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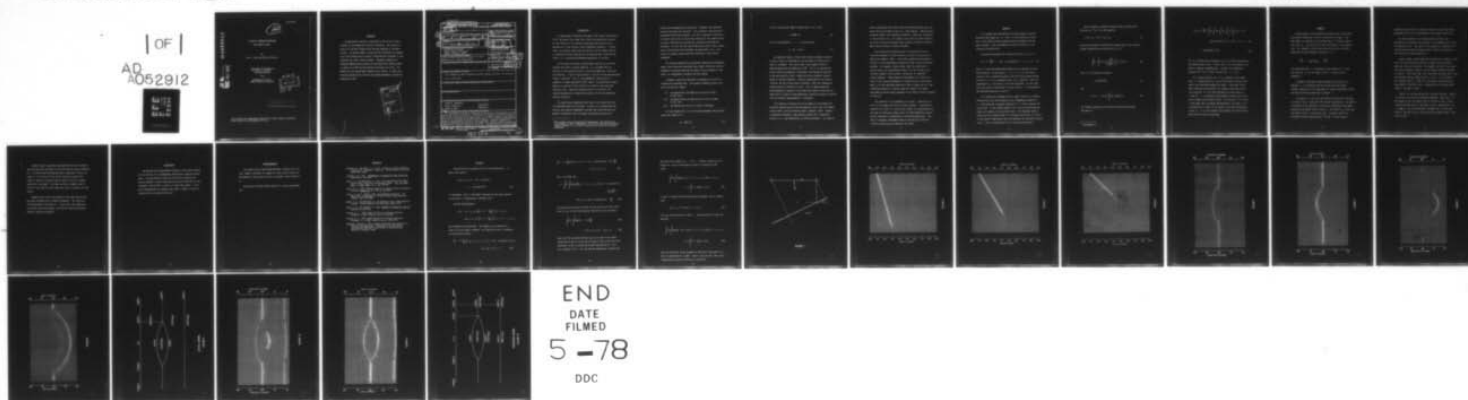
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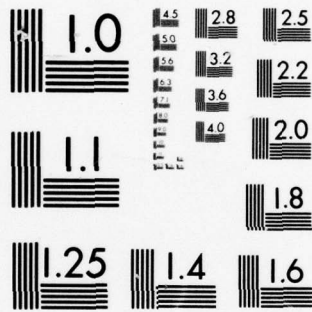
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A VELOCITY INVERSION PROCEDURE
FOR ACOUSTIC WAVES

by

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and

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Abstract

An approximate solution is presented to the seismic inverse problem for two-dimensional velocity variations. The solution is given as a multiple integral over the data observed at the upper surface. An acoustic model is used and the reflections are assumed to be sufficiently weak to allow a "linearization" procedure in the otherwise non-linear inverse problem. Synthetic examples are presented demonstrating accuracy of the method with dipping planes at angles up to 45° and with velocity variations up to 20%. The method was also tested under automatic gain control, in which case velocity estimates were lost but the method nonetheless successfully migrated the data.

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Introduction

In recent papers (Bleistein and Cohen, 1977; Cohen and Bleistein, 1977), the authors have shown that closed form approximate solutions for the velocity (or a velocity related quantity) profile can be obtained for a wide variety of wave propagation equations. "Closed form," as used here, means that the velocity profile (depth section) is obtained by direct processing on the observed data (time section) itself; i.e., by performing weighted quadratures on the data.

In this paper we present an approximate solution to an inverse problem often used in seismic modeling. It is assumed that the velocity in the subsurface varies in only two dimensions, vertically and laterally. A line of point sources is set off and the backscattered signal is observed.[†] This is the mathematical idealization of summation of common depth-point (CDP) traces. An approximate integral equation is derived for the variation in velocity from some known reference value. When that reference value is a constant, the integral equation has a closed form solution for the two-dimensional velocity variation.

The simplifying assumptions which lead to the closed form solution presented here are as follows. Firstly, it is assumed that the acoustic wave equation adequately describes the subsurface wave propagation (although in the first paper cited above inversion for

†

With a planar array of backscatter observations, the analysis can be extended to yield an approximate solution to the three-dimensional inverse problem; i.e., to find a velocity that varies in all three dimensions.

elastic wave propagation was discussed). Secondly, the subsurface velocity variations must be small. This justifies linearization of an otherwise non-linear problem. Even this "smallness" limitation is not overly restrictive as can be seen from one of the examples below in which a 20% velocity variation was successfully migrated and estimated. In fact, the real world data restrictions--noise, attenuation, discretization and finiteness of observations, etc.,--are usually of greater concern than this theoretical linearization assumption.

This second assumption has significant theoretical consequences, namely, that reflecting interfaces cause "weak" reflections and the (downward) transmitted wave may be taken to be the response to the source in a homogeneous (constant velocity) medium.

A computer program was developed to implement our result on synthetically generated data. The synthesis embodies several real world restrictions, namely:

- (i) the observations are made only at *discrete* points on the line;
- (ii) the observations are made only over a *finite* length of the line;
- (iii) the observations are *band limited* in frequency.

For the examples here, in (i) the spacing between shot-receiver points was taken to be

$$\Delta x = 100 \text{ ft.}; \quad (1)$$

in (ii), the line was taken to extend from $-L$ to L , with

$$L = 4,000 \text{ ft.}; \quad (2)$$

in (iii) the bandwidth f_- f f_+ was used with

$$f_+ = 4f_- = 24 \text{ hz.} \quad (3)$$

An important feature of the direct inverse procedure used here is that it does *not* break down as the dip angle of reflecting surfaces is increased. This can be seen in the examples below of dipping planes with dip angles up to 45° . This is so because this direct inversion procedure is formulated from propagation governed by the wave equation itself, rather than from a parabolic approximation to the wave equation. Parabolic approximations are known to "flatten" the dip of wave fronts (Claerbout, 1976) and consequently flatten the dip of profiles, as well. This is demonstrated with three-dimensional examples in the above-cited paper by Bleistein and Cohen (1977) in which a three-dimensional inversion procedure, starting from the parabolic approximation, is presented.

This feature of accuracy at all dip angles is also present for migration schemes based on the wave equation itself--see, for example, French (1975), Larner and Hatton (1976), Schneider (1970). However, as migration techniques, these methods produce only a subsurface profile--i.e., layer mapping but no velocity estimates. The velocity

profile produced by the direct inversion method described here, produces velocity estimates as well as a layer mapping. These velocity estimates require true amplitude information. When such information is not available--e.g., if automatic gain control has been applied--the direct inversion technique simply migrates the data to produce a depth section without a velocity estimate.

The linearization procedure used in the derivation of our inversion procedure is often referred to as the *Born approximation* (Morse and Feshbach, 1953). It has been successfully exploited in the past to yield approximate solutions of other inverse scattering problems. For example, Prosser (1969, 1976) has shown that in certain cases, this approximation leads to a first iterate in an iteration scheme to solve certain "refraction" or "potential" inverse problems. Three essential requirements of his proof are that (i) the scattering potential or reflectivity be "sufficiently weak;" (ii) the scattering region be finite in extent and (iii) the scattering potential or refractive index be "smooth" in a manner prescribed by certain decay estimates on its spatial Fourier transform at infinity.

The condition (i) is assumed by us, as well. Conditions (ii) and (iii) are simply not true in seismic problems. In particular, condition (ii) leads to a particularly simple first approximation of the index of refraction, namely, that its three-dimensional spatial Fourier transform is proportional to the backscattered data. The lack of transverse confinement heads to the much more difficult inversion formula given by equation (9), below.

Analysis

It is assumed that the medium to be probed supports acoustic waves with wave speed $v(x, z)$. Here x is the transverse variable and z is the depth variable (measured positive downward from the upper surface). Thus, the medium has velocity variation in one transverse direction only.

