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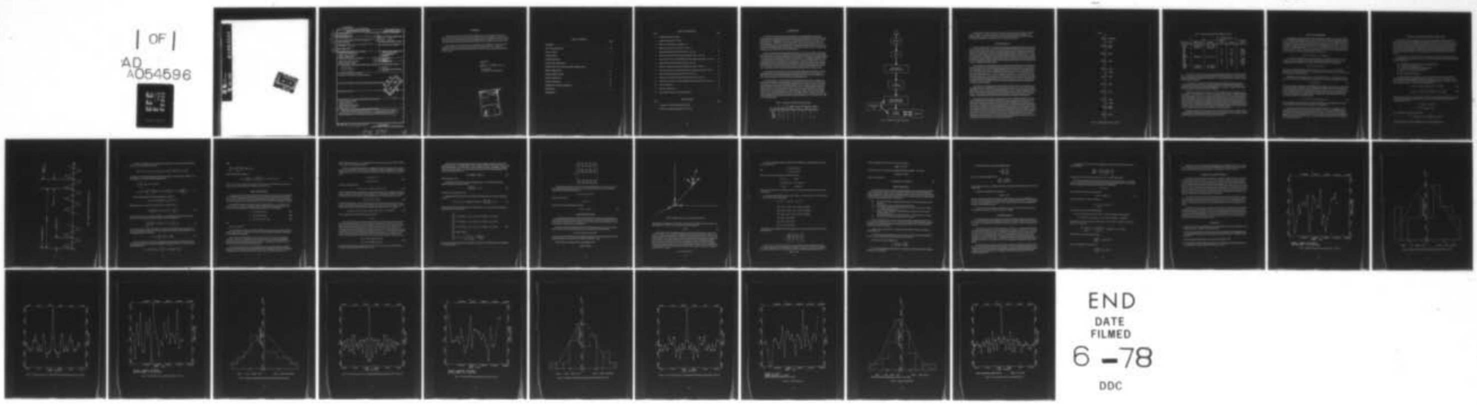

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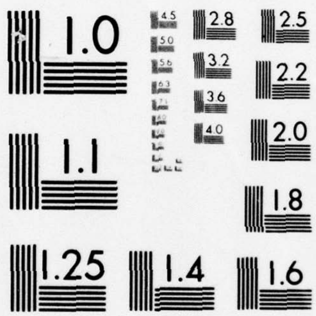


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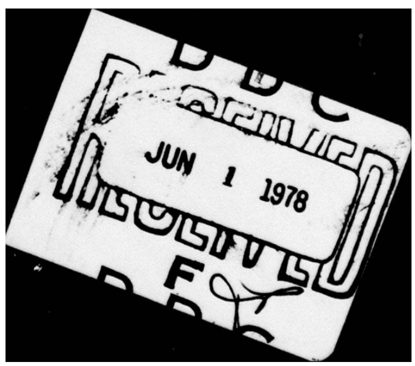
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This report describes the formulation required to convert the observations of navigation transmissions from the Navy Navigation Satellites into a two-dimensional position solution. This particular application is for the performance testing of the NAVPAC multisatellite receiver.

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## FOREWORD

The performance testing of the NAVPAC multisatellite receiver was accomplished by observing passes of particular satellites of the Navy Navigation Satellite System. These passes were stored on magnetic tape and sent to the Naval Surface Weapons Center, Dahlgren Laboratory (NSWC/DL) for reduction. This report describes the formulation which comprised this data processor, and which was current in June 1976.

This formulation is a small part of the entire NAVPAC project at NSWC/DL. The instrument itself was designed and built at the Applied Physics Laboratory under the direction of R. E. Willison. At Dahlgren the technical direction and project management were by Mr. E. Swift with programming support by Mr. A. Fisher.

Released by:

R. A. Niemann
Head, Warfare Analysis Department

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## INTRODUCTION

The NAVPAC multisatellite receiver has the capability of receiving simultaneously three dual-frequency navigation signals transmitted from the Navy Navigation Satellite System. The NAVPAC receiver was developed by the Applied Physics Laboratory of the Johns Hopkins University and consists of the following subsystems: 150/400-MHz antenna, the multisatellite receiver, a stable reference oscillator and time code generator, a data processor and core memory for storage of the data, and a tape recorder controller for final disposition of the accumulated data. This report describes the formulation used to convert the observations of Doppler shifted NAVSAT transmissions during one satellite pass into a two-dimensional navigation solution for the position of the NAVPAC antenna.

When the NAVPAC tape containing Doppler observations arrives at the processing site, the first step of reduction (Figure 1) involves unpacking the data words and separating them into their constituent parts. Table 1 illustrates the composition of each of the four types of words. The Doppler time mark word (DTM) is composed of the receiver identification (RID) and the NAVPAC time. The Doppler word (DW) contains a receiver identification, a raw Doppler count, a refraction count, and the corresponding time over count. The Doppler and refraction counts will be described in detail in a later section. The two-minute time mark word (TMTM) has the NAVSAT identification word (NID), the receiver identification, and the NAVPAC time at the instant of reception of the two-minute mark. The vehicle time mark (VTM) is the final word type. Any subset of the above information can be printed as required by the various input options.

The second step in the processing is the correction of the NAVPAC clock by comparison to the time signals transmitted by the navigation satellites and recorded as NAVPAC times in the two-minute time mark words. The corrections obtained are added to the Doppler time marks in order to establish the precise (<50-microsecond error) time reference for a particular Doppler count.

The third operation is to form passes from the chronological collection of data. A satellite pass is defined to be the duration of time during which a particular satellite is "visible" from the receiving station. For this operation a trajectory tape, containing predicted (or best fit if reduction is done at a later time) earth fixed positions of all satellites of interest, is needed as a key to establish the beginnings and ends of passes. The trajectory is especially needed to mark the ends of passes since NAVPAC gives no definitive indication when it loses lock on a satellite. A related function is to identify short, low elevation, passes which may not contain a TMTM between rise and set. Such passes cannot be identified from NAVPAC data alone since the satellite identification is present only in TMTM's (Table 1). Finally, the trajectory tape is used to establish which data will be processed. If a particular satellite is not on the trajectory file, it will not be considered further.

Table 1. Composition of NAVPAC Doppler Data Words

| Type of Word | RID | NID | Doppler Count | Time Over Count | Refraction Count | NAVPAC Clock |
|---|---|---|---|---|---|---|
| DTM | x | | | | | x |
| DW | x | | x | x | x | |
| TMTM | x | x | | | | x |
| VTM | x | | | | | x |

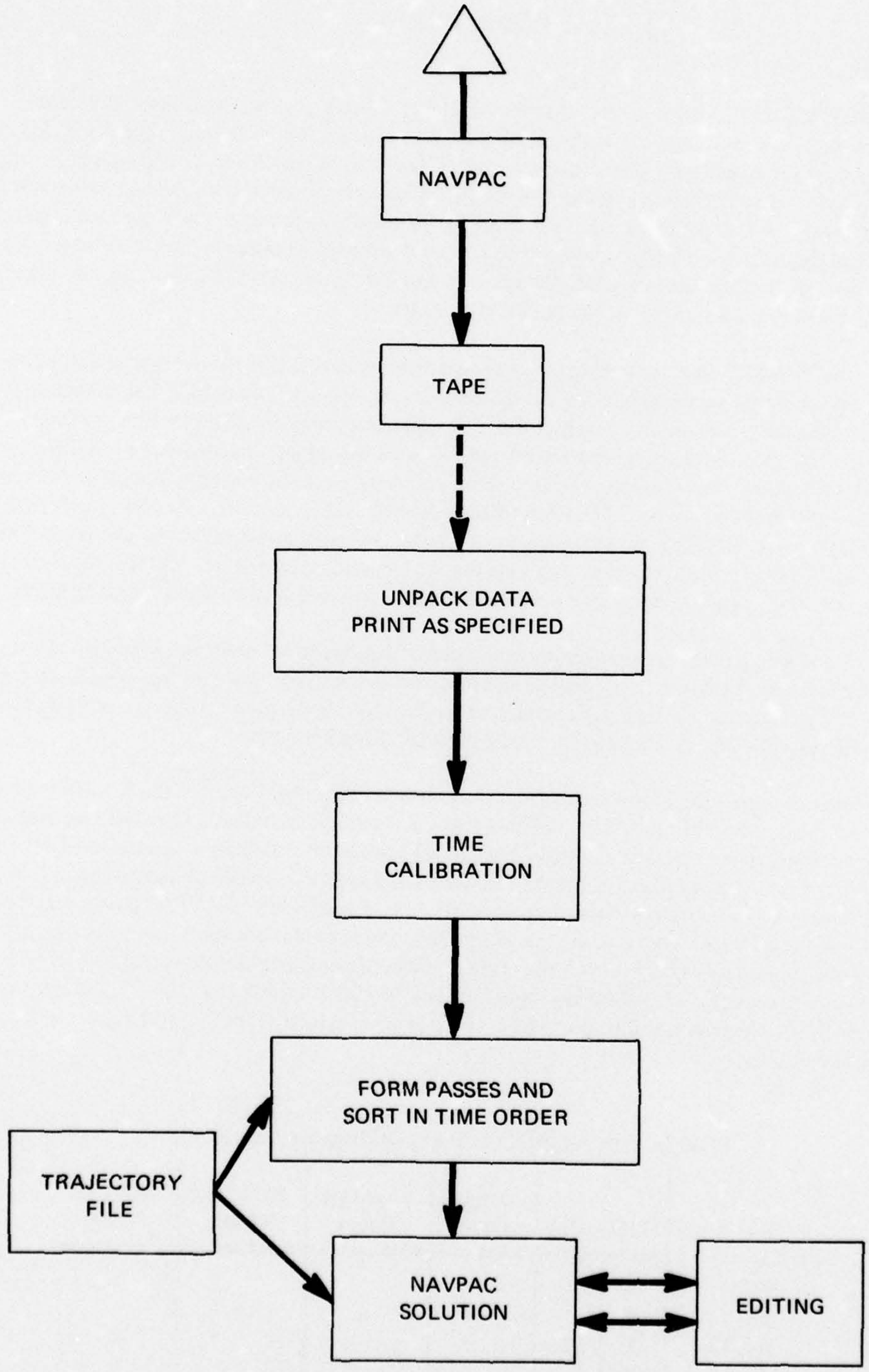


Figure 1. NAVPAC Ground Test Data Flow

The data which is accepted from each pass is written in time order in a format making it compatible with the CELEST orbit computation program.[1] Alternately, if the full capability of CELEST is not required, a smaller processor can be used as the final step. This program, which uses the data to produce a two-dimensional navigation on a pass-by-pass basis, is described in later sections.

## PASS RECOGNITION

All Navy navigation satellites are synchronized to Coordinated Universal Time (UTC) to within about 50 microseconds. Therefore the two-minute time marks received at NAVPAC provide a UTC time reference having a resolution of about this accuracy. Use is made of these time marks to establish the beginning and end time of each Doppler count. Each pass begins with a Doppler time mark, indicating that a receiver has successfully acquired a satellite signal. The time indicated by this Doppler time mark is, however, not the time at which the count began, but rather the time 30 NAVPAC seconds later. This procedure was adopted to eliminate a multitude of Doppler time marks which would have occurred at every false acquisition signal. The 30-second delay requires that NAVPAC remain locked onto a satellite signal for 30 seconds before such a fact is acknowledged by the data processor.

The first Doppler word following the Doppler time mark contains the Doppler count for the period beginning at signal acquisition and ending at the time indicated by the sum of the time over count and the time in the Doppler time mark word. Subsequent Doppler words are produced at each succeeding observation. Observations occur at intervals of 30 NAVPAC seconds plus time over counts. A NAVPAC unit of time is approximately equal to $1.2 \times 10^{-6}$ UTC seconds; more precisely, $2.5 \times 10^{7}$ NAVPAC time units equal exactly 30 NAVPAC seconds. Time over counts are always less than 128 microseconds. After eight consecutive Doppler words, another Doppler time mark is produced. The time between successive Doppler time marks should equal 240 seconds plus the sum of the intervening time over counts. Interspersed with the Doppler time marks and Doppler words are the two-minute time marks indicating the NAVPAC time at each even two-minute interval.

An example of a four-minute segment from one satellite is shown in Table 2. The time between the two Doppler time marks is 240.000162 seconds, and the sum of the time over counts for the intervening eight Doppler words is 162 microseconds, indicating agreement. Figure 2 is the same example in a different format. Figure 2 illustrates that the DTM's occur immediately before the ith and (i + 8)th Doppler words (where i = 1 in Figure 2), and includes the eight time over counts. Note that the beginning of the pass is actually at 3467.274860 seconds and therefore the first TMTM occurred after satellite acquisition.

