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BEAM DEFLECTION CAUSED BY ANTENNA PHASE ERRORS.(U)

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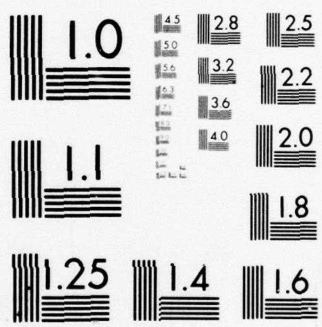

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Beam Deflection Caused by Antenna Phase Errors

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Title of Report: Beam Deflection Caused by Antenna Phase Errors

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We have calculated the mean square beam deflection, which is produced by random phase errors across an antenna. We find that the angular beam deflection $\theta_c = 17.7(\delta/\lambda)(a/L)\,\theta_B$, where δ is the tolerance error on the height of the surface imperfections, a is the transverse correlation length of the surface imperfections, L is the antenna size, λ is the wavelength, and θ_B is the beamwidth.

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Beam Deflection Caused by Antenna Phase Errors

1. INTRODUCTION

The beam deflection ρ_c caused by an arbitrary perturbation can be defined as

$$\underline{\rho}_{c} = \frac{\iint\limits_{-\infty}^{\infty} d^2\rho \; I(\underline{\rho})\,\underline{\rho}}{\iint\limits_{-\infty}^{\infty} I(\underline{\rho})\, d^2\rho} , \qquad (1)$$

where $I(\underline{\rho})$ is the intensity of the radiated beam. If we take the ensemble average of (1) we find that $\langle \rho_c \rangle = 0$, because the beam is equally likely to be deflected in all directions. The mean square deflection $\langle \rho_c^2 \rangle$ is not zero, however, and is given by

$$\langle \rho_c^2 \rangle = \frac{\iint\limits_{-\infty}^{\infty} d^2\rho \; d^2\rho' \; \underline{\rho} \cdot \underline{\rho}' \; \langle I(\underline{\rho})\, I(\underline{\rho}') \rangle}{\left[\iint\limits_{-\infty}^{\infty} d^2\rho \; \langle I(\underline{\rho}) \rangle \right]^2} . \qquad (2)$$

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We will now proceed to compute $\langle I \rangle$ and $\langle I(\underline{\rho})\, I(\underline{\rho}')\rangle$.

If depolarization effects are ignored, it is easy to show that the field radiated, in the Fraunhofer approximation by a planar aperture A with a random phase perturbation, $\Phi(\underline{\rho}_1)$, is

$$E(\rho) = \iint_{-\infty}^{\infty} d^2\rho_1\, e_o(\underline{\rho}_1) \exp\left\{ i \frac{k_o}{r} (\underline{\rho} \cdot \underline{\rho}_1) + \Phi(\underline{\rho}_1) \right\} \quad , \tag{3}$$

where $e_o(\rho_1)$ is the aperture field distribution,† $\rho_1 = (x_1, y_1)$, k_o is the vacuum wavenumber and r is the distance from the center of the aperture to the field point. In writing (3), we have ignored depolarization and neglected a factor $r^{-1} \exp(-i k_o r)$. If we now assume that Φ is a stationary gaussian random variable, it is readily shown that $\langle I \rangle = \langle EE^* \rangle$ is given by

$$\langle I(\underline{\rho})\rangle = \iint_{-\infty}^{\infty} d^2\rho_1\, d^2\rho_2\, e_o(\underline{\rho}_1)\, e_o^*(\underline{\rho}_2) \exp\left\{ i \frac{k_o}{r} \underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) - \sigma^2 + \sigma^2 R(|\underline{\rho}_1 - \underline{\rho}_2|) \right\} \quad , \tag{4}$$

where $\langle\ \rangle$ denotes an ensemble average, $\sigma^2 \equiv \langle \Phi^2 \rangle$, and

$$\sigma^2 R(|\underline{\rho}_1 - \underline{\rho}_2|) = \langle \Phi(\underline{\rho}_1)\, \Phi(\underline{\rho}_2)\rangle \quad . \tag{5}$$

