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MICROWAVE MEAT ROASTING — A COMPUTER ANALYSIS FOR CYLINDRICAL ROASTS

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July 1977

UNITED STATES ARMY
NATICK RESEARCH and DEVELOPMENT COMMAND
NATICK, MASSACHUSETTS 01760

Food Engineering Laboratory

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer analysis of microwava meat roasting for cylindrically shaped roasts is described in detail. Previous research has shown this computer program achieves good agreement with experimental data. This report describes the derivation of all heat transfer equations, assumptions and approximations used, tha numerical procedure used, and the means of incorporation of meat properties from references. The function and operation of all elaments

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20. Abstract (cont'd)

of the program are described, as well as the definition and format of all input parameters and their use in achieving considerable versatility. A variation of the program is included which provides for sequential three-dimensional time-temperature plots.

PREFACE

This report describes, in considerable detail, the computational procedure and computer model used in e computer analysis of microwave meat roasting. Several key parameters were measured experimentally for the specific oven used in this study; if this program is used with other ovens, these parameters will need to be remeasured. The computer model has shown good agreement with limited experimental data for en infinite cylinder (no end effects). Although the progrem does take end heating into account, the preliminary experimental work did not encompass measurement of end heating effects, so no computed/experimental comparison of temperature profiles of cylindrical roast ends is presented.

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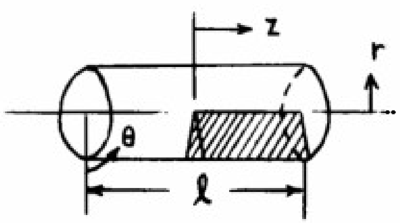
MICROWAVE MEAT ROASTING – A COMPUTER ANALYSIS FOR CYLINDRICAL ROASTS

I. INTRODUCTION

The research method and results of a program in microwave meat roasting during 1974 and 1975 has been reported in the Journal of Microwave Power.[1] The key element of this research effort was the development of a computer model of microwave meat roasting. This computer model consisted basically of the numerical solution of the heat conduction equation applied to the node network of a cylinder; the output of this program consists of node temperatures for each time increment throughout the cooking period. The solution of the temperature distribution throughout a solid cylinder has been previously obtained in closed form for various initial conditions and heating schemes.[2] The complication added by microwave heating, however, precludes the use of such closed form solutions. Also complicating the issue is the evaporative cooling at the meat surface which is both time and temperature dependent. This report describes the details and assumptions associated with the computational procedure, as well as detailed material compiled from references but not listed in reference 1. This report is intended as reference for users of the ROAST computer program.

II. HEAT TRANSFER EQUATIONS

It is assumed that microwave, convective, and radiant heating, and evaporative cooling, are uniformly distributed over the cylindrical roast surface. It is also assumed the meat is homogeneous and all properties are isotropic. Symmetry about the r and z axes is therefore assumed and temperatures in the thin wedge (shaded) are representative of temperatures throughout the entire roast. Excluding terms involving θ, the general heat conduction equation for a cylinder is:



$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial z^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \qquad (1)$$

[1] Nykvist, W. E. and Decareau, R. V., "Microwave Meat Roasting," J. Microwave Power 11 (1), 1976

[2] Schneider, P. J., *Conduction Heat Transfer*, Addison-Wesley, Reading, MA 1955

T = temperature, °C

t = Time, s

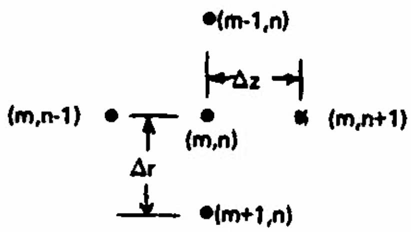
$\dot{q}$ = rate of heat addition, cal/s cm^3

k = thermal conductivity, cal/cm s°C

ρ = density, g/cm^3

c = specific heat, cal/g°C

Since the dielectric properties of meat are temperature dependent, the penetration of microwave radiation varies with temperature, greatly complicating an analytical solution of equation (1). This equation, therefore, is approximated by a finite difference technique. A network of nodes is imposed over the center plane of the thin wedge and the equation is rewritten using finite difference approximations for the differentials. Using the two-dimensional node scheme shown, equation (1) in finite difference form becomes



$$\frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{\Delta r^2} + \frac{1}{r}\,\frac{T_{m+1,n} - T_{m-1,n}}{2\Delta r} + \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{\Delta z^2}$$

$$+ \frac{\dot{q}}{k} = \frac{\rho c}{k}\left[\frac{(T_{m,n})_{new} - T_{m,n}}{\Delta t}\right] \qquad (2)$$

Rearranging, letting $\dot{q} = p/V$ and solving for the new node temperature, $(T_{m,n})_{new}$, gives:

$$(T_{m,n})_{new} = \frac{k\Delta t}{\rho c}\left[\frac{T_{m+1,n} + T_{m-1,n}}{\Delta r^2} + \frac{T_{m+1,n} - T_{m-1,n}}{2r\Delta r} + \frac{T_{m,n+1} + T_{m,n-1}}{\Delta z^2}\right.$$

$$\left. + \frac{p}{kV} + T_{m,n}\left\{\frac{\rho c}{k\Delta t} - \frac{2}{\Delta r^2} - \frac{2}{\Delta z^2}\right\}\right] \qquad (3)$$

where p = microwave power absorbed by node m,n (cal/s)

V = volume associated with node m,n (cm^3)

Holman[3] provides a detailed description of finite difference approximations.

A closer view of the thin wedge shows each node to have a volume associated with it, as shown in Figure 1.

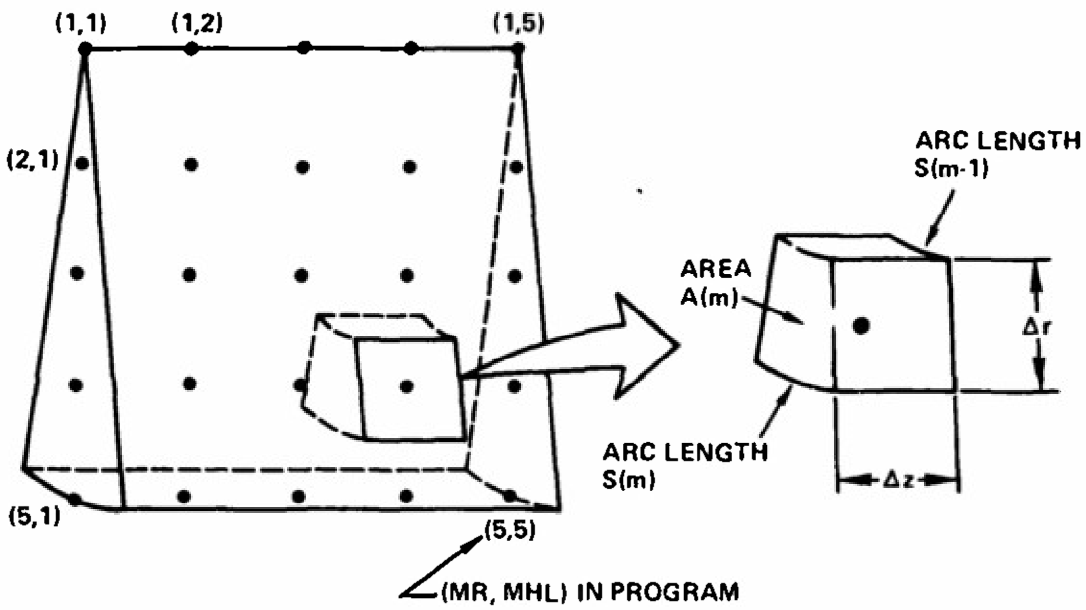


Figure 1. Roast Wedge Showing Typical Node and Associated Volume

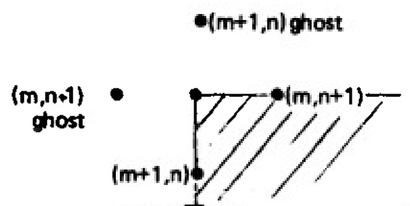
Figure 1 has a 5-by-5 network of nodes; in the ROAST computer program nodes are usually spaced every 0.5 cm, giving typically a 13-by-25 node network. Once the roast dimensions and the desired number of radial nodes is known, the "end areas" denoted by A(m) are calculated by assuming the arc length at the roast surface to be equal to Δr. Thus, associated with each node are arc lengths S(m) and S(m-1), and area A(m).

[3]Holman, J. P., *Heat Transfer*, McGraw-Hill, New York, 1968

There are two types of boundary conditions for the wedge, the inside or meat-to-meat interface and the outside or meat surface-to-air interface. The inside boundary conditions ere handled in subroutine INTER as speciel cases of equation (3). There are three special cases for insida boundery conditions:

Case 1 $m = n = 1$

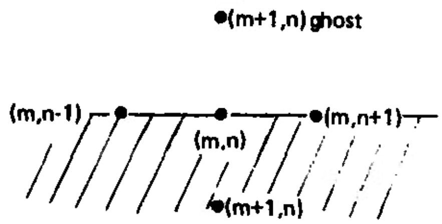
This case involves the wedge innermost corner and involves heat trensfer from only two adjacent nodes. To take into account symmetry at tha two wedge edges, "ghost" nodes ere assumed to be outside the wedge. These nodes are assumed to be at the same temperature as the real nodes opposita. The form of equation (3) then becomes



$$(T_{m,n})_{new} = \frac{k\Delta t}{\rho c}\left[\frac{2T_{m+1,n}}{\Delta r^2} + \frac{2T_{m,n+1}}{\Delta z^2} + \frac{p}{kV} + T_{m,n}\left\{\frac{\rho c}{k\Delta t} - \frac{2}{\Delta r^2} - \frac{2}{\Delta z^2}\right\}\right] \tag{4}$$

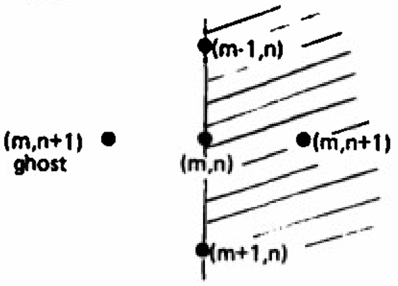
Case 2 $m = 1,\ 1 < n < MHL$

This case covers nodes along the top edge of the wedge. Using only ona ghost node, equation (3) for this special case becomes



$$(T_{m,n})_{new} = \frac{k\Delta t}{\rho c}\left[\frac{2T_{m+1,n}}{\Delta r^2} + \frac{T_{m,n+1} + T_{m,n-1}}{\Delta z^2} + \frac{p}{kV} + T_{m,n}\left\{\frac{\rho c}{k\Delta t} - \frac{2}{\Delta r^2} - \frac{2}{\Delta z^2}\right\}\right] \tag{5}$$

Case 3 $n = 1,\ 1 < m < MR$



This case covers nodes along the left edge of the wedge. The appropriate form of equation (3) becomes

$$(T_{m,n})_{new} = \frac{k\Delta t}{\rho c}\left[\frac{T_{m+1,n} + T_{m-1,n}}{\Delta r^2} + \frac{T_{m+1,n} - T_{m-1,n}}{2r\Delta r} + \frac{2T_{m,n+1}}{\Delta z^2} + \frac{p}{kV} + T_{m,n}\left\{\frac{\rho c}{k\Delta t} - \frac{2}{\Delta r^2} - \frac{2}{\Delta z^2}\right\}\right] \qquad (6)$$

