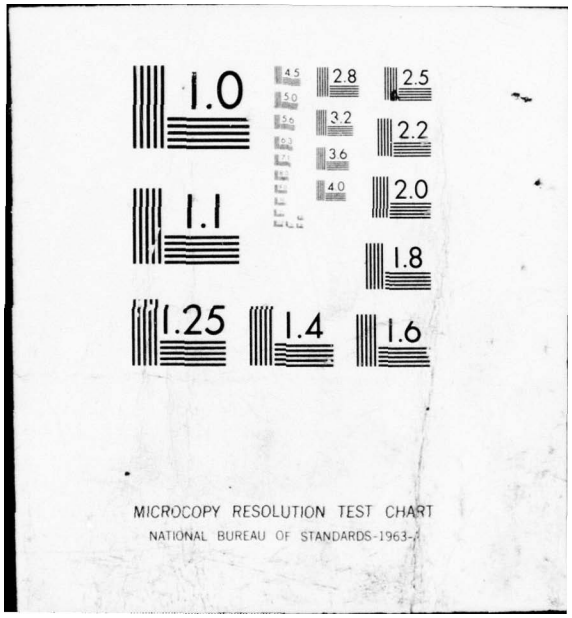
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Studies in Support of the Application of Statistical Theory to Design and Evaluation of Operational Tests. Annex C.

A COST OPTIMAL APPROACH TO SELECTION OF EXPERIMENTAL DESIGNS FOR OPERATIONAL TESTING UNDER CONDITIONS OF CONSTRAINED SAMPLE SIZE

Final report

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A THESIS

Presented to

The Faculty of the Division

of Graduate Studies

By

Sam Wallace Russ, Jr.

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in the School of

Industrial and Systems Engineering

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This report is a summary report of four studies in support of the application of statistical theory to design and evaluation of operational tests. The four topics are:

a. "A Methodology for Determining the Power of MANOVA when the Observations are Serially Correlated" by Norviel R. Eyrich, CPT, Artillery.

b. "An Application of Multiple Response Surface Optimization to the Analysis of Training Effects in Operational Test and Evaluation" by Vernon M. Bettencourt, Jr., CPT, Artillery.

c. "A Cost Optimal Approach to Selection of Experimental Designs for Operational Testing under Conditions of Constrained Sample Size" by Sam W. Russ, MAJ, Signal Corps.

d. "An Application of Bayesian Statistical Methods in the Determination of Sample Size for Operational Testing in the US Army" by Robert M. Baker, CPT, Infantry.

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A COST OPTIMAL APPROACH TO SELECTION OF EXPERIMENTAL DESIGNS FOR OPERATIONAL TESTING UNDER CONDITIONS OF CONSTRAINED SAMPLE SIZE

Approved:

Douglas C. Montgomery

Douglas C. Montgomery, Chairman

for Harrison M. Wadsworth, Jr.

Ronald L. Rardin

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A number of persons have assisted me in the conduct of this research. To all of them I express my appreciation. In particular, I thank Dr. Douglas C. Montgomery, the chairman of my thesis committee, for suggesting this problem and for his guidance and encouragement, and I thank Dr. Harrison M. Wadsworth, Jr. for his continual critical evaluations of the progress of this research. I especially thank Dr. Ronald L. Rardin for his dedicated interest and invaluable assistance in the development of the solution algorithm.

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SUMMARY

The problem of selecting the specific design structure for an operational test under conditions of constrained sample size is investigated. The research is limited to univariate, quantitative, continuous, linear response models.

Expected additional system cost (EASC) is defined and modeled as the sum of four general cost elements: fixed cost of testing; sampling cost, which is assumed to be linear; expected cost due to Type I error; and expected cost due to Type II error. The general form of each variable element is investigated as it is affected by the type of response model and by the general design structure.

The problem of selecting the significance level, the number of observations for each treatment and their allocation among the cells within treatments with limited sample sizes is formulated as a constrained, nonlinear optimization problem with EASC as the objective function. The specific formulation for the 2^n completely crossed, fixed factor design with a single covariate is developed and used to illustrate the derivation of a solution procedure based on a method of partial enumeration. However, the procedure is applicable to any factorial or ANOCOV design so long as the response variable meets the prescribed limitations. The solution algorithm is programmed in FORTRAN IV. A listing

of this program is included.

The approach is demonstrated for a hypothetical operational test based on a 2^3 completely crossed, fixed factor design with one covariate.

The proposed approach is found to be a valid method for selecting a specific design structure for operational testing within the prescribed limitations, and to be a useful decision making tool for evaluating design structure decision alternatives.

CHAPTER I

INTRODUCTION

Background

The Department of the Army acquires new major systems through a highly structured and formalized process which is prescribed in detail by a series of written directives (1, 2, 3). The development portion of this process is divided into three sequential phases: the conceptual development phase, the validation phase, and the full scale development phase. A formal review is conducted at the end of each of these phases and the Secretary of Defense must give his approval for initiation of the next phase. Alternatively, the Secretary of Defense may decide to terminate development, or extend the current phase for corrective action (9). In making these decisions, he is advised by the Defense Systems Acquisition Review Council (DSARC) which is a permanent body tasked with reviewing the progress of all major system developments at the end of each phase.

A parallel advisory body, the Army Systems Acquisition Review Council (ASARC), exists at Department of the Army level. This body reviews all major Army systems at the end of each development phase, and provides the Army's recommendations to the DSARC for incorporation into their recommendations to the Secretary of Defense.

Both the ASARC and the DSARC rely heavily on the results of tests as the basis for their recommendations. These tests, which are conducted during each of the three development phases, are of two distinct types with different but complimentary objectives. Developmental Testing (DT) is conducted primarily to determine if the system meets its technical specifications, and to identify any need for further design and development. Operational Testing (OT), on the otherhand, is primarily concerned with evaluating the operational worth of the system in terms of improved performance capabilities over current standard or competitor developmental systems. OT is conducted using typical user/operators under conditions which duplicate, as nearly as possible, those under which the system is expected to perform if adopted into the Army inventory.

Normally, each development phase is concluded with both a DT and an OT, which precede the meeting of the ASARC and DSARC. Although DT and OT may be conducted concurrently, they are required to be evaluated completely independently. Entirely separate organizations are responsible for each type of test. This research restricts itself to OT.

Operational Testing

The US Army Operational Test and Evaluation Agency is tasked with the responsibility for planning, conducting, and evaluating all OT of major Army systems. Their objectives are to evaluate, under as realistic conditions as possible,

the tested system's:

1. Military utility, operational effectiveness, and operational suitability (including reliability, availability, maintainability, compatability, interoperability, and logistic and human engineering requirements.)

2. Desirability, from the user's viewpoint, when compared to currently available and competing developmental systems.

3. The need for modifications.

4. The adequacy of organization, doctrine, operating techniques, and tactics for its employment, and the system for its maintenance support.

5. Performance in a countermeasures environment (1).

Operational testing is almost exclusively comparative in nature. Depending on the type of system, one of three standards for comparison (SFC) may be used: the current standard system, which the new system is intended to replace; a higher level system (of which the new system is a component) without the new system; or a set of performance standards which the new system must meet or exceed (26).

The Operational Test and Evaluation Agency (OTEA) must design each OT to evaluate the issue which have been prescribed by the Department of the Army. These issues are formal statements of the questions which must be answered in order to determine the operational worth of the new system. The key questions are specifically identified as critical

issues. The list of issues and their relative importance may change from one phase of OT to another, based on the results of the previous phases.

For each issue, OTEA develops an operational test criterion which specifies the issue to be evaluated, the SFC, and the measures of effectiveness (MOE) to be used to evaluate the issue (26). An MOE may be either quantitative or qualitative, depending on the nature of the issue it supports. A quantitative MOE may be either discrete or continuous.

The OT is then designed to provide the data necessary to compare the new system with the specified SFC across all of the required MOE, with particular emphasis on those MOE which support critical issues. In addition, point or interval estimates of the values for each quantitative MOE are required to determine the margin by which the new system exceeds or falls short of the SFC. The main effects of independent variables, other than the treatment (system) variables are of little interest in OT. Estimates of interactions between independent variables, other than those involving the treatment variable, are not required. Nor are OT intended to estimate functional relationships between dependent and independent variables. This sort of information is a function of other types of tests.

Problem, Objective, Scope

This research was motivated by a problem stated by OTEA:

> OTEA is continuously required to design and analyze the results of operational tests based upon small sizes whether the sample concerns numbers of prototypes, personnel, or trials. The effect (of a research project) would be directed at developing a methodology for designing, planning, and evaluating operational tests of limited sample size.

During OT, the new system is in a prototype configuration. Generally, the cost of an individual prototype of a major system dictates that only a small number be constructed. Due to other requirements, only a portion of these prototypes may be available for OT, and these may be available only for a limited amount of time. In addition, if the new system is of a type which is destroyed in use (e.g. a missile) this imposes even greater restrictions on the number of trials which may be conducted. Faced with these restrictions, OTEA must design and conduct OT which provides sufficient data to evaluate all of the required issues with an acceptable precision.

This research addresses the first part of the problem, the design of the OT. The objective of this research is to develop a methodology for selecting the design of an OT based on a criterion of minimum expected additional system cost. Recognizing that by selecting the design, the method of analysis is also determined, no designs are considered for which a well defined method of analysis does not exist.

The limited statistical information required from OT permits this research to be limited in two general areas:

1. Only linear response (MOE) models are considered.

2. For the factorial design, all factors are restricted to two levels.

These limitations are based on the assumption that OT are not required to determine functional relationships. The scope of this research is further limited in four additional areas:

1. Only two general classes of designs are considered: the completely crossed, fixed factorial design, to include fractional replications; and the analysis of covariance (ANOCOV) design. These two classes of designs are felt to be sufficient to handle a large variety of OT situations and to adaquately demonstrate how the proposed methodology may be applied to other classes of designs.

2. Only univariate response models are considered. The extension of the proposed methodology to the multivariate case is left as a topic for future research.

3. Only continuous, quantitative response variables are considered. While this limits the generality of the method with respect to OT, consideration of this requirement during selection of MOE will minimize this limitation. Future research may be able to extend this method to include both discrete and qualitative MOE.

4. The OT objective of testing the hypothesis of no

treatment main effect is used as the basis for optimization, since this is felt to be the most important information required. However, modification of the problem formulation to match another hypothesis as the basis for optimization will present no difficulties to an experimenter who is familar with the analysis of variance (ANOVA) and ANOCOV methods and who understands the proposed approach.

This research first reviews various criteria in the literature for selecting an experimental design. Various methods for improving the precision of designs for a given sample size are also reviewed, which leads to consideration of the two major classes of designs: factorial and covariate. A new criteria is then developed based on minimizing the expected additional system cost resulting from the test. Expected additional system cost is modeled by a cost equation consisting of four elements: fixed cost of experimentation; cost of sampling; cost of Type I error; and cost of Type II error. The structure of each element of the model is described for the factorial and covariate designs.

The problem of limited sample sizes is then formulated as a constrained, nonlinear optimization problem. A solution algorithm is described and demonstrated.

CHAPTER II

REVIEW OF OTHER APPROACHES AND RELATED STATISTICAL TECHNIQUES

Introduction

This chapter presents a brief review of the approaches which have been developed by previous researchers to the problem of selecting experimental designs for the linear, univariate response model experiment. Methods of improving the precision of experiments for a given sample size are also briefly reviewed. The linear, univariate response models for ANOVA and ANOCOV are described, and methods for calculating the power of tests based on these models are reviewed.

Approaches to the Selection of Efficient Experimental Designs

Two general categories of methods for measuring the efficiency of experimental designs were investigated during this research. One category consists of methods which utilize a measure of the information derived from a design as an index of its efficiency. The other category consists of methods based on the power and significance level of a design with respect to a specific set of hypotheses to be tested. With few exceptions, actual applications of both methods are directed toward determining the required number

of observations and their allocation among the experimental conditions, treating the class of design, the number and type of independent variables and their range of values as having been previously determined. With only two exceptions, methods which actually sought to determine a specific optimal experimental design for a given situation were limited to those based on some form of information criterion.

Information Based Approaches

Using a measure of the information produced by an experimental design as a measure of its efficiency was first proposed by Fisher (14). Fisher's criterion was later revised by Cox (8). This criterion is defined by

$$I_t = \text{average variance } (M_j - M_k)/(2\sigma_o^2/n) \qquad (2.1)$$

$$I_a = I_t(f + 3)/(f + 1) \qquad (2.2)$$

where $(M_j - M_k)$ is the difference between two treatment means, f is the degrees of freedom for error, σ_o^2 is the variance of the dependent variable for fixed values of the independent variable, and n is the sample size. Feldt (11) utilized this criterion to compare the relative efficiency of the two-way factorial, the analysis of covariance, and paired comparison designs, and to determine the optimum numbers of levels for the factorial design under various conditions.

A majority of information based approaches utilize the information matrix, S, defined as follows: given the linear

model

$$E(\underset{\sim}{Y}) = \underset{\sim}{Z}\ \underset{\sim}{\beta}, \tag{2.3}$$

$$\underset{\sim}{S} = \underset{\sim}{Z}^{t}\ \underset{\sim}{Z} \tag{2.4}$$

where $\underset{\sim}{Y}$ is the response vector, $\underset{\sim}{\beta}$ is a vector of unknown constants, and $\underset{\sim}{Z}$ is the matrix of independent variables expressed in standardized form.

As an example, Wald (27) proposed a criterion, W, based on the determinant of S

$$W = \Pi_{i=1}^{r}\ \lambda_i \tag{2.5}$$

where the λ_i are the eigenvalues of S, and r is the rank of S. Ehrenfeld (10), Chernoff (7), Kshirsagar (21), and Webb (37) have proposed similar criteria which are functions of the eigenvalues of S.

More recently, Neuhardt, Bradley and Henning (28) have proposed the squared Euclidean norm of S as a measure of experimental efficiency.

From the standpoint of OT, all of the information based criteria suffer two major disadvantages. First, they are difficult to interpret in terms which are meaningful to the decision makers involved in the systems acquisition process. Second, and most important, since OT only requires estimates of the treatment main effects and interactions, the total information produced by an experiment is not a valid

measure of its worth.

Error Based Approaches

Sedransk (36) and Johnson (20) are the only examples of the use of error based criteria for optimizing experimental designs based on linear response models which were found in the literature that appeared to be applicable to OT. Sedransk studied the problem of allocating a given number of samples among the cells of 2^n factorial designs in order to minimize sampling cost for a specified precision. The problem is formulated as a nonlinear optimization problem, however, Sedransk states that standard nonlinear programming algorithms are not applicable since the objective function does not meet the convexity requirements. Sedransk suggests that the problem may be solved by a graphical approach, or by a systematic use of derivatives. Johnson, on the otherhand, provides a general formulation for the problem of determining the sample size, power, and significance level for a given design in order to minimize total cost. However, Johnson carries the problem no further than the general problem formulation.

Error based cost criteria are directly applicable to OT and present two advantages which are the converse of the disadvantages associated with information based criteria. First, they are based on cost versus risk which are measures that are directly applicable to decisions involved in the acquisition process. Second, they are related to specific

hypotheses rather than to all of the possible contrasts contained within a particular experiment. Consequently, they may be used to select experimental designs based on their criterion values with respect to only those specific contrasts (or estimates) which are of most interest in OT.

The general ideas and problem formulations presented by Sedransk and Johnson provided the initial choice of direction for this research.

Methods for Improving Precision

For a given sample size, the precision of an experiment is a function of the amount of variability among the observations which are combined together to produce an estimate of a population parameter. Variability among observations may be reduced, for a given sample size, in essentially three ways: by increased physical control of the independent variables which effect the observations; by structural changes in the experimental design which result in observations occuring within smaller, more homogeneous groups; and by taking concurrent observations of the values of concomitant variables which are correlated with the response variable, and adjusting parameter estimates for the effects these variables are assumed to have on the response variable.

The entire field of experimental design is either directly or indirectly concerned with methods for reducing

this variability among observations. Only two of several basic design structures will be reviewed and applied in this research. These are the factorial design and the ANOCOV design. Detailed information may be found in any of a number of design of experiments texts. Johnson (20), Lindman (24), Scheffe' (35), and Winer (38) were found to be particularly useful for their detailed analysis procedures for more complex designs including those involving unequal sample sizes.

Factorial Designs

The factorial design structure seeks to reduce variability by grouping to produce observations with as much homogeneity as possible within groups. To the extent that the variability within groups is less than the variability between groups this structural device results in an increase in precision. This condition will occur only if the independent variables which are used as a basis for grouping are correlated with the dependent variable to be observed to a degree which will off-set the reduction in degrees of freedom for error which results from grouping. In OT independent variables are chosen because of their expected correlation with the system performance, consequently this condition should, in general, be met.

Each independent variable to be considered during the experiment is identified as a factor. The range of values for each factor may be specified as fixed levels, or by

fixed intervals on a scale with specific values for a particular observation determined randomly. All possible combinations of levels (intervals) and factors may be included as experimental conditions, yielding a completely crossed factorial design (symbolized by AxBxC for three factors), or only specified levels (intervals) of one factor may be allowed to occur with specified levels of another factor yielding some form of nested design. In addition, by assuming some higher order interaction effects to be negligible, some form of fractionalized design may be developed.

If all factors have the same number of levels (intervals), n, and there are X number of factors, the design is called an X^n factorial design. The completely crossed, fixed level, 2^n factorial design structure was selected for this research as being, in combination with the ANOCOV design, the most generally applicable to OT.

