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STABLE ADAPTIVE CONTROLLER DESIGN DIRECT CONTROL,(U)
MAR 78 K S NARENDRA, L S VALAVANI   N00014-76-C-0017
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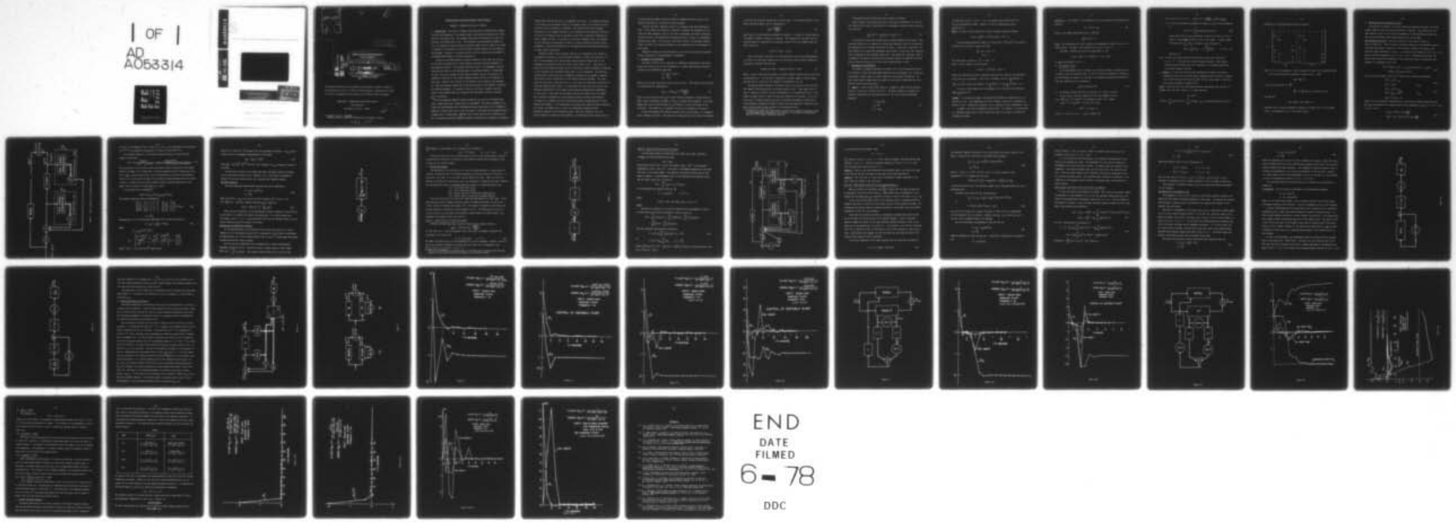

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Stable Adaptive Controller Design - Direct Control

Kumpati S. Narendra and Lena S. Valavani

1. Introduction: The theory of adaptive observers, which simultaneously estimate the state and parameters of an unknown linear time-invariant system, is well understood at the present time. The original observers were proposed in [1] and [2] and were later generalized in [3]. The extensions to multivariable systems [4], [5], the analysis of related stability questions [6], [7] and the study of the speed of response of the observer [8] have provided valuable insight into the design of such systems. Further, it has been shown that all the results can be extended in a straightforward manner to discrete systems [9].

Contrary to the expectations of research workers in the field, the solution of the control problem did not follow directly from the solutions obtained for the adaptive observer. The principal difficulty lies in the assumption that the input and the output of the observed plant are uniformly bounded. As described in [10], this is not assured in the control problem, since it is precisely the question of stability of the overall feedback system that has to be resolved in this case. Further, the structure of the observer can be chosen by the designer so that the necessary signals can be fed into any part of the observer to generate the desired estimates. In contrast to this, in the control problem, only the input to the unknown plant can be affected and this poses a serious constraint in its resolution.

In this paper, the adaptive control problem for the case of an unknown plant with a single input and a single output is considered. The plant is assumed to be linear and time-invariant and a differentiator-free controller is to be designed to generate a control input u(t) so that the output of the plant evolves asymptotically towards that of a given model. Earlier work in this field is due to Monopoli [11] who, following the approach adopted in adaptive observers [3], suggested an ingenious scheme which involves the use of an augmented error model. The approach developed in this paper has the general flavor of the method used in [11] though the actual structure of the controller is different. The latter provides the necessary motivation for the changes that have to be introduced in the controller structure as well as the adaptive laws on the basis of the prior information regarding the plant transfer function. In section 6, the various arguments used in the design of the controller are explained in terms of a single arbitrarily located control parameter to indicate the generality of the approach and its potential application in other control situations.

Perhaps the most crucial question that has to be answered in the context of the adaptive controller, as in the adaptive observer, concerns the stability of a set of non-autonomous error differential equations which assures that all the parameters are bounded and that the output error tends to zero. This point, on which the entire design is based, has not received the careful attention it deserves in [11] and hence the conclusions drawn there are not justified. The principal contributions of this paper are a systematic procedure for the design of adaptive controllers and the clarification of the related stability problems. For cases where the plant has n poles and (n-1) or (n-2) zeros, the controller is shown to be uniformly asymptotically stable. In cases where the plant has $(\leq)n-3$ zeros, the necessary modification of the method raises questions regarding the boundedness of the outputs of the model and the plant even when the output error is bounded. This problem is formulated in a more general setting in terms of the behavior of signals in a feedback loop when certain known linear transformations of these signals are bounded. It is conjectured in the paper that under the above conditions the closed loop signals will, in fact, be bounded, thus assuring uniform asymptotic stability of the adaptive system. While the validity of the conjecture remains at present an open question, its verification will justify the use of the relatively simple controller structure suggested here and result in the complete resolution of the adaptive control problem.

Indirect and direct adaptive control of an unknown plant were discussed in [10]. In the former, an observer is employed to estimate the parameters of the plant and these estimates, in turn, are used to determine the control parameters. In direct control, the parameters are adjusted directly, without an intermediate identification stage. In this paper, only direct control is discussed; indirect control as well as the relation between the two approaches will be discussed in a later paper.

Simulation results are included mainly at the end of the paper for different control situations of varying degrees of complexity.

2. Statement of the Problem:

The plant P that is to be controlled is completely represented by the input-output pair $\{u(t),y_p(t)\}$ and can be modeled by a linear time-invariant system described by the differential equations:

$$\begin{aligned} \dot{x}_p &= A_p x_p + b_p u(t) \\ y_p &= h^T x_p \end{aligned} \qquad (1)$$

where A_p is an (nxn) matrix and h and b_p are n-vectors. The transfer function $W_p(s)$ of the plant may be represented as

$$W_p(s) = h^T(sI-A_p)^{-1} \triangleq \frac{k_p Z_p(s)}{R_p(s)} \quad . \qquad (2)$$

$W_p(s)$ is strictly proper with $Z_p(s)$ a monic Hurwitz polynomial of degree $m(\leq n-1)$, $R_p(s)$ a monic polynomial of degree n, and k_p a constant gain parameter. We further assume that only m,n and the sign of k_p are known for use in the design of the controller. (The sign of k_p is assumed to be positive throughout this paper.)

A model M represents the behavior expected from the plant when it is augmented with a suitable controller. The model has a reference input r(t) which is piecewise continuous and uniformly bounded and an output $y_M(t)$. The transfer function of the model, denoted by $W_M(s)$, may be represented as

$$W_M(s) \overset{\Delta}{=} \frac{k_M Z_M(s)}{R_M(s)} \tag{3}$$

where $Z_M(s)$ is a monic Hurwitz polynomial of degree $r \leq m$, $R_M(s)$ is a monic Hurwitz polynomial of degree n and k_M is a constant. The deviation of the plant from the desired behavior is measured by the absolute value of the error between plant and model outputs as

$$|e_1(t)| \overset{\Delta}{=} |y_p(t) - y_M(t)| \tag{4}$$

Since the dynamic controller to be designed should be free of differentiators it follows that the plant together with the controller (with constant parameters) would have a transfer function with

$$\text{(number of poles)} - \text{(number of zeros)} \gtrsim (n-m) \tag{5}$$

Hence, to have a well defined problem, the model transfer function should also satisfy inequality (5). For the sake of simplicity, we shall assume in the following sections that $r = m$ and that the equality is satisfied in eq. (5).

The design method proposed in this paper calls for an operator L(s) such that the transfer function $W_M(s)L(s)$ is strictly positive real.[†] Once again, for the sake of simplicity and with no loss of generality, it is assumed that L(s), a polynomial of degree (n-m-1), exists so that this condition is satisfied. [It is obvious that a rational function $L_1(s)$ with a numerator polynomial of degree (n-1) and denominator polynomial of degree m exists such that $W_M(s)L(s)$ is positive real. Such a function can be used in place of the polynomial L(s).]

[†] Throughout the paper both differential equations and transfer functions are used in the arguments and, depending on the context, 's' is used as a differential operator or the Laplace transform variable.

The adaptive control problem may now be stated as follows:

Given a plant P with input-output pair $\{u(t),y_p(t)\}$ described by (1) and (2) and a model $M(\{r(t),y_m(t)\})$ described by (3) determine a suitable control function u(t) such that

$$\lim_{t\to\infty} |e_1(t)| = \lim_{t\to\infty} |y_p(t) - y_M(t)| = 0 \tag{6}$$

The solution to the above problem may be broadly divided into two parts. The first part (section 4) is algebraic in nature and addresses itself to the question of realizability of a suitable controller structure. Using such a structure, constant values of the controller parameters can be shown to exist such that condition (6) is satisfied for any arbitrary input r(t). The second part (sections 5 and 6) is analytical in nature and deals with the adaptive scheme for updating the parameters so that the error $e_1(t)$ in (6) evolves asymptotically to zero.

3. Mathematical Preliminaries:

The theory of adaptive observers, as shown in [3], may be developed entirely on the basis of three prototypes which indicate how the parameters of a system can be adjusted to match those of a model using only input-output information. We state below several lemmas which are useful in the control problem and which are derived from the third prototype.

a) Lemma 1: Given a stable (nxn) matrix A, a symmetric (mxm) positive definite matrix Γ, vectors $h,d \in \mathbb{R}^n$ and $\omega(t):[0,\infty) \to \mathbb{R}^m$ whose elements are bounded and piecewise continuous, the equilibrium state of the set of (n+m) differential equations

$$\begin{aligned} \dot{e} &= Ae + d\phi^T\omega \\ e_1 &= h^T e \\ \dot{\phi} &= -\Gamma e_1\omega \end{aligned} \tag{7}$$

is stable and $|e_1(t)| \to 0$ as $t \to \infty$ if the transfer function $h^T(sI-A)^{-1}d$ is strictly positive real (SPR). Further, if $\omega(t)$ is "sufficiently rich", $\lim_{t\to\infty} \|\phi(t)\| = 0$.

Proof: The proof follows directly by using a Lyapunov function candidate

$$V(e,\phi) = \frac{1}{2}[e^TPe + \phi^T\Gamma^{-1}\phi] \text{ with } P = P^T > 0.$$

From the Kalman-Yacubovich lemma it is known that if $h^T(sI-A)^{-1}d$ is strictly positive real, a matrix P exists such that

$$A^TP + PA = -qq^T - \varepsilon L$$

$$Pb = h$$

for some vector q, matrix $L = L^T > 0$ and $\varepsilon > 0$.

This, in turn, yields a time derivative

$$\dot{V}(e,\phi) = -\frac{1}{2} e^T(qq^T + \varepsilon L)e \leq 0$$

Hence, the equilibrium state is uniformly stable and $e(t)$ and $\phi(t)$ are bounded if $e(t_0)$ and $\phi(t_0)$ are bounded. Since $\omega(t)$ is bounded, from [7], $\lim_{t\to\infty} \|e(t)\| = 0$. This, in turn, yields from equation (7) $\lim_{t\to\infty} \|\dot{\phi}(t)\| = 0$ and $\lim_{t\to\infty} \phi^T(t)\omega(t) = 0$.

If $\omega(t)$ is "sufficiently rich" as defined in [7], the above two conditions imply $\lim_{t\to\infty} \|\phi(t)\| = 0$.

Comment: It is seen from Lemma 1 that $\|e(t)\|$ and $\|\phi(t)\|$ are bounded with no constraints on $\omega(t)$. The boundedness of $\|\omega(t)\|$, however, assures the convergence of $\|e(t)\|$ to zero. Further, the "richness" of $\omega(t)$ results in $\phi_i(t)$ converging to zero.

In Lemma 1 $\omega(t)$ was an independent input vector. If, however, it is generated as a feedback signal using a stable system with $e_1(t)$ as an input, we obtain the following corollary.

Corollary 1: If in Lemma 1, the condition on $\omega(t)$ is replaced by the differential equation

$$\dot{\omega} = \Lambda\, \omega(t) + be_1 \tag{8}$$

where Λ is an $(m \times m)$ stable matrix and $b \in \mathbb{R}^m$ then

$$\lim_{t\to\infty} |e_1(t)| = 0$$

Proof: The boundedness of $|e_1(t)|$ assures the boundedness of $\|\omega(t)\|$ since Λ is a stable matrix. By Lemma 1 this assures that $|e_1(t)| \to 0$ as $t \to \infty$.

Corollary 1 implies that the set of equations (7), (8)

$$\dot{e} = Ae + d\phi^T\omega; \quad \dot{\phi} = -\Gamma(h^Te)\omega; \quad \dot{\omega} = \Lambda\omega + bh^Te$$

is stable and $\|e(t)\| \to 0$ as $t \to \infty$.

b) The Operator $P_L(\theta)$:

In the development of the solution to the control problem, an operator $P_L(\theta)$ is found to play an important role. If $L(s)$ is a Hurwitz polynomial of degree n_1 in the differential operator 's' and $\theta(t)$ is a bounded differentiable function of time, the operator $P_L(\theta)$ is defined as

$$P_L(\theta) \triangleq L(s)\theta(t)L^{-1}(s) \tag{9}$$

(i) It follows directly that $P_L(\theta)$ is linear in θ. $P_L(c\theta) = cP_L(\theta)$; $P_L(\alpha\theta_1 + \beta\theta_2) = \alpha P_L(\theta_1) + \beta P_L(\theta_2)$ for all real numbers α, β and c.

(ii) When $\theta(t) = \theta^*$, where θ^* is a constant, $P_L(\theta^*) = \theta^*$. Hence, $P_L(\theta) - \theta^* = P_L(\theta-\theta^*)$. If $\theta(t) = \theta^* + \phi(t)$

$$P_L(\theta) - \theta = P_L(\phi) - \phi$$

(iii) a) If $L(s) = s + a$, $P_L(\theta) = [\theta+\dot{\theta}L^{-1}(s)]$

b) If $L(s) = s^2 + as + b$, $P_L(\theta) = [\theta + \dot{\theta} \frac{s+a/2}{L(s)} + (s+a/2)\dot{\theta} \frac{1}{L(s)}]$

c) If L(s) is any Hurwitz polynomial of degree n_1, $P_L(\theta)$ can be expressed as

$$P_L(\theta) = \theta + \sum_{i=1}^{n_1} [B_i(s)\dot{\theta}(t)A_i(s)]L^{-1}(s) \tag{10}$$

where $B_i(s)$ and $A_i(s)$ are polynomials of degree (i-1) and (i) respectively. The expression given in (10) is not unique. A useful expansion when $L(s) = \sum_{j=1}^{n_1} (s+Z_j)$ in terms of $B_i(s)$ and $A_i(s)$ is

$$B_i(s) = \prod_{j=2}^{i} (s+Z_j) \quad ; \quad A_i = \prod_{j=1}^{n_1-i} (s+Z_j) \qquad i = 1,2,\ldots,n_1$$

and $B_1(s) \equiv 1$.

