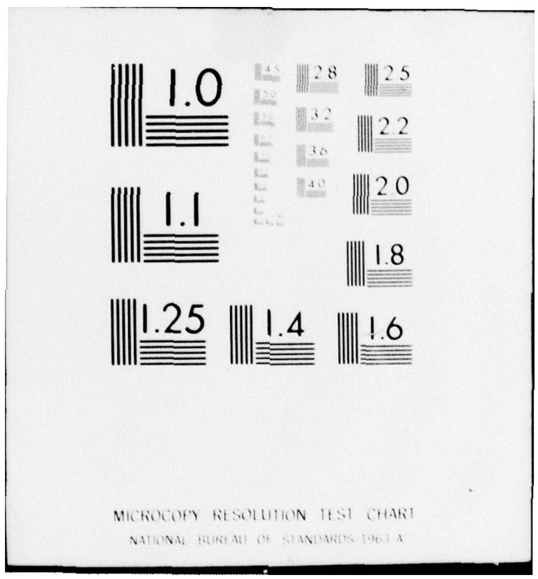
AD-A054 258 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 12/1
ON THE THEOREM OF N. V. SMIRNOV WITH RESPECT TO A COMPARISON OF--ETC(U)
NOV 77 I D KVIT
UNCLASSIFIED FTD-ID(RS)T-2015-77 NL

1 OF 1
AD
A054258

END
DATE
FILMED
6-78
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

AD-A054258

FTD-ID(RS)T-2015-77

# FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

ON THE THEOREM OF N. V. SMIRNOV WITH RESPECT TO A COMPARISON OF TWO SAMPLINGS

by

I. D. Kvit

FOREIGN TECHNOLOGY DIVISION



Approved for public release; distribution unlimited.

FTD -ID(RS)T-2015-77

# EDITED TRANSLATION

FTD-ID(RS)T-2015-77 22 November 1977

MICROFICHE NR: FTD-77-C-001463

ON THE THEOREM OF N. V. SMIRNOV WITH RESPECT TO A COMPARISON OF TWO SAMPLINGS

By: I. D. Kvit

English pages: 4

Source: Doklady Akademii Nauk SSSR, Izd-vo Moscow-Leningrad, Vol. 71, No. 1, 1950, pp 229-231

Country of origin: USSR
Translated by: Robert D. Hill
Requester: AFFDL/FBRD






PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\mathrm{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\mathrm{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

ON THE THEOREM OF N.V. SMIRNOV WITH RESPECT TO A COMPARISON OF TWO SAMPLINGS

I.D. Kvit

(Presented by Academician A.N. Kolmogorov on 12 January 1950)

In work [1] N.V. Smirnov studied the following important problem of statistics: there are two series of results of independent observations on the random quantities $\xi_1$ and $\xi_2$

$$x_1, x_2, \ldots, x_m$$

and

$$y_1, y_2, \ldots, y_n.$$

It is asked under what conditions is it possible to consider that the distribution functions $F_1(x) = P\{\xi_1 < x\}$ and $F_2(x) = P\{\xi_2 < x\}$ are equal, and when is the divergence of the experimental data so considerable that the hypothesis $F_1(x) = F_2(x)$ should be rejected.

In the present memorandum we show that the generalization of the theorems of A.N. Kolmogoorov and N.V. Smirnov, obtained by G.M. Maniya [2] for one sampling, is transferred without difficulty to the problem of N.V. Smirnov about two samplings.

Let us examine the empirical distribution functions

$$S_m(x) = \frac{k_1(x)}{m},$$

where $k_1(x)$ is the number of observed values of $\xi_1$ less than x and

$$T_n(x) = \frac{k_2(x)}{n},$$

where $k_2(x)$ is the number of observed values of $\xi_2$ less than x.

At the discontinuity points we supplement the empirical distribution functions with the vertical segments.

In the indicated work N.V. Smirnov proved that if $F_1(x)=F_2(x)$, function $F_1(x)$ is continuous and increases everywhere, $\frac{m}{n}=\text{const}$, then when

$N=\frac{mn}{m+n}\to\infty$

$$P\left\{D(m,n)\leqslant\frac{z}{\sqrt{N}}\right\}\to\sum_{k=-\infty}^{\infty}(-1)^k e^{-2k^2z^2}, \qquad (1)$$

where

$$D(m,n)=\sup_{-\infty<x<\infty}|S_m(x)-T_n(x)|.$$

Let us assume that $\theta_1$ and $\theta_2$ are arbitrary numbers, $0<\theta_1<\theta_2<1$. Let us assume that $F_1(x)=F_2(x)=F(x)$ and determine $\alpha$ and $\beta$ by means of equalities

$$F(\alpha)=\theta_1,\quad F(\beta)=\theta_2.$$

Let us denote further

$$\alpha_{mn}=\min_x[S_m(x)=\theta_1;\ T_n(x)=\theta_1],\quad \beta_{mn}=\max_x[S_m(x)=\theta_2;\ T_n(x)=\theta_2].$$