Since NAVPAC has the capability of receiving signals from three satellites simultaneously, the data from each satellite can be intermixed with the other two. Also, the possibility exists that receivers and satellites may switch without warning. For these reasons the pass recognition and sorting logic become very complex. The Doppler words give no direct information as to which satellite the present count refers, only the receiver identification. The receiver identification in turn is related to a satellite through information supplied by the TMTM's. The TMTM decoder cycles through the active receivers in sequence. Thus if all are tracking, a gap of six minutes can occur before it is confirmed that each receiver is still tracking the same satellite. If a receiver has switched, then the DTM's may indicate a time difference that is not 240 + time over seconds somewhere within the previous six minutes. Should this be the case, the number of possibilities of what anomaly may have occurred is quite large and unfortunately not unique. Some of the anomalies that can occur and which are recognized are the following:

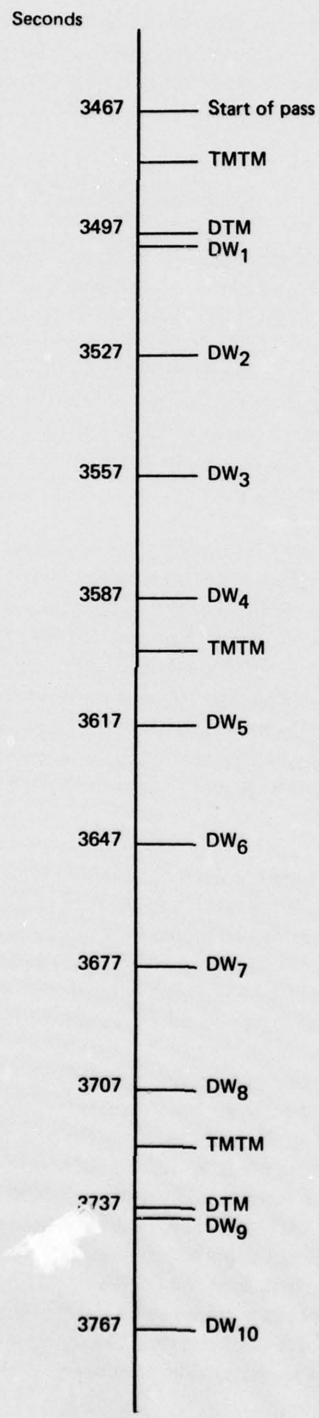


Figure 2. Graphical Representation of Table 2

Table 2. Representative Doppler Data Beginning a New Pass

| Record Type | Time | | Time Over Count | | Raw Doppler Count |
|---|---|---|---|---|---|
| | NAVPAC Units | Seconds | NAVPAC Units | μseconds | |
| TMTM | 2900728587 | 3481.114304 | | | |
| DTM | 2914395717 | 3497.274860 | | | |
| $DW_1$ | | | 30 | 36.0 | 710435 |
| $DW_2$ | | | 1 | 1.2 | 710848 |
| $DW_3$ | | | 10 | 12.0 | 711591 |
| $DW_4$ | | | 22 | 26.4 | 712731 |
| TMTM | 3000926143 | 3601.111372 | | | |
| $DW_5$ | | | 7 | 8.4 | 714363 |
| $DW_6$ | | | 19 | 22.8 | 716616 |
| $DW_7$ | | | 25 | 30.0 | 719661 |
| $DW_8$ | | | 21 | 25.2 | 723735 |
| TMTM | 3100923875 | 3721.108650 | | | |
| DTM | 3114395852 | 3737.275022 | | | |

a. Loss of Lock. A receiver once locked and tracking a NAVSAT may lose lock at some point in the pass. It is possible that this signal may be reacquired by another receiver (which was in the search mode) before it is found again by the original tracking receiver. In this case the characteristic symptom is that the NID will switch to a new RID.

b. Lock on Bogie. A receiver once locked and tracking a NAVSAT may be enticed to capture a strong but extraneous signal not having the required signal structure. The NAVPAC tracking logic will notice both the lack of correct structure and presumably little or no Doppler. The signal will be dropped, but in the meantime it has broken the continuity of the pass.

c. High-Priority Switching. Two receivers, I and II, are initially locked onto two satellite signals: a strong signal "*s*" and a weak signal "*w*." Receiver I is tracking *s* and II is tracking *w*. Suppose the NAVPAC priority logic has given II priority over I. The priority logic decides that, if two receivers acquire and lock onto the same satellite, the one that acquired first will continue, but the later receiver will be unlocked and resume its search mode. Suppose the two signals are such that the Doppler frequencies coincide at some point in the pass. It is possible that the high-priority receiver (II) may jump to the stronger signal *s* without losing lock or otherwise indicating what happened. Since II still has priority, the NAVPAC logic will kick I off *s* when it discovers that two receivers are tracking the same satellite. Thus the correct pass structure for *s* begins on receiver I and ends on II, with the pass for *w* on II being incomplete.

d. Satellite Swapping. This case is similar to the above except that now receiver I acquires the weak signal within the same 30-second interval that II switches to the strong signal. For this to happen, the signal strengths must be nearly equal and the Doppler slopes nearly identical.

## NAVPAC TIME CORRECTION

To establish the correspondence between UTC, designated by the symbol t (seconds), and NAVPAC time $\tau$ (in NAVPAC units), the reset time $t_r$ must be correct to within one minute. The reset time is defined as the UTC instant when NAVPAC clock, via a command, is reset to zero. The first TMTM received after resetting can then be used to refine the estimate of $t_r$ to an accuracy better than one second. The procedure is as follows: The time (seconds) between reception of the first TMTM and the reset time is $1.2 \times 10^{-6}\ \tau$. The UTC of the TMTM is therefore $t_t = t_r + 1.2 \times 10^{-6}\ \tau \pm$ one minute, but it is known that all TMTM's are transmitted on the even minute. Therefore a better approximation to the reset time is obtained by rounding $t_t$ to the nearest even minute $t_T$ and calculating a new reset time $t_R$

$$t_R = t_T - 1.2 \times 10^{-6}\ \tau; \qquad t_T = t_t|_{rounded}\ .$$

It is inevitable that NAVPAC time will wander slightly from the UTC standard. In order to correct for this, an expression relating the two is needed. The expression should be such that given any $\tau$, the corresponding t can be evaluated to within ± 50 $\mu$seconds. The relation chosen is a polynomial:

$$t(\tau) = t_R + a + (1 + b)\tau + c\tau^2 \tag{1}$$

The form of this polynomial models the initial offset by $t_R$ + a, a rate error by 1 + b, and the aging phenomena by c. The three parameters a, b, c are small corrections to the values $t_R$, one, and zero expected from a clock which follows UTC exactly.

All TMTM's in a preset interval are used to evaluate the parameters a, b, c in a least-squares solution. The time of reception of the ith TMTM is the observed quantity $\tau_i$. The UTC corresponding to this instant is

$$t_i = t_{Mi} + t_{pi} + t_d, \tag{2}$$

where $t_{Mi}$ is the time of transmission of the TMTM signal at the satellite (an even minute), $t_{pi}$ is the propagation time from satellite to NAVPAC, and $t_d$ is a decode delay in NAVPAC. The value for $t_{pi}$ is obtained from the trajectory file (Figure 1) by calculating the slant range at each $t_i$. Thus, $t_{pi} = 1/c$ (slant range at $t_i$). The error introduced in $t_{pi}$ by not calculating the slant range correctly and by not including refraction is less than a microsecond. This error will be ignored. The difference between Equations (1) and (2) is the error which is minimized in the least-squares sense:

$$\epsilon_i = a + (1 + b)\tau_i + c\tau_i^2 - t_{Mi} - t_{pi} - t_d + t_R \tag{3}$$

The decode delay $t_d$ is inserted in Equation (2) because this delay affects only TMTM's, and its presence must be eliminated in order to obtain a correct value for a in Equation (1).

The consistency of the solution is checked by inserting the observations $\tau_i$ into Equation (1) and calculating the corresponding $t_i$'s. If these calculated values differ from the corrected TMTM's obtained from Equation (2) by a preset tolerance, the observation is rejected. A new solution is then performed using only the observations which pass the above test. This iteration process eliminates from consideration TMTM's which do not agree with the majority. Equation (1) can now be used to convert any NAVPAC time $\tau$ to its UTC equivalent.

## SYNTHESIS OF THE RANGE DIFFERENCE OBSERVATION

The NAVPAC receivers are designed to acquire and track dual-frequency Doppler signals from the NAVSATS and convert them to Doppler counts and refraction counts at intervals of about 30 seconds. Since the relative velocity between NAVPAC and NAVSAT is continuously varying, the period of the Doppler cycles continuously varies. As illustrated in Figure 3, the time over count enters from the desire to keep the number of cycles (counts) in an observation an integer. This is accomplished by recording the time in excess of 30 seconds needed to complete the cycle which was in progress at the 30-second mark. Thus an integral number of Doppler cycles, the Doppler count, occurs in the interval (30 + time over) seconds.

The signal processing which NAVPAC performs changes the two received satellite frequencies $\nu_1$ and $\nu_2$ into a Doppler count $n_d$ and a refraction count $n_r$. In order to explain the process, the following parameters need to be defined:

$f_1$ = $150 \times 10^6$ Hz, the nominal frequency of the transmitted signal
$f_0$ = ~ 12000 Hz, the frequency offset
$f_b$ = an unknown time variable bias, NAVSAT dependent
$f_n$ = an unknown time variable bias in NAVPAC
$q$ = 8/3
$f_N$ = the ionospheric plasma frequency

For solution purposes, the two biases $f_b$ and $f_n$ are assumed to be constant over each 20-minute interval comprising a pass. Each NAVSAT transmits two frequencies: $(f_1 + f_0 + f_b)$ and $q(f_1 + f_0 + f_b)$. The frequencies received at NAVPAC which correspond to these have added to them a Doppler component and an ionospheric component. These are modeled in Equation (4):

$$\nu_1 = (f_1 + f_0 + f_b)\left[1 - \frac{v}{c}\left(1 - \frac{1}{2} f_N^2 (f_1 + f_0 + f_b)^{-2}\right)\right] \tag{4a}$$

$$\nu_2 = q(f_1 + f_0 + f_b)\left[1 - \frac{v}{c}\left(1 - \frac{1}{2} f_N^2 q^{-2} (f_1 + f_0 + f_b)^{-2}\right)\right] \tag{4b}$$

The sign convention used for the relative velocity v is that $v < 0$ if the transmission path length is decreasing. In Equations (4), the earth's magnetic field is not considered and so the index of refraction n, and the received frequency $\nu$ are[2,3]

$$n = [1 - f_N^2 (f_1 + f_0 + f_b)^{-2}]^{1/2}$$

$$\nu = (f_1 + f_0 + f_b)\left[1 - n\frac{v}{c}\right]$$

If n is expanded as a binomial series, the result is

$$n \simeq 1 - \frac{1}{2} f_N^2 (f_1 + f_0 + f_b)^{-2} - \frac{1}{8} f_N^4 (f_1 + f_0 + f_b)^{-4} + \ldots$$

The high-order terms are normally negligible and so are ignored in Equations (4).

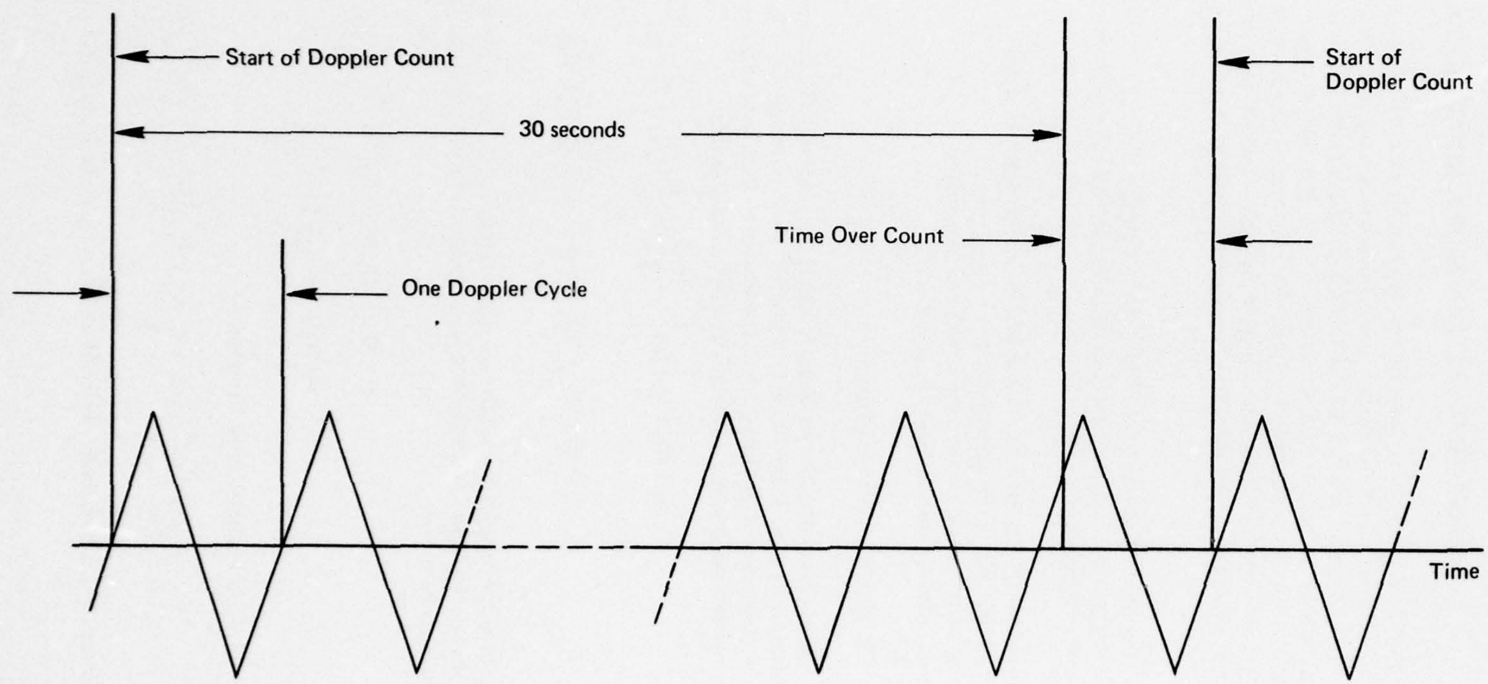


Figure 3. Diagram of the Elements of a Doppler Count

In NAVPAC, the Doppler frequency is obtained by subtracting $\nu_2$ from a locally generated frequency $q(f_1 + f_0 + f_n)$. In symbolic form, the result is

$$q(f_1 + f_0 + f_n) - \nu_2 = q(f_n - f_b) + q\frac{v}{c}(f_1 + f_0 + f_b)\left[1 - \frac{1}{2}f_N^2 q^{-2}(f_1 + f_0 + f_b)^{-2}\right]$$

