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HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS. A MATHEMATICAL MODEL--ETC(U)
MAR 78 R LUGANNANI, H C BOOKER, L E HOFF N00123-76-C-1090
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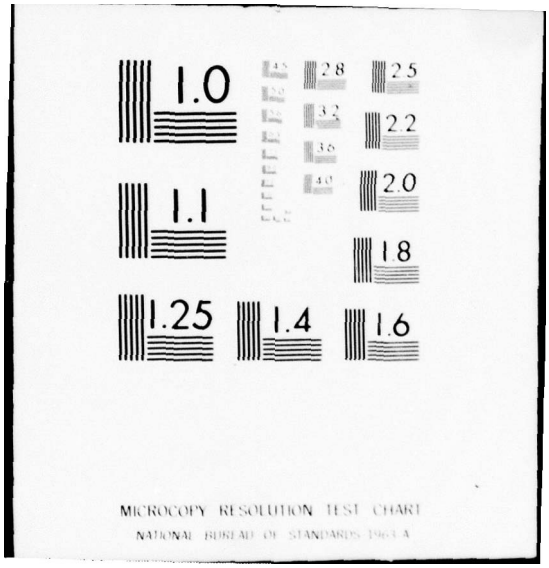
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Technical Report 208

HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS

A Mathematical Model and Computer Program for 100-kHz Bandwidth HF Channels

R Lugannani and HG Booker
LE Hoff (Contract Monitor)
31 March 1978

Final Report: February 1977 - October 1977

Prepared for
Naval Electronic Systems Command

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AN ACTIVITY OF THE NAVAL MATERIAL COMMAND

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ADMINISTRATIVE INFORMATION

Work was performed under contract N000123-76-C-1090/SEI-C-020 by Systems Exploration, Inc. for Intra-Task Force/SS Com Branch, Communications Systems and Technology Department, NOSC.

This work was performed as part of NAVELEX's Skywave Communications Task, a 6.2 Block-Funded Program. This report covers work from February 1977 to October 1977 and was approved for publication January 1978.

Released by
CA NELSON, Head
Surface/Shore Systems Division

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Communications Systems and
Technology Department

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NOSC TR-208	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): HF Channel Simulator for Wideband Signals. A Mathematical Model and Computer Program for 100-kHz Bandwidth HF Channels.	5. TYPE OF REPORT & PERIOD COVERED: Final Report • February [illegible] - October 1977
6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s): R/LUGANNANI and HC/BOOKER; LE/Hoff (Contract Monitor)	8. CONTRACT OR GRANT NUMBER(s): N000123-76-C-1090 new; SEI-C-020
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Systems Exploration, Inc. 3687 Voltaire St. San Diego, CA 92106	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62721N; F21222; XF212220 91 (B194)
11. CONTROLLING OFFICE NAME AND ADDRESS: Naval Electronic Systems Command, Code 3105, Washington, DC 20360	12. REPORT DATE: 31 March 1978
	13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*: NAVAL OCEAN SYSTEMS CENTER, San Diego, CA 92152	15. SECURITY CLASS. *(of this report)*: UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

High frequency	FORTRAN
Simulators	Broadband
Broadband Model	Channels

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A simulator has been developed that is capable of modeling the wideband HF channel. It is designed for carrier frequencies in the 2-32 MHz range and it can accommodate bandwidths up to 100 kHz and ground ranges up to 1000 km. It models the groundwave and ionosphere returns, but does not include additive noise or signal distortion produced in the transmitter or receiver. The inputs are the basic physical quantities that characterize the ionosphere (height, thickness, and maximum electron density of the E and F regions, sunspot number, time of day, etc) and those quantities that characterize the transmitter-receiver environment (carrier frequency, range, geographic location, antenna patterns, etc). Explicit values for channel parameters such as delay, attenuation, and doppler

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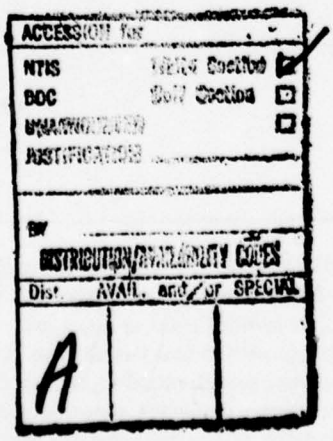
20. shift and spread are not required as these are calculated internally.

The simulator is divided into two parts. The first part is concerned with the mathematical model of the ionosphere and the computation of the channel parameters from the basic inputs. These channel parameters are used in the second part to determine the response for arbitrary inputs. A software version of the simulator has been implemented and tested, and a Fortran listing of the program is appended. The program runs in non-real time and requires a computer with a Fortran IV compiler and at least 28 K of core.



OBJECTIVE

Develop a mathematical model for the wideband HF channel and the software implementation of this model. The model must be valid for carrier frequencies in the 2-32 MHz range, for bandwidths up to 100 kHz, and for ground ranges up to 1000 km.

RESULTS

A simplified model of the ionosphere is employed to determine the various skywave returns and the effect these returns have on the overall received signal. The groundwave is modeled as a delayed, attenuated version of the transmitted signal. Inputs to the simulator consist of those basic physical quantities that characterize the ionosphere and those that characterize the transmitter-receiver environment; values of delay, attenuation, doppler shift and spread, and dispersion are calculated internally for each component of the received signal. A FORTRAN version of the simulator has been written and tested for several typical channels.

RECOMMENDATIONS

Make further refinements in the mathematical model in order to expand the scope and accuracy of the simulator. Of particular value in this regard would be the inclusion of a deviative absorption term for the skywave returns and the inclusion of an attenuation term for propagation of the groundwave over a rough, lossy ocean surface.

Investigate the relationship between the simulator inputs and the actual observed returns as measured, for example, by an ionosonde. A knowledge of this relationship would provide further verification of the mathematical model as well as a set of inputs that correspond to known operating conditions.

Finally, in view of the meager data base available in the wideband case, any measurement program aimed at enlarging this data base would represent a significant contribution to the study and modeling of HF channels.

GLOSSARY

English Letter Symbols

a	Multiplicative constant used to generate random sequence $\{Y_n\}$
$a_j(n)$	Impulse response of filter used to generate approximate random gain $\tilde{G}_j(nT_o)$
A	Multiplicative constant for doppler shift
A_g	Gain of groundwave
A_j	RMS gain of j-th ionosphere path
$A(x)$	$= 2\ G \tan \theta$
B	Multiplicative constant for doppler spread
B_r	Bearing of receiver as seen from transmitter in magnetic coordinates
BW	Two-sided bandwidth of transmitted signal
c	Velocity of light
$C_{G_j}(\tau)$	Correlation function of $G_j(t)$
$C_{\tilde{G}_j}(\tau)$	Correlation function of $\tilde{G}_j(t)$
$C_{U_j}(\tau)$	Correlation function of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$
D	Virtual path length
D_g	Delay of groundwave
$D_j^{(o)}$	$= f_c \cdot \tau_j^{(o)}$
$D_j^{(k)}$	$= \frac{1}{k!} [k\tau_j^{(k-1)} + f_c \tau_j^{(k)}]$, $k = 1,2,\ldots$
f	Frequency
f_c	Carrier frequency

f_r	Doppler reference frequency
f_{HD}	Gyrofrequency of D region
f_{HE}	Gyrofrequency of E region
f_{HF}	Gyrofrequency of F region
f_{pE}	General expression for penetration frequency of E region (ordinary or extraordinary wave)
f_{pF}	General expression for penetration frequency of F region (ordinary or extraordinary wave)
$f_{pE}^{(o)}$	Penetration frequency of E region - ordinary wave
$f_{pE}^{(x)}$	Penetration frequency of E region - extraordinary wave
$f_{pF}^{(o)}$	Penetration frequency of F region - ordinary wave
$f_{pF}^{(x)}$	Penetration frequency of F region - extraordinary wave
$G = G(f)$	Group height of signal path, oblique incidence
$G_j(t)$	Random gain of j-th path
$\hat{G}_j(t)$	Approximation for random gain of j-th path
$G_R(\theta)$	Gain of receiving antenna
$G_T(\theta)$	Gain of transmitting antenna
$G_v = G_v(f)$	Group height of signal path, vertical incidence
h	Height above earth
h_D	Height of peak electron density of D region
h_E	Height of peak electron density of E region

GLOSSARY (Continued)

h_F	Height of peak electron density of F region
h_r	Height at which refractive index equals zero at vertical incidence
$h_j(t)$	Distortion impulse response of j-th path
$\hat{h}_j(t)$	Modified distortion impulse response: $=T_o h_j(t)$
$2H_E$	Semithickness of E region electron density
$2H_F$	Semithickness of F region electron density
$H_j(f)$	Fourier transform of $h_j(t)$
I	Angle of inclination of earth's magnetic field
I(x)	Cannonical expression representing general impulse response $\hat{h}_j(t)$
$I_p(x)$	Integrals used in series representation of I(x)
k	Sunspot number multiplier for non-deviative absorption loss
K	Multiplicative constant for non-deviative absorption loss
K_j	Constant used in the generation of $\hat{G}_j(nT_o)$
L_{abs}	Absorption loss
L_{ant}	Antenna pattern loss
L_d	Spreading loss
L_j	Total loss of j-th path
M	Length of pseudo random sequence
N=N(h)	Electron density
N_E	Maximum electron density of E region

GLOSSARY (Continued)

N_F	Maximum electron density of F region
N_g	Integer multiple of T_o nearest the groundwave delay D_g
N_j	Integer multiple of T_o nearest the signal delay D_j
$P_{G_j}(f)$	Power spectral density of $G_j(t)$
$P_{\tilde{G}_j}(f)$	Power spectral density of $\tilde{G}_j(t)$
$P_{U_j}(f)$	Power spectral density of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$
$r(t)$	Received, complex, baseband signal
R	Distance between transmitter and receiver
$s(t)$	Transmitted, complex, baseband signal
$\tilde{s}_j(t)$	Distorted, complex signal returned from ionosphere via j-th path
$\hat{s}_j(t)$	Distorted, complex signal returned from ionosphere via j-th path after removal of phase and delay components: $\tilde{s}_j(t) = \exp[-2\pi i D_j^{(o)}]\ \hat{s}_j(t-D_j^{(1)})$
S	Sunspot number
$S(f)$	Fourier transform of $s(t)$
t	Time
T_o	Sampling interval (one half the Nyquist interval of the transmitted signal)
u_1, u_2	Uniform [0,1] random variables
$U_j^{(1)}(t), U_j^{(2)}(t)$	Real, independent, zero mean identically-distributed, Gaussian process used to define $G_j(t)$
$W_j(1), W_j(2), W_j(3)$	Independent, complex, Gaussian random variables used in generation of $\tilde{G}_j(nT_o)$
x	$= \cos\theta$
$\{Y_n\}$	Sequence of pseudo random integers

GLOSSARY (Continued)

Greek Letter Symbols

α	Doppler shift exponent
β	Doppler spread exponent
γ	Exponent of solar zenith angle term in expression for nondeviative absorption
$\gamma_j(1)$, $\gamma_j(2)$	Constants used in the generation of $\hat{G}_j(nT_o)$
$\Delta^{(2)}$	General amplitude distortion coefficient
$\Delta^{(3)}$	General phase distortion coefficient
$\Delta_j^{(2)}$	Normalized amplitude distortion coefficient of j-th path
$\Delta_j^{(3)}$	Normalized phase distortion coefficient of j-th path
η	Normalized frequency variable = f/BW
θ	Ray angle of signal path measured from vertical
κ	$= (e^2/4\pi^2\varepsilon_o m) \approx 80.5$
λ	Geographic longitude
λ_o	Geographic longitude of north magnetic pole
$\lambda_j(1)$, $\lambda_j(2)$	Constants used in the generation of $\tilde{G}_j(nT_o)$
μ	Refractive index
ν	General doppler shift
ν_j	Doppler shift of j-th path
ν_r	Reference doppler shift

GLOSSARY (Continued)

ξ	Complex Gaussian random variable
$\xi_j(n)$	Independent, identically-distributed Gaussian sequence
$\xi_j^{(1)}(n)$	Real part of $\xi_j(n)$
$\xi_j^{(2)}(n)$	Imaginary part of $\xi_j(n)$
ρ	Parameter used in fitting $C_{\tilde{G}_j}(\tau)$ to $C_{G_j}(\tau)$
2σ	General doppler spread
$2\sigma_j$	Doppler spread of j-th path
$2\sigma_r$	Reference doppler spread
τ	General path delay
$\tau_j(f)$	Delay of j-th path as a function of frequency
$\tau_j^{(k)}$	k-th derivative of delay of j-th path evaluated at the carrier frequency f_c
ϕ	Geographic latitude
ϕ_o	Geographic latitude of north magnetic pole
Φ	Magnetic latitude
χ	Solar zenith angle
ψ_i	Angle between earth's magnetic field and wave incident upon ionosphere
ψ_r	Angle between earth's magnetic field and wave reflected from ionosphere

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I. INTRODUCTION

The use of channel simulators to study the performance of communication systems and to assist in their design has become increasingly popular in recent years. By employing a simulator, it is possible to reproduce given channel characteristics as desired and to compare different system designs in identical environments. Moreover, the cost of simulation is usually much less than the cost of "on the air" measurements and it is quite reliable. A variety of HF channel simulators have been proposed and studied in the past [1-4]*. Sailors and Hill [5] have written an excellent survey of the field and have made several recommendations for future work. Some of these recommendations are embodied in the present report.

In the following the development of a simulator for the wideband HF channel is described. This simulator is designed for carrier frequencies in the 2-32 MHz range and it can accommodate bandwidths up to 100 kHz and ground ranges up to 1000 km. The simulator models the groundwave and all ionosphere returns (both single and multiple hops), but does not include additive noise or signal distortion produced in the transmitter or receiver. It takes into account all ionospheric phenomena that have a significant effect on wideband communications. Many of these phenomena, such as delay differences between the ordinary and extraordinary waves and signal distortion caused by variations of delay with frequency, have been neglected in the earlier simulators, resulting in severe bandwidth limitations. The inputs required are the basic parameters that characterize the physical ionosphere (height, thickness, and maximum electron density of the E and F regions, sunspot number, time of day, etc.) and those that characterize the transmitter and receiver (carrier frequency, range, geographic location, antenna patterns, etc.). Since these input parameters are the ones that are actually known in practice it is a simple matter to vary them and observe the effect they have on system performance. This

*References are listed in Section VI.

capability does not exist in the previous simulators which require as inputs not the basic quantities mentioned above but those channel parameters such as delay, attenuation, doppler shift and doppler spread derived from them.

