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PROPAGATION OF ELECTROMAGNETIC WAVES IN PARAMETRIC MEDIA IN THE--ETC(U)

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PROPAGATION OF ELECTROMAGNETIC WAVES IN PARAMETRIC MEDIA IN THE APPROXIMATION OF GEOMETRIC OPTICS

by

V. N. Krasil'nikov

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	ɑ	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	ϵ	Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	Υ	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ κ	ϰ	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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PROPAGATION OF ELECTROMAGNETIC WAVES IN PARAMETRIC MEDIA IN THE APPROXIMATION OF GEOMETRIC OPTICS.

V. N. Krasil'nikov.

§1. Posing of the problem.

During the passage of waves through media whose parameters are changed in time (subsequently let us call them simply parametric), occur special effects, for example a change in the spectral composition of signal. Therefore to their study traditional approach [1] proves to be inapplicable, and is required special examination, in order to explain the possibility of using the laws of geometric optics. In this case, systematically it is more justified to analyze directly the equations of Maxwell, but not to reduce them for different special cases to one differential equation for certain scalar function, as this frequently is done [1].

Since the approximation of geometric optics knowingly does not befit for the description of fields in the vicinity of their sources, the usual formulation of the problem of geometric approximation consists of finding of wave process of space, if is known the value of field on certain initial surface S_0. The latter can be selected in the wave zone of emitter. It can pass in those regions of space, where there is still no perturbation action of heterogeneous and parametric properties of the medium. Surface S_0 is convenient to combine with the position of wave front (in monochromatic field this is the constant-phase surface).

Outside surface S_0 let us consider the medium isotropic, heterogeneous and parametric; its properties are described by three functions: dielectric and magnetic constants $\varepsilon(\omega, r, t)$ and $\mu(\omega, r, t)$ and conductivity $\eta(\omega, r, t)$. If primary radiation (field on surface S_0) can be considered monochromatic (frequency ω_0), then due to the parametric nature of the medium the spectral composition of signal will be changed in certain interval of frequencies $\Delta\omega$ near ω_0.

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As we shall see further on, relative frequency drift in all cases of practical interest is small, and it is reasonable to disregard the dispersion of the characteristics of the medium in interval $\Delta\omega$, i.e., to take the values of functions ε, μ and η when $\omega = \omega_0$. Then in the Gaussian system of units the equations of Maxwell it is possible to write thus:

$$\begin{aligned} \text{rot}\,\mathbf{E} + \frac{1}{c} \cdot \frac{\partial}{\partial t}(\mu \mathbf{H}) &= 0, \\ \text{rot}\,\mathbf{H} - \frac{1}{c} \cdot \frac{\partial}{\partial t}(\varepsilon \mathbf{E}) &= \frac{4\pi\eta}{c}\mathbf{E}. \end{aligned} \quad (1)$$

Let us note that on the strength of the linearity of system (1) and for parametric media it is possible to utilize principle of superposition and to describe the dependence of electromagnetic vectors $\mathbf{E}$ and $\mathbf{H}$ (but not the parameters ε, μ and η !) on time with the aid of the composite factor $\exp\{-i\omega_0 t\}$ - we will consider that on the initial surface S_0 we have precisely this time behavior of field.

Let us impose on the properties of the medium limitations which make it possible to apply to the analysis of the process of the propagation of waves the method of geometric optics.

First, it is well known [1] that the medium must be weakly heterogeneous, in other words, its characteristics ε, μ and η must be changed little at distances of the order of the local wavelength λ (at a given place, at a given instant), i.e., if L is smallest of the values:

$$L_\varepsilon = \frac{|\varepsilon|}{|\nabla\varepsilon|},\ L_\mu = \frac{|\mu|}{|\nabla\mu|},\ L_\eta = \frac{|\eta|}{|\nabla\eta|},$$

then

$$\frac{\lambda}{L} \ll 1. \qquad (2)$$

Further let us consider the medium weakly-parametric, i.e., assume that its properties little are changed during a time on the order of ω^{-1}, where ω is local frequency of field. This condition takes the form

$$\frac{1}{\omega T} \ll 1, \qquad (3)$$

where T is smallest of the times:

$$T_{\varepsilon} = \left| \frac{\varepsilon}{\frac{\partial \varepsilon}{\partial t}} \right|, \quad T_{\mu} = \left| \frac{\mu}{\frac{\partial \mu}{\partial t}} \right|, \quad T_{\eta} = \left| \frac{\eta}{\frac{\partial \eta}{\partial t}} \right|.$$

