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STRAIN RATE INFLUENCES ON SHOCK WAVE PROPAGATION IN METALS.(U)

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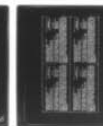
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STRAIN RATE INFLUENCES ON SHOCK WAVE
PROPAGATION IN METALS

NOVEMBER 1977

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FOREWORD

This research has been conducted by the Division of Applied Mechanics, Department of Mechanical Engineering, Stanford University, Stanford, California under Contract No. DAAG46-74-C-0057 from the Army Materials and Mechanics Research Center, Watertown, Massachusetts. Mr. J. F. Dignam of the AMMRC was project manager and Dr. S.C. Chou of the AMMRC served as technical monitor.

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1. Introduction

The paper on shock waves in elastic-plastic solids [1]* by Germain and the author of this report, is concerned in the main with the application of a generalization of classical rate-independent constitutive relations for elastic-plastic media to the analysis of the propagation of plane compressive waves. Waves exhibiting both continuous and discontinuous stress profiles were considered. Since the generation of shock waves is intimately associated with non-linear effects, the generalization of classical theory adopted incorporated finite deformation kinematics. It was found possible to formulate the theory so that the mean hydrostatic pressure component of the stress tensor and the deviator or distortional components appeared in separate terms as did the dilatation and the strain deviator. Thus the extensive literature on pressure-volume relations developed in connection with the hydrodynamic theory of shock waves in metals could be utilized directly for the analysis of wave propagation in metals when shear strength also influences the motion. The separation of pressure and shear effects was carried out on the basis of the structure of the free energy function for thermo-elastic-plastic deformation in a physically meaningful manner which also satisfied the invariance requirements of nonlinear continuum mechanics. For example, the latter links the influences of density change on direct stress and on shear stress components which follows from the structure of finite deformation thermo-elastic theory.

In the analysis of shock waves involving plastic flow using rate independent theory [1], it was found that a shock wave solution could not be determined on this basis when work hardening was considered or when part of the plastic work expended is transformed into internal energy associated with breakdown of the

* Numbers in square brackets refer to the references listed at the end of the report.

crystal lattice by the generation of dislocations. This component of energy absorption by the metal has a negligible influence on the thermo-elastic state of the element and so does not increase the entropy, in contrast to the rest of the plastic work which is dissipated into heat. Thus knowledge of the work done on the material during passage of a shock wave does not determine the new internal energy in the thermo-elastic system since part is absorbed in forming the dislocation distribution. This prevents direct evaluation of the state after passage of the wave. Similarly the work absorbed in plastic flow which determines the yield stress of the work-hardening metal after passage of the shock wave can only be calculated from a study of the shock structure which calls for a rate type plasticity law to correctly incorporate the energy dissipated in the shock wave. Plastic work is expressed by a time integral of a product of the yield stress and the plastic strain rate and cannot be determined directly in terms of variables only expressing properties before and after passage of the wave.

In view of these difficulties, a hypothetical rate-dependent plasticity law was postulated which permitted the development of a theory which would determine the shock structure for a steady wave and hence the variations of strain, temperature, plastic strain and plastic work with position through the wave and hence the thermo-elastic state of the material after passage of the wave. Moreover it was shown that the choice of rate-law for the plastic strain-rate did not influence the sequence of values taken on by these dependent variables but only their profiles in space. The plots of any three of the dependent variables e.g. temperature, plastic strain and plastic work against the other, i.e. the strain, would be independent of the rate law. In spite of this, it is

worthwhile to consider experimentally motivated rate-laws so that actual wave profiles can be evaluated. The results mentioned concerning the deduction of partial information about the solution, without knowledge of the precise rate law, is a generalization of the Rayleigh line concept for steady waves in materials defined by simpler constitutive relations. For example, in the theory of shock waves in gases, or equivalently hydrodynamic analysis, the knowledge of the work done on the material due to passage of a shock is sufficient to determine the final thermodynamic state of the material. Essentially because of the additional internal variable: plastic strain, the more complicated situation already described arises in plastic analysis.

In the next section the formulation of thermo-elastic-plastic constitutive relations for metals incorporating a rate of plastic strain influence will be discussed, and compared with the rate-independent form. Subsequently the application of these for analyzing plastic wave propagation phenomena will be considered. In discussion of the influence of rate-dependent analysis, other characteristics which generate analogous effects on wave profiles will be considered.

2. Finite Deformation Thermo-Elastic-Plastic Theory

The constitutive relations to be applied comprise a generalization of the finite deformation theory described in reference [2]. Because non-linear effects exert a major influence on shock wave generation and propagation, it is important to have a correctly formulated constitutive relation which incorporates both geometric and material nonlinearities. The kinematics [2] is based on the matrix product representation of the combined effect of elastic and plastic deformation:

$$\underline{F} = \underline{F}^e \underline{F}^p \quad (1)$$

where F_{ij} is the deformation gradient matrix $\partial x_i / \partial X_j$ where x_i is the position vector of an element in the deformed configuration and X_i for the element in its reference configuration. F^P is the deformation gradient matrix for plastic deformation relative to the initial undeformed state and F^e the elastic deformation gradient relative to a state unstressed after plastic deformation. At the current time t , $F = F(X, t)$ since in general the deformation of a body will be distributed non-homogeneously.

For the study of the propagation of plane waves of one-dimensional strain as generated in a plate slap experiment, principal directions remain fixed in space and in the body normal to and in the plane of the wave front. Thus the principal components only of F need to be introduced:

$$F = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{pmatrix} \quad (2)$$

Components λ_i are the stretch ratios of the length in the direction i after and before the deformation, and $\lambda_3 \equiv \lambda_2$ for isotropic media because of symmetry in the plane of the wave. Then taking natural logarithms of the total, elastic and plastic stretch ratios determines the natural strains:

$$\epsilon_i = \ln(\lambda_i), \quad \epsilon_i^e = \ln(\lambda_i^e), \quad \epsilon_i^p = \ln(\lambda_i^p) \quad (3)$$

and the relation (1) for total deformation reduces to additivity of the elastic and plastic strain components

$$\epsilon_i = \epsilon_i^e + \epsilon_i^p \quad (4)$$

as discussed in [3] and [1]. For plane wave analysis the strains will in general be functions of time t and of position along an axis normal to the wave-front. The latter could be the Lagrange coordinate X_1 in the reference geometry or the Euler coordinate x_1 in the current configuration.

