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ANALYTICAL STUDY OF FLOW OVER A SPILLWAY TOE CURVE.(U)

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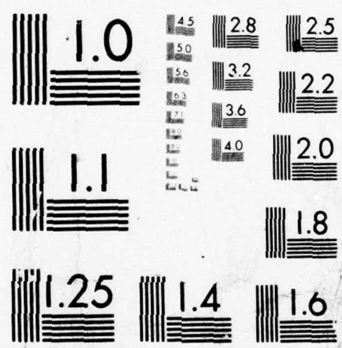

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CONTRACT REPORT H-68-1

ANALYTICAL STUDY OF FLOW OVER A SPILLWAY TOE CURVE

by

J. J. Cassidy

September 1968

Sponsored by

Office, Chief of Engineers
U. S. Army

Conducted for

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

by

Department of Civil Engineering
University of Missouri

PREFACE

A contract for an analytical study of flow over toe curves of low spillways in gravity fields was negotiated with and awarded to the Curators of the University of Missouri, Columbia, Missouri, by the U. S. Army Engineer Waterways Experiment Station. The duration of the contract was from 1 September 1966 to 30 September 1968. The Principal Investigator was Dr. John J. Cassidy, Associate Professor of Civil Engineering, Department of Civil Engineering, College of Engineering, University of Missouri. This report is the final report submitted under terms of the contract. Contract No. DACW39-67-C-0001

The study was financed by FY 1967 and 1968 funds furnished the Waterways Experiment Station by the Office, Chief of Engineers, for Engineering Studies Program, ES 804, Development of Hydraulic Design Criteria.

Directors of the Waterways Experiment Station during the contract period were COL John R. Oswalt, Jr., CE, and COL Levi A. Brown, CE. Technical Director was Mr. J. B. Tiffany. Mr. E. P. Fortson, Jr., was Chief, Hydraulics Division. Mr. F. B. Campbell was Chief, Hydraulic Analysis Branch during the initial phase of the contract study and Mr. E. B. Pickett during the final phase. Chief of the Analysis Section was Mr. R. G. Cox.

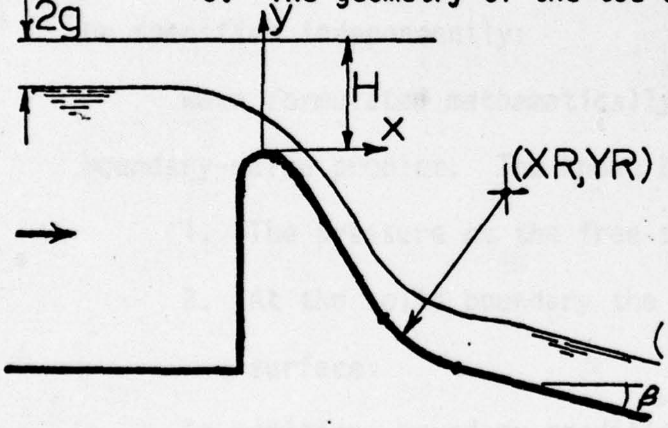
CONTENTS

Description of The Problem

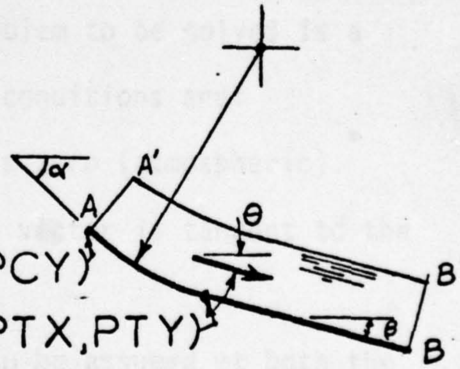
Flow over a spillway is represented graphically in Figure 1. Figure 2 shows the portion of the flow which was analyzed in this study. The object of the study was to determine the pressure distribution along the floor of the toe-curve and the coordinates of the free surface.

In analyzing the problem the following assumptions were made:

1. The flow is inviscid and irrotational.
2. The discharge per unit width Q/L is known.
3. The flow remains in contact with the lower boundary.
4. Gravity acts in the negative Y direction.
5. The elevation of the total head line is known.
6. The geometry of the toe curve is known.



Flow Over a Spillway
Fig. 1



Flow Over a Toe Curve
Fig.2

The variables represented in Figures 1 and 2 are defined as follows:

g = Acceleration due to gravity

H = Total head

R = Radius of Toe Curve

S = Slope (tan β)

Vo = Approach velocity

X,Y = Coordinates

q = Discharge per unit width of toe curve

PTX,PTY = X and Y coordinates respectively of the point of tangency

PCX,PCY = X and Y coordinates repectively of the point of curvature

XR,YR = X and Y coordinates respectively of the Center of curvature of the toe curve

V = Magnitude of the velocity

β = Angle of inclination of downstream tangent

α = Angle of inclination of tangent at the point of tangency

Unless submerged, the spillway acts as a control section. In this analysis it was assumed that the spillway controlled the flow and that the total head H and the discharge per unit width of toe curve q could be specified independently.

When formulated mathematically the problem to be solved is a boundary-value problem. The known boundary conditions are:

1. The pressure at the free surface is zero (atmospheric).
2. At the solid boundary the velocity vector is tangent to the surface.