In the course of developing the simulator a number of simplifications and approximations have been made to obtain tractable results. The most important of these assumptions concerns the electron density profile and the absorption mechanism. It is recognized that the ionosphere model used does not provide a detailed physical characterization of the ionosphere; however, it does provide an "order of magnitude" characterization suitable for communication purposes. The simulator is divided into two parts. The first part is concerned with the mathematical model of the ionosphere and the computation of the channel parameters from the basic inputs. The second part uses these parameters to determine the channel response for arbitrary inputs. Both parts of the simulator are described in detail in the following sections.

II. THE HF CHANNEL

The signal received via the HF channels considered in this report is the sum of the groundwave and the various ionosphere returns. Throughout we shall assume that the groundwave is an attenuated, delayed version of the transmitted signal but is otherwise undistorted. The ionosphere returns also suffer attenuation and delay but in addition are subject to fading and distortion. There is also time dispersion (multipath) produced by the different delays associated with each ionosphere return. The purpose of this section is to present some definitions that are used in the sequel and to introduce the mathematical model for the HF channel.

The complex transmitted baseband signal will be denoted by $s(t)$ and its Fourier transform by $S(f)$. Thus

$$s(t) = \int_{-\infty}^{\infty} S(f) \exp [2\pi i f t] df. \tag{1}$$

Distorted versions of this signal will be returned from the ionosphere for both the E region and the F region (low ray and high ray). There will be a set of these returns for the ordinary and extraordinary magnetoionic components and for as many multiple-hop paths as are present. Although many returns are possible, all but a few of these experience a large attenuation and the number of effective returns is usually small. In the following we will denote the number of effective ionosphere returns by N. This number is determined by excluding all returns that suffer an attenuation greater than some predetermined value.

Distortion of signals reflected from the ionosphere has been studied by several authors, who have attempted to characterize the impulse response for a simplified model of the ionosphere [6-10]. Unfortunately, these results have been derived using ionosphere

models that are too idealized for our purposes[1] and the impulse responses obtained are too complicated for easy implementation[2]. In order to obtain tractable expressions for the distortion we shall assume that it is caused primarily by the fact that the delay is a function of frequency and is not constant across the signal bandwidth. This characterization of the distortion mechanism allows us to make a number of approximations which result in relatively simple expressions for the distorted signal. For obvious reasons we shall refer to this particular distortion as delay distortion.

Let f_c denote the carrier frequency and let $\tau_j(f)$ denote the delay of the j-th return as a function of frequency. It follows from (1) that the distorted baseband signal returned from the ionosphere is given by

$$\tilde{s}_j(t) = \int_{-\infty}^{\infty} S(f) \exp\,[-2\pi i(f_c+f)\cdot\tau_j(f_c+f)]\cdot\exp\,[2\pi ift]\,df. \qquad (2)$$

$$j = 1,\ldots,N$$

We will characterize this distortion by the coefficients in the power series expansion of the delay about the carrier frequency. This series appears as

$$\tau_j(f_c+f) = \sum_{k=0}^{\infty} \frac{f^k}{k!}\,\tau_j^{(k)}, \quad j=1,\ldots,N \qquad (3)$$

---

[1] They allow for reflection only from a single ionospheric region and consider such simple electron density profiles as a constant and a linear function of height.

[2] In one case the impulse response is given as the solution of an integral equation.

with

$$\tau_j^{(k)} = \frac{d^k}{df^k} \tau_j(f_c+f)\Big|_{f=0} . \tag{4}$$

Substituting this series into the exponential delay term in (2) we obtain

$$\exp[-2\pi i (f_c+f)\cdot\tau_j(f_c+f)] = \exp\left[-2\pi i \sum_{k=0}^{\infty} f^k D_j^{(k)}\right] \qquad j=1,\cdots,N \tag{5}$$

where we have defined

$$D_j^{(o)} = f_c \tau_j^{(o)} \qquad j=1,\cdots N \tag{6}$$

and

$$D_j^{(k)} = \frac{1}{k!}\left[k\tau_j^{(k-1)} + f_c\tau_j^{(k)}\right] \qquad \begin{matrix} k=1,2,\cdots \\ j=1,\cdots N. \end{matrix} \tag{7}$$

Thus, for $\tilde{s}_j(t)$ we have

$$\tilde{s}_j(t) = \exp\left[-2\pi i D_j^{(o)}\right] \int_{-\infty}^{\infty} S(f) \exp\left[-2\pi i \sum_{k=2}^{\infty} f^k D_j^{(k)}\right]$$

$$\cdot \exp\left[2\pi i f (t-D_j^{(1)})\right] df. \qquad j=1,\cdots,N \tag{8}$$

For computational reasons it is convenient to separate the phase and delay terms from the actual distortion of the signal. To this end we define a function $\hat{s}_j(t)$ as follows

$$\tilde{s}_j(t) = \exp[-2\pi i D_j^{(o)}]\ \hat{s}_j(t-D_j^{(1)}) \qquad j=1,\cdots N \tag{9}$$

where

$$\hat{s}_j(t) = \int_{-\infty}^{\infty} S(f)H_j(f)\exp[2\pi ift]dt \qquad (10)$$

$$j=1,\cdots,N$$

with

$$H_j(f) = \exp\left[-2\pi i \sum_{k=2}^{\infty} f^k D_j^{(k)}\right], \quad j=1,\cdots N. \qquad (11)$$

In practice it is convenient to determine $\hat{s}_j(t)$ by performing a time domain convolution rather than by evaluating the Fourier transform (10). If the impulse response determined by $H_j(f)$ is denoted by $h_j(t)$, we have

$$h_j(t) = \int_{-\infty}^{\infty} H_j(f)\exp[2\pi ift]df, j=1,\cdots N., \qquad (12)$$

and the desired convolution appears as

$$\hat{s}_j(t) = \int_{-\infty}^{\infty} s(t-u)h_j(u)du, \quad j=1,\cdots N. \qquad (13)$$

A discussion of the approximations used in the numerical evaluation of $h_j(t)$ and $\hat{s}_j(t)$ is presented in Section IV.

In addition to the above distortion, the ionosphere returns are subject to attenuation and random fading. These two phenomena are included by multiplying the delayed, distorted signal by a random gain, which we denote by $G_j(t)$. We will assume that $G_j(t)$ is a complex, zero-mean, Gaussian process with identically-distributed real and imaginary parts. This is the Rayleigh fading model [11]. While the assumption of Rician fading [12] would provide somewhat more flexibility than the Rayleigh assumption, the available evidence [12-14] indicates that the Rayleigh model is adequate for our purposes. The random gains associated with different returns are assumed to be independent.

Let $U^{(1)}_j(t)$ and $U^{(2)}_j(t)$ denote two real, independent zero mean, identically-distributed, stationary Gaussian processes with correlation function

$$C_{U_j}(\tau) = E\, U^{(1)}_j(t+\tau)U^{(1)}_j(t) = E\, U^{(2)}_j(t+\tau)U^{(2)}_j(t). \qquad (14)$$

$$j=1,\cdots N.$$

The corresponding power spectral density is given by

$$P_{U_j}(f) = \int_{-\infty}^{\infty} C_{U_j}(\tau)\, \exp[-2\pi i f\tau]d\tau, j=1,\cdots,N. \qquad (15)$$

We define the random gain in terms of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$ as follows

$$G_j(t) = [U_j^{(1)}(t) + i\, U_j^{(2)}(t)]\cdot\exp[2\pi i\nu_j t] \qquad (16)$$

$$j=1,\cdots N.$$

where ν_j is the doppler shift for the j-th ionosphere return. The correlation and power spectral density of $G_j(t)$ are easily determined using the assumed properties of $U_j^{(1)}$ and $U_j^{(2)}(t)$. Thus

$$C_{G_j}(\tau) = EG_j(t+\tau)G_j^*(t) = 2C_{U_j}(\tau)\cdot\exp[2\pi i\nu_j\tau] \qquad (17)$$

$$j=1,\cdots N.$$

and

$$P_{G_j}(f) = 2P_{U_j}(f-\nu_j)\ ,\ j=1,\cdots N. \qquad (18)$$

Following Watterson [1] we assume that the power spectral density $P_{G_j}(f)$ has a Gaussian shape. While there is no conclusive experimental evidence for this assumption, it is a convenient model for spectra that are unimodal and that decrease rapidly. Specifically we let

$$C_{G_j}(\tau) = A_j^2 \exp[-2\pi^2\sigma_j^2\tau^2 + 2\pi i\nu_j\tau] \quad . \qquad (19)$$

$$j=1,\cdots N$$

Here A_j is the rms gain ($0 \leq A_j \leq 1$) of the j-th return and $2\sigma_j$ is the doppler spread of the return. The power spectral density associated with this correlation function is

$$P_{G_j}(f) = \frac{A_j^2}{\sqrt{2\pi}\,\sigma_j} \exp\left[-\frac{(f-\nu_j)^2}{2\sigma_j^2}\right] \quad , \; j=1,\cdots N. \qquad (20)$$

Finally, we must add the groundwave to the ionosphere returns. The groundwave is assumed to experience only delay and to have a non-random gain. If A_g denotes the gain ($0 \leq A_g \leq 1$) and D_g denotes the delay, the contribution to the received signal from the groundwave is given by $A_g\, s(t-D_g)$. The gain is a complicated function of range, sea state and frequency. Graphs of this functional relationship are given in [15].

Summarizing the above, the complex, baseband received signal can be expressed as

$$r(t) = A_g\, s(t-D_g) + \sum_{j=1}^{N} \exp[-2\pi i D_j^{(o)}]\; G_j(t)\; \hat{s}_j(t-D_j^{(1)}). \qquad (21)$$

A block diagram illustrating (21) is presented in Figure 1.

In the following sections, the quantities needed in the evaluation of r(t) are derived and the techniques employed in its numerical evaluation are described.

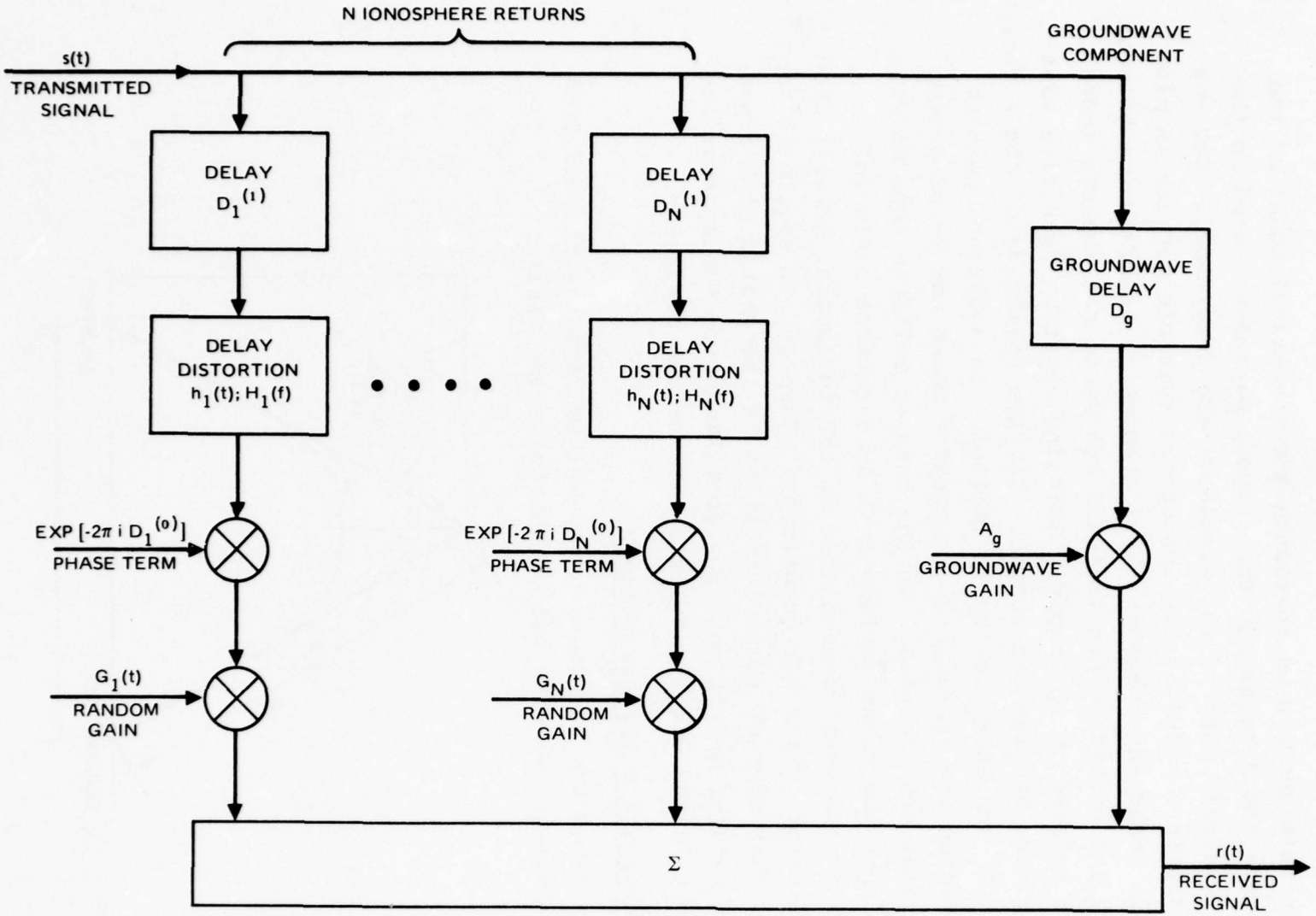

Figure 1. Block Diagram of Simulator

III. MATHEMATICAL MODEL OF THE IONOSPHERE

In this section we introduce a mathematical model of the ionosphere and discuss how the channel parameters used in the signal analysis part of the simulator are determined. Our goal is to obtain a model of the ionosphere suitable for use in mid-latitude ocean environments. Differences in transmission between daytime and nighttime are included but we do not include those effects related to the rapidly changing electron densities that occur during sunrise and sunset. Daytime splitting of the F region into an F_1 and an F_2 region is omitted. In addition, neither transmission via sporadic E or spread F modes nor M- or N-type reflections are included. In the case of multiple hops we will assume that the ocean surface acts as a perfect reflector.

As a notational convenience in the following, we will drop the subscript j from all quantities. However, it should be kept in mind that each of these quantities is associated with a particular return, and it must be identified with the appropriate subscript in the signal analysis part of the simulator.

A. Delay and Delay Derivatives

To determine the group delay of the signal reflected from the ionosphere, we make use of the following figure.