The univariate, linear response model for a completely crossed, fixed, 2^2 design (AxB) is commonly represented in the literature as

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \qquad (2.6)$$

where

Y_{ijk} = the k^{th} observation at the i^{th} level of factor A and the j^{th} level of factor B,

μ = the population mean of the observations,

α_i = the effect of factor A at level i,

β_j = the effect of factor B at level j,

$\alpha\beta_{ij}$ = the interaction effect of factors A and B at levels i and j,

ϵ_{ijk} = the error associated with the Y_{ijk} observation,

and

$$\sum_i \sum_j n_{ij} \alpha_i = \sum_i \sum_j n_{ij} \beta_j = 0 \qquad (2.7)$$

where

n_{ij} = the number of observations in the ij cell.

The error is assumed to be normally distributed with mean zero and variance, σ^2.

If only one observation is taken from each cell, the k subscript is omitted. Under this condition, the error term is confounded with the highest order interaction term and cannot be estimated separately. In order to provide a separate estimate of error, a minimum of two observations per cell is required. This constraint will be applied throughout this research. Models for larger numbers of factors are simply extensions of (2.6).

ANOVA is the method of analysis associated with the univariate factorial model.

ANOCOV Designs

The ANOCOV design structure seeks to improve the precision of an experiment through the use of concomitant vari-

ables. Such a variable is defined by a characteristic of the response variable which more or less predetermines the general range of its values and is highly correlated with it. Values of the concomitant variable, or covariate, may be determined prior to the experiment, during the experiment at the same time that observations of the response variable are taken, or even after the experiment in some cases. However, if it is possible to determine the values before the experiment, strong consideration should be given to treating the concomitant variable as a factor rather than as a covariate in view of the additional assumptions which are required for ANOCOV. Due to the added assumptions, use of covariates should generally be restricted to sources of variability which cannot feasibly be controlled by another method. Such sources may often occur in OT, consequently, this design structure was included in this research.

The univariate, linear response model for ANOCOV with two factors and a single covariate is commonly represented in the literature as

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma(Z_{ijk} - \bar{Z}) + \alpha\beta_{ij} + \epsilon_{ijk} \qquad (2.8)$$

where

Z_{ijk} = the value of the covariate for the k^{th} observation with factor A at level i and factor B at level j,

$\bar{Z}$ = the average over all values of Z_{ijk},

γ = the linear effect of the covariate at the value Z_{ijk},

and all other terms are as defined for the factorial model. The side conditions (2.7) and the assumptions for the ANOVA model also apply to ANOCOV. Four additional assumptions must generally hold for ANOCOV:

1. Z is continuous.

2. The population mean and variance of Z and the population value of the correlation coefficient, ρ_{ZY}, between Z and Y are equal for all cells.

3. Homoscedasticity in Y obtains around the regression line of Y and Z.

4. Z is independent of all factors.

Violations of ANOVA and ANOCOV Assumptions

Much research has been directed toward studying the effects of violations of the assumptions required by ANOVA. It has been found that, in most cases, ANOVA is robust with respect to all assumptions other than random sampling and to a lesser degree homogeneity of variance. Very little appears to be known about the effects of violations of assumptions with respect to ANOCOV.

The literature contains well known tests for violations of assumptions which can be applied during the analysis of data for both ANOVA and ANOCOV. Methods exist which will, in many cases, reduce violations to an acceptable level

through the application of appropriate transformations to the data.

Power of the Variance Ratio (F) Test

The Variance Ratio or F test is used for evaluating contrasts based on ANOVA and ANOCOV. The power of the F test depends on the distribution of F when the null hypothesis is false. This distribution corresponds to Fisher's distribution C (14) and is termed the noncentral F distribution, or F′. F′ depends importantly on five factors: the actual means of the treatment populations; the error variance; the significance level; the number of observations in each sample; and the degrees of freedom for error.

Feldt and Mahmond (12) have developed charts for determining the sample size required to produce a given power at a specified significance level for assumed values of the parameter

$$\phi = \sqrt{\frac{\sum_i \alpha_i^2}{I\ \sigma^2}} \tag{2.9}$$

where I is the number of treatments and α_i and σ^2 are defined as in (2.6). These charts are only available for the most commonly selected values of significance level. Another set of charts developed by Pearson and Hartley (30) determines the power given the numerator and denominator degrees of freedom, the value of ϕ in (2.9) and the significance level. Although more values are charted than by Feldt-

Mahmond, they still do not provide a wide range of values of significance level.

As given by Patnaik (29), the probability distribution of F′ is

$$\rho(F') = \sum_{j=0}^{\infty} \left[\frac{e^{-\frac{1}{2}} \left(\frac{1}{2}\lambda\right)^{j}}{j!B\left(\frac{1}{2}\nu_1+j, \frac{1}{2}\nu_2\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{1}{2}\nu_1+j} F'^{\frac{1}{2}\nu_1-1+j} \left(\frac{1+\nu_1 F'}{\nu_2}\right)^{-\frac{1}{2}(\nu_1+\nu_2)-j} \right] \tag{2.10}$$

where

ν_1 = numerator degrees of freedom,

ν_2 = denominator degrees of freedom,

λ = the noncentrality parameter.

The power of test is then defined by

$$1 - \beta = \rho(F'_{\nu_1,\nu_2} > F_c)$$

where F_c is the critical value determined by the given α_1, ν_1 and ν_2. Methods of evaluating the probability integral of F′ have been worked out by Wishart (39). They involve extensive computational effort.

Fortunately, Patnaik and Johnson have each developed suitably accurate approximations to F′ which may be used to calculate power. Johnson's method was selected for use in

this research since it is based on a central F distribution, which is conceptually more understandable than Patnaik's which is based on an incomplete Beta function.

Johnson's approximation for F′ is

$$F'_{\nu_1,\nu_2} \approx (1+\lambda)F_{(1+\lambda)^2/(1+2\lambda),\nu_2} \tag{2.12}$$

where the noncentrality parameter, λ, is defined in general by

$$\lambda = \nu_1 \frac{EMS_1}{EMS_2} - \nu_1 \tag{2.13}$$

with

ν_1 = numerator degrees of freedom (as in F′),

EMS_1 = expected mean square of the hypothesized effect given that the alternate hypothesis is true,

EMS_2 = appropriate denominator expected mean square for the type effect to be tested.

For the fixed effects model, EMS_2 is simply the design error variance, σ_o^2.

The Noncentrality Parameter, λ

Method of Weighted Squares of Means. In order to calculate the value of the noncentrality parameter, λ, the value for EMS_1 must be determined. For the case of equal sample sizes, n, this value is given by Lindman (24) and others as

$$EMS_1 = \sigma_o^2 + n \sum_i \alpha_i^2/\nu_1 \qquad (2.14)$$

However, for unequal sample sizes the computational formula for EMS_1 becomes much more complex. Scheffe (35) and Winer (38) both present methods for calculating exact values of EMS_1 for unequal sample sizes. However, Winer describes a method which is computationally much less complex for approximating EMS_1 using the harmonic mean of the sample sizes for each level of the factor of interest. This technique is called the method of weighted squares of means.

λ for the Factorial Design. Under the method of weighted squares of means as described by Winer the estimators of the main effects for a two-way ANOVA (A x B) are

$$\hat{\alpha}_i = \bar{A}_i - \bar{G} \text{ and } \hat{B}_j = \bar{B}_j - \bar{G} \qquad (2.15)$$

where

$$\bar{A}_i = \sum_j \overline{AB}_{ij}/b;\ \bar{B}_j = \sum_i \overline{AB}_{ij}/a;\ \bar{G} = \sum_i \sum_j \overline{AB}_{ij}/ab \qquad (2.16)$$

a and b being the number of levels of factors A and B respectively.

The harmonic mean of the sample sizes for the i^{th} level of factor A is defined by Winer as

$$h_i = b/\sum_j (1/n_{ij}) \qquad (2.17)$$

and the sum of squares for factor A in terms of h_i as

$$SS_A = b\left[\sum_i h_i \hat{\alpha}_i - \left(\sum_i h_i \hat{\alpha}_i\right)^2 / \sum h_i\right] \qquad (2.18)$$

Taking the expected value of the SS_A and dividing by the appropriate degrees of freedom yields

$$EMS_A = \sigma_o^2 + \sum_i h_i \alpha_i^2 / a-1 \qquad (2.19)$$

The noncentrality parameter, λ, may now be calculated by substituting (2.19) into (2.13) resulting in

$$\lambda = \sum_i h_i \alpha_i^2 / \sigma_o^2 \qquad (2.20)$$

λ for the ANOCOV Design. The expected mean square of ANOVA must be adjusted for ANOCOV as a result of the addition of the covariate, Z. This results in an adjusted mean square for the effect of interest given by the formula

$$EMS_1^* = (1-\rho_{ZY}^2)\sigma_o^2 + \sum_i h_i \alpha_i^2 / a-1 \qquad (2.21)$$

where σ_o^2 is the design error variance prior to addition of the covariate and ρ_{ZY} is the correlation coefficient between Z and the response, Y. Substituting into (2.13) yields

$$\lambda^* = \sum_i h_i \alpha_i^2 / (1-\rho_{ZY}^2)\sigma_o^2 \qquad (2.22)$$

The Design Error Variance, σ_o^2

In order to provide a standard to assist in evaluating alternative design structures, this research has adopted a method used by Feldt (11). This method expresses the design error variance of the response variable as a function of the variance of the response in a completely random design. This technique provides some useful insights into one of the principal effects that additional factors and covariates will have on the precision of the design, through their effects on the error variance. In addition, it provides a means, through variation of parameters, of empirically evaluating the effects of variations in the degree of control maintained on the levels of independent variables.

Additional Factors. Using the standard assumptions for ANOVA and ANOCOV, Feldt derived the following formula for the design error variance, σ_o^2, of a two factor design $(X_1 \times X_2)$

$$\sigma_o^2 = \sigma_Y^2 \left[1-\rho_{X_2 Y} \left(1-\frac{\bar{\sigma}_{X_2}^2}{\sigma_{X_2}^2} \right) \right] \qquad (2.23)$$

where

σ_Y^2 = variance of the response variable, Y, for a completely random design,

X_1 = the treatment factor,

X_2 = the control factor (independent variable),

ρ_{X_2Y} = coefficient of correlation between the independent variable (control factor), X_2, and the response, Y,

$\bar{\sigma}^2_{x_2}$ = average variance of X about its fixed levels,

$\sigma^2_{X_2}$ = population variance of X_2.

For additional control factors, X_3, ..., X_n, this formula can be repeatedly applied to yield the design error variance, for n factors

$$\sigma_o^{2'} = \sigma_Y^2 \left[1-\rho^2_{X_2Y}\left(1-\frac{\bar{\sigma}^2_{x_2}}{\sigma^2_{X_2}}\right)\right] \cdots \left[1-\rho^2_{X_nY}\left(1-\frac{\bar{\sigma}^2_{x_n}}{\sigma^2_{X_n}}\right)\right] \quad (2.24)$$

Addition of Covariates. The change in the design error variance as a result of adding a covariate can be observed by comparing (2.20) and 2.22). Adding a covariate to the n-factor design in (2.24) yields a design error variance, σ_o^{2*} given by

$$\sigma_o^{2*} = \sigma_o^{2'}(1-\rho^2_{ZY}) \quad (2.25)$$

where ρ_{ZY} is the coefficient of correlation between the covariate, Z, and Y, the response variable.

CHAPTER III

DEVELOPMENT OF CRITERION AND APPROACH

Introduction

This approach defines expected additional system cost (EASC) as the criterion for selecting the experimental design structure of an OT. EASC is mathematically modeled as the sum of four cost elements. The structure of each element is described with respect to OT and methods for determining cost coefficients and estimating the primary model parameters are suggested.

The design selection problem with limited sample size is formulated as a constrained, nonlinear optimization problem with EASC as the objective function. A set of constraints is established based on the characteristics of OT. The form of the objective function is investigated and an algorithm for determining the optimal solution is described based on a method of partial enumeration.

The Cost Model

General Case

Expected additional system cost is modeled in general by the equation

$$EASC = C_o + \sum_{i=1}^{N} C_i + C_\alpha \alpha + C_\beta \beta \qquad (3.1)$$

where

C_o = fixed cost of testing,

N = total number of observations,

c_i = cost of sampling for the i^{th} observation,

C_α = penalty cost of a Type I error,

C_β = penalty cost of a Type II error,

α = probability of a Type I error,

β = probability of a Type II error.

Each set of hypotheses to be tested has an EASC. Consequently, one set must be selected as the basis for optimization. The set of hypotheses with respect to the treatment main effects was used in this research. This set of hypotheses may be expressed as

$$H_o : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 - \mu_2 > 0 \qquad (3.2)$$

where

μ_i = population mean of the response variable for the i^{th} treatment ($i = 1,2$)

A Type II error is defined as failure to reject the null hypothesis, H_o, when it is false, i.e. when the alternate hypothesis, H_1, is true. In order to determine a value for the probability of a Type II error, β, the alternate hypothesis of (3.2) must be stated in the exact form

$$H_1 : \mu_1 - \mu_2 = d \qquad (3.3)$$

where d represents the minimum true difference that will be detected as significant by the test. For OT, d represents the specified performance margin by which the new system must exceed the SFC.

Fixed Cost of Testing. Fixed cost of testing, C_o, includes all costs which are independent of the choice of design structure. This includes the costs of such items as administrative and support functions, facilities, and the minimum required test control and data collection and analysis functions to include personnel and equipment.

Cost of Sampling. This includes all incremental costs associated with the collection of an observation over the fixed costs. This includes the treatment costs, and any incremental costs required to obtain the specific experimental condition under which the observation is to be taken. This research assumes that the cost per sample will be constant within treatments, but will vary between treatments. This assumption is based on the belief that, since OT are generally conducted only on major systems, the only significant differences between sampling costs will be due to differences in cost between the prototype test item and the SFC.

Errors and Penalty Costs. Based on the assumed null hypothesis (3.2), the Type I and Type II errors which may occur in OT and their interpretation with respect to the actual relative performance of the tested system and the decisions which are indicated by the results of OT are shown in

Figure 1. In addition, Figure 1 suggests a definition for the penalty cost coefficients for each type of error. It seems reasonable to use as penalties only the additional costs incurred prior to the next phase of OT, since each phase will reevaluate the new system and provide new data upon which to base decisions with respect to its acceptance, rejection, or modification.

Cost Model for the Factorial and ANOCOV Designs

For the factorial class of designs equation (3.1) becomes

$$EASC = C_o + \sum_{\varepsilon_1=1}^{L_1} \cdots \sum_{\varepsilon_K=1}^{L_K} n_{\varepsilon_1 \cdots \varepsilon_K} C_{\varepsilon_1 \cdots \varepsilon_K} + C_\alpha \alpha + C_\beta \beta(\alpha, \lambda, \nu_t, \nu_e) \qquad (3.4)$$

where

K = number of factors,

L_i = number of levels of the i^{th} factor, X_i,

ε_i = ε^{th} level of the i^{th} factor,

$n_{\varepsilon_1 \cdots \varepsilon_k}$ = number of observations in the $\varepsilon_1 \cdots \varepsilon_k^{th}$ cell,

$C_{\varepsilon_1 \cdots \varepsilon_k}$ = cost of an observation in the $\varepsilon_1 \cdots \varepsilon_k^{th}$ cell,

α = significance level,

λ = noncentrality parameter defined by (2.13),

ν_t = degrees of freedom between treatments,

ν_e = degrees of freedom for error.

