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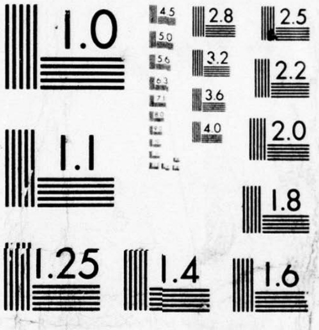
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NSWC/WOL TR 77-183

AN INVERSE REGRESSION METHOD FOR DETERMINING AN ENSEMBLE OF STATE ERROR VECTORS FROM A COVARIANCE MATRIX

BY RONALD S. BRUNSVOLD

ADVANCED WEAPONS DEPARTMENT

5 DECEMBER 1977

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A method is described to determine an ensemble of initial condition state vector errors for use in a Monte-Carlo type error model. The state vector errors are found from a given (N x N) error covarance matrix and are properly correlated with one another. The method used is an inverse regression scheme which randomly introduces a gaussion distribution of errors with determined variances about linear regression curves calculated from the given covariance Matrix. A FORTRAN coded

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implementation of the method is included.

SUMMARY

This report outlines a technique for deriving a set of initial error states from an error covariance matrix. The need for an ensemble of state errors arises in a Monte-Carlo error model for the re-entry portion of flight of a ballistic missile. Typically the boost and vacuum coast part of a trajectory is accurately modeled with a covariance propagation scheme. During reentry, however, errors become nonlinear in time and are highly cross correlated with one another and interact strongly with the dynamics of the vehicle flight. To accurately model the errors during reentry a Monte-Carlo approach is often required. The interface between a covariance propogation scheme and a Monte-Carlo model requires that an ensemble of initial condition errors be extracted from an arrival covariance matrix. The errors must be properly correlated and have the proper distribution of magnitudes. The method described herein is one technique for defining the interface between the two types of error models.

Since the original draft of this report, the author has found that a method with equivalent results has been used to determine wind profiles from a matrix of wind correlation coefficients. (e.g., Hankerson, S. H., "Wind Profiles," NWL-TN-G-4/72, Feb 1972). It is likely that other uses for the general method can be found, especially in the field of system error modeling.

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R. A. NIEMANN
By direction

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I. INTRODUCTION

Often it is desired to determine the performance of a physical system in an average or statistical sense. That is, for many repetitions of a phenomena, what are the operating limits within which a system may be expected to perform and how are deviations from nominal behavior distributed about the average of the many repetitions. A study of the statistical behavior of such physical phenomena is here referred to as system error modeling.

There are two common methods of constructing system error models. The first method is the deterministic or Monte-Carlo method. This method attempts to mathematically model the physical laws which affect a system and allow the parameters within the physical laws to vary in a particular manner within their limits of uncertainty. The parametric variations allowed have a certain randomness associated with them since the exact values of the parameters are never precisely known. Correlations between the parameter errors are permitted if they can be determined a priori. The system is allowed to operate within the framework of the Math model for many realizations of the event being studied. After exercising the model in this Monte-Carlo mode, the system states for the many realizations of an event can be analyzed and statistical variations of state amplitude and distribution can be determined. This statistical description of the system is the result being sought.

The second method of system error modeling is the covariance propagation scheme or analytic model. This model represents the uncertainties or errors in the state of a system in a covariance matrix. The elements of this matrix describe the magnitude of state errors and their correlations with one another in a statistical sense. The analytic error model propagates this covariance matrix with time to describe how the state errors change. Since all of the desired statistical information is available in the covariance matrix it need only be propogated once for each event. This method has the advantage of reduced complexity; however, it is not as versatile as the Monte-Carlo method and determination of the proper way of propagating a covariance matrix is not always a simple matter.