The stress σ and entropy s are given in terms of the density ρ , the elastic deformation F^e and temperature θ through the Helmholtz free energy function Ψ by the thermo-elastic relations:

$$\sigma = \rho F^e \partial \Psi / \partial F^e \quad (5)$$

$$s = - \partial \Psi / \partial \theta \quad (6)$$

As discussed in [4] and [5] the free energy function is taken to have the form

$$\Psi = \bar{\Psi}(F^e, \theta) + \hat{\Psi}(w) \quad (7)$$

where w is the plastic work per unit mass. The first term on the right hand side expresses the thermo-elastic response and the second term the energy stored in the dislocation distribution resulting from plastic flow. The latter has little influence on thermo-elasticity as confirmed by the insensitivity of elastic constants to plastic flow. The free energy component $\hat{\Psi}$ constitutes only a small part of the plastic work w

$$\hat{\Psi} = (1-\gamma) w, \quad \gamma \approx 0.9 \quad (8)$$

the rest being dissipated into heat.

The thermo-elastic component of the free energy is taken to have the form

$$\bar{\Psi}(\epsilon_1^e, \epsilon_2^e, \epsilon_3^e, \theta) = f(I_1, \theta) + h(J_2, J_3) \quad (9)$$

where I_1 is the first invariant of the elastic strain tensor:

$$I_1 = \epsilon_1^e + \epsilon_2^e + \epsilon_3^e \quad (10)$$

and J_2 and J_3 are the second and third invariants of the elastic strain deviator tensor e^e

$$J_2 = \frac{1}{2} [(\epsilon_1^e)^2 + (\epsilon_2^e)^2 + (\epsilon_3^e)^2] \quad (11)$$

$$J_3 = \frac{1}{3} [(\epsilon_1^e)^3 + (\epsilon_2^e)^3 + (\epsilon_3^e)^3] \quad (12)$$

Since I_1 is equal to the volumetric strain, plastic flow being incompressible, $f(I_1, \theta)$ prescribes the pressure, volume, temperature relation of the metal. Under general loading the pressure p is defined as $-\sigma_{11}/3$, the average of the normal compressive stress components, and the same dilatation law arises. The function $h(J_2, J_3)$ prescribes the deviator or distortional components of the stress and has been further simplified to $2\mu J_2/\rho_0$ in the analysis. The temperature θ is not included in this term since shear strain does not usually stimulate thermo-elastic coupling.

In plane wave experiments, prior to the arrival of release waves from the free lateral surface of the specimen, the lateral strains are zero:

$$\epsilon_2 = \epsilon_3 = 0 \quad (13)$$

and since plastic deformation is incompressible

$$\epsilon_1 = I_1 = \ln(\rho_0/\rho) \quad (14)$$

where ρ_0 is the density in the reference state.

For this special deformation the stress components can be expressed in terms of the longitudinal total and plastic strain components, ϵ_1 and ϵ_1^p respectively, by using (4) and (13) to obtain elastic strains and then applying (5) [see [1], eq.(44)]

$$\sigma_1 = -p + \frac{4\mu}{3} \frac{\rho}{\rho_0} \epsilon_1 - \frac{2\mu\rho}{\rho_0} \epsilon_1^p \quad (15)$$

$$\sigma_2 = \sigma_3 = -p - \frac{2\mu}{3} \frac{\rho}{\rho_0} \epsilon_1 + \frac{\mu\rho}{\rho_0} \epsilon_1^p \quad (16)$$

The pressure p is given by the hydrodynamic relation already mentioned and prescribed by the function $f(I_1, \theta)$. It is often convenient to express the pressure - temperature - dilatation relation in the form

$$p(s, \epsilon_1), \theta(s, \epsilon_1) \quad (17)$$

since s can be a convenient independent variable for adiabatic loading

Equations(15) and (16) indicated that the separation of dilatation and deviator terms in (9) does produce a coupling of the deviator stress components with volume change according to the factor ρ/ρ_0 .

We note that in addition to the total longitudinal strain ϵ_1 , the plastic longitudinal strain component ϵ_1^p acts as an additional state variable which permits determination of the stress for prescribed temperature or entropy. This is so without any statement concerning the laws of plasticity.

The rate independent theory of plasticity described in detail in [2] and [1] adjoins a work-hardening yield condition which must be satisfied for plastic flow to take place, in the form that an isotropic function of the deviator of $\rho^{-1}\sigma$ is prescribed by the plastic work w and the temperature θ . For our particular problem this condition is contained in the expression:

$$Y(w, \theta) \pm \rho_0 \rho^{-1} (\sigma_1 - \sigma_2) \geq 0 \quad (18)$$

where Y is the yield stress in tension or compression at the initial density and temperature θ ; the upper sign corresponds to longitudinal tensile yield and the lower to compressive. The equality sign permits plastic flow to take place and the inequality indicates that the element is inside the yield surface and hence currently subject only to elastic strain increments. When plastic flow takes place the plastic work per unit mass is determined from:

$$\dot{w} = \rho^{-1} (\sigma_1 - \sigma_2) \dot{\epsilon}_1^p \quad (19)$$

or, using (18)

$$\rho_0 \dot{w} = \pm Y(w, \theta) \dot{\epsilon}_1^p \quad (20)$$

with the same sign convention.