The form of λ and ν_e will be determined by the spe-

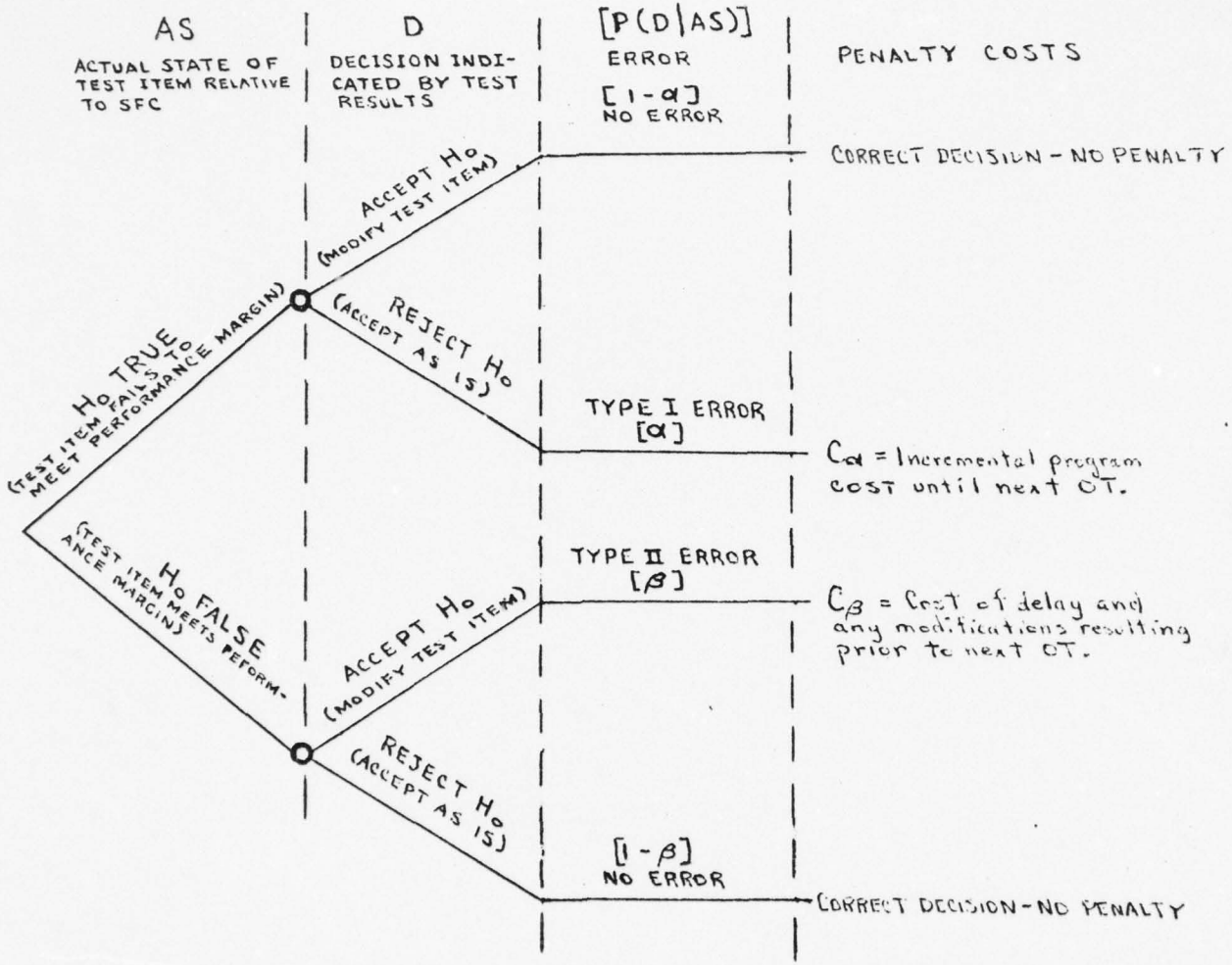


Figure 1. Errors and Penalty Costs in Operational Testing.

cific type of factors involved and the pattern in which they are to be combined. In the case of fractionalized factorial designs, the defining contrast selected will determine the cells from which observations are to be taken. Also, any higher order interactions which are validly assumed to be insignificant may be pooled with the error term to increase the error degrees of freedom.

Addition of covariates does not effect the form of the cost model. However, estimates of the sampling costs may need revision to include the additional cost per observation of the covariate. In addition, several changes in the primary parameters of the model are required. The error degrees of freedom must be reduced by the number of covariates to account for the calculation of the average value of the covariate. Also, the error variance and expected mean square of the treatment effect must be adjusted for covariance as indicated in (2.22) and (2.25). The additional assumptions required for ANOCOV must always be considered, since their violation could seriously effect the validity of the experimental results.

Estimation of Values for Primary Model Parameters

In addition to the estimates of cost coefficients, the formulation of the cost model for a specific OT requires estimates of the following primary parameters:

1. The error variance for the response variable in a completely random design.

2. The coefficients of correlation between the response variable and each factor, other than the treatment; and between the response variable and each covariate.

3. The ratio of the average variation of each factor about its fixed level to its population variance.

These estimates are generally a problem only in the first phase of OT (OT I). In subsequent phases estimates will be available from data collected in previous phases. For OT I there are several sources of data which may be available to provide a basis for estimation of the required parameters. If possible, a series of pre-tests could be conducted specifically designed to estimate these parameters. Alternatively, data from previous developmental or engineering design tests may be used, or data from previous tests of similar type systems. Population variances of factors can be estimated from analyses of both combat and field training exercise after-action reports, from studies conducted by the Training and Doctrine Command, from intelligence studies, and from simulations.

The Optimization Problem

A 2^K completely crossed ($X_1 \times X_2 \times \ldots \times X_K$) design with all factors at two fixed levels and a single covariate, Z, will be used to illustrate the optimization procedure. X_1 will always represent the treatment factor. This procedure may be applied to any factorial or ANOCOV design.

After the cost model has been determined, to include estimates of all cost coefficients and primary parameters, the problem of selecting the specific design structure for an OT with limited sample sizes may be formulated as a constrained nonlinear optimization problem with the cost model as the objective function.

Assuming that (3.2) and (3.3) are the hypotheses selected as the basis for optimization, the problem may be stated as: find $N_i = \{n_{ij}\}(i = 1,2;\ j = 1,\ldots,2^{K-1})$ and α that

Minimize:

$$EASC = C_o + \sum_{i=1}^{2} \sum_{j=1}^{2^{K-1}} c_i n_{ij} + C_\alpha \alpha + C_\beta \beta(\alpha,\lambda,\nu_t,\nu_e) \quad (3.6)$$

Subject to:

$$\sum_{j=1}^{2^{K-1}} n_{ij} \leq S_i \quad (i = 1,2) \quad (3.7)$$

$$n_{ij} \geq 2 \quad \text{(for all i and j)} \quad (3.8)$$

$$\alpha < 1 \quad (3.9)$$

$$\alpha > 0 \quad (3.10)$$

All n_{ij} integer

where

$$\nu_t = 1 \tag{3.11}$$

$$\nu_e = \sum_{i=1}^{2} \sum_{j=1}^{2^{K-1}} n_{ij} - (2^K+1) \tag{3.12}$$

$$\sigma_o^2 = \sigma_Y^2\left[1-\rho_{X_2Y}^2\left(1-\frac{\bar{\sigma}_{X_2}^2}{\sigma_{X_2}^2}\right)\right]\cdots\left[1-\rho_{X_KY}^2\left(1-\frac{\bar{\sigma}_{X_K}^2}{\sigma_{X_K}^2}\right)\right](1-\rho_{ZY}^2) \tag{3.13}$$

$$\lambda = \frac{d^2}{\sigma_o^2}\left(\frac{h_1 h_2}{h_1+h_2}\right) \tag{3.14}$$

$$h_i = \frac{2^{K-1}}{\sum_{j=1}^{2^{K-1}} 1/n_{ij}} \tag{3.15}$$

$$\beta = P\left[F_{(1+\lambda)^2/(1+2\lambda),\nu_e} < \frac{1}{1+\lambda} F_c\right] \tag{3.16}$$

S_1 and S_2 represent the separate constraints imposed on the maximum number of observations which may be taken with the new system and the SFC, respectively. Usually, $S_1 < S_2$. The second constraint (3.8) is imposed to permit a separate estimate of error. (3.11) results from the treatment factor being at two levels. (3.12) is just the formula for the error variance with unequal sample sizes and adjusted for the single covariate. (3.13) results directly from (2.24) and (2.25). F_c is the critical value determined by the given

α, ν_t and ν_e.

Equation (3.14) for the noncentrality parameter, λ, was derived by solution of the two simultaneous equations

$$\sum_{i=1}^{2} h_i \alpha_i = 0, \tag{3.17}$$

which is a necessary side condition for the fixed factor model, and

$$\alpha_1 - \alpha_2 = d, \tag{3.18}$$

which comes directly from the alternate hypothesis (3.3) since $\mu_i = \mu + \alpha_i$, for the fixed factor model. Solving (3.17) and (3.18) for the α_i's yields

$$\alpha_1 = \frac{h_2}{h_1+h_2} d \quad , \quad \alpha_2 = \frac{-h_1}{h_1+h_2} d \tag{3.19}$$

Substituting (3.19) into the formula for λ for the three-way design results in (3.14). Note that the value of d must be expressed in the same units as the response variable.

Method of Solution

The algorithm proposed by this research for solving the optimization problem as formulated in (3.6) through (3.10) was motivated by the partial enumeration method described by Neuhardt and Bradley (27). However, the two methods differ both in the form of the objective function and in

procedure.

This method utilizes a sequential analysis of the functional relationships between the constants, parameters, and variables of the objective function and constraints to eliminate within the set of feasible solutions from consideration as potentially optimal resulting in what is termed a "reduced feasible set" which is small enough to make a total enumeration over the remaining elements computationally feasible using a digital computer to perform the required calculations. Without the constraint of small sample sizes, this method is neither computationally feasible nor operationally required.

Derivation of the method of solution is described by the following steps:

1. Setting a lower bound of two observations per cell eliminates from feasibility all combinations of observations not meeting this constraint.

2. It is a well known fact in the literature that β, the probability of a Type II error, varies inversely with the noncentrality parameter, λ. This fact is illustrated in the charts by Pearson and Hartley.

3. From (3.14) it can be seen that λ varies directly with the h_i's, since $h_i \geq 1$.

4. From (3.15) it may be shown that, for a given total number of samples for the i^{th} treatment, T_i, such that $\sum_j n_{ij} = T_i$, the maximum value for h_i will occur when all of

the associated n_{ij}'s are as nearly equal as possible, considering that they are integers.

5. From the relationships described in steps 1, 2 and 3, the minimum value for β for given T_i and α will occur when all of the n_{ij}'s within each treatment are as nearly equal as possible.

6. From step 5, if all samples within a given treatment level have the same cost, c_i, which is assumed to be the case for most OT, the logical choice of allocation of the total number of samples will be the most even allocation possible, since the distribution of a given number of samples among cells has no effect on EASC except through its effect on β. This reduces the feasible set by removing all distribution patterns which do not meet this condition. It is important to note that if the sample costs per cell are not equal, this choice could not be so easily determined. Since, the increase in cost resulting from increasing β by a more imbalanced allocation of samples may be more than offset by a decrease in sampling cost resulting from taking fewer samples from high cost cells and more samples from low cost cells.

7. Since the allocation of observations to specific cells will be accomplished by a random process, after the optimal number of samples and their distribution pattern is determined for each row, for purposes of determining the optimal distribution of samples the artifical constraint,

$n_{ij} \geq n_{ik}$ for $k > j$, may be imposed in order to eliminate permutations of a specific distribution pattern from the feasible set. In otherwords, as far as the optimal solution is concerned, the order in which observations are distributed among the individual cells is immaterial since for given i, any ordering of a given set of n_{ij}'s will produce the same value of the objective function, all other conditions being fixed.

8. From (2) thru (7), for any given $T_i = \sum_j n_{ij}$ there will be a unique $N_i = \{n_{ij}\}$ within the reduced feasible set. From (3.7) and (3.8), $2^K \leq T_i \leq S_i$.

9. For any given total number of observations, N, there is only a finite set $\{(T_1,T_2) | T_1 + T_2 = N\}$.

10. If the cost of sampling for the new system, c_1, is greater than the cost for the SFC, c_2, which is generally true for OT, then no solution in which $T_1 \geq T_2$ will be optimal, since transposing the values of T_1 and T_2 produces the same value for β at a reduction in sampling cost, all other conditions being fixed. Therefore, for given N, the reduced feasible set now becomes $\{(T_1,T_2) | T_1 + T_2 = N, T_1 < T_2\}$.

11. Combining (8), (9), and (10), for given α and N, the reduced feasible set, $(S|\alpha,N)$, may be defined by

$$(S|\alpha,N) = \{(\bar{n}_1,\bar{n}_2) | \textstyle\sum_j n_{ij} = T_i, \sum_i T_i = N, T_1 < T_2\} \quad (3.20)$$

where

$$\bar{n}_i = \{n_{ij} | n_{ij} \geq 2,\ n_{ij} \geq n_{ik}(k>j)\} \qquad (3.21)$$

12. Enumeration over $(S|\alpha,N)$ will identify the optimal $(\bar{n}_1,\bar{n}_2|\alpha,N)$.

13. Enumeration over $(S|\alpha,N)$ for all possible values of N ($2^{K+1} \leq N \leq S_1 + S_2$) will identify the optimal $(\bar{n}_1,\bar{n}_2|\alpha)$.

14. Enumeration over $(S|\alpha,N)$ for all possible values of N and incrementally increasing values of α, starting with α at a lower bound close to zero, will identify the overall optimal $(\bar{n}_1,\bar{n}_2,\alpha)$. The lower bound and increment for α must be selected by the test designer. A lower bound of .01 is suggested. Initially, a relatively large increment, say .1, may be used to determine the values for α between which the optimum point lies. Then a search between these values using a smaller increment may be used to more closely approach the true optimum. One caution must be observed in this approach: if the initial increment is chosen too large, the minimum point for EASC may not be detected. To minimize this problem it is recommended that the initial increment be chosen as small as possible while still limiting the points to be evaluated to a computationally feasible number.

15. In evaluating the $(\bar{n}_1,\bar{n}_2|\alpha,N)$ the procedure is to start with the minimum value of N and the minimum value of T_1 for the given N which is the point yielding the minimum sampling cost and the maximum β cost, since λ is a minimum. For given N, increasing T_1 results in an equal decrease in T_2,

call it ΔT. It also results in an increase in λ, $\Delta\lambda$, which produces a decrease in β cost, $\Delta\beta C$; and an increase in sampling cost, ΔSC. The problem is to determine the maximum T_1 for which

$$\Delta T\left(\frac{\Delta\lambda}{\Delta T}\right)\left(\frac{\Delta\beta C}{\Delta\lambda}\right) - \Delta T\left(\frac{\Delta SC}{\Delta T}\right) > 0 \qquad (3.22)$$

where as ΔT increases, $\Delta\lambda/\Delta T$ decreases, but is always positive. Also, as λ increases, $\Delta\beta C/\Delta\lambda$ decreases continuously, reaches a minimum, and then increases, but is always negative. This minimum point may not occur within the range of λ for a given N. $\Delta SC/\Delta T = c_1 - c_2$, is constant.

The form of the non-central F probability density function precludes taking partial first and second derivatives in order to establish the exact form of the objective function as a function of $(T_1,T_2|\alpha,N)$, however, an empirical analysis indicates that it possesses a single minimum which is located either at an interior point or at one of the end points of the range defined for $(T_1,T_2|\alpha,N)$. In the next section the basis for this conclusion is presented in greater detail.

16. Based on the conclusion in the preceding paragraph, if a point $(T_1,T_2|\alpha,N)$ is reached as T_1 increases for which EASC is a local minimum, then either this point or the end point where $T_1 = T_{1min}$ for given N is optimal for the

given values of α and N, and no longer values of T_1 need be evaluated for that α and N.

17. The same rationale described for $\Delta EASC/\Delta T_1$ in the preceding paragraphs applies to $\Delta EASC/\Delta N$, for optimal $(T_1,T_2|\alpha,N)$. This is true since choosing the optimal $(T_1,T_2|\alpha,N)$ identifies the minimum point on the segment of the EASC curve for given N and α described in the following section and illustrated in Figure 5. Consequently, if a point $(T_1,T_2|\alpha,N)$ is reached as N increases for which EASC as a function of N for optimal $(T_1,T_2|\alpha,N)$ is a local minimum, then either this point or the end point, optimal $(T_1,T_2|\alpha,N_{min})$, is optimal for the given value of α, and no larger values of N need be evaluated for that value of α.

18. As indicated in step 14, the procedure for evaluating the $(\bar{n}_1,\bar{n}_2|\alpha,N)$ is to start with α at a lower bound close to zero. It is a well known fact that, all other parameters being fixed, as α increases, β decreases and approaches zero as α approaches one, its maximum value. Therefore, for incrementally $(\Delta\alpha)$ increasing values of α, there will be at most one point where for optimal $(\bar{n}_1,\bar{n}_2,N|\alpha)$

$$\frac{\Delta\ EASC}{\Delta\alpha} = 0 \qquad (3.23)$$

In other words, the objective function, EASC, as a function of α for optimal $(\bar{n}_1,\bar{n}_2,N|\alpha)$ possesses a single minimum point which is located either at an interior point or at one

of the end points of the range of α. This relationship is empirically demonstrated by Figures 10 thru 13.

19. Based on the relationship described in step 17, if, for increasing α, a value of α is reached, call it α_o, where EASC for optimal $(\bar{n}_1,\bar{n}_2,N|\alpha_o) >$ EASC for optimal $(\bar{n}_1,\bar{n}_2,N|\alpha_o-\Delta\alpha)$, then the optimal value for α, call it α^*, lies in the interval: $(\alpha_o-2\Delta\alpha) \leq \alpha^* < \alpha_o$. No larger values of α need be evaluated.

The EASC Algorithm

The EASC algorithm based on the derivation described in the preceding section consists of the following step-by-step procedure.

1. Obtain the required input data.
    a. Design structure parameters.
        (1) Number of factors, K.
        (2) Number of covariates, M.
    b. Sample size constraints.
    c. Cost coefficients.
        (1) Alpha cost coefficient, C_α.
        (2) Beta cost coefficient, C_β.
        (3) Treatment sample costs, c_1 and c_2.
    d. Primary parameters.
        (1) Random design error variance, σ_Y^2.
        (2) Correlation coefficients for control factors, $\rho_{X_2Y},\ldots,\rho_{X_KY}$.

(3) Correlation coefficients for covariates, $\rho_{Z_1Y},\ldots,\rho_{Z_MY}$.

(4) Average control variances for control factors, $\bar{\sigma}^2_{x_2},\ldots,\bar{\sigma}^2_{x_K}$.

(5) Population variances for control factors, $\sigma^2_{X_2},\ldots,\sigma^2_{X_K}$.

e. Initial value of α, α_o.

f. Increment for α, $\Delta\alpha$.

2. Calculate the value for the design error variance, σ^2_o, by (3.13) and for the error degrees of freedom, ν_e, by (3.12).