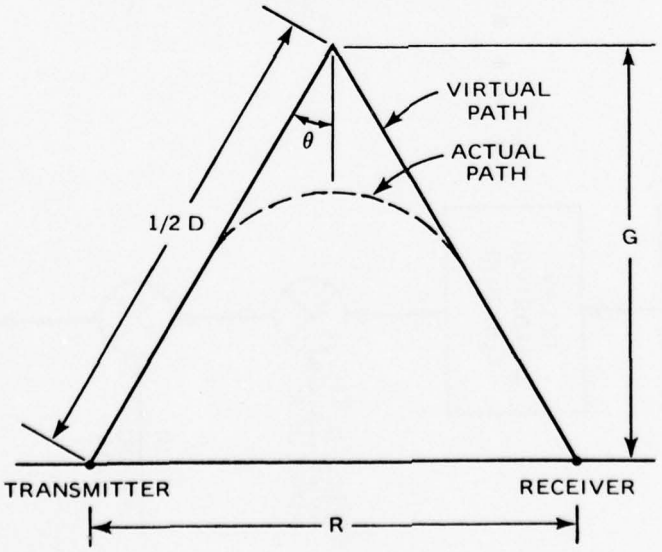

Figure 2. Virtual and Actual Signal Paths

Here the dashed line is the actual ionospheric path of the signal between the transmitter and the receiver and the solid line is the virtual path. The ground range between transmitter and receiver is denoted by R, the virtual path length by D, the ray angle measured from the vertical by θ and the group height by G. Assuming a flat earth and ionosphere, we have by simple geometry

$$D = \frac{R}{\sin\theta} = \frac{R}{(1-x^2)^{\frac{1}{2}}} \tag{22}$$

where we have defined $x = \cos\theta$. If c denotes the velocity of light, the signal delay is given by the virtual path length divided by c

$$\tau = \frac{R}{c(1-x^2)^{\frac{1}{2}}} . \tag{23}$$

It remains to determine the ray angle θ.

For vertical incidence ($\theta=0$), the group height is given by the integral [16]

$$G_v = G_v(f) = \int_o^{h_r} \frac{dh}{\mu} \tag{24}$$

where μ is the refractive index and h_r is the height at which $\mu=0$. In the absence of both collisions and the earth's magnetic field, the refractive index is given by [16]

$$\mu^2 = (1-\kappa \frac{N}{f^2}) \tag{25}$$

where $N = N(h)$ is the electron density in m^{-3}, f is the frequency in Hz and $\kappa = (e^2/4\pi^2\varepsilon_o m) \approx 80.5$. The group height for oblique incidence is the same as for vertical incidence but we must replace f by f cos θ [16]. That is $G = G(f) = G_v(f \cos\theta)$. Once the group height has been determined, we appeal to Figure 2 to obtain the relationship

$$R = 2\ G \tan\theta = 2\ G \left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} . \tag{26}$$

Defining

$$A(x) = 2\,G\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} \tag{27}$$

we see that, for a specified range R, we must solve the transcendental equation

$$R = A(x) \tag{28}$$

for x and then substitute this into (23) to obtain the delay.

We seek an approximation to the electron density for which the group height, G, can be expressed in a simple closed form. Our main concern here is with reflections from the E and F regions. At nighttime we assume that the electron density profile can be represented by two distinct parabolas as shown in Figure 3. Let N_E be the peak electron density of the E region in m^{-3}, let h_E be the height of the peak electron density of the E region in km, and let $2H_E$ be the semithickness of the E region in km; similar definitions are made for the F region. Then the nighttime electron density is given by

$$N(h) = \begin{cases} 0 & , \quad h < h_E - 2H_E \\ N_E\left[1-\left(\dfrac{h-h_E}{2H_E}\right)^2\right] & , \quad h_E - 2H_E \leq h \leq h_E + 2H_E \\ 0 & , \quad h_E + 2H_E < h < h_F - 2H_F \\ N_F\left[1-\left(\dfrac{h-h_F}{2H_F}\right)^2\right] & , \quad h_F - 2H_F \leq h \leq h_F + 2H_F \\ 0 & , \quad h_F + 2H_F < h \quad . \end{cases} \tag{29}$$

In the daytime the electron density profile is also assumed to be parabolic but allowance is made for ionization between the E and F regions. The electron density between the two regions is assumed to be constant and equal to the peak value of the E region. This is illustrated in Figure 4. Using the same definitions as before, the daytime electron density is given by

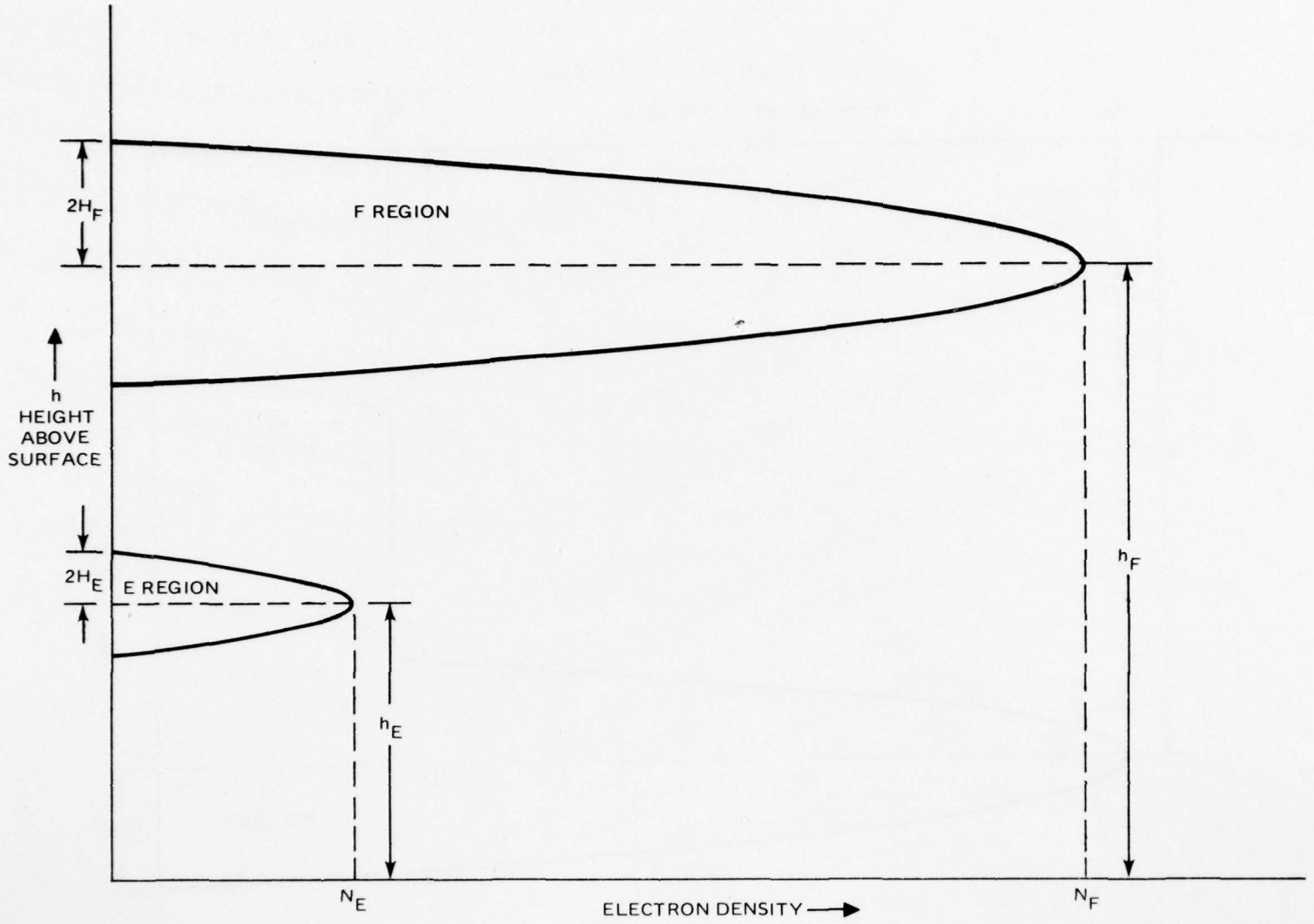

Figure 3. Nighttime Electron Density Profile

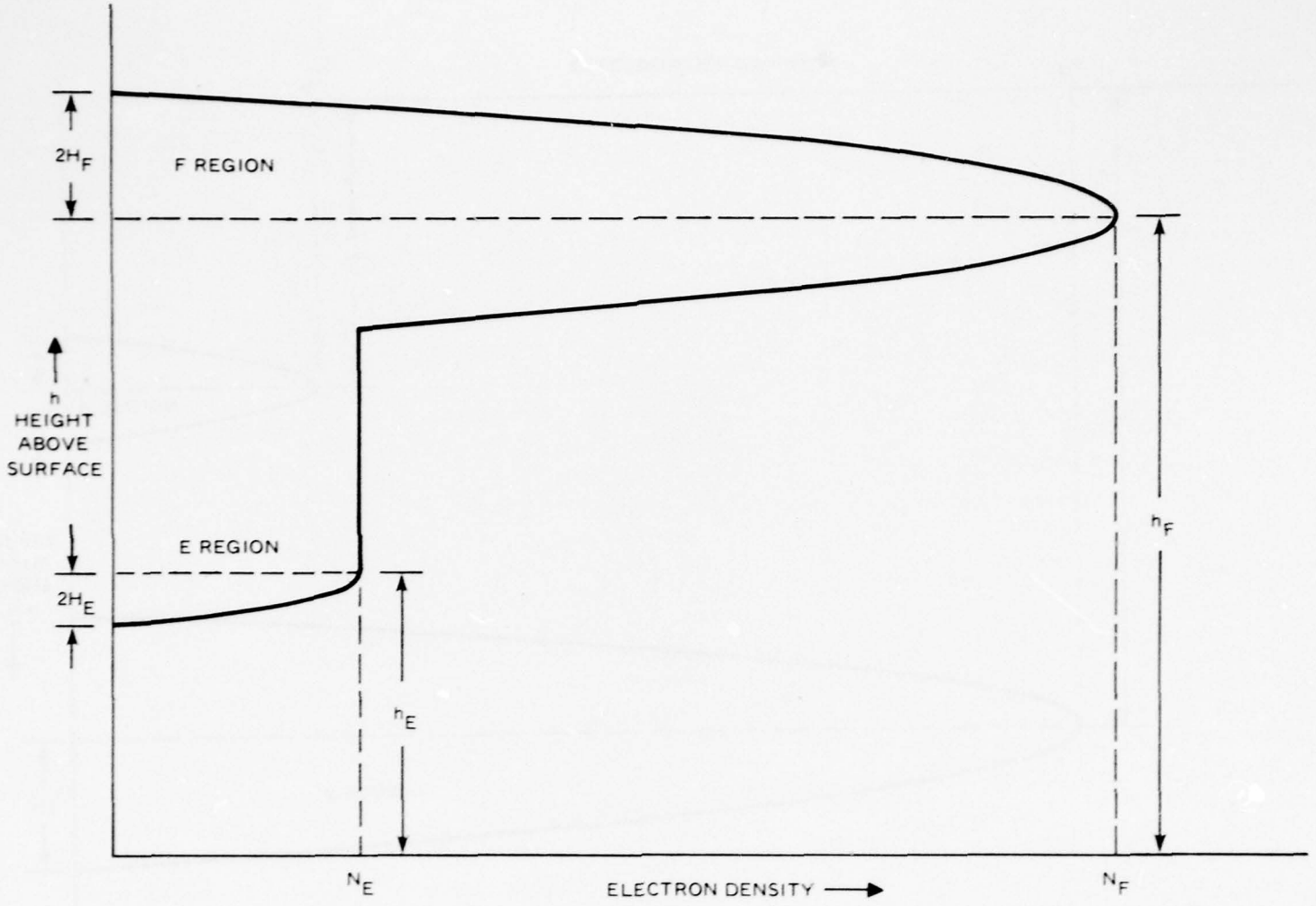

Figure 4. Daytime Electron Density Profile

$$N(h) = \begin{cases} 0, & h < h_E - 2H_E \\ N_E\left[1-\left(\frac{h-h_E}{2h_E}\right)^2\right], & h-2H_E \leq h \leq h_E \\ N_E, & h_E < h < h_F + 2H_F\left(1-\frac{N_E}{N_F}\right)^{1/2} \\ N_F\left[1-\left(\frac{h-h_F}{2h_F}\right)^2\right], & h_F+2H_F\left(1-\frac{N_E}{N_F}\right)^{1/2} \leq h \leq h_F+2H_F \\ 0, & h_F+2H_F < h. \end{cases} \tag{30}$$

Note that the constants appearing in (24) and (30) will be different owing to the different heights and ionization densities for nighttime and daytime conditions.

A quantity of interest in the present analysis is the penetration frequency of each region. This is defined as the plasma frequency at the peak electron density [16]. We have for the E and F regions

$$f_{pE}^2 = \kappa N_E \tag{31a}$$

$$f_{pF}^2 = \kappa N_F \tag{31b}$$

where $\kappa = (e^2/4\pi^2\varepsilon_o m) \approx 80.5$ and the two penetration frequencies are in Hz. The physical interpretation of f_{pE} and f_{pF} is that they represent the maximum frequencies at which signals will be reflected from the respective regions at vertical incidence.

Returning to the determination of the group height, G, it is a relatively simple matter to evaluate the integral (24) for either of the above electron densities [17]. The details of this evaluation are presented in Appendix A.

In the foregoing the effect of the earth's magnetic field has been neglected, but we take this to be an adequate approximation for the ordinary wave. For the extraordinary wave, however, the penetration frequencies are shifted relative to those for the ordinary wave, and it is essential to take this into account.

Denoting the penetration frequencies of the ordinary and extraordinary waves by the superscripts "o" and "x" we have [16]

$$f_{pE}^{(x)} = \frac{1}{2} f_{HE} + \left[f_{HE}^{2} + 4 \, (f_{pE}^{(o)})^{2} \right]^{\frac{1}{2}} \tag{32a}$$

$$f_{pF}^{(x)} = \frac{1}{2} f_{HF} + \left[f_{HF}^{2} + 4 \, (f_{pF}^{(o)})^{2} \right]^{\frac{1}{2}} . \tag{32b}$$

Here $f_{pE}^{(o)}$ and $f_{pF}^{(o)}$ are defined by (31), and f_{HE} and f_{HF} are the gyrofrequencies associated with the earth's magnetic field at the levels of maximum ionization density in the E and F regions. For the gyrofrequencies in mHz we will use the approximate expressions [16]

$$f_{HE} = 0.87 \left(\frac{6370}{6370 + h_E} \right)^{3} (1 + 3 \sin^{2} \Phi)^{\frac{1}{2}} \tag{33a}$$

$$f_{HF} = 0.87 \left(\frac{6370}{6370 + h_F} \right)^{3} (1 + 3 \sin^{2} \Phi)^{\frac{1}{2}} . \tag{33b}$$

where Φ is the latitude expressed in magnetic coordinates. If ϕ and λ are respectively the geographic latitude and longitude, and ϕ_o and λ_o represent the latitude and longitude of the north magnetic pole ($\phi_o \approx 78.3°N$, $\lambda_o \approx 69°W$), then [16]

$$\sin \Phi = \sin \phi \sin \phi_o + \cos \phi \cos \phi_o \cos (\lambda - \lambda_o) . \tag{34}$$

We assume that, to calculate delays for the extraordinary wave, we may use the expressions in Appendix A but with $f_{pE}^{(x)}$ and $f_{pF}^{(x)}$ substituted for the ordinary wave penetration frequencies. Otherwise the calculations of delay are assumed to take the same form as those for the ordinary wave.