Furthermore, it is reasonable to consider that the attenuation of waves at a distance of the order of wavelength is slight (otherwise the very problem of propagation does not appear). Then it is possible to introduce the third low parameter of the problem

$$\frac{4\pi\eta}{\varepsilon\omega} \ll 1, \qquad (4)$$

characterizing the relative role of the bias currents and conduction currents (latter are small). Condition (4) is not in principle necessary, but it makes it possible to obtain the results in a more compact and visible form.

Sometimes inequalities (3) and (4) are conveniently written in the form of the ratio of wavelength λ to certain large graphic scale. So, if v - the velocity of propagation of electromagnetic wave, then (3) is equivalent to the inequality

$$\frac{\lambda}{L_T} \ll 1, \quad (3a)$$

where $L_T = vT$ while (4) it is possible to present thus:

$$\frac{\lambda}{L_\eta} \ll 1. \quad (4a)$$

$L_\eta = \frac{v\varepsilon}{4\pi\eta}$ determines the distance at which the wave amplitude decreases due to losses in the medium by a factor of e.

Let us search for the solution of system of equations (1) in the form

$$\begin{aligned} E(r, t) &= E_0(r, t)\, e^{i\varphi(r, t)}, \\ H(r, t) &= H_0(r, t)\, e^{i\varphi(r, t)}, \end{aligned} \quad (5)$$

where are clearly isolated the common for fields **E** and **H** rapidly changing phase factor and the slowly changing amplitude vectors $\mathbf{E}_0$ and $\mathbf{H}_0$. Let us assume that from the existence of the small parameters (2), (3) and (4) ensues the possibility of the separation of all entering the equations expressions into two categories:

1) the values of basic order; to them are related $\mathbf{E}_0$, $\mathbf{H}_0$, $\nabla\varphi$ and $\frac{\partial\varphi}{\partial t}$;

2) the small first-order quantity, at least one of our small parameters. This: $\text{rot}\,\mathbf{E}_0$, $\text{rot}\,\mathbf{H}_0$; $\frac{\partial \mathbf{E}_0}{\partial t}$; $\frac{\partial \mathbf{H}_0}{\partial t}$; $\eta \mathbf{E}_0$; $\frac{\partial \varepsilon}{\partial t}\mathbf{E}_0$; $\frac{\partial \mu}{\partial t}\mathbf{H}_0$.

At the basis of this classification lie evaluations of the type

$$|\mathrm{rot}\,\mathbf{E}_0| = O\left(\frac{|\mathbf{E}_0|}{L}\right) \ll |\nabla\varphi|\cdot|\mathbf{E}_0|$$

and obvious determinations for local frequency $\omega = \frac{\partial\varphi}{\partial t}$ and local wavelength $\lambda = \frac{2\pi}{|\nabla\varphi|}$. Let us emphasize that we still must show that the taken separation of values into orders actually follows from the restrictions placed on the properties of the medium.

§2. Phase relationships

Let us apply the method of successive approximations to the system of equations of Maxwell.