3. Calculate lower and upper bounds for total number of observations, N as follows:

a. $N_{min} = 2^{K+1}$.

b. $N_{max} = S_1 + S_2$

4. Starting with $\alpha = \alpha_o$, do the following to determine the optimal $(\bar{n}_1,\bar{n}_2,N|\alpha)$:

a. Starting with $N = N_{min}$, do the following to determine the optimal $(\bar{n}_1,\bar{n}_2|N,\alpha)$:

(1) Determine the initial values of $(T_1,T_2|N)$ as follows:

(a) If $N - S_2 < 2^K$, $T_{1min} = 2^K$, otherwise, $T_{1min} = N - S_2$.

(b) $T_2 = N - T_{1min}$.

(2) Determine the $\bar{n}_i|T_i)$ which results in the most even allocation possible, considering that the n_{ij}'s are integers, and where, $\bar{n}_i = \{n_{ij}|n_{ij} > 2, n_{ij} > n_{ik}\ (k > j)\}$, as follows:

(a) For i = 1,2 and j = 1,...,2^{K-1}: let $n_{ij} = (T_i/2^{K-1}) - (2^{K-1}\text{Modulo } T_i)$.

(b) For i = 1,2 and j = 1,..., $(2^{K-1}\text{Modulo } T_i)$: add 1 to n_{ij}.

(3) Calculate the $(h_i|\bar{n}_i)$ by (3.15).

(4) Calculate the $(\lambda|h_1,h_2)$ by (3.14).

(5) Determine $(F_c|\alpha,\nu_t,\nu_e)$ by using tables or, if using a computer program, by a prepackaged subroutine.

(6) Determine $(\beta|\lambda,F_c,\nu_e)$ from (3.16) by interpolation from tables or by computer subroutine.

(7) Calculate (EASC $\bar{n}_1,\bar{n}_2,N,\alpha,\beta$) from (3.6) and record this value.

(8) If $T_1 = T_{1min}$, set the minimum value for (EASC N,α) equal to the value for EASC just calculated in (7), record this value and skip to (10), otherwise, continue with (9).

(9) Compare the value of EASC just calculated in (7) with the current minimum value of (EASC N,α). If it is less, make it the new minimum value of (EASC N,α), record this value, and continue with (10), otherwise, check to de-

termine if EASC for the previous value of (T_1,T_2) is a local minimum. If it is, skip to (11), otherwise continue with (10).

(10) If $T_1 < S_1$ and $T_2 > 2^{K-1}$, add one to T_1 and subtract one from T_2 then return to step 4a(2), otherwise, continue with (11).

(11) Set optimal (EASC N,α) equal to the current minimum value of (EASC|N,α), record this value and continue with 4b.

b. If $N = N_{min}$, set the minimum value of (EASC|α) equal to the current (EASC|N,α), record this value and skip to 4d, otherwise, continue with 4c.

c. Compare the optimal (EASC|N,α) just determined in 4a(11) with the current minimum value of (EASC|α). If it is less, make it the new minimum value of (EASC|α) and continue with 4d, otherwise, check to determine if optimal EASC for the previous value of N is a local minimum. If it is, skip to 4e, otherwise, continue with 4d.

d. If $N < N_{max}$, increase N by one and return to step 4a(1), otherwise continue with 4e.

e. Set optimal (EASC|α) equal to the current minimum value of (EASC|α), record this value and continue with 5.

5. If $\alpha = \alpha_o$, set overall minimum value of EASC equal to the current optimal (EASC|α), record this value and skip to 7, otherwise, continue with 6.

6. Compare the optimal (EASC$|\alpha$) just determined in 4e with the current overall minimum value of EASC. If it is less, make it the new overall minimum value of EASC and continue with 7, otherwise, skip to 8.

7. If $\alpha < 1.0 - \Delta\alpha$, increase α by $\Delta\alpha$ and return to step 4a, otherwise, continue with 8.

8. Set the overall optimal EASC equal to the current minimum value of EASC. This is the last step of the algorithm. By recording the values of T_1, T_2, N and α for each value of EASC, the algorithm yields the overall optimal $(\bar{n}_1, \bar{n}_2, \alpha)$.

As previously stated, the actual assignment of observations to specific cells and the sequence of observations must be determined by a random process constrained to the optimal distribution pattern determined by the EASC algorithm.

This algorithm was programmed in FORTRAN IV for the Georgia Institute of Technology's CDC CYBER 70 computer. A computer listing of this program and description of the output options is contained in the Appendices.

Empirical Analysis of Objective Function

The computer programmed solution algorithm was used to generate data for a 2^3 completely crossed design with one covariate based on hypothetical values of the cost coefficients and the primary parameters in order to test the pro-

gram and empirically investigate the functional relationships between the objective function and the decision variables, $\bar{n}_1, \bar{n}_2, N$, and α. With the exception of Figure 4, all illustrations in this section are based on this data. The values for cost coefficients and primary parameters not indicated on the figures themselves are contained in Appendix B.

EASC as a Function of $(T_1, T_2 | \alpha, N)$

Each $(T_1, T_2 | \alpha, N) \varepsilon$ S, defines a specific value for the noncentrality parameter, λ, which increases monotonically with T_1 as illustrated in Figure 2. The values of β as a function of λ derived from the test data are shown in Figure 3. The variations in λ for given ν_e are caused by the different values for $(T_1, T_2 | \nu_e)$. Consequently, β cost, as a function of T_1, for continuous T_1, is of the general form shown in Figure 4.

The form of the derivative of β cost with respect to T_1, $\partial \beta C / \partial T_1$, is also shown in Figure 4. The partial derivative of sampling cost with respect to T_1, $\partial SC / \partial T_1$, is a constant $(c_1 - c_2)$. Note since $c_1 > c_2$ as discussed in step 10 of the previous section, $\partial SC / \partial T_1 > 0$. However, to more clearly show the relationships between $\partial \beta C / \partial T_1$, Figure 4 illustrates the negative of $\partial SC / \partial T_1$. For continuous T_1

$$\lim_{T_1 \to 0} \frac{\partial \beta C}{\partial T_1} = 0 \quad \text{and} \quad \lim_{T_1 \to \infty} \frac{\partial \beta C}{\partial T_1} = 0 \qquad (3.24)$$

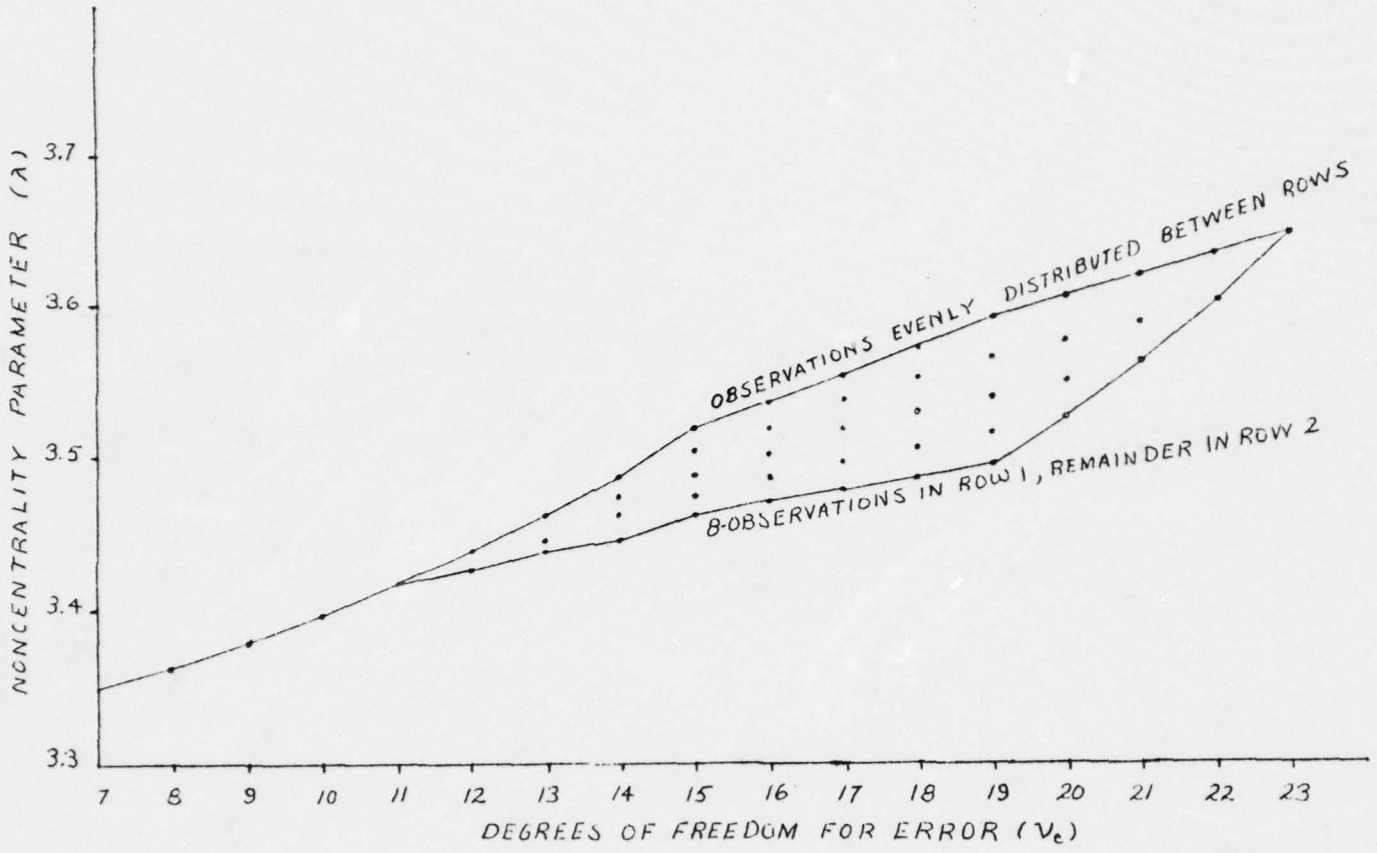


Figure 2. λ as a Function of ν_e and T_1.

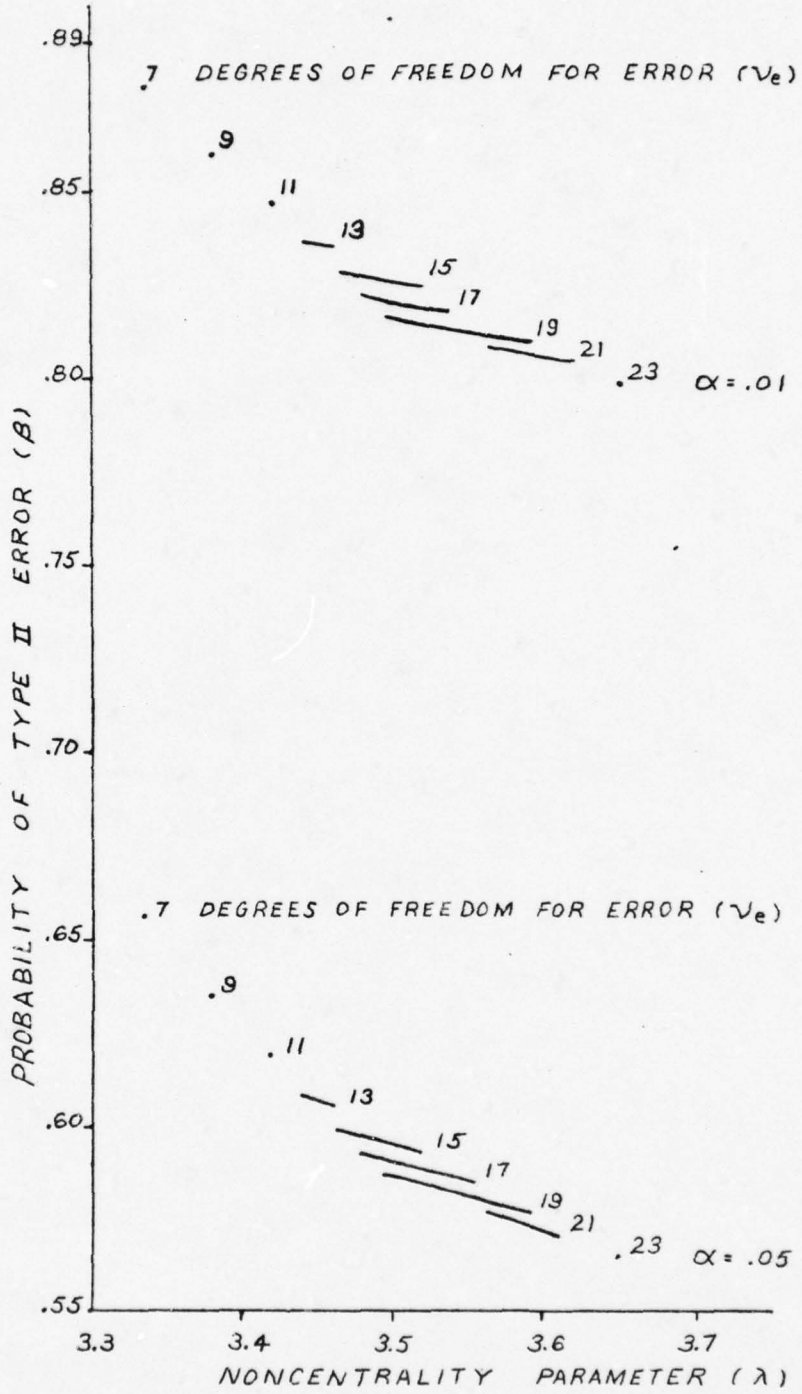


Figure 3. β as a Function of λ.

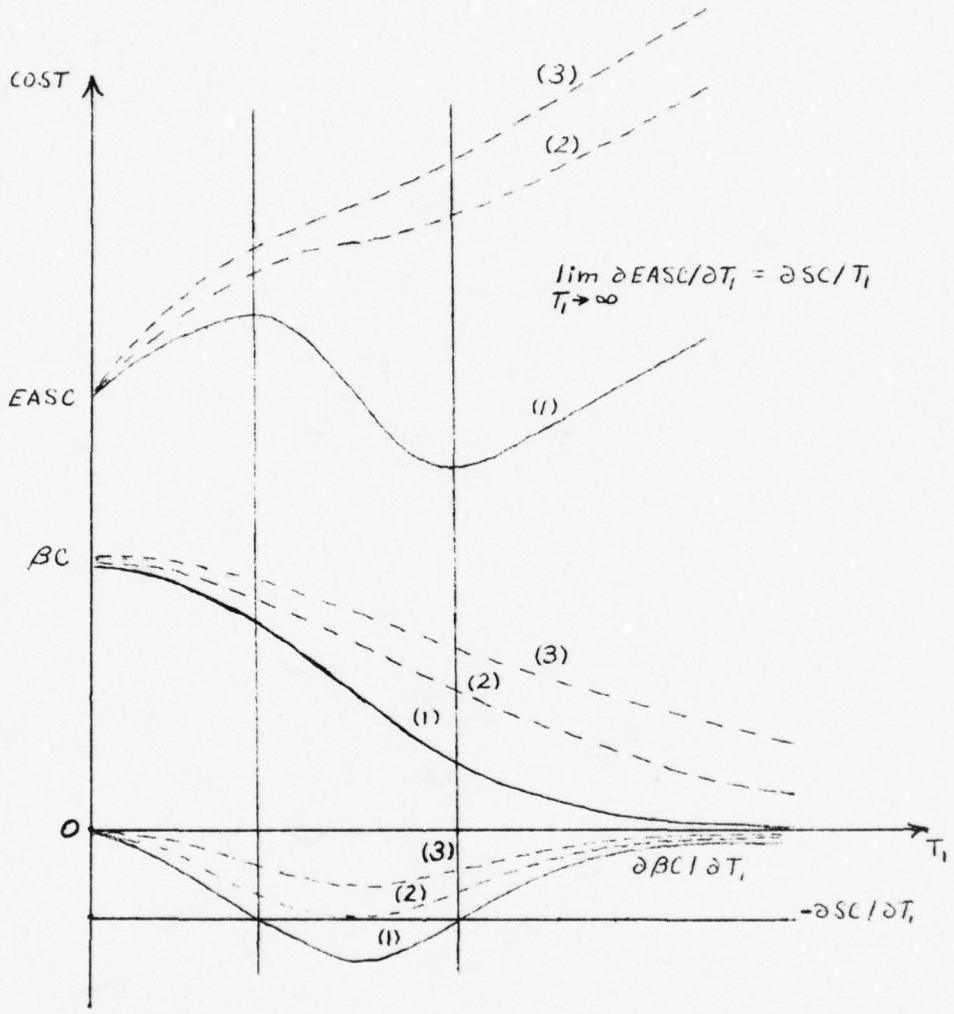


Figure 4. EASC as a Function of $(T_1, T_2 | \alpha, N)$ - General Form.

Based on these relationships, EASC as a function of (T_1, $T_2|\alpha,N$) will have one of the three general forms shown, depending on the relationship between the minimum value of $\partial\beta C/\partial T_1$ and $-\partial SC/\partial T_1$

$$\min \partial\beta C/\partial T_1 < -\partial SC/\partial T_1 \Longrightarrow \text{EASC (1)}$$

$$\min \partial\beta C/\partial T_1 = -\partial SC/\partial T_1 \Longrightarrow \text{EASC (2)}$$

$$\min \partial\beta C/\partial T_1 > -\partial SC/\partial T_1 \Longrightarrow \text{EASC (3)}$$

However, since T_1 is bounded from above and below, only a segment of the curves shown in Figure 4 will actually occur. Also, since T_1 is integer, only discrete points within that segment will be generated. Figure 5 illustrates these segment of the EASC curve which were obtained from the example data for several different values of N. Note how increasing the value of N shifts the segment of the EASC curve which actually occurs from right to left with respect to Figure 4.

