK                                                     BLST3280
  303 ZA1 = ZA11(JJ) + LAM1*ZA12(JJ)                                    BLST3290
      ZA2 = ZA21(JJ) + LAM2*ZA22(JJ)                                    BLST3300
      ZAP1 = ZAP11(JJ) + LAM1*ZAP12(JJ)                                 BLST3310
      ZAP2 = ZAP21(JJ) + LAM2*ZAP22(JJ)                                 BLST3320
      ZAT = ZA(JJ) + ZA1*Y + ZA2*Y**2                                   BLST3330
      ZAPT = ZAP(JJ) + ZAP1*Y + ZAP2*Y**2                               BLST3340
      V = ALP*ZAT**L*ZAPT                                               BLST3350
      VEL = ZAT - ALP*ZM(JJ)*ZAPT + LAMDA*Y*(ZA1+2.D0*ZA2*Y)            BLST3360
      ZGT = ZG(JJ) + (ZG11(JJ) + LAM1*ZG12(JJ))*Y +(ZG21(JJ)+LAM2*      BLST3370
     1 ZG22(JJ))*Y**2                                                   BLST3380
      IF(KK.GT.1) GO TO 305                                             BLST3390
      ZGTF = ZGT                                                        BLST3400
      VF = V                                                            BLST3410
      VELF = VEL                                                        BLST3420
  305 RHO = VF/V                                                        BLST3430
      ZGT = ZGT/ZGTF                                                    BLST3440
      VELR = VEL/VELF                                                   BLST3450
      WRITE(6,448) ZM(JJ),ZAT,ZGT,RHO,VELR                              BLST3460
  350 CONTINUE                                                          BLST3470
      GO TO 10                                                          BLST3480
  440 FORMAT('  Y =',F6.3,'  OMEGA =',E11.5,'  Z =',E11.5,'  J =',F10.5 BLST3490
     1,'  KE/J =',F10.5,'  LAMDA =',F10.5)                              BLST3500
  442 FORMAT(/,' SUMMARY',/,' ALFA=',F10.5,2X,'BETA=',F10.5,2X,'JO=',   BLST3510
     1F10.5,2X,'J1=',F10.5,2X,'J2=',F10.5,/,' LAMO=',F10.5,2X,'LAM1=',  BLST3520
     2F10.5,2X,'LAM2=',F10.5,2X,'LAM3=',F10.5,2X,'LAM4=',F10.5,/,' J11='BLST3530
     3,F10.5,2X,'J12=',F10.5,2X,'J21=',F10.5,2X,'J22=',F10.5,/,' GAMMA='BLST3540
     4,F10.5,2X,'STEP=',F10.5)                                          BLST3550
  445 FORMAT(/,'  Y =',F5.2,2X,'LAMDA =',F10.5)                         BLST3560
  446 FORMAT(4X,15X,'MASS',13X,'RADIUS',11X,'PRESSURE',12X,'DENSITY'    BLST3570
     1,11X,'VELOCITY',//)                                               BLST3580
  448 FORMAT(4X,5F19.5)                                                 BLST3590
  999 STOP                                                              BLST3600
      END                                                               BLST3610
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      SUBROUTINE FCT(Z,ZP,C,M)                                          FCT   10
      IMPLICIT REAL*8 (A-H,O-Z)                                         FCT   20
      REAL*8 LAMN,KN,M,LAM1,K1,LAMP1                                    FCT   30
      DIMENSION Z(15),ZP(15),C(16)                                      FCT   40
C AND Z TRANSFERS                                                       FCT   50
      IF(M.NE.1.DO) GO TO 10                                            FCT   60
      LAMN = C(1)                                                       FCT   70
      KN = C(2)                                                         FCT   80
      K1 = C(3)                                                         FCT   90
      ALP = C(4)                                                        FCT  100
      GAMM = C(5)                                                       FCT  110
      GMCG = C(6)                                                       FCT  120
      AK11 = C(7)                                                       FCT  130
      AK21 = C(8)                                                       FCT  140
      RLAM = C(10)                                                      FCT  150
      LAM1 = C(11)                                                      FCT  160
      ALF = C(12)                                                       FCT  170
      GAM = C(13)                                                       FCT  180
      LAMP1 = LAMN + 1.DO                                               FCT  190