The relations prescribed are appropriate, for example, to determine the stresses generated by a continuous adiabatic process defined by a given variation of $\epsilon_1(t)$ from a prescribed state. If the element is initially subjected to stress inside the yield surface, so that the inequality in (18) is satisfied, only elastic increments of strain can occur so that

$$\dot{\epsilon}_1^p = \dot{w} = \dot{s} = 0 \quad (21)$$

and (15) and (16) with p determined by (17) give the stress variation since ϵ_1^p is known for the initially prescribed state. When plastic flow takes place the equality (18) with (15) and (16) yield

$$3\epsilon_1^P/2 = \epsilon_1 + Y/(2\mu) \quad (22)$$

The part $\dot{\gamma}\dot{w}$ of the plastic power which is dissipated into heat causes an entropy increase according to

$$\dot{\theta} \dot{s} = \dot{\gamma} \dot{w} \quad (23)$$

Combining this with (20) and changing the independent variable from t to ϵ_1 yields the differential equation system

$$\rho_0 \dot{\theta} \frac{ds}{d\epsilon_1} = \rho_0 \dot{\gamma} \frac{dw}{d\epsilon_1} = \pm Y \dot{\gamma} \frac{d\epsilon_1^P}{d\epsilon_1} \quad (24)$$

The variable ϵ_1^P can be eliminated using (22), and utilizing the known functions $\theta(s, \epsilon_1)$ from (17) and $Y(w, \theta)$ gives a pair of differential equations to determine $w(\epsilon_1)$ and $s(\epsilon_1)$ using the known initial state to provide initial conditions. The plastic strain component ϵ_1^P is given by (22), $Y(w, \theta)$ being obtained using $\theta(s, \epsilon_1)$, and then the stresses can be evaluated from (15) and (16). Note that the time variable does not appear explicitly in the solution for either elastic or elastic-plastic deformation, in conformity with the adoption of the rate independent plasticity constitutive relation. It might, for example, be convenient to utilize the variable ϵ_1 as a pseudo "time" in all rate relations considered so far in this section, for example equations (19) and (20), and evaluate the stresses without introducing real time t .

Consider now a modification of the hypothetical rate dependent plasticity law of [1] to bring it into conformity with experimental findings. Equations (15) and (16) show that the stress can be determined in terms of

total and plastic longitudinal strain components. These are deduced from the thermo-elastic stress law (5) after expressing the elastic strain components in terms of ϵ_1 and ϵ_1^P by means of (4), (13) and the incompressibility of plastic flow. These stresses can be substituted into the yield condition and a rate of plastic strain prescribed in terms of the extent to which the yield condition has been violated. If the stress point lies inside or on the yield surface the rate of growth of plastic strain will be zero.

Whatever the law of plasticity (15) and (16) yield the relation:

$$\epsilon_1 - \frac{3}{2} \epsilon_1^P = \frac{\rho_0}{\rho} \frac{\sigma_1 - \sigma_2}{2\mu} \quad (25)$$

so that the yield condition (18) can be written:

$$Y(w, \theta) + 2\mu(\epsilon_1 - \frac{3}{2} \epsilon_1^P) \geq 0 \quad (26)$$

Thus a convenient means of expressing over-stress is through the variable z , defined by

$$z = (\epsilon_1 - \frac{3}{2} \epsilon_1^P)^2 - \frac{Y^2}{4\mu^2} \quad (27)$$

This will be positive when the yield condition is violated and negative if the stress point lies inside the yield surface. Thus an over-stress condition of the Malvern type can take the form

$$\dot{\epsilon}_1^P = (\epsilon_1 - \frac{3}{2} \epsilon_1^P) k(z) \quad (28)$$

where k is an appropriate function. The factor $(\epsilon_1 - \frac{3}{2} \epsilon_1^P)$ is introduced to ensure the correct sign of the deduced plastic strain rate. The

yield stress function $Y(w, \theta)$ now has a changed significance since at yield the plastic strain-rate will be zero, and the yield stress must be exceeded for non-zero plastic strain-rate to occur. Thus k is a monotonically increasing function with $k(0) = 0$.

Using (25) the rate of plastic work is now given by

$$\dot{w} = \frac{2\mu}{\rho_0} \left(\epsilon_1 - \frac{3}{2} \epsilon_1^p \right) \dot{\epsilon}_1^p \quad (29)$$

The adiabatic entropy growth relation (23) can still be expected to apply, and (28), (29) and (23) combined with the thermo-elastic relation specify the corresponding thermo-elastic-plastic constitutive relation.

If a prescribed variation $\epsilon_1(t)$ from a known state is imposed under adiabatic conditions, (23), (28) and (29) constitute a system of three differential equations to determine w , ϵ_1^p and s , when use is made of the known function $\theta(s, \epsilon_1)$ from (17).

The introduction of a strain-rate effect in combination with a work hardening law introduces "a strain-rate history effect" of the type demonstrated by Klepaczko [6] and mentioned by Clifton [7] (p.103) in his article on plastic waves. Thus, for a given plastic strain increment, more plastic work is expended if the straining is carried out at higher strain-rate since the stress is higher according to (28), (27) and (25). If straining is first carried out with an initial strain-rate, $\dot{\epsilon}_1$, until a prescribed strain is achieved, and thereafter the strain-rate is changed to $\dot{\epsilon}_f$; for a particular strain during the second part of the stepped strain-rate test, the stress would be higher had the initial strain-rate $\dot{\epsilon}_1$ been bigger, since the plastic work w and hence the equilibrium yield stress $Y(w, \theta)$ would be larger. Qualitatively, such behavior has

been observed by Klepaczko [6]. As discussed in the following section, such a strain-rate history dependent type law is incorporated into the analysis of shock wave structure presented in [1].

By taking the yield stress Y to depend on ϵ_1^p , instead on w , which in the case considered in this report of longitudinal monotonic straining is equivalent to the commonly used generalized plastic strain $\bar{\epsilon}^p = \sqrt{\frac{2}{3}} (\int (\dot{\epsilon}_1^p \dot{\epsilon}_1^p)^{\frac{1}{2}} dt)$ (see Hill [8] p. 30), the appearance of a rate history effect is avoided, since then the rate at which the stress history is carried out does not affect the equilibrium yield stress Y . As described in the following section this change does not greatly modify the analysis of straining in wave propagation based exactly on the elastic-plastic theory presented, although a good approximation is more easily obtained. A somewhat similar plastic strain-rate and total plastic-strain dependent thermo-elastic-plastic constitutive relation has been discussed by Clifton [7](p.131 ff.). This also contained a vector of parameters which characterized the internal structure of the material. The plastic work w could be such an internal variable and the work-hardening theory can be considered to be analogous to such a law.

3. Wave Propagation Phenomena

As mentioned in the Introduction, certain difficulties in obtaining shock wave solutions for strain-rate independent elastic-plastic materials were encountered for work-hardening plasticity laws. Related rate-dependent laws were therefore studied which result in dual shock structure of an elastic wave exhibiting discontinuous properties attached to a steady wave associated with the development of plastic flow. This corresponds to an over-driven wave forced to travel at a speed higher than elastic wave velocity (see [7], p. 148 or Herrmann [9], p. 10). It is not uncommon to refer to the combination as a

shock-wave, terming the variations of properties in the steady wave the study of shock structure. We shall term these two components the shock wave and the steady wave with invariant profile.