(iv) If W(s) is a stable transfer function with m poles and n zeros and $n_1 \leq n - m - 1$, $W(s)P_L(\theta)$ is a linear bounded operator. This follows directly from the facts that L(s) is Hurwitz, θ is bounded and W(s)L(s) is a stable transfer function.

c) Lemma 2: If Q(s) and T(s) are monic polynomials of degree n and $m(\leq n-1)$ respectively which are relatively prime, polynomials P(s) and R(s) of degree (n-1) with P(s) monic exist such that PQ + RT can be made equal to any arbitrary monic polynomial of degree (2n-1).

Proof: Since Q(s) and T(s) are relatively prime, polynomials A(s) and B(s) of degrees (m-1) and (n-1), where $m \geq 1$, exist such that

$$AQ + BT = 1$$

If $Q(s) = \sum_{i=0}^{n} q_i s^i$ and $T(s) = \sum_{i=0}^{m} t_i s^i$, $(q_n = t_m = 1)$, the above equation can be satisfied if the (n+m-2)x(n+m-2) matrix C given by

$$C = \left[\underbrace{\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ q_{n-1} & 1 & & & \cdot \\ \cdot & q_{n-1} & & & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & 1 \\ q_0 & \cdot & & & q_{n-1} \\ 0 & q_0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & q_0 \\ 0 & 0 & & & 0 \end{array}}_{m-1} \middle| \underbrace{\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ t_{m-1} & 1 & & & & 0 \\ \cdot & t_{m-1} & \cdot & & & 0 \\ \cdot & \cdot & & & & 0 \\ t_0 & \cdot & & & & 0 \\ 0 & t_0 & & & & 1 \\ 0 & 0 & & & & t_{m-1} \\ 0 & 0 & & & & t_{m-2} \\ 0 & 0 & & & & t_{m-k} \end{array}}_{n-1} \right]$$

where $0 \leq k \leq m$, is nonsingular. This is assured if Q and T are relatively prime.

Let M be the arbitrary polynomial of degree $(\leq)2n-1$. Then

$$MAQ + MBT = M$$

If N is the proper part of $\frac{BM}{Q}$

$$\frac{BM}{Q} = N + \frac{R}{Q} \text{ where R is of degree } (\leq)n-1.$$

We then have

$$(MA + NT)Q + (MB - NQ)T = M$$

where MA + NT is a monic polynomial of degree n-1 and MB - NQ = R is of degree $\leq(n-1)$. Choosing MA + NT = P, the result follows.

4. The Structure of the Adaptive System:

As mentioned in section 1, we first require the adaptive controller to possess enough freedom so that the control problem has a solution. This implies that for some constant values of the controlled parameters the corresponding transfer function of the controlled system exactly matches that of the model.

The Controller Structure: The basic structure of the adaptive system is shown in Figure (1). This structure is suitably modified for the different control situations considered in section 5. The controller consists of a gain k_0, and auxiliary signal generators F_1 and F_2. F_1 contains (n-1) parameters $c_i (i = 1,2,...n-1)$ and F_2 contains n parameters $d_i (i = 0, 1,2,...,n-1)$. Together with k_0 these constitute the 2n adjustable parameters of the controller, which are denoted by the elements of a parameter vector $\theta(t)$.

$$\begin{aligned} \theta^T(t) &\triangleq [k_0(t),c_1(t),\dots,c_{n-1}(t),d_0(t),d_1(t)\dots d_{n-1}(t)] \\ &\triangleq [k_0(t),c^T(t),d_0(t),d^T(t)] \end{aligned} \qquad (11)$$

F_1 and F_2 are described by the $(n-1)^{th}$ order vector differential equations

$$\begin{aligned} \dot{v}^{(1)} &= \Lambda v^{(1)} + bu \\ w^{(1)} &= c^T v^{(1)} \qquad (F_1) \end{aligned} \qquad (12a)$$

$$\begin{aligned} \dot{v}^{(2)} &= \Lambda v^{(2)} + by_p \\ w^{(2)} &= d_0 y_p + d^T v^{(2)} \qquad (F_2) \end{aligned} \qquad (12b)$$

$$\text{and } b^T = [0,0,\dots 1].$$

where Λ is an ((n-1)x(n-1)) stable matrix, F_1 and F_2 have transfer functions $W_1(s)$ and $W_2(s)$ respectively for constant values of the parameters c_i and d_j $(i = 1,2,...n-1; j = 0,1,...n-1)$.

$$\begin{aligned} W_1(s) &= c^T(sI-\Lambda)^{-1}b \triangleq \frac{C(s)}{N(s)} \\ W_2(s) &= d_0 + d^T(sI-\Lambda)^{-1}b \triangleq d_0 + \frac{D(s)}{N(s)} \end{aligned} \qquad (13)$$

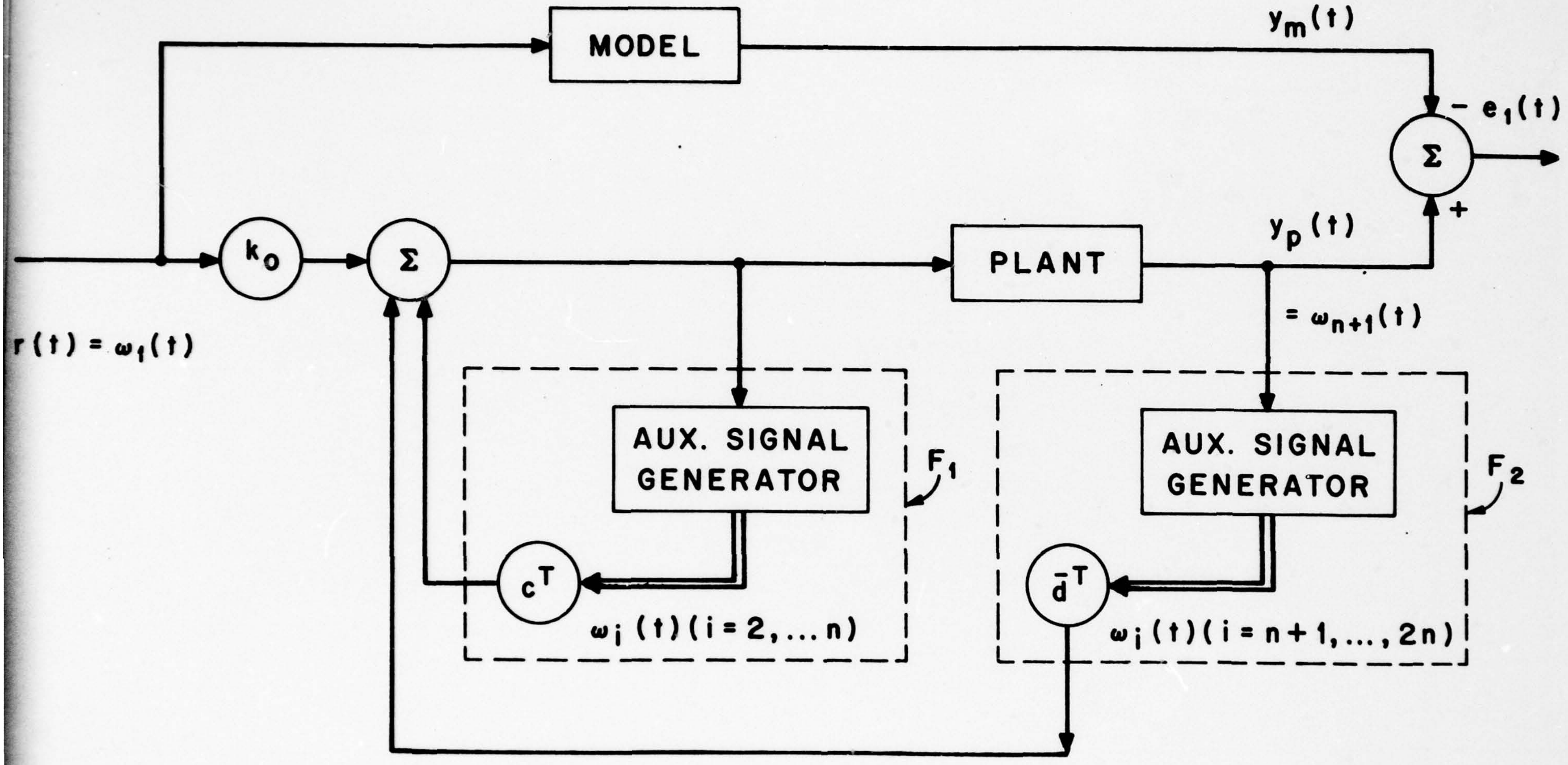


Figure 1. Basic Structure of the Adaptive System.

If (Λ,b) is in companion form, c_i and d_i $(i = 1,2,\ldots,n-1)$ represent the coefficients of s^{n-i-1} of the numerator polynomials of $c^T(sI-\Lambda)^{-1}b$ and $d^T(sI-\Lambda)^{-1}b$.

For constant values of θ, the overall transfer function of the controlled system is $W(s)$ where

$$W(s) = \frac{k_0 W_p(s)}{1 + W_1(s) + W_p(s)W_2(s)} = \frac{k_0 k_p Z_p(s)N(s)}{[N(s) + C(s)]R_p(s)+k_p Z_p(s)[D(s)+d_0 N(s)]} \qquad (14)$$

$W(s)$ as given in (14) has $(3n-2)$ poles and $(2n+m-2)$ zeros of which $(n-1)$ are common. Further, by lemma 2 it is known that a constant parameter vector θ^* exists such that $W(s) = W_M(s)$, which implies there are $(n-1)$ additional pole and zero cancellations. From (14) it is seen that the poles of the auxiliary signal generators are also zeros of $W(s)$. In order that the overall transfer function have the same zeros as the model, $N(s)$ is chosen to contain $Z_M(s)$ as a factor.

If the vector $\omega(t)$ is defined as

$$\omega^T(t) \triangleq [r(t),v^{(1)}(t),y_p(t),v^{(2)}(t)]$$

the overall system can also be represented as

$$\left[\begin{array}{c} \dot{x}_p \\ \hline \dot{v}^{(1)} \\ \hline \dot{v}^{(2)} \end{array}\right] = \left[\begin{array}{c|c|c} A_p & 0 & 0 \\ \hline 0 & \Lambda & 0 \\ \hline bh^T & 0 & \Lambda \end{array}\right] \left[\begin{array}{c} x_p \\ \hline v^{(1)} \\ \hline v^{(2)} \end{array}\right] + \left[\begin{array}{c} b_p \\ \hline b \\ \hline 0 \end{array}\right] [\theta^T(t)\omega] \qquad (15)$$

$$y_p = h^T x_p.$$

Denoting $\theta(t) = \theta^* + \phi(t)$ the state equations (15) can also be written as

$$\dot{x} = A_c x + b_c [k_0^* r + \phi^T(t)\omega] \qquad (16)$$

where

$$x^T = [x_p{}^T, v^{(1)^T}, v^{(2)^T}]$$

$$A_c = \left[\begin{array}{c|c|c} A_p + d_0{}^* b_p h^T & +b_p c^{*T} & +b_p d^{*T} \\ \hline +bd_0{}^* h^T & \Lambda+bc^{*T} & +bd^{*T} \\ \hline bh^T & 0 & \Lambda \end{array}\right], \quad b_c = \left[\begin{array}{c} b_p \\ \hline b \\ \hline 0 \end{array}\right],$$

and $b^T = [0,0,\ldots,1]$ is an $(n-1)^{th}$ order vector.

When $\phi(t) \equiv 0$ and $\theta(t) = \theta^*$, equation (16) also represents the model. If x_{mc} is the state vector of a nonminimal representation of the model,

$$\dot{x}_{mc} = A_c x_{mc} + b_c k_0^* r \tag{17}$$

where $x_{mc}^T = \{x_m^T, v_m^{(1)^T}, v_m^{(2)^T}\}$ and the (3n-2) elements of x_{mc} correspond to those of the plant.

We note that the plant is not known and, hence, the model cannot be realized in the form shown in equation (17). However, (17) is used only for purposes of analysis of the error equations and not to generate any adaptive signals.

The Error Equation:

The error equations between model and plant may now be expressed as

$$\begin{aligned} \dot{e} &= A_c e + b_c[\phi^T(t)\omega] \\ e_1 &= h_c^T e \end{aligned} \tag{18}$$

where $e(t) \overset{\Delta}{=} x(t) - x_{mc}(t)$, h_c and b_c are (3n-2)-vectors, $h_c^T = [1,0,0,\ldots,0]$, $b_c^T = [b_p^T, b^T, 0]$. The error transfer function $W_e(s)$ is given by

$$W_e(s) = h_c^T(sI-A_c)^{-1}b_c = \frac{k_p}{k_M} W_M(s). \tag{19}$$

Hence the error equations can be represented as shown in Figure 2, with $\phi^T(t)\omega(t)$ as the input to a system with transfer function $W_e(s)$. It is worth noting that the signals $\omega_i(t)$, $(i = 1,2,\ldots,2n)$, are derived from $r(t)$, the reference input, $u(t)$, the control input, and $y_p(t)$, the output of the plant.

Modification of Controller Structure:

In the representation of the controller that we have used thus far, if every element $\theta_i(t)$ of the parameter vector $\theta(t)$ is replaced by $P_L(\theta_i)$ (with L-a polynomial of degree (n-m-1) in 's'), it follows directly that (i) $\theta(t) = \theta^*$ would make the overall transfer function W(s) equal to $W_M(s)$,

and (ii) the error equations can be represented by a linear time-invariant dynamical system with (3n-2) + 2n(n-m-1) states, a single output e_1 and a single input $u_e(t) = \sum_{i=1}^{2n} \phi_i(t)\xi_i(t)$. The transfer function from $u_e(t)$ to $e_1(t)$ is now

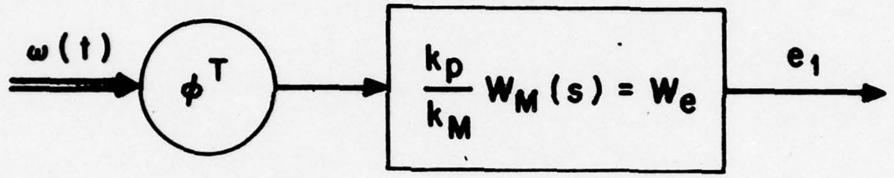


Figure 2.

$\frac{k_p}{k_M} L(s)W_M(s)$ and the signals $\xi_i(t)$ in Figure (3) are given by

$$\xi_i(t) = L^{-1}(s)\omega_i(t) \qquad (i = 1,2,\ldots,2n). \tag{20}$$

The proper choice of L(s), the realization of $P_L(\theta)$ in the system without using differentiators and the proof of stability of the modified system form the essence of the solution to the control problem.

5. The Control Problem:

The complexity of the solution to the control problem depends to a large extent on the prior information that is available regarding the transfer function of the plant - in particular, the order 'n' of the plant and the number m of zeros. While our aim is to develop a controller for the general case, for convenience of exposition we shall consider the following four cases of increasing generality where it is known that $W_p(s)$ has:

(i) (n-1) zeros and n poles.
(ii) (n-2) zeros and n poles.
(iii) $m(\leq n-3)$ zeros and n poles and a known gain k_p.
(iv) $m(\leq n-3)$ zeros, n poles and an unknown gain k_p.