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Consideration of the terms of basic order leads to the relationships

$$\nabla\varphi \times \mathbf{E}_0 = -\frac{\mu}{c}\left(\frac{\partial\varphi}{\partial t}\right)\mathbf{H}_0,$$
$$\nabla\varphi \times \mathbf{H}_0 = \frac{\varepsilon}{c}\left(\frac{\partial\varphi}{\partial t}\right)\mathbf{E}_0, \qquad (6)$$

whence for phase function ϕ follows the generalized equation of the eikonal

$$(\nabla\varphi)^2 - \frac{\varepsilon\mu}{c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2 = 0, \qquad (7)$$

physical sense of which is very simple: it describes the propagation of constant-phase surface ϕ = const at the local speed $v=c/\sqrt{\varepsilon\mu}$ in directions $\pm\nabla\varphi$. It is not difficult to interpret relationships (6): vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ are oriented orthogonally to each other and to the direction of propagation of wave [1].

FOOTNOTE [1]. Here and subsequently we for definiteness will analyze the waves which go in direction $+\nabla\varphi$.ENDFOOTNOTE.

Between their values there is connection through the local impedance of the medium. Introducing unit vector in direction of beam $\mathbf{l}=\frac{\nabla\varphi}{|\nabla\varphi|}$, from equations (6) and (7) we have

$$\mathbf{E}_0=\sqrt{\frac{\mu}{\varepsilon}}\,[\mathbf{l}\times\mathbf{H}_0]. \qquad (8)$$

In other words, with an accuracy to the small first-order quantities the local structure of the field of electromagnetic wave coincides with the same for the case of plane wave in the medium with constants ε and μ.

We already mentioned that typical for geometric-optical calculations is this posing of the question, when field is considered assigned at certain phase wave front S_0. For the slightly inhomogeneous and weakly-parametric media the field of concentrated source at distances of several wavelengths from it will still little differ from the field which it would create in the homogeneous medium with the time-constant parameters. Therefore with sufficient for practical calculations accuracy, the field in the vicinity of emitter can be considered known. In this case, it is obvious that in the near zone of the source where play role static and induction fields, condition (8) does not obtain; this corresponds to the known fact of the inapplicability of the geometric-optical approximation for the calculation of the local fields of emitter.

Thus, let the constant-phase surface lie in the wave zone of source, and at certain moment of time t_0 the value of the phase in S_0 is $\phi_0 = \phi(t_0)$. From the generalized equation of the eikonal (7) it follows that this phase front propagates at a rate of v in the direction of the normal to itself (the latter, by definition, is the direction of beam).

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Having in principle radiation picture, we can trace in detail the motion of this phase of wave $\phi = \phi_0$ in time. In this case, the inconstancy in time of speed $v(r, t)$ forces us during calculation of the phase picture, which corresponds to Fig. 1, to take at each point of space the value of velocity at that moment of time when the phase front in question passes through this point. Let us agree this delaying value of the velocity subsequently to designate [v]. It is obvious that the spatial distribution of [v] in parametric media is a function of the starting time t_0, and, generally speaking, the radiation picture of the transfer of another phase of the wave ϕ'_0, "starting" from the initial surface S_0 at another moment of time t'_0, will be different. Under such unsteady conditions it is convenient to construct distributions of the beam, recording not current time t, but the starting time t_0 in the manner shown in Fig. 1.

It is easy to see that propagation time of this phase of wave from point M_0 in initial surface to certain point M, which lies on the beam, which left M_0 at the moment of time t_0, is equal

$$t-t_0=\int_{M_0}^{M}\frac{ds}{[v]}, \tag{9}$$

where the integral is taken on the beam, and the path, passed by phase front, i.e., the "length" of beam,

$$s_{MM_0}=\int_{t_0}^{t}[v]\,dt, \tag{10}$$

where the function [v] also must be taken at the appropriate points of beam.

During the calculation of beam trajectories, can prove to be useful the differential equation of beam. For its derivation let us examine at moment of time t so small a section of phase front $\phi = \phi_0$ that it would be possible to consider it plane. After differentially short time dt, phase front will be shifted in space in direction in distance ds = vdt (Fig. 2).