      L = ALF + .01DO                                                   FCT  200
   10 AN = Z(1)                                                         FCT  210
      ANP = Z(2)                                                        FCT  220
COMPUTE COMMON TERMS                                                    FCT  230
      ANL = AN**L                                                       FCT  240
      FM1 = M**(LAMN/ALP)                                               FCT  250
      GN = KN/(ANL*ANP)**GAM/FM1                                        FCT  260
      FAJ = AN - ALP*M*ANP                                              FCT  270
      AA = ALP*GAM*(ANL*GN/ANP - ALP*M**2)                              FCT  280
COMPUTE ZERO ORDER DERIVATIVES                                          FCT  290
      B = ALP*GAM*(2.DO*ALF + LAMN)*M/2.DO                              FCT  300
      CO = GAM*(LAMN/2.DO + GN*ALF*ALP*ANL*ANP/AN**2)                   FCT  310
      ZP(1) = ANP                                                       FCT  320
      ZP(2) = (B*ANP - CO*AN - GN*ANL*LAMN/M)/AA                        FCT  330
      EKO = GAM*FAJ**2/ALP/2.DO                                         FCT  340
      ZP(3) = EKO + GN*ANL*ANP/GAMM                                     FCT  350
COMPUTE FIRST ORDER DERIVATIVES                                         FCT  360
      A11 = Z(4)                                                        FCT  370
      AP11 = Z(5)                                                       FCT  380
      A12 = Z(6)                                                        FCT  390
      AP12 = Z(7)                                                       FCT  400
      ANPP = ZP(2)                                                      FCT  410
      GNP = -GN*(LAMN/ALP/M + GAM*(ALF*ANP/AN+ANPP/ANP))                FCT  420
      BB = ALP*GAM*(ANL*(GN*ANPP/ANP**2 - GNP/ANP - ALF*GN/AN)          FCT  430
     1 -2.DO*LAMN*M) + B                                                FCT  440
      CC = CO + GAM*LAMN**2/2.DO - GAMM*ALF*ALP*ANL*GNP/AN              FCT  450
      DD = AK11*FM1*ANL*(GNP*ALP + GN*LAMN/M)                           FCT  460
      EE = (ANL*ALP*GNP - DD/AK11)/LAMN - FAJ*GAM/2.DO                  FCT  470
      ZP(4) = AP11                                                      FCT  480
      ZP(5) = (BB*AP11+CC*A11+DD)/AA                                    FCT  490
      ZP(6) = AP12                                                      FCT  500
      ZP(7) = (BB*AP12+CC*A12+EE)/AA                                    FCT  510
      IF(K1.NE.0.DO) GO TO 100                                          FCT  520
      EK11 =  GAM*FAJ*(A11*LAMP1-ALP*M*AP11)/ALP                        FCT  530
      EK12 =  GAM*FAJ*(A12*LAMP1-ALP*M*AP12)/ALP                        FCT  540
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      ZP(8) = EK11 + GN*ANL*(AK11*ANP*FM1/GAMM-AP11-ALF*ANP*A11/AN)      FCT  550
      ZP(9) = EK12 + GN*ANL*(ANP*(1.D0-FM1)/GAMM/LAMN - AP12             FCT  560
     1 - ALF*ANP*A12/AN)                                                 FCT  570
      ZP(10) = EK0                                                       FCT  580
      ZP(11) = EK11                                                      FCT  590
      ZP(12) = EK12                                                      FCT  600
      RETURN                                                             FCT  610
COMPUTE SECOND ORDER DERIVATIVES                                         FCT  620
  100 A1 = A11 + LAM1*A12                                                FCT  630
      AP1 = AP11 + LAM1*AP12                                             FCT  640
      APP1 = ZP(5) + LAM1*ZP(7)                                          FCT  650
      G1 = GN*(RLAM - GAM*(ALF*A1/AN + AP1/ANP) + K1*FM1 )               FCT  660
      GP1 = G1*GNP/GN + GN*(K1*LAMN*FM1/M/ALP - GAM*(ALF*(AP1/AN-A1      FCT  670
     1*ANP/AN**2) + (APP1/ANP-AP1*ANPP/ANP**2)))                         FCT  680
      A21 = Z(10)                                                        FCT  690
      AP21 = Z(11)                                                       FCT  700
      A22 = Z(12)                                                        FCT  710