In some instance it may be desirable or necessary to mix the two methods of error modeling for a given physical system. If, say, the initial part of an event is modeled with the analytic error model and the latter stages with a Monte-Carlo model, then how are the two models interfaced at the point of transition? What technique will be used to calculate the ensemble of initial deterministic state vectors from a terminal covariance matrix in order to begin the Monte-Carlo process? The following analysis describes one method of defining the interface.

II. LINEAR REGRESSION

A. Two Variable Regression

Given an (n x n) symmetric error covariance matrix:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nn} \end{bmatrix}$$

The diagonal terms of this matrix represent the variance of error in a state property. If a state property is quantified by a known part Y_k plus an uncertain zero mean error in the state property say Y_u, then the property Y may be expressed as:

$$Y = Y_k + Y_u \qquad (1a)$$

The S_{11} term of the error covariance matrix then represents the statistical variance of Y_u for many realizations of the state.

$$S_{11} = \frac{\sum Y_{u_1}^{\;2}}{N} \qquad (1)$$

N is the large number of realizations of Y_u. Similar definitions exist for the other (N - 1) states implicit in the covariance matrix. The off-diagonal terms of the matrix are the covariances of the (N) states defined by:

$$S_{12} = \frac{\sum Y_{u_1} \; Y_{u_2}}{N} \qquad (2)$$

The magnitude of the covariances indicate the degree to which state errors are dependent on one another. To put this dependence in a nondimensional form a correlation coefficient can be written:

$$r_{12} = \frac{S_{12}}{(S_{11}\ S_{22})^{\frac{1}{2}}} \qquad (3)$$

In this nondimensional form, a correlation of ± 1 indicates perfect correlation between errors Y_{u_1} and Y_{u_2}, and a correlation of 0 indicates no correlation exists. This definition of correlation coefficient is applicable only for linear correlation analysis. A non linear correlation may exist for $r_{12} = 0$. The present method assumes only linear correlations exist among the state errors.

If the state errors of two state variables are known for many realizations of the two states, then a least square regression line (see Figure (1)) can be described by

$$\left(Y_{u_2}\right)_{\text{est.}} = \left(\frac{\sum Y_{u_1}\ Y_{u_2}}{\sum Y_{u_1}{}^2}\right) \left(Y_{u_1}\right) \text{ actual} \qquad (4)$$

The mean values of the errors are clearly assumed to be zero (i.e., no biases exist).

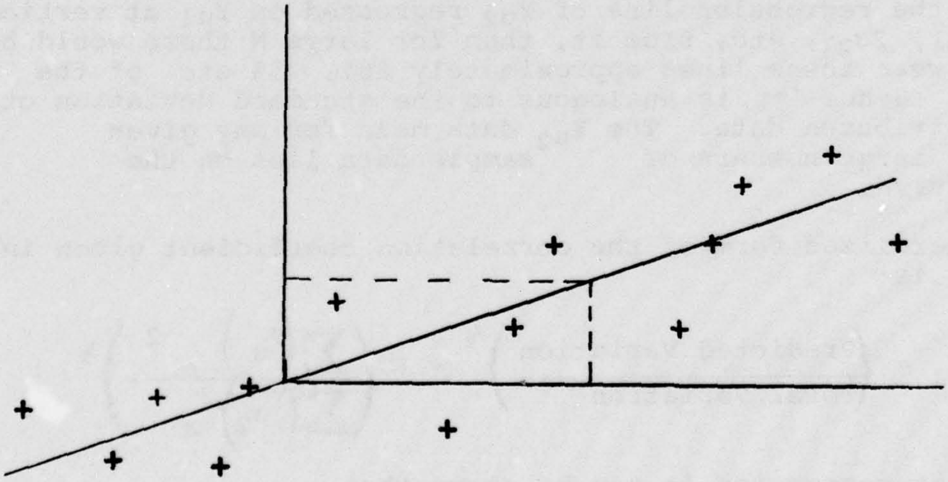

FIGURE 1. LEAST SQUARES REGRESSION LINE.