In addition, boundary conditions had to be assumed at both the upstream limit (Line A-A') and at the downstream limit (Line B-B'). At the downstream end the velocity was assumed to be constant (Hydrostatic pressure distribution). At the upstream end it was assumed that the pressure was zero and, further, that a potential line there was a straight line.

Mathematical Statement

Because the pressure is zero along the free surface the velocity there is given by

$$V = \sqrt{2g(H-Y)} \qquad (1)$$

On the solid boundary the pressure head will be given by

$$\frac{P}{\gamma} = H - Y - \frac{V^2}{2g} \tag{2}$$

In order to analyze the problem it is desirable to map the physical plane shown in Figure 2 into the complex-potential plane shown in Figure 3. In Figure 3 ψ and ϕ represent the two-dimensional stream function and potential function respectively.

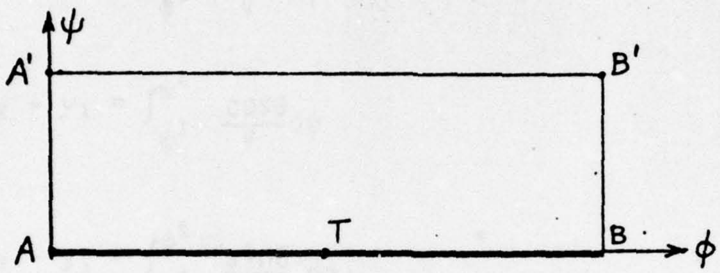


Fig. 3. Complex-Potential Plane

Within the complex-potential plane the problem is to determine X, Y and V as functions of ϕ and ψ. The velocity vector has a magnitude V and an inclination θ which can be expressed as the real and imaginary parts of the complex velocity [1]. The natural logarithm of the magnitude of the velocity lnV and the inclination θ must satisfy the laplace equations

$$\frac{\partial^2\theta}{\partial\psi^2} + \frac{\partial^2\theta}{\partial\phi^2} = 0 \tag{3}$$

$$\frac{\partial^2 \ln V}{\partial\psi^2} + \frac{\partial^2 \ln V}{\partial\phi^2} = 0 \tag{4}$$

within the complex-potential plane [2]. The parts lnV and θ are also related by the Cauchy-Riemann conditions [3].

$$\frac{\partial \ln V}{\partial\phi} = -\frac{\partial\theta}{\partial\psi} \tag{5}$$

$$\frac{\partial \ln V}{\partial\psi} = \frac{\partial\theta}{\partial\phi} \tag{6}$$

In addition, connections between the physical-plane coordinates and lnV and θ are provided by the following integral equations valid only along lines of constant ϕ or ψ [2].

$$x_2 - x_1 = \int_{\psi^1}^{\psi^2} -\frac{\sin\theta}{V}\,d\psi \qquad (7)$$

$$x_2 - x_1 = \int_{\phi^1}^{\phi^2} \frac{\cos\theta}{V}\,d\phi \qquad (8)$$

$$y_2 - y_1 = \int_{\psi^1}^{\psi^2} \frac{\cos\theta}{V}\,d\psi \qquad (9)$$

$$y_2 - y_1 = \int_{\phi^1}^{\phi^2} \frac{\sin\theta}{V}\,d\phi \qquad (10)$$

The system of equations (3), (4), (5), (6), (7), (8), (9), and (10) must be solved in accordance with the proper boundary conditions. Equation (1) represents one of the boundary conditions.

At the upstream end the boundary condition must be formulated in equation form. With the pressure equal to zero along A-A' in Figures 2 and 3, the velocity must be given by

$$V = -\frac{\partial\psi}{\partial r} = \sqrt{2g(H-Y)}$$

where r is defined in Figure 4.

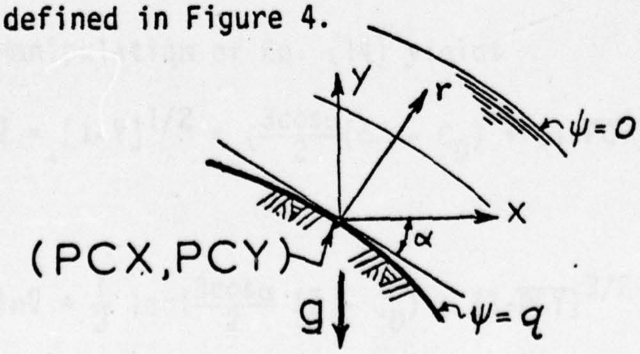


Fig. 4. Definition Sketch for Upstream Boundary Condition

Integrating Eq. 11 with $dr = dy/\cos\alpha$

$$\int -\frac{\partial\psi}{\partial r}\, dr = \frac{1}{\cos\alpha} \int\sqrt{2g(H-Y)}\, dy + C$$

$$-\psi = -\frac{2}{\bar{o}g\cos\alpha}\,[2g(H-Y)]^{3/2} + C \qquad (12)$$

where C is a constant of integration which is evaluated by setting $\psi = q$ at $Y = PCY$.