Figure 6 illustrates the effect of increasing the significance level, α. Note that as α increases the segment of the EASC curve which occurs appears to shift from left to right with respect to Figure 4. What is actually occuring is that the increasing α causes all of the curves in Figure 4 to be compressed to the left. This is due to the fact that as α increases for given N the rate of change of β with respect to T_1 increases. Note also in Figure 6 that increasing

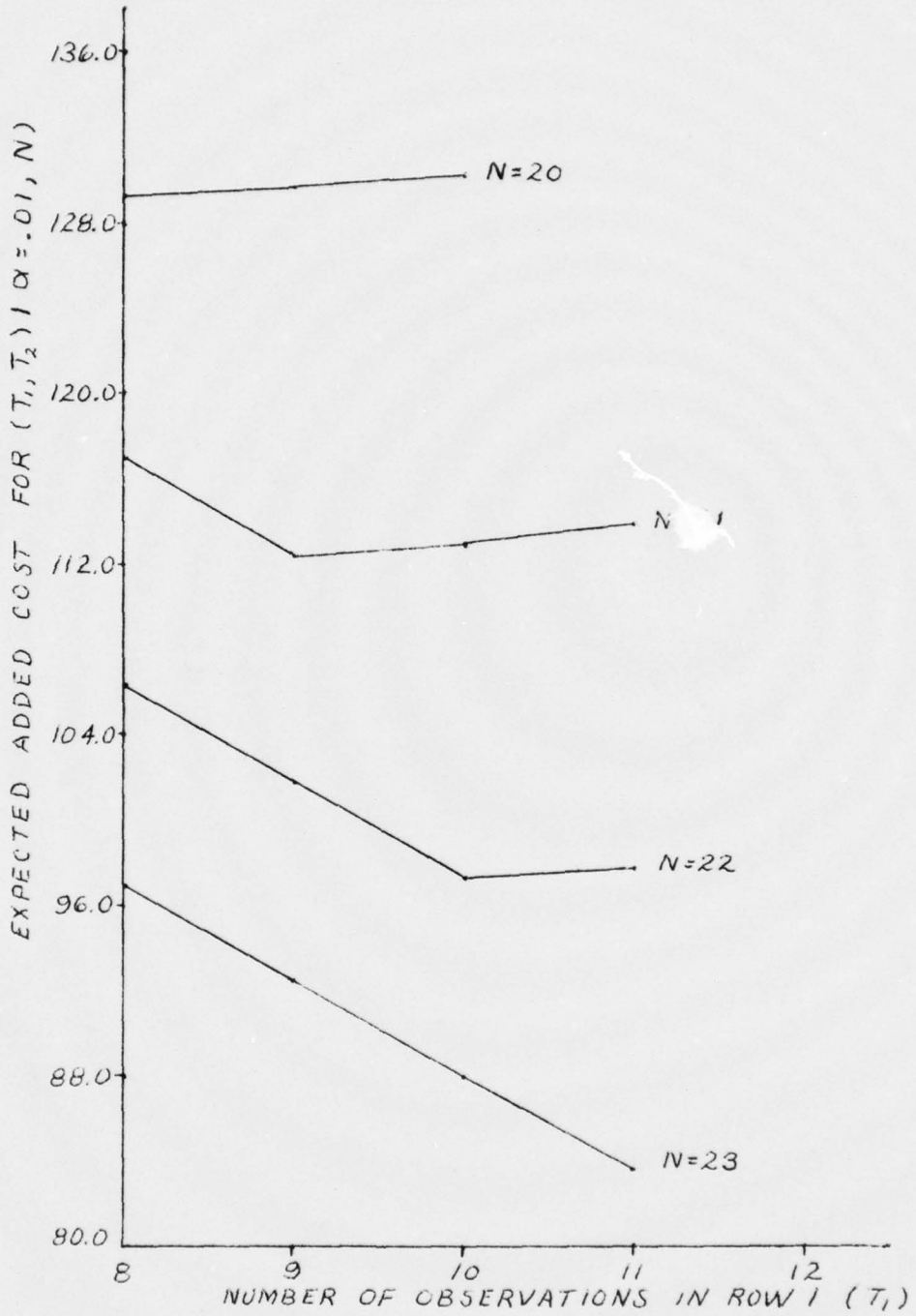


Figure 5. EASC for $(T_1, T_2 | \alpha = .01, N)$.

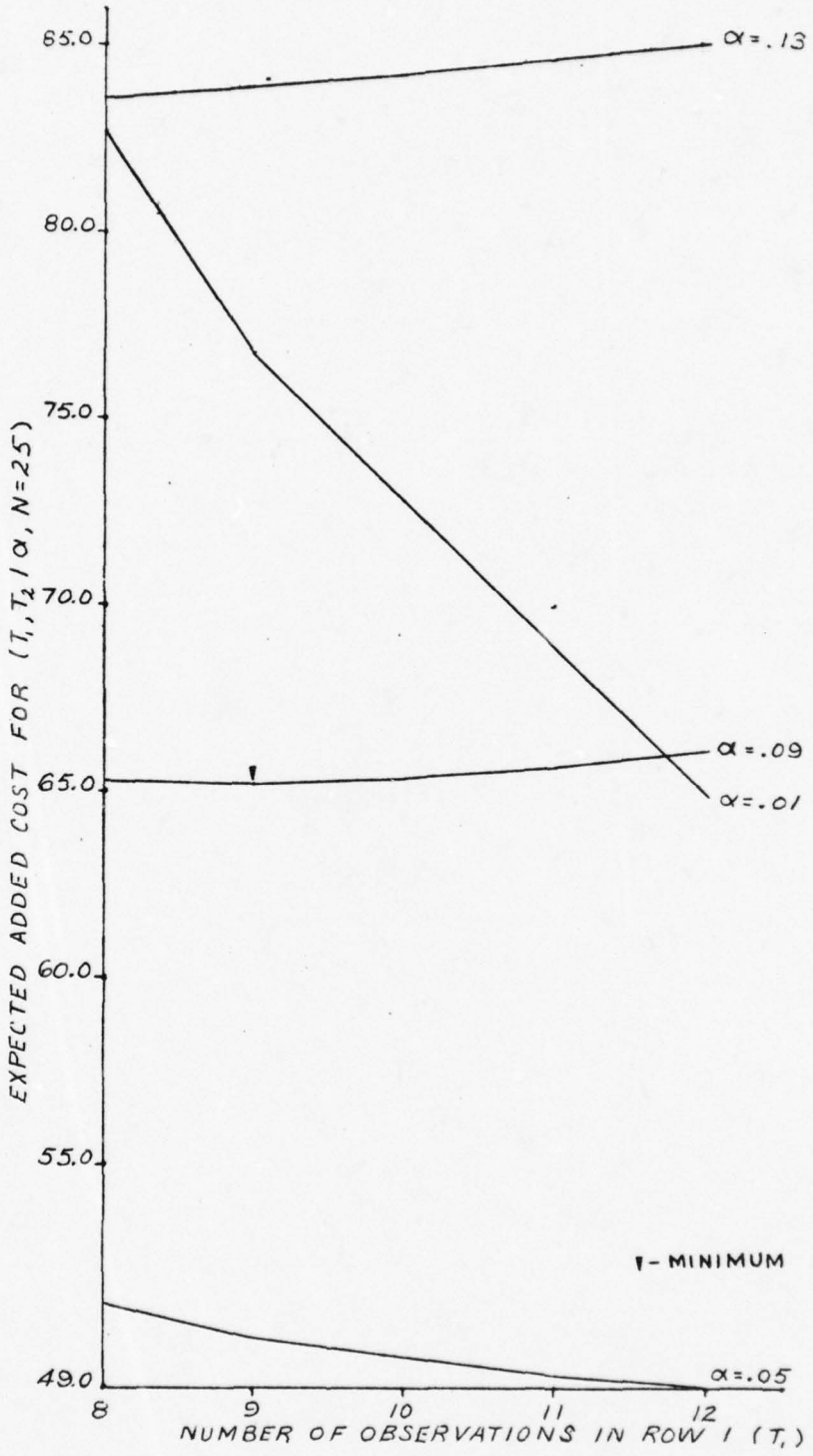


Figure 6. EASC as a Function of $(T_1, T_2 | \alpha, N)$ - Computed Example.

α causes wach value of EASC to be increased by a constant, $C_\alpha \Delta\alpha$.

EASC as a Function of N for Optimal $(\bar{n}_1, \bar{n}_2 | \alpha, N)$

Selecting for each value of N the optimal allocation of observations, $(\bar{n}_1, \bar{n}_2)$, results in the EASC values shown in the EASC values shown in Figure 7. Note that as the significance level increases, the optimal number of observations initially increases, then decreases. This is the result of the variations in the rate of change of β with respect to N for given values of α and N. Where this rate is high enough to off-set the increase in sampling cost, increasing N will reduce EASC. Once this rate decreases to the point where

$$C_\beta \frac{\Delta\beta}{\Delta N} < \frac{\Delta SC}{\Delta N} \tag{3.25}$$

then increasing N will increase EASC. Figures 8 and 9 show this relationship and how it is effected by the magnitude of C and by the ratio

$$r = d^2/\sigma_o^2 \tag{3.26}$$

Note that as r increases (r = .022 to .086), initially the optimal $(T_1, T_2, N | \alpha)$ increases, but as r continues to increase (r = .086 to 8.67), it decreases. This is due to the effect that variations in r have on the rate of change of β with respect to N for given α. Initially, as r increases, it pro-

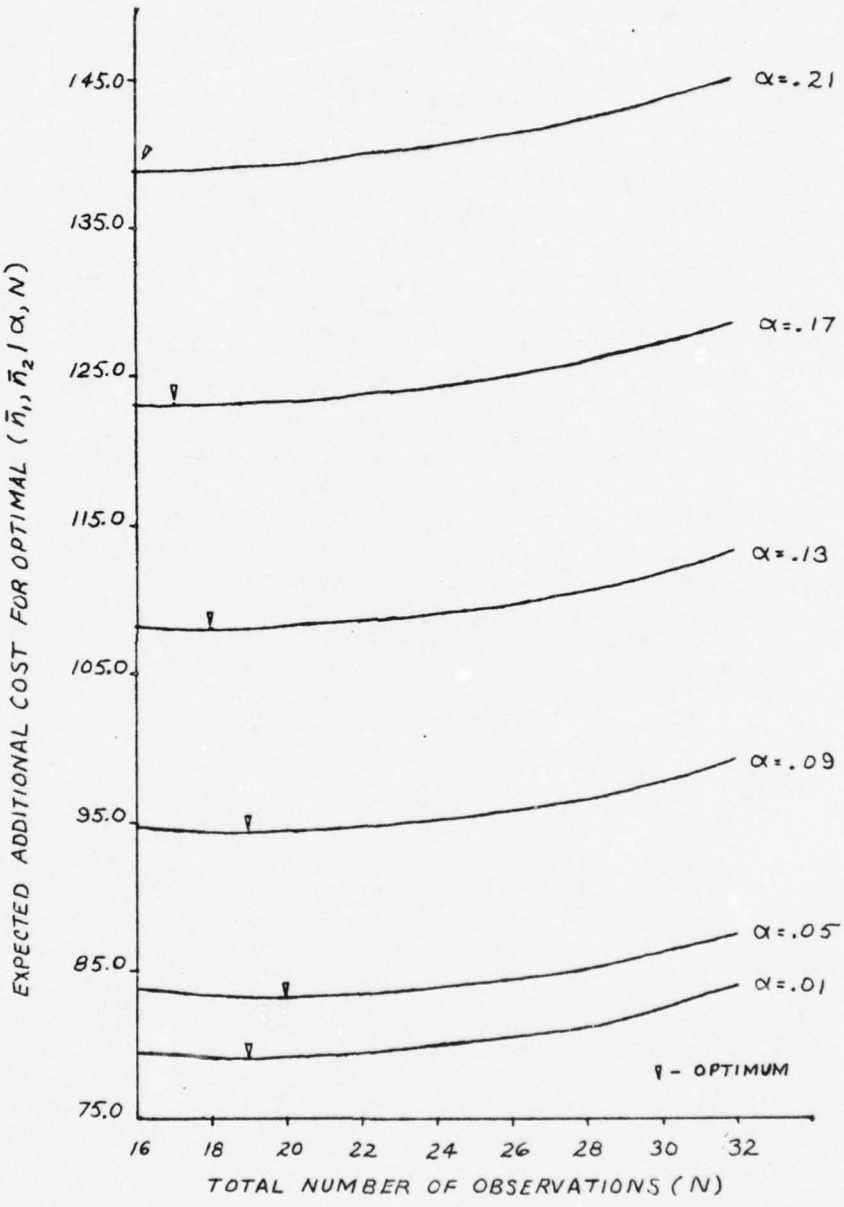


Figure 7. EASC as a Function of N for Optimal $(T_1, T_2 | \alpha, N)$.

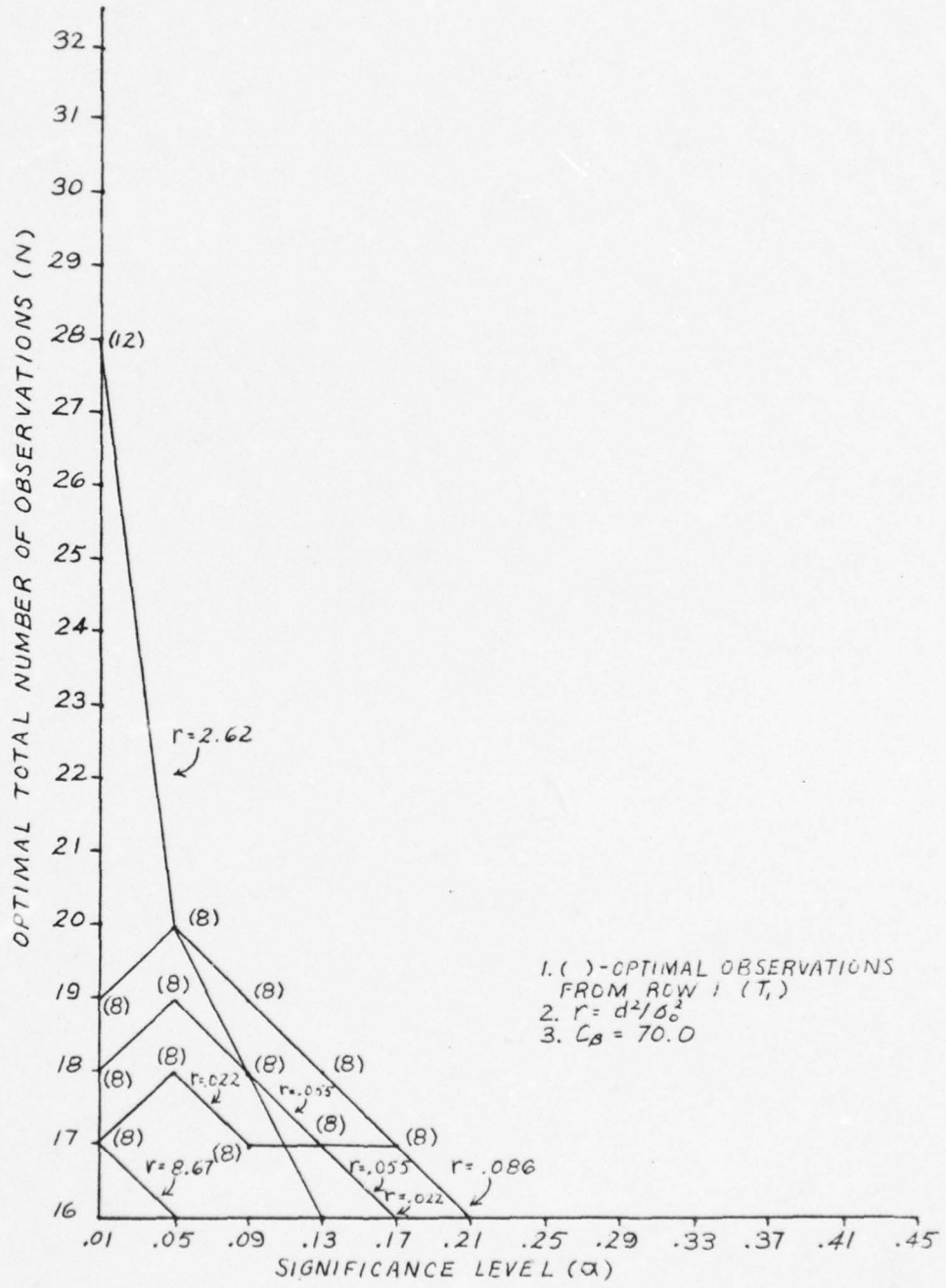


Figure 8. Optimal N as a Function of α ($C_\beta = 70.0$).