The governing equation for the wave field $U(t, x, y, z)$ is

$$\nabla^2 U - v^{-2} \frac{\partial^2 U}{\partial t^2} = - \delta(x - \xi) \delta(y) \delta(z) \delta(t), \quad U \equiv 0, \quad t < 0. \quad (4)$$

Here ∇^2 is the three-dimensional Laplacian and δ denotes the Dirac delta function. At each point, $x = \xi, y = z = 0$, on the source-receiver line, an impulsive source is set off and the backscattered field at $(t, \xi, 0, 0)$, denoted by $U_S(t, \xi)$, is observed. Each such experiment (i.e., for each ξ) is set off separately. In each case, the "time clock of observation" is reset so that $t = 0$ corresponds to the time when the pulse is set off.

This formulation dispenses with the upper surface as a boundary. Alternatively, one could replace (4) by a homogeneous equation in $z > 0$ and prescribe a boundary condition at $z = 0$ which contains the source term. In the simplest model of this type one would prescribe $\partial U / \partial z$ as an impulse at each surface point. The consequent change induced by this simplest model is to introduce multipliers of 2 and 4 in the results stated below, while not changing the substantive results at all. Thus, we proceed with (4) as the governing equation.

With c denoting a constant reference speed, we define the variation in $v^{-2}(x, z)$ by the equation

$$v^{-2}(x, z) = c^{-2}[1 + \alpha(x, z)] . \quad (5)$$

By using the methods in Bleistein and Cohen (1977), the following integral equation may be derived for $\alpha(x, z)$:*

$$\int_0^{\infty} dz \int_{-\infty}^{\infty} dx \alpha(x, z) \frac{H(\frac{c\tau}{2} - \rho)}{\sqrt{(\frac{c\tau}{2})^2 - \rho^2}} = \frac{\tau}{2} \Theta(\tau, \xi) . \quad (6)$$

Here, H is the Heaviside function,

$$\rho = \sqrt{(x-\xi)^2 + z^2} \quad (7)$$

and

$$\Theta(\tau, \xi) = -(4\pi c)^2 \int_0^{\tau} dt (\tau-t) U_S(t, \xi) . \quad (8)$$

The integral equation can be solved by transform techniques.

The result is

* See Appendix A.

$$\alpha(x, z) = \frac{8ic^3}{\pi} \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_3 \int_0^{\infty} d\tau \int_0^{\tau} dt k_3 (\tau^2 - \tau t) \cdot U_S(t, \xi) \exp\{2ik_1(x - \xi) - 2ik_3 z + i\omega\tau\} ; \quad (9)$$

$$\omega = c [\operatorname{sgn} k_3] \sqrt{k_1^2 + k_3^2} \quad (10)$$

This is a formula which reproduces $\alpha(x, z)$ by direct processing of the observed data itself. In fact, this is *not* the formula to be used for computer implementation. Instead, the integrand is multiplied by $-2ik_3$ to yield a formula for $\alpha' = \partial\alpha/\partial z$.

For a layered medium, $\alpha(x, z)$ is a constant in each layer while α' is proportional to a sum of Dirac delta functions which peak on the interfaces between the layers. Since band limited delta functions are much easier to recognize than band limited step functions, it is far more desirable to process data for α' than for α . An analytical development of these ideas can be found in Mager and Bleistein (1976). In particular, it is shown in that paper how to estimate the magnitude of the jump in α at a discontinuity in terms of the output of the band limited derivative, α' . It is this analysis that provides our velocity estimates from this band limited data.

Examples

To take account of the constraints cited earlier in this paper, the multiple integration in (9) is carried out as follows. The time domain is truncated at the maximum time of "reliable" observations $U_S(t, \xi)$. The step size in the time interval is the sampling rate. The integrals in wave vector are restricted to an annulus consistent with the frequency constraint, i.e.,

$$\frac{2\pi f_-}{c} < \sqrt{k_1^2 + k_3^2} < \frac{2\pi f_+}{c} .$$

The integration over ξ is truncated to the interval $(-L, L)$ and the step size $\Delta\xi$ (in this paper, 100 ft.) is used for this quadrature.

A computer algorithm has been developed by the authors to generate α' in accordance with (9) and the discussion above. Synthetic data $U_S(t, \xi)$ was generated for various subsurface profiles and then the data was processed to produce α' .

In Figures 2, 3, and 4 are the results for profiles of the type in Figure 1 with θ being 15° , 30° , and 45° respectively. The angle of inclination of the interface is accurately reproduced in the region bounded by the dashed lines of Figure 1. The velocity c in this case was 5,000 ft./sec., Δc was 250 ft./sec. A typical value of Δc estimated from the output was 249.7 ft./sec. In this type of

configuration, the error is virtually linear in Δc and hence these percentage errors will be maintained for much higher values of Δc .

For these examples, one can generate the synthetic data analytically and calculate the integral in (9) asymptotically, under the constraint (11). The result of that analysis is completely consistent with the numerical output, both as regards the location of the interface and the estimate of Δc .

Figure 5 shows a time section for an anticline. Figure 6 is the result of direct inversion and gives virtually the exact model, which was a circular anticline. For example, the top of the actual circle was located at 1,500 ft. and our output yielded an estimate of 1,502 ft. The actual jump Δc at this point was again 250 ft./sec.; the estimate from our output was 247.1 ft./sec. In the middle of the flanking plane ($x = 2,700$ ft.), the actual location and jump were 2,000 ft. and 250 ft./sec., respectively; our estimates were 2,000 ft. and 248.1 ft./sec.

Figure 7 is the time section for a circular syncline. Figure 8 is the result of our direct inversion procedure. Depth and Δc at the bottom of the circle were 3,472 ft. and 250 ft./sec.; direct inversion yielded 3,472 ft. and 249.3 ft./sec., respectively. At $x = 3,600$ ft. (on the flank plane) the depth and Δc values were 2,000 ft. and 250 ft./sec.; direct inversion yielded 2,010 ft. and 253.6 ft./sec.

Figure 9 shows a subsurface configuration for which synthetic data was graciously provided to us by the research group at Marathon Oil. The time section provided by them is depicted in Figure 10; Figure 11 is the result of our direct inversion procedure and Figure 12 shows our estimate from the output of various relevant quantities in the model. The lower two sets of numbers exhibit errors of less than 4%, while above that level, the errors are less than 1%.

Automatic gain control was applied to the input and the data was again processed by our inversion procedure. The output was indistinguishable from Figure 11. In this case, the output does not provide velocity estimates, and thus our inversion procedure becomes a migration procedure.

Conclusions

We have derived an approximate solution to the inverse problem for the velocity in an inhomogeneous medium which supports acoustic waves. The approximations made are often used in modeling the inverse problem in seismic exploration and can be found also in the references cited earlier, as well as in many other papers. A computer implementation on synthetic data under a number of realistic constraints has also been carried out.

Acknowledgments

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We also wish to thank Willene Grady for a superb programming job.

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APPENDIX

The derivations of equation (6) will be presented here. To begin, the function

$$U_0(t, x, y, z) = \delta(t - r/c)/(4\pi r),$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad (A1)$$

is introduced. This is the Green's function for the "wave operator" in (4) when v is replaced by c , defined in (5).