      AP22 = Z(13)                                                       FCT  720
      BBB = BB - 2.D0*ALP*GAM*M*LAMN                                     FCT  730
      CCC = CC + GAM*LAMN*(1.D0+2.5D0*LAMN)                              FCT  740
      DDD = AK21*LAMN*FM1**2*(GNP/LAMN+2.D0*GN/ALP/M) - K1*LAM1*FM1*     FCT  750
     1(GNP/LAMN+GN/ALP/M) + GNP*(GAM*(ALF*(A1/AN)**2+(AP1/ANP)**2) -     FCT  760
     2(G1/GN)**2-RLAM**2)/2.D0+GP1*(G1/GN+ALF*A1/AN)+GAM*GN*(ALF*A1*     FCT  770
     3(AN*AP1-A1*ANP)/AN**3 + (ANP*AP1*APP1-AP1**2*ANPP)/ANP**3)         FCT  780
      DDD = DDD*ALP*ANL + GAM*LAM1*(A1*(1.D0+4.D0*LAMN) - 3.D0*ALP*M*    FCT  790
     1 AP1)/2.D0 + ALP*ALF*(ALF-1.D0)*A1**2*GNP/2.D0                     FCT  800
      EEE = .5D0*ALP*(ANL*GNP*(1.D0-FM1**2)/LAMN + GAM*M*ANP) -          FCT  810
     1 ANL*GN*FM1**2/M - GAM*AN/2.D0                                     FCT  820
      ZP(10) = AP21                                                      FCT  830
      ZP(11) = (BBB*AP21+CCC*A21+DDD)/AA                                 FCT  840
      ZP(12) = AP22                                                      FCT  850
      ZP(13) = (BBB*AP22+CCC*A22+EEE)/AA                                 FCT  860
      G21 =(((G1/GN)**2-RLAM**2)/2.D0 + AK21*FM1**2 - RLAM*K1*FM1        FCT  870
     1 - GAM*(ALF*(A21/AN-(A1/AN)**2/2.D0) + AP21/ANP - (AP1/ANP)**2     FCT  880
     2/2.D0))*GN                                                         FCT  890
      G22 = GN*((1.D0-FM1**2)/LAMN/2.D0 - GAM*(ALF*A22/AN+AP22/ANP))     FCT  900
      EK21 = GAM*(FAJ*(LAM1*A1 + (LAMP1+LAMN)*A21 - ALP*M*AP21)          FCT  910
     1 + (LAMP1*A1 - ALP*M*AP1)**2/2.D0)/ALP                             FCT  920
      ZP(14) = EK21 +ANL*ANP*((ALF*A21/AN + AP21/ANP + (AP1/ANP+(ALF     FCT  930
     1-1.D0)*A1/AN/2.D0)*ALF*A1/AN)*GN+G1*(AP1/ANP+ALF*A1/AN)+G21)/GAMM  FCT  940
      EK22 = GAM*(FAJ*((LAMP1+LAMN)*A22 - ALP*M*AP22))/ALP               FCT  950
      ZP(15) = EK22 + ANL*ANP*(GN*(ALF*A22/AN+AP22/ANP) + G22)/GAMM      FCT  960
      ZP(8) = EK21                                                       FCT  970
      ZP(9) = EK22                                                       FCT  980
      RETURN                                                             FCT  990
      END                                                                FCT 1000
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      SUBROUTINE RUNKUT(Z,X,C,H,N)                                          RNKT  10
CONSTANT STEP, FOURTH ORDER RUNGE-KUTTA INTEGRATION ALGORITHM               RNKT  20
      IMPLICIT REAL*8 (A-H,O-Z)                                             RNKT  30
      DIMENSION Z(15),C(16),Y(15),RK1(15),RK2(15),RK3(15),RK4(15)           RNKT  40
      H2 = H/2.DO                                                           RNKT  50
      CALL FCT(Z,RK1,C,X)                                                   RNKT  60
      DO 10 J=1,N                                                           RNKT  70
   10 Y(J) = Z(J) + H2*RK1(J)                                               RNKT  80
      X = X+ H2                                                             RNKT  90
      CALL FCT(Y,RK2,C,X)                                                   RNKT 100
      DO 11 J=1,N                                                           RNKT 110
   11 Y(J) = Z(J) + H2*RK2(J)                                               RNKT 120
      CALL FCT(Y,RK3,C,X)                                                   RNKT 130
      DO 12 J=1,N                                                           RNKT 140
   12 Y(J) = Z(J) + H *RK3(J)                                               RNKT 150
      X = X+ H2                                                             RNKT 160
      CALL FCT(Y,RK4,C,X)                                                   RNKT 170
      DO 13 J=1,N                                                           RNKT 180
   13 Z(J) = Z(J) + H*(RK1(J)+2.DO*(RK2(J)+RK3(J))+RK4(J))/6.DO             RNKT 190
      RETURN                                                                RNKT 200
      END                                                                   RNKT 210
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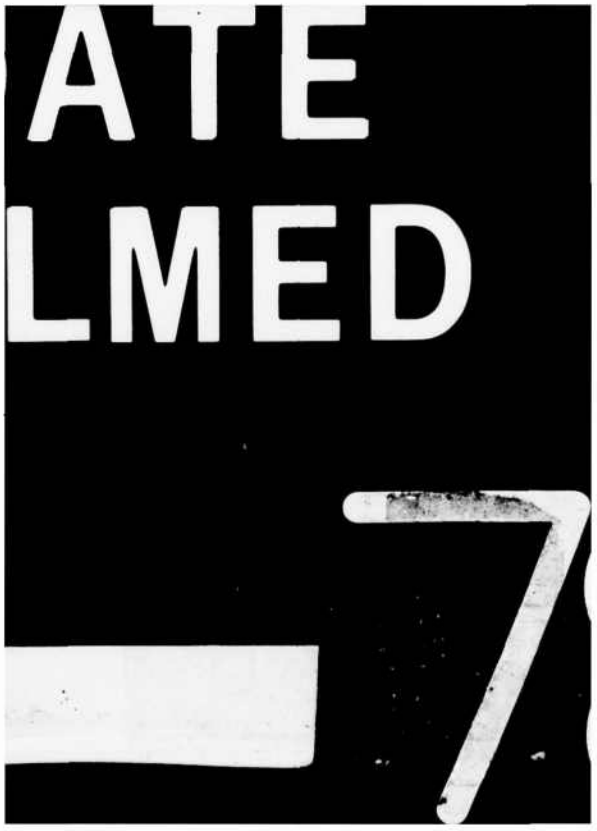
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