At this point it is convenient to introduce dimensionless parameters. Equation (12) is made dimensionless by dividing through by $(\sqrt{2g}\,H^{3/2})$. Thus, Eq. 12 becomes

$$-\frac{\psi}{\sqrt{2g}\,H^{3/2}} = \frac{2}{3\cos\alpha}\,[1 - Y/H]^{3/2} + C_1 \qquad (13)$$

Other variables expressed in dimensionless form are

$$\frac{X}{H},\ \frac{Y}{H},\ \frac{R}{H},\ \frac{V}{\sqrt{2gH}},\ \frac{q}{\sqrt{2g}\,H^{3/2}} = C_D,\ \frac{\phi}{\sqrt{2g}\,H^{3/2}},\ \frac{P}{\gamma H}$$

Setting $\psi/\sqrt{2g}\,H^{3/2} = C_D$ at $Y/H = PCY/H$, Eq. 13 becomes

$$-\bar{\psi} = -\frac{2}{3\cos\alpha}([1-\bar{Y}]^{3/2} + [1-\overline{PCY}]^{3/2}) - C_D \qquad (14)$$

where the bar over the letter indicates the dimensionless form of the variable. Manipulation of Eq. (14) yields

$$\bar{V} = [1-\bar{Y}]^{1/2} = \{\frac{3\cos\alpha}{2}(\bar{\psi} - C_D) + (1-\overline{PCY})^{3/2}\}^{1/3} \qquad (15)$$

And, hence

$$\ln\bar{V} = \frac{1}{3}\ln\{\frac{3\cos\alpha}{2}(\bar{\psi} - C_D) + (1-\overline{PCY})^{3/2}\} \qquad (16)$$

Equation (16) was used to express $\ln\bar{V}$ in terms of $\bar{\psi}$ along line A-A' in Figure 3. However, it was used in the form

$$\ln\bar{V} = \ln \bar{V}_T + \frac{1}{3} \ln [\{(\bar{V}_B/\bar{V}_T)^3 - 1\} \frac{\bar{\psi}}{C_D} + 1] \qquad (17)$$

where $\bar{V}_B$ and $\bar{V}_T$ are the velocity at A and A' respectively.

Numerical Solution

Solution of the equations presented in the previous section was accomplished in the complex-potential plane because the geometry is regular there. An iterative procedure was used.

Along the solid boundary $\bar{Y} = \bar{Y}(\bar{x})$ is known at the outset but $\bar{Y} = \bar{Y}(\phi)$ is required. A short FORTRAN program was written to generate initial values for $\bar{X}(\phi,\psi)$, $\bar{Y}(\phi,\psi)$, $\ln\bar{V}(\phi,\psi)$, and $\theta(\phi,\psi)$. This program requires that a value be assumed for the distance A-A' in Figure 3. Using the known values of C_D, H, and geometry, a free surface was generated by assuming that the depth of flow was the same at the downstream limit and that both $\bar{X}(\phi,\psi)$ and $\bar{Y}(\phi,\psi)$ varied linearly in the direction of flow. This is admittedly a crude approximation, but it appeared to be satisfactory as a first approximation. A copy of this program is given in the appendix. The dimensionless distance A-A' is called D.

Using the initial approximations the iterative procedure was as follows:

1. Values of $\theta(\phi,\psi)$ were calculated on the free surface by numerical differentiation ($dy/dx = \tan^{-1}\Theta$).
2. Values of $\theta(\phi,\psi)$ were calculated on the solid boundary according to the known geometry.
3. The θ field was settled using a numerical approximation to Eq. 3.
4. Values of $\ln\bar{V}$ were calculated on the free surface using Eq. (1).
5. Values of $\ln\bar{V}$ on the solid boundary were calculated by numerical integration using a numerical approximation to Eq. (6).

6. The $\ln\bar{V}$ field was settled using a numerical approximation to Eq. (4).
7. New values of $\bar{X}$ were computed on the solid boundary by numerically integrating Eq. (8) starting at the PC.
8. New values of $\bar{Y}$ on the solid boundary were computed in accordance with the $\bar{X}$ values obtained in step (7).
9. New values of $\bar{X}$ and $\bar{Y}$ on the free surface were obtained by numerically integrating Eqs. (7) and (9) respectively, starting at the solid boundary.
10. The new values of $\bar{X}$ and $\bar{Y}$ were compared with those used in steps (1) and (2). If a significant change was noted a new iteration was begun at step (1). Successive iterations were performed until the $\bar{X}$ and $\bar{Y}$ values remained constant.

The iterative procedure outlined in steps (1) through (10) was programmed for processing by computer. The program is included in the appendix.

A second program was written to calculate pressures once a solution was obtained. This program is also contained in the appendix.

Example Solution

A numerical solution was performed for the spillway toe curve shown on plate 34 of EM 1110-2-1603, Hydraulic Design of Spillways, March 1965. The following table lists the variables used in the solution, their equivalent form in the Fortran language, and the magnitude assigned in the example solution.

