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THE OCCURRENCE OF FINGERPRINT CHARACTERISTICS AS A TWO-DIMENSIO--ETC(U)
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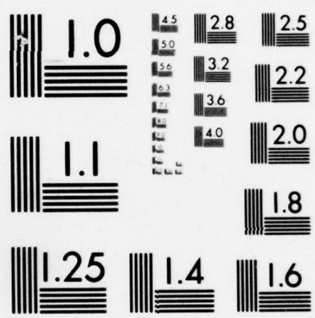
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OCCURRENCE OF FINGERPRINT CHARACTERISTICS AS A TWO-DIMENSIONAL PROCESS

by

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THE OCCURRENCE OF FINGERPRINT CHARACTERISTICS AS A TWO-DIMENSIONAL PROCESS*

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OUTLINE

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*This report (February, 1978) is a revision of a report bearing the same title and written in July, 1976. The present report renders the earlier report obsolete.

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ABSTRACT

The model for occurrence of fingerprint characteristics in terms of multinomial trials on a grid of cells is extended to consider dependence between the cells. The occurrence of the characteristics is modelled as a two-dimensional Markov process.

KEY WORDS: Fingerprints; Identification; Criminalistics; Multinomial model; Two-dimensional stochastic process; Markov process.

(Author's Footnote)

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AS A TWO-DIMENSIONAL PROCESS

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1. INTRODUCTION

The individuality of a fingerprint is based on the pattern of occurrence of the ridge-line details. These minutiae, called Galton [2] characteristics, are of ten types: islands, bridges, spurs, dots, ridge endings, forks (bifurcations), lakes trifurcations, double bifurcations, and deltas. (See [3] for diagrams and detailed descriptions.) In [3] the estimation of fingerprint probabilities based on Galton characteristics was treated according to the following model:

Assumption 1. A fingerprint is considered in terms of a grid of one millimeter cells.

Assumption 2. For each cell of the grid there are 13 possibilities: either the cell is empty, or one of the following 12 possibilities has occurred: island, bridge, spur, dot, ending ridge, fork, lake, trifurcation, double bifurcation, broken ridge (two ridge endings), or some other multiple occurrence.

Assumption 3. There is statistical independence between cells.

Under this model the probability distribution for a given cell is the point multinomial

$$p_0^{z_0} p_1^{z_1} \cdots p_{12}^{z_{12}},$$

where, for $i = 0,1,2,\ldots,12$, $z_i = 0$ or 1 according as the i-th possibility occurs or not, so $z_0 + z_1 + \ldots + z_{12} = 1$. Under Assumption 3--independence among cells--the probability P of a given configuration of t cells is the product over cells,

$$P = \prod_{c=1}^{t} p_0^{z_{0c}} p_1^{z_{1c}} \cdots p_{12}^{z_{12,c}}, \tag{1.1}$$

where, for $i = 0,1,2,\ldots,12$, $z_{ic} = 1$ or 0 according as the i-th possibility occurs in the c-th cell or not.

Thus the probability for a configuration with k_0 empty cells, k_1 cells containing an island, k_2 cells containing a bridge, ..., k_{10} cells containing a delta, k_{11} cells containing two ridge endings, and k_{12} cells containing some other multiple occurrence is

$$P = p_0^{k_0} p_1^{k_1} \cdots p_{12}^{k_{12}},$$

where, for $i = 0,1,2,\ldots,12$, $k_i = \Sigma_{c=1}^{t} z_{ic}$, the number of occurrences of possibility i across the t cells. The estimates of the p_i's (from [3]) are given in Table 1.

[INSERT TABLE 1 HERE.]

For example, consider the configuration of Figure 1. It has 43 cells, 37 of them empty, the other six being occupied by 4 ridge endings and 2 forks. The estimated probability is

$$\hat{P} = \hat{p}_0^{37} \hat{p}_5^{4} \hat{p}_7^{2} = .766^{37} .0832^{4} .0382^{2}; \quad -\log_{10}\hat{P} = 11.4 .$$

[INSERT FIGURE 1 HERE.]