The Doppler count is obtained by integrating this Doppler frequency over the observation interval $(t_i - t_{i-1})$. This is equivalent to counting the number of Doppler cycles.

$$n_d = \int_{t_{i-1}}^{t_i} [q(f_1 + f_0 + f_n) - \nu_2]\, dt$$

$$n_d = q(f_n - f_b)\int_{t_{i-1}}^{t_i} dt + \left[\frac{q}{c}(f_1 + f_0 + f_b) - \frac{1}{2}f_N^2\,\frac{q^{-1}}{c}\,(f_1 + f_0 + f_b)^{-1}\right]\int_{t_{i-1}}^{t_i} v\,dt \tag{5}$$

The refraction frequency is obtained by subtracting $\nu_1$ from $q^{-1}\nu_2$.

$$q^{-1}\nu_2 - \nu_1 = \frac{1}{2}f_N^2\frac{v}{c}(f_1 + f_0 + f_b)^{-1}(q^{-2} - 1)$$

Integrating this over the same time interval results in the refraction count $n_r$,

$$n_r = 4\left(\frac{1}{2}\right)f_N^2\frac{1}{c}(f_1 + f_0 + f_b)^{-1}(q^{-2} - 1)\int_{t_{i-1}}^{t_i} v\,dt \tag{6}$$

In Equation (6), the factor 4 is inserted by NAVPAC. If $n_r$ is multiplied by an appropriate constant the result can be used to simplify Equation (5). This form is written as Equation (7):

$$\left(\frac{q^{-1}}{q^{-2} - 1}\right)\frac{n_r}{4} = \frac{1}{2}f_N^2\,\frac{q^{-1}}{c}\,(f_1 + f_0 + f_b)^{-1}\int_{t_{i-1}}^{t_i} v\,dt \tag{7}$$

The constant was chosen so that the last term on the right in Equation (5) is the same as in Equation (7). If these two expressions are summed, the effects of refraction are eliminated from the Doppler count. The corrected Doppler count is therefore $n_c$:

$$n_c = n_d + \frac{q^{-1}}{q^{-2} - 1}\,\frac{n_r}{4} = n_d - \frac{6}{55}n_r \tag{8}$$

These raw counts are now converted to range differences so they can be used as observations in the remainder of the formulation. Performing the integration in Equation (5), where $\rho$ is the range from NAVSAT to NAVPAC, gives

$$n_c = q(f_n - f_b)(t_i - t_{i-1}) + \frac{q}{c}(f_1 + f_0 + f_b)[\rho(t_i) - \rho(t_{i-1})]\,,$$

where

$$\int_{t_{i-1}}^{t_i} v dt = \int_{t_{i-1}}^{t_i} \frac{d\rho}{dt} dt = \rho(t_i) - \rho(t_{i-1}).$$

Finally, the observation expression is

$$O_i = \frac{cn_c}{q(f_1 + f_0 + f_b)} = \frac{c(f_n - f_b)}{f_1 + f_0 + f_b} (t_i - t_{i-1}) + \rho(t_i) - \rho(t_{i-1}) \quad (9)$$

which is in units of meters and is the range difference expression $O_i$, for the NAVPAC corrected count $n_c$ ending at time $t_i$. When Equation (9) is evaluated, $f_n$ is taken to be zero.

## MODEL FORMULATION

The primary result desired from this formulation is an indication of the level of performance possible with the NAVPAC hardware compared with the performance of other comparable systems. To this end, this formulation must be similar to that used to process satellite data on a routine basis. Therefore many of the techniques described here were chosen because of their use in the CELEST orbit computation program.[1]

The NAVPAC antenna location can be expressed in either earth-fixed $x_g$, $y_g$, $z_g$ coordinates or in geodetic latitude $\phi_g$, longitude $\lambda_g$, and height $h_g$. Given an ellipsoid of revolution about the z axis with eccentricity e and semimajor axis $\alpha$, the relationship between the two sets of coordinates is

$$x_g = (Q + h_g) \cos \phi_g \cos \lambda_g \quad (10a)$$

$$y_g = (Q + h_g) \cos \phi_g \sin \lambda_g \quad (10b)$$

$$z_g = (Q + h_g) \sin \phi_g - Qe^2 \sin \phi_g \quad (10c)$$

where

$$Q = \alpha[1 - e^2 \sin^2 \phi_g]^{-1/2}$$

In the formulation that follows, earth fixed coordinates are used; however, in many cases the station location is given as geodetic coordinates on an ellipsoid. Equations (10) are included here to indicate how the transformation is accomplished.

The previous section described how range difference information is obtained from the Doppler observations. The value of range difference at each observation time $t_i$ is $O_i$. The corresponding value calculated from information residing on the trajectory file is $C_i$. The difference of these two $(O_i - C_i)$ is the error which is to be minimized in a least-squares sense.

To form the expression for the computed range difference, the range from satellite to NAVPAC is necessary at the instant of transmission at the beginning and end of a Doppler count interval. Knowledge of the propagation time is needed to obtain both values. The trajectory file provides estimates of the

NAVSAT earth fixed position $x_s$, $y_s$, $z_s$ at equal intervals in time, but positions at other times are available by use of eight-point LaGrangian interpolation.

In order to establish the propagation time at the beginning and end of a Doppler count, an iterative procedure is used as follows. The slant range from NAVPAC to NAVSAT is the magnitude of the difference of their respective vectors $\bar{r}_g$ and $\bar{r}_s$. In earth fixed coordinates, the vector to NAVPAC is $\bar{r}_g$, and to NAVSAT $\bar{r}_s$.

$$\bar{r}_s = x_s\hat{x} + y_s\hat{y} + z_s\hat{z}$$

$$\bar{r}_g = x_g\hat{x} + y_g\hat{y} + z_g\hat{z} \tag{11a}$$

The range is the magnitude of $\bar{\rho}$

$$\bar{\rho} = \bar{r}_s - \bar{r}_g = (x_s - x_g)\hat{x} + (y_s - y_g)\hat{y} + (z_s - z_g)\hat{z} \tag{12}$$

The proper propagation time is obtained when $\bar{r}_s$ is evaluated at the instant of transmission of the Doppler signal $t_{Mi}$, while $\bar{r}_g$ is evaluated at the instant of reception $t_i$. If each observation is labeled by the subscript of the time at the instant of reception, then the propagation time for the ith observation is $t_{pi}$, where

$$t_{pi} = \frac{1}{c}\,|\bar{r}_s(t_i - t_{pi}) - \bar{r}_g(t_i)| \tag{13}$$

The iteration is begun with $t_{pi} = 0$ on the right, and $\bar{r}_s(t_i)$ is calculated by the interpolation formula. The result is a nonzero value for $t_{pi}$, which is used to calculate $\bar{r}_s(t_i - t_{pi})$ for the next round. When $t_{pi}$ changes by less than one microsecond, the procedure is terminated. The range for the time $t_i$ is therefore

$$\rho_i = [(x_s(t_i - t_{pi}) - x_g(t_i))^2 + (y_s(t_i - t_{pi}) - y_g(t_i))^2 + (z_s(t_i - t_{pi}) - z_g(t_i))^2]^{1/2}, \tag{14}$$

and the range difference calculated for the observation $O_i$ at $t_i$ is

$$\rho_i - \rho_{i-1}.$$

One subtle point that must be mentioned is the following. The satellite position is calculated on the basis of an earth fixed coordinate system (one which rotates with the earth). In this frame, any location which is fixed to the earth has time invariant coordinates. It is clear, however, that the signal transmitted from the satellite travels in inertial space not in a frame rotating with the earth. Thus each time $\bar{r}_g$ is calculated, a small correction must be made to the station position to account for the station motion during the propagation interval. Given that the angular velocity of the earth is $\omega_g = 7.292115855 \times 10^{-5}$ radians/second, in a time interval $t_p$ the earth will have rotated through an angle $\Delta\lambda = \omega_g t_p$. With the station location as defined by Equations (10), the corrections to $x_g$ and $y_g$ due to $\Delta\lambda$ ($z_g$ is independent of $\lambda$) are

$$\Delta x_g = -(\Delta\lambda \sin\lambda_g)(Q + h_g)\cos\phi_g$$

$$\Delta y_g = (\Delta\lambda \cos\lambda_g)(Q + h_g)\cos\phi_g \tag{15}$$

The corrected station positions are therefore $x'_g = x_g + \Delta x_g$, $y'_g = y_g + \Delta y_g$ and $z_g$. In the NAVSAT-to-ground case the maximum station shift is about 7 meters.

A small correction to the range difference (Equation (15)) is added to compensate for the effect of tropospheric refraction. The Hopfield Model[4] is used to calculate the range correction, $R_H(t)$, based on zenith angle of the NAVSAT as seen from the station, and local temperature, pressure, and humidity. The range correction is scaled by a factor $1 + C_R$, which is adjusted in the solution for each pass independently. The correction for each observation $O_i$ is

$$(1 + C_R)[R_H(t_i) - R_H(t_{i-1})] \tag{16}$$

with $C_R$ initially set to zero.

Also included in the solution is a frequency bias parameter $f_n - f_b$. The nature of this term is explained in the previous section; its form is

$$\frac{c(f_n - f_b)}{f_1 + f_0 + f_b} (t_i - t_{i-1}), \tag{17}$$

with the sum $f_n - f_b$ initially set to zero.

Summing Equations (15), (16), and (17) provides the expression for $C_i$ used to form the observation matrix D.

$$C_i = (\rho_i - \rho_{i-1}) + (1 + C_R)[R_H(t_i) - R_H(t_{i-1})] + \frac{c(f_n - f_b)}{f_1 + f_0 + f_b} (t_i - t_{i-1}) \tag{18}$$

Differentiation of Equation (18) with respect to the parameters $x_g$, $y_g$, $z_g$, $C_R$, $f_b$ forms the following five partial derivatives; $f_n$ is assumed zero.

$$\begin{aligned}
\frac{\partial C_i}{\partial x_g} &= \rho^{-1}(t_{i-1})[x_s(t_{i-1} - t_{pi-1}) - x_g(t_{i-1})] - \rho^{-1}(t_i)[x_s(t_i - t_{pi}) - x_g(t_i)] \\
\frac{\partial C_i}{\partial y_g} &= \rho^{-1}(t_{i-1})[y_s(t_{i-1} - t_{pi-1}) - y_g(t_{i-1})] - \rho^{-1}(t_i)[y_s(t_i - t_{pi}) - y_g(t_i)] \\
\frac{\partial C_i}{\partial z_z} &= \rho^{-1}(t_{i-1})[z_s(t_{i-1} - t_{pi-1}) - z_g(t_{i-1})] - \rho^{-1}(t_i)[z_s(t_i - t_{pi}) - z_g(t_i)] \\
\frac{\partial C_i}{\partial C_R} &= R_H(t_i) - R_H(t_{i-1}) \\
\frac{\partial C_i}{\partial f_b} &= -c(t_i - t_{i-1}) \frac{f_1 + f_0}{(f_1 + f_0 + f_b)^2} = \frac{-c(t_i - t_{i-1})}{f_1 + f_0}
\end{aligned} \tag{19}$$

When these are evaluated at the time of each observation, a matrix A is formed where each row corresponds to one observation time.

$$A = \begin{vmatrix} \frac{\partial C_1}{\partial x_{g1}} & \frac{\partial C_1}{\partial y_{g1}} & \frac{\partial C_1}{\partial z_{g1}} & \frac{\partial C_1}{\partial C_{R1}} & \frac{\partial C_1}{\partial f_{b1}} \\ \frac{\partial C_2}{\partial x_{g2}} & \frac{\partial C_2}{\partial y_{g2}} & \frac{\partial C_2}{\partial z_{g2}} & \frac{\partial C_2}{\partial C_{R2}} & \frac{\partial C_2}{\partial f_{b2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial C_n}{\partial x_{gn}} & \frac{\partial C_n}{\partial y_{gn}} & \frac{\partial C_n}{\partial z_{gn}} & \frac{\partial C_n}{\partial C_{Rn}} & \frac{\partial C_n}{\partial f_{bn}} \end{vmatrix}$$

Corresponding to each observation, $O_i$, there is a computed range difference, $C_i$, and a linear correction consisting of partial derivatives from a row of A and incremental variations in the five parameters previously listed and henceforth denoted by X.

$$X = |\delta x_g \; \delta y_g \; \delta z_g \; \delta C_R \; \delta f_b|^T.$$

Therefore in matrix form

$$O = C + AX,$$

which can be rearranged by defining the matrix D:

$$D = O - C = AX. \tag{20}$$

## MATRIX MANIPULATIONS

Each satellite pass observed by a ground station will produce a system of linear equations like those represented in Equation (20). Solution of Equation (20) for the vector X will in principle provide the corrections needed to establish the true values for $x_g$, $y_g$, $z_g$, $C_R$, and $f_b$. The matrix manipulations necessary to obtain the least-squares solution are described next.