We form the expression,

$$\begin{aligned} & U_0(\tau - t, x - \xi, y, z) \left[\nabla^2 - c^{-2} \frac{\partial^2}{\partial t^2} \right] U(t, x, y, z) \\ & - U(t, x, y, z) \left[\nabla^2 - c^{-2} \frac{\partial^2}{\partial t^2} \right] U_0(\tau - t, x - \xi, y, z), \end{aligned}$$

and integrate over space-time. The integral can be shown to be equal to zero by Green's theorem. An alternative result is obtained by noting the following:

$$\left[\nabla^2 - c^{-2} \frac{\partial^2}{\partial t^2} \right] U_0(\tau - t, x - \xi, y, z) = -\delta(x - \xi)\delta(y)\delta(z) \delta(\tau - t),$$

$$U_0 = U_{0,t} = 0, \quad t = \tau; \quad (A2)$$

$$\left[\nabla^2 - c^{-2} \frac{\partial^2}{\partial t^2} \right] U(t, x, y, z) = -\delta(x - \xi)\delta(y)\delta(z)\delta(t) + \frac{\alpha}{c^2} \frac{\partial^2 U}{\partial t^2},$$

$$U = U_t = 0, t = 0. \quad (A3)$$

Thus, it follows that

$$0 = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx dy dz \left\{ U_0(\tau - t, x - \xi, y, z) \left[-\delta(x - \xi)\delta(y)\delta(z)\delta(t) + \frac{\alpha}{c^2} \frac{\partial^2 U}{\partial t^2} \right] + U(t, x, y, z)\delta(x - \xi)\delta(y)\delta(z)\delta(\tau - t) \right\}. \quad (A4)$$

By using the Dirac delta functions and the initial and final conditions in (A2, 3) this result may be simplified to the following:

$$\int_0^{\tau} dt \int_{-\infty}^{\infty} dx dy dz \propto U_0 \frac{\partial^2 U}{\partial t^2} \quad (A5)$$

$$= -c^2 U(\tau, \xi, 0, 0) = -U_S(\tau, \xi). \quad (A5)$$

Here, the last expression defines $U_S(\tau, \xi)$ as used in this paper.

Integration by parts in (A5) and use again of the initial and final conditions in (A2, 3) places the second time derivative $\partial^2/\partial t^2$ on U_0 instead of on U . This can then be replaced by a second time

derivative with respect to τ : $\partial^2/\partial\tau^2$. A double integration with respect to τ and an interchange of orders of integration then yields

$$\begin{aligned} \int_0^\tau dt \int_{-\infty}^\infty dx dy dz \alpha U_0(\tau - t, x - \xi, y, z) U(t, x, y, z) \\ = -c^2 \int_0^\tau (\tau - t) U_S(t, \xi) dt. \end{aligned} \quad (A6)$$

If (A3) is formally solved by perturbation methods, then to leading order

$$U(t, x, y, z) = U_0(t, x - \xi, y, z) \quad (A7)$$

with the correction term of order α . Substituting (A7) yields the equation,

$$\begin{aligned} \int_0^\tau dt \int_{-\infty}^\infty dx dy dz \alpha(x, z) U_0(\tau - t, x - \xi, y, z) U_0(t, x - \xi, y, z) \\ = -c^2 \int_0^\tau (\tau - t) U_S(t, \xi) d\xi, \end{aligned} \quad (A8)$$

with the correction to the integrand on the left of the order of α^2 , which is negligible for α small. With U given by (A1), the t and y integrations can now be carried out to yield (6).