The purpose of the present paper is to study the extent of departure from Assumption 3 and to refine the model according to that departure. Accordingly, the occurrence of fingerprint characteristics is modelled as a two-dimensional process to take into account the dependence between cells.

Appendix A gives a data-analysis relating to dependence between cells. It shows that the probability that a cell is occupied increases monotonically with the number of neighbors occupied.

2. MODELLING DEPENDENCE AMONG CELLS

Let the cells be numbered in some fixed order, say, as one reads English, starting with the top row and moving from left to right within each row. Let X_c be a random vector giving the outcome in the c-th cell,

$$X_c = (Z_{0c}, Z_{1c}, Z_{2c}, \ldots, Z_{12,c}), \quad c = 1,2,\ldots,t.$$

Then

$$P = P(X_1=x_1)P(X_2=x_2|X_1=x_1)P(X_3=x_3|X_2=x_2,X_1=x_1)$$
$$\ldots P(X_t=x_t|X_{t-1}=x_{t-1},\ldots,X_1=x_1) \ . \tag{2.1}$$

Under Assumption 3, independence among cells, expression (2.1) simplifies to

$$P = P(X_1=x_1)P(X_2=x_2)\ldots P(X_t=x_t),$$

which is the same as (1.1) since

$$P(X=x) = P[(Z_0,Z_1,\ldots,Z_{12})=(z_0,z_1,\ldots,z_{12})] = p_0^{z_0}p_1^{z_1}\ldots p_{12}^{z_{12}} \ .$$

As a step toward modelling dependence, we introduce

Assumption 3'. The outcome in the c-th cell depends upon the outcomes in the other cells only through the outcomes in the adjacent cells.

Due to the fact that the probability (2.1) forces one to use a linear ordering of the cells, one must write things in terms of the four preceding adjacent cells rather than all eight adjacent cells. More precisely, under Assumption 3', the conditional probability $P(X_c=x_c|X_{c-1},X_{c-2},\ldots,X_1)$ will not depend upon all of X_{c-1}, X_{c-2}, ..., X_1 but only upon four of these variables, namely, those corresponding to the cell to the left (west) of cell c, the cell above (north of) cell c, the cell just northwest of cell c, and the cell just northeast of cell c. [If the configuration were rectangular and indexed as (i,j), then the four cells upon which the outcome in cell (i,j) would depend would be cells (i,j-1), (i-1,j-1), (i-1,j), and (i-1,j+1).] If W_c denotes the matrix whose columns are these four neighbors of X_c, then Assumption 3' is

$$P(X_c=x_c|X_{c-1},X_{c-2},\ldots,X_1) = P(X_c=x_c|W_c) \ . \tag{2.2}$$

Assumption 3' may be viewed as an assumption that the process is a Markov process; see Appendix B.

2.1. Data Analysis

Sets of 5 cells were examined to study the dependence of X_c on its four preceding neighbors. According to the model, these sets were of the form $\begin{matrix} x & x & x \\ x & y & \end{matrix}$, where y denotes the dependent cell and the x's its four preceding neighbors.

For any such set s, let the variable $B_s = 1$ or 0 according as the fifth ("y") cell in the s-th set is occupied or not, and let A_s be the number of preceding adjacent cells ("x" cells) which are occupied; A_s is between 0 and 4. Table 2 gives, for each value of A, the proportion of B's that are equal to 1. We say

[INSERT TABLE 2 HERE.]

that a cell has one <u>adjacency</u> for each of the four preceding adjoining cells which is occupied. We shall refer to the variable A as the <u>number of adjacencies</u>. The probability of occupancy increases monotonically with the number of adjacencies. Such absolute consistency was not expected, firstly because perfect consistency seems so rare in data analyses and secondly because it was thought that occurrences in most of the four adjacent cells might crowd out occurrence in the fifth cell.

Combining (2.2) with the multinomial gives

$$P(X_c = x_c | W_c) = \prod_{i=0}^{12} [p_i(W_c)]^{z_i}, \qquad (2.3)$$

where $z = (z_0 . z_1, \ldots, z_{12})$. The model we shall use for $p_i(W_c)$ is that it depends on W_c only through the number of cells occupied. That is,

$$p_i(W_c) = p_i(a),$$

where

$$a = a(W_c)$$

is the number of adjacencies for cell c -- the number of occupied cells among the four cells preceding and adjacent to cell c; the quantity a is either 0,1,2,3 or 4. We now have

$$P = \prod_{c=1}^{t} \prod_{i=0}^{12} [p_i(a_c)]^{z_{ic}}. \qquad (2.4)$$

Some adjustments are necessary for border cells -- cells in the first row, first column, or last column. See the next section.