The first case with (n-1) zeros and n poles has been known for a long time. It has been explicitly used in adaptive observer theory by Narendra and Kudva [3] and in the control context by Monopoli and Gilbart [13]. The results presented for the second case are new although a less general version appears in [11]. The modification of the controller structure as discussed in the previous section enables the same concepts to be extended to the general cases (iii) and (iv).

Case (i): $W_p(s)$ with (n-1) zeros and n poles:

A model is chosen* which has a strictly positive real transfer function:

$$W_M(s) = h^T(sI-A_m)^{-1}b_m = \frac{k_M Z_M(s)}{R_M(s)}$$

In this case $L(s) \equiv 1$ and the parameter error vector $\phi(t)$ is updated in equation (8) according to the control law

$$\dot{\phi} = \dot{\theta} = -\Gamma e_1(t)\omega(t) \qquad \Gamma = \Gamma^T > 0 \tag{21}$$

By lemma 1 the state error e(t) and parameter error $\phi(t)$ are bounded. Further, since the state of the model is bounded, it follows that the vector $\omega(t)$ is bounded so that $e(t) \to 0$ as $t \to \infty$ or $|e_1(t)| \to 0$ as $t \to \infty$.

---

* This is for convenience of analysis only; if the model is not positive real the same analysis can be used by proper prefiltering of the reference signal.

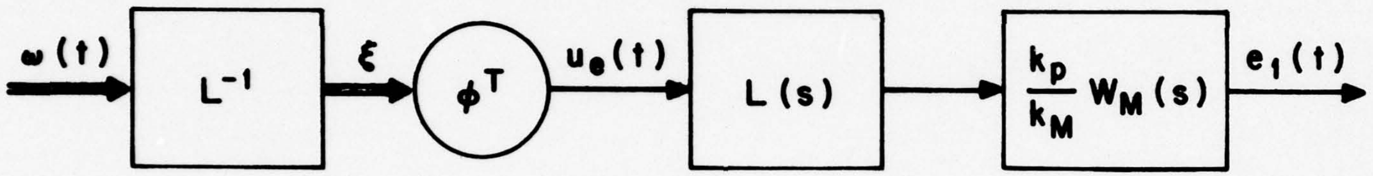


Figure 3.

Case ii: $W_p(s)$ with (n-2) zeros and n poles:

In this case, a model is chosen with (n-2) zeros and n poles such that $L(s)W_M(s)$ is strictly positive real with

$$L(s) = (s+\rho)$$

From section 3b with $L(s) = (s+\rho)$, the operator $P_L(\phi) = L\phi L^{-1}$ is functionally equivalent to $\{\phi(t) + \dot{\phi}(t)\, L^{-1}\}$. Since $\dot{\phi}(t)$ is determined completely by the adaptive law, it is a known signal. The adaptive controller structure has the form shown in Figure 4. Every parameter $\theta_i(t)$ is now replaced by the operator $P_L(\theta_i)$ and the control input u(t) is given by

$$u(t) = \sum_{i=1}^{2n} [\theta_i(t) + \dot{\theta}_i\, L^{-1}]\omega_i(t).$$

If the parameters are updated using the law

$$\dot{\theta} = -\Gamma\, e_1(t)\xi(t) \qquad \Gamma = \Gamma^T > 0$$

where

$$L^{-1}\omega(t) = \xi(t), \text{ the error } e_1(t) \to 0 \text{ as } t \to \infty.$$

Proof:

The proof of stability of the overall system and the convergence of $|e_1(t)|$ to zero follow along lines very similar to those in case (i).

$$u(t) = \sum_{i=1}^{2n} P_L(\theta_i)\omega_i(t) = \sum_{i=1}^{2n} P_L(\theta_i^*)\omega_i(t) + \sum_{i=1}^{2n} [P_L(\phi_i)\omega_i(t)]$$

$$= \sum_{i=1}^{2n} \theta_i^*\omega_i(t) + \sum_{i=1}^{2n} P_L(\phi_i)\omega_i(t).$$

The error equation (18) may now be written as

$$\dot{e} = A_c e + b_c [\sum_{i=1}^{2n} P_L(\phi_i)\omega_i(t)];\ e_1 = h^T e \tag{22}$$

or

$$\dot{e} = A_c e + b_{PR} \sum_{i=1}^{2n} \phi_i \xi_i \quad ;\ e_1 = h_c^T e$$

where $\{h_c^T(sI-A_c)^{-1}b_c\}\ L(s) = W_M(s)L(s) = \{h_c^T(sI-A_c)^{-1}b_{PR}\}$ is strictly positive real and $L^{-1}(s)\omega_i(t) = \xi_i(t)$.

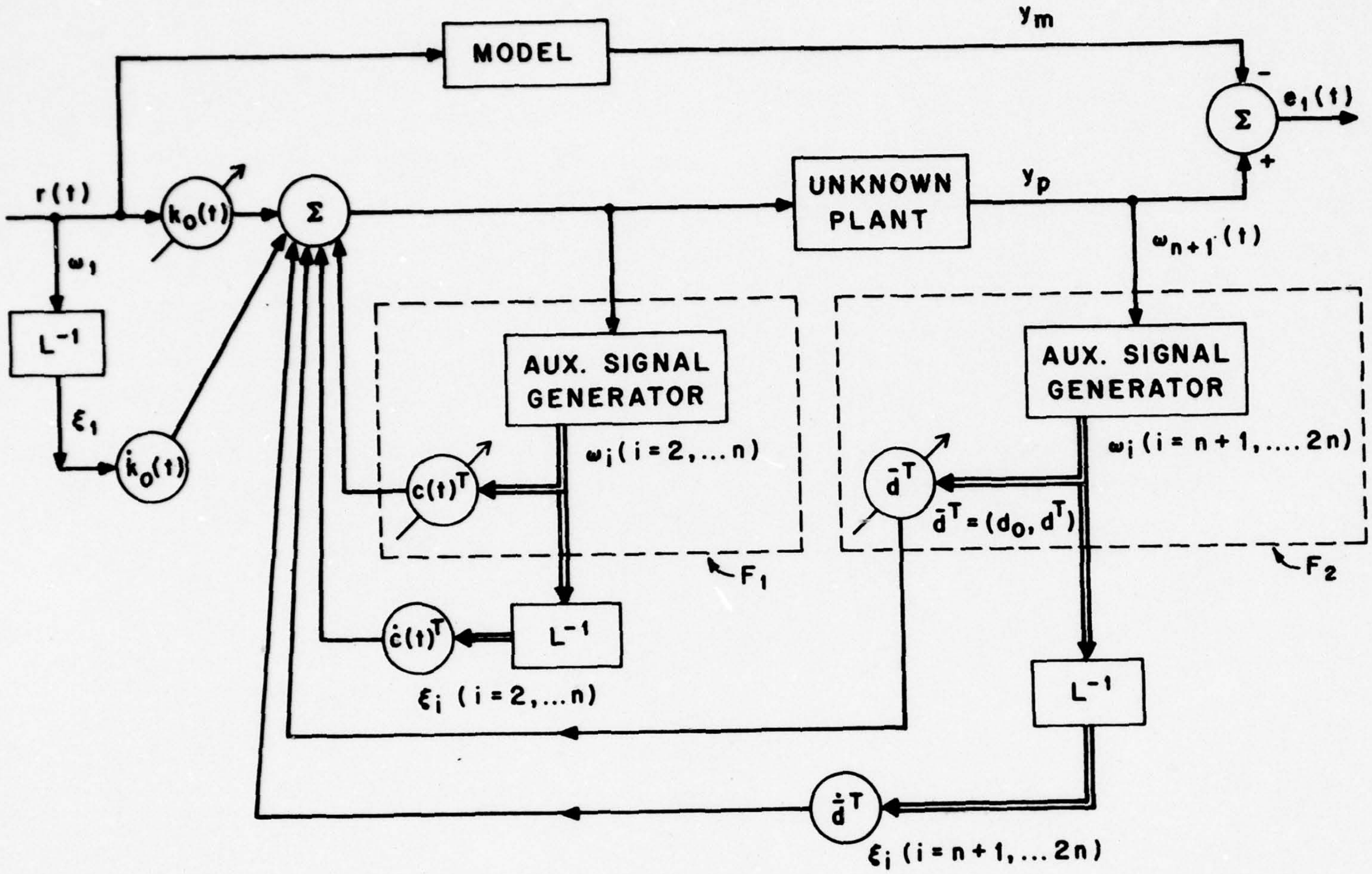


Figure 4. System Structure for Plant with n-2 Zeros.

It follows directly from Lemma 1 that

$$\dot{\phi} = -\Gamma e_1 \xi$$

will result in $|e_1(t)| \to 0$ as $t \to \infty$. Since $\|\xi\|$ is bounded, this also implies that $\|\dot{\phi}(t)\| \to 0$ as $t \to \infty$. Hence the auxiliary signals $\dot{\phi}_i L^{-1} \omega_i(t)$ $(i = 1,2,\ldots,2n)$ into the plant also tend to zero as $t \to \infty$.

Comment 1: While in the above discussion the auxiliary input is fed into the plant, it can also be fed into the model as described in the following section.

Comment 2: For this case it is seen that 2n additional transfer functions of the first order are needed in the controller.

Case iii: With $m(\leq n-3)$ zeros and n poles and known gain k_p:

When k_p is known, by including a gain $\frac{k_M}{k_p}$ in series with the plant, its gain may be adjusted to have any desired value k_M. Hence without loss of generality we can assume that the model gain k_M is chosen to be equal to that of the plant i.e. $k_p = k_M$.

In case (ii) the derivative $\dot{\theta}(t)$ of the parameter vector is available from the adaptive laws and it is this fact that allows the use of the procedure outlined. Unfortunately, this method cannot be directly extended to the present case since higher derivatives of $\theta(t)$ are not available.

Faced with this problem we develop an alternative approach which involves the use of the model in the generation of the adaptive signals. Since a successful adaptive scheme will result in the plant transfer function evolving to that of the model and in the derivatives of $\theta_i(t)(i=1,2,\ldots,2n)$ tending to zero, the new algorithm calls for an additional input to the model, as in [11], which depends on these derivatives. Specifically, the signals fed into the model are such that the form of the previous error equation (18) is preserved.

If the state equations of the plant together with the controller are given by

$$\dot{x} = A_c x + b_c[k_0^* r + \phi^T(t)\omega(t)] \tag{23}$$

the auxiliary signals $[\phi - P_L(\phi)]\omega(t)$ (or $[\theta - P_L(\theta)]\omega(t)$) are used as inputs to the model in equation (17) resulting in the model state equations

$$\begin{aligned} \dot{x}_{mc} &= A_c\, x_{mc} + b_c[k_0^* r + \{\phi - P_L(\phi)\}^T \omega(t)] \\ y_m &= h_c^T\, x_{mc} \end{aligned} \tag{24}$$

Since $\phi - P_L(\phi) = \phi - L\phi L^{-1} = L[L^{-1}\phi - \phi L^{-1}]$, if $L(s)$ is chosen to be a polynomial in 's' of degree n-m-1 so that

$$\{h_c^T(sI-A_c)^{-1}b_c\}L(s) = W_M(s)L(s) = \{h_c^T(sI-A_c)^{-1}b_{PR}\}$$

is strictly positive real, the auxiliary signals can be realized without the use of differentiators.

The model state equations (24) are modified to

$$\dot{x}_{mc} = A_c\, x_{mc} + b_c k_0^* r + b_{PR}[L^{-1}\phi(t) - \phi(t)L^{-1}]^T \omega(t).$$

If

$$[L^{-1}\phi_i(t) - \phi_i(t)L^{-1}]\omega_i(t) = \zeta_i(t) \tag{25}$$

the controller requires two transfer functions of order (n-m-1) corresponding to each parameter that is adjusted. Further, since $k_M = k_p$ in this case, $k_0 = 1$ and only (2n-1) parameters need to be adjusted.

The error equations corresponding to (23) and (24) are

$$\begin{aligned} \dot{e} &= A_c e + b_c P_L(\phi)^T \omega(t) \\ e_1 &= h_c^T e \end{aligned} \tag{26}$$

which are identical to (22) in case (ii). From (26) it follows that the adaptive law

$$\dot{\phi} = -\Gamma e_1(t)\xi(t)$$

where $L^{-1}(s)\omega(t) = \xi(t)$, as before, results in bounded output error $e_1(t)$ and parameter errors $\phi_i(t)$ $(i = 1,2,\ldots,2n)$.

It is at this point that we encounter a new stability problem which did not arise in the previous two cases. The boundedness of $e_1(t)$ does not assure the boundedness of the plant and model outputs. The model output now consists of two components - the desired bounded output and the output due to the auxiliary signals. Since the latter signals may be unbounded, the adaptive scheme may result in an overall unstable system. This question, whose resolution is essential to the completion of the adaptive control problem, is discussed later in this section in greater detail.

Case iv: $W_p(s)$ has $m(\leq n-3)$ zeros and n poles; k_p unknown:

In (iii) the gain k_p was assumed to be known. Here, since the auxiliary signal corresponding to each parameter $\theta_i(t)$ is fed into the model (whose gain k_M is different from k_p), additional compensatory gains $k_i(t)$ $(i = 1,2,\ldots,2n)$ are needed in the adaptive controller. While the plant equations remain the same as in (iii) the model state equations are modified to

$$\begin{aligned}\dot{x}_{mc} &= A_c x_{mc} + b_c[k_0^* r + k_M \sum_{i=1}^{2n} L(s)k_i(t)\{L^{-1}\phi_i(t)-\phi_i(t)L^{-1}\}\omega_i(t)] \\ &= A_c x_{mc} + b_c k_0^* r + b_{PR} \sum_{i=1}^{2n} k_M k_i(t)\zeta_i(t) \qquad (27)\end{aligned}$$

The error equations may be expressed as:

$$\begin{aligned}\dot{e} &= A_c e + \frac{1}{k_p} b_{PR}[\sum_{i=1}^{2n} k_p\phi_i(t)\omega_i(t)] - b_{PR}[\sum_{i=1}^{2n} k_M k_i(t)\zeta_i(t)] \\ &= A_c e + b_{PR}[\sum_{i=1}^{2n} k_p L^{-1}\phi_i(t)\omega_i(t) - k_M k_i(t)\zeta_i(t)] \qquad (27a)\end{aligned}$$

Defining $[1 - \frac{k_M}{k_p} k_i(t)] \triangleq \psi_i(t)$, (27a) reduces to

$$\dot{e} = A_c e + b_{PR}[\sum_{i=1}^{2n} \psi_i(t)\zeta_i(t) + \phi_i(t)\xi_i(t)]$$
$$e_1 = h_c^T e \tag{28}$$

Hence the adaptive laws for the 4n parameters are

$$\dot{\phi}_i(t) = -e_1\xi_i(t) \qquad (i = 1,2,\ldots,2n)$$
$$\dot{\psi}_i(t) = -e_1\zeta_i(t) \qquad (i = 1,2,\ldots,2n)$$

where by (20) and (25) $L^{-1}\omega_i(t) = \xi_i(t)$; $[L^{-1}\phi_i(t) - \phi_i(t)L^{-1}]\omega_i(t) = \zeta_i(t)$ and hence 4n parameters and 4n transfer functions $L^{-1}(s)$ are needed in this case.

The questions of stability raised in (iii) also apply here and are considered in the following.