Outside boundary conditions involve heat transfer at the meat surface. These conditions are handled by making a heat balance about each surface node instead of modifying equation (3). The three basic formulas used are

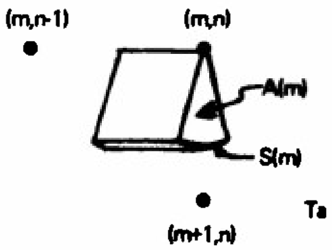
(a) heat conducted = k [conduction area] $\frac{\Delta t}{\text{[path length]}}$

(b) heat convected = h [surface area] $[T_{m,n}\text{-}T_a]$

(c) increase in internal energy = ρc [node volume] $\left[\frac{(T_{m,n})_{new} - T_{m,n}}{\Delta t}\right]$

A special subroutine, BNDRY, handles the following five special cases:

Case 4 $m = 1,\ n = MHL$



This case covers tha node at the upper right corner of the wedge. No ghost node need be considered in this case as a direct heat balance is being carried out. This node can receive radiant and convective energy and can lose heat by evaporative cooling at tha outside surface designated by A(m). The heat balance is

heat from (m,n-1) to (m,n)	+	heat from (m+1,n) to (m,n)	+	convected heat	+

$$kA(m)\left[\frac{T_{m,n-1} - T_{m,n}}{\Delta z}\right] + kS(m)\,\frac{\Delta z}{2}\left[\frac{T_{m+1,n} - T_{m,n}}{\Delta r}\right] + hA(m)\left[T_a - T_{m,n}\right] +$$

radiant heat	+	microwave heat	−	evaporative heat	=	increase in internal energy

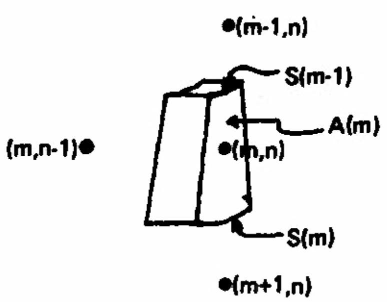
$$\dot{q}_r A(m) + p - h_e A(m) = \rho c A(m)\,\frac{\Delta z}{2}\left[\frac{(T_{m,n})_{new} - T_{m,n}}{\Delta t}\right] \quad (7)$$

This equation is solved for $(T_{m,n})_{new}$

$$(T_{m,n})_{new} = \frac{2\Delta tk}{\rho c A(m)\,\Delta z}\left[\frac{A(m)}{\Delta z}\,T_{m,n-1} + \frac{S(m)\Delta z}{2\Delta r}\,T_{m+1,n} + \frac{hA(m)}{k}\,T_a + \frac{\dot{q}_r A(m)}{k}\right.$$

$$\left. + \frac{p}{k} - \frac{h_e A(m)}{k} + T_{m,n}\left\{\frac{\rho c A(m)\,\Delta z}{2\,\Delta\,tk} - \frac{A(m)}{\Delta z} - \frac{S(m)\,\Delta z}{2\,\Delta r} - \frac{hA(m)}{k}\right\}\right] \quad (8)$$

Case 5 $\quad 1 < m < MR,\ n = MHL$

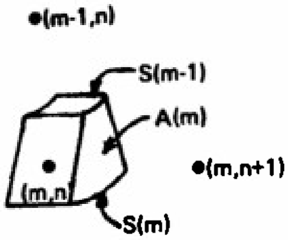


This case covers the nodes on the right side of the wedge. Again, the surface with area A(m) is subject to radiant, convective and evaporative heat transfer.

Writing tha heat balance and solving for $(T_{m,n})_{new}$ gives

$$(T_{m,n})_{new} = \frac{2\Delta tk}{\rho cA(m)\Delta z}\left[\frac{A(m)}{\Delta z}T_{m,n-1} + \frac{S(m)\Delta z}{2\Delta r}T_{m+1,n} + \frac{S(m-1)\Delta z}{2\Delta r}T_{m-1,n} + \frac{hA(m)T_a}{k} + \frac{\dot{q}_r A(m)}{k} + \frac{p}{k} - \frac{h_e A(m)}{k} + T_{m,n}\left\{\frac{\rho cA(m)\Delta z}{2\Delta tk} - \frac{A(m)}{\Delta z} - \frac{S(m-1)\Delta z}{2\Delta r} - \frac{hA(m)}{k}\right\}\right] \qquad (9)$$

Case 6 m = MR, m = 1

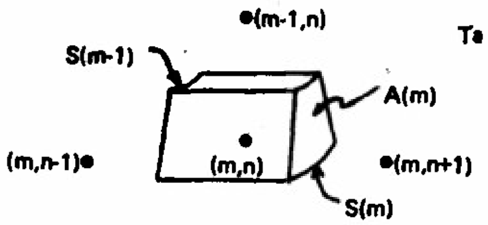


This case covars the lower left corner of the wedge. Hara tha curved surface with area $S(m)\Delta z/2$ is the outer heat exchange surface. The haat balance becomes

$$(T_{m,n})_{new} = \frac{2\Delta tk}{\rho cA(m)\Delta z}\left[\frac{A(m)}{\Delta z}T_{m,n+1} + \frac{S(m-1)\Delta z}{2\Delta r}T_{m-1,n} + \frac{hS(m)\Delta zT_a}{2k} + \frac{\dot{q}_r S(m)\Delta z}{2k} + \frac{p}{k} - \frac{h_e S(m)\Delta z}{2k} + T_{m,n}\left\{\frac{\rho cA(m)\Delta z}{2\Delta tk} - \frac{A(m)}{\Delta z} - \frac{S(m-1)\Delta z}{2\Delta r} - \frac{hS(m)\Delta z}{2k}\right\}\right] \qquad (10)$$

Case 7 $m = MR,\ 1 < n < MHL$

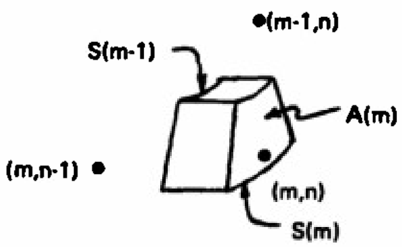
This case covers the bottom surface of the wedge. The heat balance is



$$(T_{m,n})_{new} = \frac{\Delta tk}{\rho c A(m)\Delta z}\left[\frac{A(m)}{\Delta z}T_{m,n-1} + \frac{A(m)}{\Delta z}T_{m,n+1} + \frac{S(m-1)\Delta z}{\Delta r}T_{m-1,n} + \frac{hS(m)\Delta z T_a}{k} + \frac{\dot{q}_r S(m)\Delta z}{k} + \frac{p}{k} - \frac{h_e S(m)\Delta z}{k} + T_{m,n}\left\{\frac{\rho c A(m)\Delta z}{\Delta tk} - \frac{2A(m)}{\Delta z} - \frac{S(m-1)\Delta z}{\Delta r} - \frac{hS(m)\Delta z}{k}\right\}\right] \quad (11)$$

Case 8 $m = MR,\ n = MHL$

This final case covers the right lower corner node, the only node with two outside surfaces. The heat balance is



$$(T_{m,n})_{new} = \frac{2\Delta tk}{\rho c A(m)\Delta z}\left[\frac{A(m)}{\Delta z}T_{m,n-1} + \frac{S(m-1)\Delta z}{2\Delta r}T_{m-1,n} + \frac{h}{k}\left[A(m) + S(m)\frac{\Delta z}{2}\right]T_a + \frac{\dot{q}_r}{k}\left[A(m) + S(m)\frac{\Delta z}{2}\right] + \frac{p}{k} - \frac{h_e}{k}\left[A(m) + S(m)\frac{\Delta z}{2}\right] + T_{m,n}\left\{\frac{\rho c A(m)\Delta z}{2\Delta tk} - \frac{A(m)}{\Delta z} - \frac{S(m-1)\Delta z}{2\Delta r} - \frac{h}{k}\left[A(m) + S(m)\frac{\Delta z}{2}\right]\right\}\right] \quad (12)$$

The choice of numerical velues for Δr, Δz end Δt is made according to the convergence criteria of Holmen (see reference 3). The coefficient of $T_{m,n}$ in equetion (3) should be equal to or greater than zero. This gives

$$\frac{\rho c}{k\Delta t} - \frac{2}{\Delta r^2} - \frac{2}{\Delta z^2} \geqslant 0 \qquad (13)$$

Any combination of Δt, Δz end Δr which satisfies (13) will insure convergence of the numerical solution.

III. MICROWAVE HEAT ABSORPTION

Due to the exponentiel decay of microwave energy as it penetrates the roast, each node absorbs a different amount of energy. In tha ensuing discussion microwave energy will be discussed in terms of individuel "beams" of energy. In any microwave oven multiple reflections of energy from oven wells will cause incident microwave beams to strike the roast surface with engles from the surface normal varying between 0 and 90°. Microwave beams that are incident et any angle greeter than 0° ere strongly refracted since the eir/meat interface has an index of refraction of about 7 as seen in reference 1. Thus a beam with en incident angle of 70° would actually penetrate the meat et about 8° from the surface normal. Consider a roast cross section with only 5 radiel nodes as shown in Figure 2.

A circle is drawn halfway between each node defining e "ring" that is associated with eech node. To define the microwave power of incident beam P_o one must consider first the total power, P_T, ebsorbed by the roast. This power is assumed to be equally distributed over the roast surface. If the cross section shown in Figure 2 is assumed to be a disc with thickness Δz, the fraction of totel incident power received by this disc, denoted by RT, is

$$RT = \left[\frac{2\pi r l}{2\pi r l + 2\pi r^2}\right]\left[\frac{\Delta z}{l}\right] = \frac{\Delta z}{l + r} \qquad (14)$$

Then the microwave power incident upon the disc would be

$$P_o = (P_T)(RT) \quad \text{cal/s} \qquad (15)$$

As seen in Figure 2, en incident beam with power P_o is refracted end travels obliquely through rings 5, 4, 3 end 2; it does not trevel through ring 1 end for this specific beem ring 1 does not receive eny energy. Microweve power ebsorbed is calculated according to

$$P = P_o e^{-2\alpha d} \quad (16)$$

where P = remaining power at depth d (cm)

P_o = power transmitted through surface

α = dielectric attenuation coefficient

The calculation of α is discussed in reference 1.