For a given actual value of Y_{u1}, an estimate of Y_{u2}, can be made from the equation of the least square regression line according to equation (4). The "goodness" of the estimate depends on how well Y_{u2} is correlated with Y_{u1}. If the correlation were perfect (i.e., $r_{12} = 1$) then there would be no scatter in the data points in Figure (1). All the data would be directly on the linear regression line. Conversely, a poor correlation would be seen as much scatter of the data about the regression line in Figure (1), and the estimate of Y_{u2} from the regression equation would not be expected to be very good.

Equation (4) can be rewritten in terms of the covariance from equations (1) and (2) as:

$$\left(Y_{u_2}\right)_{\text{est.}} = \frac{S_{12}}{S_{11}} \left(Y_{u_1}\right)_{\text{actual}} \qquad (5)$$

A quantitative measure of the data scatter about the regression curve is defined as a "Standard Error of Estimate":

$$\sigma_{21} = \left(\frac{\sum\left(\left(Y_{u_2}\right)_a - \left(Y_{u_2}\right)_e\right)^2}{N}\right)^{1/2} \qquad (6)$$

The difference of the two terms in equation (6) is the difference between each data point and the regression curve approximating the data. The standard error of estimate is similar to a deviation of the data about the regression curve. If lines are constructed parallel to the regression line of Y_{u2} regressed on Y_{u1} at vertical distances σ_{21}, $2\sigma_{21}$, etc., from it, then for large N there would be included between these lines approximately 68%, 95% etc. of the data points. Hence σ_{21}, is analogous to the standard deviation of normally distributed data. The Y_{u2} data mean for any given Y_{u1} and very large numbers of sample data lies on the regression curve.

A generalized form of the correlation coefficient given in equation (3) is:

$$r_{21} = \left(\frac{\text{Predicted Variation}}{\text{Total Variation}}\right)^{1/2} = \pm\left(\frac{\sum\left(Y_{u_2}\right)_e^2}{\sum\left(Y_{u_2}\right)_a^2}\right)^{1/2} \qquad (7)$$

and for linear regression it can be shown that

$$\sum\left[\left(Y_{u_2}\right)_a - \left(Y_{u_2}\right)_e\right]^2 = \sum\left(Y_{u_2}\right)_a^2 - \sum\left(Y_{u_2}\right)_e^2 \qquad (8)$$

hence with (8), (7) and (6)

$$\sigma_2 = S_{22}^{\frac{1}{2}} (1 - r_{21}^{\;2})^{\frac{1}{2}} \qquad (9)$$

B. Multiple Linear Regression

When multiple correlation exist among a set of random variables as is the case when given an N x N covariance matrix, multi-dimensional regression equations can be constructed. Generalizing equation (5) to the multi-dimension equation:

$$\left(\frac{Y_{u_N}}{S_{NN}^{\frac{1}{2}}}\right)_e = a_1\left(\frac{Y_{u_1}}{S_{11}^{\frac{1}{2}}}\right)_a + a_2\left(\frac{Y_{u_2}}{S_{22}^{\frac{1}{2}}}\right)_a + \ldots + a_{N-1}\left(\frac{Y_{u_{n-1}}}{S_{(n-1)(n-1)}^{\frac{1}{2}}}\right)_a \qquad (10)$$

Where the a_n coefficients are found from simultaneous solution of the algebraic equations:

$$\begin{array}{llllll}
a_1 r_{11} & + a_2 r_{12} & + a_3 r_{13} & + \ldots + a_{n-1} r_{1(n-1)} & = r_{1n} \\
a_1 r_{21} & + a_2 r_{22} & + a_3 r_{23} & + \ldots + a_{n-1} r_{2(n-1)} & = r_{2n} \\
\vdots & \vdots & \vdots & \quad\vdots & \vdots \\
a_1 r_{(n-1)1} & + a_2 r_{(n-1)2} & + a_3 r_{(n-1)3} & + \ldots + a_{(n-1)} r_{(n-1)(n-1)} & = r_{(n-1)n}
\end{array}$$

and the standard error of estimate about the multiple regression line is an expanded form of equation (9):

$$\sigma_n = S_{nn}^{\;1/2} \left(1 - r_{n,1}\, a_1 - r_{n,2}\, a_2 - \ldots r_{n,(n-1)}\, a_{n-1}\right)^{1/2} \qquad (12)$$