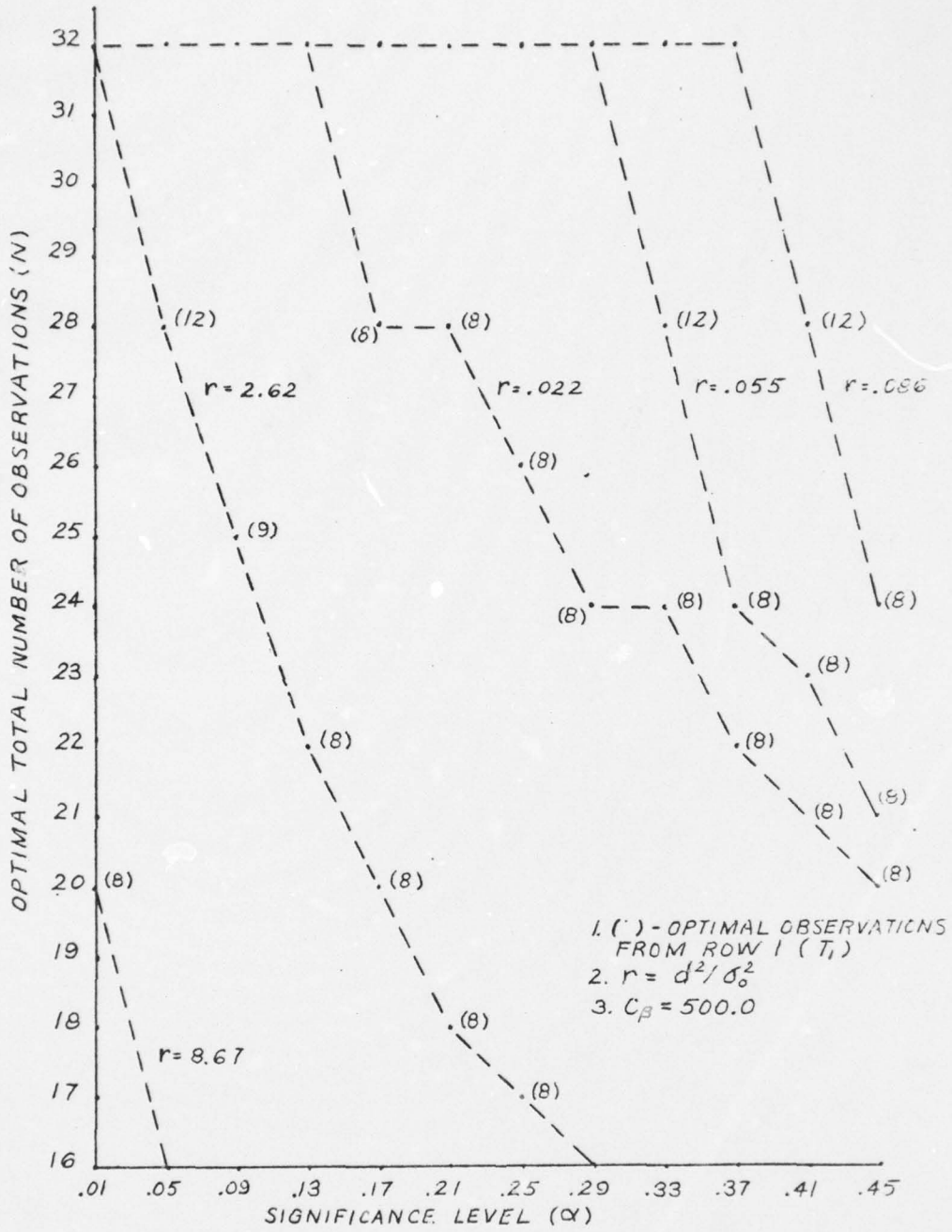


Figure 9. Optimal N as a Function of α (C_β = 500.0).

duces large changes in β with respect to N, however, as it continues to increase, β rapidly approaches zero for given α, consequently, the condition in (3.24) occurs at increasingly smaller values of $(N|\alpha)$. The effect of r on β is indirect through its effect on λ. This can be seen by rewriting (3.14) in terms of r

$$\lambda = 4\,r\,\frac{h_1 h_2}{h_1+h_2} + 3 \qquad (3.27)$$

The effect of the value for C_β on the optimal value of N is also through its effect in (3.24).

EASC as a Function of α for Optimal $(\bar{n}_1,\bar{n}_2,N|\alpha)$

Figure 10 shows the optimal value of the objective function, EASC, for given α. This figure clearly shows that the optimal value of $(EASC|\alpha)$ is sensitive to variations in r. This sensitivity increases rapidly as the ratio C_β/C_α increases. Also, the optimal value of α becomes increasingly sensitive to variations in r as C_β/C_α increases. These effects are directly a result of the relationships described in the preceding section. From the standpoint of OT, this illustrates that, in general, the larger the required performance margin, the easier the experimenter's job will be.

Summary of Procedure

The basic procedure for the design of an OT described during this research is briefly summarized as:

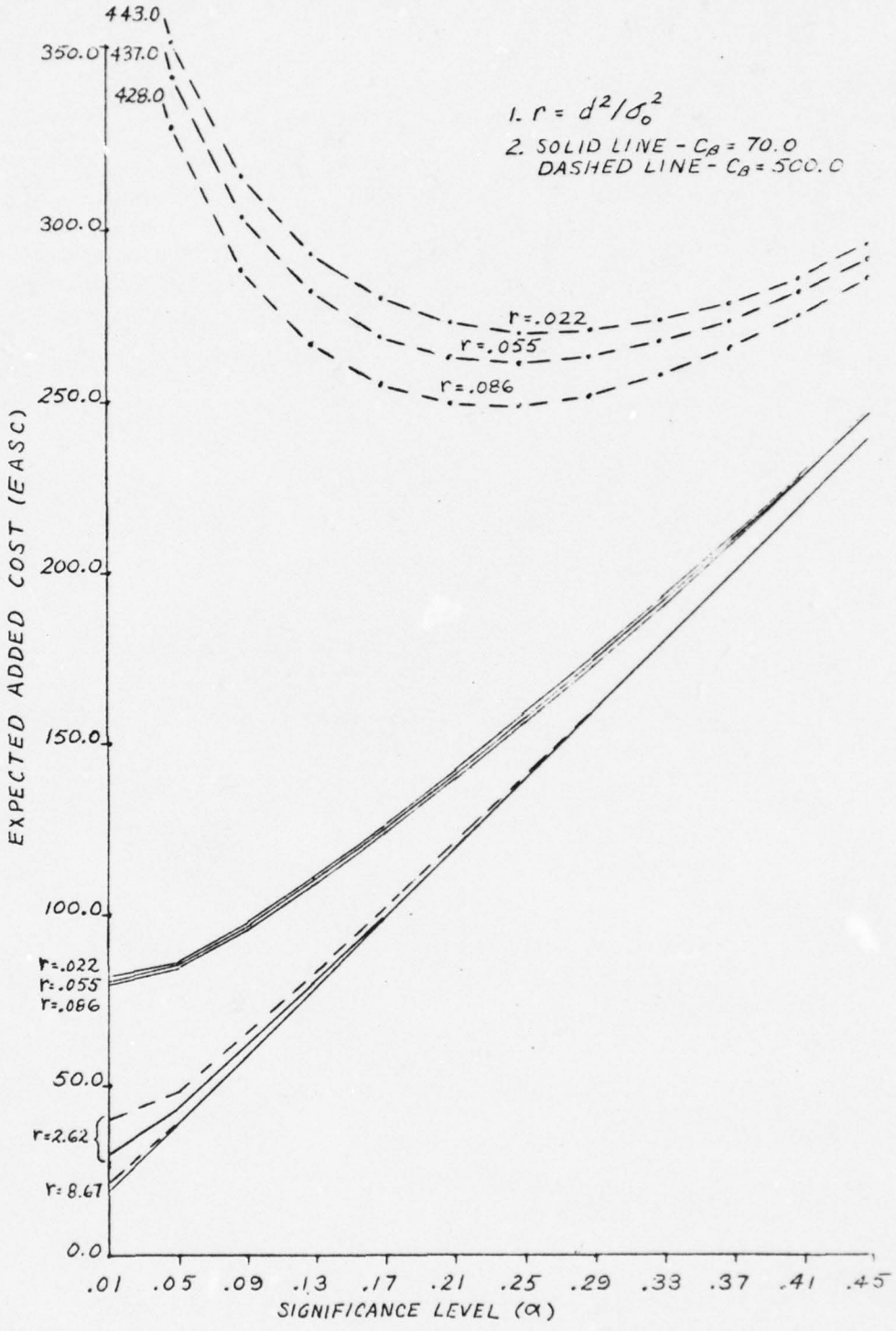


Figure 10. EASC as a Function of α for Optimal $(T_1, T_2, N|\alpha)$.

1. Determine minimum number and type of factors to be considered and how they are to be combined to determine the conditions under which observations will be taken. The minimum number of factors will generally be dictated by the test issues.

2. Determine response variable to be measured (MOE). This must be a continuous variable.

3. Formulate the appropriate response model based on Steps 1 and 2.

4. Select the set of exact hypotheses to be used as the basis for optimizatation. Normally, this will be the null hypothesis of no treatment effect versus an exact form of the alternate hypothesis: the tested system exceeds the SFC by the required performance margin.

5. Determine the cost model to include estimates of all cost coefficients and primary parameters.

6. Formulate the optimization problem to include all constraints.

7. Apply the EASC algorithm to determine the number of observations to be taken in each row and their distribution, the level of significance, and the power of the test.

8. Use a random process to assign observations to specific cells and to determine the sequence in which observations are to be taken.

9. Vary the control limits on the levels of factors to determine the optimum control required if control is an-

ticipated to become a problem.

10. Repeat Steps 5, 6, and 7 for any alternatives which may be of interest to the experimenter such as addition of a blocking factor or covariate; an increase in the number of observations, if the previous optimal solution occurred at the upper limit of this constraint for one or both treatments; or fractional replication.

11. Select the optimal feasible alternative.

12. Begin experimentation.

13. Correct estimates of input parameters as test data becomes available.

14. Repeat Step 7 and other steps as necessary to determine the effect, if any, of the corrected parameter estimates on the optimal solution.

CHAPTER IV

DEMONSTRATION OF THE APPROACH

Introduction

This chapter presents a brief demonstration of the approach developed in Chapter III. This demonstration illustrates the iterative use of this methodology as more accurate estimates of input parameters become available, cost coefficients change, and/or new alternatives become available to the test designer. Variation of the input parameters and cost coefficients permits the evaluation of any number of alternatives. This demonstration includes only a few of these as illustrations. A hypothetical OT requirement is used as a basis for this demonstration.

The Requirement

The Commander, US Army Operational Test and Evaluation Agency (OTEA) has been directed to conduct operational tests to evaluate the overall military worth of the new ground-to-air tactical missile system, TAAMS, which is under development as a replacement for the current standard HAWK missile system.

The plan of test calls for separate sub-tests of the major subsystems of TAAMS. One of these subsystems is the missile guidance system. The plan of test states that,

since the missile warhead is detonated by the proximity of the missile to the target aircraft, the most critical issue for evaluation of the guidance system is its accuracy. The ambient temperature, altitude of the target, and speed of the target are indicated as the most likely factors to have a significant effect on the accuracy of the guidance system.

The project manager for TAAMS has notified OTEA that a maximum of 12 missiles may be fired in each phase of OT to evaluate the guidance system. In addition, the Department of the Army has specified that no more than 20 HAWK missiles may be fired during each phase of OT for evaluation of the guidance system.

For the guidance system sub-test, Commander, OTEA has directed that a comparative test be conducted using live firings of the two missile systems against drone aircraft targets. He further specified that the standard for comparison (SFC) will be the HAWK missile and guidance system, and that the measure of effectiveness (MOE) for accuracy of the guidance systems will be the mean miss distance from the target as measured by a radar mounted on the drone and recorded by telemetry.

Although point and interval estimates of the values of the MOE are required, as well as tests for interaction effects, the principal purpose of the sub-test is to determine if the accuracy of TAAMS exceeds the HAWK by the required performance margin. Consequently, the following hy-

potheses were chosen as the basis for selecting an optimal design through use of the EASC approach:

$$H_O : \mu_T - \mu_H = 0$$

versus

$$H_1 : \mu_T - \mu_H = d$$

where T indicates TAAMS and H indicates HAWK.

Test Design for OT I

After evaluating the test directive, the guidance system sub-test designer recommended a 2^3 completely crossed factorial design with ambient temperature, Z, to be treated as a covariate, and recorded at the launch site just prior to each firing. The two independent variables to be treated as control factors are: speed of target, X_3 and altitude of target, X_2. Ambient temperature was selected as a covariate because of the difficulty in maintaining any control over it. Also, the test designer feels that it will sufficiently meet the ANOCOV assumptions.

The test designer now wishes to determine the number of firings which should be used for each type of missile, the distribution of these observations among the cells of the 2^3 design, the level of significance to be set for the test, and the resultant power of the test with respect to H_1. Using all available data he develops estimates of the

cost coefficients and primary parameters required for the EASC approach. These estimates are shown in Table 1. He further determines that, since tests of interaction effects are required, a minimum of two observations will be allocated to each design cell.

Inputing the data contained in Table 1 and the sample size constraints imposed by the project manager and the Department of the Army into the EASC program produced the values shown in Figure 11. Note that the values for EASC beyond the optimal value of α are shown throughout this demonstration in order to further illustrate the form of EASC as a function of α for optimal $(\bar{n}_1, \bar{n}_2, N/\alpha)$. The actual EASC program contained in Appendix A does not evaluate these points as explained in step 18 of Chapter III. A modified version of the program was used to obtain this data.

During a planning meeting with the instrumentation group, the fact was mentioned that a new control unit costing $7,000 was available for the target drones which would reduce variations in altitude by 50 percent. Commander, OTEA asked the test designer to determine if this new unit would be worth the additional cost. In order to evaluate this alternative, the test designer input the new value of the control variance for altitude $\bar{\sigma}^2_{x_2}$, into the EASC program maintaining all other parameters at their initial values. The new optimal solution was EASC = $8.897 M, a reduction of $10,000 from the initial value. Based on this expected re-

Table 1. Initial Input Data for OT I

Cost Coefficients (million dollars)	Primary Parameters
C_o = 1.000	σ_Y^2 = 4.000
C_α = 10.000	d = .200
C_β = 10.000	$\rho_{X_2Y}^2$ = .500
c_1 = .250	$\rho_{X_3Y}^2$ = .500
c_2 = .100	ρ_{ZY}^2 = .500
	$\bar{\sigma}_{x_2}^2$ = 2.000
	$\bar{\sigma}_{x_3}^2$ = 2.000
	$\sigma_{X_2}^2$ = 20.000
	$\sigma_{X_3}^2$ = 10.000

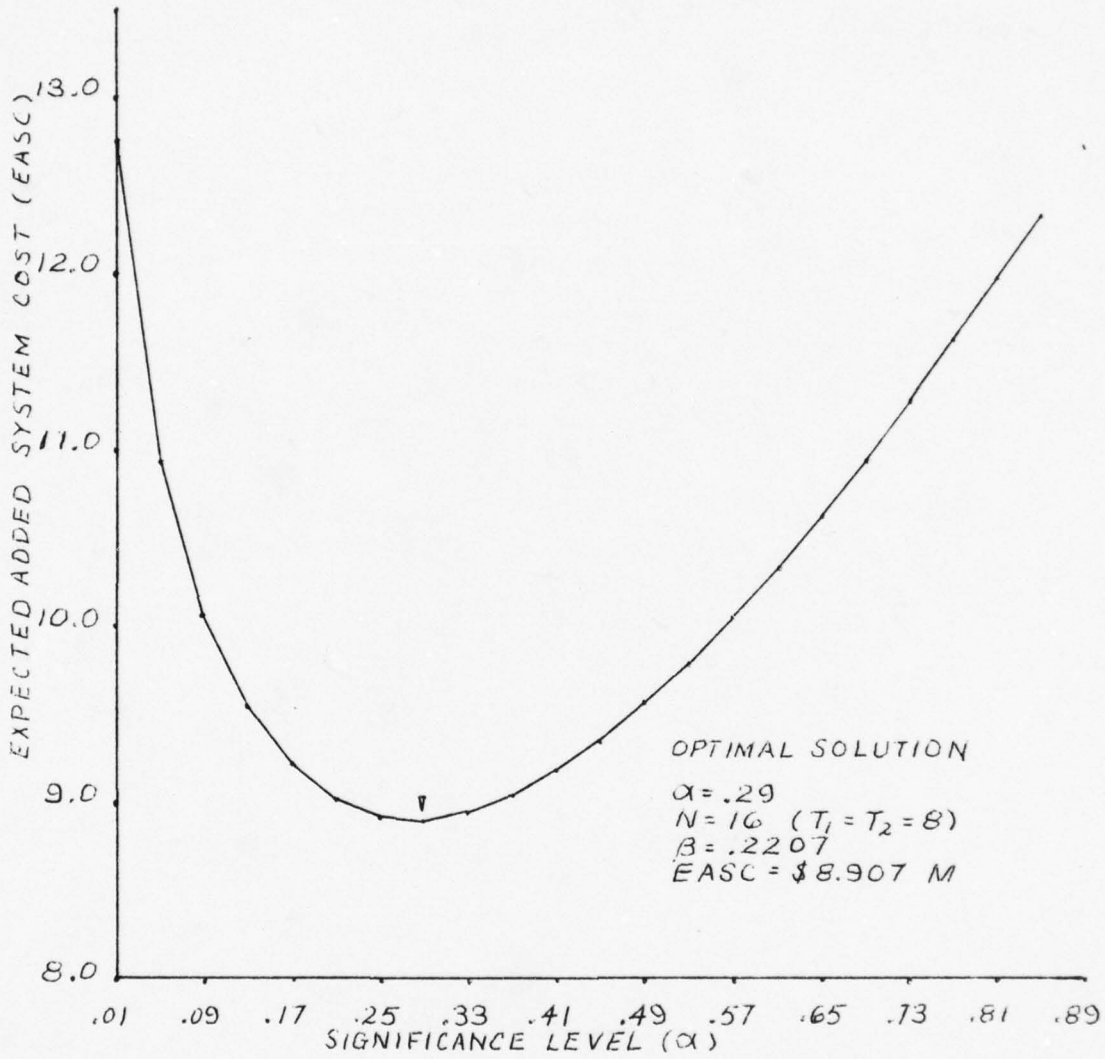


Figure 11. Optimal (EASC/α) for Initial OT I Design.

duction in EASC, the new unit was purchased and OT I was conducted.