A number of peripheral questions arise in the theoretical analysis of the phenomenon and in connection with experimental measurements. For example it was commonly the practice to analyze steady waves as having an invariant profile for measurements taken at a fixed point in space, that is to measure, say, the velocity $v(x_1, t)$ for varying t and fixed $x = x_1$. The Euler coordinate x_1 denotes a fixed point in space as the material moves across it. It is now realized that measuring devices are commonly fixed to a material element and hence yield the data $v(X_1, t)$ where the Lagrange coordinate X_1 defines a position in the reference configuration, that is at a particular material element. It seems plausible that a wave which has an invariant profile at each point in space would also have one at each point in the body. However, since the constitutive relation for elastic plastic media is highly non-linear, it seems unlikely, without further consideration, that the transformation from one coordinate to the other could maintain the invariance.

The motion along the axis x of the cross-section labelled X in the reference configuration in Fig. 1 is given by

$$x = x(X, t) \quad (30)$$

where the subscript to the coordinates has been dropped for one-dimensional waves. The stretch ratio λ is given by

$$\lambda = \frac{\partial x}{\partial X} \quad (31)$$

Hence

$$\frac{\partial \lambda}{\partial t} = \frac{\partial^2 x}{\partial t \partial X} = \frac{\partial v}{\partial X} \quad (32)$$

where v is the particle velocity

$$v = \frac{\partial x}{\partial t} \quad (33)$$

If there is a steady wave in the Lagrange frame which travels without change of profile when considered plotted in the reference configuration, then dependent variables σ, v and λ are functions of $(X - C_L t)$ only, when C_L is a constant - the wave speed in the Lagrange sense, i.e. along the reference configuration. Thus

$$\lambda = \lambda(X - C_L t), \quad v = v(X - C_L t) \quad (34)$$

Relation (32) then takes the form

$$- C_L \lambda' = v' \quad (35)$$

where the prime denotes differentiation with respect to the single argument.

Integrating (35) gives

$$v + C_L \lambda = \text{constant} \quad (36)$$

Now, corresponding to propagation velocity C_L along the reference frame, the propagation velocity along the deformed body (i.e. in space) is

$$C_L \lambda + v \quad (37)$$

since travel dX in the reference state is travel relative to the deformed material through the distance λdX , the material meanwhile moving with velocity v . This is the Euler wave speed

$$C_E = C_L \lambda + v \quad (38)$$

and by (36) is therefore constant. Since constant values of λ, v and σ propagate with velocity C_L in the reference frame with coordinate X , they travel with C_E in space, so that

$$\lambda = \lambda(x - C_E t), \dots \text{etc.} \quad (39)$$

Thus (34) corresponds to a steady wave in space (Euler coordinates).

Note that for a steady wave traveling into an undisturbed body (e.g. as discussed in [7] p. 148 and [9] p. 10) $\lambda = 1$ and $v = 0$ at the wave front so that $C_E = C_L$ by (38). Of course the profiles of stress σ , velocity v or stretch λ are not the same expressed as functions of X and x . Since, for a steady wave to develop, it must be propagating into material homogeneously stressed and at uniform velocity, if axes are chosen which bring the material ahead of the wave to rest, and if material uniformly

strained as that ahead of the wave is used as the reference state, the Euler and Lagrange wave speeds will be the same for all cases.

To extend the consideration slightly, following Clifton's development [7] p. 149), the equation of motion in the reference frame is

$$\frac{\partial \sigma}{\partial X} = \rho_0 \frac{\partial v}{\partial t} \quad (40)$$

Since there is no lateral motion in one-dimensional strain situations, cross-sectional areas are identical in the reference and current configurations so that the nominal stress in (40) is equal to the usual Cauchy or "true" stress in the deformed body. For a steady wave with velocity C_L in the reference frame; (40) becomes

$$\sigma' = - \rho_0 C_L v' \quad (41)$$

which combined with (35) gives

$$\sigma' = \rho_0 C_L^2 \lambda' \quad (42)$$

which integrated is:

$$\sigma = \rho_0 C_L^2 \lambda + \text{constant} \quad (43)$$

Thus the stress and stretch ratio are linearly related in a steady wave, this relationship commonly being called the Rayleigh line. Thus in spite of the non-linear irreversible constitutive relation governing rate dependent elastic-plastic response at finite strain, the constraint of a

steady wave forces such simple connection between the variables. It is important to observe that these results depend only on the compatibility of the kinematic variables (32) and the equation of motion (40) and their form does not depend on the constitutive relation of the material.

It is thus clear that these results follow directly from the conservation laws as, for example, developed in [1] Section 2. The discussion of this aspect is related to the analogous studies by Duvall [10] and Johnson and Barker [11]. For the x -axis chosen so that the wave is propagating in the direction of increasing x into material at rest, this coordinate being denoted by x , let $v(x,t)$ be the particle velocity. The relation for conservation of mass for a wave of one-dimensional strain is ([1], section 2):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0 \quad (44)$$

If the Euler wave velocity, C_E , reversed, thus now directed towards decreasing x , is superposed on the whole configuration, the modified x coordinate will be:

$$\bar{x} = x - C_E t \quad (45)$$

and the particle velocity relative to this coordinate system will be:

$$\bar{v} = v - C_E \quad (46)$$

The steady wave will be transformed into a stationary wave, so that

$$\bar{v} = \bar{v}(\bar{x}) = v(x,t) - c_E \quad (47)$$

The other dependent variables ρ, σ and λ will not change but will become functions of $\bar{x}$ only for the stationary wave:

$$\rho(x,t) = \rho(\bar{x}), \sigma(x,t) = \sigma(\bar{x}), \lambda(x,t) = \lambda(\bar{x}) \quad (48)$$

The equation for conservation of mass will now be

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \bar{x}} (\rho \bar{v}) = 0 \quad (49)$$

which, since $\rho = \rho(\bar{x})$, reduces to

$$\frac{\partial}{\partial \bar{x}} (\rho \bar{v}) = 0 \text{ or } \rho \bar{v} = m \quad (50)$$

where m is a constant of the motion. Since ahead of the wave $\bar{v} = -c_E$ and $\rho = \rho_0$, the density of the undisturbed material,

$$\rho \bar{v} = m = -\rho_0 c_E \quad (51)$$

Note that this integral of the continuous motion in the steady wave takes the same form as the jump condition across a surface of discontinuities in the dependent variables (see [1], Section 2). The reason for this is that between any two cross-sections which propagate with the wave, an invariant distribution of density, velocity, stress and stretch is maintained so that no contributions to the conservation relations are introduced by the material between the sections.