Fortran Variable	Corresponding Variable in Figs. 1 and 2	Magnitude Assigned
	g	32.2
R	R/H	2.0
S	S	0.0
X	X/H	
Y	Y/H	
Q	$q/\sqrt{2g}\,H^{3/2}$	0.445
PTX	PTX/H	1.893
PTY	PTY/H	-0.676
PCX	PCX/H	0.724
PCY	PCY/H	-0.299
XR	XR/H	1.893
YR	YR/H	1.324
BETA	β	0.0 radians
ALPH	α	-0.6242 radians
TH	θ	
V	$\ln(V/\sqrt{2gH'})$	

In the computer programs TH, V, X, and Y are stored in arrays and are dimensioned as TH(I.J), V(I,J), X(I,J), and Y(I,J) respectively. The subscript I refers to the row (I=1 refers to the uppermost streamline) while J=1 refers to the upstream-most equipotential line -- A-A' in Fig. 3). I and J have maximum values designated as N and M respectively for the variables TH(I,J) and V(I,J). However, in X(I,J) and Y(I,J) I is either 1 or 2 while J has a maximum value equal to M. In the example problem

N = 9 and M = 60. Thus X(I,J) and Y(I,J) are 2 x 60 arrays while V(I,J) amd TH(I,J) are both 9 x 60 arrays in the example. Values of TH(I,J) must be in radians and TH(I,J) = 0 corresponds to the positive X axis.

Results

The pressure-head distribution and free-surface profile obtained in the example solution are shown in Fig. 5. Considerable difficulty was encountered in the vicinity of the upstream end. As can be seen by the computed free-surface profile, some numerical error still exists at the upstream end. However, the pressure distribution obtained is smooth and was not influenced by the irregularity at the free surface.

In the use of the programmed solution it is necessary to decide what error will be tolerated in the free surface coordinates. In the FORTRAN program this is labeled DEL. In the example solution DEL was set equal to 0.01. Thus, the probable error in X/H or Y/H is of the order 0.01.

In the relaxation procedure used to settle the TH and V fields, a tolerable error must also be specified. This error has been called ERR. Experience with the numerical procedure indicates that ERR should never be greater than DEL/10 if stability in the solution is to be obtained.

A total of 33 iterations were required to obtain the solution. On the IBM 7040 this required a total time of 6 minutes.

Numerical accuracy is undoubtedly closely related to the curvature of the streamlines. Experience in this solution indicated that the number of streamlines used (the value of N) should be such that the increment in the streamfunction between streamlines (C_D/N-1) should be less than 0.05. On geometry where R is *smaller* than 2.0 (as used in the example) it may be necessary to use more streamlines.

Use of the Program Decks

Schematic representations of the program decks are contained in the appendix. To use the program for obtaining initial approximations for TH, V, X, and Y one must choose a value for the dimensionless distance A-A'. This is called D. The program will punch out cards containing the required initial approximation to TH, V, Y, and X.

In the main program a value for LIMIT is read in to control the number of iterations the computer will perform (it may be desirable to punch out intermediate values of V, TH, X, and Y because of time limitations). A statement DONE will be typed out when a solution has been obtained.

The pressure computation deck uses the output of the main deck and prints out pressure heads and free-surface coordinates.

APPENDIX

References:

1. Rouse, H. (Editor), *Advanced Mechanics of Fluids*, John Wiley and Sons, New York, 1959.

2. Cassidy, J. J., "Irrotational Flow Over Spillways of Finite Height," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 4591, EM6, Dec. 1965.