Also

$$\langle I(\underline{\rho})\, I(\underline{\rho}')\rangle = \iiiint_{-\infty}^{\infty} d^2\rho_1\, d^2\rho_2\, d^2\rho_3\, d^2\rho_4\, e_o(\underline{\rho}_1)\, e_o^*(\underline{\rho}_2)\, e_o(\underline{\rho}_3)\, e_o^*(\underline{\rho}_4) \cdot \exp\left\{ i \frac{k_o}{r} \left[\underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) + \underline{\rho}' \cdot (\underline{\rho}_3 - \underline{\rho}_4) \right] - \sigma^2 \left[2 - R_{12} + R_{13} - R_{14} - R_{23} + R_{24} - R_{34} \right] \right\} \quad , \tag{6}$$

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†Footnote: The form in (3) is also appropriate for Fresnel zone calculations of $I(\rho)$, provided we replace r by z and replace $e_o(\rho_1)$ by $e_o(\rho_1) \exp(-i k_o \rho_1^2 / 2z)$, where z is the distance normal to the aperture surface. This same statement applies also to (5) and (6).

where $R_{st} = R(|\underline{\rho}_s - \underline{\rho}_t|)$. It is quite difficult to evaluate the integrals in (4) and (6) for an arbitrary value of σ^2; when $\sigma^2 \ll 1$ and $\sigma^2 \gg 1$, however, it is possible to obtain approximate expressions for $\langle I \rangle$ and $\langle II' \rangle$.

2. WEAK PHASE FLUCTUATIONS

When $\sigma^2 \ll 1$, we may expand $\exp(\sigma^2 R_{st})$ in a Taylor series. We then obtain, correct to second order in σ

$$\langle I(\underline{\rho}) \rangle = e^{-\sigma^2} |h(\underline{\rho})|^2 + \sigma^2 e^{-\sigma^2} C(\underline{\rho}) \quad , \tag{7}$$

$$\begin{aligned} \langle I(\underline{\rho})\, I(\underline{\rho}') \rangle = e^{-2\sigma^2} |h(\underline{\rho})|^2 |h(\underline{\rho}')|^2 + \sigma^2 e^{-2\sigma^2} \Big\{ & |h(\underline{\rho})|^2 C(\underline{\rho}') \\ & + |h(\underline{\rho}')|^2 C(\underline{\rho}) + 2 \operatorname{Re} \left[h(\underline{\rho}') h^*(\underline{\rho}) D(\underline{\rho}, \underline{\rho}') \right] \\ & - 2 \operatorname{Re} \left[h^*(\underline{\rho}) h^*(\underline{\rho}') F(\underline{\rho}, \underline{\rho}') \right] \Big\} + o(\sigma^4) \quad , \end{aligned} \tag{8}$$

where Re denotes "real part of" and

$$h(\underline{\rho}) = \iint_{-\infty}^{\infty} d^2\rho_1 \, e_o(\underline{\rho}_1) \exp\left[i \frac{k_o}{r} \underline{\rho} \cdot \underline{\rho}_1 \right] \quad , \tag{9}$$

$$C(\underline{\rho}) = \iint_{-\infty}^{\infty} d^2\rho_1 \, d^2\rho_2 \, e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) R(|\underline{\rho}_1 - \underline{\rho}_2|) \exp\left[i \frac{k_o}{r} \underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) \right] \quad , \tag{10}$$

$$\begin{aligned} D(\underline{\rho}, \underline{\rho}') = \iint_{-\infty}^{\infty} & d^2\rho_1 \, d^2\rho_2 \, e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) R(|\underline{\rho}_1 - \underline{\rho}_2|) \\ & \exp\left[i \frac{k_o}{r} (\underline{\rho} \cdot \underline{\rho}_1 - \underline{\rho}' \cdot \underline{\rho}_2) \right] \quad , \end{aligned} \tag{11}$$

$$F(\underline{\rho},\underline{\rho}') = \iint_{-\infty}^{\infty} d^2\rho_1\, d^2\rho_2\, e_o(\underline{\rho}_1)\, e_o(\underline{\rho}_2)\, R\left(|\underline{\rho}_1 - \underline{\rho}_2|\right)$$

$$\exp\left[i \frac{k_o}{r} (\underline{\rho} \cdot \underline{\rho}_1 + \underline{\rho}' \cdot \underline{\rho}_2) \right] . \tag{12}$$