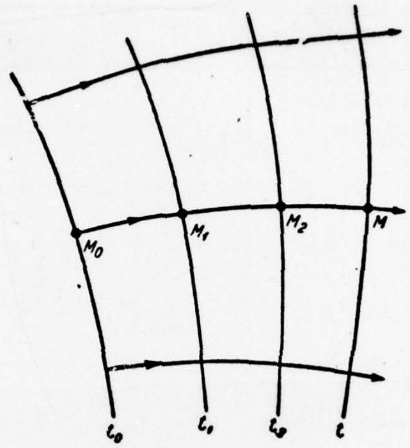

Fig. 1.

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If speed v(t) is not identical for all points of front, then its position at moment of time t + dt will not be parallel to the initial one. Being limited to the small first-order quantities, we will obtain the rotation of phase front (and, this means, the unit vector of beam $\mathbf{l}$) through certain angle dα:

$$\mathbf{l}'=\mathbf{l}+d\mathbf{l},\quad |d\mathbf{l}|=d\alpha,$$

where $d\mathbf{l}\perp\mathbf{l}$ is caused by difference from zero of gradient of speed v in the direction orthogonal to $\mathbf{l}$. Simplest geometric calculation gives

$$d\mathbf{l}=-\frac{\nabla_{\perp}v}{v}\,ds,$$

where $\nabla_\perp v = \nabla v - \frac{dv}{ds}\mathbf{l}$ is a transverse to $\mathbf{l}$ part ∇v. Introducing refractive index n = c/v, we come to the following differential equation of beam:

$$\frac{d}{ds}(n\mathbf{l}) = \nabla n. \qquad (11)$$

It is interesting to note that precisely to the same equation are subordinated the beams in the medium with constant parameters [2], but in our case, as can be seen from the given derivation, it is correct only during the analysis of the family (or distribution) of beams united by the total starting time t_0.

In concluding this paragraph, recall that sometimes during the construction of the equations of geometric optics as the basis of analysis is used the postulate about the extreme character of beams [3] - the so-called Fermat principle. The equations of the eikonal and beam are obtained as a consequence of this principle as a result of the solution to variational problem.

In the case of parametric media, it is logical to require the minimality of integral (9), that determines the propagation time of the recorded phase of the wave between two points of space. Then it is not difficult to ascertain that the equations of Euler for this variational problem in accuracy coincide with the differential equation of beam. In other words, Fermat's principle retains its force.

The extremality of beams makes it possible to obtain a convenient formula for an instantaneous signal frequency from passage by the latter of path s from the starting surface on which we considered it monochromatic (with frequency ω_0), i.e., $\phi = \omega_0 t_0 + \delta$ on S_0 (δ - the certain phase shift). By definition of instantaneous frequency at the moment of time t

$$\omega = \left(\frac{\partial\varphi}{\partial t}\right)_s = \omega_0 \left(\frac{\partial t_0}{\partial t}\right)_s.$$

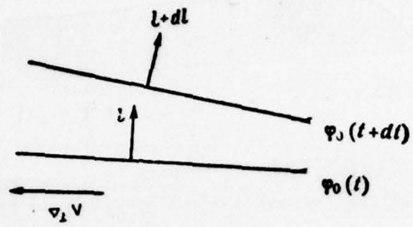

Fig. 2.

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The calculation of the last partial derivative (on the basis of formula (9)) substantially is facilitated on the strength of equality to zero of first variation in functional (9) with the attached upper limit and the "natural" [4] conditions at the lower end of the path of integration. Finally is obtained the following formula:

$$\omega = \omega_0 \exp\left\{\int_{M_0}^{M} \left[\frac{v'}{v^2}\right] ds\right\}, \qquad (12)$$

where V' - derivative of the wave propagation velocity in terms of time. The "departure" of frequency in the parametric medium is an integral effect, which is accumulated along the entire route of the propagation of beam. But with the limited dimensions of parametric region and smallness of relative changes in the speed (most probable in practical sense situation) frequency change is small.

§3. Amplitude of electromagnetic field.