III. INVERSE REGRESSION

A. Method Description

If it is desired to find an ensemble of state vectors which include random errors and if it is also necessary that the random errors included in the state vectors have the proper statistical cross correlations, then it may be possible to construct such an ensemble, given the covariance matrix of errors. The process would be an inverse regression scheme as follows.

Making the assumption that all errors are normally distributed about linear regression lines it is possible to construct a set of regression lines and then statistically "scatter" the errors about the line using a zero mean normally distributed random number generator with a variance equal to σ_n^2 (equation 12). The errors so generated would be added to the mean value of the state vector and repeated many times to build an ensemble of state vectors with properly correlated errors.

A consistent scheme of selecting the order of regression should be used. One such scheme would be as follows:

1. Select the two elements of the state vector that are least correlated (i.e., smallest $|S_{nm}|$, $N \neq M$).

2. Continue selecting elements in a monotonically increasing order of correlation.

3. Choose the selected initial two elements (from step 1) and construct a regression curve from equation (5) similar to Figure (1).

3a. Find the so-called actual value $\left(Y_{u1}\right)_a$ from a zero mean normal random generator (NRNG) with variance S_{11}.

3b. Solve for the estimated value of Y_{u2}

4. Introduce scatter to $\left(Y_{u2}\right)_e$ by adding an error term found from (NRNG) using variance σ_2 as found from equation (9).

Solve for the so-called actual value of Y_{u_2}

$$\left(Y_{u_2}\right)_a = \left(Y_{u_2}\right)_e + (\text{NRNG})\ \sigma_2 \qquad (13)$$

5. Repeat the process to calculate actual values for all the state errors.

Equation (13) is a defined value of a single realization of the error in the Y_2 element of the state vector. When this process is repeated for each element of the state vector, it is necessary to use the expanded multiple regression equation (10) and variance, equation (12).

For example when finding the third error term, $\left(Y_{u3}\right)$, from (10):

$$\left(\frac{Y_{u_3}}{S_{33}^{1/2}}\right)_e = a_1\left(\frac{Y_{u_1}}{S_{11}^{1/2}}\right)_a + a_2\left(\frac{Y_{u_2}}{S_{22}^{1/2}}\right)_a \qquad (14)$$

where $(Y_{u_1})_a$ and $(Y_{u_2})_a$ are the same values as found in the preceding steps. The variance used to introduce "scatter" into the (Y_{u_3})est. of equation (14) is:

$$\sigma_3^{\;2} = \left[S_{33}^{\;1/2} \left(1 - r_{31}\,a_1 - r_{32}\,a_2\right)^{1/2}\right]^2 \qquad (15)$$

The process is complete when all N values of the state vector have been altered by an error term as in equation (1a).

For a given covariance matrix of errors, the A_N coefficients of equation (11) need only be calculated once. They change only when the covariance matrix changes. However, the value of the "A_N" coefficient are different for each value of N in equation 10. A set of coefficients correspond to each value of N which is the order of the given covariance matrix. As many realizations as desired of the state vector can be generated for a given covariance matrix. Each new realization is constructed from a new (NRNG) value for the first error term Y_{u_1}.