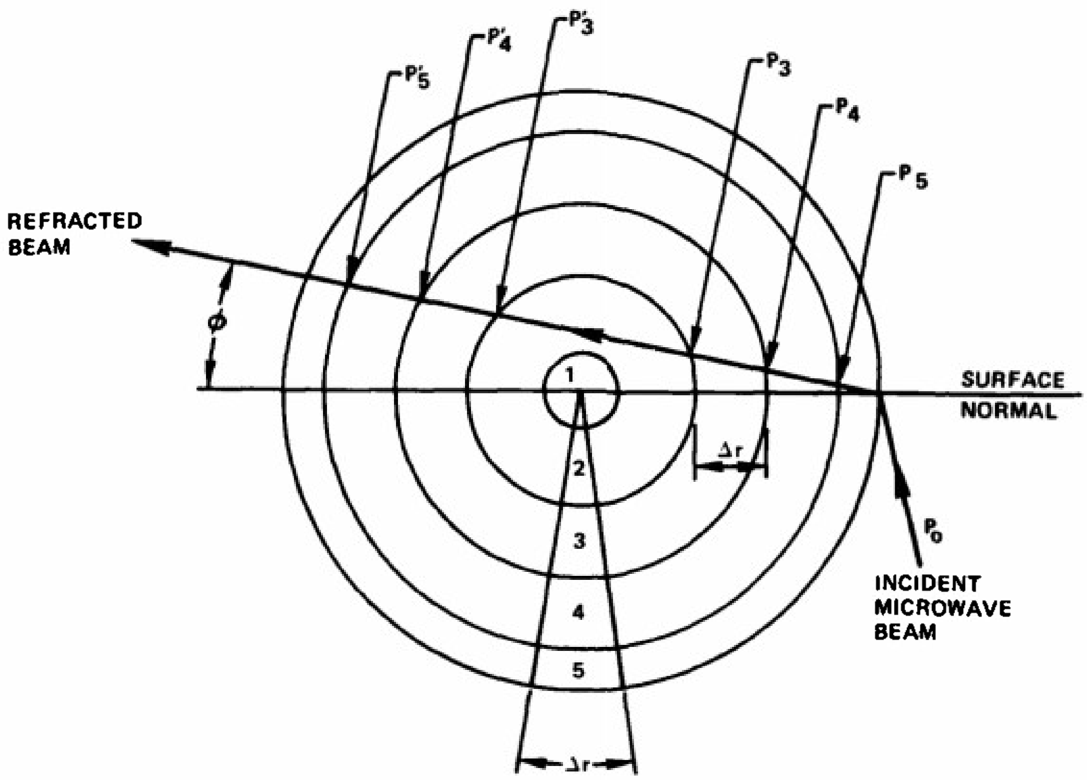
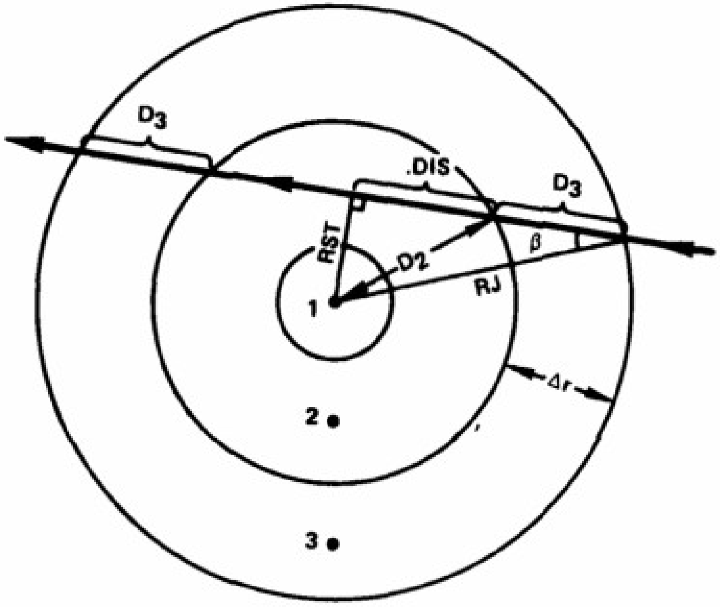


Figure 2. Roast Cross Section Showing Typical Microwave Beam Penetration

Since the angle ϕ is known, geometric considerations give the distance the beam travels through each ring. The beam cuts through ring 5, 4 and 3 twice, but ring 2 only once. Distances traveled ere calculated as follows:



$$\cos\beta = \frac{\sqrt{RJ^2 - RST^2}}{RJ}$$

$$DIS = \sqrt{D_2^2 - RST^2}$$

$$D_3 \cos\beta = \Delta r \text{ (APPROX.)}$$

$$D_3 = \frac{\Delta r}{\cos\beta}$$

Figure 3. Calculation of Distance Traveled in Each Ring

In the computer program, prior to any power absorption calculations, the value of $\cos\beta$ for each ring is calculated as shown in Figure 3. To calculate power absorbed, one must start at the surface (outermost ring) using equation (16). Then

$$P_5 = P_o e^{-2\alpha D_5}$$

and

$$P_4 = P_5 e^{-2\alpha D_4} = P_o e^{-2\alpha D_5} e^{-2\alpha D_4} = P_o e^{[-2\alpha D_4 - 2\alpha D_5]}$$

accordingly

$$P_3 = P_o e^{-2[\alpha D_3 + \alpha D_4 + \alpha D_5]} \quad (17)$$

Equation 17 cannot be further simplified as α is a function of temperature, and each ring (or each node) is at a different temperature. The microwave power absorbed in the first pass through ring 3, ΔP_3, is

$$\Delta P_3 = P_4 - P_3 = P_o \left[e^{-2(\alpha D_4 + \alpha D_5)} - e^{-2(\alpha D_3 + \alpha D_4 + \alpha D_5)}\right] \quad (18)$$

In a similar manner, the power absorbed in the second pass through ring 3, $\Delta P'_3$ is calculated as

$$\Delta P'_3 = P'_3 - P'_4 = P_o \left(e^{-2(\alpha D_5 + \alpha D_4 + \alpha D_3 + 2\alpha D1S)} - e^{-2(\alpha D_5 + \alpha D_4 + \alpha D_3 + 2\alpha D1S)}\right] \quad (19)$$

Then the total energy absorbed in ring 3 is the sum of energy absorbed in the two passes, and is written as

$$P_{3T} = \Delta P_3 + \Delta P'_3 \quad (20)$$

The power absorbed by each ring can be calculated in a similar manner. Referring to Figure 1, it is seen that only a fraction of the power absorbed in each ring is absorbed by the wedge-shaped nodal area. This fraction is the wedge outer arc length (assumed to be Δr) divided by the roast outer circumference (here $8\pi\Delta r$, see Figure 2), or $1/8\pi$. Then the radial component of power absorbed by node 3 would be

$$P_{node\ 3} = P_{3T}/8\pi \quad \text{cal/s} \quad (21)$$

In actual operation the POWER subroutine calculates power absorbed in a similar manner but somewhat modified to make computations in the computer more efficient. To get good computed/experimental data agreement, several beam incidence angles (ϕ_I) were assumed with a certain percentage of total power associated with each. In reference 1 a typical set of ϕ_I with associated percentages, called the m/w distribution, was 40% power at $\phi_1 = 0°$, 30% power at $\phi_2 = 10°$, and 30% power at $\phi_3 = 20°$. For each ϕ_I, an array COB(I, J) is computed (where J = 1,MR) which stores the value of $\cos\beta$ for each ring (J) for each incidence angle (I).

Since the heat transfer portion of the program, subroutine INTER, begins with node (1,1) and increments radially out to node (MR,1), then node (1,2) to (MR,2), etc., the POWER subroutine first calculates the power remaining at the most interior node the beam passes through and works its way to the surface by doing things in reverse.

Microwave power entering the roast ends is a much simpler case to handle. A procedure utilizing a direct application of equetion (16) is used for energy propagating in the z-direction. Again microwave beams are incident in a random manner, with angles from the surface normal varying between 0° and 90°. Due to the large refraction at the roast surface end the plane surface involved, it is assumed that the microwave energy is in the form of a plane wave. Then the simplifying assumption that all microwave power is incident perpendicularly to the surface can be made. Energy entering both ends is taken into account. In a procedure similar to that already discussed, the program first steps through equation (16) from the surface to tha center, and works through the iteration process in reverse.

IV. SURFACE PHENOMENA

A. Convective Heat Transfer

The temperature difference between the roast surface and surrounding air causes natural convection currents to transfer some heat from (or to) the roast. The complex process of convective heat transfer is summed up in one term, h, the convectiva heat transfer coefficient. The measurement of h is difficult as both h and h_e (evaporative cooling coefficient, discussed in the next section) are surface cooling phenomena present during microwave cooking. Early research on meat roasting in a heated oven (no microwaves) with the roast suspended from a scale provided evaporative cooling data, typically as shown in Figure 4. Using this experimental data, a trial-and-error process was used to determine h, by matching the computed and experimental time-temperature profiles. For the case shown in Figure 4, the corresponding temperature profila is shown in Figure 5. The trial-and-error process gave the value of h to be 0.00011 cal/g°Ccm². This value of h was confirmed by several more tests, and was used throughout all research.

B. Evaporative Cooling

The evaporative cooling coefficient, h_e (called HEVAP in computer program) has been a difficult parameter to work with. Without the benefit of surface evaporation and

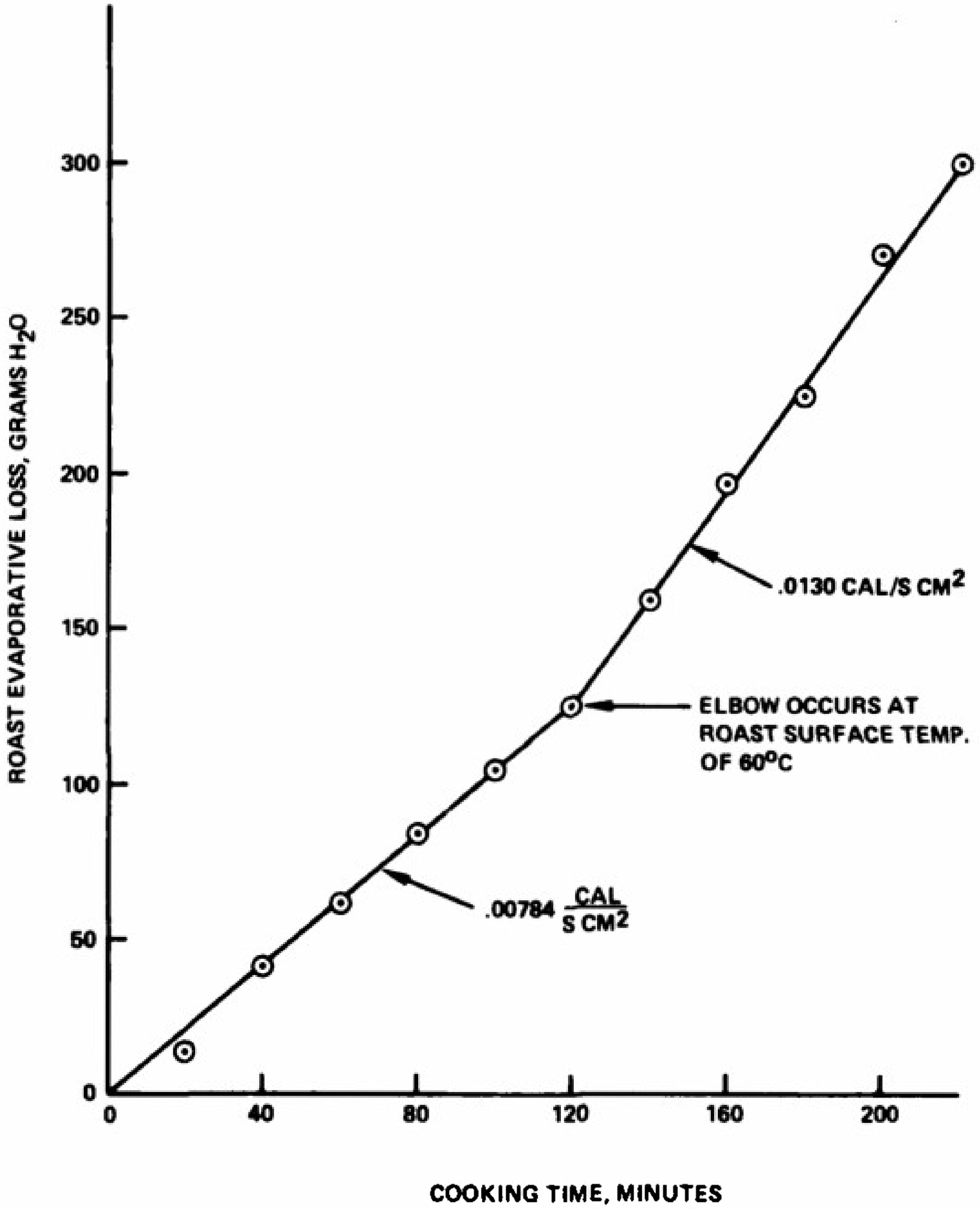


Figure 4. Evaporative Loss in 121°C Oven, Beef Roast 3380 g, Surface Area 1186 cm^2