To characterize delay distortion we need the derivatives of the delay evaluated at the carrier frequency. Actually, only the first three derivatives are required for the approximation discussed in Section IV. The determination of these three derivatives is straightforward although the resulting expressions are quite complicated. They are listed in Appendix A.

For multiple hop transmissions we assume that the ionosphere parameters are identical for each hop. In the two-hop case the foregoing analysis is applied with the original range multiplied by 1/2. The angle is then correct as calculated but the path length, delay and delay derivatives must be multiplied by 2. Higher numbers of hops are treated in a similar manner using appropriate multipliers.

B. Attenuation

Three main sources of attenuation are included in the present model. They are: (1) nondeviative absorption, (2) antenna gain, and (3) spreading ($1/r^2$ loss). Other factors such as focusing and polarization mismatch will also affect the received signal strength, but these are assumed to be of secondary importance and they have accordingly been omitted.

B.1 Absorption

Absorption is assumed to take place only in the D region of the ionosphere and only in the daytime. The absorption is assumed to be nondeviative, i.e., it takes place in a region where the refractive index is near unity. The attenuation will be characterized by the expression [16]

$$L_{abs} = K \sec\theta \cdot (1+kS) \cdot (\cos\chi)^{\gamma} \cdot \left\{ \frac{1}{(f \pm f_{HD}|\cos\psi_i|)^2} + \frac{1}{(f \pm f_{HD}|\cos\psi_r|)^2} \right\} . \quad (35)$$

Here L_{abs} is the attenuation due to absorption in decibels per hop, θ is the ray angle measured from the vertical, S is the sunspot number, χ is the solar zenith angle, f is the frequency in mHz, f_{HD} is the gyrofrequency at the D region level in mHz and ψ_i [resp., ψ_r] is the angle between the earth's magnetic field and the incident [resp., reflected] wave. The plus sign is used for the ordinary wave and the minus sign is used for the extraordinary wave.

The relative importance of the sunspot number and solar zenith angle depends upon the constants K, k and γ; following Davies [16, p. 235] we will set K = 215, k = 0.0035 and γ = 0.75. The gyrofrequency, f_{HD}, is given by [16]

$$f_{HD} = 0.87 \left(\frac{6370}{6370+h_D} \right)^3 (1+3 \sin^2 \Phi)^{\frac{1}{2}} . \quad (36)$$

where h_D is the height of the D region in km and Φ is the latitude in magnetic coordinates. In this expression we will take h_D to be 70 km and use (34) to determine sin Φ. The angles ψ_i and ψ_r are determined from the relationships

$$\cos\psi_i = \sin\theta \cos B_r \cos I + \cos\theta \sin I \quad (37a)$$

$$\cos\psi_r = \sin\theta \cos B_r \cos I - \cos\theta \sin I \quad (37b)$$

where B_r is the bearing of the receiver as seen from the transmitter in magnetic coordinates and I, the angle of inclination of the earth's magnetic field, is given by

$$\tan I = -2 \tan \Phi. \tag{38}$$

For multiple-hop paths, the expression (35) must be multiplied by the number of hops.

B.2 Antenna Gain

No attempt has been made in this model to provide a variety of antenna patterns. While any desired antenna gain can be easily included in the program, we shall use the gain of a short vertical dipole for both transmitter and receiver. If G_T and G_R denote the transmitter and receiver power gains relative to isotropic antennas we have

$$G_T(\theta) = G_R(\theta) = \frac{3}{2} \sin^2 \theta . \tag{39}$$

Thus for the overall attenuation in decibels we have

$$L_{ant} = -10 \log_{10} G_T(\theta) - 10 \log_{10} G_R(\theta) = -20 \log_{10}[\tfrac{3}{2} \sin^2 \theta] . \tag{40}$$

B.3 Spreading Loss

The decrease in signal strength attributable to the length of the transmission path is given by

$$L_d = 20 \log_{10} \left(\frac{4\pi Df}{0.3} \right) \tag{41}$$

where L_d is the path loss in decibels, D is the total path length in km, taking into account any multiple hops, and f is the frequency in mHz.

The total attenuation for a particular ionosphere return is obtained by adding (35), (40) and (41), after making any necessary adjustments for multiple hops. Denoting the total attenuation for the j-th return by L_j we have

$$L_j = L_{abs} \times (\text{No. of Hops}) + L_{ant} + L_d. \tag{42}$$

The rms gain A_j is thus given by

$$A_j = 10^{-(L_j/20)} \quad . \tag{43}$$

C. Doppler

The remaining quantities needed to characterize the random gains, $G_j(t)$, are the doppler shifts and the doppler spreads. In this report we are not concerned with the relatively large doppler values that occur during sunrise and sunset, but with the smaller values that are typical of the ionosphere for the majority of a typical twenty-four hour period. What is desired is the shift and spread associated with each of the propagation modes. Unfortunately, measurements that have been made concerning doppler values usually have not been concerned with the shift and spread of the individual propagation modes; instead they have recorded composite doppler values involving many modes [18]. Moreover, most theoretical studies have not been concerned with communication system performance and are of marginal value for present purposes [19-22].

What is required for our model is an expression relating the doppler shifts and spreads to frequency for each ionosphere return. This expression will depend on the electron density profile and its time rate of change as well as on various other ionosphere parameters [23]. We assume that the doppler values can be characterized adequately by the following power laws

$$\nu = A\ f^{\alpha} \tag{44}$$

and

$$\sigma = B\ f^{\beta} \tag{45}$$

where ν and 2σ are the doppler shift and doppler spread; f is the frequency in mHz; and A, B, α, and β are constants that characterize the region from which the wave is reflected and the magnetoionic

component involved. In general we need an expression of the above type for the low and high ray returns from each region and for each magnetoionic component.

It is convenient to express the constants A and B appearing in (44) and (45) in terms of a reference value measured at a known frequency. Thus the above expressions can be rewritten as

$$\nu = \nu_r (f/f_r)^{\alpha} \tag{46}$$

$$\sigma = \sigma_r (f/f_r)^{\beta} \tag{47}$$

where ν_r and $2\sigma_r$ are the doppler shift and spread measured at the reference frequency f_r. For multiple hops we assume that the doppler effects combine incoherently so that

$$\nu = (\text{No. of Hops}) \cdot \nu_r \cdot (f/f_r)^{\alpha} \tag{48}$$

and

$$\sigma = (\text{No. of Hops})^{\frac{1}{2}} \cdot \sigma_r \cdot (f/f_r)^{\beta} \quad . \tag{49}$$

Given the appropriate reference values and exponents, the above expressions provide the doppler shift and spread for each ionosphere return. However, since the reference values and exponents usually are not known, we must make some assumptions to obtain "default" values for use in the absence of more accurate data. It will be assumed that the doppler shift and spread are the same for the high ray and low ray and for both magnetoionic components of the E and F region returns. The following table gives the "default" reference values and exponents to be used for the E and F regions. These values are based upon measurements made by Watterson, et. al. [1].

Region	ν_r(Hz)	$2\sigma_r$(Hz)	f_r(mHz)	α	β
E	0.01	0.02	9.30	1.0	1.0
F	0.01	0.15	9.30	1.0	1.0

Table 1. Default Values for Doppler Shift and Doppler Spread

D. Computer Program

The Fortran program used to calculate the foregoing channel parameters is presented in Appendix B. The basic output of this program is a listing of the input parameters and a tabular presentation of the computed channel parameters. A typical output is shown in Table 2. The indexing convention used for the returns is presented in Table 3 and a glossary of the terms used in the output is presented in Table 4.

A modified version of this output can be produced in which all returns whose attenuation exceeds some threshold value above the minimum attenuation are omitted. Those modes for which there is no return are likewise omitted. Using this modified output one can identify those returns that are of primary interest for communication purposes. An example is shown in Table 5 using the same inputs that produced Table 2 and an attenuation threshold of 40 dB.

A synthetic ionogram also has been produced using the program. The resulting curves are presented in Figure 5 and include returns for one, two and three hops together with the corresponding attenuations.

Table 2. Typical Output Listing for Computed Channel Parameters (Includes All Returns for Six or Less Hops)

SUMMARY OF TRANSMISSION PARAMETERS

SIGNAL PARAMETERS

CARRIER FREQ. = 5.000 MHZ. BANDWIDTH = 100.00 KHZ.

TRANSMITTER/RECEIVER PARAMETERS

LOCATION OF TRANSMITTER: LONGITUDE = 150.00 DEGREES (+ FOR WEST, - FOR EAST)
LATITUDE = 30.00 DEGREES (+ FOR NORTH, - FOR SOUTH)
DISTANCE BETWEEN TRANSMITTER AND RECEIVER = 500.00 KM.
DIRECTION FROM TRANSMITTER TO RECEIVER = 0.00 DEGREES (MEASURED FROM TRUE NORTH)
TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
SEA STATE = 1

IONOSPHERE PARAMETERS

SUNSPOT NUMBER = 100.00
SOLAR ZENITH ANGLE = 85.00 DEGREES
DAYTIME ELECTRON DENSITY PROFILE
E REGION: HEIGHT OF MAX. ELECTRON DENSITY = 110.00 KM.
SEMITHICKNESS OF PARABOLA = 20.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 2.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 2.630 MHZ.
F REGION: HEIGHT OF MAX. ELECTRON DENSITY = 250.00 KM.
SEMITHICKNESS OF PARABOLA = 50.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 8.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 8.537 MHZ.
NONDEVIATIVE ABSORPTION CONSTANTS
MULTIPLICATIVE CONSTANT = 215.00, SUNSPOT NUMBER MULTIPLIER = 0.003500, SOLAR ZENITH ANGLE EXPONENT = 0.750
DOPPLER REFERENCE VALUES
E REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.0000 SPREAD = 0.0200 HZ, SPREAD EXP. = 1.0000 REF. FREQ. = 9.300 MHZ.
F REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.0000 SPREAD = 0.1500 HZ, SPREAD EXP. = 1.0000 REF. FREQ. = 9.300 MHZ.

COMPUTED CHANNEL PARAMETERS

GROUNDWAVE: DELAY = 1.687E-03 SEC. ATTENUATION = 1.200E+02 DB.
IONOSPHERE RETURNS:

MODE	SOLUTION INDICATOR (0=NO SOLN)	RAY ANGLE (DEGREES)	PATH LENGTH (KM)	CARRIER DELAY (SEC)	CARRIER PHASE (CYCLES)	SIGNAL DELAY (SEC)	AMPLITUDE DISTORTION (SEC/MHZ)	PHASE DISTORTION (SEC/MHZ**2)	ATTENUATION (DB)	DOPPLER SHIFT (HZ)	DOPPLER SPREAD (HZ)
1E0 LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1E0 HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1F0 LOW	1.0	46.91	684.68	2.282E-03	4.094E-01	2.096E-03	5.760E-05	1.346E-05	1.265E+02	5.376E-03	8.065E-02
1F0 HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1FX LOW	1.0	67.53	541.07	1.809E-03	7.755E-01	1.933E-03	1.098E-04	9.042E-05	1.619E+02	5.376E-03	1.075E-02
1FX HIGH	1.0	58.39	587.19	1.957E-03	4.458E-01	1.161E-03	7.306E-05	1.595E-04	1.405E+02	5.376E-03	1.075E-02
1FX LOW	1.0	44.11	718.30	2.394E-03	6.671E-01	1.824E-03	1.434E-04	2.562E-05	1.401E+02	5.376E-03	8.065E-02
1FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2E0 LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2E0 HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2F0 LOW	1.0	28.55	1046.08	3.487E-03	6.699E-01	3.583E-03	1.991E-04	4.136E-05	1.471E+02	1.075E-02	1.140E-01
2F0 HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.

Table 2. Typical Output Listing for Computed Channel Parameters (Includes All Returns for Six or Less Hops). (Contd)

2EX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2FX LOW	1.0	27.51	1082.65	3.609E-03	1.263E-01	3.196E-03	2.474E-04	5.244E-05	1.745E+02	1.075E-02	1.140E-01
2FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO LOW	1.0	19.86	1471.76	4.906E-03	3.858E-01	5.305E-03	3.744E-04	7.501E-05	1.674E+02	1.613E-02	1.397E-01
3FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX LOW	1.0	19.26	1515.47	5.052E-03	9.167E-01	4.737E-03	4.052E-04	8.120E-05	2.060E+02	1.613E-02	1.397E-01
3FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO LOW	1.0	15.12	1916.91	6.390E-03	4.819E-01	7.068E-03	5.453E-04	1.080E-04	1.864E+02	2.151E-02	1.613E-01
4FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX LOW	1.0	14.71	1949.42	6.565E-03	7.092E-01	6.310E-03	5.655E-04	1.101E-04	2.403E+02	2.151E-02	1.613E-01
4FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO LOW	1.0	12.18	2364.95	7.900E-03	1.331E-01	8.839E-03	7.112E-04	1.401E-04	2.044E+02	2.688E-02	1.803E-01
5FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5EX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FX LOW	1.0	11.86	2432.10	8.107E-03	9.682E-01	7.890E-03	7.249E-04	1.389E-04	2.715E+02	2.688E-02	1.803E-01
5FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6EO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FO LOW	1.0	10.19	2826.92	9.423E-03	2.850E-01	1.061E-02	8.736E-04	1.716E-04	2.217E+02	3.226E-02	1.975E-01
6FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX LOW	1.0	9.93	2899.17	9.664E-03	4.629E-01	9.471E-03	8.806E-04	1.675E-04	3.020E+02	3.226E-02	1.975E-01
6FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.

ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THAN 40.00 DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED

Table 3. Index Convention for Ionosphere Returns

Index	Number of Hops	Region	Ordinary or Extraordinary	Low Ray or High Ray
1	1	E	O	LOW
2	1	E	O	HIGH
3	1	F	O	LOW
4	1	F	O	HIGH
5	1	E	X	LOW
6	1	E	X	HIGH
7	1	F	X	LOW
8	1	F	X	HIGH
9	2	E	O	LOW
10	2	E	O	HIGH
11	2	F	O	LOW
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
44	6	F	O	HIGH
45	6	E	X	LOW
46	6	E	X	HIGH
47	6	F	X	LOW
48	6	F	X	HIGH

Table 4. Glossary of Terms Used
for Computed Channel Parameters

Mode	Identifies particular ionosphere return (e.g., 2FO LOW is the two-hop, F region, ordinary wave, low ray return)
Solution Indicator	This equals 1 if there is a return for the particular mode and equals 0 if there is no return
Ray Angle	θ_j
Total Path Length	D_j
Carrier Delay	$\tau(f_c)$
Carrier Phase	The fractional part of $D_j^{(o)}$
Signal Delay	$D_j^{(1)}$
Amplitude Distortion	$D_j^{(2)}$
Phase Distortion	$D_j^{(3)}$
Attenuation	L_j
Doppler Shift	ν_j
Doppler Spread	$2\sigma_j$

Table 5. Typical Output Listing For Computed Channel Parameters

(Includes Only Those Returns Having Attenuations Within 40dB Of The Smallest Nonzero Attenuation)

SUMMARY OF TRANSMISSION PARAMETERS

SIGNAL PARAMETERS

CARRIER FREQ. = 5.000 MHZ. BANDWIDTH = 100.00 KHZ.