The Stability of the Adaptive Loop:

The arguments in (iii) and (iv) lead us to the central stability question connected with the adaptive algorithm suggested in this paper. We discuss this question here and make a conjecture whose verification will complete the solution to the adaptive control problem.

The proof of boundedness of the output error $e_1(t)$ is relatively straightforward in all the four cases discussed earlier. In (i) and (ii), since the output of the model is bounded, the output of the plant $y_p(t)$ $(= y_M(t) + e(t))$ is also bounded, in which case $e_1(t) \to 0$ as $t \to \infty$. In (iii) and (iv) the introduction of auxiliary signals to the model precludes such an immediate conclusion. If it can be shown that the plant output is bounded, then the error $e_1(t)$ will tend to zero asymptotically and, hence, the auxiliary signals also tend to zero as $t \to \infty$. The model output then evolves to the desired output and is followed by the plant.

The plant state equations (29) and the system error equations (30) are:

$$\dot{x} = A_c x + b_c[k_0^* r + \phi^T\omega]$$
$$y_p = h_c^T x \tag{29}$$

$$\begin{aligned} \dot{e} &= A_c e + b_c [P_L(\phi)]^T \omega \\ e_1 &= h_c^T e \end{aligned} \qquad (30)$$

where the components of $\omega(t)$ are all state variables of the plant. Hence (29) represents a feedback loop with (2n-1) time-varying parameters while (30) is an open-loop system containing the same parameters. The fundamental stability problem is then to conclude the boundedness of $x(t)$ from the boundedness of $e(t)$ and $\phi(t)$. Since A_c is a stable matrix and only the boundedness of $x(t)$ is of interest, it is sufficient to treat the homogeneous equation (i.e. $r(t) \equiv 0$ in (29)).

The principal stability problem which has to be resolved is now stated as a conjecture.

A Conjecture: Let two systems be described by the differential equations:

$$\begin{aligned} \dot{x} &= Ax + b\phi(t)^T Cx \\ \dot{e} &= Ae + b[P_L(\phi)]^T Cx \end{aligned}$$

where A is an (nxn) stable matrix, b is an n-vector and $\phi(t)$ is an m-vector ($m \leq n$) of bounded continuous time functions. If $e(t)$ is bounded, then $x(t)$ is also bounded.

A better understanding of the implications of the conjecture can be obtained by considering only a single time-varying parameter as shown in Figures 5a and b. In this case, the feedback system has a stable transfer function $W(s)$ in the forward path and a single bounded time-varying gain $\phi(t)$ in the feedback path. The output of $W(s)$ is $y(t)$ and is the input to the bounded linear operator $[W(s)L(s)\phi(t)L^{-1}(s)]$ whose output is $e(t)$ as shown in Figure 5a. The special case when $W(s)L(s) = \frac{1}{s+\alpha}$ is shown in Figure 5b. Given that $e(t)$ is bounded, then, according to the conjecture, $y(t)$ is also bounded.

If $y(t)$ is unconstrained, its boundedness cannot be inferred from the boundedness of $e(t)$, where $e(t) = \{WL\phi L^{-1}\}y(t)$. The basis for the conjecture lies in the fact that $y(t)$ has to belong to the class of signals generated by the feedback loop shown in Fig. 5. If Ω_1 is the class of all unbounded signals defined for all $t \in [0,\infty)$

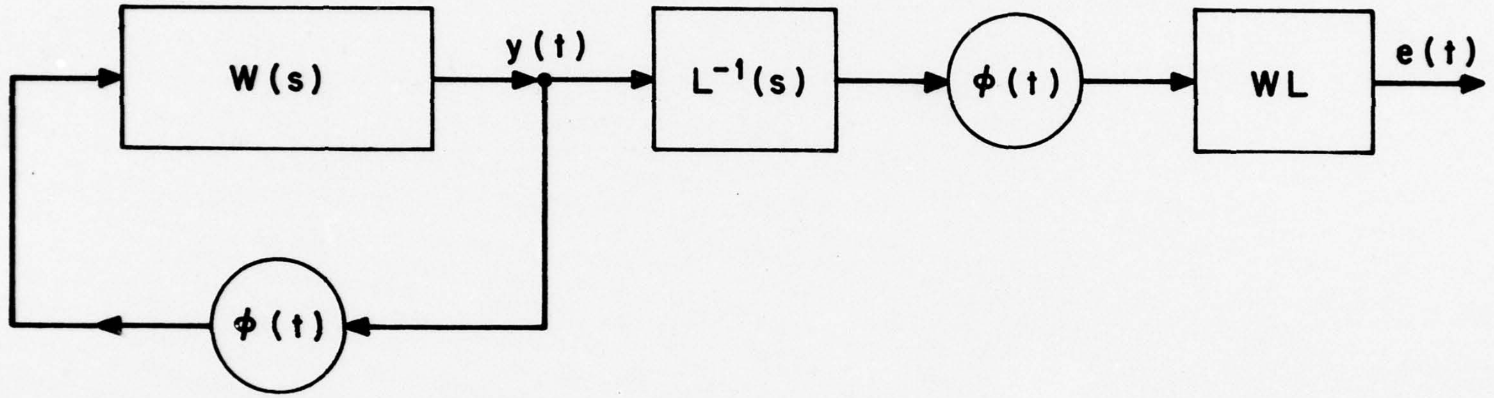


Figure 5a.

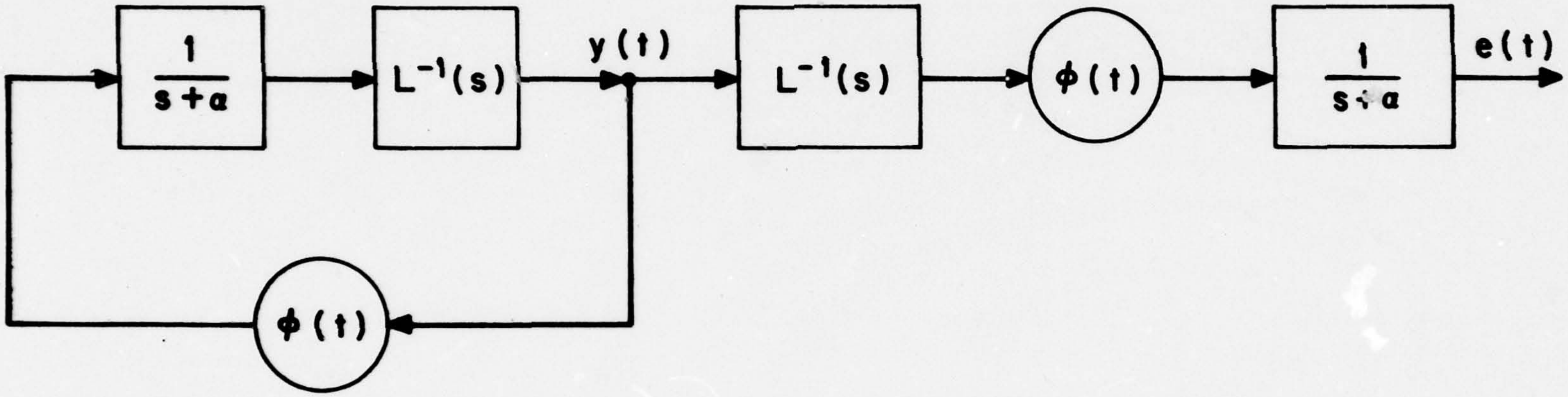


Figure 5b.

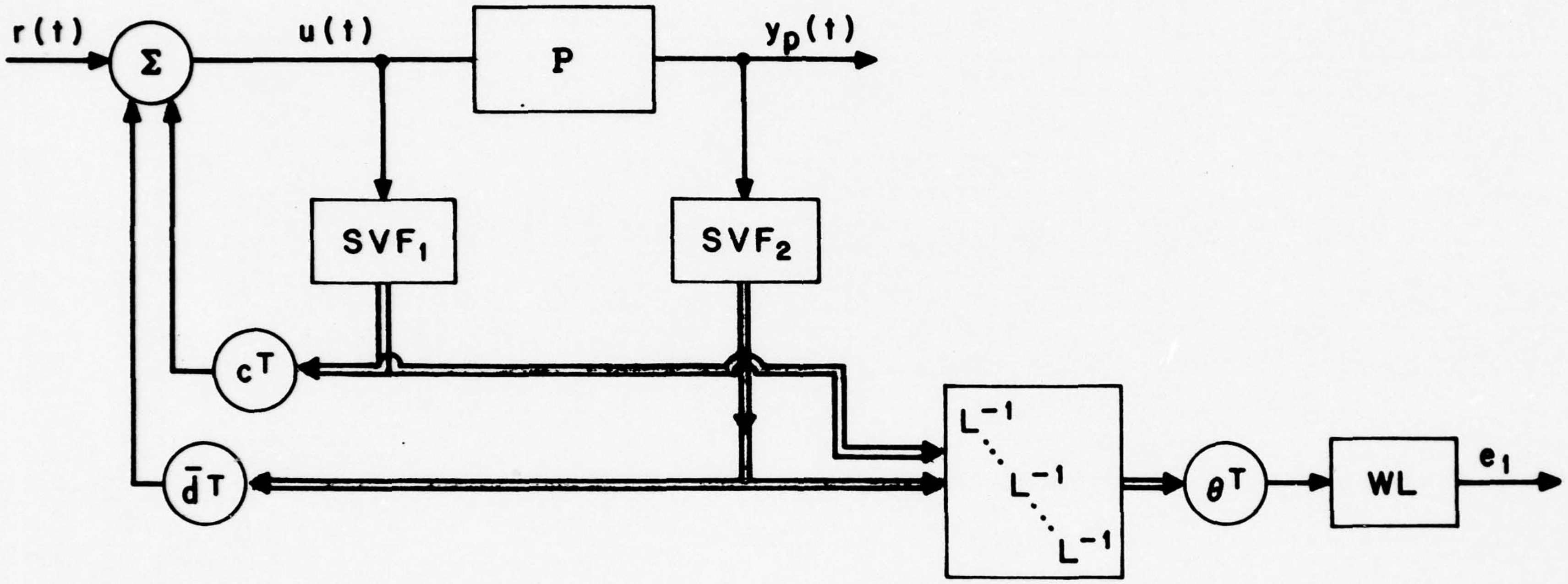
such that $\{WL\phi(t)L^{-1}\}y$ is bounded for $y \varepsilon \Omega_1$ and Ω_2 is the set of all solutions of an n^{th} order linear differential equation with a single bounded time-varying parameter $\phi(t)$, the conjecture states that $\Omega_1 \cap \Omega_2$ is the null set.

The application of the conjecture to the adaptive control problem may be described using Figure 5c. According to the conjecture if $e_1(t)$ is bounded, so is the output of the plant $y_p(t)$.

6. Generalized Adaptive Procedure:

The method presented in the previous section can now be generalized to arbitrarily located control parameters. Since the same procedure can be used for any control parameter, we shall merely consider the case of a single adjustable parameter in the plant. The following development is motivated in part by a procedure used for the generation of sensitivity functions as described in [12].

Let M represent the model and P the plant in Figure 6a and let $\theta(t)$ be a control parameter. It is assumed that when $\theta(t) = \theta^*$, a constant, the transfer function of the plant is identical to that of the model. The plant with $\theta(t) = \theta^*$ is denoted by P^*; if $\theta(t) = \theta^* + \phi(t)$, the plant can be represented as shown in Figure 6b by P^* together with a parameter $\phi(t)$. For the purposes of analysis four points 1,2,3,4 in Figure 6b are of interest and we assume that the corresponding points 1',2',3' and 4' in the model are also accessible. The open loop transfer function $W_{14}(s)$ between points 1 and 4 is equal to $W_{1'4'}(s)$ in the model. The aim of the adaptive procedure is to determine the rule for updating $\dot{\phi}(t)$(or equivalently $\dot{\theta}(t)$) so that $\lim_{t\to\infty} |e_1(t)| = 0$. The four cases that we considered in the previous section can now be summarized for this general problem as shown below. It is worth pointing out that the transfer function $W_{14}(s)$(or $W_{1'4'}(s)$) is of direct interest in this case and not the transfer functions $W_{24}(s)$(or $W_{2'4'}(s)$). However, for ease of exposition, we shall assume that points 1 and 2 coincide (i.e. the output of the adjusted parameter is fed back to the input) so that $W_{14}(s) = W_{24}(s)$. In this case all the conditions can be stated in terms of $W_{2'4'}(s)$, or the model transfer function. For the more general case where points 1 and 2 do not coincide, $W_M(s)$ in the following discussion must be replaced by $W_{1'4'}(s)$.



$$\theta^T = (c^T, \bar{d}^T)$$

Figure 5c.

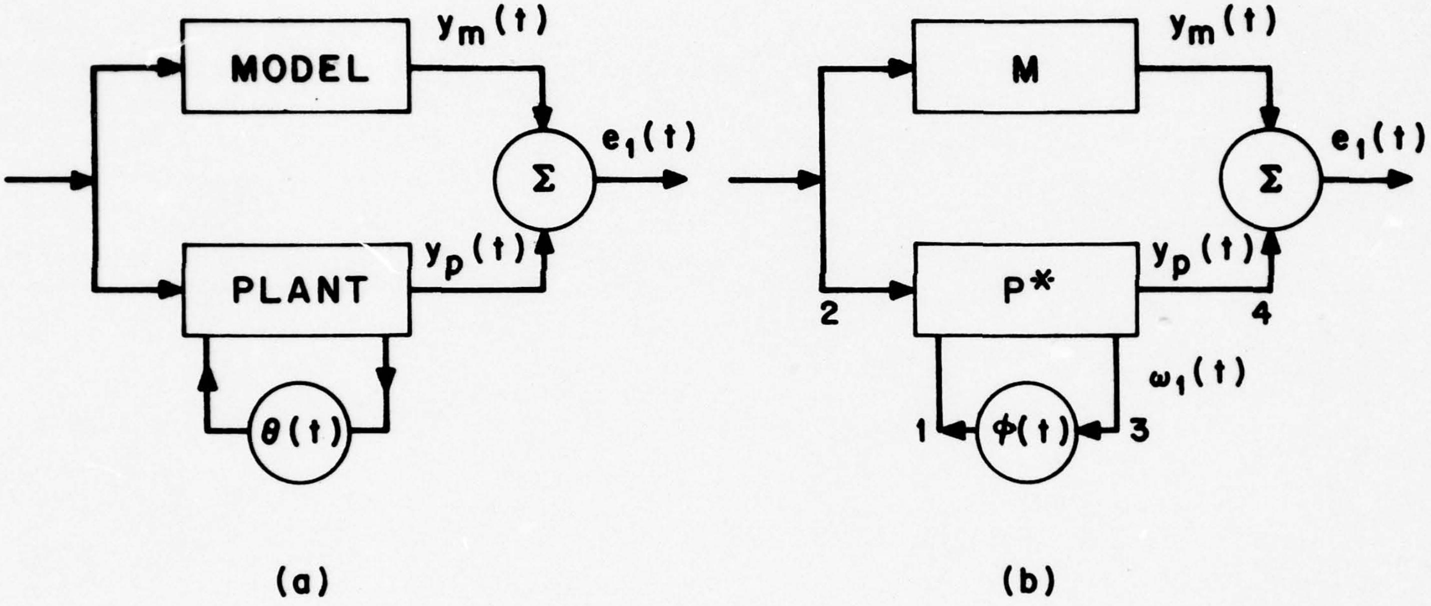

MODEL
PLANT
$y_m(t)$
$y_p(t)$
Σ
$e_1(t)$
$\theta(t)$
(a)
M
P*
$y_m(t)$
$y_p(t)$
Σ
$e_1(t)$
$\omega_1(t)$
$\phi(t)$
1
2
3
4
(b)

Figure 6.