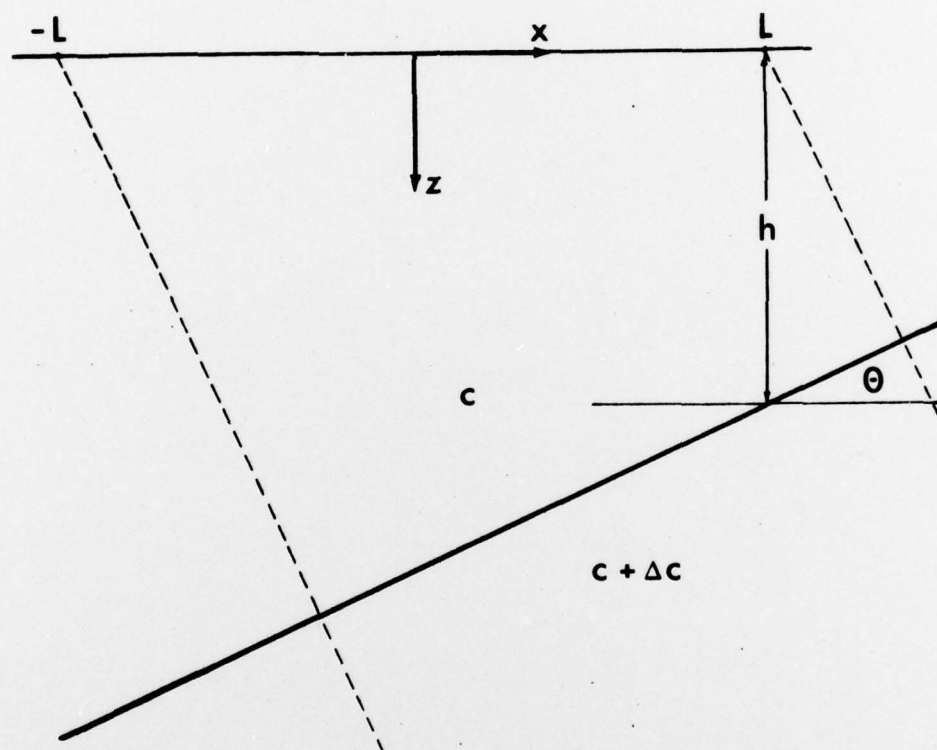


FIGURE 1

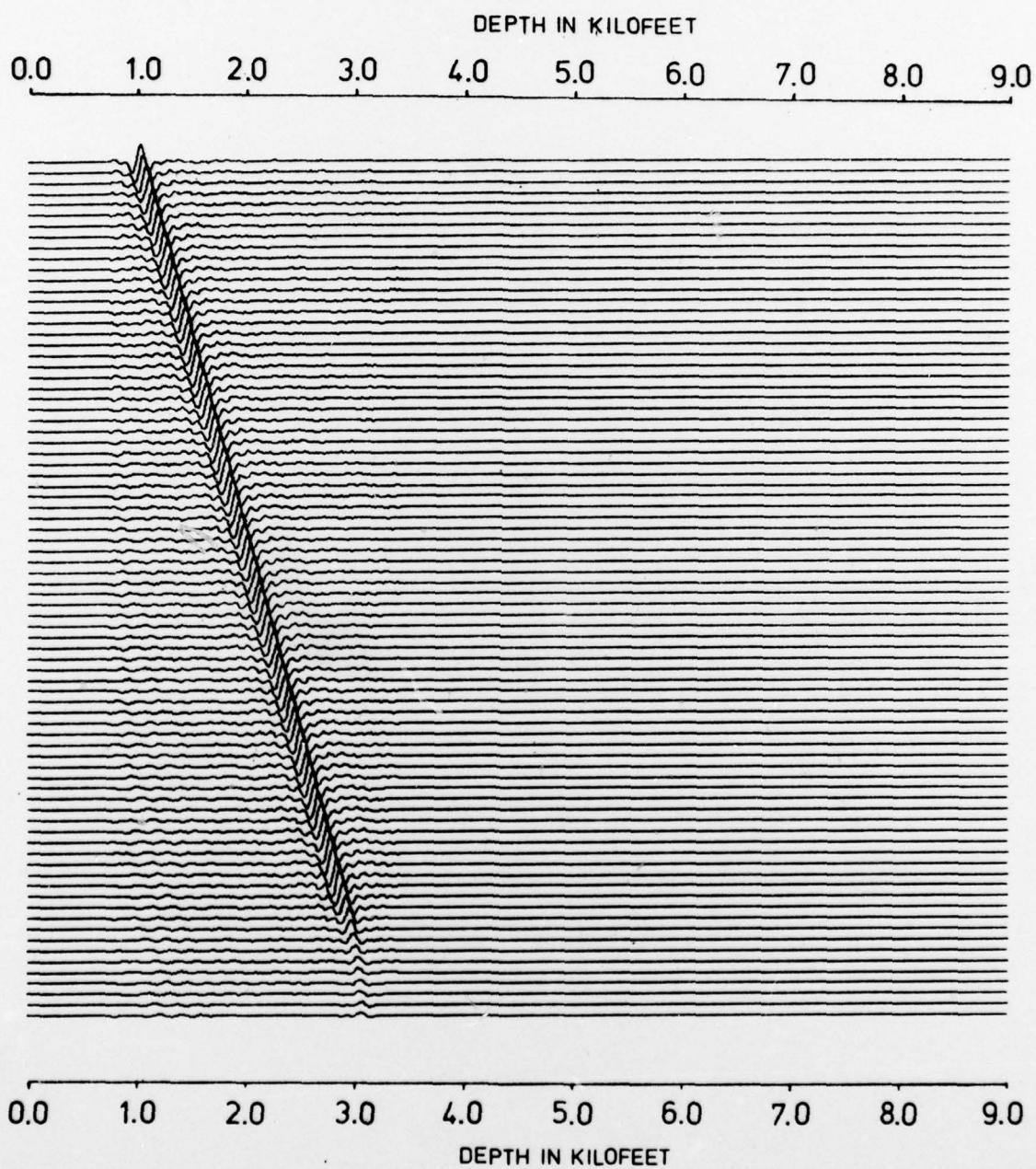
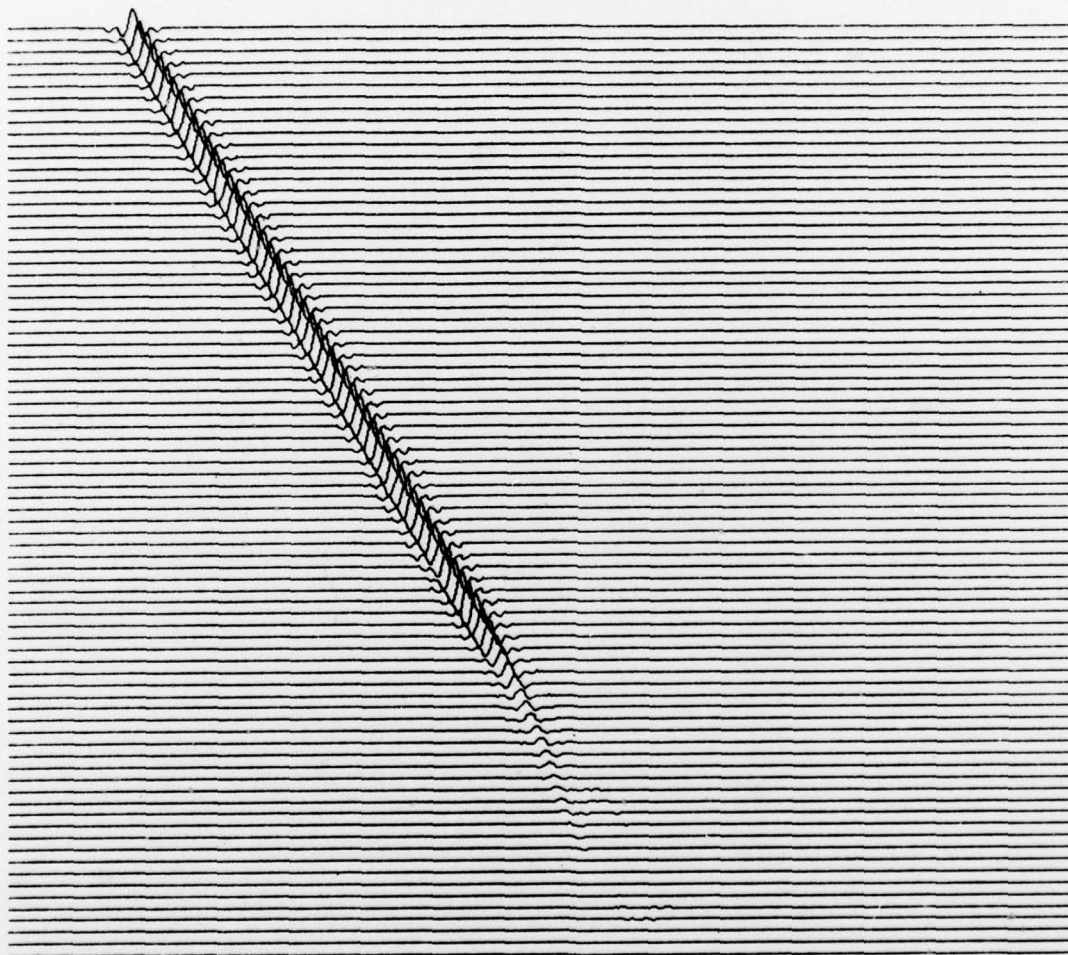


FIGURE 2

DEPTH IN KILOFEET

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0



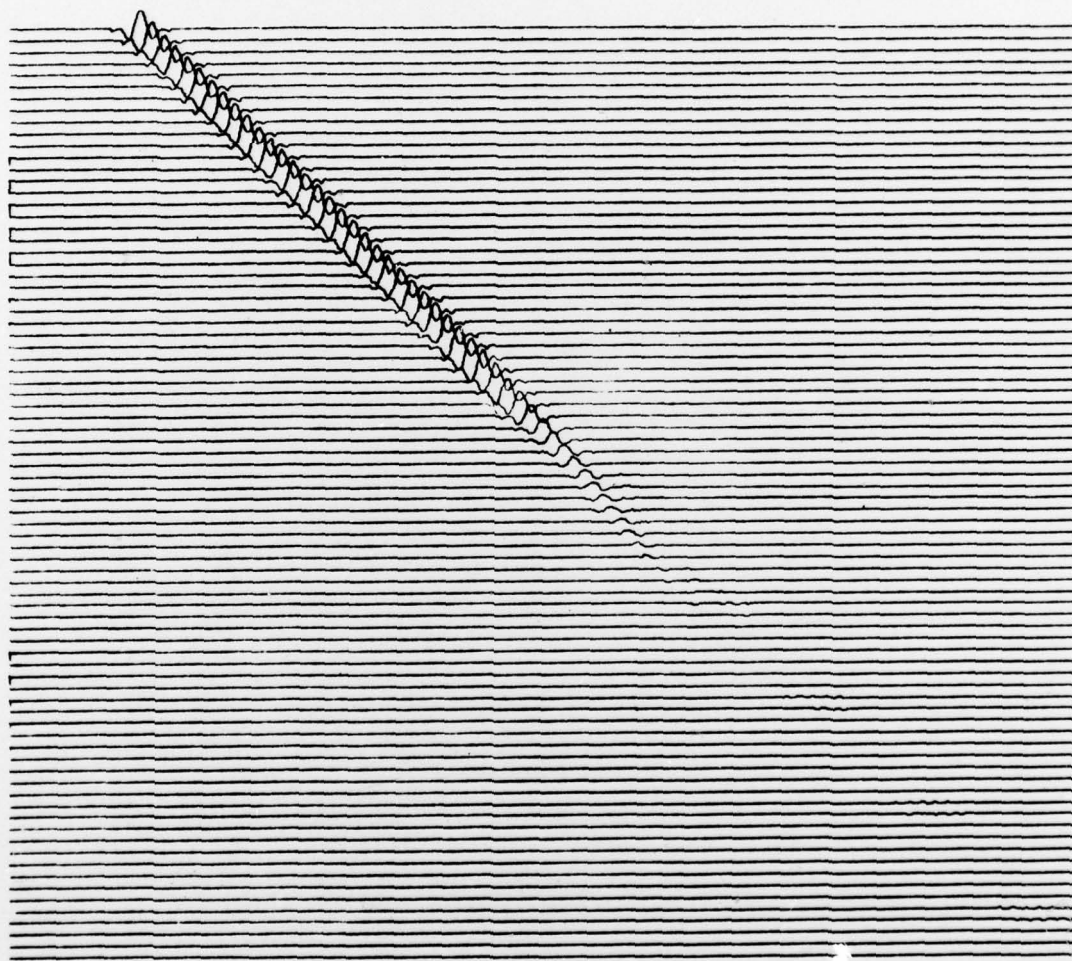
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0