Let E be the event that a given cell is occupied. We shall assume that the probability of Possibility i in any given cell, which was p_i in the model of [3], is

Assumption 4: $p_i(a) \;=\; p_i P(E|A=a)/P(E), \quad i = 1,2,\ldots,12.$

Thus

$$p_0(a) = 1 - \sum_{i=1}^{12} p_i(a)$$

$$= 1 - [P(E|A=a)/P(E)] \sum_{i=1}^{12} p_i \,.$$

Note that, for $i = 1,2,\ldots,12$, $p_i/P(E)$ is simply the conditional probability of occurrence of possibility i, given that the cell is occupied, so that the effect of Assumption 4 is simply to allocate $P(E|A=a)$ to the twelve different possibilities in the same way, regardless of the value of A.

This gives

$$P \;=\; \prod_{c=1}^{t} \prod_{i=0}^{12} [p_i P(E|A_c=a_c)/P(E)]^{z_{ic}}$$

$$= [\prod_{a=0}^{4} p_0(a)^{k_0(a)}] [\prod_{i=1}^{12} p_i^{k_i}] P(E)^{-(k_1+k_2+\ldots+k_{12})} [\prod_{a=1}^{4} P(E|A=a)^{m_a}] \,,$$

where, for $a = 0,1,2,3,4$,

$k_0(a)$ = number of empty cells with exactly a adjacencies,

for $i = 1,2,\ldots,12$,

k_i = number of cells containing possibility i

(as above), and for $a = 0,1,2,3,4$,

m_a = number of occupied cells with exactly a adjacencies.

2.2. Border Cells

Cells at the border, not being touched by the full complement of four preceding adjacent cells, require some special treatment. One could take the results in border cells as given and take the probabilities for the other cells conditionally on the outcomes in the border cells, but this would result in considerable reduction in the effective sample size. (E.g., 18 of the 43 cells in Figure 1 are border cells.)

We wish to use such information as is present. I.e., we shall make use of the fact that some but not all 4 preceding adjacent cells are represented. E.g., if there are 3 adjacent cells and 2 are occupied, then we know that if the 4 cells were present, then 2, 3 or 4 of them would be occupied: we need to know $P(E|2\leq A\leq 4)$, where E is the event that the given cell is occupied. In general, we need to know $P(E|a_1\leq A\leq a_2)$, $0\leq a_1\leq a_2\leq 4$. We have

$$\begin{aligned} P(E|a_1\leq A\leq a_2) &= P(E \text{ and } a_1\leq A\leq a_2)/P(a_1\leq A\leq a_2) \\ &= \sum_{a=a_1}^{a_2} P(E \text{ and } A=a) \Big/ \sum_{a=a_1}^{a_2} P(A=a) \\ &= \sum_{a=a_1}^{a_2} P(E|A=a)P(A=a) \Big/ \sum_{a=a_1}^{a_2} P(A=a)\ . \end{aligned}$$

For this we need the marginal distribution of A, given in Table 3.

[TABLE 3]

2.3. Example

Let us now apply the formula to the configuration of Figure 1. Recall from the Introduction that for this configuration the method of [3] gave $-\log_{10}\hat{P} = 11.4$.