TRANSMITTER/RECEIVER PARAMETERS

LOCATION OF TRANSMITTER: LONGITUDE = 150.00 DEGREES (+ FOR WEST, - FOR EAST)
LATITUDE = 30.00 DEGREES (+ FOR NORTH, - FOR SOUTH)
DISTANCE BETWEEN TRANSMITTER AND RECEIVER = 500.00 KM.
DIRECTION FROM TRANSMITTER TO RECEIVER = 0.00 DEGREES (MEASURED FROM TRUE NORTH)
TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
SEA STATE = 1

IONOSPHERE PARAMETERS

SUNSPOT NUMBER = 100.00
SOLAR ZENITH ANGLE = 85.00 DEGREES
DAYTIME ELECTRON DENSITY PROFILE
E REGION: HEIGHT OF MAX. ELECTRON DENSITY = 110.00 KM.
SEMITHICKNESS OF PARABOLA = 20.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 2.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 2.630 MHZ.
F REGION: HEIGHT OF MAX. ELECTRON DENSITY = 250.00 KM.
SEMITHICKNESS OF PARABOLA = 50.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 8.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 8.537 MHZ.
NONDEVIATIVE ABSORPTION CONSTANTS
MULTIPLICATIVE CONSTANT = 215.00, SUNSPOT NUMBER MULTIPLIER = 0.003500, SOLAR ZENITH ANGLE EXPONENT = 0.750
DOPPLER REFERENCE VALUES
E REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.000, SPREAD = 0.0200 HZ, SPREAD EXP. = 1.000, REF. FREQ. = 9.300 MHZ.
F REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.000, SPREAD = 0.1500 HZ, SPREAD EXP. = 1.000, REF. FREQ. = 9.300 MHZ.

COMPUTED CHANNEL PARAMETERS

GROUNDWAVE: DELAY = 1.667E-03 SEC. ATTENUATION = 1.200E+02 DB.
IONOSPHERE RETURNS:

MODE	SOLUTION INDICATOR (0=NO SOLN)	RAY ANGLE (DEGREES)	PATH LENGTH (KM)	CARRIER DELAY (SEC)	CARRIER PHASE (CYCLES)	SIGNAL DELAY (SEC)	AMPLITUDE DISTORTION (SEC/MHZ)	PHASE DISTORTION (SEC/MHZ**2)	ATTENUATION (DB)	DOPPLER SHIFT (HZ)	DOPPLER SPREAD (HZ)
1FO LOW	NA	46.91	684.68	2.282E-03	4.094E+01	2.096E-03	5.760E-05	1.346E-05	1.265E+02	5.376E-03	8.065E-02
1FX LOW	NA	67.53	541.07	1.804E-03	7.755E+01	1.933E-03	1.098E-04	9.042E-05	1.619E+02	5.376E-03	1.075E-02
1EX HIGH	NA	58.38	587.19	1.957E-03	4.458E+01	1.161E-03	7.306E-05	-1.595E-04	1.465E+02	5.376E-03	1.075E-02
1FX LOW	NA	44.11	716.30	2.394E-03	6.671E+01	1.824E-03	1.434E-04	2.562E-05	1.401E+02	5.376E-03	8.065E-02
2FO LOW	NA	28.55	1046.08	3.487E-03	6.699E+01	3.583E-03	1.991E-04	4.136E-05	1.471E+02	1.075E-02	1.140E-01

ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THAN 40.00 DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED

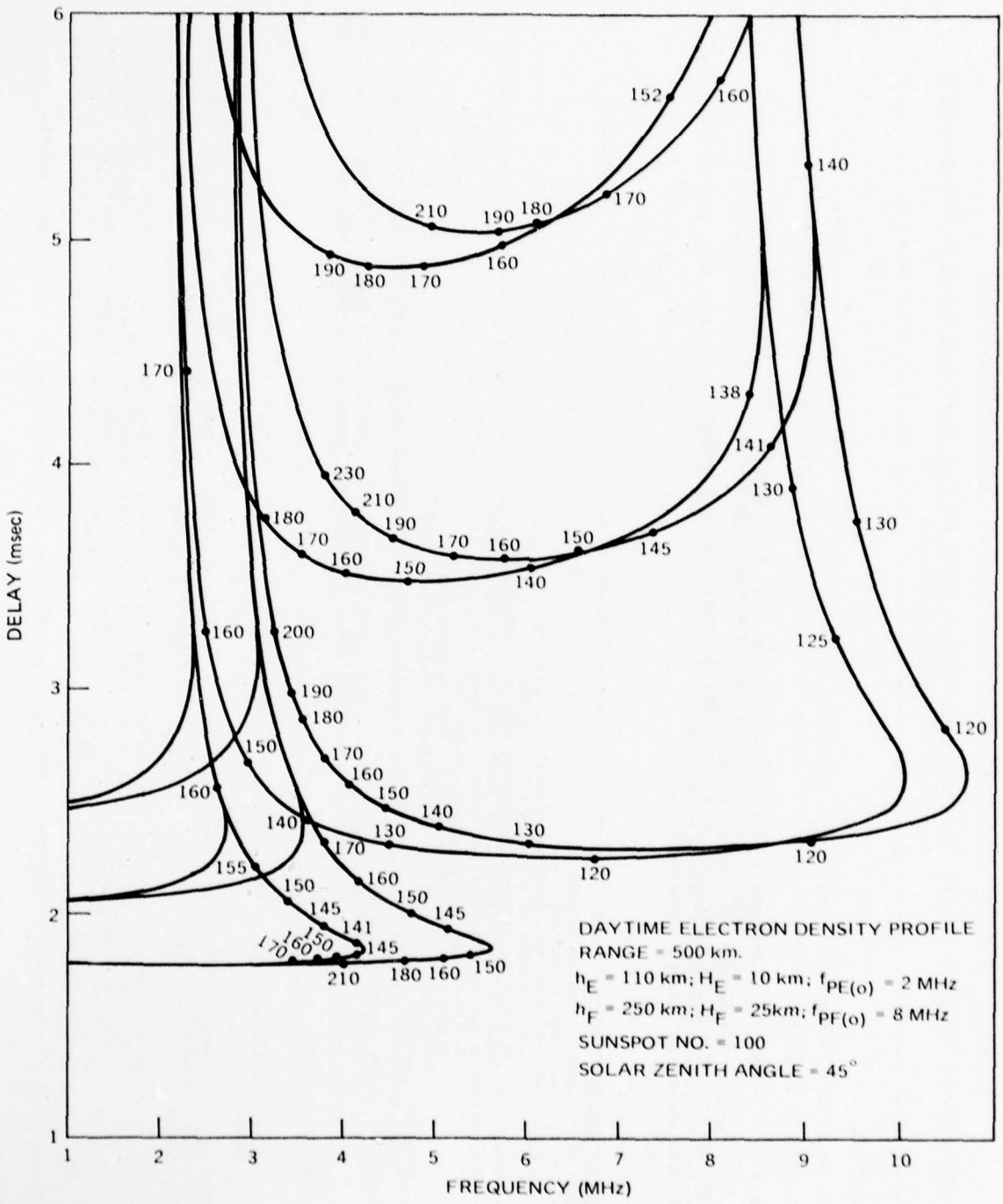

Figure 5. Computed Ionosphere Returns for One, Two and Three Hops
(Numbers on Curves are Attenuation in dB)

IV. SIGNAL ANALYSIS

This section is concerned with the signal analysis part of the simulator and the approximations used in its implementation. We will assume that the transmitted signal, s(t), is bandlimited with (two sided) bandwidth BW. The received signal, r(t), is not strictly bandlimited because it is multiplied by the random gains, which are not bandlimited. However, since the doppler spreads are small compared with the signal bandwidth, the received signal can also be assumed to be bandlimited but with a slightly larger bandwidth than the transmitted signal. The error introduced by this assumption is negligible as long as the assumed bandwidth of the received signal is large compared with the sum of the transmitted signal bandwidth and the largest doppler spread present. In the following we will assume that all signals have a bandwidth equal to twice that of the transmitted signal. This is sufficient to reduce aliasing errors to a negligible value. Denoting the sampling interval associated with this larger bandwidth by T_o we have

$$T_o = \frac{1}{2 \cdot BW} \quad . \tag{50}$$

As a consequence of the cardinal series representation for bandlimited functions, we require a knowledge of the signals involved only at the sample times nT_o, $n = 0, \pm 1, \cdots$. This greatly simplifies the computational and storage requirements. From (21) the received, sampled signal appears as

$$r(nT_o) = A_g \, s(nT_o - N_o T_o) + \sum_{j=1}^{N} \exp[-2\pi i D_j^{(o)}] \cdot G_j(nT_o) \, \hat{s}_j(nT_o - N_j T_o) \qquad n=0,1,\cdots \tag{51}$$

where the various delays have been replaced by the nearest integer multiple of T_o. That is, N_g is defined as the integer satisfying

$$D_g - \frac{1}{2} T_o \leq N_g T_o < D_g + \frac{1}{2} T_o \tag{52}$$

and the N_j's are defined as the integers satisfying

$$D_j^{(1)} - \frac{1}{2} T_o \leq N_j T_o < D_j^{(1)} + \frac{1}{2} T_o, \quad j=1,\cdots N. \tag{53}$$

It remains to determine the sampled, distorted signals $\hat{s}_j(nT_o)$ and the sampled random path gains $G_j(nT_o)$.

A. Delay Distortion

The transmitted signal has the representation

$$s(t) = \sum_{n=-\infty}^{\infty} s(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)} \tag{54}$$

and the impulse response $h_j(t)$, which we also assume to be band-limited, has the representation

$$h_j(t) = \sum_{n=-\infty}^{\infty} h_j(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)}, \quad j=1,\cdots N. \tag{55}$$

We obtain the distorted waveform, $\hat{s}_j(t)$, by convolving $s(t)$ and $h_j(t)$. But $\hat{s}_j(t)$ also has an expansion of the above type and

$$\hat{s}_j(t) = \sum_{n=-\infty}^{\infty} \hat{s}_j(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)}, \quad j=1,\cdots N. \tag{56}$$

We are interested here in the sample values $\hat{s}_j(nT_o)$ and these are given by the convolution

$$\hat{s}_j(nT_o) = \sum_{m=-\infty}^{\infty} s(nT_o-mT_o)\, \hat{h}_j(mT_o), \quad j=1,\cdots,N. \tag{57}$$

where

$$\hat{h}_j(mT_o) = T_o h_j(mT_o) = T_o \int_{-BW}^{BW} H_j(f) \exp[2\pi i f m T_o] df. \qquad (58)$$

$$j=1,\cdots N$$

The transfer function $H_j(f)$ is given by (11), and it will be approximated with the first two terms of the series appearing in the exponent, that is

$$H_j(f) \simeq \exp[-2\pi i f^2 D_j^{(2)} - 2\pi i f^3 D_j^{(3)}], \; j=1,\cdots N. \qquad (59)$$

After the change of variable $\eta = f/BW$, we obtain from (58)

$$\hat{h}_j(mT_o) \simeq \frac{1}{2} \int_{-1}^{1} \exp[-i\Delta_j^{(2)} \eta^2 - i\Delta_j^{(3)} \eta^3] \cdot \exp[im\pi\eta] d\eta \qquad (60)$$

$$j=1,\cdots N$$

with

$$\Delta_j^{(2)} = 2\pi (BW)^2 D_j^{(2)} \qquad (61)$$

$$\Delta_j^{(3)} = 2\pi (BW)^3 D_j^{(3)} . \qquad (62)$$

Closed form expressions for (60) are not known and further approximations are required for the numerical determination of $\hat{h}_j(mT_o)$. Let $I(x)$ represent the general integral defining the impulse responses, that is

$$I(x) = \frac{1}{2} \int_{-1}^{1} \exp[-i\Delta^{(2)} \eta^2 - i\Delta^{(3)} \eta^3] \cdot \exp[ix\eta] d\eta . \qquad (63)$$

Expanding $\exp[-i\Delta^{(3)} \eta^3]$ in a power series we have the representation

$$I(x) = \sum_{p=0}^{\infty} \frac{(-i\Delta^{(3)})^p}{p!} I_p(x) \tag{64}$$

where we have defined

$$I_p(x) = \frac{1}{2} \int_{-1}^{1} \eta^{3p} \exp[-i\Delta^{(2)}\eta^2] \cdot \exp[ix\eta] d\eta \quad . \tag{65}$$

The integral $I_o(x)$ is readily expressed in terms of Fresnel integrals [24]. For the higher order integrals the following recurrence relation has been derived

$$I_p(x) = \frac{i}{4\Delta^{(2)}} \exp[-i\Delta^{(2)}] \left\{ \exp[ix] + (-1)^p \exp[-ix] \right\}$$
$$+ \frac{x}{2\Delta^{(2)}} I_{p-1}(x) - i \frac{(p-1)}{2\Delta^{(2)}} I_{p-2}(x), \; p \geq 1. \tag{66}$$

where we have adopted the convention that $I_{-1}(x)=0$.

The impulse responses for the ionosphere returns can now be evaluated with relative ease. Starting with $I_o(x)$ and using (66) we determine as many of the terms $I_p(x)$ as are needed to adequately approximate the infinite sum (64). The integral $I_o(x)$ is evaluated using the rational approximation presented in [25]. Care must be exercised in implementing this procedure, however. The error incurred by using the rational approximation for $I_o(x)$ is approximately 1.0×10^{-9}. For large arguments ($x>5$) nearly equal functions are subtracted in (66) and this reduces the number of significant figures in the resulting expression. For the higher order terms this reduction in the number of significant figures is important and the computed values may become inaccurate. It is evident from (64) that this effect will be more noticeable when $\Delta^{(3)}$ is large. Experience has shown that values of x as large as 20-25 can be tolerated in most applications before the errors become excessive.