For finding the amplitude vectors E_0 and H_0 we have the following system of equations, which ensues from (1), (5) and (6):

$$\begin{cases} \operatorname{rot} \mathbf{E}_0 + \frac{\mu}{c} \cdot \frac{\partial \mathbf{H}_0}{\partial t} = -\frac{4\pi\sigma_H}{c} \mathbf{H}_0, \\ \operatorname{rot} \mathbf{H}_0 - \frac{\varepsilon}{c} \cdot \frac{\partial \mathbf{E}_0}{\partial t} = \frac{4\pi\sigma_E}{c} \mathbf{E}_0, \end{cases} \quad (13)$$

where we have introduced the designation:

$$\sigma_E = \eta + \frac{1}{4\pi} \cdot \frac{\partial \varepsilon}{\partial t}; \quad \sigma_H = \frac{1}{4\pi} \cdot \frac{\partial \mu}{\partial t}. \quad (14)$$

The first of values (σ_E) let us call electrical, and the second (σ_H) - permeance of the parametric medium.

Utilizing standard transformations [4], it is possible to pass from (13) to the quadratic relationship which characterizes the energy balance in terms of amplitude vectors (let us emphasize the substantiality of the latter):

$$\operatorname{div} [\mathbf{E}_0 \times \mathbf{H}_0] + \frac{\mu}{c} \left(\mathbf{H}_0 \cdot \frac{\partial \mathbf{H}_0}{\partial t} \right) + \frac{\varepsilon}{c} \left(\mathbf{E}_0 \cdot \frac{\partial \mathbf{E}_0}{\partial t} \right) + \frac{4\pi}{c} \left(\sigma_H H_0^2 + \sigma_E E_0^2 \right) = 0. \quad (15)$$

From a mathematical point of view in the extracted formula enter the terms only of first (and higher) order of smallness from any of three low parameters of our problem.

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Let us compose the scalar function

$$u(r,\ t)=\frac{c}{4\pi}(\mathbf{E}_0,\ [\mathbf{l}\times\mathbf{H}_0]),$$

numerically equal to the energy current density in beam direction. Taking into account that with an accuracy to terms of first-order smallness the vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ satisfy relationships (8) that determine the structure of the field of plane electromagnetic wave, we can write (15) in the following form (for the wave, which goes in direction $+\mathbf{l}$):

$$\operatorname{div}(u\mathbf{l})+\frac{1}{v}\cdot\frac{\partial u}{\partial t}+\frac{4\pi}{c}\left(\sigma_H\sqrt{\frac{\mu}{\varepsilon}}+\sigma_E\sqrt{\frac{\varepsilon}{\mu}}\right)u=0. \quad (15a)$$

Equation (15a) is distinguished from precise formula (15) only by terms of the second order of smallness and reflects the law of conservation of energy, which takes place along beam tubes.

Actually, let us examine the dynamic picture of beams, which corresponds to the recorded starting time t_0, and let us isolate beam tube with as small a cross section as is convenient (Fig. 3). Let σ_1 and σ_2 be two of its differentially close sections, which actually exist in times t and t + dt respectively and distant from each other at a distance ds = vdt. Integrating (15a) at moment of time t in connection with of the volume of the beam tube, included between these sections, utilizing the theorem of Gauss and retaining only the first-order terms of smallness with respect to dt, we will obtain

$$\int_{\sigma_2} u d\sigma - \int_{\sigma_1} u d\sigma + \frac{1}{v} \int_{\sigma_1} \frac{\partial u}{\partial t} d\sigma ds + \alpha \int_{\sigma_1} u d\sigma ds = 0, \qquad (16)$$

where

$$\alpha \equiv \frac{4\pi}{c}\left(\sigma_E \sqrt{\frac{\varepsilon}{\mu}} + \sigma_H \sqrt{\frac{\mu}{\varepsilon}}\right).$$

If $w = \int_{\sigma} u d\sigma$ is the total flux of the energy through a certain section σ of beam tube, distant at a distance s with respect to the beam from starting surface, then (16) will take the form

$$w(t, s+ds) - w(t, s) + \left(\frac{\partial w}{\partial t}\right)_{s,t} \frac{ds}{v} + \alpha w(t, s) ds = 0.$$

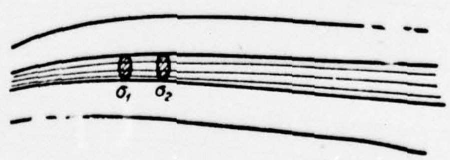

Fig. 3.