B. Proof of Proper Correlation

It remains to show that the order chosen for regressing the variables does not affect the statistical cross correlations of the errors in the final ensemble of state vectors. If we accept equations (13) as a valid method of introducing "scatter" or randomness to regression generated errors, then it is sufficient to show that cross correlations of error states are correct, independent of the order of their regression.

$$\left(Y_{u_n}\right)_a = \left(Y_{u_n}\right)_e + (\text{NRNG})_{\sigma n} \qquad (16)$$

The term $(\text{NRNG})_{\sigma n}$ is a single realization of a normal random number generator with standard deviation, σ_n.

Let us arbitrarily choose two elements of an error state vector Y_{u_1} and Y_{u_2} and regress Y_{u_2} on Y_{u_1}:

$$\left(Y_{u_2}\right)_e = \frac{S_{12}}{S_{11}}\left(Y_{u_1}\right)_a \qquad (17)$$

Where $\left(Y_{u_1}\right)_a$ is chosen from $(\text{NRNG})_{(S_{11})^{1/2}}$

Then $\left(Y_{u_2}\right)_a = \left(Y_{u_2}\right)_e + (\text{NRNG})_{\sigma_2}$ (18)

σ_2 being found from (9).

The cross correlation between $(Y_{u_1})_a$ and $(Y_{u_2})_a$ for a large number of realizations of equation (18) is:

$$(CC)_{12} = \sum \left(Y_{u_1}\right)_a \left(Y_{u_2}\right)_a \Big/ N$$

from (17) and (18)

$$(CC)_{12} = \sum \left(Y_{u_1}\right)_a \left[\frac{S_{12}}{S_{11}} \left(Y_{u_1}\right)_a + (NRNG)\,\sigma_2\right] \Big/ N$$

$$= S_{12} + \sum \left(Y_{u_1}\right)_a (NRNG)\,\sigma_2 \Big/ N \qquad (19)$$

The term on the right is equal to zero in the limit since each term of the product has zero mean and the terms are uncorrelated. Hence as expected:

$$(CC)_{12} = S_{12} = \sum \left(Y_{u_1}\right)_a \left(Y_{u_2}\right)_a \Big/ N \qquad (20)$$

From (10) the regression equation for a third variable is:

$$\left(Y_{u_3}\right)_e = \frac{S_{33}^{1/2} A_1}{S_{11}^{1/2}} \left(Y_{u_1}\right)_a + \frac{S_{33}^{1/2} A_2}{S_{22}^{1/2}} \left(Y_{u_2}\right)_a \qquad (21)$$

$$\left(Y_{u_3}\right)_a = \left(Y_{u_3}\right)_e + (NRNG)\,\sigma_3 \qquad (22)$$

Now check to see if $(Y_{u_3})_a$ generated by (22) is properly correlated with $(Y_{u_1})_a$:

$$\frac{\sum \left(Y_{u_1}\right)_a \left(Y_{u_3}\right)_a}{N} = \qquad (23)$$

$$\frac{\sum \left(Y_{u_1}\right)_a \left[\frac{S_{33}^{1/2} A_1}{S_{11}^{1/2}} \left(Y_{u_1}\right)_a + \frac{S_{33}^{1/2} A_2}{S_{22}^{1/2}} \left(Y_{u_2}\right)_a + (NRNG)\,\sigma_3\right]}{N}$$

This quickly reduces to:

$$= S_{11}^{1/2} S_{33}^{1/2} A_1 + \frac{S_{33}^{1/2} S_{12}}{S_{22}^{1/2}} A_2 + \frac{\sum \left(Y_{u_1}\right)_a (NRNG)\,\sigma_3}{N} \tag{24}$$

Since the last term is zero in the limit for a product of uncorrelated zero mean variables, (24) is:

$$\frac{\sum \left(Y_{u_1}\right)_a \left(Y_{u_3}\right)_a}{N} = S_{11}^{1/2} S_{33}^{1/2} A_1 + \frac{S_{33}^{1/2} S_{12}}{S_{22}^{1/2}} A_2 \tag{25}$$