B. Random Gains

The random gains, $G_j(t)$, in the ionosphere returns are complex valued, zero mean, Gaussian processes whose correlation function and power spectral density are given by (cf. Eqns. 19 and 20)

$$C_{G_j}(\tau) = A_j^2 \exp[-2\pi^2\sigma_j^2\tau^2 + 2\pi i\nu_j\tau] \ , \ j=1,\cdots N \tag{67}$$

and

$$P_{G_j}(f) = \frac{A_j^2}{\sqrt{2\pi}\sigma_j} \exp\left[-\frac{(f-\nu_j)^2}{2\sigma_j^2}\right] \ , \ j=1,\cdots N. \tag{68}$$

We wish to generate a random sequence with the above correlation function and we wish to do this directly in the time domain and not rely on Fourier transform techniques. Because the doppler spreads, σ_j, are small relative to the signal bandwidths, the time domain approach requires a large memory. To avoid these large memory requirements we will replace the original process with an approximation that can be generated more easily.

The approximate random gain will be denoted by $\tilde{G}_j(t)$ and it will be chosen to have the power spectral density

$$P_{\tilde{G}_j}(f) = \frac{8A_j^2}{3\pi\rho\sigma_j} \frac{1}{\left[1+\left(\frac{f-\nu_j}{\rho\sigma_j}\right)^2\right]^3} \ , \ j=1,\cdots,N \tag{69}$$

and corresponding correlation function

$$\begin{aligned} C_{\tilde{G}_j}(\tau) = A_j^2 &\left[1 + 2\pi\rho\sigma_j|\tau| + \frac{1}{3}\left(2\pi\rho\sigma_j|\tau|\right)^2\right] \\ &\cdot\exp\left[-2\pi\rho\sigma_j|\tau| + 2\pi i\nu_j\tau\right] \ , \ j=1,\cdots,N. \end{aligned} \tag{70}$$

The parameter ρ is chosen so that (70) agrees with (67) over a reasonable range of arguments. The value $\rho = 2.1$ has been determined to be a good choice and this is the value that will be used

throughout the following. Normalized plots of $C_{G_j}(\tau)$ and $C_{\tilde{G}_j}(\tau)$ are compared in Figure 6.

It is obvious from (69) that the approximate process $\tilde{G}_j(t)$ is not bandlimited. However, the error introduced by assuming that the process has bandwidth BW will be negligible since $(\sigma_j/BW)<<1$. Accordingly we will treat $\tilde{G}_j(t)$ as if it is bandlimited, in which case we need only evaluate it at the sample times nT_o. Thus, the original problem has been reduced to the generation of a complex valued, zero mean, Gaussian sequence with correlation function

$$C_{\tilde{G}_j}(nT_o) = A_j^2\left[1 + 2\pi\rho\sigma_j T_o|n| + \frac{1}{3}\left(2\pi\rho\sigma_j T_o|n|\right)^2\right] \cdot\exp\left[-2\pi\rho\sigma_j T_o|n| + 2\pi i\nu_j T_o n\right] \quad (71)$$

$$n=0,\pm1,\ldots \qquad j=1,\cdots N$$

The technique used for the computer generation of the approximate process is discussed in Appendix C. A plot of the normalized, empirical correlation function of the computer-generated process is also displayed in Figure 6. Agreement between the computer-generated correlation function and the approximate correlation function (71) is excellent for the range of arguments considered.

C. Computer Program

A listing of the Fortran program used to implement the signal analysis part of the simulator is presented in Appendix D. Channel parameter information is read into the program via data cards that have been produced by the first part of the simulator. The sampled input values are read from a tape, the program calculates the sampled output values according to Eqn. (51) and writes the output on another tape. As an alternate, printed output also can be obtained, but, because the quantity of output data is large, the usefulness of this mode of presentation is severely limited.

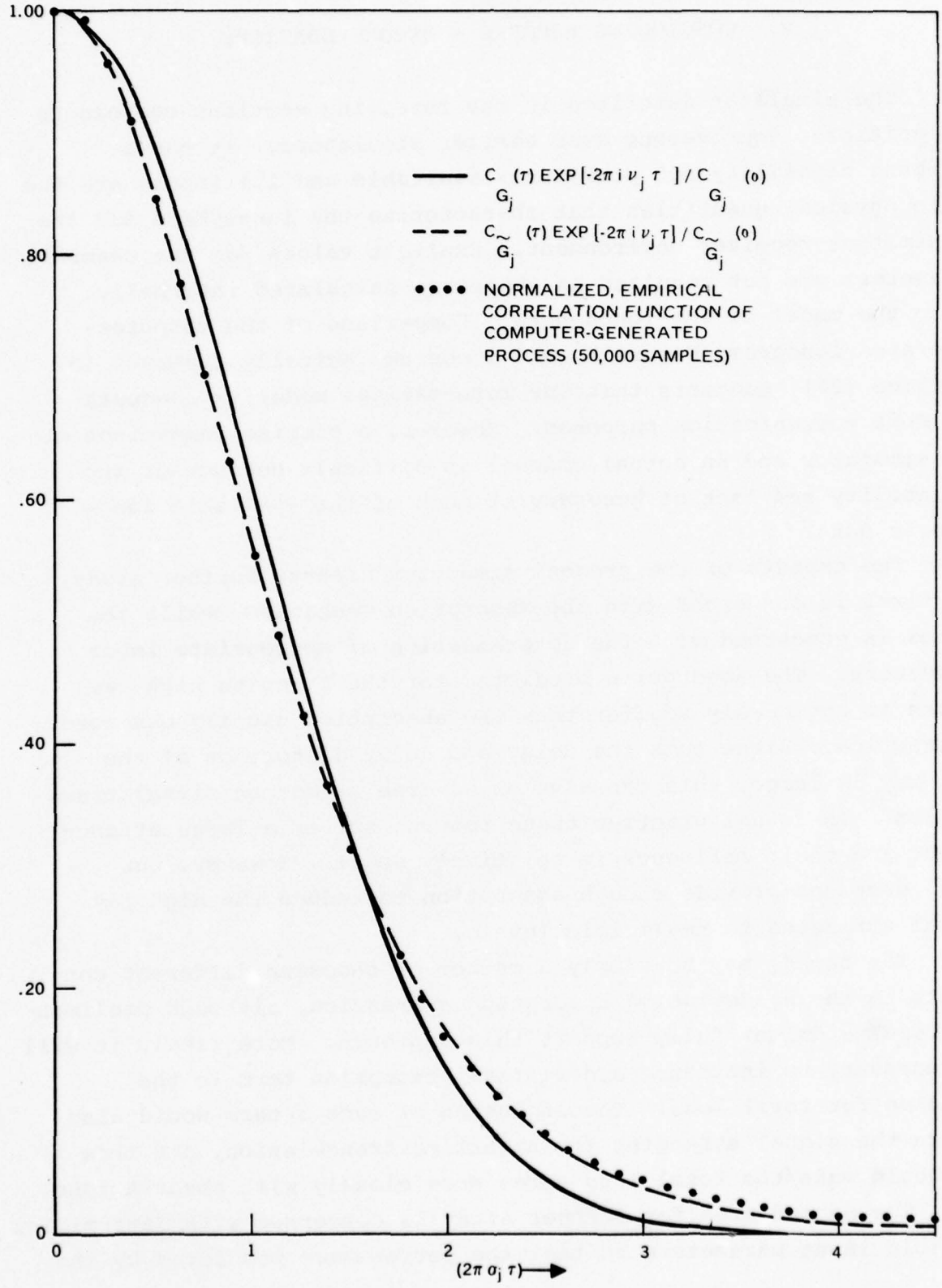

Figure 6. Normalized Correlation Function for the Random Gains

V. CONCLUDING REMARKS - RECOMMENDATIONS

The simulator described in the foregoing sections represents a significant improvement over earlier simulators. It has a wideband capability not previously available and its inputs are the basic physical quantities that characterize the ionosphere and the transmitter-receiver environment. Explicit values for the channel parameters are not required as these are calculated internally using the model of the ionosphere. Comparison of the computer-generated ionogram, Figure 5, and ionograms actually observed in practice [27], suggests that our mathematical model is adequate for most communication purposes. However, a precise comparison of the simulator and an actual channel is difficult because of the variability and lack of accuracy of much of the available ionospheric data.

Two aspects of the present simulator deserve further study. The first is concerned with the absorption mechanism, while the second is concerned with the determination of appropriate input parameters. The absorption predicted for the F region high ray return is noticeably smaller than the absorption usually observed in practice. Since both the delay and delay distortion of the high ray is large, this can have an adverse affect on signal transmission. In actual practice these returns suffer a large attenuation, and their influence is relatively small. However, our model does not provide enough absorption to reduce the high ray signal strengths to negligible levels.

The remedy may be simply a matter of choosing different constants in the nondeviative absorption expression, although preliminary results do not fully support this approach. More likely it will be necessary to introduce a deviative absorption term in the equation for total loss. The inclusion of such a term would also reduce the signal strengths for nighttime transmission, and this too would make the total loss agree more closely with observations.

The second area for further study is concerned with determining suitable input parameters so that the performance predicted by the

mathematical model agrees as well as possible with the actual observed performance. This involves matching actual and computer-generated observables such as doppler shift, doppler spread, attenuation and delay vs. frequency plots (ionograms) to determine the input parameters. Besides the obvious advantage of providing further justification of the assumptions used in the model, this parameter determination would provide the user with a set of inputs that correspond to a variety of actual environments.

The primary difficulty encountered in the determination of the input parameters is that an explicit functional relationship does not exist between the input parameters and the computed channel parameters; the channel parameters are determined by numerically solving a transcendental equation. As a consequence, techniques must be developed that are tailored to the ionosphere model. One approach is to select the most significant observable features and to determine the relationship between these features and the input parameters. For example, the nearly horizontal portions of an observed ionogram would be affected most by the heights of the regions and to a lesser degree by the thickness of the regions and their peak electron densities. On the other hand, the maximum operating frequency is likely to be more affected by the peak electron densities than the heights or thicknesses of the regions. A detailed study of these and other pertinent relationships would provide the user with the ability to determine input parameters corresponding to a variety of observed operating conditions. These same conditions could then be reproduced by the channel simulator as required for system studies.

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APPENDIX A
DELAY AND DELAY DERIVATIVES

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DELAY AND DELAY DERIVATIVES

In this appendix we present the equations for the delay and the first three delay derivatives. Recalling the definition of A(x) we have

$$A(x) = 2G\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} \tag{A.1}$$

where $x = \cos\theta$. We must solve the equation

$$R = A(x) \tag{A.2}$$

with R equal to the distance between transmitter and receiver. The solution then yields the delay via the equation

$$\tau = \frac{R}{c\,(1-x^2)^{\frac{1}{2}}} \tag{A.3}$$

with c the velocity of light. The delay derivatives are given by

$$\frac{d\tau}{df} = \frac{x\tau}{(1-x^2)}\,\frac{dx}{df} \tag{A.4}$$

$$\frac{d^2\tau}{df^2} = \frac{1+2x^2}{x^2\tau}\left(\frac{d\tau}{dx}\right)^2 + \frac{x\tau}{(1-x^2)}\,\frac{d^2x}{df^2} \tag{A.5}$$

and

$$\frac{d^3\tau}{df^3} = -\frac{3(1+x^2+2x^4)}{x^4\tau^2}\left(\frac{d\tau}{df}\right)^2 + \frac{3(1+2x^2)}{x^2\tau}\left(\frac{d\tau}{df}\right)\left(\frac{d^2\tau}{df^2}\right)$$

$$+ \frac{x\tau}{(1-x^2)}\,\frac{d^3x}{df^3} \quad . \tag{A.6}$$

Thus to determine the delay and delay derivatives we must solve (A.2) for x and then evaluate the first three derivatives of x with respect to f for use in (A.4) - (A.6). In the following these functions are listed for the two electron density profiles. The group height G(x) is obtained by evaluating the integral (24).

1. Nighttime

For the nighttime electron density the integral for the group height is evaluated and used in (A.1) to yield

$$A(x) = \begin{cases} 2\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}}\left\{h_E - 2H_E + H_E\,\hat{f}_E\,x\,\ln\left(\frac{1+\hat{f}_E x}{1-\hat{f}_E x}\right)\right\}, & \hat{f}_E x \leq 1 \\ 2\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}}\left\{h_F - 2H_F - 4H_E + 2H_E\,\hat{f}_E x\,\ln\left(\frac{\hat{f}_E x+1}{\hat{f}_E x-1}\right)\right. & \\ \left. \quad +H_F\,\hat{f}_F x\,\ln\left(\frac{1+\hat{f}_F x}{1-\hat{f}_F x}\right)\right\}, & \hat{f}_E x > 1 \end{cases} \tag{A.7}$$

where

$$\hat{f}_E = \frac{f}{f_{pE}} \text{ and } \hat{f}_F = \frac{f}{f_{pF}} .$$

For $\hat{f}_E x \leq 1$ the derivatives of x with respect to frequency are:

$$\frac{dx}{df} = \left(\frac{x}{f}\right)\cdot\frac{R\cdot\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}} - \left[2h_E - \frac{4H_E}{1-\hat{f}_E^2 x^2}\right]}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{\frac{3}{2}} + \left[2h_E - \frac{4H_E}{1-\hat{f}^2_E x^2}\right]}, \quad \hat{f}_E x \leq 1 \tag{A.8}$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\left(\frac{dx}{df}\right)$$

$$+ \left(\frac{x}{f}\right)\cdot\frac{R\cdot\left(\frac{1}{1-x^2}\right)^{3/2}\left(\frac{dx}{df}\right)\left[1-\frac{3xf}{1-x^2}\frac{dx}{df}\right]+\frac{8H_E\, f\, \hat{f}^2_E\left(\frac{x}{f}+\frac{dx}{df}\right)}{\left(1-\hat{f}^2_E x^2\right)^2}}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}^2_E x^2}\right]}, \quad \hat{f}_E x \leq 1 . \tag{A.9}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\left(\frac{dx}{df}\right) - \frac{1}{f}\left(\frac{d^2x}{df^2}\right)$$

$$- \frac{1}{x}\left(\frac{x}{f}-\frac{dx}{df}\right)\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]$$

$$- \frac{\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}^2_E x^2}\right]}\left\{\frac{3R\cdot x^2\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}} - \frac{8H_E x\hat{f}^2_E\left(\frac{x}{f}+\frac{dx}{df}\right)}{(1-\hat{f}^2_E x^2)^2}\right\}$$

$$+ \frac{\left(\frac{x}{f}\right)}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}^2_E x^2}\right]}\left\{\frac{R}{(1-x^2)^{5/2}}\left[1-\frac{3xf}{1-x^2}\frac{dx}{df}\right]\right.$$

$$\cdot\left[3x\left(\frac{dx}{df}\right)^2 + (1-x^2)\frac{d^2x}{df^2}\right]$$

$$- \frac{3Rf\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}\left[\frac{1+x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 + \left(\frac{x}{f}\right)\frac{dx}{df} + x\frac{d^2x}{df^2}\right]$$

$$+ \frac{32H_E\, x\, f\, \hat{f}^4_E\left(\frac{x}{f}+\frac{dx}{df}\right)^3}{(1-\hat{f}^2_E x^2)^3}$$

$$\left. + \frac{8H_E\,\hat{f}^2_E\left(\frac{x}{f}+\frac{dx}{df}\right)}{(1-\hat{f}^2_E x^2)^2}\left[\frac{x}{f}+5\frac{dx}{df}+2f\frac{d^2x}{df^2}\right]\right\}, \quad \hat{f}_E x \leq 1 . \tag{A.10}$$