Fig. 3.

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After replacing w(t, s + ds) - w(t, s) by $\left(\frac{\partial w}{\partial s}\right)_{t,s} ds$, we will obtain

$$dw + \alpha w ds = 0, \qquad (16a)$$

where dw is the total differential of energy flow between sections σ_1 and σ_2, taking into account both space and time changes.

Equation (16a) is a differential first-order equation, describing the change in the energy flow along the beam tube. It is substantial, that it is derived only for the family of beams united by the total starting time t_0. The energy flow itself is given by the formula

$$w(t, s) = w_0(t_0, 0)e^{-\int_0^s [\alpha] ds'}, \qquad (17)$$

where $w(t_0, 0)$ - energy flow in the beginning of beam tube, and fading is determined by the delaying values of electrical conductivity and permeance (14). An increase in the permeability of the medium leads to the supplementary energy absorption of wave, and a decrease in values ε and μ is connected with the release of energy by the medium.

For tubes with differentially narrow cross section $d\sigma$, we will obtain $w = ud\sigma$ and energy current density in certain section s of the beam tube

$$u(t, s)=u_0(t_0, 0)\frac{d\sigma(t_0, 0)}{d\sigma(t, s)}e^{-\int_0^s [\alpha]ds'}. \quad (18)$$

In formula (18) is clearly expressed the amplitude reduction of field because of the divergence of beams. Since it is obvious that

$$u=\frac{c}{4\pi}\sqrt{\frac{\varepsilon}{\mu}}E_0^2=\frac{c}{4\pi}\sqrt{\frac{\mu}{\varepsilon}}H_0^2, \quad (19)$$

the values of the strength of fields E_0 and H_0 easily are located through (18).

For explaining the polarization of wave, let us introduce unit vectors $\mathbf{e}$ and $\mathbf{h}$ determining at each point of space the sense of the vector $\mathbf{E}_0$ and $\mathbf{H}_0$. Then from relationship (6) ensues the connection of these unit vectors with the unit vector of beam $\mathbf{l}$:

$$\mathbf{l}\times\mathbf{e}=-\mathbf{h};\ \mathbf{l}\times\mathbf{h}=\mathbf{e}. \quad (20)$$

Differentiating the extracted relationship with respect to s, we will obtain

$$\begin{aligned}\frac{d\mathbf{l}}{ds}\times\mathbf{e}+\mathbf{l}\times\frac{d\mathbf{e}}{ds}&=-\frac{d\mathbf{h}}{ds},\\ \frac{d\mathbf{l}}{ds}\times\mathbf{h}+\mathbf{l}\times\frac{d\mathbf{h}}{ds}&=\frac{d\mathbf{e}}{ds}.\end{aligned} \quad (21)$$

Assuming the polarization of the wave to be linear (which will not be fundamental limitation) and by taking into account the easily proven fact of the retention of polarization in the isotropic homogeneous medium, we will search for such solutions (21) which satisfy obvious condition $\frac{d\mathbf{h}}{ds}=\frac{d\mathbf{e}}{ds}=0$, if $\frac{d\mathbf{l}}{ds}=0$.