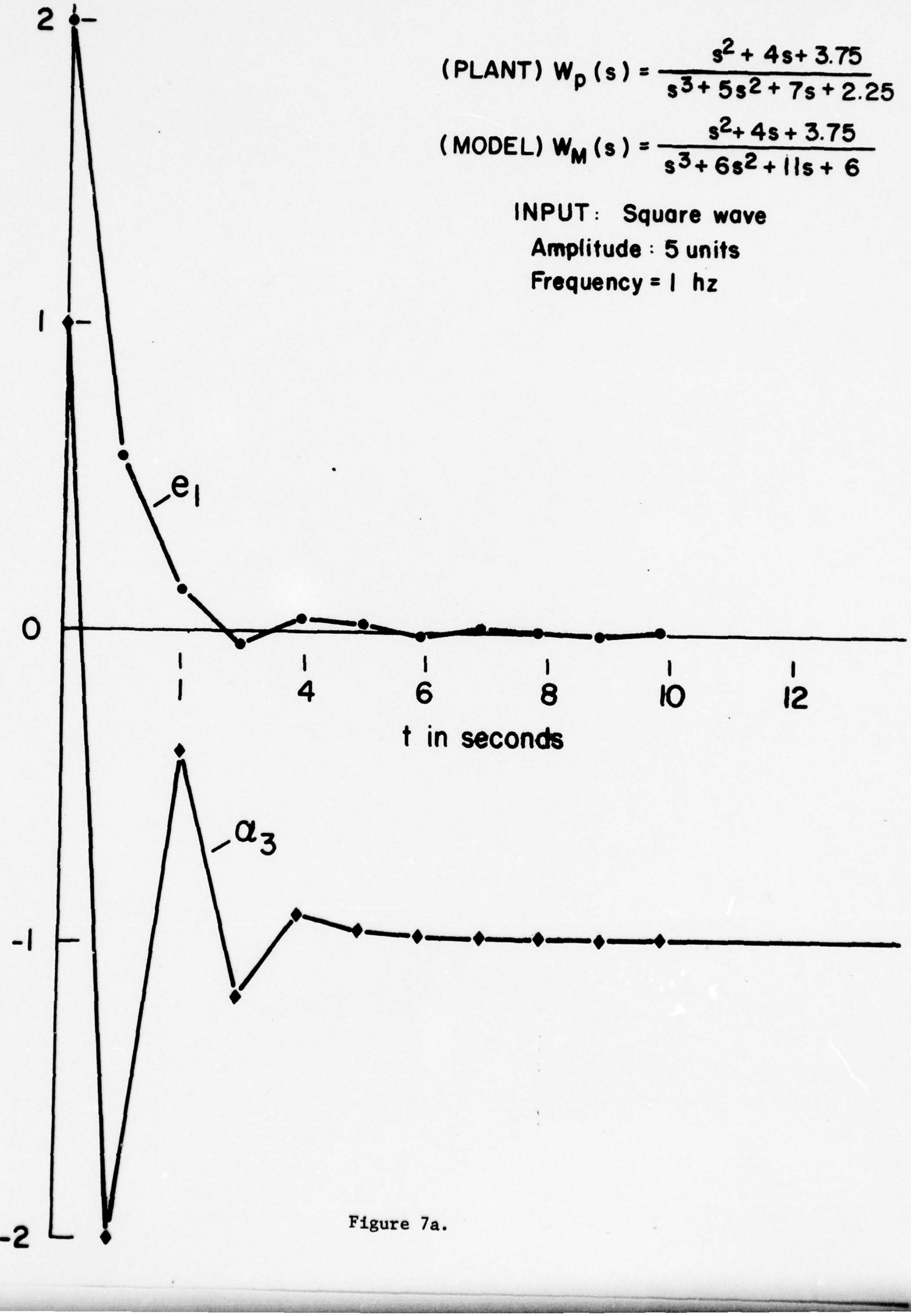

(PLANT) $W_p(s) = \frac{s^2 + 4s + 3.75}{s^3 + 5s^2 + 7s + 2.25}$
(MODEL) $W_M(s) = \frac{s^2 + 4s + 3.75}{s^3 + 6s^2 + 11s + 6}$
INPUT: Square wave
Amplitude : 5 units
Frequency = 1 hz
2
1
0
-1
-2
e_1
α_3
1
4
6
8
10
12
t in seconds

Figure 7a.

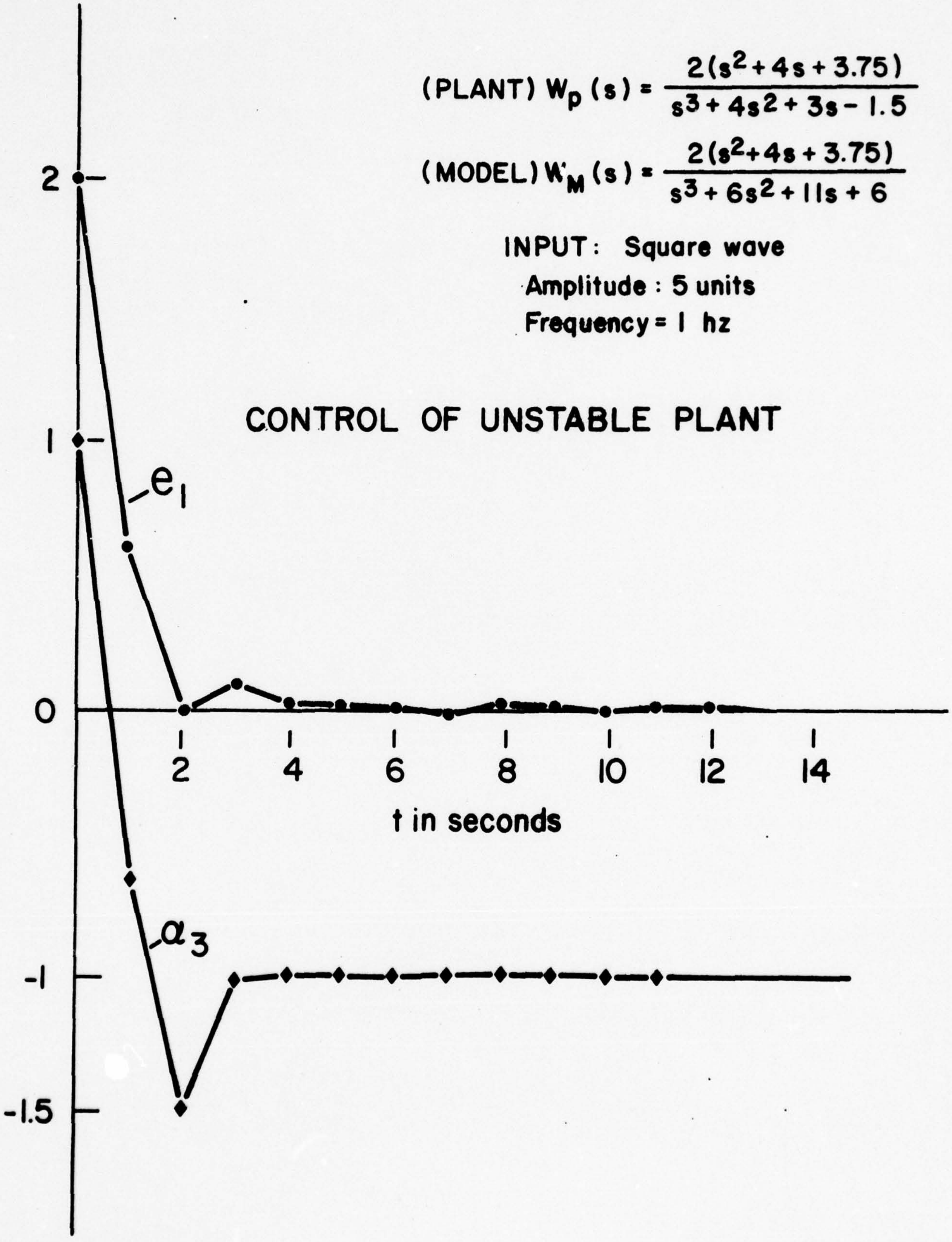

(PLANT) $W_p(s) = \frac{2(s^2+4s+3.75)}{s^3+4s^2+3s-1.5}$
(MODEL) $W_M(s) = \frac{2(s^2+4s+3.75)}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude : 5 units
Frequency = 1 hz
CONTROL OF UNSTABLE PLANT
2
1
0
-1
-1.5
e_1
α_3
2
4
6
8
10
12
14
t in seconds

Figure 7b.

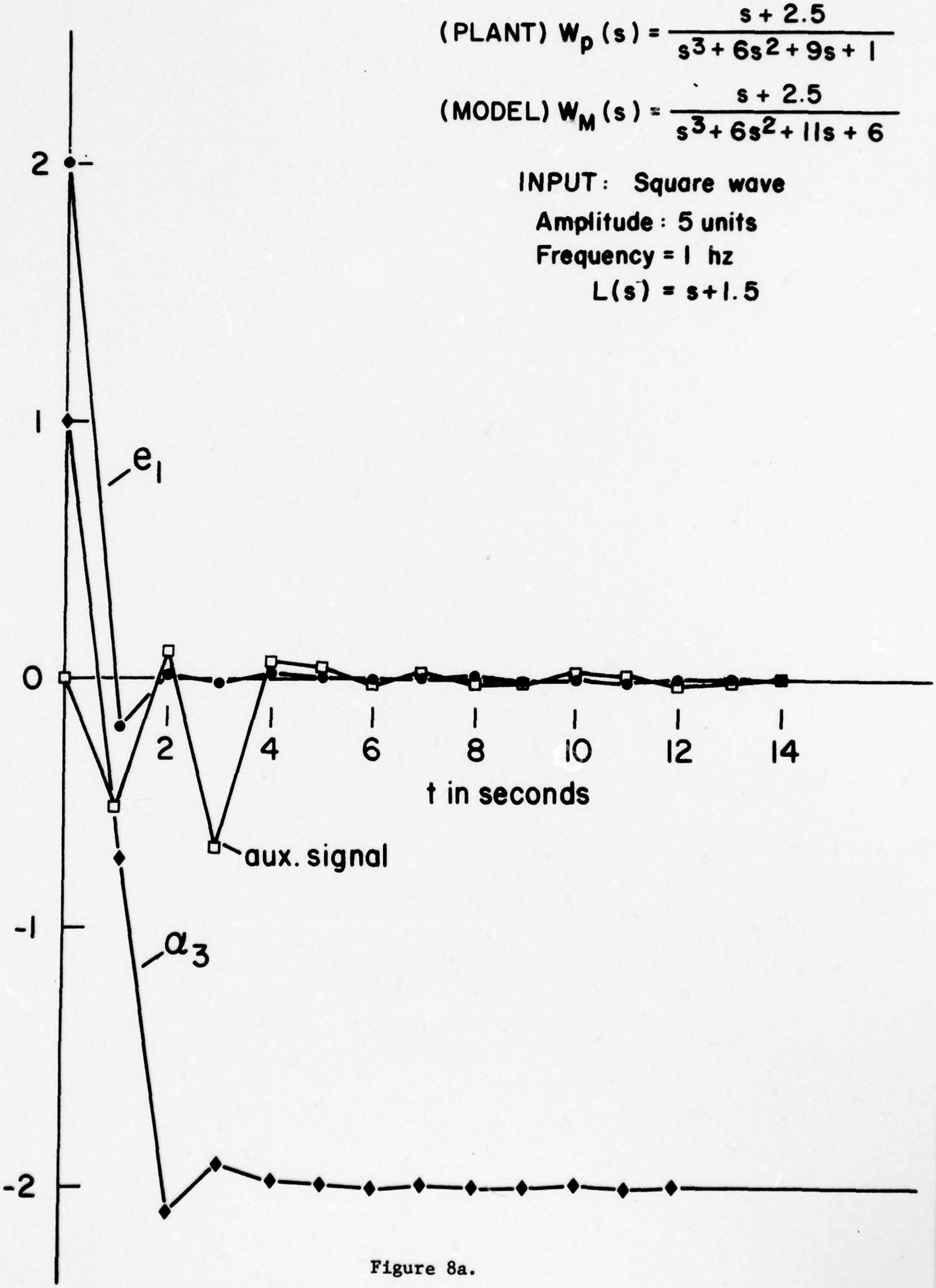

(PLANT) $W_p(s) = \frac{s + 2.5}{s^3 + 6s^2 + 9s + 1}$
(MODEL) $W_M(s) = \frac{s + 2.5}{s^3 + 6s^2 + 11s + 6}$
INPUT: Square wave
Amplitude: 5 units
Frequency = 1 hz
L(s) = s+1.5
2
1
0
-1
-2
e_1
α_3
aux. signal
2
4
6
8
10
12
14
t in seconds

Figure 8a.

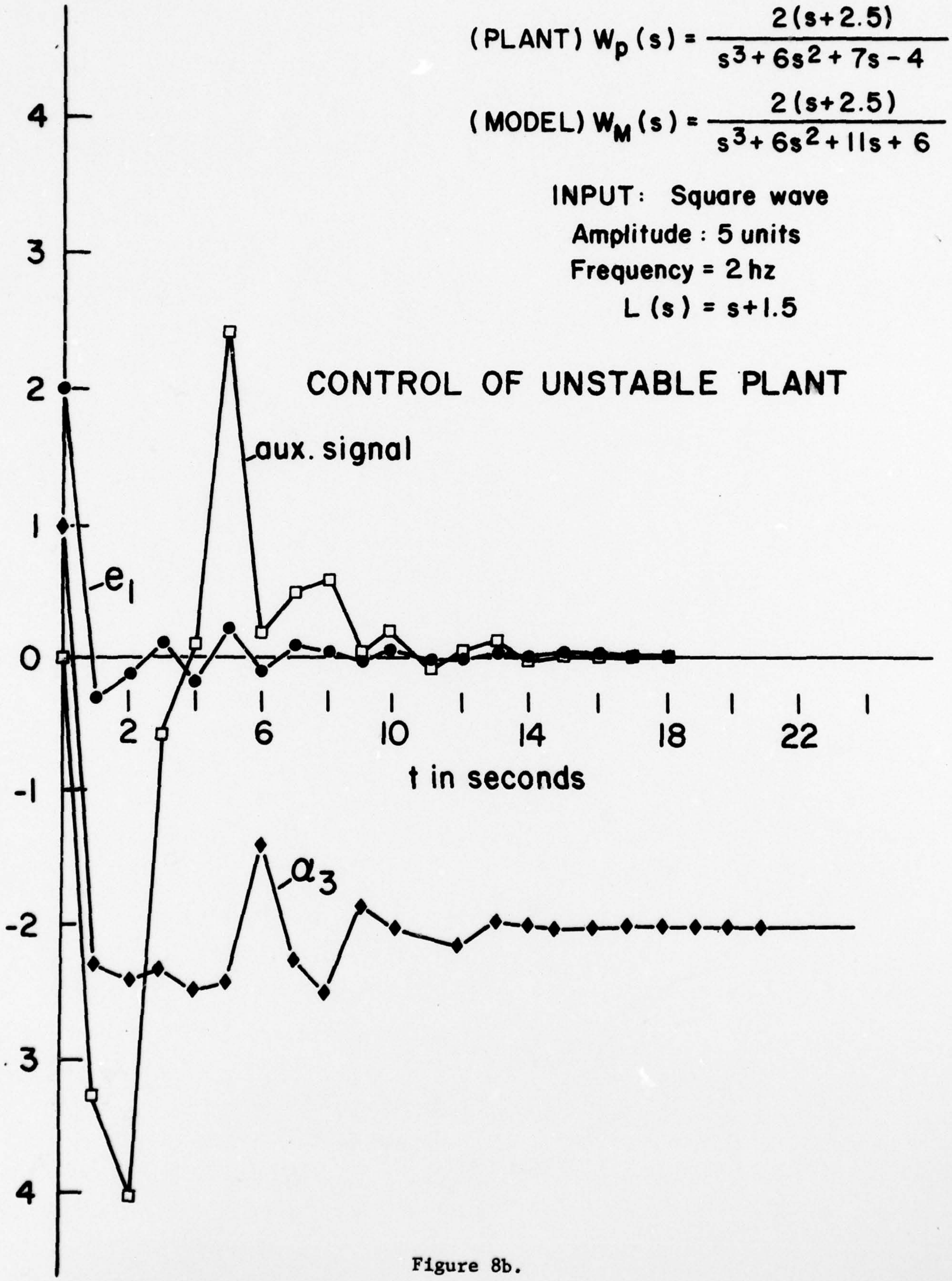

(PLANT) $W_p(s) = \frac{2(s+2.5)}{s^3+6s^2+7s-4}$
(MODEL) $W_M(s) = \frac{2(s+2.5)}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude : 5 units
Frequency = 2 hz
L (s) = s+1.5
CONTROL OF UNSTABLE PLANT
aux. signal
e_1
α_3
4
3
2
1
0
-1
-2
3
4
2
6
10
14
18
22
t in seconds

Figure 8b.