For $\hat{f}_E x > 1$ these derivatives appear as

$$\frac{dx}{df} = \left(\frac{x}{f}\right) \cdot \frac{F \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}} - \left[2h_F + \frac{8H_E}{\hat{f}^2_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2_F x^2}\right]}{F \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{3}{2}} + \left[2h_F + \frac{8H_E}{\hat{f}^2_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2_F x^2}\right]}, \quad \hat{f}_E x > 1 \qquad \text{(A.11)}$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\left(\frac{dx}{df}\right)$$

$$+\left(\frac{x}{f}\right) \cdot \frac{\frac{R \cdot \left(\frac{dx}{df}\right)}{(1-x^2)^{\frac{3}{2}}}\left[1 - \frac{3xf}{1-x^2}\frac{dx}{df}\right] + \frac{16H_E f \hat{f}^2_E \left(\frac{x}{f} + \frac{dx}{df}\right)^2}{\left(\hat{f}^2_E x^2 - 1\right)^2} + \frac{8H_F f \hat{f}^2_F \left(\frac{x}{f} + \frac{dx}{df}\right)^2}{\left(1-\hat{f}^2_F x^2\right)^2}}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{3}{2}} + \left[2h_F + \frac{8H_E}{\hat{f}^2_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2_F x^2}\right]}, \quad \hat{f}_E x > 1 \qquad \text{(A.12)}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\left(\frac{dx}{df}\right) - \frac{1}{f}\left(\frac{d^2x}{df^2}\right)$$

$$- \frac{1}{x}\left(\frac{x}{f} - \frac{dx}{df}\right)\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]$$

$$- \frac{\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{3}{2}} + \left[2h_F + \frac{8H_E}{\hat{f}^2_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2_F x^2}\right]} \left\{ \frac{3Rx^2\left(\frac{dx}{df}\right)}{(1-x^2)^{\frac{5}{2}}} \right.$$

$$\left. - \frac{16H_E x \hat{f}^2_E \left(\frac{x}{f} + \frac{dx}{df}\right)}{\left(\hat{f}^2_E x^2 - 1\right)^2} - \frac{8H_F x \hat{f}^2_F \left(\frac{x}{f} + \frac{dx}{df}\right)}{\left(1-\hat{f}^2_F x^2\right)^2} \right\}$$

(cont.)

$$+ \frac{\frac{x}{f}}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2} + \left[2h_F + \frac{8H_E}{\hat{f}^2_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2_F x^2}\right]}$$

$$\cdot \left\{ \frac{R\cdot\left[1 - \frac{3xf}{1-x^2}\frac{dx}{df}\right]}{(1-x^2)^{5/2}} \left[3x\left(\frac{dx}{df}\right)^2 + (1-x^2)\left(\frac{d^2x}{df^2}\right)\right] \right.$$

$$- \frac{3Rf\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}} \left[\frac{1+x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 + \left(\frac{x}{f}\right)\frac{dx}{df} + x\frac{d^2x}{df^2}\right]$$

$$- \frac{64H_E x f \hat{f}^4_E \left(\frac{x}{f} + \frac{dx}{df}\right)^3}{(\hat{f}^2_E x^2 - 1)^3} + \frac{32H_F x f \hat{f}^4_F \left(\frac{x}{f} + \frac{dx}{df}\right)^3}{(1-\hat{f}^2_F x^2)^3}$$

$$+ \frac{16H_E \hat{f}^2_E \left(\frac{x}{f} + \frac{dx}{df}\right)\left[\frac{x}{f} + 5\frac{dx}{df} + 2f\frac{d^2x}{df^2}\right]}{(\hat{f}^2_E x^2 - 1)^2}$$

$$+ \frac{8H_F \hat{f}^2_F \left(\frac{x}{f} + \frac{dx}{df}\right)\left[\frac{x}{f} + 5\frac{dx}{df} + 2f\frac{d^2x}{df^2}\right]}{(1-\hat{f}^2_F x^2)^2}, \quad f_E x > 1. \qquad \text{(A.13)}$$

2. Daytime

The daytime electron density profile differs from the nighttime profile only for heights greater than h_E. Thus the daytime equations for $\hat{f}_E \leq 1$ are the same as the nighttime equations. In this case

$$A(x) = \begin{cases} 2\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_E - 2H_E + H_E \hat{f}_E x \ \ln\left(\frac{1+\hat{f}_E x}{1-\hat{f}_E x}\right)\right\}, & \hat{f}_E x \leq 1 \\ 2\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_E - 2H_E + H_E \hat{f}_E x \ \ln\left(\frac{\hat{f}_E x+1}{\hat{f}_E x-1}\right)\right. \\ \quad + \left[h_F - h_E - 2H_F (1-r^2)^{\frac{1}{2}}\right]\left(\frac{\hat{f}_E^2 x^2}{\hat{f}_E^2 x^2 - 1}\right)^{\frac{1}{2}} \\ \quad \left. + 2H_F \hat{f}_F x \ln\left[\frac{(1-r^2)^{\frac{1}{2}} + (\hat{f}_F^2 x^2 - r^2)^{\frac{1}{2}}}{(1-\hat{f}_F^2 x^2)^{\frac{1}{2}}}\right]\right\}, & \hat{f}_E x > 1 \end{cases} \qquad (A.14)$$

where

$$r^2 = \frac{f_{pE}^2}{f_{pF}^2} \quad .$$

For $\hat{f}_E x \leq 1$ the derivatives of x with respect to frequency are given by (A.8) - (A.10). For $\hat{f}_E x > 1$ we have

$$\frac{dx}{df} = \left(\frac{x}{f}\right) \frac{R \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}} - B(x)}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{\frac{3}{2}} + B(x)}, \quad \hat{f}_E x > 1 \qquad (A.15)$$

with B(x) defined by

$$B(x) = 2h_E + \frac{4H_E}{\hat{f}_E^2 x^2 - 1} + 2\left(h_F - h_E - 2H_F\sqrt{1-r^2}\right) \cdot \left(\frac{\hat{f}_E^2 x^2}{\hat{f}_E^2 x^2 - 1}\right)^{\frac{3}{2}}$$

$$- \frac{4H_F \hat{f}_F^3 x^3}{\sqrt{1-r^2}\ (\hat{f}_F^2 x^2 - r^2)^{\frac{1}{2}} + (\hat{f}_F^2 x^2 - r^2)} - \frac{4H_F \hat{f}_F^3 x^3}{1-\hat{f}_F^2 x^2} \qquad (A.16)$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\frac{dx}{df}$$

$$+ \frac{\frac{R\left(\frac{x}{f}\right)\frac{dx}{df}}{(1-x^2)^{3/2}}\left[1-\frac{3xf}{1-x^2}\frac{dx}{df}\right] - \left(\frac{x}{f}+\frac{dx}{df}\right)\frac{dB(x)}{df}}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2} + B(x)}, \quad \hat{f}_E x > 1 \qquad \text{(A.17)}$$

$$\frac{dB(x)}{df} = \left(\frac{x}{f}+\frac{dx}{df}\right)\cdot\left\{\frac{8H_E\hat{f}^2_E x}{(\hat{f}^2_E x^2-1)^2} - 6\left(h_F-h_E-2H_F\sqrt{1-r^2}\right)\frac{\hat{f}^3_E x^2}{(\hat{f}^2_E x^2-1)^{5/2}}\right.$$

$$- \frac{12H_F\hat{f}^3_F x^2}{\sqrt{1-r^2}\,(\hat{f}^2_F x^2-r^2)^{1/2} + (\hat{f}^2_F x^2-r^2)}$$

$$+ \frac{4H_F\hat{f}^5_F x^4\left[2+\sqrt{1-r^2}\,(\hat{f}^2_F x^2-r^2)^{-1/2}\right]}{\left[\sqrt{1-r^2}\,(\hat{f}^2_F x^2-r^2)^{1/2} + (\hat{f}^2_F x^2-r^2)\right]^2}$$

$$\left. - \frac{4H_F\hat{f}^3_F x^2(3-\hat{f}^2_F x^2)}{(1-\hat{f}^2_F x^2)^2}\right\} \qquad \text{(A.18)}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\frac{dx}{df} - \frac{1}{f}\frac{d^2x}{df^2}$$

$$+ \frac{1}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2} + B(x)} \left\{ - \frac{3R\ x\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}\left[\frac{1+4x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 + \left(\frac{x}{f}\right)\left(\frac{dx}{df}\right) + 3x\frac{d^2x}{df^2}\right]\right.$$

$$+ \frac{R}{f\cdot(1-x^2)^{3/2}}\left[\frac{1+2x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 - \left(\frac{x}{f}\right)\frac{dx}{df} + x\frac{d^2x}{df^2}\right]$$

$$- \frac{1}{f}\frac{dB(x)}{df}\left[2\frac{dx}{df} - \frac{x}{f} + 2f\frac{d^2x}{df^2} - \frac{f}{x}\left(\frac{dx}{df}\right)^2\right]$$

$$\left. - \left(\frac{x}{f} + \frac{dx}{df}\right)\frac{d^2B(x)}{df^2}\right\}, \quad \hat{f}_E x > 1 \qquad \text{(A.19)}$$

$$\frac{d^2B(x)}{df^2} = \frac{1}{f}\ \frac{\frac{dB(x)}{df}}{\left(\frac{x}{f} + \frac{dx}{df}\right)}\left[\frac{dx}{df} - \frac{x}{f} + f\frac{d^2x}{df^2}\right]$$

$$+ \left(\frac{x}{f} + \frac{dx}{df}\right)\left\{ - \frac{8H_E\hat{f}^2_E}{(\hat{f}^2_E x^2 - 1)^3}\left[\hat{f}^2_E x^2\left(2\frac{x}{f} + 3\frac{dx}{df}\right) + 2\left(\frac{x}{f} + \frac{dx}{df}\right)\right]\right.$$

$$\left. + \frac{6\hat{f}^3_E\ x\left(h_F - h_E - 2H_F\sqrt{1-r^2}\right)}{(\hat{f}^2_E x^2 - 1)^{1/2}}\left[\hat{f}^2_E x^2\left(2\frac{x}{f} + 3\frac{dx}{df}\right) + \left(3\frac{x}{f} + 2\frac{dx}{df}\right)\right]\right.$$

(cont.)

$$+\frac{12H_F\hat{f}^3{}_F x\left[-\left(3\frac{x}{f}+2\frac{dx}{df}\right)+\hat{f}^2{}_F x^2\left(\frac{x}{f}+\frac{dx}{df}\right)\left[2+\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{-\frac{1}{2}}\right]\right]}{\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{\frac{1}{2}}+\left(\hat{f}^2{}_F x^2-r^2\right)}$$

$$+\frac{4H_F\hat{f}^5{}_F x^3\left(5\frac{x}{f}+4\frac{dx}{df}\right)\left[2+\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{-\frac{1}{2}}\right]}{\left[\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{\frac{1}{2}}+\left(\hat{f}^2{}_F x^2-r^2\right)\right]^2}$$

$$-\frac{4H_F\hat{f}^7{}_F x^5\sqrt{1-r^2}\left(\frac{x}{f}+\frac{dx}{df}\right)\cdot\left(\hat{f}^2{}_F x^2-r^2\right)^{-\frac{3}{2}}}{\left[\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{\frac{1}{2}}+\left(\hat{f}^2{}_F x^2-r^2\right)\right]^2}$$

$$-\frac{8H_F\hat{f}^7{}_F x^5\left(\frac{x}{f}+\frac{dx}{df}\right)\left[2+\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{-\frac{1}{2}}\right]^2}{\left[\sqrt{1-r^2}\left(\hat{f}^2{}_F x^2-r^2\right)^{\frac{1}{2}}+\left(\hat{f}^2{}_F x^2-r^2\right)\right]^3}$$

$$-\frac{16H_F\hat{f}^5{}_F x^3\left(\frac{x}{f}+\frac{dx}{df}\right)\left(3-\hat{f}^2{}_F x^2\right)}{\left(1-\hat{f}^2{}_F x^2\right)^3}$$

$$-\frac{4H_F\hat{f}^3{}_F x\left[3\left(3\frac{x}{f}+2\frac{dx}{df}\right)-\hat{f}^2{}_F x^2\left(5\frac{x}{f}+4\frac{dx}{df}\right)\right]}{\left(1-\hat{f}^2{}_F x^2\right)^2}\Bigg\} \qquad \text{(A.20)}$$