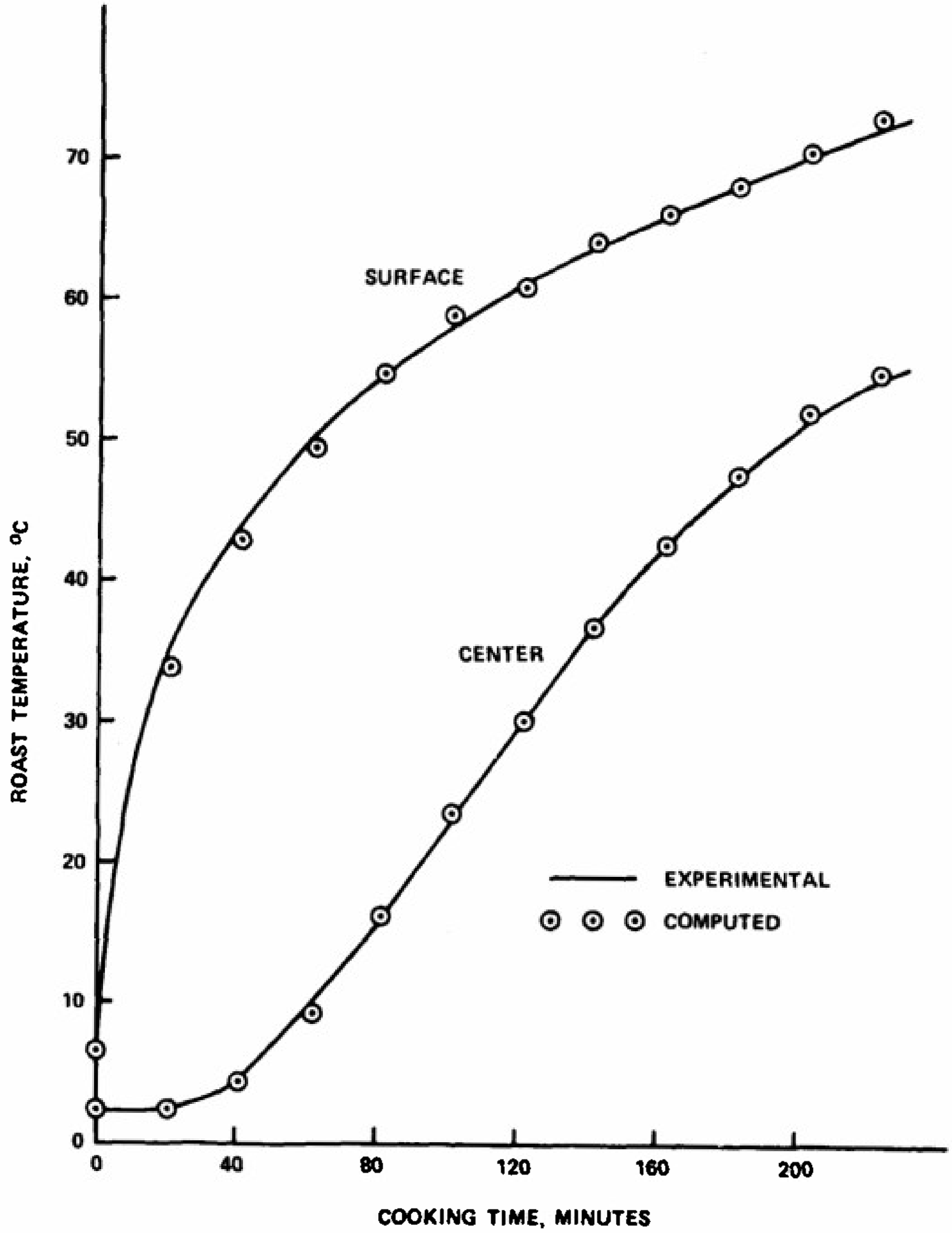


Figure 5. Roast Temperature History in 121°C Oven, Beef Roast 3380 g

its cooling effect, the meat near the surface would overcook resulting in toughening, hardening, and unpalatability. Acquisition of evaporative cooling data during microwave cooking was not possible as the roast had to be rotated during microwave heating and could not be suspended from a scale. Using $h = 0.00011$ cal/g°Ccm² as previously discussed, a trial-and-error procedure was used to determine the value of h_e, again by matching the experimental and computed temperature profiles. For several roasting conditions, h_e is shown as a function of surface temperature in Figure 6. The data of Figure 6 was graphed to show both slope and constant value of HEVAP v.s. wattage, providing a linear relationship between these variables. This information was directly programmed in the EVAP subroutine so that proper values of the slope and the constant value of HEVAP are automatically assigned given the frequency and wattage of microwave power used. For a heated oven without use of microwave power, or combinations of microwave heating and radiant heating, values of variables HP and HMX are read in as input data, and HEVAP = HP * TO(M,N) for roast surface temperatures less than 40°C, and HEVAP = HMX for surface temperatures greater than 40°C.

C. Radiant Heat Transfer

A detailed analysis of radiant energy transfer was performed. Four surfaces were involved: (1) the roast, (2) the oven ends, (3) the heating elements, and (4) the curved oven wall. The size and shape of each surface was measured, and shape factors between all surfaces were calculated. Then a set of 4 equations in 4 unknowns, the radiosities J_i, were written for the measured surface temperatures for each oven setting. Using the calculated radiosities, radiant heat transfer to the roast was calculated by multiplying the difference in radiosities by the shape factor. Appendix A shows the details, and the slopes and intercepts of the curves of Figure A3 are included in element MAIN 2.

D. Index of Refraction

Incident microwave radiation experiences a large refraction as it enters the roast surface. Assuming meat has a negligible magnetic loss and a permeability equivalent to that of a vacuum, Von Hippel[4] gives the index of refraction, n_r, as

[4]Von Hippel, A. R., *Dielectrics and Waves*, MIT Press, Cambridge, MA, 1954.

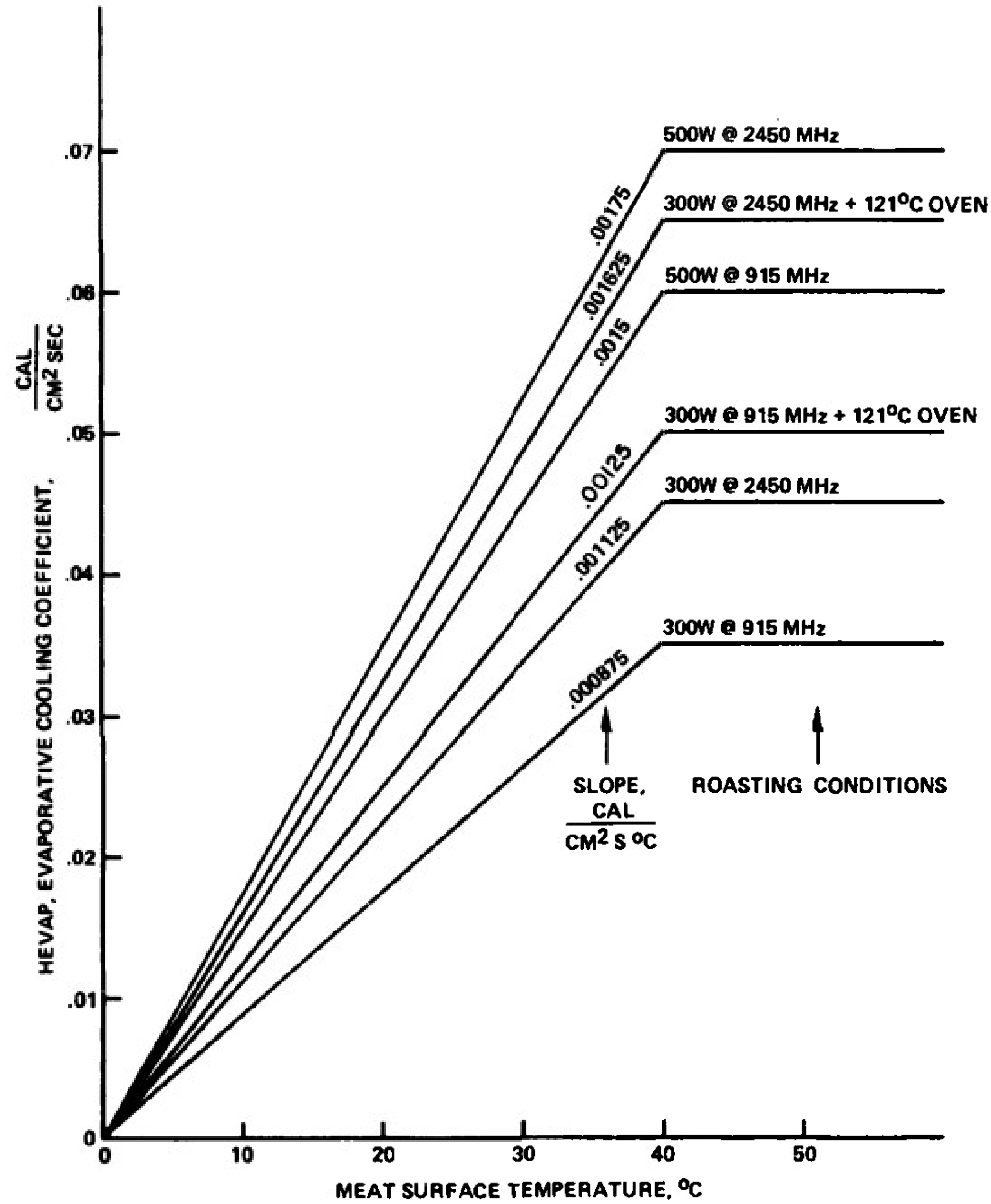


Figure 6. **Evaporative Cooling Coefficient Determined by Computed/Experimental Data Matching, with h = 0.00011 cal/cm² s°C**

$$n_r = \left[\frac{\epsilon'_r}{2}\left\{\sqrt{1 + \left(\frac{\epsilon''_r}{\epsilon'_r}\right)^2} + 1\right\}\right]^{1/2} \qquad (22)$$

where ϵ'_r = dielectric constant

ϵ''_r = dielectric loss factor

For fresh raw beef the value of n_r over the range of 0–100°C ranges from 6.5 to 7.7 for frequencies from 900 to 2800 MHz. For frozen beef, the value of n_r is 2.1 to 2.2 for the above frequency range. For example, if a microwave "beam" was incident at 45° from the roast surface normal, it would penetrate the meat at approximately 19° for frozen meat, and approximately 6° for thawed meat. Thus the microwave distribution and amount of focusing of microwaves is much less for a frozen roast than a thawed one. It is important to note that the index of refraction depends on the surface temperature, and is an important consideration when using microwave energy to thaw roasts.