DEPTH IN KILOFEET

FIGURE 3

DEPTH IN KILOFEET

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0



0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0

DEPTH IN KILOFEET

FIGURE 4

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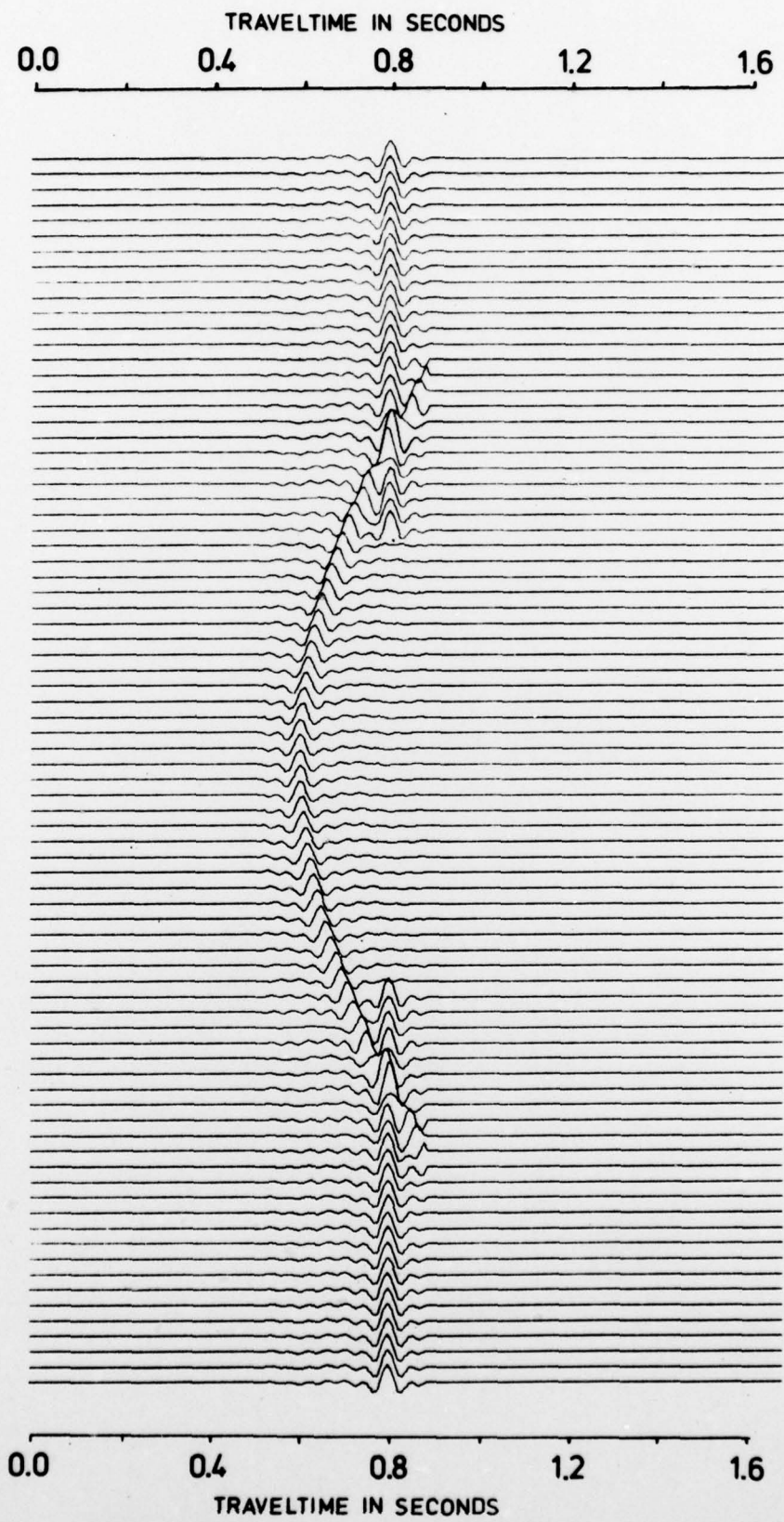