Each observation $O_i$ which makes up Equation (20) is assigned a variance $\sigma^2_{obs\,i}$, which is dependent upon the duration of that particular Doppler count. The expression adopted is the following:

$$\sigma^2_{obs} = (30 + \text{time over})^{-1} \; (\text{seconds})^{-1}$$

The reciprocal of each individual variance is assigned to the diagonal of a square weight matrix W. The off diagonal elements are set to zero while the ith diagonal element is $w_{ii} = \sigma^{-2}_{obs\,i}$.

The first operation on Equation (20) is to premultiply by $A^TW$

$$A^TWAX = A^TWD$$

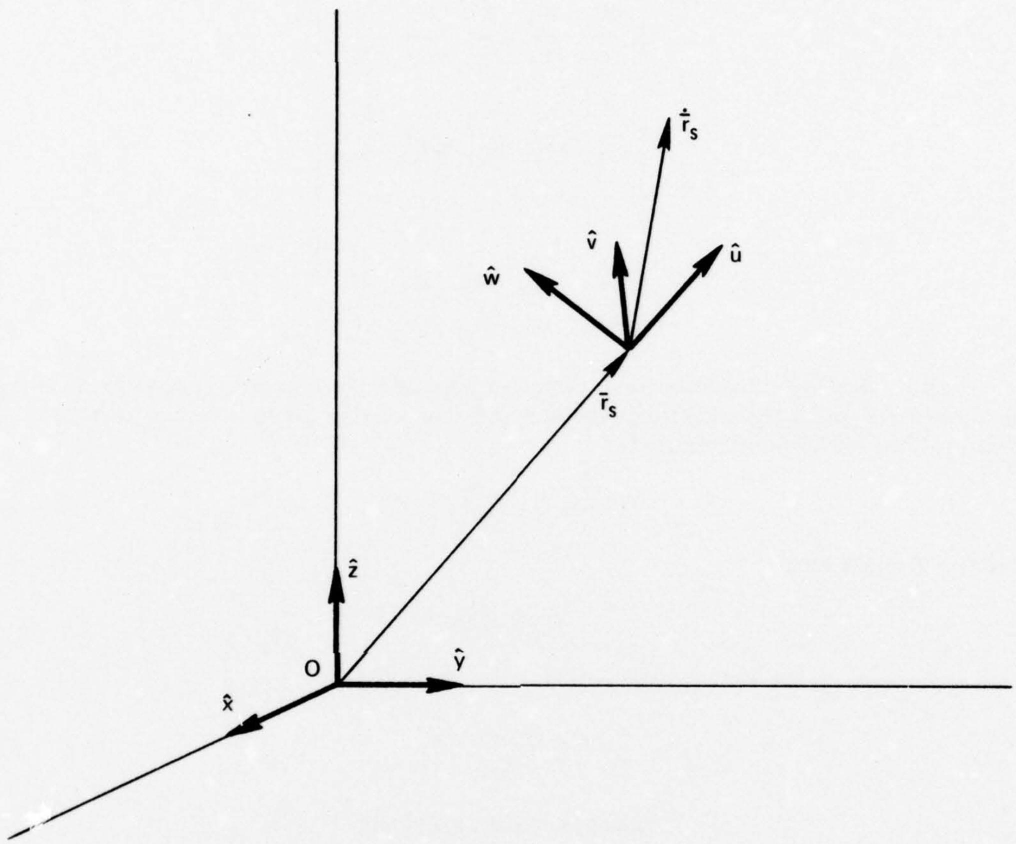


Figure 4. Radial, Along Track, and Cross Track Unit Vectors

This produces the normal equations denoted by the square matrix $B = A^T WA$. The matrix product on the right-hand side is $E = A^T WD$. The above expression is therefore simplified to

$$BX = E \tag{21}$$

If a solution for X were attempted at this point, the result would be indeterminate. The geometry involved in a single pass will not permit a three-dimensional solution for station position. However, a two-dimensional solution is possible and can be obtained from the present formulation by rotation to another coordinate system. The coordinate frame to choose is one which will isolate the geometric ambiguity to a single coordinate. Such a system is denoted by $\hat{u}$, $\hat{v}$, $\hat{w}$ (radial, along track, cross track), illustrated in Figure 4. The cross track direction, $\hat{w}$, is the geometrically weak coordinate, which can be deweighted, forcing the least-squares procedure to ignore it as a possible degree of freedom. The transformation matrix R, evaluated at the time of minimum slant range, operates on the matrices in Equation (21) changing the solution vector X to Y, where

$$Y = |\delta u \; \delta v \; \delta w \; \delta C_R \; \delta f_b|^T.$$

The matrix R is derived as follows. The vector to the NAVSAT $\bar{r}_s$ is as given in Equation (11), while the velocity vector is $\dot{\bar{r}}_s$

$$\dot{\bar{r}}_s = \dot{x}_s\hat{x} + \dot{y}_s\hat{y} + \dot{z}_s\hat{z},$$

and

$$r_s = (x_s^2 + y_s^2 + z_s^2)^{1/2}.$$

The three unit vectors defining the new coordinate system are

$$\hat{u} = \bar{r}_s \div r_s \qquad \text{radial,}$$

$$\hat{w} = (\bar{r}_s x \dot{\bar{r}}_s) \div |\bar{r}_s x \dot{\bar{r}}_s| \qquad \text{cross track, and}$$

$$\hat{v} = \hat{w} x \hat{u} \qquad \text{along track}$$

These definitions show that $\hat{w}$ is perpendicular to the plane containing the radial and velocity components of the satellite.

The rotation matrix R which transforms any vector X(x, y, z) into a vector Y(u, v, w) by the equation Y = RX consists of the following elements:

$$R_{11} = \frac{x_s}{r_s}, \quad R_{12} = \frac{y_s}{r_s}, \quad R_{13} = \frac{z_s}{r_s}$$

$$R_{21} = [(z_s\dot{x}_s - x_s\dot{z}_s)z_s - (x_s\dot{y}_s - y_s\dot{x}_s)y_s] \div |(\bar{r}_s x \dot{\bar{r}}_s) x \bar{r}_s|$$

$$R_{22} = [(x_s\dot{y}_s - y_s\dot{x}_s)x_s - (y_s\dot{z}_s - z_s\dot{y}_s)z_s] \div |(\bar{r}_s x \dot{\bar{r}}_s) x \bar{r}_s|$$

$$R_{23} = [(y_s\dot{z}_s - z_s\dot{y}_s)y_s - (z_s\dot{x}_s - x_s\dot{z}_s)x_s] \div |(\bar{r}_s x \dot{\bar{r}}_s) x \bar{r}_s|$$

$$R_{31} = (y_s\dot{z}_s - z_s\dot{y}_s) \div |\bar{r}_s x \dot{\bar{r}}_s|$$

$$R_{32} = (z_s\dot{x}_s - x_s\dot{z}_s) \div |\bar{r}_s x \dot{\bar{r}}_s|$$

$$R_{33} = (x_s\dot{y}_s - y_s\dot{x}_s) \div |\bar{r}_s x \dot{\bar{r}}_s|.$$

The parameters for tropospheric refraction and frequency bias are not altered by the rotation, thus the full rotation matrix is

$$R = \begin{vmatrix} R_{11} & R_{12} & R_{13} & 0 & 0 \\ R_{21} & R_{22} & R_{23} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

After rotation, a matrix P containing $n_p \lesssim 5$ a-priori variances is added to appropriately initialize the rotated covariance matrix by deweighting the out-of-plane coordinate w. The rotation is performed by pre- and postmultiplying the covariance matrix by R and $R^{-1}$, and premultiplying the E matrix by R.

$$RBR^{-1}Y = RE$$

Now P can be added to the rotated covariance matrix resulting in

$$(RBR^{-1} + P)Y = RE$$

The solution vector Y can be freed by premultiplying both sides by $(RBR^{-1} + P)^{-1}$ leaving

$$Y = (RBR^{-1} + P)^{-1}\ RE.$$

In terms of A and W this is

$$Y = (RA^TWAR^{-1} + P)^{-1}\ RA^TWD \tag{22}$$

## EDITING PROCEDURE

The observations belonging to each pass are edited independently of all other passes, first by absolute tolerance tests and then by evaluation of the residuals after solution of Equation (22). The adjusted D matrix, or residual matrix is then calculated to establish the noise level. If any element of $D_{adj}$ exceeds the established tolerance, that observation is eliminated from the next iteration of the solution. The parameters which are used for the editing decisions are the following:

- $\sigma^2_{obs}$ – The observation variance which appears in the W matrix. The value will change with each edit cycle.
- $\sigma^2_t$ – The initial value of $\sigma^2_{obs}$ as it appears on the observation file.
- $\eta_m$ – The multiplication factor used to change $\sigma_{obs}$ from one edit cycle to the next. Initially $\eta_m = 1$ and consequently $\sigma_{obs} = \sigma_t$.
- $\eta_t$ – A tolerance factor indicating the maximum deviation allowed before an observation is rejected. Initially $\eta_t >> 1$.
- $z_a$ – The zenith angle tolerance. Any observations obtained at zenith angles $> z_a$ are rejected.
- $\eta_a$ – If any elements of the D matrix exceed $\eta_a$, they are rejected.

One edit cycle (which is concurrent with a solution cycle) begins by comparing each element of D with $\eta_a$ and each zenith angle with $z_a$. If $z > z_a$ or if $|D_i| > \eta_a$, the observation is eliminated from further consideration. The number of observations remaining in a pass after these tests is $n_0$. The steps which follow are numbered for convenience.

1. If $|D_i| > \eta_t \eta_m \sigma_t$, the observation is eliminated, and the number of observations remaining is decremented by one.

2. If no observations are rejected in Step 1 of this cycle, the processing of this pass is completed. If this is the first cycle through the editing procedure, continue on to Step 3.

3. Recalculate $\eta_t$ using the empirical form

$$\eta_t = \left[\frac{n_0}{n_0 + 50} + 2\right]\left(\frac{n_0}{n_r}\right).$$

4. Form the matrix (Equation (20)), using only the observations remaining and continue through the manipulations to Equation (22).

5. Calculate the parameter V using the following formula

$$V = \sum_{i=1}^{n_r} \frac{D_i^2}{\eta_m^2 \sigma_t^2},$$

then use V to calculate the signal to noise

$$\left(\frac{S}{N}\right)^2 = \frac{V - E^T X}{n_0 + n_p - 5}$$

Finally recalculate $\eta_m$ using $\eta_m = (S/N)\eta_m$ where the $\eta_m$ on the right is the old value, and $\eta_m$ on the left is the new value.

6. Compute the adjusted D matrix, $D_{adj}$, where

$$D_{adj} = D - AX$$

Since AX is the linear approximation to D (Equation (20)), the difference of these should ideally be zero. However, the noise in $O_i$ will prevent a null result. What is desired is that the elements of the matrix $D_{adj}$ be zero mean, white Gaussian noise. The variance of this noise is the quantity of interest and establishes the performance of the NAVPAC hardware.

7. The last step is a return to Step 1 to begin the next edit cycle. In all cycles other than the first, the $D_{adj}$ matrix is used in place of the D matrix in Steps 1 through 5.

## EDITING RATIONALE

The editing procedure outlined above has accreted partly empirically and partly theoretically over several years of satellite processing at NSWC/DL. The present form adaptively adjusts itself to the peculiarities of each pass. The adjustments are applied by the parameters $\eta_t$ and $\eta_m$, which are recalculated in each edit cycle.