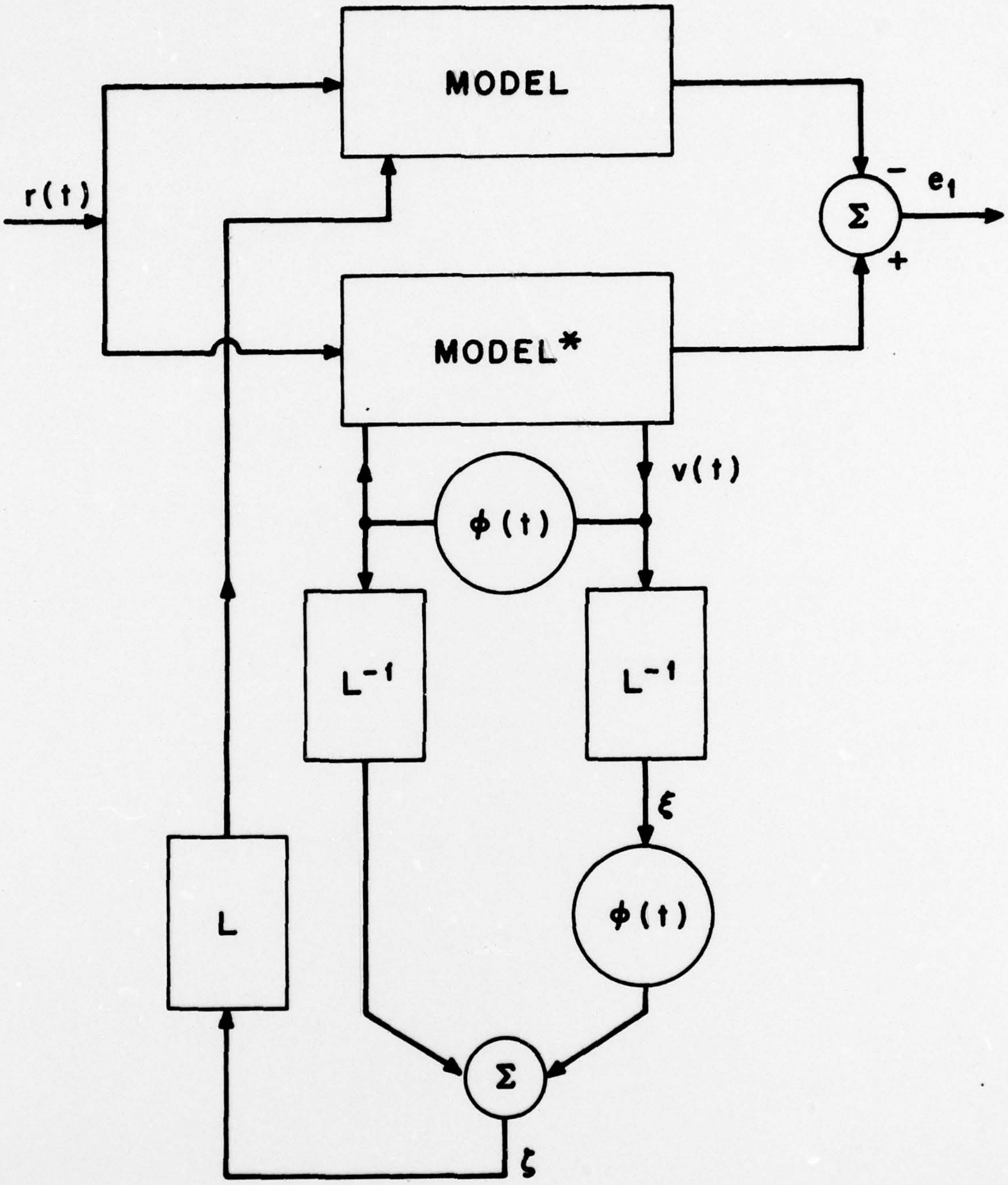


Figure 9.

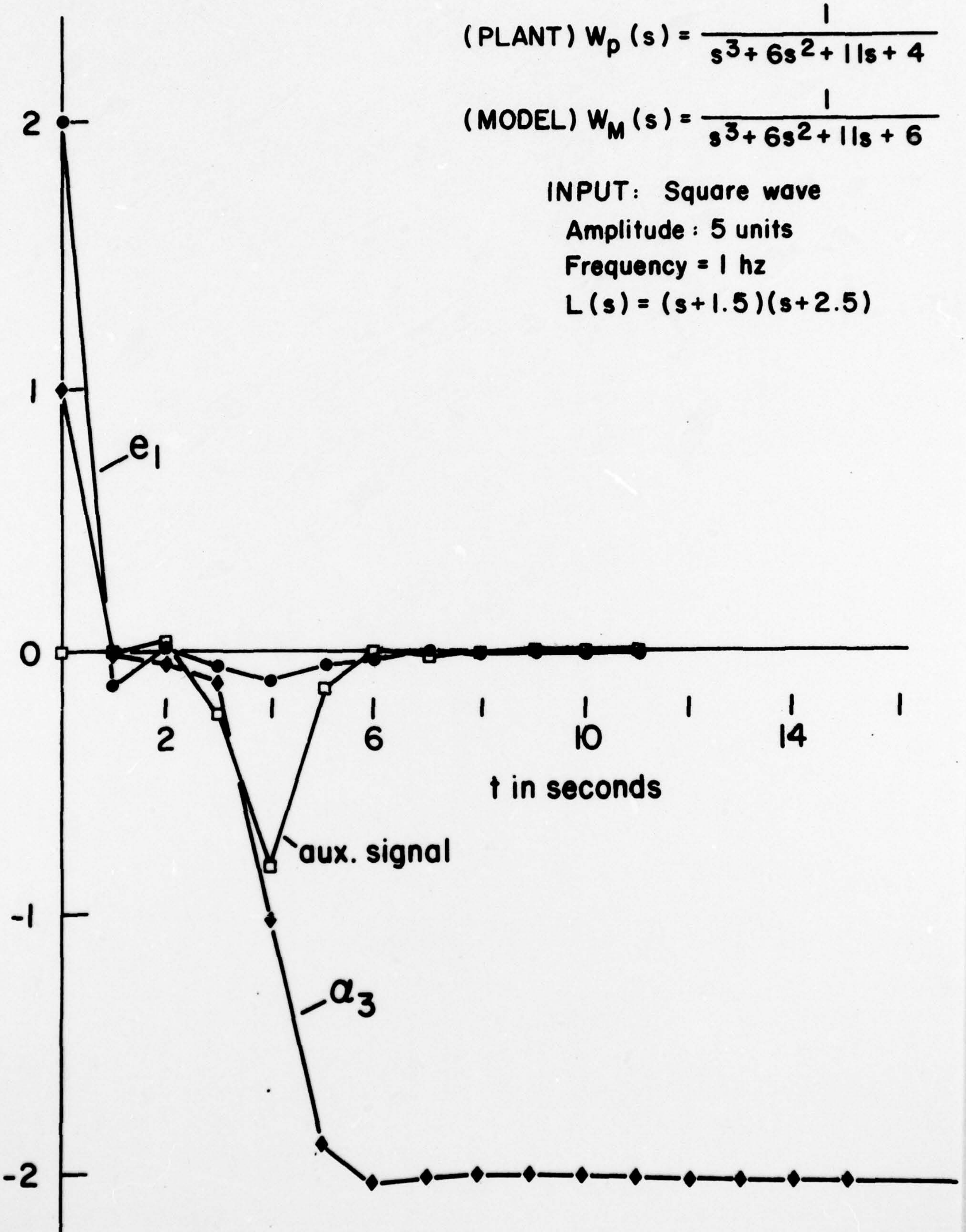

(PLANT) $W_p(s) = \frac{1}{s^3+6s^2+11s+4}$
(MODEL) $W_M(s) = \frac{1}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude : 5 units
Frequency = 1 hz
L(s) = (s+1.5)(s+2.5)
2
1
0
-1
-2
2
6
10
14
t in seconds
e_1
aux. signal
α_3

Figure 10a.

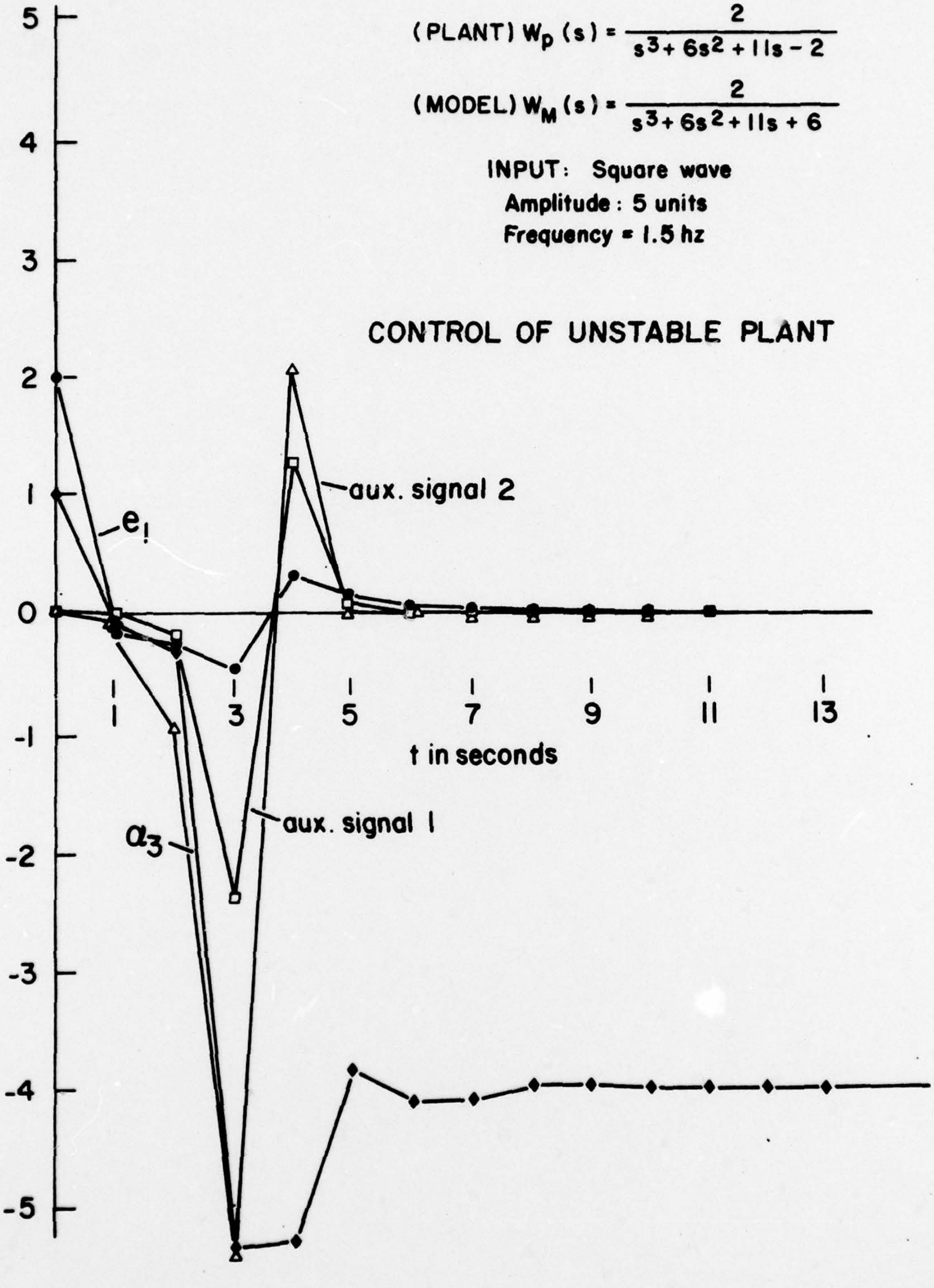

(PLANT) $W_p(s) = \frac{2}{s^3+6s^2+11s-2}$
(MODEL) $W_M(s) = \frac{2}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude: 5 units
Frequency = 1.5 hz
CONTROL OF UNSTABLE PLANT
5
4
3
2
1
0
-1
-2
-3
-4
-5
1
3
5
7
9
11
13
t in seconds
e_1
aux. signal 2
aux. signal 1
a_3

Figure 10b.