Then when $m\to\infty$, $n\to\infty$ in virtue of the theorem of Glivenko, there should be

$$\alpha_{mn}\to\alpha,\quad \beta_{mn}\to\beta.$$

Let us introduce the notations

$$D^+_{mn}(\theta_1,\theta_2)=\sup_{(\alpha_{mn},\beta_{mn})}\{S_m(x)-T_n(x)\},$$

$$D_{mn}(\theta_1,\theta_2)=\sup_{(\alpha_{mn},\beta_{mn})}|S_m(x)-T_n(x)|.$$

The obtained results can be formulated in the form of the following two theorems.

Theorem 1. If when $n \to \infty$ $\theta_1^{(n)} = \theta_1 + o\left(\frac{1}{\sqrt{n}}\right)$ and $\theta_2^{(n)} = \theta_2 + o\left(\frac{1}{\sqrt{n}}\right)$, $0 < \theta_1 < \theta_2 < 1$ then

$$P\left\{D_{mn}^{+}(\theta_1^{(n)}, \theta_2^{(n)}) \leqslant \frac{z}{\sqrt{N}}\right\} \xrightarrow[N \to \infty]{} \Phi^{+}(\theta_1, \theta_2; z)$$

where

$$\Phi^{+}(\theta_1, \theta_2; z) = \frac{1}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{a}\int_{-\infty}^{b} e^{-\frac{1}{2}\overline{Q}(z_1, z_2)}\, dz_1 dz_2 -$$

$$- \frac{e^{-2z^2}}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{a'}\int_{-\infty}^{b'} e^{-\frac{1}{2}Q(z_1, z_2)} dz_1 dz_2,$$

$$a = \frac{z}{\sqrt{\theta_1(1-\theta_1)}}, \qquad b = \frac{z}{\sqrt{\theta_2(1-\theta_2)}},$$

$$a' = \frac{z - 2z\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad b' = \frac{z - 2z(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}}, \quad R = \sqrt{\frac{\theta_1(1-\theta_2)}{\theta_2(1-\theta_1)}},$$

$$Q(z_1, z_2) = \frac{1}{1-R^2}[z_1^2 + 2Rz_1z_2 + z_2^2],$$

$$\overline{Q}(z_1, z_2) = \frac{1}{1-R^2}[z_1^2 - 2Rz_1z_2 + z_2^2].$$

Hence, in particular,

$$P\left\{D_{mn}^{+}(0, 1) \leqslant \frac{z}{\sqrt{N}}\right\} \xrightarrow[N \to \infty]{} 1 - e^{-2z^2}.$$

We see thus that the theorem of N.V. Smirnov about the one-sided deviations of the empirical function from the theoretical is carried over to the case of the maximum of one-sided deviations of two empirical functions.

Theorem 2. In the suppositions of theorem 1

$$P\left\{D_{mn}(\theta_1^{(n)}, \theta_2^{(n)}) \leqslant \frac{z}{\sqrt{N}}\right\} \xrightarrow[N \to \infty]{} \Phi(\theta_1, \theta_2; z),$$

where

$$\Phi(\theta_1, \theta_2; z) = \frac{1}{2\pi\sqrt{1-R^2}} \int_{-a}^{a} \int_{-b}^{b} e^{-\frac{1}{2}Q(z_1, z_2)} dz_1 dz_2 -$$

$$- \frac{1}{\pi\sqrt{1-R^2}} \sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2z^2} \int_{-a_k}^{a_k} \int_{-b_k}^{b_k} e^{-\frac{1}{2}Q(z_1, z_2)} dz_1 dz_2,$$

$$a_k = \frac{z - 2kz\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad b_k = \frac{z - 2kz(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}}.$$

Hence when $\theta_1 = 0, \theta_2 = 1$ we get (1).

Similar to the theorems of G.M. Maniya, the results discussed make it possible to use that interval of the observed values in which the results of the observations are more reliable.

In the work we used the method of Laplace transforms applied by W. Feller [3] for proofs of the theorems of A.N. Kolmogorov and N.V. Smirnov.

In conclusion I wish to express my deep thanks to Prof. B.V. Gnedenko for the statement of the problem and his guidance in the solving of it.

L'vov State University im. I. Franko Submitted 12 January 1950

Bibliography

[1] Н. В. Смирнов, Бюлл. НИИМ МГУ, 2, в. 2 (1939). [2] Г. М. Мания, ДАН, 69, № 4 (1949). [3] W. Feller, Ann. of Math. Statistics, 19, 2, 177 (1948).