FIGURE 5

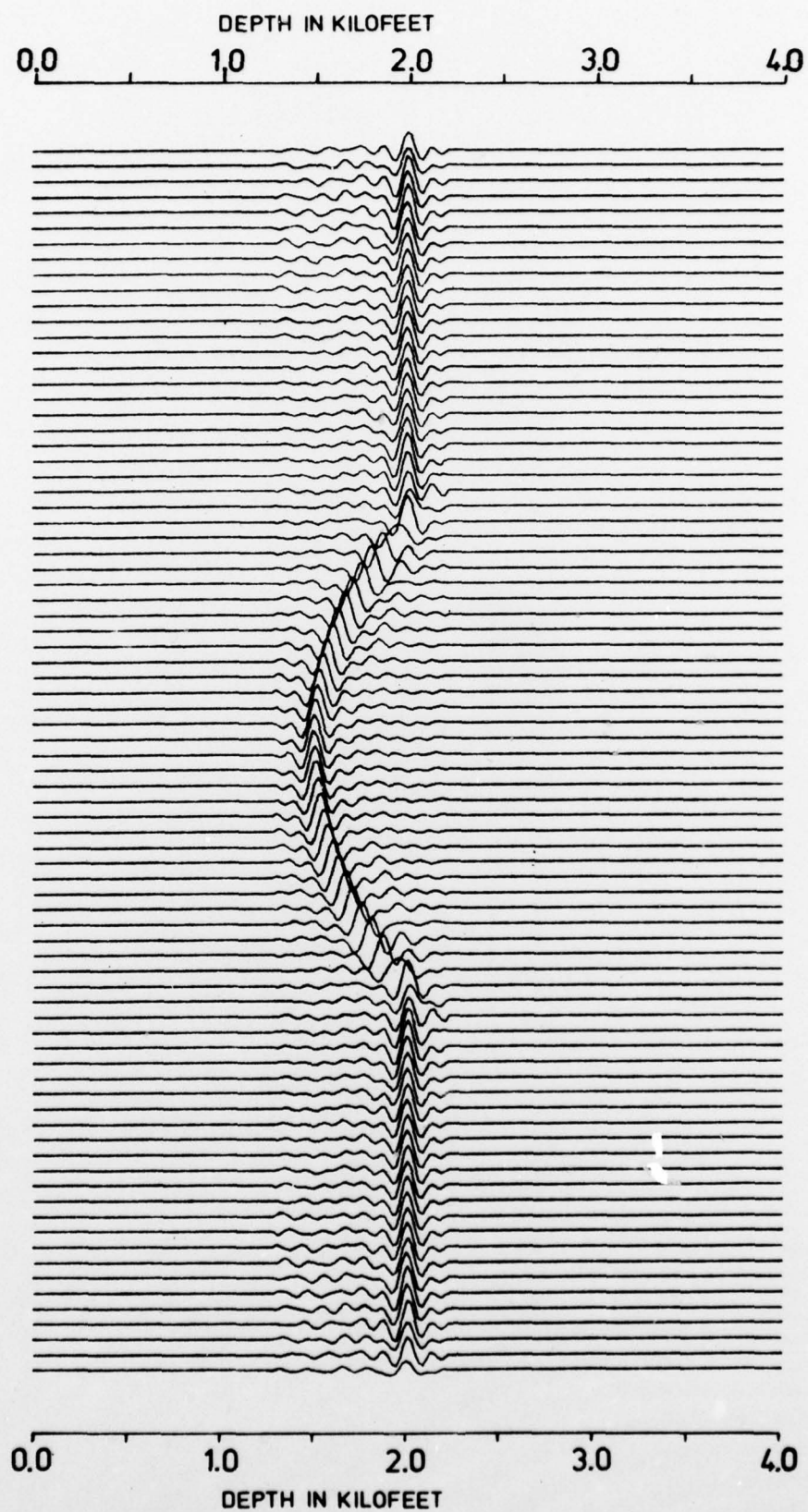


FIGURE 6

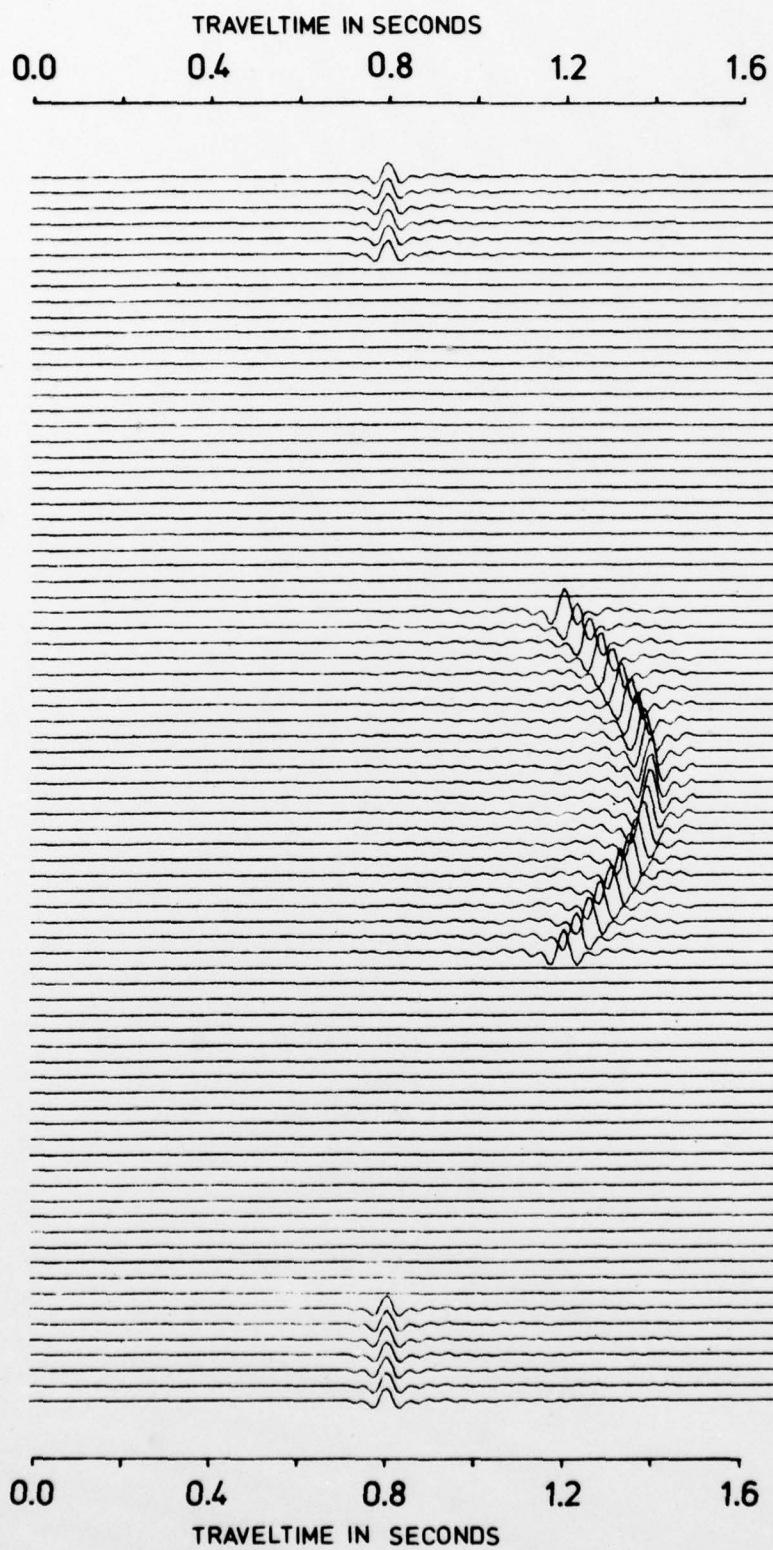