To make the adjustment for border cells, $p_i(a) = P(\text{Possibility } i|A=a)$ is replaced by the relevant probability of the form $P(\text{Possibility } i|a_1\leq A\leq a_2)$. We insert the relevant cell probabilities on Figure 2, a copy of Figure 1. For this we need the following numbers, obtained from the tables:

for ending ridges:--

$.356\ P(E|A=0) = .356(.199) = .0708$

$.356\ P(E|0\leq A\leq 2) = .356(.261) = .0929$

for forks:--

$.163\ P(E|A=0) = .163(.199) = .0324$

$.163\ P(E|A=1) = .163(.291) = .0474$

for empty cells:--

$$1 - P(E|0\leq A\leq 4) = 1 - .280 = .720$$

$$1 - P(E|0\leq A\leq 3) = 1 - .273 = .727$$

$$1 - P(E|0\leq A\leq 1) = 1 - .239 = .761$$

$$1 - P(E|1\leq A\leq 2) = 1 - .309 = .691$$

$$1 - P(E|1\leq A\leq 3) = 1 - .322 = .678$$

[FIGURE 2]

The estimated probability is

$$\hat{P} = .0324^{1}.0474^{1}.0708^{3}.0929^{1}.667^{1}.678^{1}.691^{9}.709^{12}.720^{1}.727^{5}.761^{2}.801^{6};$$

the negative log of this is $-\log_{10}\hat{P} = 12.5$. Compare this with the figure of 11.4 given by the approximation based on an assumption of independence between cells. The difference in logarithms is 1.1; the ratio of the two estimates is thus 12.6. This difference is unimportant since we are interested only in order of magnitude. Note further that the estimate based on independence is a larger probability, i.e., it is conservative, in the sense of giving the suspect the benefit of the doubt. In general, independence gives too much weight (too low a probability) to configurations with a lot of clustering of occurrences. In the configuration of Figure 1 there is some but not a great deal of clustering.

3. DISCUSSION

3.1. Upper Bounds

Often we are concerned primarily with *upper* bounds, as these are conservative, i.e., they err in favor of the suspect. An upper bound on the probability that is highly conservative within the context of the present model can be obtained by multiplying the relative frequency of each occurrence by .714, the occupancy probability for 4 adjacencies, and replacing the probabilities for empty cells by .801, the probability a cell is empty given that all its neighbors are empty. This gives the upper bound

$$\overline{P} = .801^{k_0}\,\hat{p}_1^{k_1}\cdots\hat{p}_{12}^{k_{12}}(.714/.234)^{k_1+\cdots+k_{12}}$$

$$= (.801/.766)^{k_0}(.714/.234)^{k_1+\cdots+k_{12}}\,\hat{P}$$

$$= 1.06^{k_0}\,3.05^{k_1+\cdots+k_{12}}\,\hat{P}\,,$$

where $\hat{P}$ is the estimate based on independence. The decrease in information is at most $\log_{10}\overline{P} - \log_{10}\hat{P} = k_0 \log_{10} 1.06 + (k_1+\ldots+k_{12}) \log_{10} 3.05 = 0.0253\,k_0 + 0.484(k_1+\ldots+k_{12}) = 0.0253(t-c) + 0.484c$, where $c = k_1+\ldots+k_{12}$ is the number of occupied cells, $t = k_0 + c$ is the total number of cells (and $t-c = k_0$ is the number of empty cells). For example, for the configuration of Figure 1, we have $\log_{10}\overline{P} - \log_{10}\hat{P} = 0.0253(37) + 0.484(6) = 3.84$, so the ratio of $\overline{P}$ to $\hat{P}$ is about 7,000. For the estimate based on dependence (denote it here by $\tilde{P}$), we have $\log_{10}\overline{P} - \log_{10}\tilde{P} = 15.2 - 12.5 = 2.7$, corresponding to a ratio of about 500.

3.2. Pattern Area vs. Non-pattern Area

The pattern area of a full print is defined as the central area, delineated in terms of the positions on the ridge lines where they change concavity. The non-pattern area is the border area, outside of the pattern area. In [3] it was mentioned that the density of occurrences in the non-pattern area is only about 60% as great as in the pattern area. The model of the present paper provides an effective means of dealing with this inhomogeneity. Among the reasons for favoring the model of this paper to a model based on the pattern-non-pattern dichotomy are the following: (i) the definition of pattern area is not entirely precise, and even if it were, crime-scene partial prints do not always permit identification of the pattern and non-pattern area; (ii) the density of occurrences is not a function of the pattern/non-pattern dichotomy, rather it decreases as the distance from the core increases, with a possible increase in density around deltas.