The A_1 and A_2 coefficients are found from (10) to be:

$$A_1 = \frac{r_{13} - r_{12} r_{23}}{1 - r_{12}^{2}} \; ; \; A_2 = \frac{r_{23} - r_{12} r_{13}}{1 - r_{12}^{2}} \tag{26}$$

Substituting (3) and (26) into (25):

$$\frac{\sum \left(Y_{u_1}\right)_a \left(Y_{u_3}\right)_a}{N} = S_{13}$$

Hence, even though Y_{u_3} was the third variable chosen it is properly correlated with the first variable. In a similar fashion, the third variable can be shown to be properly correlated with the second variable. This process can be continued for all subsequent elements of the state vector regardless of their order of regression. Therefore, the order chosen to regress the state vector errors is arbitrary and has no adverse effect on the cross correlations of the ensemble of state vectors so generated.

IV. CODE DESCRIPTION

The inverse regression method described has been coded for use in a trajectory program. If a state error covariance matrix is available at some point in a vehicle's trajectory and it is desired to complete the trajectory with a Monte-Carlo error analysis, then it is necessary to determine an ensemble of initial state vectors. One state vector from the ensemble is used as an initial condition for each trajectory of the Monte-Carlo series.

A listing is enclosed as Appendix B. for the subroutines that read in the covariance matrix and then determine the coefficients and standard deviation required to solve for state error vectors. Subroutine COVSET(M) stores in common the "A" coefficients and "S" variances of equation (10). Also stored are the standard deviations of equation (12) for the M-dimensional square covariance matrix. These stored parameters along with a normal random number generator are sufficient to form a state error vector by repeatedly solving equation (10) for M values of the state (Y_u) est. This error state determines the initial condition for one trajectory realization. Additional realizations of the Monte-Carlo series can be found with the same stored coefficients (i.e., these need not be calculated again). The normal random number generator with previously calculated standard deviation supplies the required variation of the state vector for subsequent trajectories.

Subroutine FILL uses a system supplied function (MAM) to solve the set of simultaneous equations (11). Any simultaneous algebraic equation solver may be substituted here. Subroutine IMOD actually generates the desired error vectors.

V. CONCLUSIONS

A method has been described which allows mixed modes of error modeling for a single event. The method defines the interface between a covariance propagation error model and a Monte-Carlo deterministic error model. Given an arrival uncertainty covariance matrix for a physical system at some point in time, it is possible to construct an ensemble of state error vectors which may be used as initial conditions for the error state in order to exercise a Monte-Carlo model of the system. The error states so generated have been shown to be properly cross correlated with each other and have the correct distribution of magnitudes. A FORTRAN computer program has been written to facilitate implementation of the method within trajectory codes.

APPENDIX A

SYMBOLS

a	Regression coefficients
cc	Error covariance
N	Number of samples or summation index
NRNG	Zero mean normal random number generator
r	correlation coefficients
S_{NN}	Variance of error
S_{jk}	Covariance of errors
Y	Element of state vector
Y_k	Known part of state vector element
Y_u	Unknown part of state vector element
σ	Standard deviation for gaussian distribution
Σ	Summation symbol

<u>Subscripts</u>

est. or e	Regression estimated value of an error
actual or a	Determined actual value of an error