The product $(\eta_m \sigma_t)^2$ is the observation variance $\sigma_{obs}^2$. At first $\eta_m = 1$, which makes the observation variance just the initial estimate $\sigma_t^2$. Each succeeding edit cycle should see $\eta_m$ decrease, thus restricting the number of observations which pass Step 1. The parameter $\eta_t$ multiplies $\sigma_{obs}$ by some factor greater than two, and thus allows observations to pass Step 1 if they deviate less than $\eta_t \sigma_{obs}$ from the mean. The calculation of $\eta_t$ in Step 3 depends upon the number of observations, $n_0$, in the pass, and the number remaining at the current edit cycle step $n_r$. A NAVSAT pass as seen from the ground will rarely contain 30 observations; thus when $n_r = n_0$ the value of $\eta_t \lesssim 2.375$. As observations are eliminated, $\eta_t$ will increase slightly.

The calculation of $\eta_m$ measures how well the present solution has performed as compared with the previous. This is expressed by the signal to noise in Step 5. Contrary to most definitions of signal to noise, the object here is to make the ratio unity. The signal in this case represents dynamics which have not been removed by the least-squares model. If the model truly represents the observations, then only white Gaussian noise can be left.

Following O'Toole[1], the value for the multiplying factor $\eta_m$ is adjusted so that the signal to noise squared, defined by

$$\left(\frac{S}{N}\right)^2 = \frac{1}{n}\left[\frac{(O - C')^T(O - C')}{(\eta_m \sigma_t)^2}\right],$$

is approximately unity. In the above, n is the sum $n_0 + n_p - 5$, and $C'$ is defined below.

To show that this form of $(S/N)^2$ is the same as that given in the preceding section, it is necessary to reintroduce the computed vector C, the observation vector O, plus the weight matrix W. A new computed vector $C'$, containing the computed values after the solution, is defined by

$$C' = C + AX$$

An expanded form of equation (21) is also needed

$$BX = E = A^T W(O - C), \tag{23}$$

along with the variance V rewritten in matrix form

$$V = \frac{1}{(\eta_m \sigma_t)^2}(O - C)^T(O - C).$$

It is assumed that $\sigma_t$ is the same for all observations.

In order to procede, the quantity $(O - C')^T(O - C')$ must be expanded by substituting for $C'$.

$$(O - C')^T(O - C') = (O - C)^T(O - C) - (O - C)^T AX - (AX)^T(O - C) + (AX)^T AX$$

Dividing the above by $(\eta_m \sigma_t)^2$ allows the weight matrix to be introduced. For simplicity it would be defined as $W = (1/(\eta_m \sigma_t)^2)I$.

$$\frac{(O - C')^T(O - C')}{(\eta_m \sigma_t)^2} = \frac{(O - C)^T(O - C)}{(\eta_m \sigma_t)^2} - 2(AX)^T W(O - C) + X^T A^T WAX$$

Substituting for the variance and using (23) gives

$$n\left(\frac{S}{N}\right)^2 = V - 2X^T E + X^T E,$$

which can be simplified to the desired form:

$$\left(\frac{S}{N}\right)^2 = \frac{1}{n}[V - E^T X]$$

The signal to noise is used to scale the multiplying parameter $\eta_m$, which operates on $\sigma_t$ to set $\sigma_{obs}$ in Step 1 of the editing cycle. After several cycles, this feedback allows the number of observations rejected to reach a steady state and makes the tolerance testin·· less sensitive to the value chosen for $\sigma_t$.

## EXAMPLES OF ADJUSTED RESIDUALS

Examples of adjusted residuals are shown in Figures 5 through 16. Adjusted D matrices obtained by processing real data are shown in Figures 5, 8, and 11. For comparison, Figure 14 is computer-generated zero mean Gaussian white noise with unit standard deviation. Figure 15 illustrates a histogram generated from this sample of white noise with the mean and standard deviation calculated from the histogram. The dashed line is a Guassian function having the same mean, standard deviation, and maximum as the histogram. Figure 16 is the autocorrelation function calculated from this same sample. If the computer-generated noise were perfectly white, there would be no correlation and the autocorrelation function would consist of an impulse at zero lag. Figures 5 through 13 are plots similar to those described above but obtained from the single pass NAVPAC solution and editing procedure outlined here.

An interesting feature of the autocorrelation functions is that they are distinctly different. Figure 7 shows two definite secondary correlations at lag steps 9 and 16. Figure 10 does not distinguish itself with obvious correlations but is more like the white noise result than the other two. The function plotted as Figure 13 is intermediate between the first two showing possible regions of correlation both positive and negative.

The different autocorrelation results illustrated seem to indicate time variable phenomena influencing the data acquisition portion of NAVPAC. Since all three cases were processed in the same way using the same model, any variation in the results must be due to either propagation effects or peculiar time variable correlations introduced by the NAVPAC electronics. These autocorrelation functions are a quick visual guide to the quality of the individual pass. Strong correlations indicate a poor fit of the observed data to the model; weak or rapid variations indicate a good fit.

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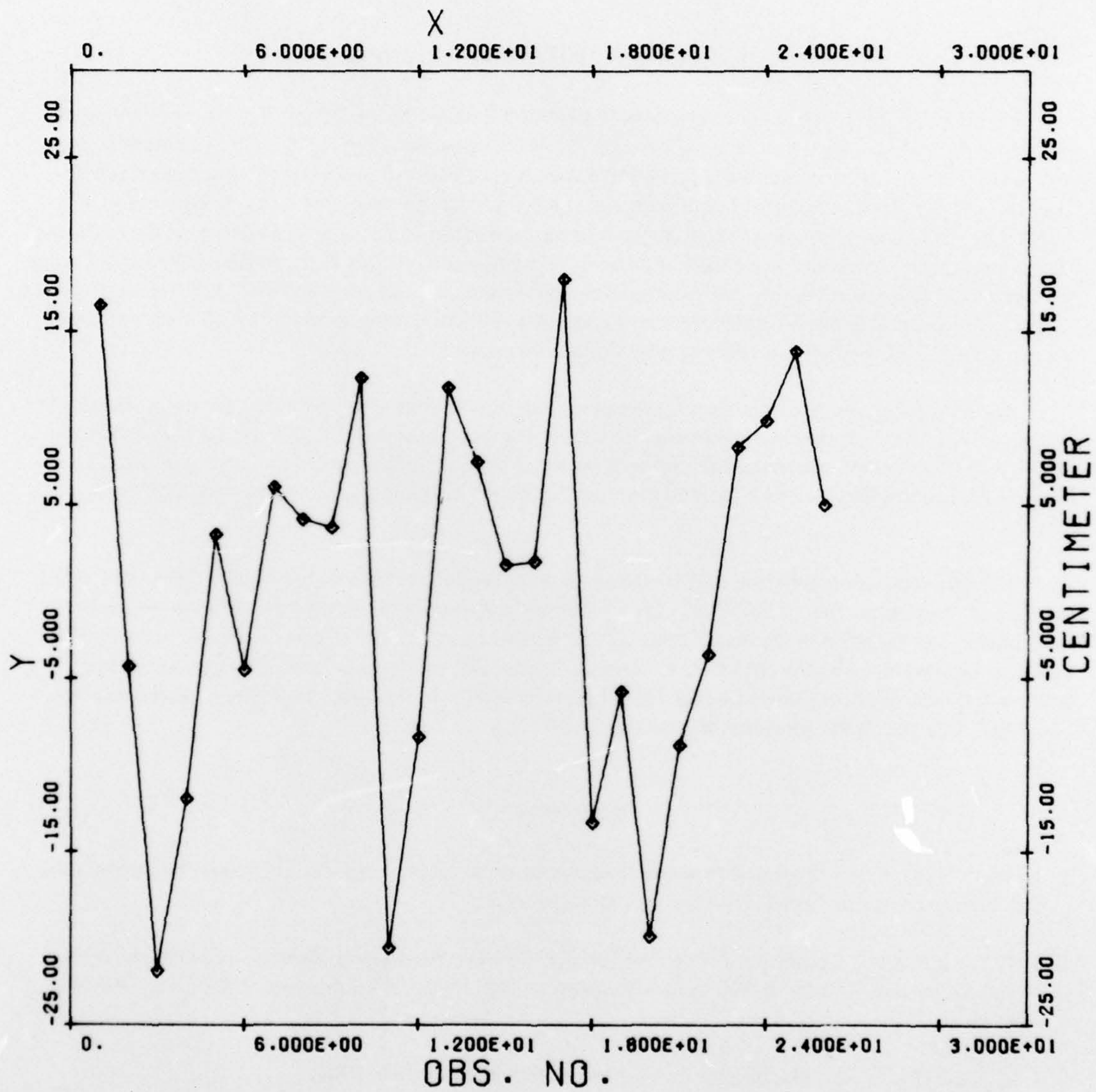
3. J. M. Kelso, *Radio Ray Propagation In the Ionosphere.* McGraw-Hill, 1964.

4. H. S. Hopfield, *A Two-Quartic Refractivity Profile for the Troposphere, for Correcting Satellite Data,* The John Hopkins University, Applied Physics Laboratory. 1969.




AVERAGE = 0.0000015 CENTIMETERS
ROOT MEAN SQUARE = 10.80 CENTIMETERS

Figure 5. Adjusted (Observed-Computed) Values (APL 11, Pass 36)

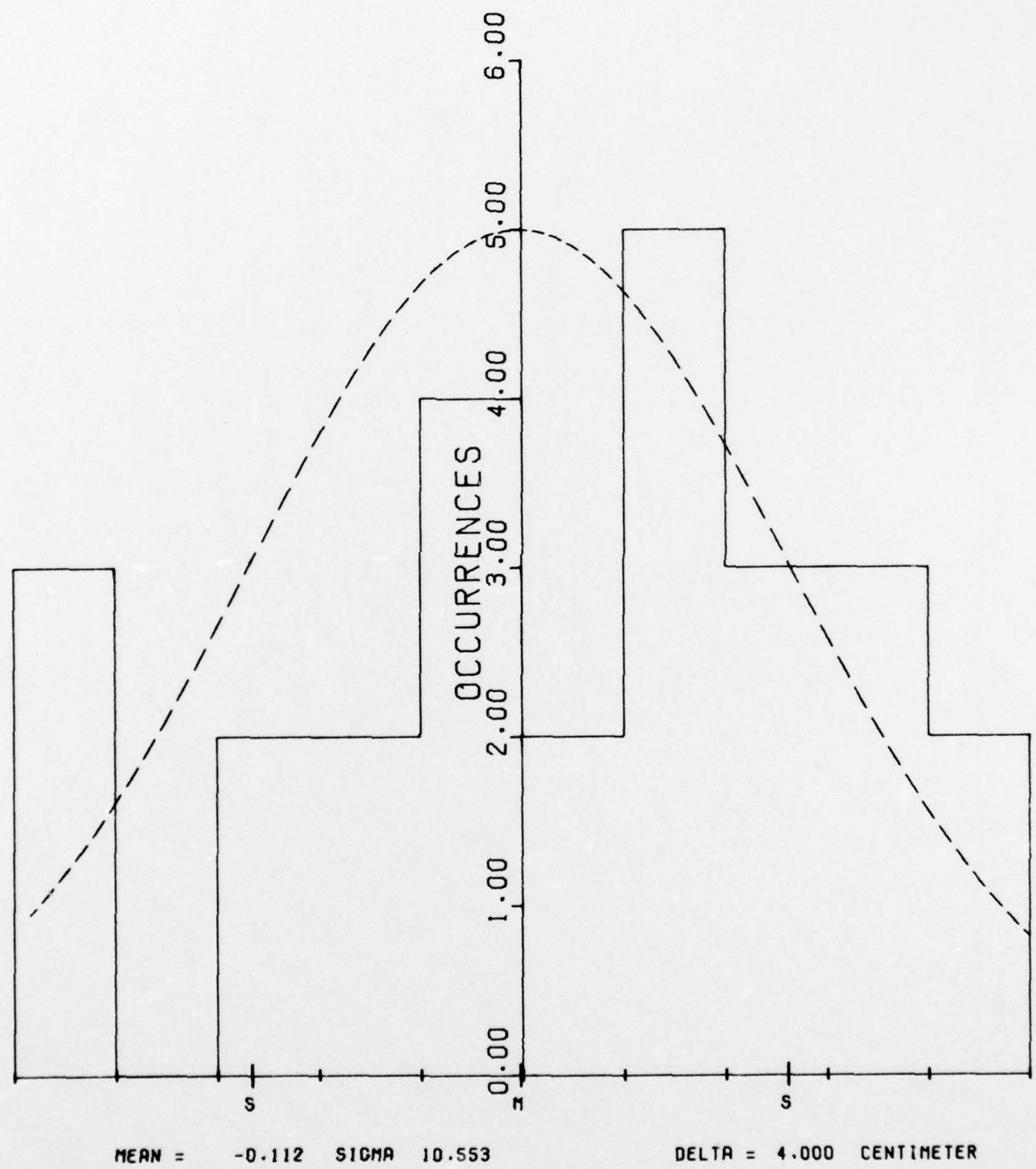



Figure 6. Histogram of Adjusted (Observed-Computed) Values (APL 11, Pass 36)