V. SUPPORTING DATA

A. Dielectric Properties

The dielectric properties of meat differ for the two microwave frequencies used, 915 and 2450 MHz. From the values of ϵ'_r and ϵ''_r reported by Goldblith and Wang[5] for 915 MHz, the value of α, the attenuation coefficient, was calculated according to

$$\alpha = \frac{2\pi\nu}{C_v}\left[\frac{\epsilon'_r}{2}\left\{\sqrt{1 + \epsilon''_r/\epsilon'_r} - 1\right\}\right]^{1/2} \qquad (23)$$

where ν = microwave frequency, cycles/s

C_v = speed of light in vacuum, cm/s

The value of α was plotted as a function of temperature (see reference 1), divided up into 10°C segments, and a median value of α assumed constant over each 10°C interval. Thus the variation of α with temperature is approximated by a stepped curve; this information is stored in an array in computer function ALPHA. When the value of α

[5] Goldblith, S. A., and Wang, D.I.C., "Dielectric Properties of Foods," US Army Netick R&D Command, Technical Report TR 76–27–FEL, 1975.

for a particuler node is required in computer computations, function ALPHA is called. The node temperature is divided by 10, this result truncated giving en integer, end the value of α associated with that integer is used.

For 2450 MHz, dielectric data from Bengtsson and Risman[6] and Ohlsson and Bengtsson[7] were used. Although obtained for 2800 MHz, the data were used for 2450 MHz (see reference 1 for the explanation). Equetion (23) is used to obtain values of α, and the same step-epproximation previously described is used tn digitize the data. Here function ALPHF is used to store the data and obtain the curr t value of α (according to the node temperature) when called.

B. **Thermel Conductivity**

The variation of the thermal conductivity of meat, k, with temperature and percent moisture was accounted for in this research. Data from Hill[8] was linearized with respect to percent moisture, and nonlinear equations were derived for k as a function of both temperature and percent moisture. Figure 7 shows the raw data plotted and Figure 8 shows the variation of the slope and intercept of the Figure 7 curves plotted as a function of temperature. Thus for a given temperature, from the equations of the Figure 8 a value of slope and intercept is found which in essence defines a straight line on Figure 7; then given the percent moisture, a unique value of k is identified in Figure 7. Subroutine COND contains the equations illustrated in Figures 7 and 8.

[6] Bengtsson, N. E., and Risman, P. O., "Dielectric Properties of Foods at 3 GHZ as Determined by a Cavity Perturbation Technique II Measurements on Food Meteriels," J. Microwave Power, 6 (2), 1971.

[7] Ohlson, T., and Bengtsson, N. E., "Dielectric Food Data for Microwave Sterilizetion Processing," J. Microwave Power, 10 (1), 1975.

[8] Hill, J. E., Leitmen, J. D., and Sunderland, J. E., "Thermal Conductivity of Various Meets," Food Tech., 21 (8), 1967.

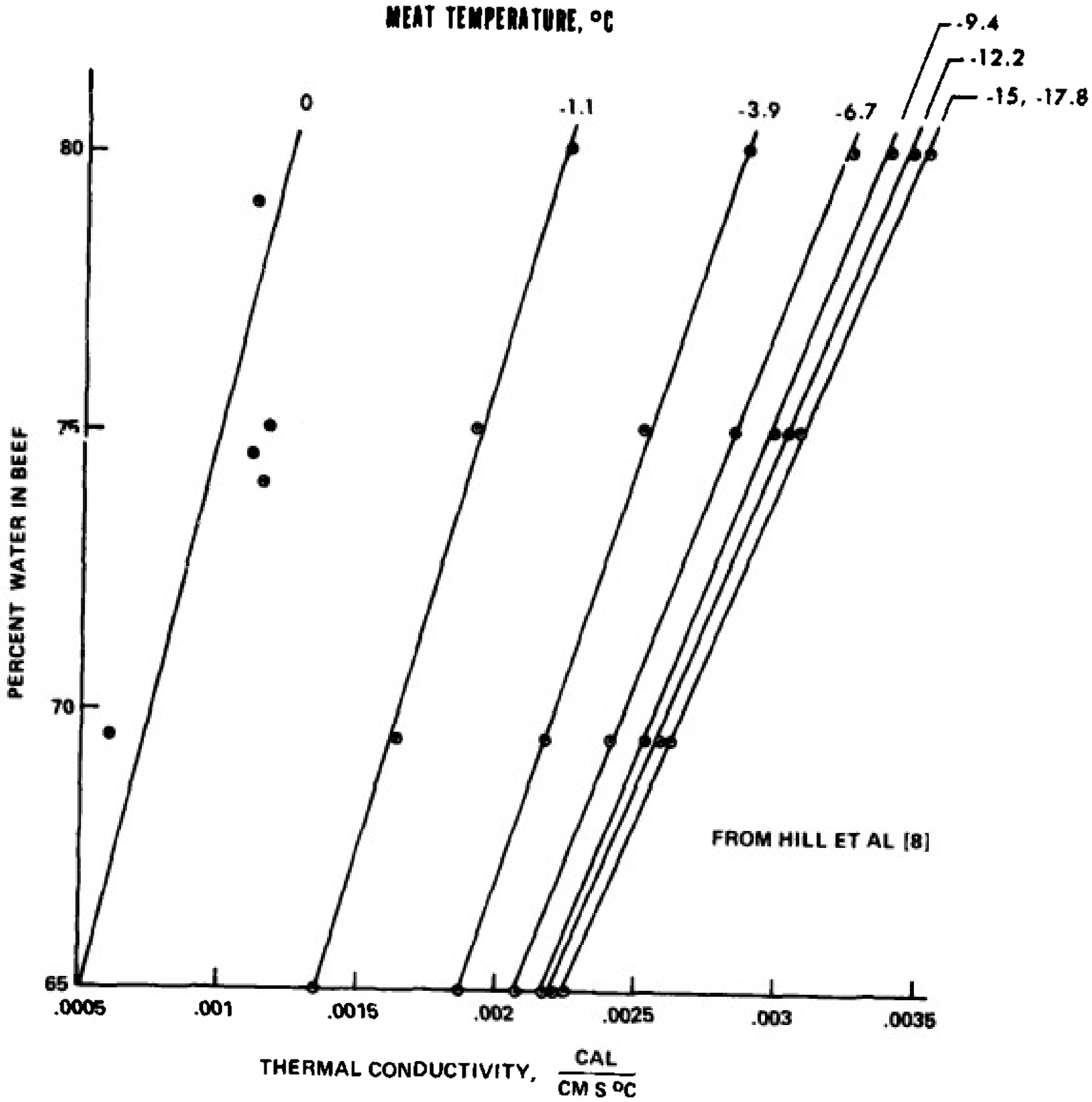


Figure 7. Thermal Conductivity of Beef Below Freezing vs Percent Water

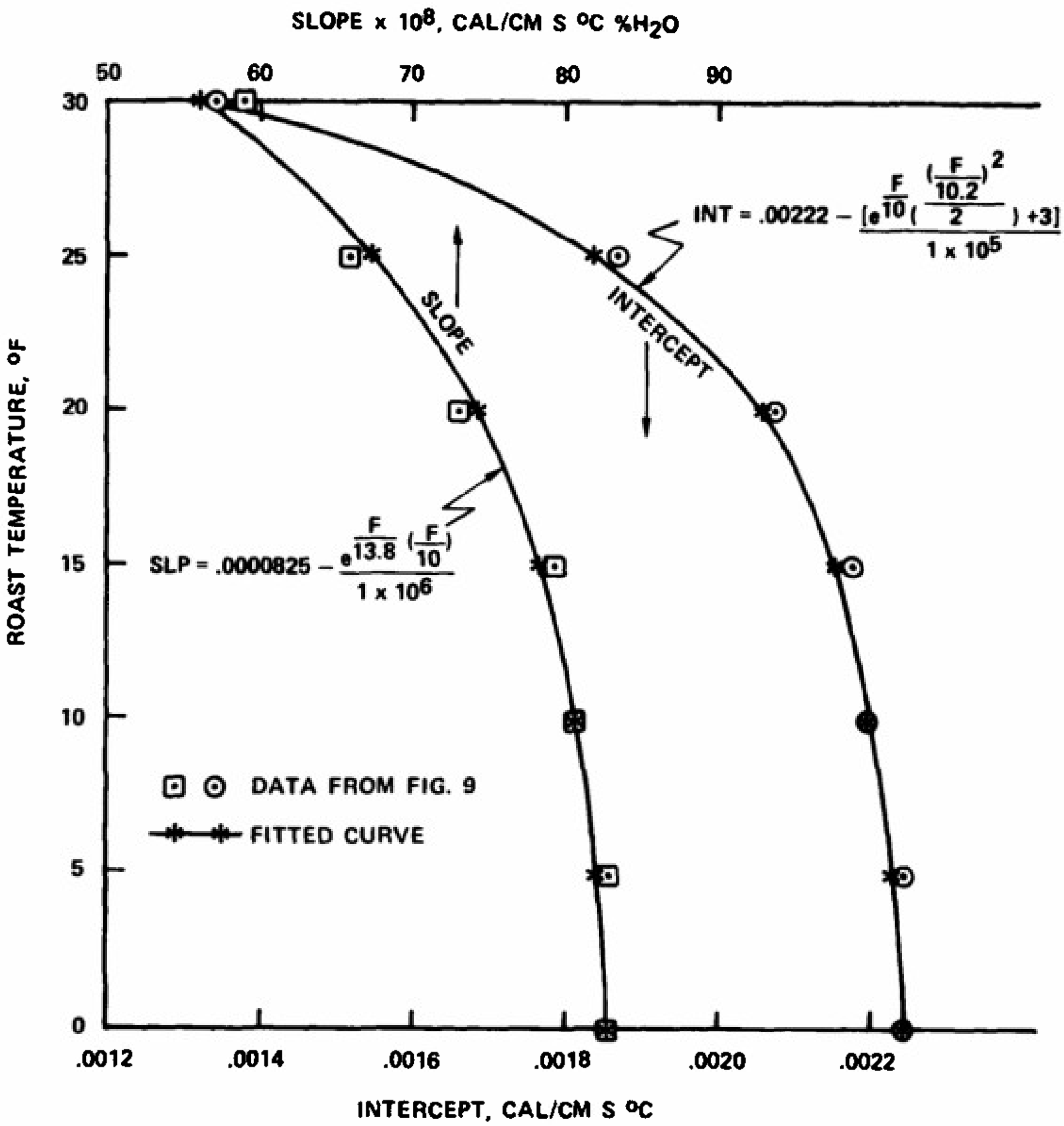


Figure 8. Fitted Curves for Slope and Intercepts from Figure 7 vs Roast Temperature

C. Enthalpy and Specific Heat

Enthalpy is a term used to describe the change in energy required to heat meat from a frozen state to a thawed state (meat is over 70% water) which occurs ovar a tamperatura range of about −10° to 0°C. Dickerson[9] determined values of enthalpy (units cal/g) for meat over the temperature range −40° to +10°C. The instantaneous slope of this curve is tha valua of specific heat, c, (units cal/g°C) at tha corresponding temperature. Tha enthalpy curva, shown in Figure 9, was linaarized over 9 temperature intervals, each interval with a different slope, or specific heat. This temperature and slope data is incorporated into subroutine SPHT. This subroutine, when called, identifies the proper interval for tha given node temperature and chooses the proper value for the specific heat.