3. Thom, A., and C. J. Apelt, *Field Computations in Engineering and Physics*, Van Nostrand, 1960.

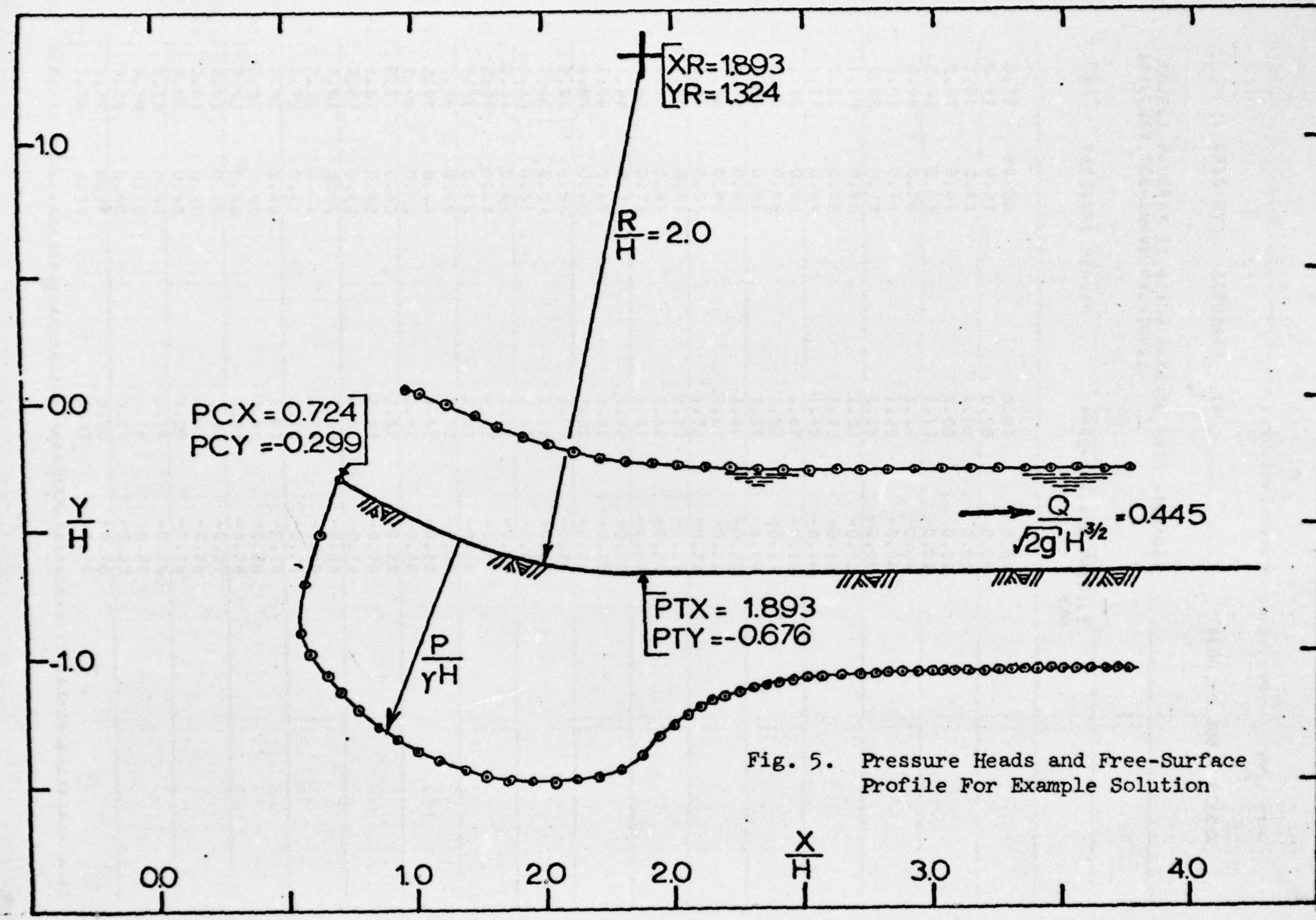


Fig. 5. Pressure Heads and Free-Surface Profile For Example Solution

J. CASSIDY 21207672 14 IBLDR -- JOB CCCOOC

OBJECT PROGRAM IS BEING ENTERED INTO STORAGE.
SPILLWAY TOE-CURVE PRESSURES

X	PRESSURE HEAD/H	FREE-SURFACE PROFILE X/H	Y/H
0.72	-0.00	0.97	0.04
0.77	0.23	0.98	0.04
0.81	0.43	1.02	0.03
0.87	0.61	1.08	0.01
0.92	0.65	1.13	-0.01
0.98	0.70	1.18	-0.04
1.03	0.74	1.23	-0.06
1.09	0.77	1.28	-0.08
1.15	0.80	1.32	-0.10
1.21	0.82	1.37	-0.12
1.27	0.84	1.42	-0.14
1.33	0.85	1.47	-0.16
1.40	0.86	1.52	-0.17
1.46	0.87	1.57	-0.19
1.52	0.87	1.62	-0.20
1.58	0.86	1.67	-0.21
1.64	0.85	1.72	-0.22
1.70	0.83	1.77	-0.23
1.77	0.81	1.82	-0.24
1.82	0.78	1.87	-0.25
1.88	0.72	1.92	-0.25
1.94	0.65	1.97	-0.26
1.99	0.60	2.02	-0.26
2.05	0.56	2.07	-0.26
2.10	0.53	2.12	-0.27
2.15	0.51	2.17	-0.27
2.21	0.49	2.22	-0.27
2.26	0.47	2.27	-0.27
2.31	0.46	2.32	-0.28
2.36	0.45	2.37	-0.28
2.41	0.44	2.42	-0.28
2.46	0.43	2.47	-0.28
2.51	0.43	2.52	-0.28
2.57	0.42	2.57	-0.28
2.62	0.42	2.62	-0.28
2.67	0.41	2.68	-0.28
2.72	0.41	2.72	-0.28
2.77	0.41	2.78	-0.28
2.82	0.40	2.83	-0.28
2.88	0.40	2.88	-0.28
2.93	0.40	2.93	-0.28
2.98	0.40	2.98	-0.28
3.03	0.40	3.03	-0.28
3.08	0.40	3.09	-0.28
3.13	0.40	3.14	-0.28
3.19	0.40	3.19	-0.28
3.24	0.40	3.24	-0.28
3.29	0.39	3.30	-0.28
3.34	0.39	3.35	-0.28