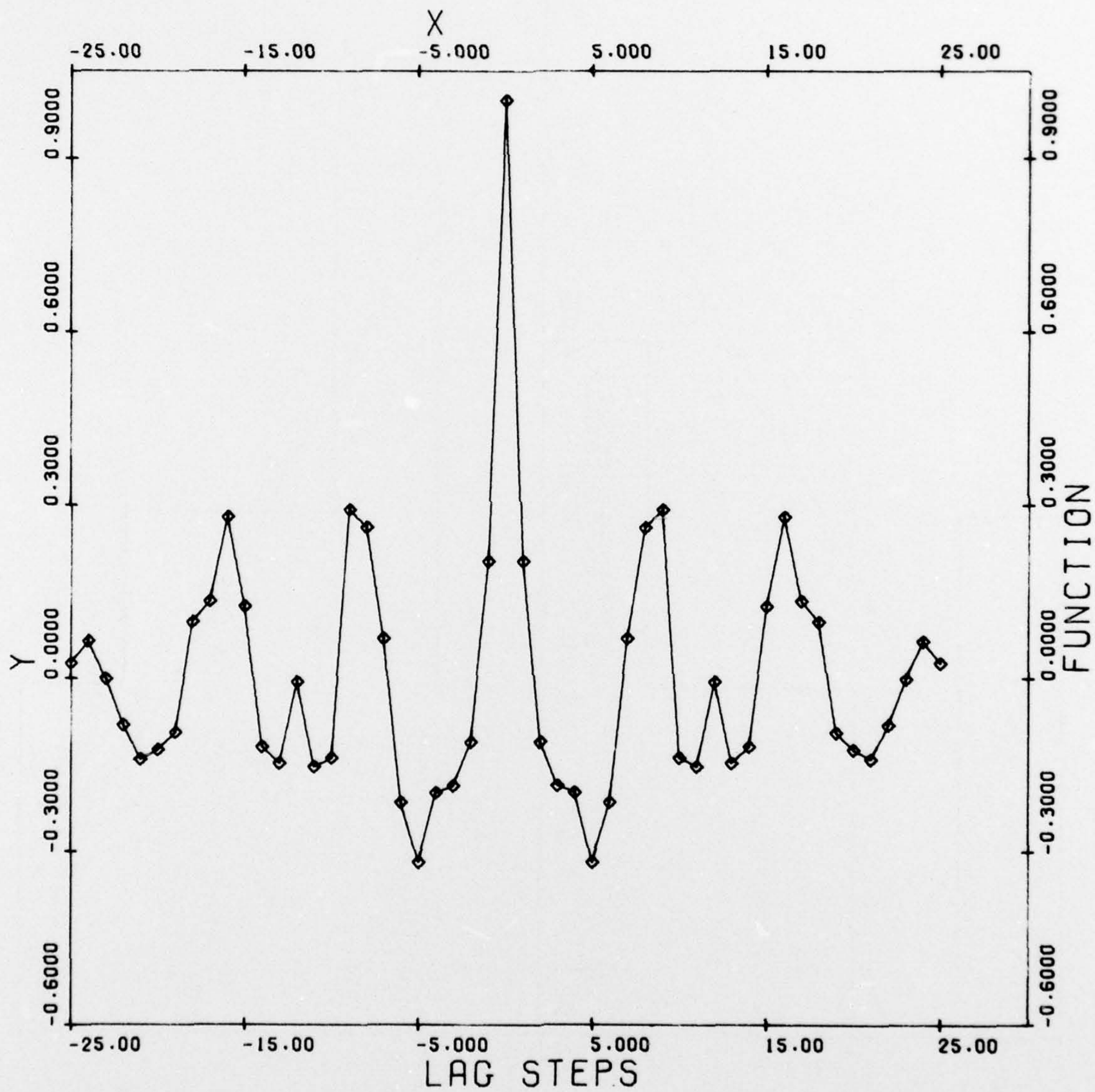



Figure 7. Autocorrelation Function of Adjusted (Observed-Computed) Values (APL 11, Pass 36)

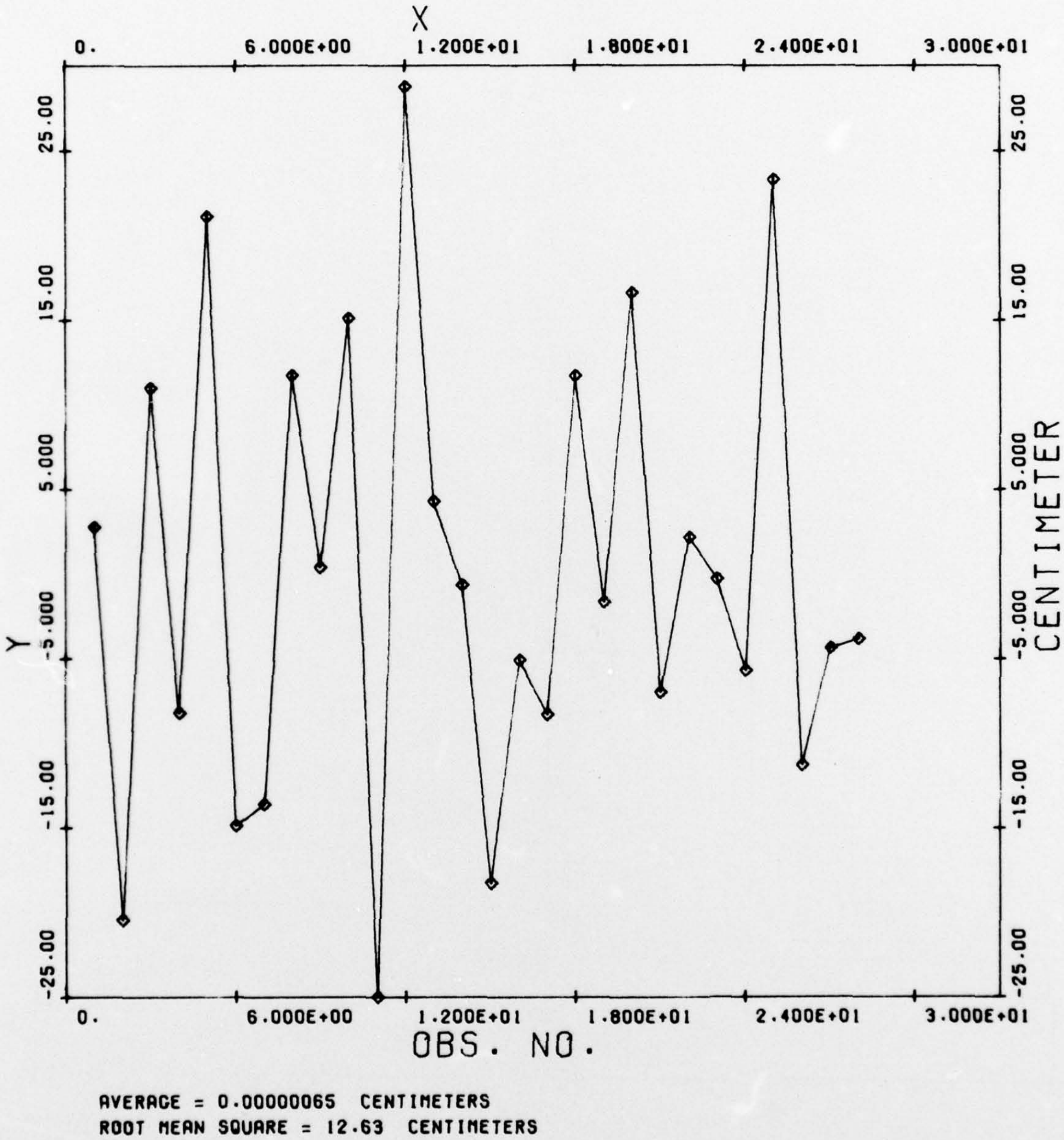



Figure 8. Adjusted (Observed-Computed) Values (APL 15, Pass 26)

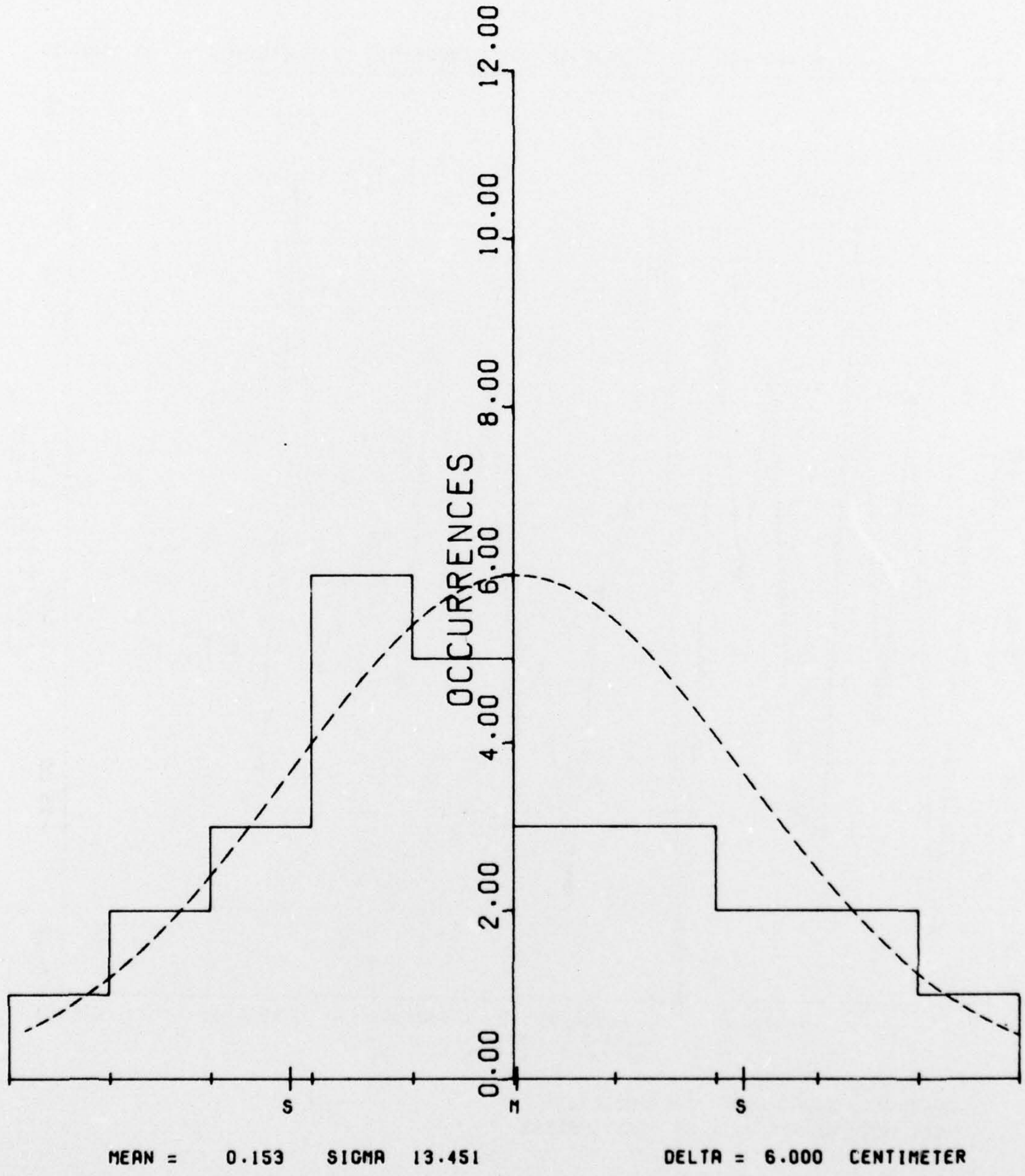



Figure 9. Histogram of Adjusted (Observed-Computed) Values (APL 15, Pass 26)