FIGURE 7

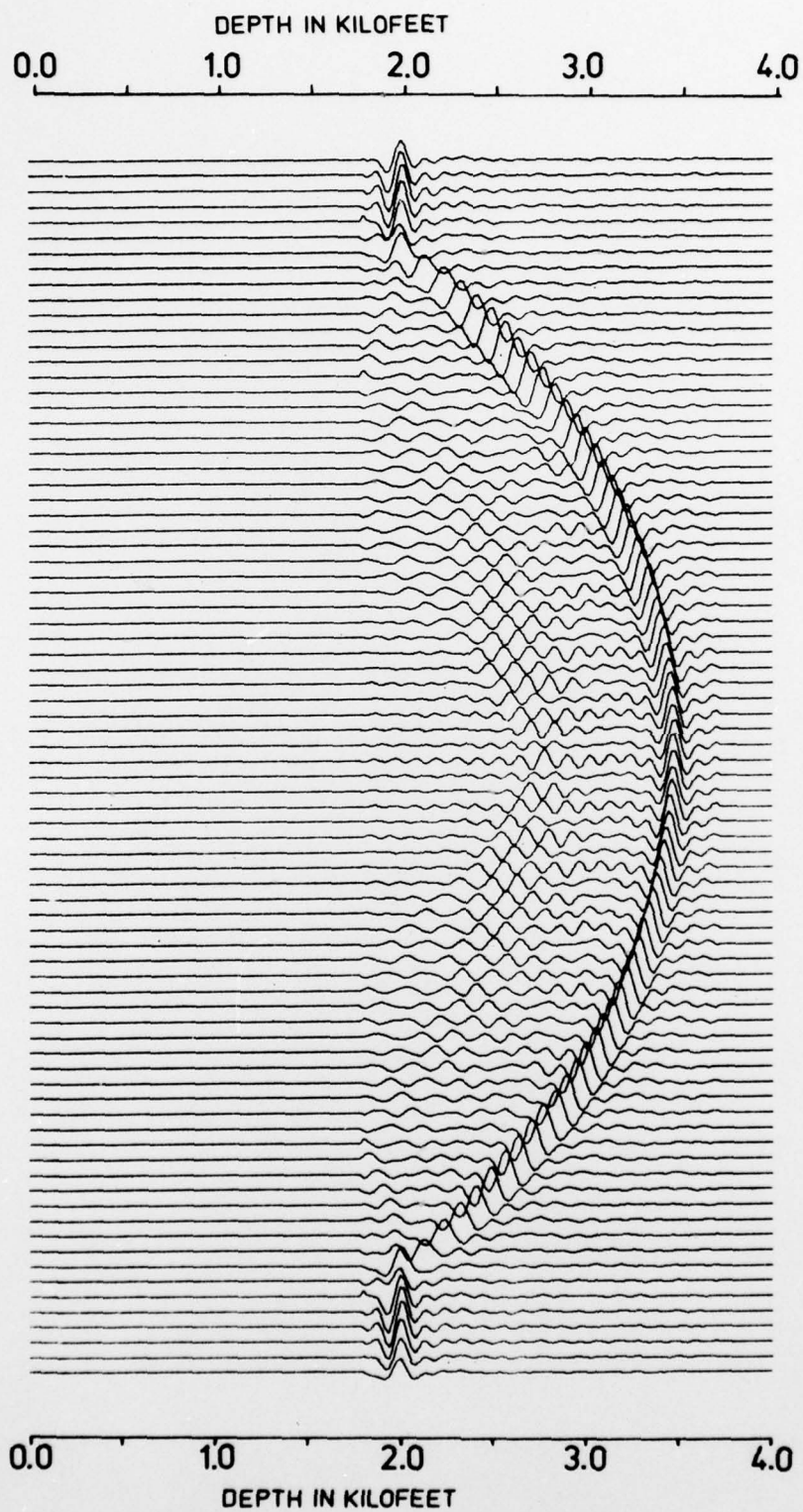
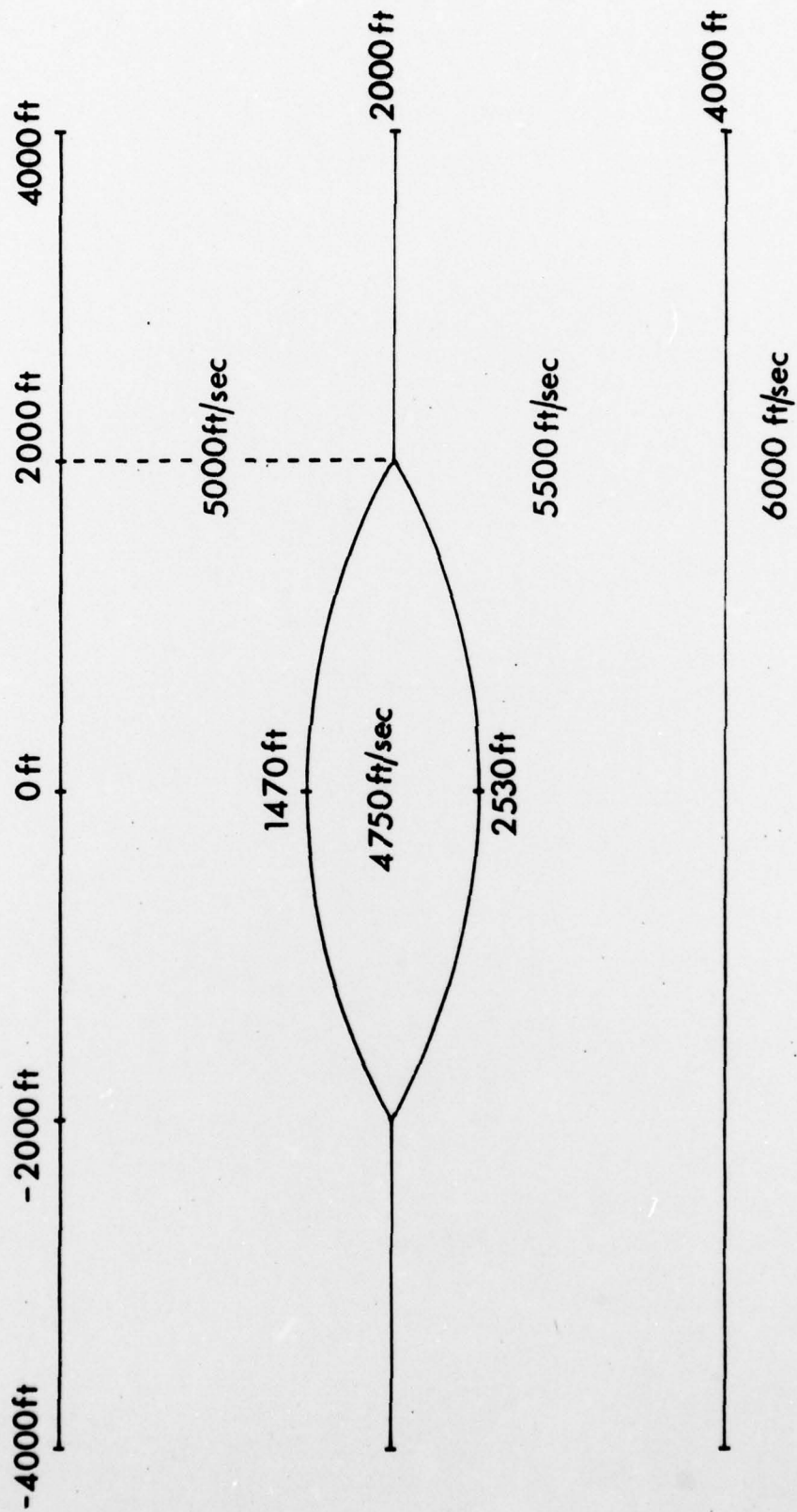