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From a physical point of view, this means that the rotation of vectors **E** and **H** along beam is caused only by bending of the latter, but formally forces from all possible solutions of the system

$$\mathbf{l}\times\frac{d\mathbf{e}}{ds}=-\frac{d\mathbf{h}}{ds},\ \mathbf{l}\times\frac{d\mathbf{h}}{ds}=\frac{d\mathbf{e}}{ds}$$

to select only the zero one. Therefore instead of the complete equations in formula (21) we can examine the "shortened" relationships

$$\frac{d\mathbf{l}}{ds}\times\mathbf{e}=-\frac{d\mathbf{h}}{ds},\ \frac{d\mathbf{l}}{ds}\times\mathbf{h}=\frac{d\mathbf{e}}{ds}.$$

Utilizing (20), we pass to two differential equations:

$$\frac{d\mathbf{h}}{ds}=-\mathbf{l}\left(\frac{d\mathbf{l}}{ds}\cdot\mathbf{h}\right),\ \frac{d\mathbf{e}}{ds}=-\mathbf{l}\left(\frac{d\mathbf{l}}{ds}\cdot\mathbf{e}\right), \qquad (22)$$

either of which can be utilized for finding the polarization of wave with known radiation picture.

Specifically, if beam lies in certain plane L, then from (22) it follows that the electromagnetic vector, orthogonal to this plane, retains its direction. In this case there is no need to integrate equations (22), and it suffices to separate the studied field into two waves, in one of which the vector $\mathbf{E}$ is orthogonal to L, and in the other - vector $\mathbf{H}$.

Equations (22) can be written somewhat otherwise; taking into account (11) they take the form

$$\frac{d\mathbf{h}}{ds}=-\frac{1}{n}(\nabla n\cdot\mathbf{h}),\quad \frac{d\mathbf{e}}{ds}=-\frac{1}{n}(\nabla n\cdot\mathbf{e}). \qquad (22a)$$

§4. Conditions of the applicability of geometric-optical observation.

Thus, we obtained the following picture of the propagation of waves in a slightly inhomogeneous and weakly-parametric medium:

1) phase distribution of wave in space is given by the generalized equation of the eikonal (7); the geometry of beams is changed in the course of time; it is physically reasonable to examine the families of beams which correspond to one starting time;

2) the energy, which flows along beam tubes, changes its value only due to losses in the medium (the latter sometimes can be negative), i.e., the reverse reflected waves do not appear.

The only inaccuracy in our reasonings was the replacement of formula (15) by expression (15a). From a physical point of view, this error is wholly caused by the difference between the structure of our electromagnetic field and the structure of the plane of wave in the homogeneous medium.

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For plane electromagnetic waves, as is well known, the laws of geometric optics are precise.

From formal point of view, we disregarded terms of the second order of smallness. However, we have not yet shown that the satisfaction of inequalities (2), (3) and (4) provides the accepted by us separation of values by order of smallness (see §1). Now this is not difficult to do, since we have formulas (15) and (19), that make it possible to produce the necessary estimations [1].

FOOTNOTE [1]. The detailed conducting of all estimations would require much space; therefore we will be restricted to the fundamental observations, the bases for which are easy to perceive in formulas

(18) and (19). ENDFOOTNOTE.

For example, it is necessary to show that

$$\left|\frac{\partial E_0}{\partial x_i}\right| \ll \frac{|E_0|}{\lambda}, \quad \left|\frac{\partial E_0}{\partial t}\right| \ll \omega |E_0|. \qquad (23)$$

The conditions of weak heterogeneity, weak parametricity, and smallness of losses in the medium assure satisfaction of these inequalities, but with one supplementary limitation, namely:

$$|\nabla (d\sigma)| \ll \frac{d\sigma}{\lambda}, \quad \frac{\partial}{\partial t}(d\sigma) \ll \omega d\sigma, \qquad (24)$$

i.e. we must examine only such fields for which a relative change in the value of cross section $d\sigma$ of beam tube is small at distances of the order of wavelength and at times on the order of the period of the fluctuations of field. A similar requirement will be carried out, if the radii of curvature of phase fronts are great in comparison with wavelength. In other words, relationships (24) forbid the use of geometric-optical approximations near concentrated sources (necessary compulsorily to leave into their wave zone), and also in the vicinity of foci and caustic surfaces of wave field (where $d\sigma \longrightarrow 0$, i.e., the beam tubes boundlessly become narrow). Let us note at the same time that geometric optics is inapplicable in those regions of space (generally speaking, moving with time), where $\lambda \longrightarrow \infty$, i.e., the permeability of the medium (ε or μ) vanishes.