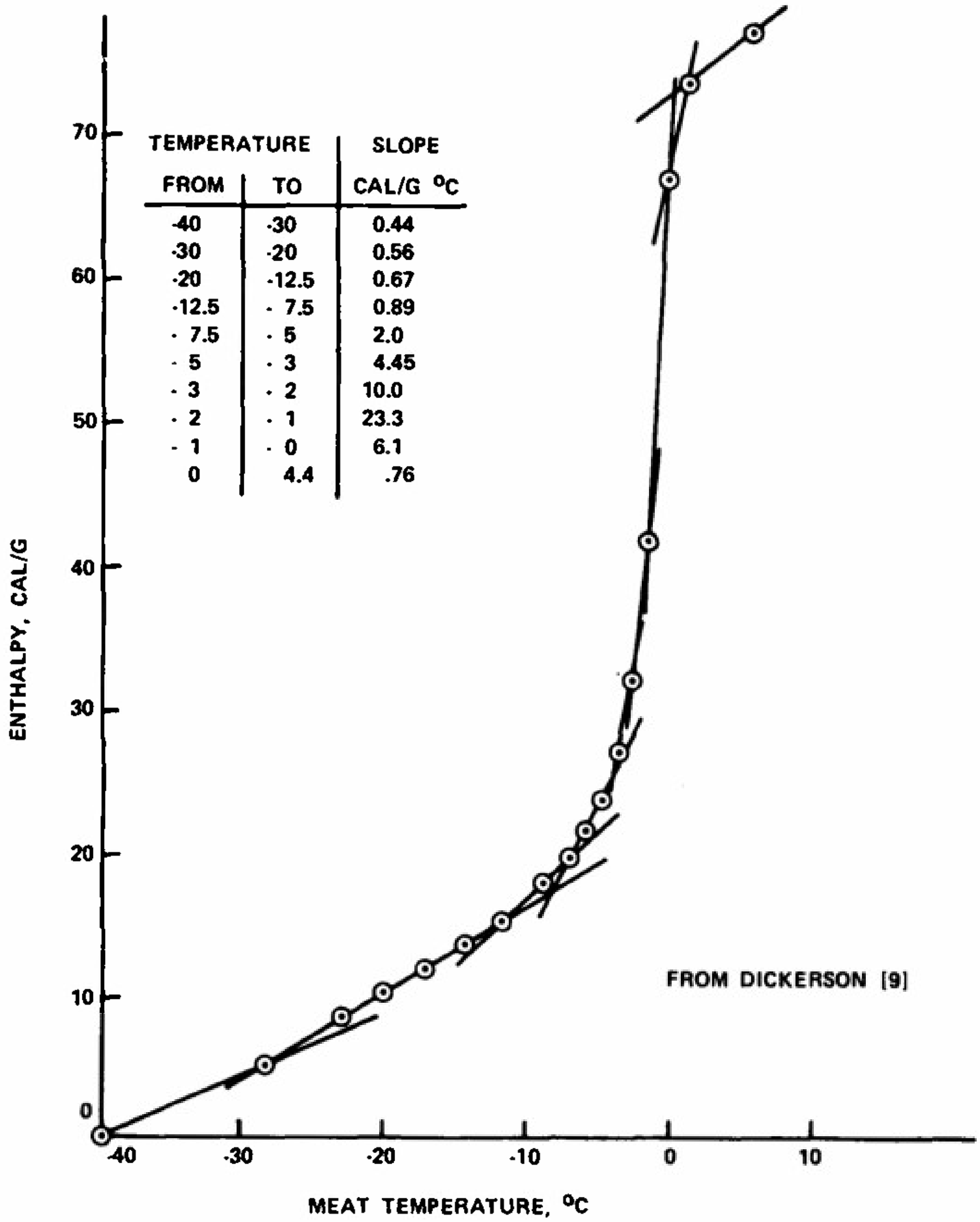
VI. COMPUTER PROGRAM SUMMARY

A. Elements

The program file is named MEAT*ROAST. and consists of a main program and 8 sub-programs. The elements are described as follows:

Element Name	Subroutine/Function Name	Description/Function
MAIN2	——	Main program which calls all subprograms, carries out iterative process, reads input data, prints output.
MAIN1	——	Main program which is used to obtain 3-dimensional plots – same MAIN2 except in way output data handled.
INTER	INTR	Calculates intarior node temperatures using finite differance approximation.
BNDRY	BNDR	Calculates boundary node temperatures taking into account radiant and microwave energy.

[9] Dickarson, R. W., Jr., "Tharmal Properties of Foods," in *The Freezing Preservation of Foods*, 4th edition, Avi Publishing Co., Westport, CT, 1968.



TEMPERATURE FROM	TEMPERATURE TO	SLOPE CAL/G °C
-40	-30	0.44
-30	-20	0.56
-20	-12.5	0.67
-12.5	- 7.5	0.89
- 7.5	- 5	2.0
- 5	- 3	4.45
- 3	- 2	10.0
- 2	- 1	23.3
- 1	- 0	6.1
0	4.4	.76

Figure 9. Enthalpy of Frozen Beef, 74.5% Water

Element Name	Subroutine/Function Name	Description/Function
POWER	POWRF	Calculates microwave power absorbed by each interior node.
ALFA	ALPHA	Calculates attenuation coefficient for 915 MHz.
ATTEN	ALPHF	Calculates attenuetion coefficient for 2450 MHz.
CONDCT	COND	Calculetes thermal conductivity.
ENTHPY	SPHT	Calculates slope of enthalpy curve, or specific heat.
EVAP	EVAP	Calculates HEVAP, the evaporative cooling coefficient.
DATA	––	Input data file described in next section.

B. Input Data Format

There ere 10 lines of input data necessary to completely define roasting conditions. These 10 lines ere contained in a data file called DATA, and are in the free format; i.e., real numbers separated by a comme. DATA variables must be in the following order:

1. P2450, P915, TIN, TAM, ENDPCT, NSHLD

2. D1, D2, XL, ΔZ, ΔT, TMAX, KDELO

3. RHO, H, PH20

4. TAM1, TAM2

5. T2450A, P2450A, T2450B, P2450B, T915, PN915, TOVEN, TMPOVN

6. HP, HMX

7. NBT

8. TT(I), PCTT(I)

9. NBF

10. TF(I), PCTF(I)

Definition of the above variables is as follows:

Variable	Definition
P2450	Oven power at 2450 MHz, watts
P915	Oven power at 915 MHz, watts
TIN	Initial temperature of roast, °C
TAM	Oven ambient temperature, °C
ENDPCT	Fraction of total microwave energy entering roast through shield
NSHLD	Number of node where shield overlap begins
D1	Diameter of raw roast, cm
D2	Diameter of cooked roast, cm
XL	Roast length, cm
ΔZ	Axial node spacing, cm
ΔT	Time increment, s
TMAX	Maximum cooking time, s
KDELO	Printout interval in number of time increments
RHO	Meat density, g/cm^3
H	Convective heat transfer coefficient, cal/s°C cm^2
PH20	Percent moisture in meat
TAM1, TAM2	Used to compute oven temperature rise (about 20–50° C) When cooking with microwaves, TAM = TAM2 + TAM1 * TMIN

Variable	Definition	
T2450A	Time at which P2450 is to change, min	
P2450A	New value of P2450, watts, at T2450A min	
T2450B	Time at which P2450A is to change, min	
P2450B	New value of P2450B, watts, at T2450B min	
T915	Time at which P915 is to change, min	
PN915	New value of P915, watts, at T915 min	
TOVEN	Time at which TAM is to change, min	
TMPOVN	New value of TAM, °C, at TOVEN min	
HP	Up to 40°C roast surface temperature, HEVAP = HP * TO(M,N)	
HMX	Greater than 40°C roast surface temperature, HEVAP = HMX	
NBT	Number of incident angles of incoming radiation	thawed roast
TT(I)	Angle of incoming radiation to surface normal, rad.	thawed roast
PCTT(I)	Percent of total radiation at angle TT(I)	thawed roast
NBF	Same as NBT, TT(I) and PCTT(I) except for frozen roast	
TF(I)	Same as NBT, TT(I) and PCTT(I) except for frozen roast	
PCTF(I)	Same as NBT, TT(I) and PCTT(I) except for frozen roast	

Several features of the program not previously mentioned can be conveniently explained now.

(1) Partial or full shielding of the roast ends is possible using the ENDPCT input. Shield overlap can also be specified using NSHLD.

(2) Radial shrinkage of the roast during cooking is accounted for by inputs D1 and D2. Twenty-four equal diametral divisions are assumed and linear shrinkage from diameter D1 at start to diameter D2 at finish is accounted for by modifying Δr each iteration.

(3) Chenges in microwave power or oven temperature can be made et e specified time using input line 5.

(4) If the roast is initielly frozen (i.e., TIN < 0°C), proper choice of property variebles and microwave distribution is made automatically by IF statements within the program.

(5) An optionel 30 minutes cooling (thermal equilibrium) period has been provided for in MAIN2. During this time the roast is outside the oven, the temperature profile tends to even out. An outright guess for values of h end h_e was used; for relieble results, values for h end h_e need to be determined experimentally.

C. Three-Dimensional Plotting

The CALCOMP plotter is used to plot a three-dimensional surface representing temperetures over e center exiel cross-section of the entire roest. After performing all the operations contained in MAIN2, element MAIN1 manipulates the temperature data for the 1/4 cross-section, using symmetry considerations to create a temporary file (called 15.) with temperatures representative of e full axiel cross-section. The THREE-D/II Calcomp software system is used; this system requires, for our case, e deta input of 11 lines which specify ell characteristics of the finel 3-D graph. The MAIN1 program is set up to store temperature profile at preset intervels, typically 5 minutes, in the file 15. When the roast is finished cooking (mathematically), file 15. is quite large, containing typically 10 to 20 sets of temperature profiles to be plotted. In the THREE-D/II data input, line A tells the plotter how many plots to expect (elong with other information) end Lines B through M contain graph cheracteristics. Thus two elements need to be used to edd the THREE-D/II input data. Element ROAST.B-M contains 10 lines of input for lines B through M; element ROAST.ADDS contains line A input, followed by statement "@ ADD ROAST.B-M" 10 to 20 times, one time for each plot desired. Thus the THREE-D/II softwere will produce e three-dimensionel temperature plot every preset interval of cooking time. An exemple plot is shown in Figure 10. An animeted film showing sequential tamperature profile plots, from start to finish of cooking, has been made for several heating methods. This film shows very cleerly the difference between heating methods throughout the entire roast.