If we now substitute† (7) and (8) into (2) and ignore terms of order σ^4 and smaller, we find that

$$\langle \rho_c^2 \rangle = \frac{\sigma^2 \iint_{-\infty}^{\infty} d^2\rho\, d^2\rho'\, \underline{\rho} \cdot \underline{\rho}'\, M(\underline{\rho},\underline{\rho}')}{\left[\iint_{-\infty}^{\infty} d^2\rho\, |h(\underline{\rho})|^2 \right]^2} , \tag{13}$$

where

$$M(\underline{\rho},\underline{\rho}') = 2\,R_e \left[h(\underline{\rho}')\, h^*(\underline{\rho})\, D(\underline{\rho},\underline{\rho}') \right] - 2\,\mathrm{Re} \left[h^*(\underline{\rho})\, h^*(\underline{\rho}')\, F(\underline{\rho},\underline{\rho}') \right] . \tag{14}$$

In order to easily evaluate the integrals for h, D, and F, we shall assume that

$$e_o(\underline{\rho}_1) = \exp\left(- \frac{\rho_1^2}{L^2} \right) , \tag{15}$$

where

$$R(|\underline{\rho}_1 - \underline{\rho}_2|) = \exp\left(- \frac{|\underline{\rho}_1 - \underline{\rho}_2|^2}{a^2} \right) . \tag{16}$$

We next substitute (15) and (16) into (9), (11) and (12) and evaluate the integrals. The results for h, D, and F are then substituted into (13), which can be evaluated using the result

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†Footnote: We do not calculate $\langle \rho_s^2 \rangle$ for the case when $\sigma^2 \ll 1$ because there is generally very little beam broadening.

$$\iint_{-\infty}^{\infty} d^2\rho \, d^2\rho' \, \underline{\rho} \cdot \underline{\rho}' \exp\left[-\beta(\rho^2 + \rho'^2) \pm \gamma \underline{\rho} \cdot \underline{\rho}'\right] = \frac{\pm \pi^2 \gamma}{2\left(\beta^2 - \frac{\gamma^2}{4}\right)^2} . \tag{17}$$

The final answer is

$$\langle \sin^2 \theta_c \rangle = \frac{\langle \rho_c^2 \rangle}{r^2} = \left[\frac{2\sigma\left(\frac{a}{L}\right)}{k_o L \left(1 + \frac{a^2}{L^2}\right)}\right]^2 \tag{18}$$

where θ_c is the angular deflection of the beam.

3. DISCUSSION AND CONCLUSIONS

Note that in deriving (18), we have not made any assumptions about the relative sizes of a and L. When $a \gg L$ we see that $\theta_c \simeq 2\sigma/(k_o a)$, so that $\theta_c \to 0$ for $a \to \infty$, as expected. When a = L we find that θ_c is a maximum, and has the value $\theta_c = \sigma/(k_o L)$. Finally when $a \ll L$, we find $\theta_c \simeq 2a\sigma/(k_o L^2)$. This last result is useful in reflector antenna design, because in that case we can set† $\sigma = 2 k_o \delta$ where δ is the tolerance error on the height of the surface imperfections. We can then rewrite (18) as

$$\frac{\theta_c}{\theta_B} = 5.64\,\pi \left(\frac{\delta}{\lambda}\right)\left(\frac{a}{L}\right) \tag{19}$$

where λ is the signal wavelength and θ_B is the beamwidth of the main beam of the antenna, and is defined by $\theta_B^2 = 2/(k_o L)^2$. Note that although $\theta_c/\theta_B \ll 1$, the deflection θ_c can still be important especially for high-accuracy monopulse systems.

As a numerical example, suppose $\delta = \lambda/32$ and $a/L = 1/5$. Then $\theta_c/\theta_B = 0.11$, which is a significant deflection.

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†Footnote: When the phase fluctuations are produced by a turbulent medium over the antenna it is appropriate to replace $k_o^2/4 \int dz \int dz' \langle n(z)\, n(z') \rangle$ where n is the index of refraction of the medium.