Test Design for OT II

Based on data collected during OT I, estimates of the primary parameters are revised. C_α and C_β are also increased to reflect the expected increased expenditure of funds which will occur between OT II and OT III as a result of the ASARC and DSARC recommendations which will be based to a large extent on the results of OT II. C_α has now become larger than C_β, since, if the new system is approved for transition into Phase III, an advanced prototype will be constructed. While if the results of OT II indicate that modifications to the guidance system are necessary, their costs are expected to be relatively minor and the system will be required to undergo further limited testing, after their completion and prior to approval for transition into Phase III.

The initial OT II input data to the EASC program is shown in Table 2 and the resultant output values are graphed in Figure 12.

After initiation of OT II testing, OTEA is notified by the project manager that the required performance margin, d, has been reduced from .200 to .150. Commander, OTEA directs that the current design for OT II be reevaluated in light of this change and revised if indicated. Running the

Table 2. Initial Input Data for OT II

Cost Coefficients (million dollars)	Primary Parameters
$C_o = 1.000$	$\sigma_Y^2 = 2.500$
$C_\alpha = 20.000$	$d = .200$
$C_\beta = 15.000$	$\rho^2_{X_2Y} = .700$
$c_1 = .250$	$\rho^2_{X_3Y} = .600$
$c_2 = .100$	$\rho^2_{ZY} = .650$
	$\bar{\sigma}^2_{x_2} = .800$
	$\bar{\sigma}^2_{x_3} = 1.400$
	$\sigma^2_{X_2} = 20.000$
	$\sigma^2_{X_3} = 10.000$

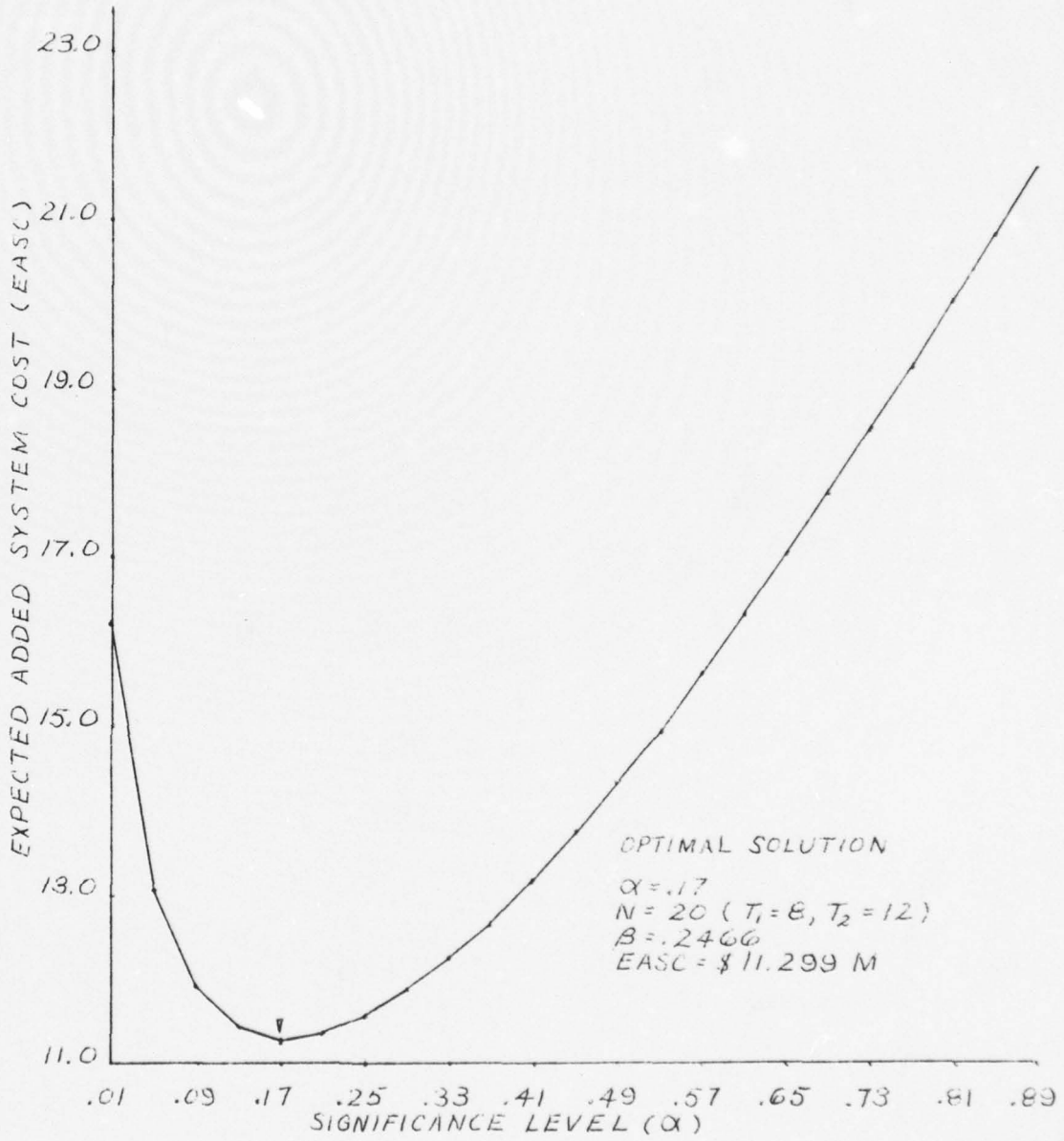


Figure 12. Optimal (EASC/α) for Initial OT II Data.

EASC program with the revised value for d results in the new optimal solution

$$\alpha = .21$$

$$N = 18 \ (T_1=8, T_2=10)$$

$$\beta = .2583$$

$$EASC = \$12.074 \text{ M}$$

Decreasing d results in a decrease in the ratio, r, defined in Chapter III as d^2/σ_o^2. Note the changes that this decrease in r produces in the optimal solution. N decreases from 20 to 18, α increases from .17 to .21, and EASC increases from \$11.299 M to \$12.074 M. All of these changes are examples of the relationships described in Chapter III and illustrated in Figures 9 and 10. A decrease in d results in an overall increase in β for all values of $(T_1, T_2 | \alpha, N)$, which accounts for the increase in the optimal value for EASC. Any change in d will result in an opposite effect on the optimal value for EASC. It will also result in changes in the optimal value of α in the opposite direction but these changes may not be detected if the change is smaller than the increment between values of α which are evaluated. Changes in the optimal value for N may result, depending on the effect that the resulting change in r has on $\Delta\beta/\Delta N$ for given α. Note also that if a change in N results which produces a change in sampling cost large enough to off-set the change in EASC produced by d, the overall

effect will be a reduction in EASC. This was not the case in this example.

As a result of the new data, the number of firings for the HAWK missile are reduced from 12 to 10. In addition, the value for the significance level to be used during analysis is increased from .17 to .21. OT II is then completed.

Test Design for OT III

Estimates of the primary parameters are again revised based on the combined data from OT I and II. C_α and C_β are increased significantly since the results of OT III will be used to decide if the new system should be placed into production. Again, however, C_α is larger than C_β due to the different costs which are expected to result from the occurrence of a Type I error versus a Type II error.

The initial OT III input data to the EASC program is shown in Table 3. The output values are graphed in Figure 13.

Prior to the start of testing, OTEA is informed by the Army Material Command (AMC) that a new type of target drone is available which has an improved speed control system. Based on the data furnished by AMC for the new drone, the test designer calculates that the control variance for speed could be reduced by 28.5 percent. The effect of this reduction is evaluated using the EASC program. Inputing the reduced value for $\bar{\sigma}^2_{x_3}$ produces the new optimal solution

Table 3. Initial Input Data for OT III

Cost Coefficients (million dollars)	Primary Parameters
C_o = 1.000	σ^2 = 2.500
C_α = 500.000	d = .150
C_β = 150.000	$\rho^2_{X_2Y}$ = .600
c_1 = .350	$\rho^2_{X_3Y}$ = .600
c_2 = .100	ρ^2_{ZY} = .550
	$\bar{\sigma}^2_{x_2}$ = .800
	$\bar{\sigma}^2_{x_3}$ = 1.400
	$\sigma^2_{X_2}$ = 20.000
	$\sigma^2_{X_3}$ = 10.000

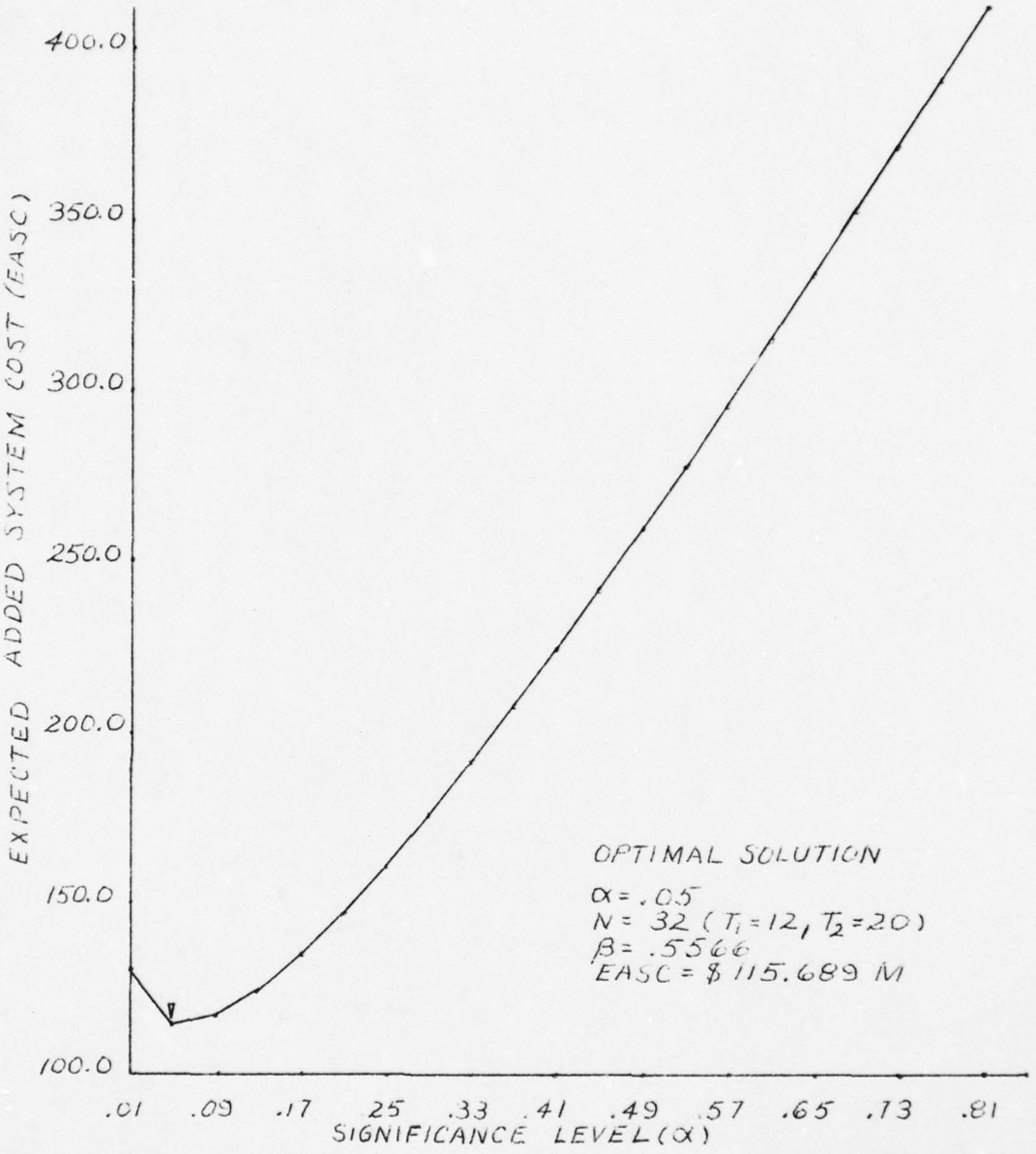


Figure 13. Optimal (EASC/α) for Initial OT III Data.

$$\alpha = .05$$

$$N = 32 \ (T_1=12, T_2=20)$$

$$\beta = .5527$$

$$EASC = \$115.107 \text{ M}$$

The expected reduction in EASC is $.582 M. The total increase in cost to obtain the required 32 new drones is $.320 M. Therefore, the purchase of the new drone is justified by the EASC approach.

Nothing that the optimal solution calls for the maximum number of firings for both missile systems, the Commander, OTEA asks the test designer to evaluate the effect on the optimal EASC of increasing the maximum number of HAWK firings by one. Inputing this change into the EASC program yields the new optimal solution

$$\alpha = .05$$

$$N = 33 \ (T_1=12, T_2=21)$$

$$\beta = .5502$$

$$EASC = \$114.831 \text{ M}$$

Based on a comparison of the $.276 M reduction in EASC with the $.100 M cost of the HAWK missile, a request for one additional missile was submitted.

Although the input data used for this example is completely hypothetical, it serves to demonstrate the application of the EASC approach to OT. The types of decision

alternatives which may be evaluated using EASC are not limited to those contained in this example. Through variation of parameters, numerous other alternatives may be evaluated to include, for example, the addition or deletion of a factor or covariate, estimates of the effects of inaccuracies in estimation of primary parameters, and the effects of variations of cost coefficients.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Limitations of the Research

This research is limited in application to univariate, quantitative, continuous, linear response models. Also it is limited by the additional assumptions required for ANOVA to the extent that a particular departure from those assumptions effects the validity of the analysis and cannot be corrected for by the application of appropriate techniques. The more restrictive assumptions required by ANOCOV are only applicable when this particular design is actually employed.

Another apparent limitation is the requirement that reasonably accurate estimates of the required input parameters and cost coefficients be available. However, some form of estimate, conscious or unconscious, of these values is inherent in any method for selecting experimental designs, although it may not be explicitly stated. Use of the EASC approach forces the designer to consciously formulate these estimates and allows him, by variation of parameters, to evaluate the sensitivity of the optimal solution to the accuracy of initial estimates.

Conclusions

The criterion of expected additional system cost

(EASC) as defined in this research is a valid measure of the relative efficiency of alternative experimental design structures for operational tests, subject to the limitations previously described.

The EASC approach represents a feasible and systematic method for selecting specific experimental design structures for operational testing under conditions of constrained sample size.

As the upper bound on sample size increases, this approach quickly becomes impractical due to the rapid increase in the size of the feasible solution set which negates the practicality of the partial enumeration solution procedure employed.

Recommendations

This research is limited in application as previously described. Operational testing generally involves multiple MOE, many of which are either discrete or qualitative. Extension of this approach to optimization over multiple responses or the development of a multivariate analog to this approach are suggested as subjects for future research. Extension of this method to discrete and/or qualitative MOE is another area in which future research is recommended.

Based on an empirical analysis of the form of the objective function with respect to the decision variables, some form of nonlinear programming technique may be applica-

ble and more efficient for solving the optimization problem as formulated in Chapter III. This is an area in which future research might significantly extend the applicability of this approach by allowing a much larger size of feasible solution set to be handled.

Finally, it is recommended that the US Army Operational Test and Evaluation Agency adopt, on a trial basis, the EASC approach as an adjunct to current selection procedures for situations which fall within the limitations of this approach in its current infant state.

APPENDICES

APPENDIX A

This appendix contains a FORTRAN IV listing of the main program for the EASC approach algorithm. The program receives input data from a user created data file which is given a local file name, TAPE5. The program utilizes two subroutines from the CDC CYBER 70 Math-Science Library Package to calculate the Type II error: PIFDIS determines the critical value for the F test given the value of significance level and the numerator and denominator degrees of freedom: PFDIST determines the value for the Type II error given the adjusted critical value and the numerator and adjusted denominator degrees of freedom as indicated in Chapter II.