0	1	2	3	4
3.39	0.39		3.40	-0.28
3.44	0.39		3.45	-0.28
3.50	0.39		3.51	-0.28
3.55	0.39		3.56	-0.28
3.61	0.39		3.61	-0.28
3.65	0.39		3.66	-0.28
3.71	0.39		3.72	-0.28
3.76	0.39		3.77	-0.28

```
C PROGRAM TO GENERATE INITIAL X,Y,V,TH  FOR TOE-CURVE PROGRAM
      DIMENSION TH)9,60*,V)9,60*,X)2,60*,Y)2,60*
      READ(5,100)R,PCX,PCY,PTX,PTY,Q,LIMIT
  100 FORMAT(6F10.5,I5)
      READ(5,101)M,N,BETA,S,ERR,DEL,XR,YR,ALPH
  101 FORMAT(2I5,4F5.3,3F10.5)
      READ(5,107)D
  107 FORMAT(F10.5)
      MN=M-1
      NM=N-1
      A=NM
      DX=D/A
      X(2,1)=PCX
      Y(2,1)=PCY
      DO 1 J=2,M
    1 X)2,J*=X)2,J-1*.DX
      X)1,1*=X)2,1*-D*SIN)ALPH*
      X)1,M*=X)2,M*
      A=MN
      DX=)X)1,M*-X)1,1**/A
      DO 2 J=2,MN
    2 X)1,J*=X)1,J-1*.DX
      Y)1,1*=PCY.D*COS)ALPH*
      DO 5 J=2,M
      IF)PTX-X)2,J*)4,3,3
    3 Y)2,J*=-SQRT)R*R-)X)2,J*-XR***2*.YR
      GO TO 5
    4 Y)2,J*=PTY.S*)X)2,J*-PTX*
    5 CONTINUE
      Y)1,M*=Y)2,M*.D
      DY=)Y)1,1*-Y)1,M**/A
      DO 6 J=2,MN
    6 Y)1,J*=Y)1,J-1*-DY
      DO 7 J=1,M
    7 V)1,J*=)ALOG)H-Y)1,J***/2.
      V)N,1*=)ALOG)H-Y)2,1***/2.
      A=NM
      DV=)V)N,1*-V)1,1**/A
      DO 8 I=2,N
      V)I,M*=V)1,M*
    8 V(I,1)=V(I-1,1)+DV
      A=MN
      DV=(V(N,M)-V(N,1))/A
      DO 20 J=2,MN
   20 V(N,J)=V(N,J-1)+DV
      DO 10 J=2,MN
      DV=)V)N,J*-V)1,J**/A
      DO 10 I=2,NM
   10 V)I,J*=V)I-1,J*.DV
      DO 15 J=1,M
      IF)X)2,J*-PTX*12,12,11
   12 TH)N,J*=ATAN)-)X)2,J*-XR*/)Y)2,J*-YR**
      GO TO 13
   11 TH)N,J*=BETA
   13 DO 14 I=1,NM
   14 TH)I,J*=TH)N,J*
   15 CONTINUE
      WRITE)6,103*)X)1,J*,Y)1,J*,X)2,J*,Y)2,J*,J=1,M*
      WRITE)6,104*))V)I,J*,I=1,N*,J=1,M*
```

```
      WRITE)6,104*))TH)I,J*,I=1,N*,J=1,M*
  104 FORMAT)9F9.5*
      WRITE)7,102*))TH)I,J*,I=1,N*,J=1,M*
      WRITE)7,102*))V)I,J*,I=1,N*,J=1,M*
      WRITE)7,103*)X)1,J*,Y)1,J*,X)2,J*,Y)2,J*,J=1,M*
  102 FORMAT)9F8.5*
  103 FORMAT)2F10.5,F15.5,F10.5*
      END
```

```
C SPILLWAY TOE-CURVE ANALYSIS
      DIMENSION TH(9,60),V(9,60),X(2,60),Y(2,60)
C READ IN DATA
      READ(5,100)R,PCX,PCY,PTX,PTY,Q,LIMIT
  100 FORMAT(6F10.5,I5)
      READ(5,101)M,N,BETA,S,ERR,DEL,XR,YR,ALPH
  101 FORMAT(2I5,4F5.3,3F10.5)
      READ(5,102)((V(I,J),I=1,N),J=1,M)
      READ(5,102)((TH(I,J),I=1,N),J=1,M)
  102 FORMAT(9F8.5)
      READ(5,103)(X(1,J),Y(1,J),X(2,J),Y(2,J),J=1,M)
  103 FORMAT(2F10.5,F15.5,F10.5)
C COMPUTE PARAMETERS
      MK=M-2
      ML=M-1
      NM=N-1
      V(N,1)=ALOG(SQRT(1.0-Y(2,1)))
C INITIALIZE COUNT OF ITERATIONS
      NTIME=0
C COMPUTE NEW THETA ON SOLID BOUNDARY
    6 DO 34 J=1,M
      IF(X(2,J)-PTX)32,33,33
   32 TH(N,J)=ATAN(-(X(2,J)-XR)/(Y(2,J)-YR))
      GO TO 34
   33 TH(N,J)=BETA
   34 CONTINUE
C COMPUTE NEW UPPER FREE-SURFACE THETAS
      I=1
      A=NM
      DO  9 J=2,ML
      DYDX=Y(I,J-1)*(X(I,J)-X(I,J+1))/((X(I,J-1)-X(I,J))*(X(I,J-1)-X(I,J
     1+1)))+Y(I,J)*(2.*X(I,J)-X(I,J+1)-X(I,J-1))/((X(I,J)-X(I,J+1))*(X(I
     1,J)-X(I,J-1)))+Y(I,J+1)*(X(I,J)-X(I,J-1))/((X(I,J+1)-X(I,J-1))*(X
     1(I,J+1)-X(I,J)))
    9 TH(I,J)=ATAN (DYDX)
      TH(1,M)=TH(1,M-1)
      DYDX=Y(1,1)*(2.*X(1,1)-X(1,2)-X(1,3))/((X(1,1)-X(1,2))*(X(1,1)-
     1X(1,3)))+Y(1,2)*(X(1,1)-X(1,3))/((X(1,2)-X(1,1))*(X(1,2)-
     1X(1,3)))+Y(1,3)*(X(1,1)-X(1,2))/((X(1,3)-X(1,1))*(X(1,3)-X(1,2)))
      TH(1,1)=ATAN(DYDX)
      B=(TH(N,M)-TH(1,M))/A
      DO 10 I=1,NM
   10 TH(I,1)=TH(N,1)
      DO 11 I=2,NM
   11 TH(I,M)=TH(I-1,M)+B
C SETTLE TH FIELD
   12 BOX=0.
      DO 14 I=2,NM
      DO 14 J=2,ML
      B=TH(I,J)
      TH(I,J)=(TH(I-1,J)+TH(I,J-1)+TH(I,J+1)+TH(I+1,J))/4.
      IF(ABS(B-TH(I,J))-ERR)14,14,13