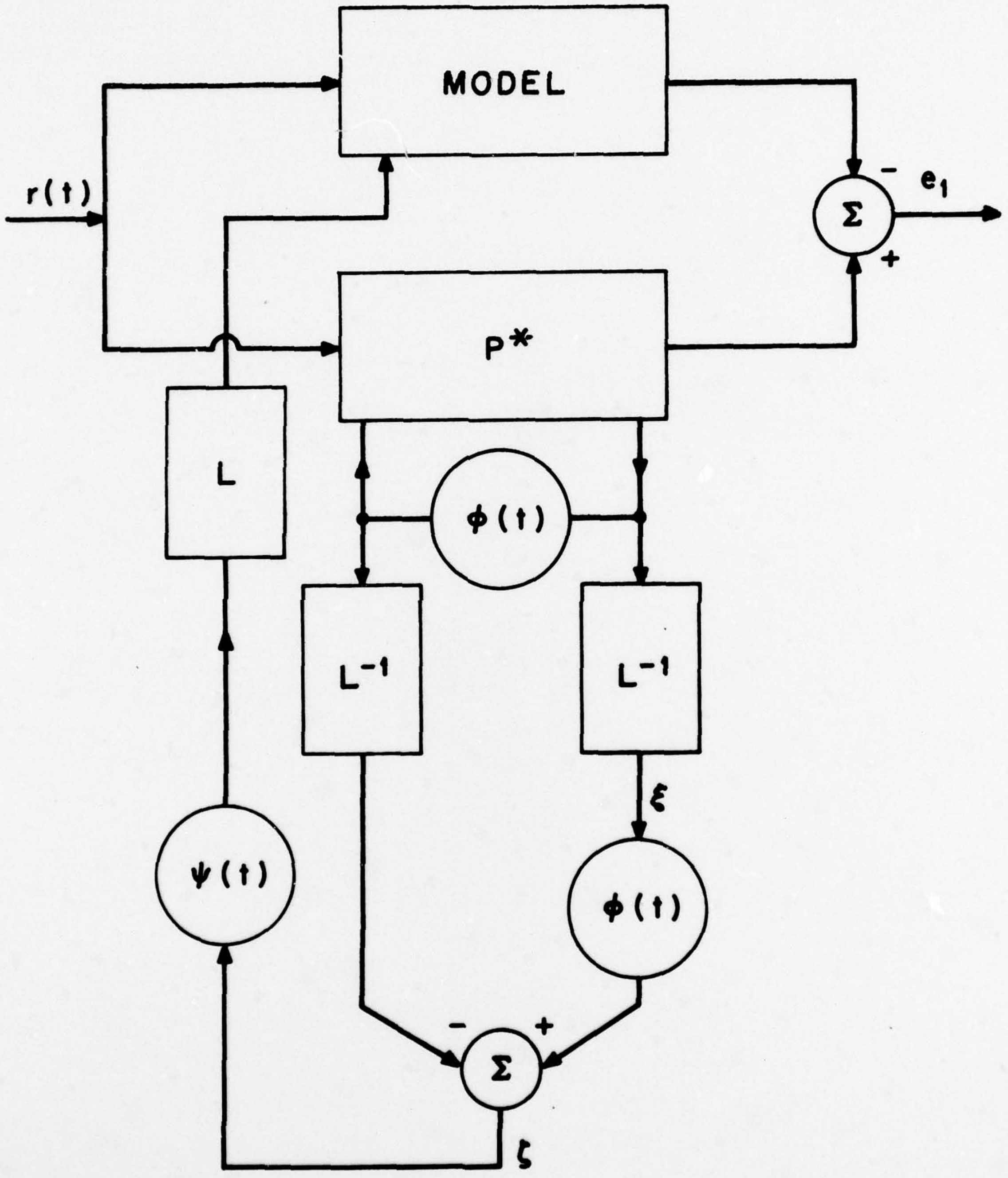


Figure 11.

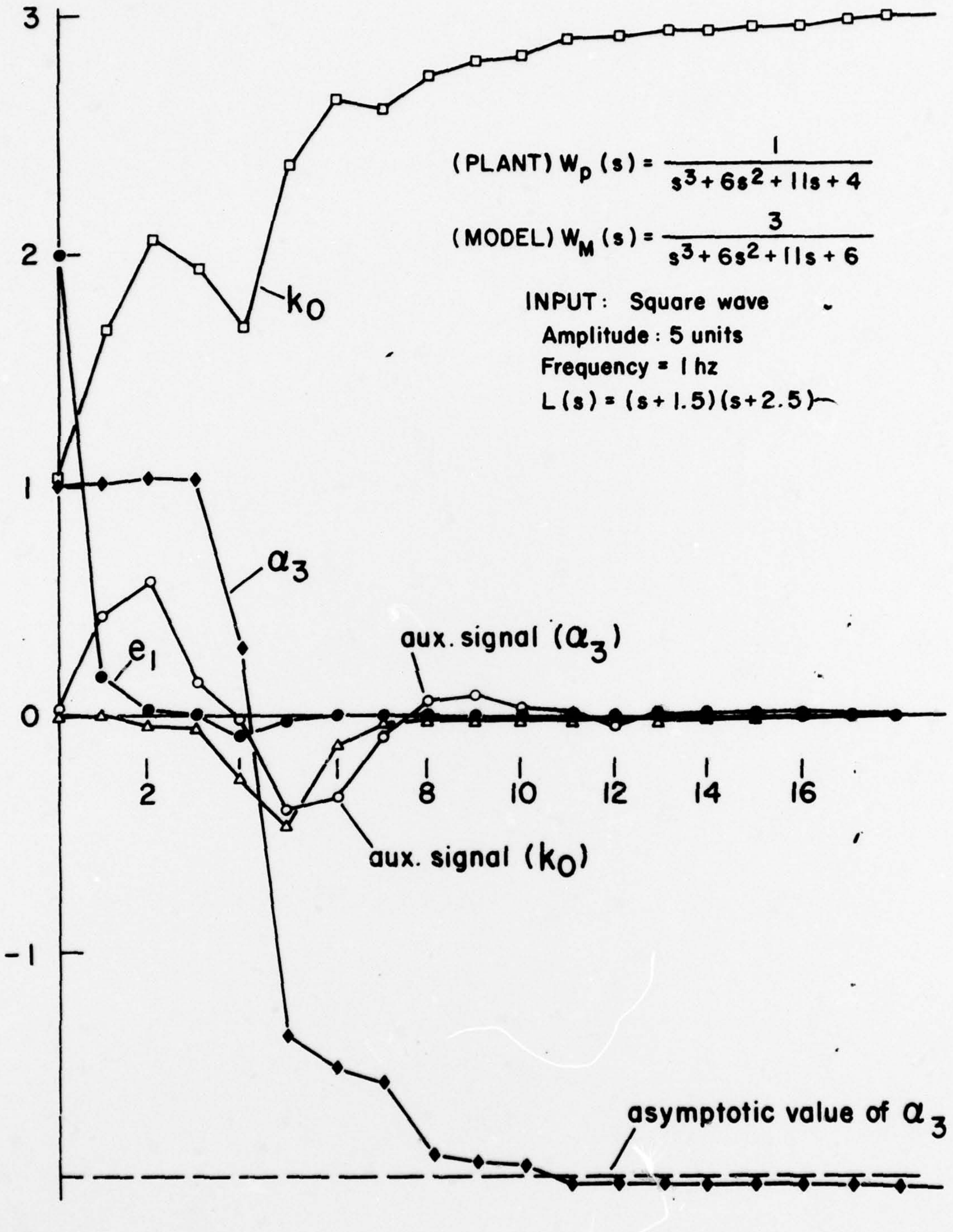

3
2
1
0
-1
2
8
10
12
14
16
k_0
α_3
e_1
aux. signal (α_3)
aux. signal (k_0)
asymptotic value of α_3
(PLANT) $W_p(s) = \frac{1}{s^3+6s^2+11s+4}$
(MODEL) $W_M(s) = \frac{3}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude: 5 units
Frequency = 1 hz
$L(s) = (s+1.5)(s+2.5)$

Figure 12a.

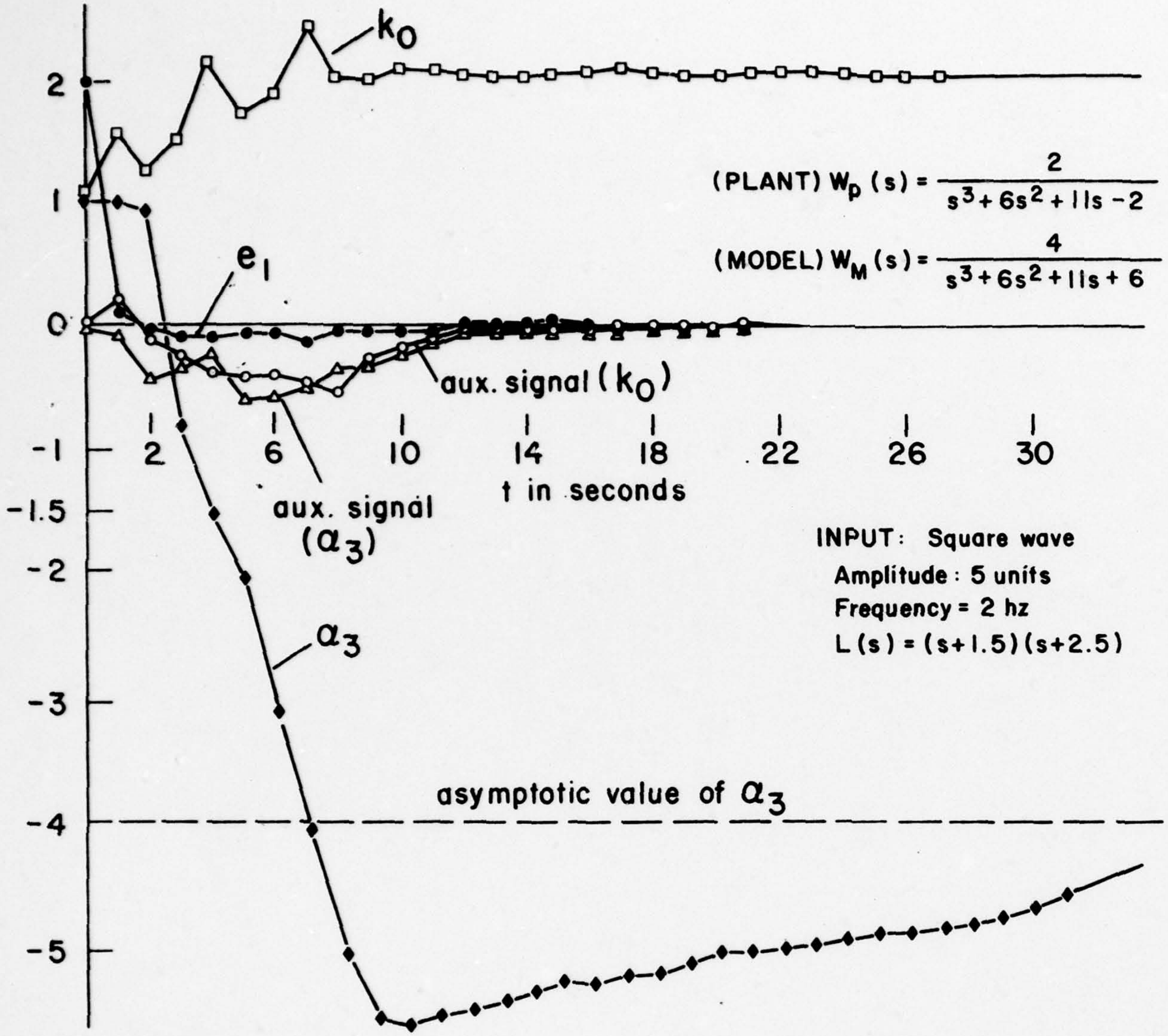

k0
e1
aux. signal (k0)
aux. signal (α3)
α3
asymptotic value of α3
(PLANT) $W_p(s) = \frac{2}{s^3+6s^2+11s-2}$
(MODEL) $W_M(s) = \frac{4}{s^3+6s^2+11s+6}$
INPUT: Square wave
Amplitude : 5 units
Frequency = 2 hz
L(s) = (s+1.5)(s+2.5)
t in seconds
2
6
10
14
18
22
26
30
2
1
0
-1
-1.5
-2
-3
-4
-5

Figure 12b.

(i) $W_M(s) \in \{SPR\}$:

The adaptive rule

$$\dot{\phi}(t) = -e_1(t)\omega_1(t)$$

where $\omega_1(t)$ is the input to the parameter being adjusted assures that $|e_1(t)| \to 0$ as $t \to \infty$ by the direct application of lemma 1. The evolution of the parameter to its final value and the error $e_1(t)$ to zero for stable and unstable plants is shown in Figs. 7a,b.

(ii) $W_M(s)(s+\rho) \in \{SPR\}$:

When $W_M(s)$ is such that $W_M(s)(s+\rho)$ can be made strictly positive real by the proper choice of a constant ρ, an additional stabilizing signal is fed into the plant as shown in Figure 4. The signal is proportional to $\dot{\phi}(t)$ and tends to zero as the adaptation progresses. The adjustment of a single parameter using this scheme is shown in Figs. 8a,b for both stable and unstable plants.

(iii) $W_M(s)L(s) \in \{SPR\}$:

When a polynomial L(s)(of degree ≥ 2) exists such that W_ML is strictly positive real, the stabilizing signal $\{\theta-P_L(\theta)\}\omega_1$ is fed into the model as shown in Fig. 9. The model is suitably modified so that this can be accomplished without the use of differentiators. The evolution of the feedback parameter, the stabilizing signal and the output error is shown in Figures 10a,b for both stable and unstable plants.

(iv) $W_p(s) = \frac{k_p}{k_M} W_M(s); W_M(s)L(s) \in \{SPR\}$:

If the transfer function between points 1 and 4 in the plant is the same as that of the model except for a constant gain, an additional gain is needed in the path of the stabilizing signal in (iii),(Fig. 11). The evolution of the feedback parameter $\theta(t)$ to the value θ^*, the stabilizing signal $\zeta(t)$ and the output error is shown in Figure 12a,b for both stable and unstable plants.

7. Further Simulation Results:

Extensive simulations of the control problem on the digital computer indicate that the procedures outlined in the previous sections do result in a robust controller with the output error $|e_1(t)|$, as well as the stabilizing signal $|\zeta(t)|$, tending to zero at rates which are practical. In view of the fundamental theoretical questions that arise in the stability analysis of the adaptive system, these simulation studies prove invaluable in providing insight into the nature of the adaptive controller. In this section we present simulation results for a third order system for the four cases described in section 5. The plant and model transfer functions for the four cases are given in Table I.

TABLE I

CASE	PLANT $W_p(s)$	$W_M(s)$
(i)	$\frac{s^2 + 5s + 4}{s^3 + 9s^2 + 24s + 20}$	$\frac{2(s^2 + 4s + 3.75)}{s^3 + 6s^2 + 11s + 6}$
(ii)	$\frac{s + 4}{s^3 + 9s^2 + 24s + 20}$	$\frac{2(s + 1.5)}{s^3 + 6s^2 + 11s + 6}$
(iii)	$\frac{1}{s^3 + 9s^2 + 24s + 20}$	$\frac{1}{s^3 + 6s^2 + 11s + 6}$
(iv)	$\frac{1}{s^3 + 9s^2 + 24s + 20}$	$\frac{2}{s^3 + 6s^2 + 11s + 6}$

In cases (i) and (ii) six parameters are adjusted while in case (iii) only five control parameters are needed. However, in the last case, twelve parameters have to be adjusted, six of which represent the extra gains discussed in section 5. To generate the auxiliary signals, F_1 and F_2 are used with characteristic polynomials

$$\Lambda(s) = s^2 + 4s + 3.75$$

The reference input r(t) in each case was a square wave with an amplitude of 5 units and frequencies ranging from .5 hz to 6 hz. (Figure 13).