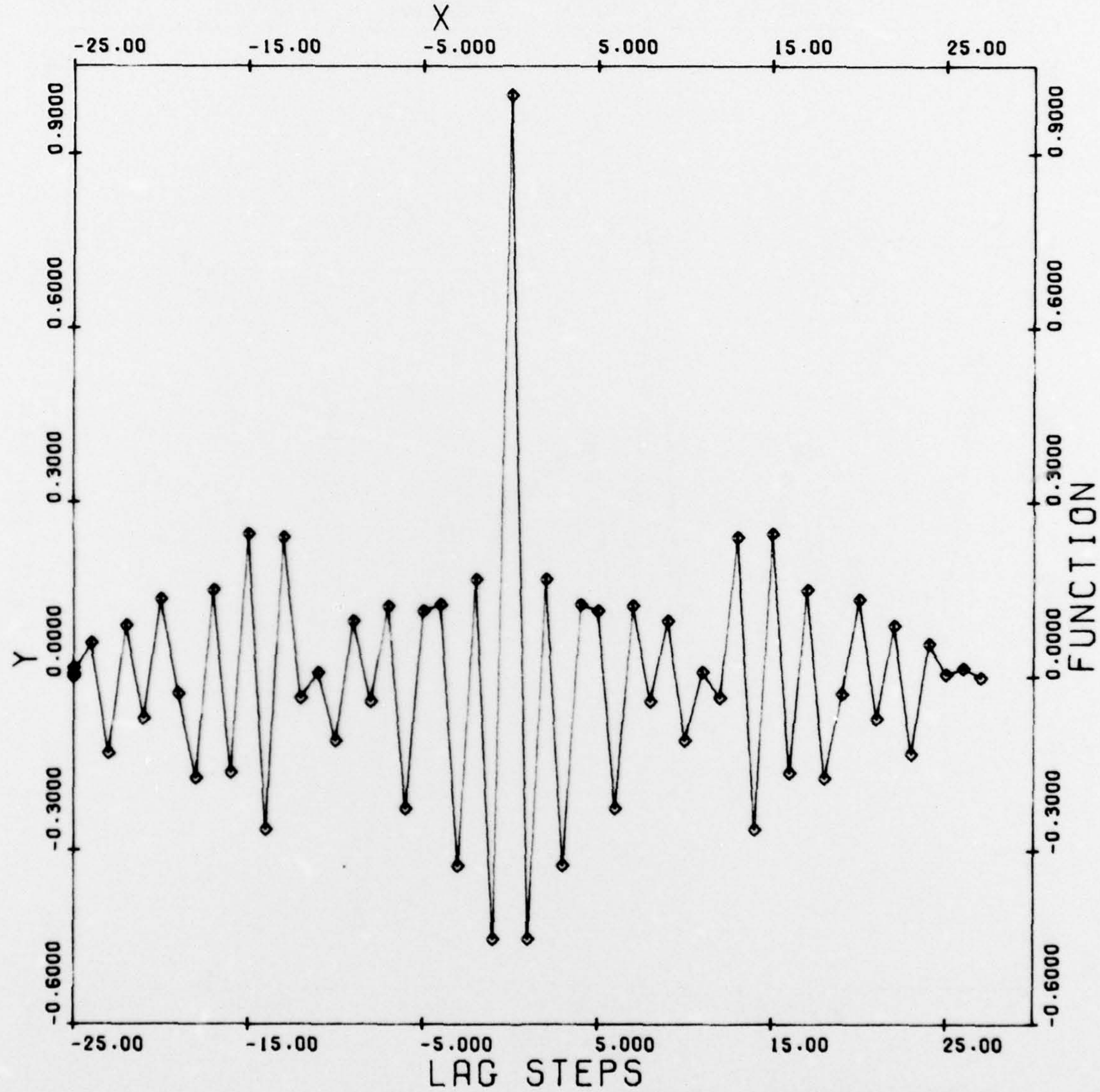



Figure 10. Autocorrelation Function of Adjusted (Observed-Computed) Values (APL 15, Pass 26)

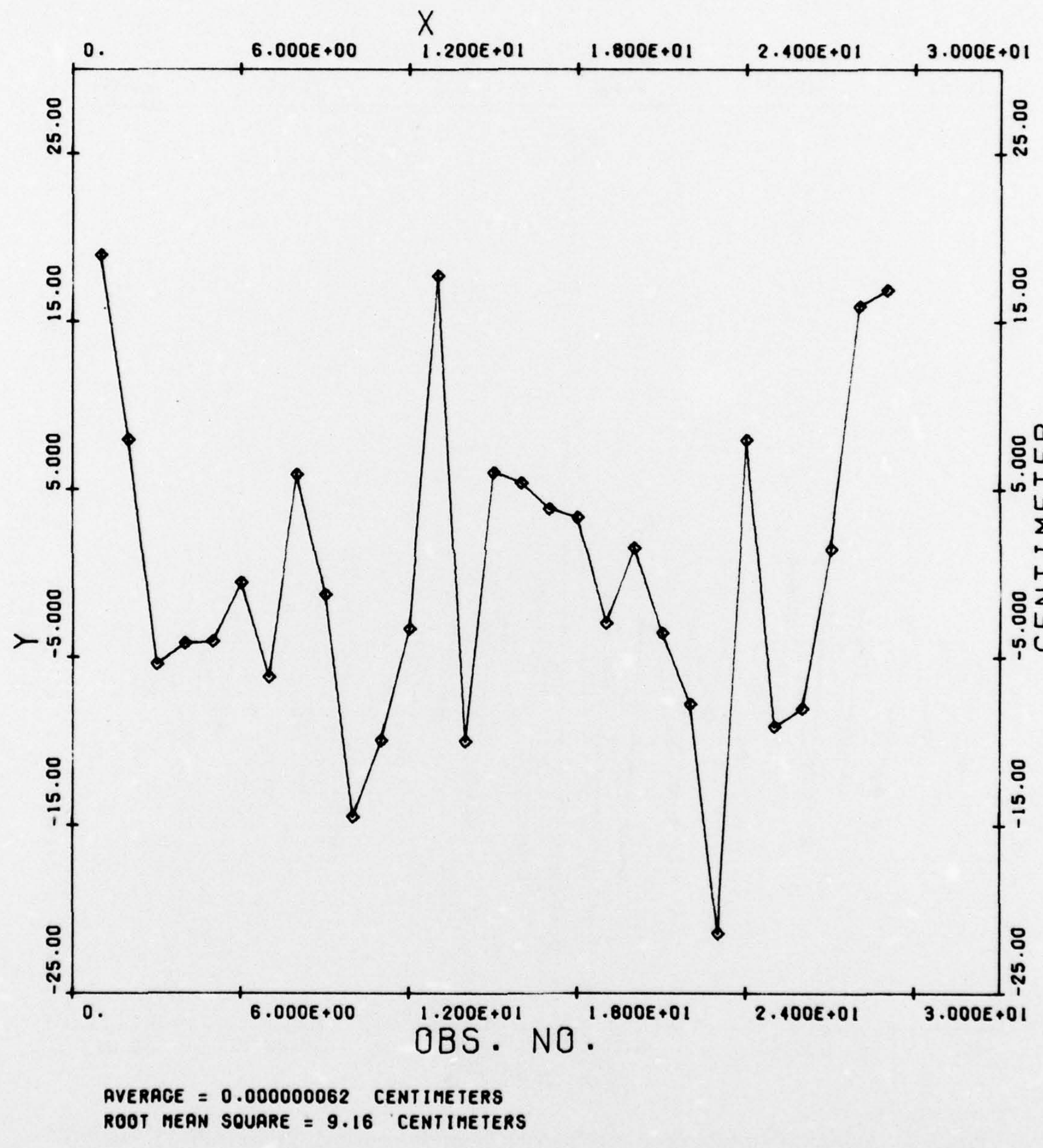



Figure 11. Adjusted (Observed-Computed) Values (APL 16, Pass 12)

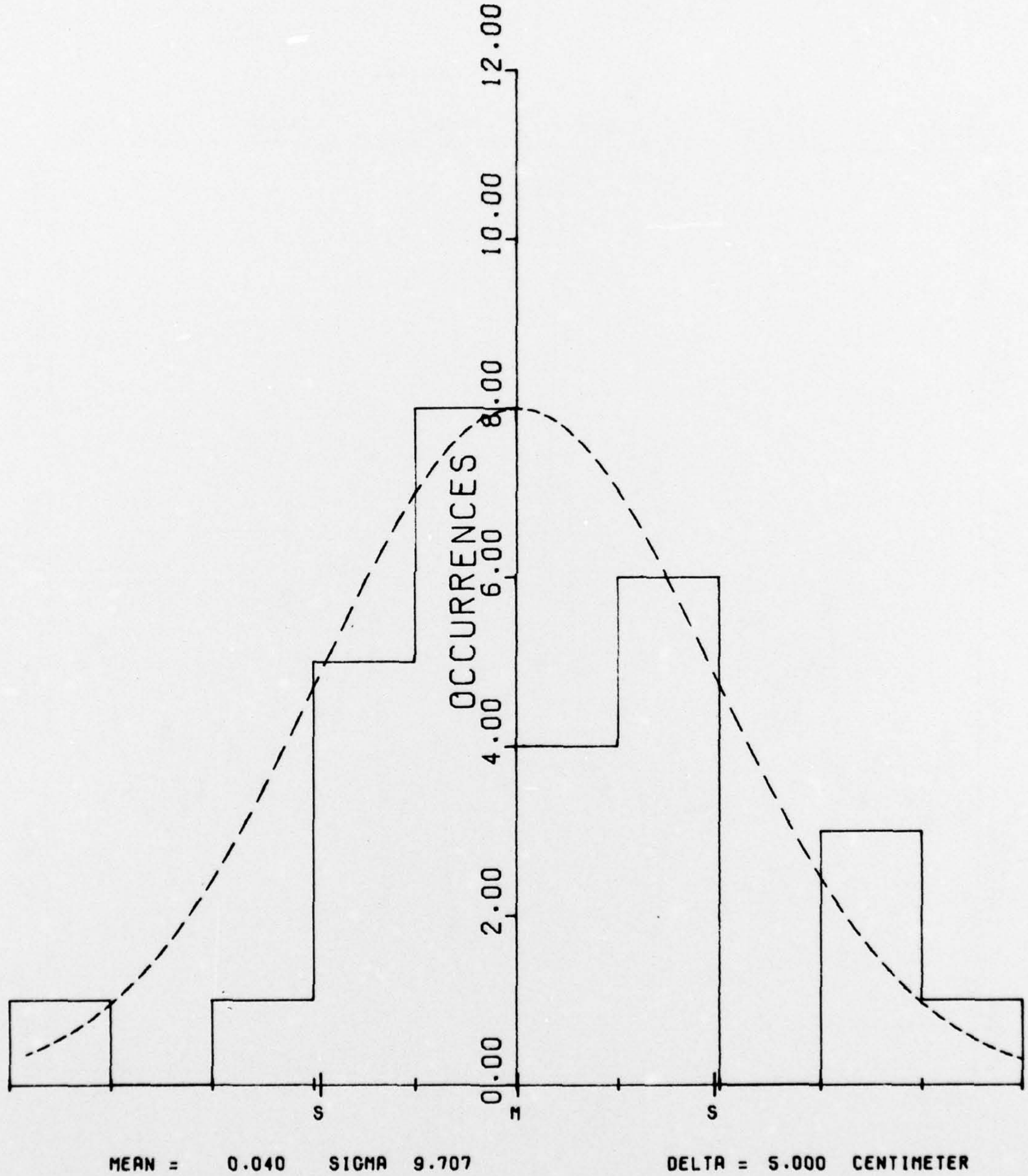



Figure 12. Histogram of Adjusted (Observed-Computed) Values (APL 16, Pass 12)

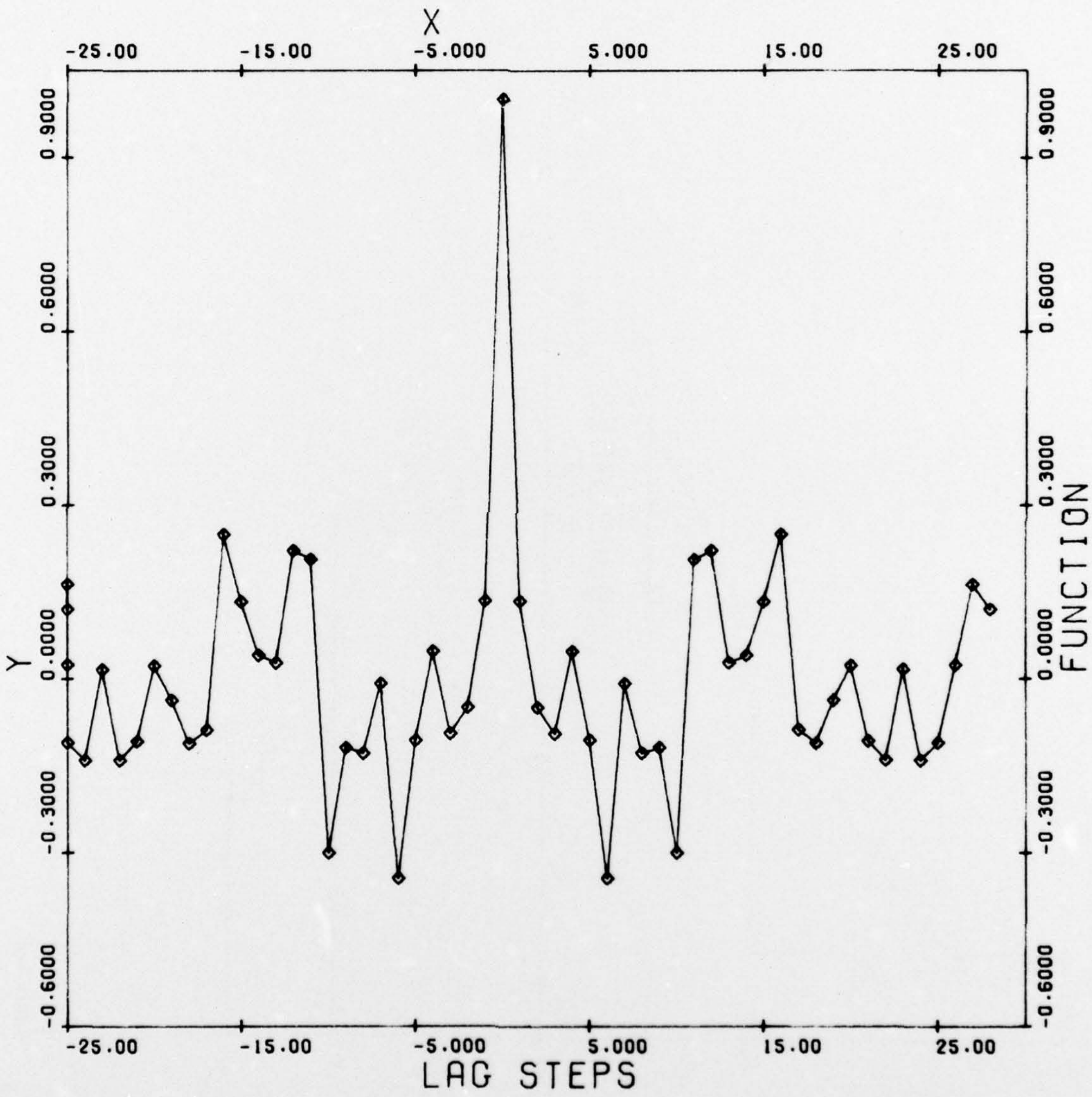



Figure 13. Autocorrelation Function of Adjusted (Observed-Computed) Values (APL 16, Pass 12)