   13 BOX=BOX+1.
   14 CONTINUE
      IF(BOX)15,15,12
C COMPUTE INCREMENT IN STREAM FUNCTION
   15 A=NM
      DQ=Q/A
C COMPUTE NEW V ON FREE SURFACE
```

```
      SI=0.
      VB=EXP(V(N,1))
      VT=EXP(V(1,1))
      VBVT=(VB/VT)**3-1.
      DO 29 J=1,M
   29 V(1,J)=(ALOG(1.0-Y(1,J)))/2.
C COMPUTE NEW V AT ENDS
      DO 30 I=2,N
      SI=SI+DQ
      V(I,1)=V(1,1)+ALOG((VBVT*SI/Q)+1.)/3.
   30 V(I,M)=V(1,M)
C COMPUTE NEW V ON SOLID BOUNDARY
      DO 31 J=3,MK
      J1=J-1
      J2=J-2
      J3=J+1
      J4=J+2
      DO 31 I=2,N
      I1=I-1
      I2=I-2
      A=14.*(TH(I1,J3)+TH(I,J3)-TH(I1,J1)-TH(I,J1))
      B= TH(I1,J2)+TH(I,J2)-TH(I1,J4)-TH(I,J4)
   31 V(I,J)=V(I1,J)-(A+B)/48.
      V(N,2)=(V(N,3)+V(N,1))/2.
      V(N,ML)=(V(N,M)+V(N,MK))/2.
C SETTLE V FIELD
    3 BOX=0.
      DO 5 I=2,NM
      DO 5 J=2,ML
      B=V(I,J)
      V(I,J)=(V(I-1,J)+V(I+1,J)+V(I,J+1)+V(I,J-1))/4.
      IF(ABS(B-V(I,J))-ERR)5,5,4
    4 BOX=BOX+1.
    5 CONTINUE
      IF(BOX)7,7,3
C  COMPUTE X ON SOLID BOUNDARY
    7 A=COS(TH(N,1))/EXP(V(N,1))
      XBOX=0.
      DO 16 J=3,M,2
      C=A
      B=COS(TH(N,J-1))/EXP(V(N,J-1))
      A=COS(TH(N,J))/EXP(V(N,J))
      D=X(2,J)
      BX=(5.*A+8.*B-C)*DQ/12.
      X(2,J-1)=(X(2,J-1)+X(2,J-2)+BX)/2.
      BX=(A+4.*B+C)*DQ/3.
   16 X(2,J)=(X(2,J-2)+BX+D)/2.
C COMPUTE NEW Y ON SOLID BOUNDARY
      DO 22 J=2,M
      B=Y(2,J)
      IF(PTX-X(2,J))19,18,18
   18 Y(2,J)=-SQRT(R*R-(X(2,J)-XR)**2)+YR
      GO TO 20
   19 Y(2,J)=PTY+S*(X(2,J)-PTX)
   20 IF(ABS(Y(2,J)-B)-DEL)22,22,21
   21 XBOX=XBOX+1.
   22 CONTINUE
C  COMPUTE Y AND X ALONG FREE SURFACE
      DO 26 J=1,M
      P=EXP(V(N,J))
```

```
      D=SIN(TH(N,J))/P
      A=COS(TH(N,J))/P
      DX=0.
      DY=0.
      AB=X(1,J)
      BA=Y(1,J)
      DO 23 I=3,N,2
      K=N-I+3
      C=A
      P=EXP(V(K-1,J))
      PP=EXP(V(K-2,J))
      B=COS(TH(K-1,J))/P
      A=COS(TH(K-2,J))/PP
      F=D
      E=SIN(TH(K-1,J))/P
      D=SIN(TH(K-2,J))/PP
      DX=DX+(F+4.*E+D)*DQ/3.
   23 DY=DY+(C+4.*B+A)*DQ/3.
      Y(1,J)=(Y(1,J)+Y(2,J)+DY)/2.
      X(1,J)=(X(1,J)+X(2,J)-DX)/2.
C COMPARE NEW AND OLD COORDINATES
      IF(ABS(X(1,J)-AB)-DEL)40,40,41
   40 IF(ABS(Y(1,J)-BA)-DEL)26,26,41
   41 XBOX=XBOX+1.
   26 CONTINUE
      IF(XBOX)27,36,27
   27 NTIME=NTIME+1
      IF(NTIME-LIMIT)6,37,37
   36 WRITE(6,105)
  105 FORMAT(5H DONE)
   37 WRITE(6,107)NTIME
  107 FORMAT(I5)
      WRITE(7,102)((V(I,J),I=1,N),J=1,M)
      WRITE(7,102)((TH(I,J),I=1,N),J=1,M)
      WRITE(7,103)(X(1,J),Y(1,J),X(2,J),Y(2,J),J=1,M)
      WRITE(6,103)(X(1,J),Y(1,J),X(2,J),Y(2,J),J=1,M)
      WRITE(6,104)((V(I,J),I=1,N),J=1,M)
      WRITE(6,104)((TH(I,J),I=1,N),J=1,M)
  104 FORMAT(9F10.5)
      END
```

```
C SPILLWAY TOE-CURVE PRESSURES
      DIMENSION TH(9,60),V(9,60),X(2,60),Y(2,60)
C READ IN DATA
      WRITE(6,106)
  106 FORMAT(29H SPILLWAY TOE-CURVE PRESSURES//)
      READ(5,100)R,PCX,PCY,PTX,PTY,Q,LIMIT
  100 FORMAT(6F10.5,I5)
      READ(5,101)M,N,BETA,S,ERR,DEL,XR,YR,ALPH
  101 FORMAT(2I5,4F5.3,3F10.5)
      READ(5,102)((V(I,J),I=1,N),J=1,M)
      READ(5,102)((TH(I,J),I=1,N),J=1,M)
  102 FORMAT(9F8.5)
      READ(5,103)(X(1,J),Y(1,J),X(2,J),Y(2,J),J=1,M)
  103 FORMAT(2F10.5,F15.5,F10.5)
C WRITE OUT HEADINGS
      WRITE(6,104)
  104 FORMAT(52H   X       PRESSURE HEAD/H          FREE-SURFACE PROFILE)
      WRITE(6,107)
  .07 FORMAT(52H                                          X/H         Y/H//)
C COMPUTE AND PRINT OUT PRESSURES
      DO 3 J=1,M
      P=1.0-(EXP(V(N,J))**2)-Y(2,J)
    3 WRITE(6,105)X(2,J),P,X(1,J),Y(1,J)
  105 FORMAT(F6.2,F9.2,F20.2,F12.2)
      END
```