An exactly similar deduction applies for conservation of momentum and energy, leading to the integrals of the motion; identical to the jump relations ([1] Sections 2 and 8)

$$-\sigma + \rho \bar{v}^2 = P \quad (52)$$

and

$$e + \bar{v}^2/2 - \sigma v/m = E \quad (53)$$

where e is the specific internal energy, and P and E are constants. In (53) adiabatic loading is considered as commonly acceptable for the analysis of wave propagation. Since no lateral strain occurs in plane waves, mass conservation demands

$$\rho_0 = \rho \lambda, \text{ or } \lambda = \rho_0/\rho \quad (54)$$

and (51) yields

$$\bar{v} + \lambda C_E = 0 \quad (55)$$

for all $\bar{x}$. Thus

$$[\bar{v}] + [\lambda] C_E = 0 \quad (56)$$

where $[]$ denotes the difference between the values of the enclosed variable at any two cross-sections propagating with the wave. Now since

$$[\bar{v}] \equiv [v] \quad (57)$$

because the two velocity variables differ only by the superposed $-C_E$,

(56) becomes, for arbitrary choice of axes in the direction of wave propagation:

$$[v] = - [\lambda] C_E \quad (58)$$

which is equivalent to (38) when $C_L = C_E$ as has been established for the case when the material ahead of the wave is at rest.

In a similar way (52) with (51) can be written

$$-\sigma - \rho_0 C_E \bar{v} = P$$

hence

$$[\sigma] + \rho_0 C_E [v] = 0 \quad (59)$$

and using (58)

$$[\sigma] = \rho_0 C_E^2 [\lambda] \quad (60)$$

which is equivalent to the Rayleigh line expression (43). A similar manipulation on the energy balance equation (53), using the relation ([1], eq. (9))

$$[ab] = \bar{a} [b] + [a] \bar{b} \quad (61)$$

where the superposed bar indicates the arithmetic mean of the enclosed variable values, gives the well known energy relation:

$$\rho_0 [e] = \bar{\sigma} [\lambda] = \frac{\sigma_1 + \sigma_2}{2} (\lambda_2 - \lambda_1) \quad (62)$$

Note that if values of the stresses and stretch ratios are known before and after passage of the wave, the work done in deforming the material and the increase in internal energy is given by (62) without introducing additional information about the constitutive law of the material.

In view of the remarkably confining constraints which the existence of a steady wave imposes on the dependent variables: stress σ , velocity v ,

and stretch ratio λ , one may wonder whether such waves are produced in practice. That a close approximation to such a configuration is achieved in plate slap impact experiments has been demonstrated experimentally by Johnson and Barker [11]. Duvall [12], (p. 102), gave a qualitative description of the stability of a steady wave in terms of the distortion of the Rayleigh line and the influence introduced by this towards re-establishing the steady configuration. Bland [13] investigated analytically the development of a steady wave for a Kelvin type viscoelastic solid with a nonlinear equilibrium response. He showed by asymptotic methods that any monotonically increasing loading pulse would tend towards a steady wave configuration, and that for a step pulse, the time of formation is of the order of five times the shock thickness divided by the excess convective velocity. Particular cases of elastic-plastic waves were evaluated by Clifton [14] who demonstrated the generation of steady waves. The matter has been reviewed recently by Herrmann [9] and can clearly play an important role in the investigation of rate effects, particularly at the higher end of the range.

The fact that the work done on the material as a shock wave traverses it, (62), is not based on the constitutive equation of the material demands that the equation adopted, or equivalently the physical mechanism envisaged, must be flexible enough to permit the required absorption of energy. Part of the work is expended in producing pure volume compression which is reversible on pressure release, the rest is associated with distortion and involves both elastic and plastic deformation. The latter is irreversible and involves dissipation of mechanical work into heat according to (23). A rate-dependent or visco-plastic law incorporates the required flexibility to absorb mechanical

energy since an increase in the rate of strain requires the stress driving it to increase and hence also the absorption of work for a given strain increment. Since the shock-wave propagation speed for a material is insensitive to changes in loading and temperature, for an approximately constant wave amplitude the strain rate increases as the wave becomes thinner, and such a change in configuration can accommodate the energy absorption requirement. If the coefficient in the strain-rate dependent expression for stress is large, only moderate thinning will be required to provide a certain energy absorption, but a smaller coefficient in the strain-rate term will call for a narrower shock wave profile, and hence a higher strain rate. A simple means of evaluating this situation for a simpler elastic-plastic law than that considered in [1] has been presented by Kelly and Gillis [15].

The rate independent law suggested in [1] to permit the analysis of elastic-plastic shock waves in a work hardening material, was to replace the plastic work generation relation (20) by an incremental relation across the shock:

$$\rho[w] = \bar{Y}[\pm \epsilon_1^p] \quad (63)$$

where, as in Section 2, the square brackets indicate the jump in the enclosed variable across the shock, and the bar, the average of the values of the variable on the two sides of the shock. It does not appear that the plastic work defined in this way could be made consistent with the total work defined by (62), since there is no mechanism to increase the flow stress above the static yield to achieve the required energy absorption and to meet the constraint of the Rayleigh line relation. The fact that plastic flow must provide the mechanism for the dissipation of energy was discussed in Section 8 of [1].

The steady wave analysis developed in that Section, based on a strain-rate dependent work-hardening plasticity law, includes full consideration of thermo-mechanical coupling according to the elastic-plastic relations described in Section 2 of the present report. Although implemented in [1] for the hypothetical rate law which permits some plastic flow below the static yield, it applies for general rate laws which incorporate a strict static yield stress.