3.3. Control for Variation in Finger Size

This study and [3] are based on the use of one millimeter squares for all prints. A print-dependent metric might be more appropriate; e.g., the cell size could be taken equal to, say, the average distance to cross six ridge lines. This would be comparatively difficult to work with. Use of the model of the present paper, conditioning on occurrences in adjacent cells, provides at least a partial control for variation in density of occurrences associated with variation in finger sizes.

APPENDIX A Data Analysis of Dependence between Cells

Square blocks of 9 cells, 3 cells by 3 cells, were examined to determine the extent of inter-cell dependence. The data set of [3] yielded 845 separate such blocks of cells. For $i = 1, 2, \ldots, 845$ blocks, let the variable $y_i =$ 1 or 0 according as the center cell of th i-th block is occupied or not, and let x_i be the number of adjacent cells which are occupied; x_i is between 0 and 8. Table 5 gives the cross-tabulation of y and x. Table 6 gives, for each value of x, the proportion of y's that are equal to 1, i.e., the proportion of center cells which are occupied.

[INSERT TABLES 5 AND 6 HERE.]

The probability of occupancy increases monotonically with x. Such absolute consistency was not expected, firstly because it seems so rare in data analyses and secondly because it was thought that occurrences in most of the adjacent cells might crowd out occurrence in the center cell.

Table 7, based on Table 5, gives observed and expected values of the number of blocks with center cells occupied.

[TABLE 7]

The value of the chi-square statistic for testing independence based on Table 7 is 18.77 (6 d.f., $P < .005$). The decomposition of this overall value based on the value 0.14 of the correlation coefficient between x and y is given in Table 8.

[TABLE 8]

APPENDIX B Some Remarks on Two-Dimensional Processes

The idea of order of a two-dimensional process that we use is essentially that discussed by P.K. Bhattacharya [2]. Consider a stochastic process with a two-dimensional parameter (s,t). In order to calculate probabilities we shall need to define a linear ordering on $\{(s,t)\}$. For simplicity assume the array $\{(s,t)\}$ is rectangular: $s = 1, 2, \ldots, S$, $t = 1, 2, \ldots, T$. We shall use the ordering $(1,1), (1,2), \ldots, (1,T), (2,1), (2,2), \ldots, (S,T)$, which might be called "English-language ordering." [Bhattacharya uses the "lexicographic ordering," $(1,1), (2,1), \ldots, (S,1), (1,2), (2,2), \ldots, (S,T)$.] We shall say that a two-dimensional process is of order r if the conditional distribution of $X_{s',t'}$, given all the random variables preceding it in the ordering, depends only upon those random variables X_{st} such that $(s - s')^2 + (t - t')^2 \leq r^2$. (In some problems it may prove meaningful to use a metric different from Euclidean distance.) Thus in a first-order process, X_{st} depends upon $X_{s-1,t}$ and $X_{s,t-1}$. In a process of order $2^{1/2}$, X_{st} depends upon the four variables $X_{s-1,t-1}$, $X_{s-1,t}$, $X_{s-1,t+1}$, and $X_{s,t-1}$; this is the order of process employed in the present paper.

APPENDIX C Infinitely Divisible Random Vectors

Let $Y_c = (Y_{1c}, Y_{2c}, \ldots, Y_{10,c})$ be the observation in the c-th cell, where, for $v = 1, 2, \ldots, 10$ Galton characteristics, Y_{vc} = number of occurrences of the v-th characteristic in the c-th cell. The family of random variables $\{Y_c, c=1,2,\ldots,t$ cells$\}$ may be considered as a stochastic process in the plane, a multivariate point process. It is reasonable to model this process as an infinitely divisible process. Such processes have been studied under AFOSR Grant 76-3050 (see [4]) and are being studied under AFOSR Grant 77-3454. Now underway is a study of the occurrences of fingerprint characteristics as a multivariate Poisson process, a special case of an infinitely divisible process. Upon completion of that study, infinitely divisible processes will be further studied and will be applied to other pattern recognition problems and to other phenomena, as outlined in the proposal for AFOSR Grant 77-3454.