The second remarkable fact is the existence of the integral inequalities, which ensure evaluations of the type of (23). For example, the condition of the slowness of the rotation of the plane of polarization, which is reduced on the strength of §3 to the requirement for the slow rotation of beam, taking into account (11) takes the form

$$\left|\frac{\partial l}{\partial t}\cdot\frac{1}{\omega}\right| = \left|\frac{\partial}{\partial t}\int_0^s \left[\frac{\tau n}{n}\right] ds'\right| \ll 1. \qquad (25)$$

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Analogous character bears the inequality, which limits the gradients of amplitude between adjacent beam tubes and requiring that the difference in fading between them would be little:

$$\int_0^s \left[\frac{\partial \alpha}{\partial x_i}\right] ds' \ll \frac{1}{\lambda}. \qquad (26)$$

If (26) does not obtain, then it is not possible to disregard the effect of transverse diffusion [5] between beam tubes.

It would seem that inequalities (25) and (26), ensuring the possibility of neglect of some terms in the differential equations of problem, make the conditions of the applicability of geometric optics more rigid as compared with the case of the nonconducting and nonparametric medium. However, this is not so. It is well known that the error in solution, caused by an inaccuracy in the equations, can be accumulated during considerable changes in the independent variables. Actually, if we instead of approximation (15a) write the precise relationship

$$\operatorname{div}(u\mathbf{l}) + \frac{1}{v}\cdot\frac{\partial u}{\partial t} + \alpha u = F(s, t), \qquad (27)$$

where the standing in right side function F(s, t) - the value of the 2nd order of smallness, is equal to

$$F(s, t) = \operatorname{div}\{u\mathbf{l} - \mathbf{E}_0 \times \mathbf{H}_0\} + \\ + \frac{1}{v}\left\{\frac{\partial u}{\partial t} - \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}}\frac{\partial H_0^2}{\partial t} - \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\cdot\frac{\partial E_0^2}{\partial t}\right\} + \\ + \frac{4\pi}{c}\left\{\sigma_H\sqrt{\frac{\mu}{\varepsilon}}(u - H_0^2) + \sigma_E\sqrt{\frac{\varepsilon}{\mu}}(u - E_0^2)\right\},$$

to evaluate corrections to the approximation of geometric optics on the basis of formula (27) it is possible to write the integral relationship

$$u_1(s, t)\,d\sigma(s, t) = \int_0^s [F(s')\,d\sigma(s')]\exp\left\{-\int_{s'}^{s}[\alpha]\,ds''\right\}ds' + \\ + \int_s^{\infty}[F(s')\,d\sigma(s')]\exp\left\{-\int_{s}^{s'}[\alpha]\,ds''\right\}ds', \qquad (28)$$

where u is the difference between the solutions to precise (27) and geometric-optical (15a) equations.

To ensure the relative smallness of this correction (in the general case) is possible only under the condition of the examination of the localized heterogeneities whose linear dimensions D are commensurable (but not greater!) in order of magnitude with the scales, which characterize the degree of heterogeneity or parametricity of the medium.

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It is obvious that $F(s, t) = O(u\ \lambda/L^2)$, and the integral in formula (28) can be estimated as

$$u_1(s, t) = O\left(\frac{d\sigma(0, t_0)}{d\sigma(s, t)} u(s, t) \frac{\lambda D}{L^2}\right). \quad (29)$$

where D it is determined by the length of that part of the beam on which is essential the account of correction $F(s, t)$, i.e., D is the dimension of heterogeneous region. The smallness of ratio $\lambda D/L^2$ provides simultaneously the execution of integral inequalities (25) and (26), since in them integrand also has the 2nd order of smallness.

Everything said in this section makes it possible to assert that the fundamentally new limitations for applying the method of geometric optics to the analysis of the propagation of electromagnetic waves in the parametric medium does not appear.

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