Since steady waves which propagate into material at rest are normally considered, we denote the wave speed based on laboratory or Euler coordinates by C for, as already shown, it is equal to the Lagrange wave speed defined on the reference configuration. Then, according to (45), the stationary wave coordinate $\bar{x}$ is given by

$$\bar{x} = x - Ct \quad (64)$$

in terms of which the stress, velocity and strain can be expressed. The material derivative of a function $g(\bar{x})$ is given by $v(dg/d\bar{x})$, so that the strain-rate law (28) takes for form

$$\bar{v} \frac{d\epsilon_1^p}{d\bar{x}} = (\epsilon_1 - \frac{3}{2} \epsilon_1^p) k(z) \quad (65)$$

and the plastic work relation (29):

$$\bar{v} \frac{dw}{d\bar{x}} = \frac{2\mu}{\rho_0} (\epsilon_1 - \frac{3}{2} \epsilon_1^p)^2 k(z) \quad (66)$$

The variable $\bar{v}$ can be eliminated from (65) and (66) by means of the mass conservation relation (51), $\bar{v} = m/\rho$, and the conservation relations (52)

and (53) can be used to provide expressions for the plastic strain and the plastic work ([1], eqs. (71) and (72)):

$$\epsilon_1^P = \frac{2}{3} \epsilon_1 + \frac{\rho_0}{2\mu\rho} \left(P - \left(p + \frac{m^2}{\rho} \right) \right) \quad (67)$$

$$(1-\gamma)w = E - \left(\tilde{e} + \frac{p}{\rho} + \frac{m^2}{2\rho} \right) - \frac{3\rho_0}{8\mu\rho} \left(P - \left(p + \frac{m^2}{\rho} \right) \right) \left(P - \left(p + \frac{m^2}{\rho} \right) + \frac{8\mu\rho}{\rho_0} \right) \quad (68)$$

where $\tilde{e}(\rho, \theta)$ is the specific internal energy for pure dilatation, and is associated with the corresponding free energy component f in (9). Eqs. (65) to (68) form a system of four equations for the variables which define the state of the material ϵ_1 , θ , ϵ_1^P and w , which are all functions of position $\bar{x}$ in the stationary wave. The fact that two of the equations are differential and two algebraic gives a particular structure to the system which is discussed at some length in Section 8 of [1]. This permits a solution in the form of an elastic shock combined with a steady plastic wave in which the variables change continuously. This combination corresponds to the over-driven wave discussed, for example, in [9], p. 10. The strength of the wave can be expressed by the mass flow m , which determines the elastic shock front across which ϵ^P and w do not change. The post elastic-shock values of the variables then form the initial conditions for solution of the system of equations which determines the plastic steady wave. Eqs. (67) and (68) can be used to eliminate ϵ_1^P and w from (65) and (66) which results in a pair of differential equations for ϵ_1 and θ . Integration from the initial conditions is continued until the rate of plastic strain drops to zero, and

this condition determines the equilibrium stress after passage of the wave.

If the wave profile as a function the space variable $\bar{x}$ is not needed but only the ranges of stress, strain etc. occurring in the wave, the quotient of (66) and (65)

$$\frac{dw}{d\epsilon_1^p} = \frac{2\mu}{\rho_0} \left(\epsilon_1 - \frac{3}{2} \epsilon_1^p \right) \quad (69)$$

can be used to obtain a single differential equation which determines the ranges of these variables. Substitution into (69) for ϵ_1^p and w from (67) and (68) yields an ordinary differential equation in the (ϵ_1, θ) plane with the post elastic-shock values as initial condition. The solution values (ϵ_1, θ) give an image of the steady wave in this plane, and calculation of the corresponding values of ϵ_1^p and w from (67) and (68) will determine the extent of the wave profile since it terminates when plastic deformation ceases. It will be observed that in forming (69) as the quotient of (66) and (65), the strain-rate response function $(\epsilon_1 - 3\epsilon_1^p/2)k(z)$ cancels, and thus does not influence the stress and strain range of the wave. This is consistent with the Rayleigh line concept which is not dependent on the precise strain rate law, although the space distribution of the wave is.

For a smaller value of mass flow, m , a precursor elastic wave will be deduced which is propagated ahead of the plastic wave. The latter extends from the yield stress, where the plastic strain rate is zero through a region of plastic flow until an equilibrium stress is reached for which the rate of plastic flow is again zero. Such a wave corresponds to the underdriven wave discussed in [9], p. 11.

These analyses incorporated complete thermo-mechanical coupling according to the thermo-elastic-plastic theory presented in Section 2. The major contributor in this regard is the dilatational component of the internal energy $\tilde{e}$ in (68), which is perhaps best represented by the mie-Gruneisen equation of state deduced from hydrodynamic analysis. Other analyses of the problem have been limited to the adoption of a purely mechanical theory without explicit thermo-mechanical coupling. Clifton's theory, [7], is limited to moderate stresses involving little entropy change, which is assumed to be zero in the analyses of plane waves ([17], pages 135 and 137). Herrmann formally neglects thermo-mechanical coupling ([9] p. 2) but in his analysis of plane waves in an aluminum alloy [16] he uses the Hugoniot relation to formulate the equation of state, which incorporates thermo-mechanical coupling in the dilatational component of the deformation, which, as already mentioned normally comprises the major component.

As mentioned in Section 2, work-hardening rate-dependent elastic-plastic analysis introduces a strain-rate history effect, since for a given strain increment, the plastic work and hence hardening is greater at higher strain rates. This is of the nature of such influences observed, for example, by Klepaczko [6], but may be too pronounced, since Herrmann [16] cites evidence that strain hardening provides an adequate basis for analysis. However, the structure of the wave analyses described above including thermo-mechanical coupling with an internal variable, plastic work, w , suggests how such a variable can be incorporated into steady wave analysis, when it is not tied to the physical quantity: plastic work. A functional relation not so constrained could, perhaps be selected to provide an adequate representation of the strain-rate history influence.

It is well known (Hill [8] p. 39) that for rate-independent elastic-plastic isothermal theory based on the Mises yield condition, work hardening and strain hardening give identical responses. For rate dependent analysis this is not so, and in particular the rate history influence of work hardening already discussed does not arise if strain hardening is assumed. For a prescribed plastic-strain increment in monotonic loading, the equilibrium hardening will be independent of the rate at which the strain was imposed. A strain dependent version of the theory already described is therefore of interest. If the complete thermodynamic structure is maintained this change will not modify the theory appreciably since evaluation of plastic work is needed to express the entropy increase due to the irreversibility of plastic flow (23).