1. Estimates of Probabilities and Relative Frequencies of the Possibilities

i		Estimated probability, $\hat{p}_i$	Estimated relative frequency, $\hat{p}_i/(1-\hat{p}_0) = \hat{p}_i/.234$
0	Empty cell	.766	
1	Island	.0177	.0756
2	Bridge	.0122	.0521
3	Spur	.00745	.0318
4	Dot	.0151	.0645
5	Ending ridge	.0832	.356
6	Fork	.0382	.163
7	Lake	.00640	.0274
8	Trifurcation	.000582	.00249
9	Double bifurcation	.00140	.00598
10	Delta	.00198	.00846
11	Broken ridge	.0139	.0594
12	Other multiple occurrence	.0355	.152
		1.0	1.0

2. Number of Adjacencies and Probability of Occupancy

Number of adjacencies	0	1	2	3	4
Probability of occupancy	.199	.291	.333	.400	.714

3. Distribution of A,

the Number of Adjacencies

a	0	1	2	3	4
P(A=a)	.391	.296	.214	.084	.015

4. Estimates for Border Cells:

Estimates of Probability of Occupancy,

given Partial Information about Adjacent Cells

$$P(E|a_1 \leq A \leq a_2)$$

a_1	a_2				
	0	1	2	3	4
0	.199	.239	.261	.273	.280
1		.291	.309	.322	.331
2			.333	.352	.369
3				.400	.447
4					.714

5. Cross-Tabulation of

Occupancy of Center Cell

and Number of Adjacent Cells Occupied

x	y = 0	y = 1	Total
0	152	28	180
1	170	45	215
2	163	45	208
3	97	29	126
4	44	23	67
5	23	13	36
6	7	5	12
7	0	0	0
8	0	1	1
	656	189	845 blocks of cells

y = 1 if given (center) cell is occupied

= 0 if it is empty

x = number of adjacencies (number of adjacent cells occupied)

6. Probability of Occupancy as a Function of Number of Adjacent Cells Occupied

x:	0	1	2	3	4	5	6	7	8	Total
Frequency:	180	215	208	126	67	36	12	0	1	845
Percent of blocks with center cells occupied:	15.6	20.9	21.6	23.0	34.3	36.1	41.7	----	100	22.4

x = number of adjacencies

7. Observed and Expected Values
of Number of Blocks with Center Cells Occupied

	Observed (Expected)		
x	y		
	0	1	Total
0	152 (139.7)	28 (40.3)	180
1	170 (166.9)	45 (48.1)	215
2	163 (161.5)	45 (46.5)	208
3	97 (97.8)	29 (28.2)	126
4	44 (52.0)	23 (15.0)	67
5	23 (27.9)	13 (8.1)	36
6 or more	7 (10.1)	6 (2.9)	13
Total	656	189	845

8. Decomposition of Chi-Square
according to Correlation between x and y

Source of variation	d.f.	Value of chi-square	
Overall	6	18.77	($P<.005$)
nr^2	1	16.65	($P<.005$)
Residual	5	2.12	($.80<P<.85$)

	1	2	3	4	5	6
A	O	O	O	O	O	O
B	E	O	O	O	E	O
C	O	O	F	O	O	O
D	O	O	O	O	O	O
E	O	O	O	E	O	O
F	O	O	O	O	F	O
G	O	E	O	O	O	O
H			O			

Figure 1. Configuration of 43 cells with 4 ending ridges and 2 forks. O = empty cell, E = ending ridge, F = fork.

	1	2	3	4	5	6
A	.720	.727	.727	.727	.727	.727
B	<u>.0929</u>	.709	.801	.801	<u>.0708</u>	.691
C	.678	.709	<u>.0324</u>	.667	.709	.691
D	.691	.709	.709	.709	.801	.761
E	.691	.801	.801	<u>.0708</u>	.709	.761
F	.691	.801	.709	.709	<u>.0474</u>	.691
G	.691	<u>.0708</u>	.709	.709	.709	.691
H			.691			

<u>Figure 2</u>. Cell probabilities for configuration of Figure 1.

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The model for occurrence of fingerprint characteristics in terms of multinomial trials on a grid of cells is extended to consider dependence between cells. The occurrence of the characteristics is modelled as a two-dimensional Markov process. The relationship with infinitely divisible random vectors is indicated.

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