APPENDIX B
COMPUTER CODE LISTING

```
      SUBROUTINE COVSET(M)
C      BRUNSVOLD    MAY  77
C
C     FOR A GIVEN MONTE-CARLO RUN SET. THE FOLLOWING INFORMATION
C     NEED BE CALCULATED ONLY ONCE.
C
C     CALCULATE CERTAIN COEFFICIENTS FROM ARRIVAL UNCERTAINTY
C     COVARIANCE MATRIX  (INITIAL CONDITIONS FOR EACH RUN)
C
C     STORAGE IN THE X ARRAY BEGINS AT  X(1)   AND IS OPEN ENDED
C     STORAGE LOCATIONS = *A* STORAGE + SIGMA STORAGE + S(N) STORAGE
C     =((M-1)+(M-2)+.....+(2))+(M-1)+M
C     X ARRAY NOW SUFFICIENT TO HANDLE A 12 X 12
C
      COMMON X(90)
      DIMENSION BMM(3).SMM(3.3).ARRAY(2.2).IC(2)
  200 FORMAT(5F15.7.5X)
  201 FORMAT(1H ,5F15.7.5X)
  202 FORMAT(1H1)
      X(2)=FLOAT(M)
C     READ IN COVARIANCE MATRIX
      READ(5,200)SMM
      WRITE(6,202)
      WRITE(6,201)SMM
C     TO SIMPLIFY COMPUTATIONS. DIAGONAL ELEMENTS OF SMM MATRIX ARE
C     SQRT OF ORIGINAL COVARIANCE DIAGONAL ELEMENTS.
C     CALCULATE CORRELATION COEFFICIENTS MATRIX FROM ARRIVAL COVARIANCE
      DO 10 K=1.M
      DO 10 J=1.M
      IF(K.GE.J)GO TO 10
C      CALCULATE THE *R* VALUES      (CORRELATION COEFFICIENTS)
C     CORRELATION COEFFICIENT MATRIX BECOMES SMM AS WE WRITE ON TOP OF
C     ORIGINAL COVARIANCE MATRIX TO SAVE STORAGE.
      SMM(J.K)=SMM(J.K)/(SMM(J.J)*SMM(K.K))
      SMM(K,J)=SMM(J.K)
   10 CONTINUE
      DO 11 J=1,M
C     PUT THE S(N) VALUES IN STORAGE. THESE ARE THE DIAGONAL ELEMENTS OF
C     THE COVARIANCE MATRIX EQUAL TO THE STANDARD DEVIATION. FIRST
C     LOCATION IS X(3).
      X(2+J)=SMM(J,J)
C     SET THE DIAGONAL ELEMENTS OF THE CORRELATION MATRIX TO 1.0
   11 SMM(J,J)=1.
C     X(2)=M AND X(1)=S(1,2)/S(1.1). ARE COEFFICIENTS FOR YU2 ESTIMATE
      X(1)=SMM(1.2)*X(4)/X(3)
      N=1 $ NB=MC=M+2 $ M1=M-1
   12 N=N+1
C     CALCULATE THE *A*S AND PUT INTO STORAGE.  A(1) AND A(2) FIRST.
C     THESE ARE THE UNKNOWN VARIABLES OF THE SIMULTANEOUS ALGEBRAIC
C     EQUATIONS.     SUBROUTINE FILL SOLVES THESE EQUATIONS.
C     BMM=SOLUTION VECTORS=*A*S.,  SMM=CORRELATION COEFFICIENT MATRIX,
C     M=ORDER OF SMM,  N=NUMBER OF SIMULTANEOUS EQUATIONS TO BE SOLVED
C     IN THIS PASS THROUGH FILL.  ARRAY AND IC=DUMMY MATRIX THAT MUST BE
C     DIMENSIONED IN COVSET. DIMENSION TO M-1
```

```
      CALL FILL(BMM,SMM,M,N,ARRAY,IC)
      DO 13 ILO=1,N
      MM=ILO+MB
C     STORE SOLUTION VECTORS FROM FILL.  FIRST LOCATION IS X(M+3).
      X(MM)=BMM(ILO)
   13 CONTINUE
      IF(N.EQ.M1)GO TO 14
      MB=MM
      GO TO 12
C     CALCULATE M-1 PERMANENT VALUES OF SIGMA. SIGMA(2) IS FIRST AT