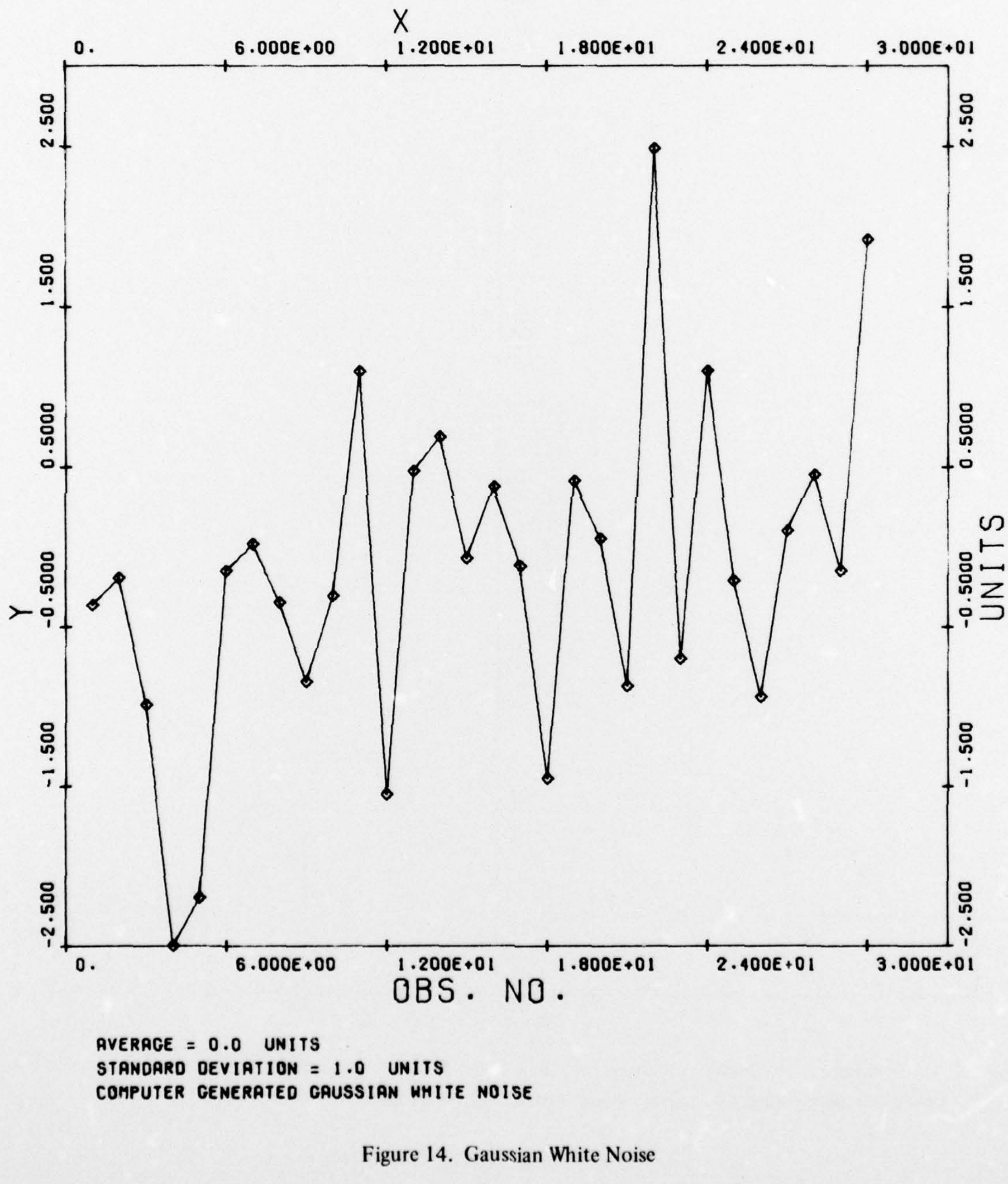



Figure 14. Gaussian White Noise

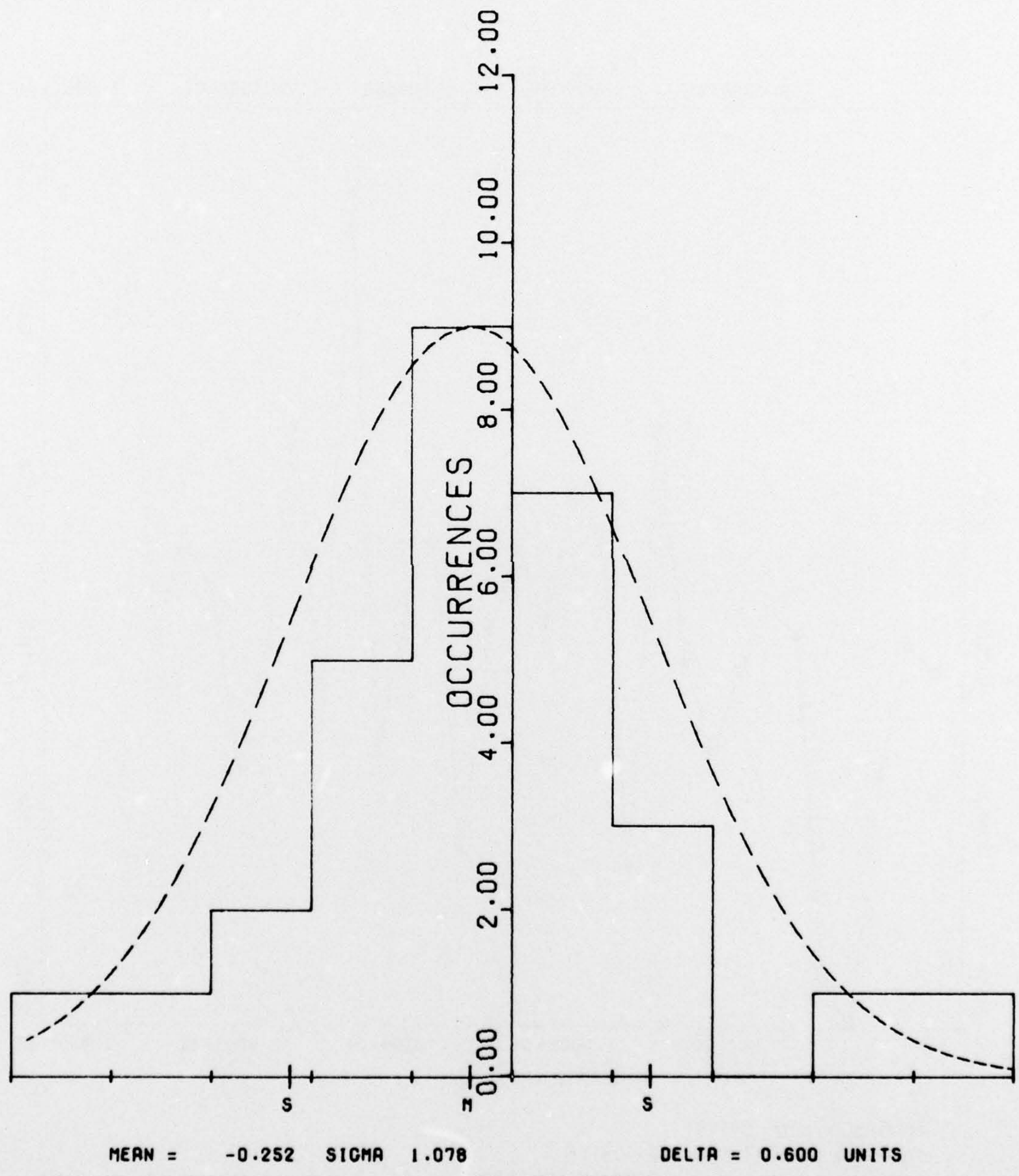



Figure 15. Histogram of White Noise

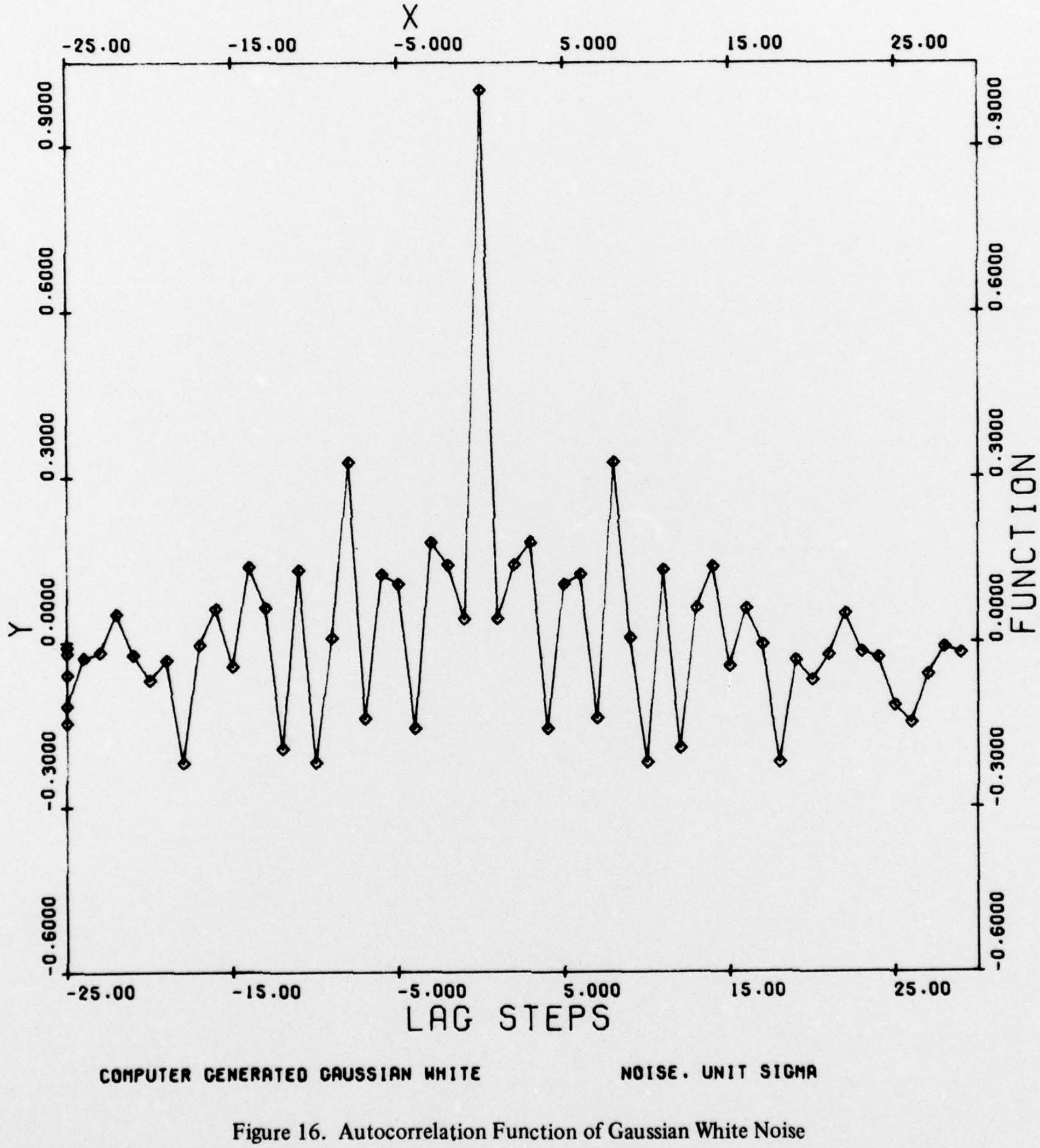



Figure 16. Autocorrelation Function of Gaussian White Noise