For a strain hardening material, the yield strain γ in (18) would in general be a function of the generalized plastic strain, $\bar{\epsilon}^P$, and temperature, but for monotonic loading in a plane wave, $\bar{\epsilon}^P = \epsilon_1^P$, so that:

$$\gamma = \gamma(\epsilon_1^P, \theta) \quad (70)$$

In the rate independent theory applied to shock wave evaluation, the problem encountered with work hardening, that the plastic work absorbed as the shock wave traverses an element cannot be determined without analysing the shock structure still arises, for plastic work appears in the energy conservation equation. A rate of strain term is needed to determine the shock structure. Similarly in the plastic steady wave structure evaluation, equations (65) to (68) are still all needed, since plastic work appears on the left hand side

of (68). Thus the form of solution is the same for a strain-hardening material as for a work-hardening one.

However, the term $(1 - \gamma)w$ on the left hand side of the energy balance equation (68) is likely to be small compared with other terms in the equation if the impact stress is many times larger than the yield stress which is usually the case in plate slap experiments. The term $(1 - \gamma)w$ represents the part of the plastic work which is stored in the material as dislocations and lattice defects following plastic flow. The factor $(1 - \gamma) \sim 0.1$, for most of the plastic work is dissipated directly into heat. The plastic work itself will be much smaller than other terms in the energy balance equation since, for example, it is given by the yield stress times the plastic strain increment while the energy absorbed in volume compression is of the order of the total stress multiplied by the strain increment. Thus very little error will be involved in neglecting the term $(1 - \gamma)w$ and then the analysis simplifies since plastic work does not appear as a variable. Eq. (68) then gives the image of the shock structure in the (ϵ_1, θ) plane and (67) determines the plastic strain. The differential relation (65) is needed only to obtain the wave profile in space. Thus a much simpler solution for the steady wave results. In view of this simple structure, for practical calculations it would be advantageous to consider waves of increasing and decreasing strain separately and use a direct overstress form for the plastic strain rate instead of (28) in terms of the variable z , which, because of its quadratic nature applies to both type of waves. In that case the factor $(\epsilon_1 - 3\epsilon_1^P / 2)$ would not be needed to generate the appropriate sign for $\dot{\epsilon}_1^P$, which could be included separately.

4. Discussion and Conclusions

The analysis of shock wave propagation through elastic-plastic materials which exhibit work-hardening, strain-rate dependent response to stress requires the solution of a differential equation associated with the structure of a steady plastic wave. The theory includes thermo-mechanical coupling, and about 10% of the plastic work is assumed absorbed in the resulting dislocation and lattice-defect distribution as measured by Quinney and Taylor [17]. This component of the plastic work absorbed is small compared with the total work done as the wave traverses a section, so that a negligible effect on the process will arise from assuming this component of the absorbed energy to be dissipated into heat as is the rest of the plastic work. In the case of strain-hardening, this minor change in the thermo-mechanical coupling process permits the stress generated across a plastic shock wave to be evaluated using the conservation relations only, which are algebraic, rather than differential in nature. This offers the possibility of much more convenient evaluation of the stress range and total and plastic strain generated. For rate dependent elastic-plastic materials work-hardening and strain-hardening laws lead to qualitatively different behavior, which is not so for the corresponding rate-independent theory. Strain-hardening appears to reproduce experimental findings and has been used by Herrmann in his studies of wave propagation [9, 16]. Since the relations are algebraic, the jumps in the dependent variables can be determined without evaluating the shock structure, and it may be adequate in many cases to utilize this information in design calculations.

Much of the experimental work on the measurement of stress-wave profiles has been concerned with elucidating the strain-rate response law governing

plastic flow. As described in Section 3, the relationship between wave profiles and the rate of strain law is associated with the increase in stress needed to lift the equilibrium stress-strain relation up to the dynamically imposed Rayleigh line including the thermal influences on these relations. However, possible difficulties of interpretation arise in the case of materials having a marked microstructure since this itself will also tend to broaden the wave front and thus generate a lower average strain-rate. In the case of poly-crystalline metals, this can arise from anisotropy of the crystallites, and particularly in the case of initiation of plastic flow, because of the concomitant residual stress in the crystallites resulting from the process used to form the specimen. These influences have been commented on by Herrmann [9]. For porous media the effect of inhomogeneity can be much more marked, and the local micro-motion of the collapsing pores for an element in compression can have an appreciable influence on the average stress-strain relation. It was shown in [18] that the inertia forces of the material adjacent to the collapsing pores necessitate an increase in the average compressive stress according to a strain-rate dependent relation. This is so since even at constant average rate of strain, the surface of the closing pores must accelerate as the pore surface decreases in area in order to accommodate the effectively incompressible deformation due to plastic flow. This requires additional compressive stress proportional to the strain rate. This constitutes a system rate of strain influence superposed on whatever material rate of strain characteristic exists. Such an effect will contribute to the wave profile generated. It may therefore be important to incorporate such effects into investigations of basic material strain-rate influences. Moreover

the existence of inhomogeneity will modify wave profiles evaluated on the assumption of homogeneous material, which may dictate against placing reliance on computed wave profiles. This may lend additional encouragement to attempts to satisfy design requirements on the basis of calculations of jumps of stress and strain across the wave, while reducing the significance of the detailed stress profile to a minimum.