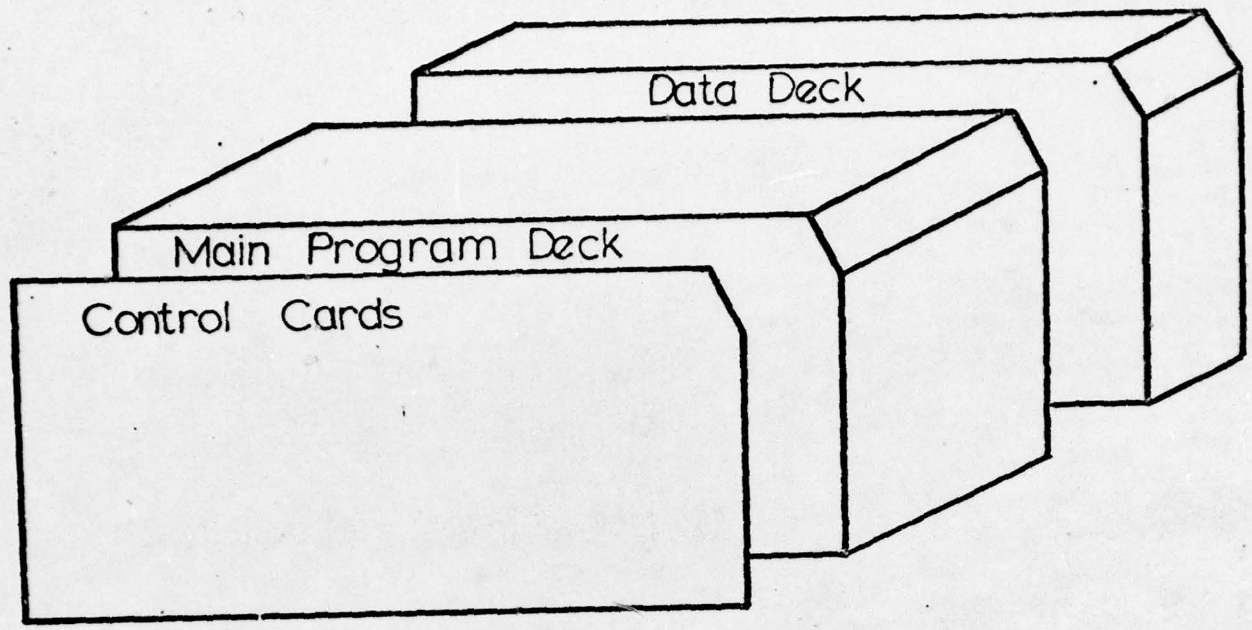


Fig. 6 Computer Program Arrangement

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