C     LOCATION X(MM+1)=X(3+M+(M-1)+(M-2)+...+2)
C     CALCULATE SIGMA(2)
   14 X(MM+1)=X(4)*(1.-SMM(1,2)**2)**.5
C     CALCULATE SIGMA(3) TO SIGMA(M)
      DO 21 K=2,M1
      REST=0.
      DO 22 L=1,K
      REST=SMM(K+1,L)*X(MC+L)+REST
   22 CONTINUE
      MC=MC+K
C     PUT CALCULATED SIGMA S INTO X ARRAY
      X(MM+K)=X(K+3)*(1.-REST)**.5
   21 CONTINUE
C     WITH SIGMA S, *A*S, AND S(N) S, ERROR VECTORS MAY BE GENERATED
C     WITH NORMAL RANDOM NUMBER GENERATOR.  A SUBROUTINE FOR THIS
C     CALLED "IMOD" IS INCLUDED.
      RETURN
      END
C
C
      SUBROUTINE FILL(B,BMM,M,N,ARRAY,IC)
C     BRUNSVOLD   MAY  77
C     THIS SUBROUTINE FINDS THE SOLUTION VECTOR FOR THE "A" COEFFICIENTS
C
      DIMENSION BMM(M,M),ARRAY(N,N),B(N),IC(N)
      DO 15 K=1,N
C     GENERATE RIGHT SIDE OF ALGEBRAIC EQUATIONS FROM CORRELATION MATRIX
      B(K)=BMM(K,N+1)
      DO 15 J=1,N
C     GENERATE COEFFICIENTS OF ALGEBRAIC EQUATIONS FROM CORRELATION
C     MATRIX AND PUT INTO ARRAY.
      ARRAY(J,K)=BMM(J,K)
      ARRAY(K,J)=ARRAY(J,K)
   15 CONTINUE
C     MAM IS SYSTEM SUBROUTINE THAT SOLVES LINEAR SET OF ALGEBRAIC EQNS.
      CALL MAM(ARRAY,N,N,B,1,IC,ID)
      IF(ID.EQ.2)GO TO 20
      GO TO 16
   20 WRITE(6,100)
  100 FORMAT(1H ,26HFAILURE IN SUBROUTINE FILL)
      STOP
C     RETURN TO COVSET WITH SOLUTION VECTOR B.
   16 RETURN $ END
```

```
      SUBROUTINE IMOD
C     THIS INITIAL MODULE SOLVES FOR THE INITIAL ERRORS IN THE STATE VEC
      COMMON X(90)
      DIMENSION Y(9)
      M=IFIX(X(2)) $ OA=0.
C     FIND THE PROPER INDEX IN THE X ARRAY
      J=0 $ M1=M-1
      DO 5 I=2,M
      J=I+J
    5 CONTINUE
      MM=J+2
C     GENERATE A NORMALLY DISTRIBUTED RANDOM NUMBER, (RN), OF
C     SIGMA=1. AND MEAN=0.
   25 CALL RANNUM(OA,1.,0.,R,RN)
C     GENERATE THE FIRST STATE ERROR Y(1).
      Y(1)=RN*X(3)
      CALL RANNUM(OA,1.,0.,R,RN)
C     GENERATE THE SECOND STATE ERROR.
      Y(2)=X(1)*Y(1)+RN*X(MM+1)
      MC=M+2
C     SOLVE FOR THE REMAINING Y ERRORS
      DO 10 K=2,M1
      REST =0.
      DO 20 L=1,K
      REST=(X(MC+L)*Y(L)/X(L+2))+REST
   20 CONTINUE
      CALL RANNUM(OA,1.,0.,R,RN)
      Y(K+1)=X(K+3)*REST+RN*X(MM+K)
      MC=MC+K
   10 CONTINUE
C     THE Y VECTOR GENERATED IS ONE ERROR VECTOR REALIZATION TO BE ADDED
C     TO MEANS.  ADDITIONAL REALIZATIONS CAN BE GENERATED WITH OTHER SET
C     OF RANDOM NUMBERS, RN'S.
      WRITE(6,100)Y
  100 FORMAT(1H0,9F10.6)
      RETURN $ END
```

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