VII. SUMMARY

A versatile heet transfer program for computer simuletion of redient/convective heeting of cylindrical food substances hes been presented. Provisions for microwave end shielding, radiel shrinkage, chenges in heating method during cooking, thawing, and e temperature equilibretion period have been included. Detailed heat trensfer equetions and computationel techniques have been shown.

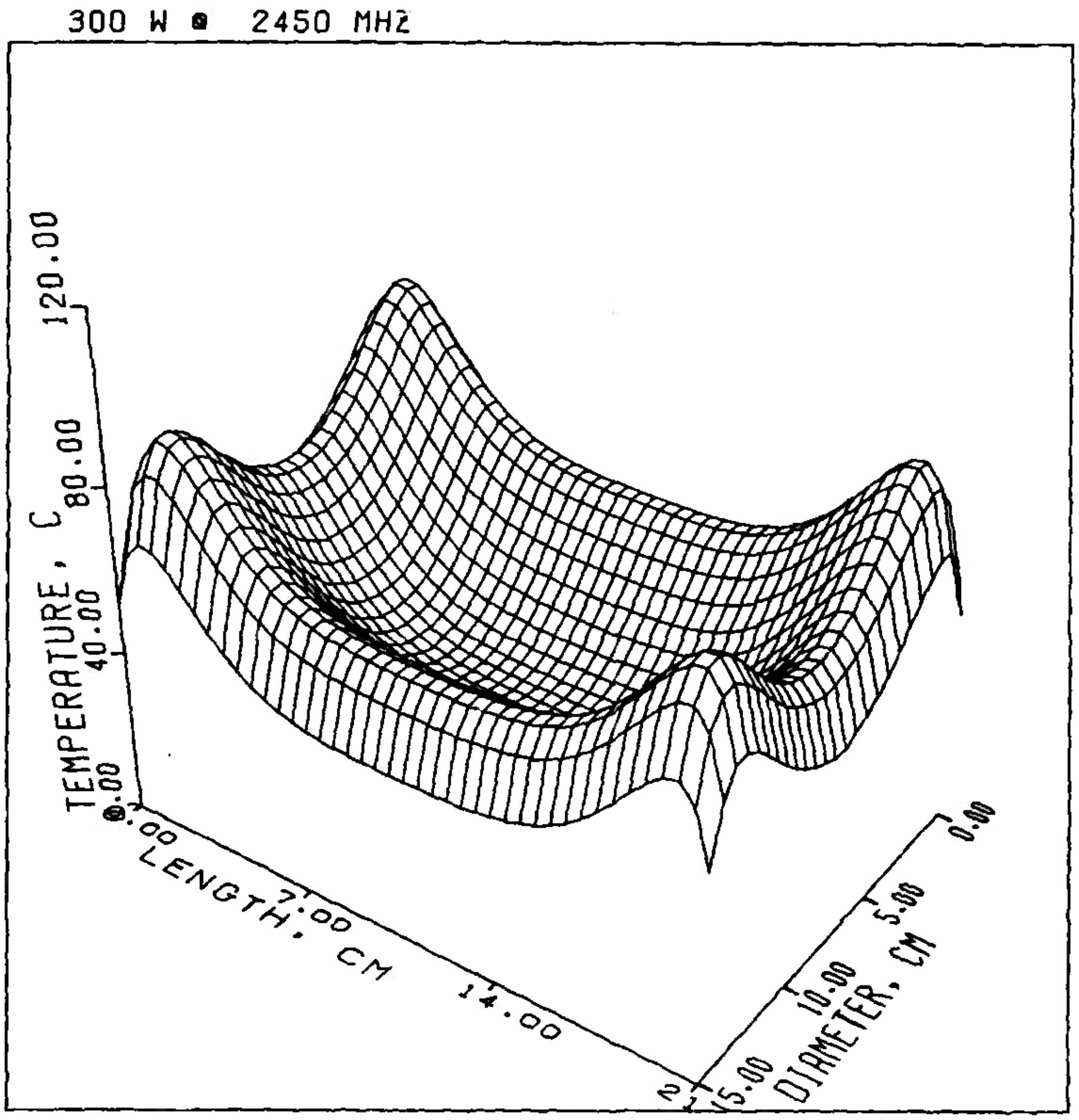


Figure 10. Three-Dimensional Temperature Plot for Microwave Heating at 300 Watts at 2450 MHz, 60 Minutes Heating Time

REFERENCES

1. Nykvist, W. E., and Decareau, R. V., "Microwave Meat Roasting," J. Microwave Power, 11 (1), 1976.

2. Schneider, P. J., *Conduction Heat Transfer*, Addison-Wesley, Reading, MA, 1955.

3. Holman, J. P., *Heat Transfer*, McGrew-Hill, New York, 1968.

4. Von Hippel, A. R., *Dialectrics and Waves*, MIT Press, Cambridge, MA, 1954.

5. Goldblith, S. A., end Wang, D. I. C., "Dielectric Properties of Foods," US Army Natick R&D Command, Technical Report TR 76-27-FEL, 1975.

6. Bengtsson, N. E., end Rismen, P. O., "Dielectric Properties of Foods at 3 GHz as Determined by e Cavity Perturbation Technique II Measurements on Food Materials," J. Microwave Power, 6 (2), 1971.

7. Ohlsson, T., and Bengtsson, N. E., "Dielectric Food Data for Microwave Sterilization Processing," J. Microwave Power, 10 (1), 1975.

8. Hill, J. E., Leitmen, J. D., and Sunderland, J. E., "Thermal Conductivity of Various Meats," Food Tech., 21 (8), 1967.

9. Dickerson, R. W., Jr., "Thermel Properties of Foods," in The Freezing Preservetion of Foods, 4th edition, Avi Publishing Co., Westport, CT, 1968.

A1. Kreith, F., *Principles of Heat Transfer*, Internetionel Textbook Co., Scranton, PA, 1967.

APPENDIX A

RADIATION HEAT TRANSFER

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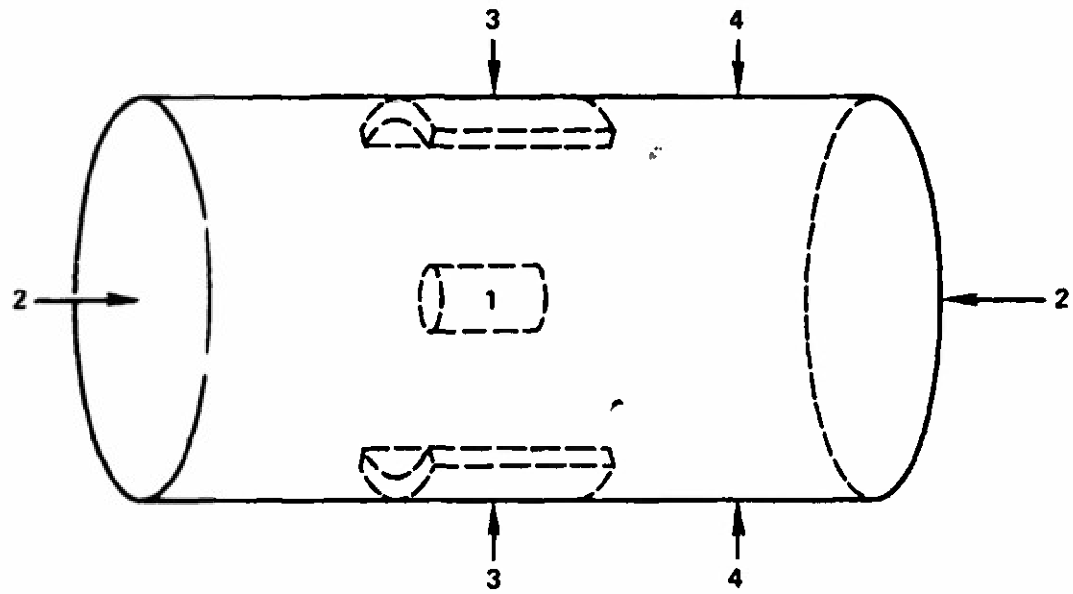
RADIATION HEAT TRANSFER

In order to carry out a radiation heat transfer analysis, the shape factors for tha various heat transfer surfaces need to be computed. These are obtained from a consideration of the oven and roast geometry, as shown in Figure A-1:



A1:	ROAST	0.117 m^2
A2:	OVEN ENDS	0.280 m^2 EACH.
A3:	HEATING ELEMENTS	0.155 m^2 EACH
A4:	OVEN CURVED SURFACE (HEATED)	1.452 m^2

Figure A1. Oven cavity, 59.7 cm dia by 94 cm long, showing heat transfer surface areas.

The shape factors are then computed as follows:

$$F_{1-4} = \frac{A_4}{A_2+A_3+A_4} = 0.63 \qquad F_{2-4} = \frac{A_2}{A_1+A_3+A_4} = 0.78$$

$$F_{1-2} = \frac{A_2}{A_2+A_3+A_4} = 0.24 \qquad F_{1-3} = \frac{A_3}{A_2+A_3+A_4} = 0.13$$

$$F_{3-2} = \frac{A_2}{A_1+A_2+A_4} = 0.26 \qquad F_{3-4} = \frac{A_4}{A_1+A_2+A_4} = 0.68$$

To calculate the heat transfer within the oven cavity, the electrical analogy of Kreith[A1] is used. To use the electrical analogy, values of the emissivity, ϵ, and reflectivity, ρ, are needed. Values of ρ and ϵ for each surface are estimated to be:

$\rho_1 = 0.1$ $\qquad \epsilon_1 = 0.9$

$\rho_2 = 0.5$ $\qquad \epsilon_2' = 0.5$

$\rho_3 = 0.1$ $\qquad \epsilon_3 = 0.9$

$\rho_4 = 0.5$ $\qquad \epsilon_4 = 0.5$

The net rate of radiation leaving surface 1 is:

$$\dot{q}_{net} = \frac{\epsilon_1}{\rho_1} A_1 \left(E_{b_1} - J_1\right) \tag{1A}$$

A1. Kreith, F., *Principles of Heat Transfer*, International Textbook Co., Scranton, PA, 1967.