Acknowledgement

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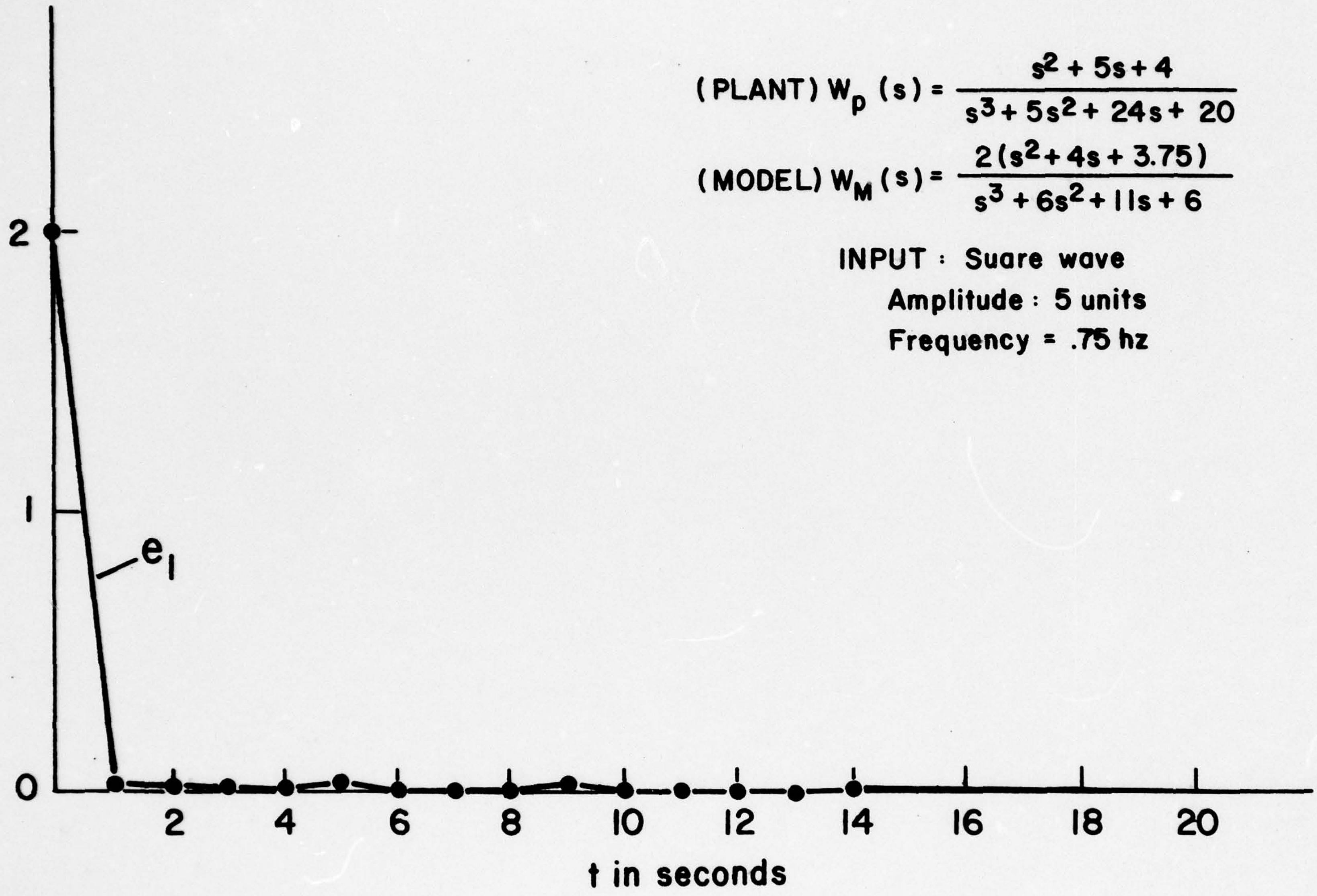


$$(\text{PLANT})\ W_p\,(s) = \frac{s^2 + 5s + 4}{s^3 + 5s^2 + 24s + 20}$$

$$(\text{MODEL})\ W_M\,(s) = \frac{2(s^2 + 4s + 3.75)}{s^3 + 6s^2 + 11s + 6}$$

INPUT : Suare wave
Amplitude : 5 units
Frequency = .75 hz

Figure 13(i).

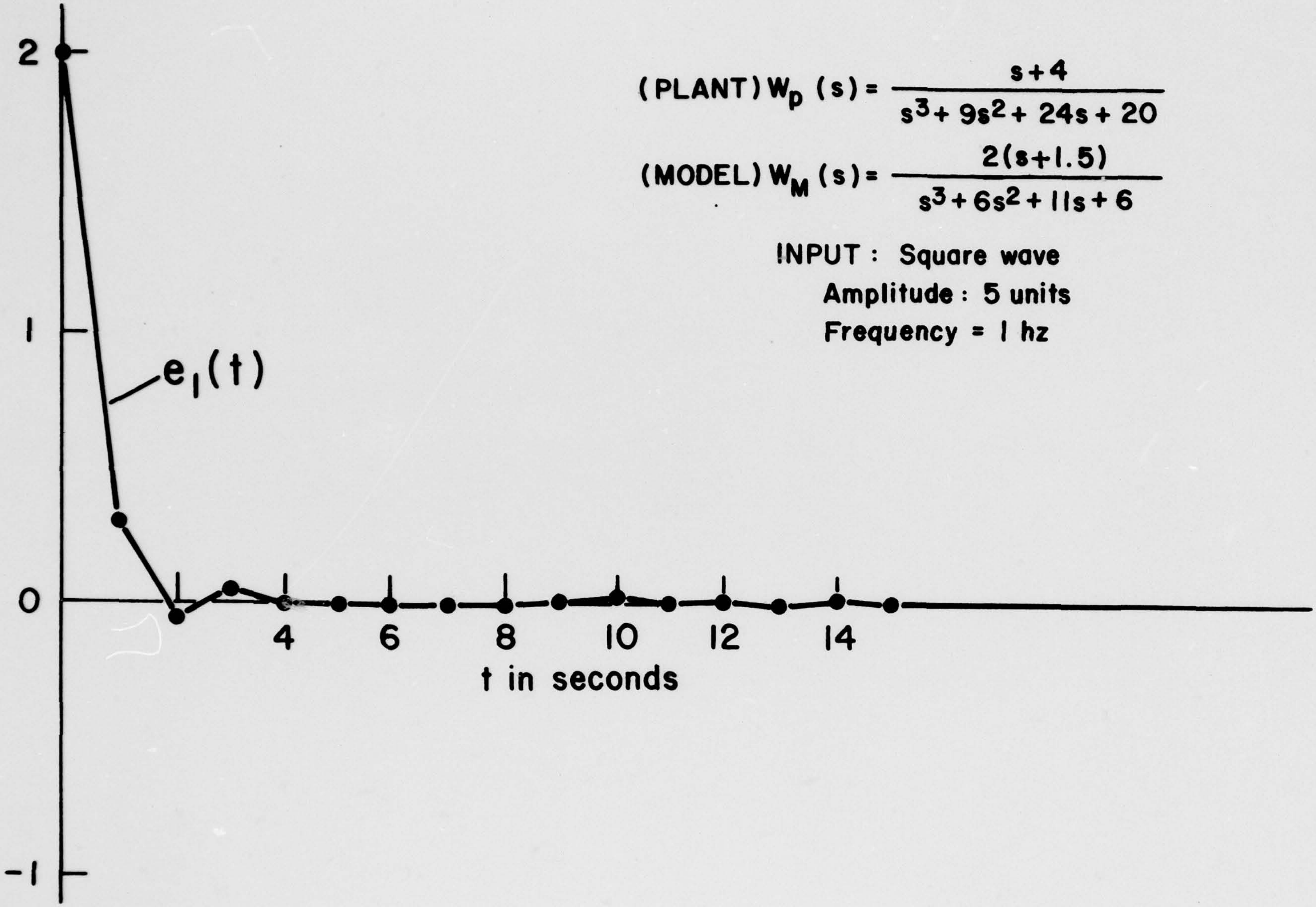

2
1
0
-1
$e_1(t)$
4
6
8
10
12
14
t in seconds
(PLANT) $W_p(s) = \frac{s+4}{s^3+9s^2+24s+20}$
(MODEL) $W_M(s) = \frac{2(s+1.5)}{s^3+6s^2+11s+6}$
INPUT : Square wave
Amplitude : 5 units
Frequency = 1 hz

Figure 13(ii).

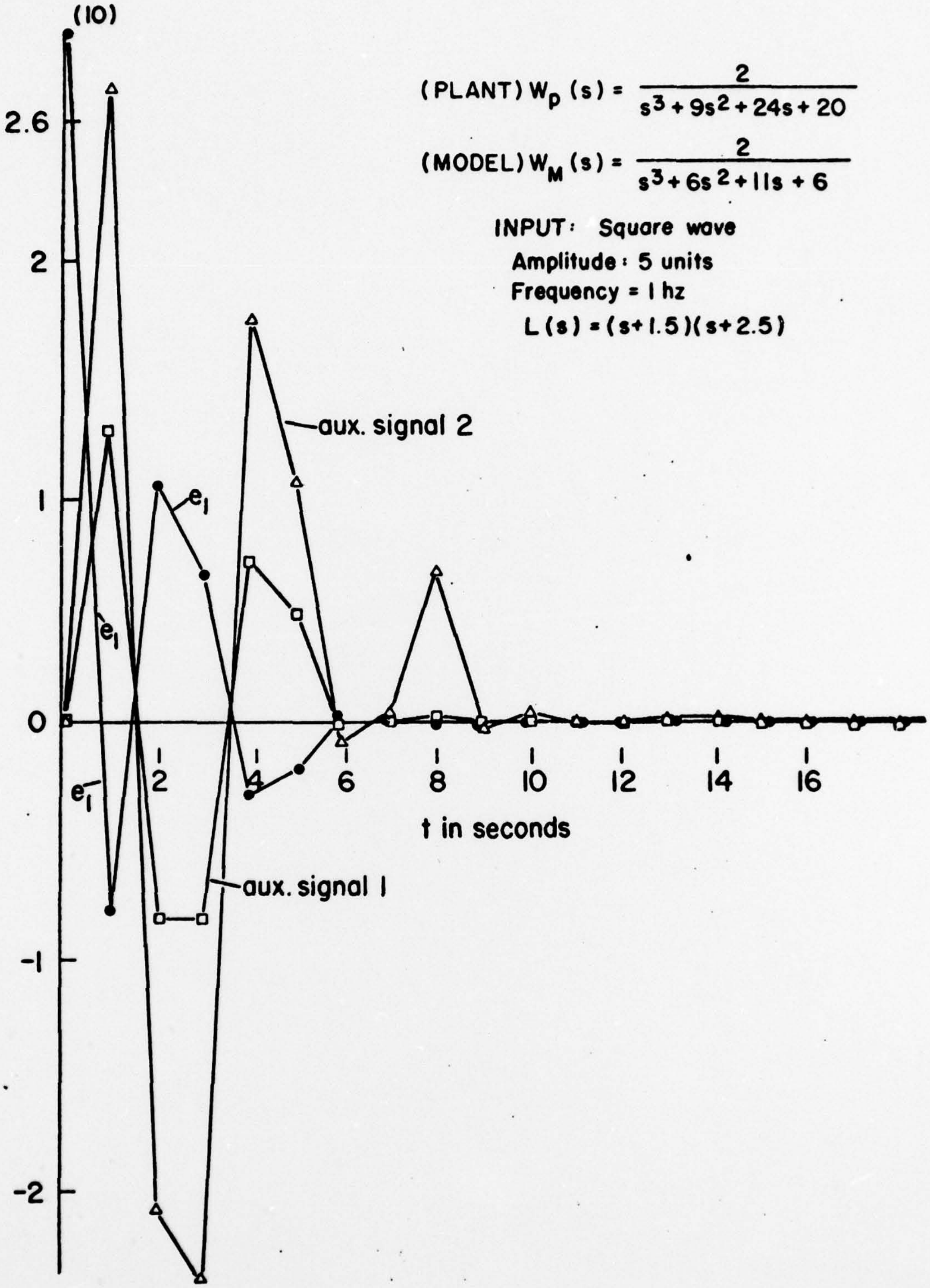
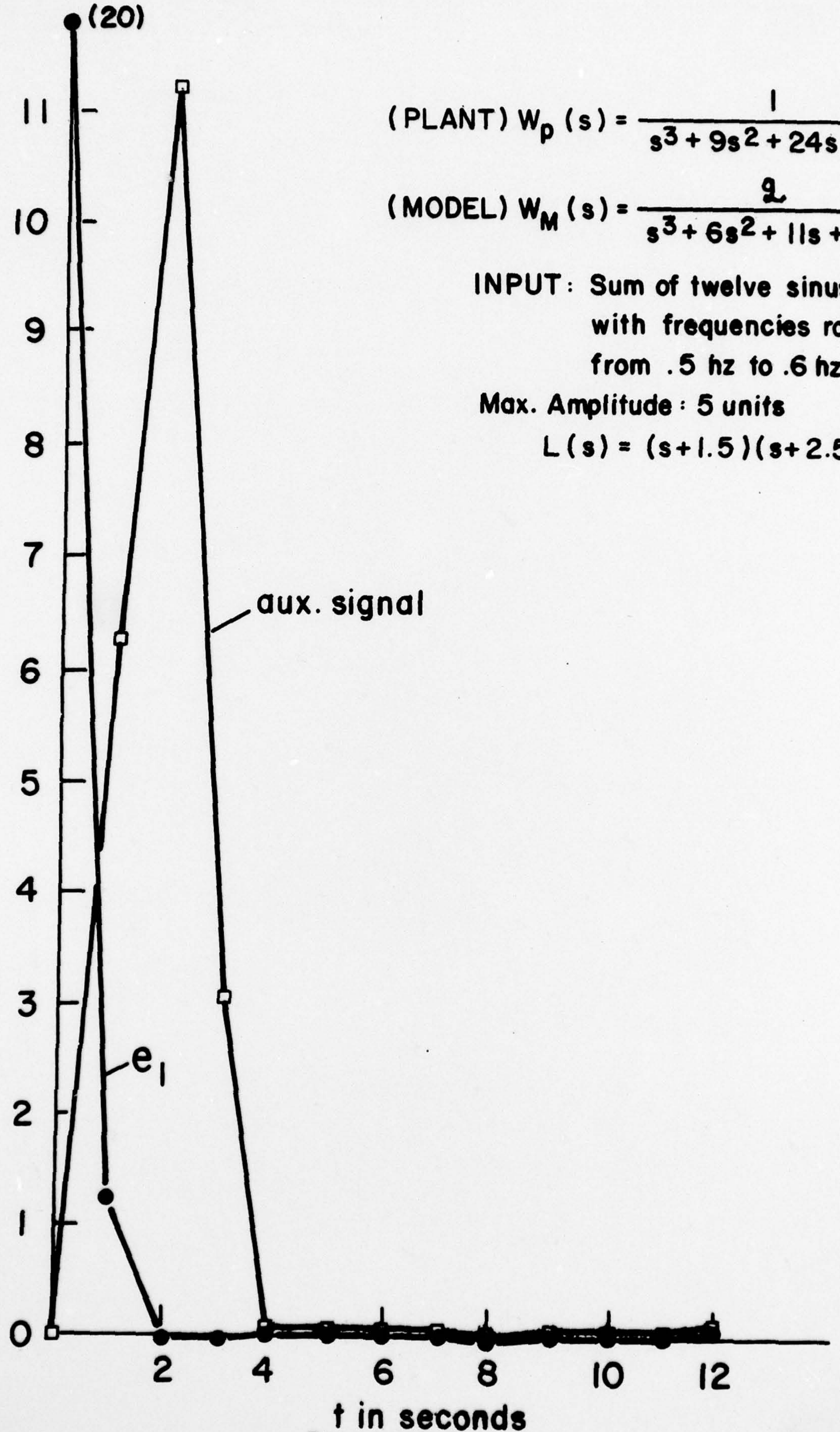


Figure 13(iii).



$$\text{(PLANT) } W_p(s) = \frac{1}{s^3 + 9s^2 + 24s + 20}$$

$$\text{(MODEL) } W_M(s) = \frac{2}{s^3 + 6s^2 + 11s + 6}$$

INPUT: Sum of twelve sinusoids with frequencies ranging from .5 hz to .6 hz

Max. Amplitude: 5 units

$L(s) = (s+1.5)(s+2.5)$

Figure 13(iv).

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