The user may select three different levels of output by specifying the value for PARAM. PARAM = 1 will provide a complete list of all points evaluated by the program as well as identifying the optimal values for each value of significance level and the overall optimum. PARAM = 2 lists all points evaluated and the optimum for only one value of significance level. PARAM = 3 lists only the optimal points for each value of significance level and the overall optimum. Appendix B contains an example of the output format for each option.

```
      PROGRAM COSTOPT(INPUT,OUTPUT,TAPE5,TAPE5=INPUT,TAPE6)
      DIMENSION RHOF(6)
      DIMENSION RHOX(6)
      DIMENSION AFVAR(6),FVAR(6)
      DIMENSION N1MIN(100),N2MIN(100),XTCOST(50)
      DIMENSION TCMIN(100),SCMIN(100), BMIN(100),BCMIN(100)
      DIMENSION XLAMIN(100),ALAM(50,50),SCOST(50,50),VTADJ(50,50)
      DIMENSION N(2,50),XN(2,50),C(2),NN(2)
      DIMENSION MAXN1(100)
      DIMENSION PARM(2),H(2),SUMXN(2)
      READ(5,500) NFACT,NX,N1,N2,ACOF,BCOF,FCOST
      READ(5,501) VAR,D
      READ(5,502) ALPHA,CI
      READ(5,503) RHOF
      READ(5,503) RHOX
      READ(5,504) AFVAR
      READ(5,504) FVAR
      READ(5,505) C
      READ(5,506) IPARAM
  500 FORMAT(213,214,2F10.0,F6.0)
  501 FORMAT(F7.0,F8.0)
  502 FORMAT(2F6.0)
  503 FORMAT(6F4.0)
  504 FORMAT(6F7.0)
  505 FORMAT(2F6.0)
  506 FORMAT(12)
  599 FORMAT(* NUM OF FACTORS=*,I3,/,* NUM OF COV=*,I3)
  600 FORMAT(1H1)
  601 FORMAT(1H0,* RUN NUMBER- *,I4,//)
  602 FORMAT(* PARAMETER VALUES ARE- *)
  603 FORMAT(T2,*ALPHA COEFF=*,F10.3,/,T2,*BETA COEFF=*,F10.3)
  604 FORMAT(T1,*SAMPLE SIZE CONSTRAINTS:*,/,T26,*TESTED ITEM=*,
     +I3)
 6041 FORMAT(T26,*STANDARD ITEM=*,I4)
  605 FORMAT(T2,*VAR FOR RANDOM DESIGN=*,F7.2)
  606 FORMAT(T2,*REQ PERFORMANCE MARGIN=*,F7.2)
  607 FORMAT(T2,*CORRELATION COEFFICIENTS:*)
  608 FORMAT(T27,*FACTOR *,I3,*=*,F4.2)
  609 FORMAT(T27,*COVAR *,I3,*=*,F4.2)
  610 FORMAT(T12,*AVG VAR ABOUT FIXED LEVEL*,T48,*POP VAR*)
  611 FORMAT(T2,*FACTOR*,I3,T21,F7.3,T51,F7.3)
  612 FORMAT(T2,*DESIGN ERROR VAR=*,F10.4,/)
  613 FORMAT(T2,*SAMPLE COSTS:*,/,T16,*TESTED ITEM= *,F6.3)
  614 FORMAT(T16,*STANDARD ITEM= *,F6.3)
  700 FORMAT(///,* ALPHA= *,F6.4)
  701 FORMAT(* ALPHA COST= *,F10.3)
  702 FORMAT(1HO,T2,*NO OBS*,/,T2,*ROW1*,I7,*ROW2*,T14,*SCOST*,
     +T22,*LAMBDA*,T31,*BETA*,T39,*BCOST*,T55,*TCOST*,/)
  704 FORMAT(T2,I3,I7,I3,T14,F7.3,T22,F7.3,T31,F6.4,T39,F10.3,
     +T55,F11.3)
  708 FORMAT(//,* FOR ALPHA= *,F6.4,* OPTIMAL VALUES ARE:*,/)
```

```
710 FORMAT(//,* OVERALL OPTIMAL ALPHA= *,F6.4)
711 FORMAT(* MINIMUM TOTAL COST= *,F11.3)
712 FORMAT(//,* END OF RUN NUMBER: *,I3)
722 FORMAT(2H0,* BEGIN OUTPUT DATA*)
723 FORMAT(T2,I2)
NRUN=0
IZ=2
IPROD=(IZ)**(NFACT)
LIMCN=(IZ)**(NFACT-1)
YLIMCN=FLOAT(LIMCN)
NRUN=NRUN+1
WRITE(6,600)
WRITE(6,601) NRUN
WRITE(6,599) NFACT,NX


WRITE(6,602)
WRITE(6,603) ACOF,BCOF
WRITE(6,613) C(1)
WRITE(6,614) C(2)
WRITE(6,604) N1
WRITE(6.6041) N2
WRITE(6,605) VAR
WRITE(6,606) D
WRITE(6,607)
199 DO 299 I=2,NFACT
299 WRITE(6,608) I, RHOF(I)
198 DO 298 I=1,NX
298 WRITE(6,609) I, RHOX(I)
WRITE(6,610)
197 DO 297 I=2,NFACT
297 WRITE(6,611) I, AFVAR(I), FVAR(I)
BIGM=(10.0)**(6.0)
GTC=BIGM
MINVE=IPROD=NX
MAXVE=(N1+N2)-(IPROD+NX)
EVAR=VAR
201 DO 301 I=2,NFACT
301 EVAR=EVAR*(1.0-(RHOF(I)*(1.0-(AFVAR(I)/FVAR(I)))))
202 DO 302 I=1,NX
302 EVAR=EVAR*(1.0-RHOX(I))
WRITE(6,612) EVAR
WRITE(6,722)
ICOUNT=1
999 CONTINUE
P=1.0-ALPHA
IF(P) 1000,1000,11
11 ACOST=ACOF*ALPHA
WRITE(6,700) ALPHA
WRITE(6,701) ACOST
COFFL=(D**2)/EVAR
```

```
203 DO 303 K=1,2
2031 DO 303 J=2,LIMCN
303 N(K,J)=2
IF(IPARAM .EQ. 3) GO TO 46
WRITE(6,702)
46 TCA=BIGM
888 DO 800 I=MINVE,MAXVE
TCLOW=BIGM
PARM(2)=FLOAT(I)
NOBS=I+IPROD+NX
IF(IPARAM .EQ. 3) GO TO 45
WRITE(6,723) NOBS
45 NDIF=NOBS-(IPROD+N2)
NEQO=NOBS/2
NM1=IPROD
N1LIM=MAXN1(NOBS)
IF(N1LIM .GE. NM1) GO TO 19
N1LIM=N1
19 IF(NDIF) 21,21,20
20 NM1=NM1+NDIF
21 NMLIM=MINO(NEQO,N1,N1LIM)
899 DO 900 IJ=NM1,NMLIM
NN(1)=IJ
NN(2)=NOBS-NN(1)
IF(ICOUNT .GE. 2) GO TO 805
SCOST(NN(1),NN(2))=(NN(1)*C(1))+(NN(2)*C(2))
207 DO 307 K=1,2
2071 DO 3071 J=1,LIMCN
3071 N(K,J)=NN(K)/LIMCN


IMOD=MOD(NN(K),LIMCN)
IF(IMOD) 307,307,208
208 DO 308 J=1,IMOD
308 N(K,J)=N(K,J)+1
307 CONTINUE
SUMXN(1)=0.0
SUMXN(2)=0.0
205 DO 305 K=1,2
DO 3051 J=1,LIMCN
XN(K,J)=FLOAT(N(K,J))
3051 SUMXN(K)=SUMXN(K)+(1.0/XN(K,J))
305 H(K)=YLIMCN/SUMXN(K)
ALAM(NN(1),NN(2))=((COFFL*H(1)*H(2))/(H(1)+H(2)))
VTADJ(NN(1),NN(2))=(1.0+ALAM(NN(1),NN(2)))**2/(1.0+(2.0
+*ALAM(NN(1),NN(2))))
805 CONTINUE
PARM(1)=1.0
FC=PIFDIS(P,PARM,IR)
X=FC/(1.0+ALAM(NN(1),NN(2)))
PARM(1)=VTADJ(NN(1),NN(2))
```

```
      BETA=PFDIST(X,PARM)
      BCOST=BCOF*BETA
      TCOST=FCOST+SCOST(NN(1),NN(2))+ACOST+BCOST
      IF(IPARAM .EQ. 3) GO TO 44
      WRITE(6,704) NN(1),NN(2),SCOST(NN(1),NN(2)),ALAM(NN(1),NN(2)),
     +BETA,BCOST,TCOST
      44 XTCOST(IJ)=TCOST
      IF(IJ-(NM1+1)) 24,24,23
      23 TC1=XTCOST(IJ-2)
      TC2=XTCOST(IJ-1)
      TC3=XTCOST(IJ)
      IF(TC2 .LE. TC1 .AND. TC2 .LE. TC3) GO TO 901
      24 IF(TCOST .GE. TCLOW) GO TO 900
      SCMIN(NOBS)=SCOST(NN(1),NN(2))
      XLAMIN(NOBS)= ALAM(NN(1),NN(2))
      TCMIN(NOBS)=TCOST
      BMIN(NOBS)=BETA
      BCMIN(NOBS)=BCOST
      N1MIN(NOBS)=NN(1)
      N2MIN(NOBS)=NN(2)
      TCLOW=TCMIN(NOBS)
      900 CONTINUE
      GO TO 902
      901 MAXN1(NOBS)=IJ-1
      902 ATC=TCMIN(NOBS)
     *IF(TCA-ATC) 801,800,25
      801 AC1=TCMIN(NOBS-2)
      AC2=TCMIN(NOBS-1)
      AC3=TCMIN(NOBS)
      IF(AC2 .LE. AC1 .AND. AC2 .LE. AC3) GO TO 802
      25 TCA=ATC
      N1A=N1MIN(NOBS)
      N2A=N2MIN(NOBS)
      NTA=NOBS
      SCA=SCMIN(NOBS)
      BA=BMIN(NOBS)
      BCA=BCMIN(NOBS)
      XLA=XLAMIN(NOBS)
      800 CONTINUE
      802 IF(IPARAM .EQ. 3) GO TO 47
      WRITE(6,708) ALPHA
      47 WRITE(6,702)
      WRITE(6,723) NTA
      WRITE(6,704) N1A,N2A,SCA,XLA,BA,BCA,TCA
      IF(IPARAM .EQ. 2) GO TO 1005
    **IF(GTC-TCA) 1000,50,26
      26 GTC=TCA


      GALPHA=ALPHA
      50 ALPHA=ALPHA+CI
```

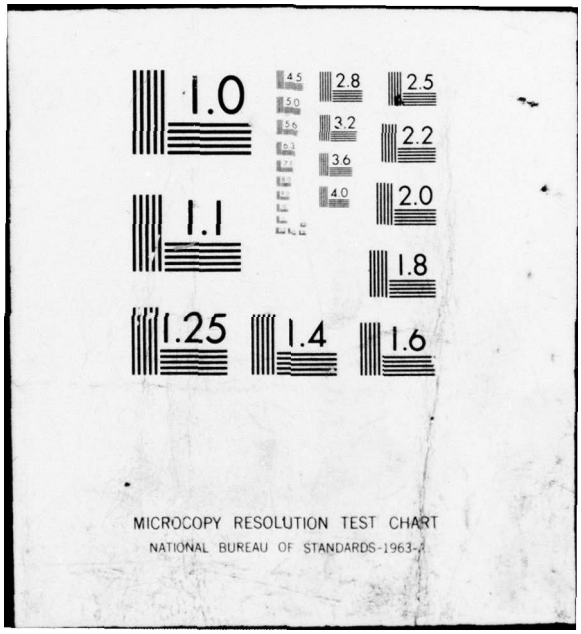
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4.5
5.0
5.6
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
      ICOUNT=ICOUNT+1
      GO TO 999
 1000 CONTINUE
      WRITE(6,710) GALPHA
      WRITE(6,711) GTC
      WRITE(6,712) NRUN
 1005 STOP
      END
```

*Changing 801 to 800 in this statement will cause the program to evaluate and print the appropriate output for all values of (N/α) rather than stopping one increment beyond the local minimum $(\bar{n}_1, \bar{n}_2, N/\alpha)$.

**Changing 1000 to 50 in this statement will cause the program to evaluate and print the appropriate output for all values of α rather than stopping one increment beyond the optimal $(\bar{n}_1, \bar{n}_2, \alpha)$ value.

APPENDIX B

This appendix contains an example illustrating the output options which are available from the EASC program, COSTOPT, by specifying the value of the input parameter, PARAM, as described in Appendix A. Unless otherwise indicated on the figures themselves, the input data listed for this example was used to obtain the data for all of the figures contained in Chapter III, with the exception of Figure 4.

```
 RUN NUMBER-     1


NUM OF FACTORS=  3
NUM OF COV=  1
PARAMETER VALUES ARE-
ALPHA COEFF=    500.000
BETA COEFF=     70.000
SAMPLE COSTS:
                TESTED ITEM=  1.000
                STANDARD ITEM=    .500
SAMPLE SIZE CONSTRAINTS:
                          TESTED ITEM= 12
                          STANDARD ITEM=  20
VAR FOR RANDOM DESIGN=    2.00
REQ PERFORMANCE MARGIN=     .10
CORRELATION COEFFICIENTS:
                           FACTOR   2= .70
                           FACTOR   3= .60
                           COVAR    1= .70
          AVG VAR ABOUT FIXED LEVEL              POP VAR
FACTOR  2              2.000                        20.000
FACTOR  3              2.000                        10.000
DESIGN ERROR VAR=      .1154   (This value is calculated within
                                the program)

. BEGIN OUTPUT DATA


ALPHA=  .0100
ALPHA COST=       5.000

NO OBS
ROW1 ROW2    SCOST    LAMBDA    BETA     RCOST              TCOST

16
  8    9      12.000    3.347    .8781       61.469             79.469
17
  8    9      12.500    3.362    .9687       60.906             79.306
18
  8   10      13.000    3.378    .9605       60.234             79.234
  9    9      13.500    3.378    .9605       60.234             79.734
19
  8   11      13.500    3.396    .8533       59.731             79.231
  9   10      14.000    3.396    .8533       59.731             79.731
20
  8   12      14.000    3.416    .8469       59.281             79.281
  9   11      14.500    3.416    .8469       59.281             79.781
 10   10      15.000    3.416    .8469       59.281             80.281
21
```

8	13	14.500	3.426	.9416	58.914	79.414
9	12	15.000	3.438	.9410	58.870	79.370
10	11	15.500	3.438	.9410	58.870	80.370
22						
8	14	15.000	3.438	.9369	58.585	79.585
9	13	15.500	3.450	.9362	58.537	80.037
10	12	16.000	3.462	.9355	58.487	80.487
11	11	16.500	3.462	.9355	58.487	80.387
23						
8	15	15.500	3.450	.9327	58.286	79.786
9	14	16.000	3.462	.9319	58.235	80.235
10	13	16.500	3.475	.9311	58.190	80.690
11	12	17.000	3.488	.9303	58.123	81.123
24						
8	16	16.000	3.462	.9287	58.011	80.011
9	15	16.500	3.475	.9279	57.956	80.456
10	14	17.000	3.489	.9271	57.897	80.897
11	13	17.500	3.504	.9262	57.835	81.335
12	12	18.000	3.520	.9253	57.769	81.769
25						
8	17	16.500	3.476	.9254	57.780	80.280
9	16	17.000	3.489	.9242	57.697	80.697
10	15	17.500	3.504	.9233	57.633	81.133
11	14	18.000	3.520	.9224	57.566	81.566
12	13	18.500	3.537	.9213	57.494	81.994
26						
8	18	17.000	3.478	.9224	57.566	80.566
9	17	17.500	3.499	.9211	57.479	80.979
10	16	18.000	3.520	.9198	57.385	81.385
11	15	18.500	3.537	.9187	57.311	81.811
12	14	19.000	3.554	.9176	57.233	82.233
27						
8	19	17.500	3.496	.9196	57.369	80.869
9	18	18.000	3.507	.9182	57.277	81.277
10	17	18.500	3.530	.9168	57.177	81.677
11	16	19.000	3.554	.9153	57.068	82.068
12	15	19.500	3.574	.9140	56.983	82.483
28						
8	20	18.000	3.495	.9169	57.185	81.185
9	19	18.500	3.517	.9155	57.088	81.588
10	18	19.000	3.540	.9140	56.983	81.983
11	17	19.500	3.566	.9124	56.868	82.368
12	16	20.000	3.594	.9106	56.741	82.741
29						
9	20	19.000	3.526	.9130	56.910	81.910
10	19	19.500	3.551	.9114	56.800	82.300
11	18	20.000	3.578	.9097	56.679	82.679
12	17	20.500	3.607	.9078	56.544	83.044
30						

```
 10    20      20.000    3.562    .8049       56.625          82.625
 11    19      20.500    3.590    .8071       56.497          82.997
 12    18      21.000    3.621    .8051       56.355          83.355
31
 11    20      21.000    3.603    .8046       56.322          83.322
 12    19      21.500    3.635    .8025       55.173          83.673
32
 12    20      22.000    3.650    .7999       55.995          83.995

FOR ALPHA=  .0100  OPTIMAL VALUES ARE:

NO OBS
ROW1 ROW2    SCOST    LAMBDA    BETA     BCOST           TCOST

19
  8   11     13.500    3.396    .8533       59.731          79.231
```

The above represents the output format for PARAM = 2 PARAM = 1 produces the same output as above for all values of significance level considered, and in addition includes the following at the end of each run:

```
OVERALL OPTIMAL ALPHA=  .0100
MINIMUM TOTAL COST=      79.231

END OF RUN NUMBER:   1
```

PARAM = 3 produces only the optimal value for each value of significance level considered and the overall optimum.

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