References:

1. P. Germain and E. H. Lee, "On Shock Waves in Elastic-Plastic Solids", J. Mech. Phys. Solids, 21, 359-382, 1973.
2. E. H. Lee, "Elastic-Plastic Deformation at Finite Strains", Jl. of Appl. Mech., 36, 1-6, 1969.
3. E. H. Lee, and T. Wierzbicki, "Analysis of the Propagation of Plane Elastic-Plastic Waves at Finite Strain", Jl. of Appl. Mech., 34, 931-936, 1967.
4. E. H. Lee and D. T. Liu, "Finite Strain Elastic-Plastic Theory Particularly for Plane Wave Analysis", Jl. Appl. Phys., 38, 19-27, 1967.
5. E. H. Lee, "Constitutive Relations for Dynamic Loading and Plastic Waves", Inelastic Behavior of Solids, Eds. M. F. Kanninen, W. F. Adler, A. R. Rosenfield and R. I. Jaffee, 423-439, McGraw-Hill, 1970.
6. J. Klepaczko, "Discussion of the Incremental Wave Effects in a Bar", Archives of Mechanics, 24, 187-202, 1972.
7. R. J. Clifton, Plastic Waves: Theory and Experiment, Mechanics Today, Vol. 1, 1972, Ed. S. Nemat-Nasser, Pergamon Press, 102-167, 1974.
8. R. Hill, "The Mathematical Theory of Plasticity", the Clarendon Press, Oxford, 1950.
9. W. Herrmann, "Some Recent Results in Elastic-Plastic Wave Propagation", Propagation of Shock Waves in Solids, Ed. Eric Varley, ASME, 1-26, 1976.
10. G. E. Duvall, "Propagation of Plane Shock Wave in a Stress-Relaxing Medium", Stress Waves in Anelastic Solids, Springer-Verlag, 21-32, 1964.
11. J. N. Johnson and L. M. Barker, "Dislocation Dynamics and Steady Plastic Wave Profiles in 6061-T6 Aluminum", Jl. in Appl. Phys., 40, 4321-4334, 1969.
12. G. E. Duvall, "Shock Waves and Equations of State". Dynamic Response of Materials to Intense Impulsive Loading, Ed. P. C. Chou and A. K. Hopkins, Air Force Materials Lab, 89-121, 1973.
13. D. R. Bland, "On Shock Structure in a Solid", Jl. of Inst. of Math. and Its Applic., 1, 56-75, 1965.
14. R. J. Clifton, "On the Analysis of Elastic/Visco-Plastic Waves of Finite Uniaxial Strain", Shock Waves and the Mechanical Properties of Solids, Eds. J. J. Burke and V. Weiss, Syracuse University Press, Syracuse, 73-119, 1971.

15. J. M. Kelly and P. P. Gillis, "Shock Thickness in Viscoplastic Solids", J. of Applied Mech., 37, 163-170, 1970.
16. W. Herrmann, "Development of a High Strain Rate Constitutive Equation for 6061-T6 Aluminum", International Conference on the Behavior of Materials, Kyoto, Japan, 1970.
17. G. I. Taylor and H. Quinney, "The Latent Energy Remaining in a Metal After Cold Working", Proc. Roy. Soc. Lond., 143, 307-326, 1934.
18. E. H. Lee, R. L. Mallett, T. C. T. Ting and W. H. Yang, "Dynamic Analysis of Structural Deformation and Metal Forming", Comp. Meth. in Appl. Mech. and Engg., 5, 69-82, 1975.

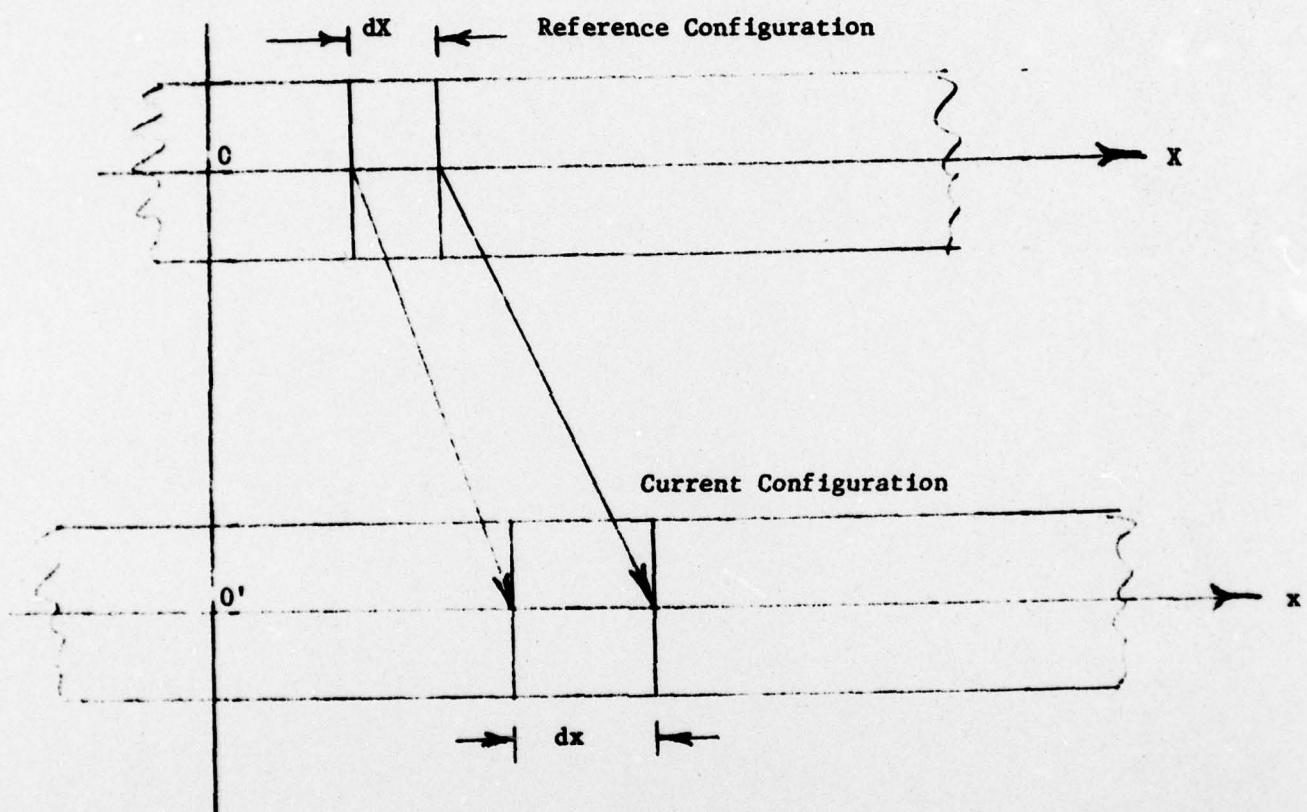


Fig. 1. Lagrange (X) and Euler (x) coordinates systems.

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Watertown, Massachusetts 02172
STRAIN-RATE INFLUENCES ON SHOCK
WAVE PROPAGATION IN METALS
E. M. Lee, Division of Applied Mechanics
Department of Mechanical Engineering,
Stanford University, Stanford, Ca. 94305

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