In the electrical analogy, this equation is interpreted as a rate of current flow between nodes E_{b1} and J_1 connected by a resistance $\rho_1/A_1\epsilon_1$. Connecting nodes J_1 through J_4 by resistances due to shape factors gives the electrical analogy shown in Figure A2:

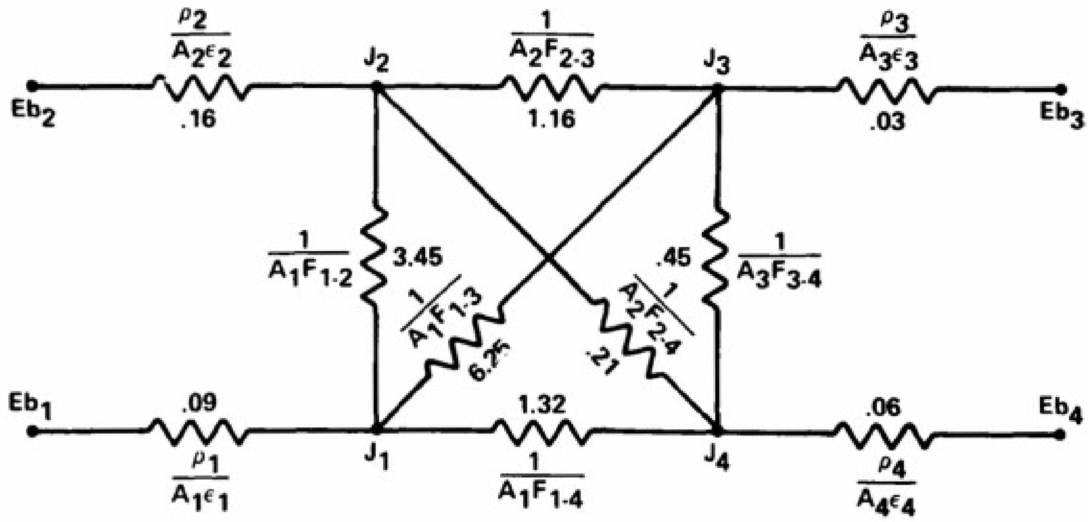


Figure A2. Electrical Analogy

Here E_{b_1} is the black body emissive power of surface 1 and is equal to $\sigma T_1{}^4$ where T_1 is the absolute temperature of surface 1 and σ is the Stefan-Boltzmann constant. The radiosity, J_1, is defined as the net rate at which radiation leaves surface 1 per unit area.

To reduce the electrical analogy network to a set of equations to solve, an equation balancing the currents at each radiosity node is written. For node J_1 this is

$$\frac{E_{b_1} - J_1}{0.09} + \frac{J_2 - J_1}{3.45} + \frac{J_3 - J_1}{6.25} + \frac{J_4 - J_1}{1.32} = 0 \qquad (2A)$$

This reduces to

$$1.11J_1 - .026J_2 - .014J_3 - .068J_4 = E_{b_1} \tag{3A}$$

Similar equations written for the other nodes J_2, J_3, and J_4 can be combined with equation (3A) into matrix form, as

$$\begin{bmatrix} 1.11 & -.026 & -.014 & -.068 \\ -.046 & 1.95 & -.14 & -.77 \\ -.005 & -.026 & 1.10 & -.067 \\ -.046 & -.29 & -.133 & 1.47 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} E_{b_1} \\ E_{b_2} \\ E_{b_3} \\ E_{b_4} \end{bmatrix} \tag{4A}$$

Then for known values of E_{b_1} through E_{b_4}, which are calculated from the surface temperatures, solution of the four simultaneous equations yields values of the radiosities. Then the radiant heat transfar received by the roast is calculated by

$$\frac{q}{A} \text{ roast} = (J_2 - J_1)F_{1-2} + (J_3 - J_1)F_{1-3} + (J_4 - J_1)F_{1-4} \tag{5A}$$

Surface tamperatures T_2, T_3, and T_4 for various oven temperatura settings ware determined by experiment. Table A1 depicts typical values obtained:

Table A1. Surface Tamperatures for Various Oven Settings

OVEN SETTING °C	OVEN SETTING K	T_2 K	T_3 K	T_4 K
121	394	389	458	333
149	422	417	497	355
177	450	444	550	378
204	478	478	611	403

Three representativa roast temperatures are chosen, 15.6°C, 54.4°C, and 93.3°C. For these roast temperatures, a valua of radiosity for each oven setting is found by solving the set of equations (4A). Tabla A2 summarizes the results.

Oven Temperature Setting (°C)	Node	Radiosity $\frac{cal}{S\ cm^2}$ For Roast Temperature (°C) 15.6	54.4	93.3
121	J_1	.0117	.0174	.0246
	J_2	.0254	.0257	.0260
	J_3	.0570	.0570	.0571
	J_4	.0316	.0319	.0322
149	J_1	.0129	.0185	.0250
	J_2	.0336	.0338	.0342
	J_3	.0787	.0788	.0789
	J_4	.0420	.0422	.0425
177	J_1	.0145	.0200	.0281
	J_2	.0450	.0452	.0455
	J_3	.1173	.1173	.1174
	J_4	.0559	.0561	.0564
204	J_1	.0169	.0225	.0305
	J_2	.0619	.0621	.0624
	J_3	.1781	.1781	.1782
	J_4	.0769	.0771	.0775

Table A2. Radiosity Values for Various Oven Temperature Settings

Using equation (5A) and the radiosity values of Table A2, tha net radiant heat received by tha roast is calculated. Values ara summarized in Table A3 and plotted in Figure A3.

Oven Temperature (°C)	Roast Temperature (°C)	Radiant Energy Received by Roast cal/s cm^2
121	15.6	.0216
	54.4	.0162
	93.3	.0093
149	15.6	.0319
	54.4	.0264
	93.3	.0203
177	15.6	.0468
	54.4	.0414
	93.3	.0336
204	15.6	.0695
	54.4	.0641
	93.3	.0564

Table A3. Radiant Energy Received by the Roast for Various Oven and Roast Temperatures

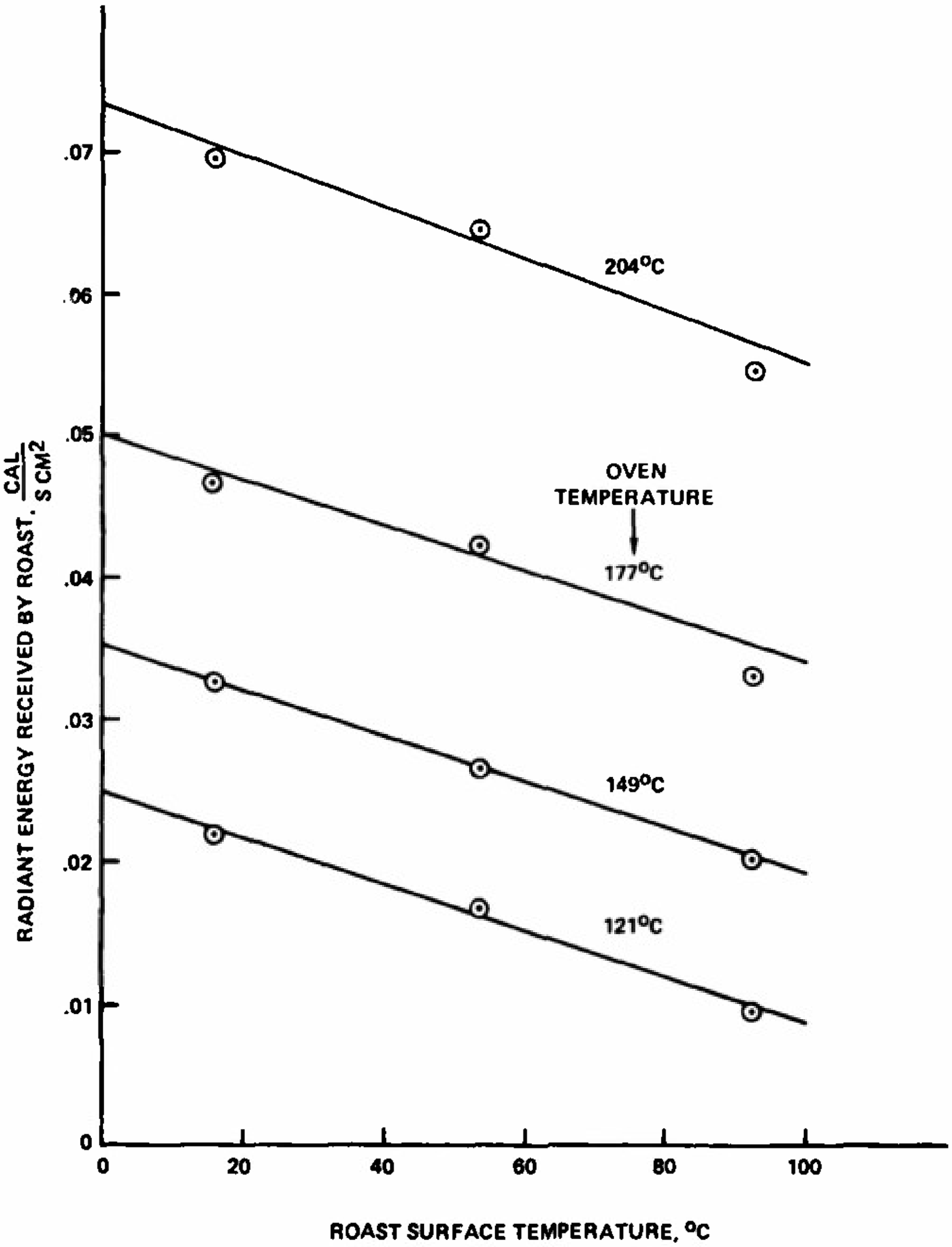


Figure A3. Radiant Energy Received by Roast for Several Oven Temperature Settings.

NOMENCLATURE

A(m)	Area associated with node (m,n), cm^2
$A_1 - A_4$	Heet trensfer surface areas in oven, m^2
c	Specific heat, cal/g°C
C_v	Speed of light in e vacuum, m/s
d	Depth measured from meat surface, cm
D_m	Distance microwave beam travels through ring m, cm
$E_{b_1} - E_{b_4}$	Black body emmissive power of surfeces 1 through 4
$F_{1.4}$	Shepe factor for surface 1 to surface 4
h	Convective heat transfer coefficient, cal/s°C cm^2
h_e	Eveporative cooling coefficient, cal/s cm^2
$J_1 - J_4$	Radiosities of surfaces 1 through 4
k	Thermel conductivity, cal/cm s°C
m	Redial node component
MR	Maximum numerical value of rediel node
MHL	Maximum numerical value of axial node
n	Axlel node component
n_r	Index of refraction
p	Microwave power ebsorbed in volume associated with node (m,n), cal/s
P	Microwave power remaining at depth d in roast, cal/s
P_o	Microwave power trensmitted through roast surface, cal/s
P_m	Microwave power remaining in beam efter first pass through ring m, cal/s
P'_m	Microwave power remaining in beam after second pass through ring m, cal/s

P_{mT}	Total microwave power lost in ring m (any integer), cal/s
P_T	Microwave power absorbed by entire roast, cal/s
$\dot{q}$	Rate of heat addition, cal/s cm^3
$\dot{q}_r$	Rate of radiant heat addition, cal/s cm^2
r	Radial axis
RT	Fraction of total power received by disc with thickness Δz
S(m)	Outer arc length of area surrounding node (m,n), cm
S(m-1)	Inner arc length of area surrounding node (m,n), cm
t	Time, s
T	Temperature, °C
$T_1 - T_4$	Absolute temperatures of surfaces 1 through 4, K
T_e	Ambient temperature surrounding roast, °C
$T_{m,n}$	Temperature of node (m,n), °C
V	Volume associated with node (m,n), cm^3
z	Axial axis
α	Attenuation coefficient, cm^{-1}
β	Angle the microwave beam makes with radial line in given ring
Δ	Increment; used as a prefix, e.g., Δr
ϕ	Refracted beam angle, degrees from normal
θ	Angular axis
ρ	Meat density, g/cm^3
$\rho_1 - \rho_4$	Reflectivity of surfaces 1 through 4
ϵ'_r	Dielectric constant
ϵ''_r	Dielectric loss factor

$\epsilon_1 - \epsilon_4$	Emissivity of surfaces 1 through 4
ν	Microwave frequency, cycles/s
σ	Stephan – Boltzmann constant