FIGURE 8

2x



ACTUAL MODEL

FIGURE 9

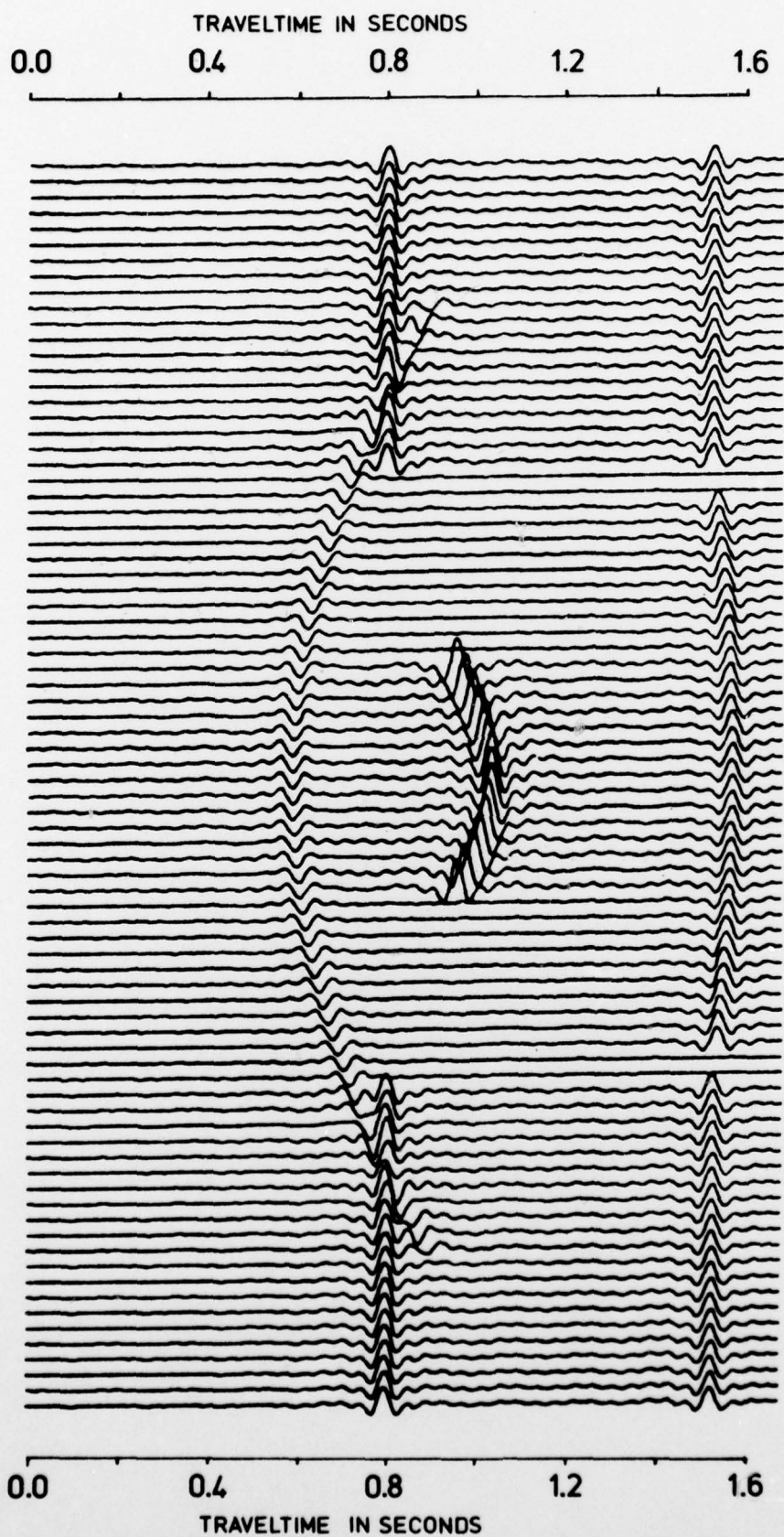


FIGURE 10

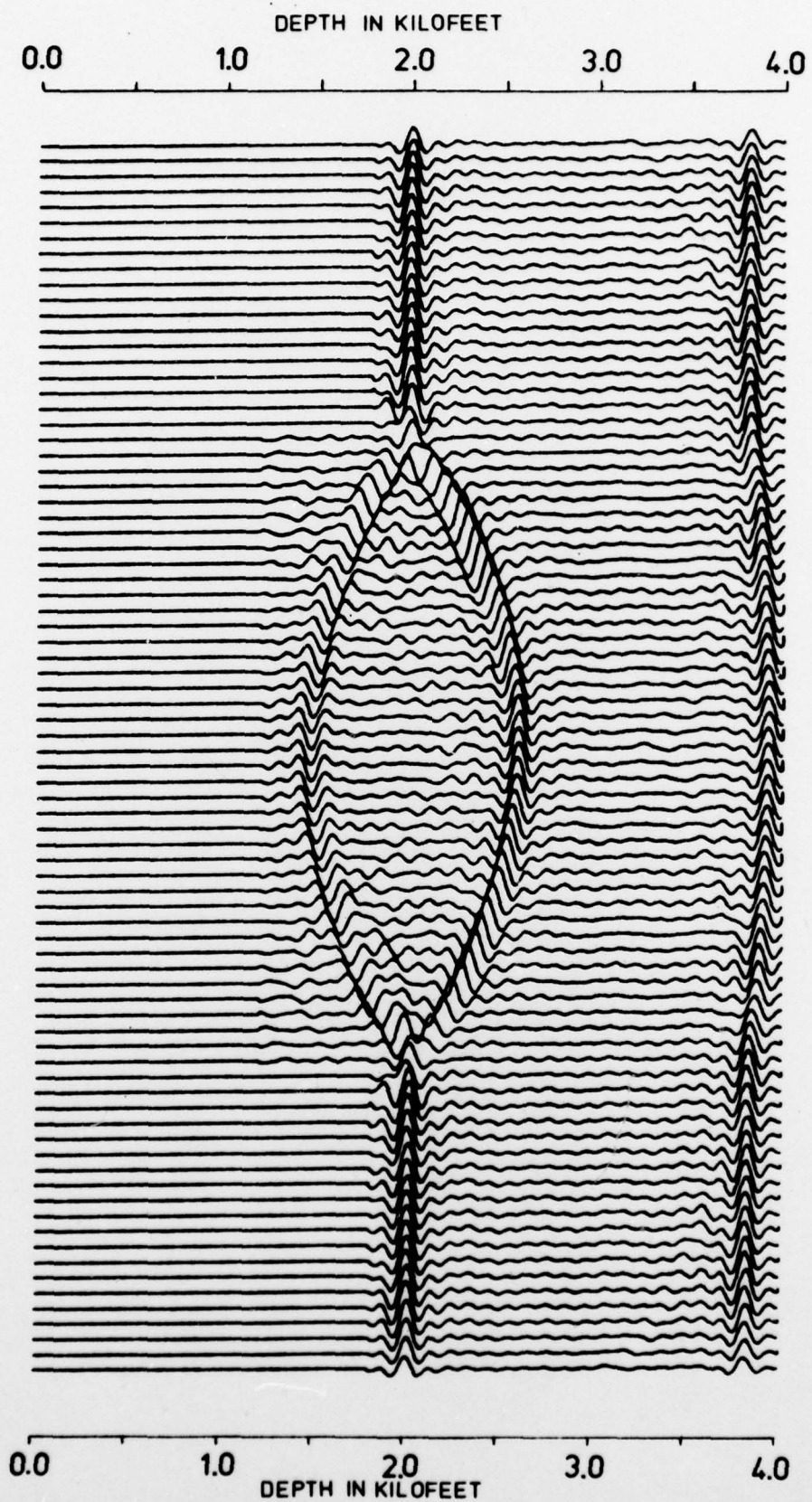
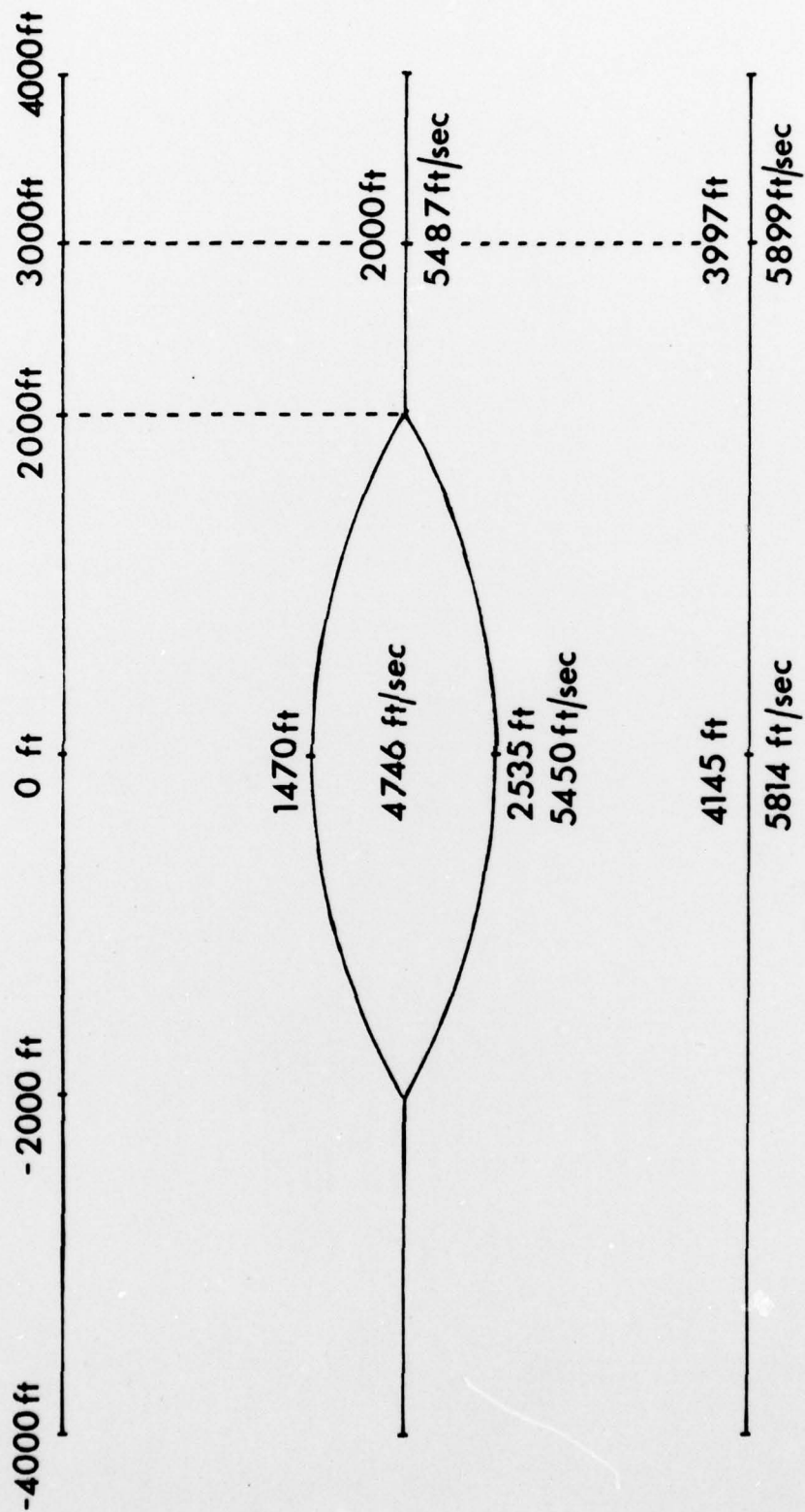


FIGURE 11



COMPUTED MODEL

FIGURE 12