APPENDIX B

PROGRAM FOR CALCULATING CHANNEL PARAMETERS

```
B6700/B7700  FORTRAN COMPILATION  MARK 2.8.080   MONDAY, 09/26/77   03:00 PM

                                                        START OF SEGMENT 002
                                                        FORMAT SEGMENT IS 0006 LONG
                                                        FORMAT SEGMENT IS 0000 LONG
                                                        FORMAT SEGMENT IS 0003 LONG
      DIMENSION T(48,14),P(4,8),A(3)                    C  0021000010
      COMMON HE,HEM,HF,HFM,TIME                         C  0021000010
                                                        C  0021000010
C          THIS PROGRAM COMPUTES CHANNEL PARAMETERS FROM BASIC INPUTS   C  0021000010
                                                        C  0021000010
C          GLOSSARY OF MOST IMPORTANT VARIABLES         C  0021000010
C              FC    = CARRIER FREQUENCY (MHZ.)         C  0021000010
C              BW    = BANDWIDTH (KHZ.)                 C  0021000010
C              XLONG = LONGITUDE OF TRANSMITTER (DEGREES) + FOR WEST,  C  0021000010
C                      - FOR EAST)                       C  0021000010
C              XLAT  = LATITUDE OF TRANSMITTER (DEGREES) + FOR NORTH,  C  0021000010
C                      - FOR SOUTH)                      C  0021000010
C              R     = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM.)  C  0021000010
C              RAG   = DIRECTION FROM TRANSMITTER TO RECEIVER (DEGREES)  C  0021000010
C                      MEASURED IN GEOGRAPHIC COORDINATES)   C  0021000010
C              RAM   = DIRECTION FROM TRANSMITTER TO RECEIVER (DEGREES)  C  0021000010
C                      MEASURED IN MAGNETIC COORDINATES)     C  0021000010
C              ISEA  = SEA STATE                         C  0021000010
C              SPI   = SUNSPOT NUMBER                    C  0021000010
C              ZEN   = SOLAR ZENITH ANGLE (DEGREES)      C  0021000010
C              TIME  = TIME OF DAY ('DAY' OR 'NITE')     C  0021000010
C              HEM   = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)  C  0021000010
C              AHE   = THICKNESS OF E REGION (KM.)       C  0021000010
C              HFM   = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)  C  0021000010
C              AHF   = THICKNESS OF F REGION (KM.)       C  0021000010
C              FPEO  = PENETRATION FREQ. OF F REGION O-WAVE (MHZ.)  C  0021000010
C              FPEO  = PENETRATION FREQ. OF F REGION O-WAVE (MHZ.)  C  0021000010
C              FPEX  = PENETRATION FREQ. OF E REGION X-WAVE (MHZ.)  C  0021000010
C              FPFX  = PENETRATION FREQ. OF F REGION X-WAVE (MHZ.)  C  0021000010
C              ABSMLT= MULTIPLICATIVE CONST. FOR NONDEVIATIVE ABSORPTION  C  0021000010
C              SPIMLT= SUNSPOT NO. MULTIPLIER FOR NONDEVIATIVE ABSORPTION  C  0021000010
C              SOLEXP= SOLAR ZEN. ANG. EXPNT. FOR NONDEVIATIVE ABSORPTION  C  0021000010
C              SHIFFE= E REGION REFERENCE DOPPLER SHIFT (HZ.)  C  0021000010
C              EXSHFE= E REGION DOPPLER SHIFT EXPONENT   C  0021000010
C              SPRDE = E REGION REFERENCE DOPPLER SPREAD (HZ.)  C  0021000010
C              EXSPRE= E REGION DOPPLER SPREAD EXPONENT   C  0021000010
C              REFFQE= E REGION DOPPLER REFERENCE FREQ. (MHZ.)  C  0021000010
C              SHIFTF= F REGION REFERENCE DOPPLER SHIFT (HZ.)  C  0021000010
C              EXSHIF= F REGION DOPPLER SHIFT EXPONENT   C  0021000010
C              SPRDF = F REGION REFERENCE DOPPLER SPREAD (HZ.)  C  0021000010
C              EXSPRF= F REGION DOPPLER SPREAD EXPONENT   C  0021000010
C              REFFQF= F REGION DOPPLER REFERENCE FREQ. (MHZ.)  C  0021000010
C              ATT   = ATTENUATION (DB.)                 C  0021000010
C              ATTDIF= ATTENUATION DIFFERENCE BETWEEN STRONGEST RETURN  C  0021000010
C                      FROM IONOSPHERE AND WEAKEST RETURN RETAINED  C  0021000010
C              NPATH = NO. OF IONOSPHERE RETURNS RETAINED  C  0021000010
C              GDELAY= GROUNDWAVE DELAY (SEC.)           C  0021000010
C              GATTEN= GROUNDWAVE ATTENUATION (DB)       C  0021000010
C              TPRINT= PRINT SELECTOR FOR T(I,J) ARRAY ('ALL' OR 'NET')  C  0021000010
C              NPUNCH= PUNCH SELECTOR. PUNCHES DATA CARDS FOR USE IN  C  0021000010
C                      SIGNAL ANALYSIS PART OF SIMULATION IF NPUNCH=1.  C  0021000010
C                      DOES NOT PUNCH DATA CARDS IF NPUNCH=0.  C  0021000010
                                                        C  0021000010
C          ARRAY T(I,J) CONTAINS THE FOLLOWING PARAMETERS  C  0021000010
```

```
C               I = N: INDEX NUMBER FOR PATH                                    0021000U10
C               J = 1: NUMBER OF HOPS                                           0021000U10
C                 = 2: REGION AND MAGNETOIONIC INDICATOR (ED = E REGION =       0021000U10
C                      ORDINARY WAVE) SIMILARLY FOR EX, FO, AND FX)             0021000U10
C                 = 3: 'HIGH' (FOR HIGH RAY) OR 'LOW' (FOR LOW RAY)             0021000U10
C                 = 4: SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION)           0021000U10
C                 = 5: THETA = RAY ANGLE MEASURED FROM VERTICAL (DEGREES)       0021000U10
C                 = 6: PATH LENGTH (KM.)                                        0021000U10
C                 = 7: DELAY AT CARRIER FREQUENCY (SEC.)                        0021000U10
C                 = 8: RECEIVED CARRIER PHASE (CYCLES)                          0021000U10
C                 = 9: SIGNAL DELAY (SEC.)                                      0021000U10
C                =10: AMPLITUDE DISTORTION (SEC/MHZ)                            0021000U10
C                =11: PHASE DISTORTION (SEC/MHZ**2)                             0021000U10
C                =12: ATTENUATION (DB.)                                         0021000U10
C                =13: DOPPLER SHIFT (HZ.)                                       0021000U10
C                =14: DOPPLER SPREAD (HZ.)                                      0021000U10
                                                                                0021000U10
      PI=3.14159265359                                                          0021000U10
      TN='NITE'                                                                 0021000510
      PRFT='RET'                                                                0021000713
C           READ DATA                                                           0021000913
      READ/,FC,BW                                                               0021000913
                                                                                  FIB IS 0006 LONG
      READ/,XLONG,XLAT                                                          0021001310
      READ/,R                                                                   0021001U10
      READ/,RAG                                                                 0021002510
      READ/,ISEA                                                                0021002U10
      READ/,SPT                                                                 0021003510
      READ/,ZEN                                                                 0021003U10
      READ(5,909)TIME                                                           0021004510
      READ/,HEM,HE,FPEO                                                         0021004U12
      READ/,HFM,HF,FPFO                                                         0021005A10
      READ/,AMSMLT,SMTMLT,SOLEXP                                                0021006810
      READ/,SHIFTE,EXSHTR,SPRDE,EXSPRE,RFRFQE                                   0021007410
      READ/,SHIFTF,EXSHTR,SPRDF,EXSPRF,RFRFQF                                   0021008410
      READ/,ATTDIF                                                              0021009410
      READ/,GDELAY,GATTEN                                                       0021009C10
      READ(5,909)TPRINT                                                         0021009A10
      READ/,NPUNCH                                                              0021009U12
C           END READ DATA                                                       0021009B10
C           COMPUTE PENETRATION FREQUENCIES FOR EXTRAORDINARY WAVES             0021009B10
      CALL GETGMA(XLAT,XLONG,XLATM,XLONGM)                                      0021009B10
      SINPHI=SIN((PI/180.)*XLATM)                                               0021009B14
      XMLAT=.87*SQRT(1.+3.*(SINPHI**2))                                         0021009BA11
      FHFX=XMLAT*((6370./(6370.+HFM))**3)                                       0021009BE15
      FHEX=XMLAT*((6370./(6370.+HEM))**3)                                       0021009C210
      FPEX=.5*FHEX+SQRT((FPEO**2)+(.25*(FHEX**2)))                              0021009C514
      FPFX=.5*FHFX+SQRT((FPFO**2)+(.25*(FHFX**2)))                              0021009C815
C           END COMPUTATION OF PENETRATION FREQS. FOR EXTRAORDINARY WAVES       0021009D15
C           COMPUTE BEARING OF RECEIVER IN MAGNETIC COORDINATES                 0021009D15
      CALL ANGMAG(XLAT,XLONG,N,RAG,RAM)                                         0021009D15
C           END COMPUTATION OF RECEIVER BEARING                                 0021009D10
      WRITE(6,910)                                                              0021009D10
                                                                                  FIB IS 0006 LONG
      WRITE(6,911)                                                              0021009V12
      WRITE(6,912)FC,BW                                                         0021009DD12
      WRITE(6,913)                                                              0021009E512
      WRITE(6,915)XLONG                                                         0021009EV12
      WRITE(6,916)XLAT                                                          0021009FU12
      WRITE(6,917)R                                                             0021009F712
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      WRITE(6,918)MAG                                                  C  002100FE12
      WRITE(6,919)                                                     C  0021010512
      WRITE(6,920)                                                     C  0021010C12
      WRITE(6,921)ISEA                                                 C  0021010D12
      WRITE(6,922)                                                     C  0021011412
      WRITE(6,923)SPT                                                  C  0021011812
      IF(TIME.IS.IN)GOTO9                                              C  0021011F12
      WRITE(6,932)ZEN                                                  C  0021012110
      GOTO10                                                           C  0021012712
    9 WRITE(6,933)                                                     C  0021012F15
   10 IF(TIME.IS.IN)GOTO11                                             C  0021012C12
      WRITE(6,924)                                                     C  0021012E10
      GOTO12                                                           C  0021013212
   11 WRITE(6,925)                                                     C  0021013415
   12 WRITE(6,926)HEM                                                  C  0021013712
      SE=2.*HE                                                         C  0021013E12
      WRITE(6,927)SE                                                   C  0021013F15
      WRITE(6,928)FPEO                                                 C  0021014012
      WRITE(6,930)FPEX                                                 C  0021014D12
      WRITE(6,929)HFM                                                  C  0021015412
      SF=2.*HF                                                         C  0021015B12
      WRITE(6,927)SF                                                   C  0021015D10
      WRITE(6,928)FPFO                                                 C  0021016312
      WRITE(6,930)FPFX                                                 C  0021016A12
      WRITE(6,953)                                                     C  0021017112
      WRITE(6,954)ABSHIFT,SPTHLT,SOLEXP                                C  0021017512
      WRITE(6,945)                                                     C  0021017F12
      WRITE(6,946)SHIFTE,FXSHTF,SPRDE,EXSPRE,KFREQE                    C  0021018312
      WRITE(6,947)SHIFTF,FXSHTF,SPRDF,EXSPRF,KFREQF                    C  0021019012
      WRITE(6,900)                                                     C  0021019D12
      WRITE(6,934)                                                     C  002101A112
      WRITE(6,900)                                                     C  002101A512
      WRITE(6,935)GDELAY,GAITEN                                        C  002101A912
      WRITE(6,936)                                                     C  002101B112
C          DO 60 COMPUTES THE ARRAY T(I,J)                             C  002101B512
      DO60NHOP=1,6                                                     C  002101B512
      XNHOP=FLOAT(NHOP)                                                C  002101B810
      RH=R/XNHOP                                                       C  002101B815
      NNHOP=8*(NHOP-1)                                                 C  002101B881
      DO35I=1,8                                                        C  002101BA10
      II=I+NNHOP                                                       C  002101BB10
      T(11,I)=NHOP                                                     C  002101BC13
   35 CONTINUE                                                         C  002101BD15
      I=I+NNHOP                                                        C  002101C010
C          ASSIGN LABELS TO EACH TRANSMISSION MODE                     C  002101C112
      T(I,2)='EO'                                                      C  002101C112
      T(I,3)='LOW '                                                    C  002101C411
      I=I+1                                                            C  002101C711
      T(I,2)='EO'                                                      C  002101C813
      T(I,3)='HIGH'                                                    C  002101CB11
      I=I+1                                                            C  002101CE11
      T(I,2)='FO'                                                      C  002101CF13
      T(I,3)='LOW '                                                    C  002101D211
      I=I+1                                                            C  002101D511
      T(I,2)='FO'                                                      C  002101D813
      T(I,3)='HIGH'                                                    C  002101DB11
      I=I+1                                                            C  002101DE11
      T(I,2)='EX'                                                      C  002101DD13
      T(I,3)='LOW '                                                    C  002101E011
      I=I+1                                                            C  002101E311
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      T(I,2)='EX'                                                    C 0021E483
      T(I,3)='HIGH'                                                  C 0021E711
      I=I+1                                                          C 0021EA11
      T(I,2)='FX'                                                    C 0021EB13
      T(I,3)='LOW '                                                  C 0021EE11
      I=I+1                                                          C 0021F111
      T(I,2)='FX'                                                    C 0021F213
      T(I,3)='HIGH'                                                  C 0021F511
C          END LABEL ASSIGNMENT                                      C 0021F811
C          NMAG = 1 FOR ORDINARY WAVE, NMAG = 2 FOR EXTRAORDINARY WAVE C 0021F811
      DO50NMAG=1,2                                                   C 0021F811
      IF(NMAG-1)36,36,37                                             C 0021F910
C          SUBROUTINE 'PATH' RETURNS ARRAY P(I,J) CONTAINING SIGNAL  C 0021FA14
C          PATH LENGTH, DELAY AND DELAY DERIVATIVES                  C 0021FA14
   36 CALL PATH(RH,FREQ,FDFO,FC,P)                                   C 0021FA14
      GOTO38                                                         C 0021FE10
   37 CALL PATH(RH,FREX,FDFX,FC,P)                                   C 0021FE13
   38 DO40I=1,4                                                      C 00210201E5
      II=I+4*(NMAG-1)+NNHOP                                          C 0021020410
      T(II,4)=P(I,1)                                                 C 0021020515
      T(II,5)=(180./PI)*ARCOS(P(I,3))*P(I,1)                         C 0021020810
      T(II,6)=XNHOP*P(I,4)                                           C 0021020D13
      T(II,7)=P(I,5)*XNHOP                                           C 0021021014
      T(II,8)=FC*P(I,5)*XNHOP*(1.E+06)                               C 0021021410
      INT=I(II,8)                                                    C 0021021912
      T(II,8)=T(II,8)-FLOAT(INT)                                     C 0021021A10
      T(II,9)=(P(I,5)+FC*P(I,6))*XNHOP                               C 0021021E13
      T(II,10)=(P(I,6)+.5*FC*P(I,7))*XNHOP                           C 0021022114
      T(II,11)=((.5*P(I,7))+((FC*P(I,8))/6.))*XNHOP                  C 0021022814
C          COMPUTE ATTENUATION                                       C 0021023012
      XIND=P(1,1)-(1.E-08)                                           C 0021023012
      IF(XIND)21,21,20                                               C 0021023314
   20 TH=T(II,5)                                                     C 0021023413
      DIST=T(II,8)                                                   C 0021023611
      XMAG=3.-2.*FLOAT(NMAG)                                         C 0021023715
      A(1)=ARSMLT                                                    C 0021023914
      A(2)=SPIMLT                                                    C 0021023A15
      A(3)=SOLEXP                                                    C 0021023C10
      CALL ATTEN(XLAT,TH,RAM,SPT,ZEN,TIME,DIST,NHOP,FL,XMAG,A,ATT)   C 0021023D12
      T(II,12)=ATT                                                   C 0021024413
      GOTO22                                                         C 0021024613
   21 T(II,12)=0.                                                    C 0021024710
C          END COMPUTATION OF ATTENUATION                            C 0021024815
C          COMPUTE DOPPLER SHIFTS AND SPREADS                        C 0021024815
   22 XIND=FLOAT(I)-2.5                                              C 0021024815
      IF(XIND)23,23,24                                               C 0021024814
   23 T(II,13)=XNHOP*SHIFTE*((FC/REFREQE)**EXSHTE)*P(I,1)            C 0021024C13
      T(II,14)=SQRT(XNHOP)*SPRDE*((FC/REFREQE)**EXSPRE)*P(I,1)       C 0021025213
      GOTO25                                                         C 0021025911
   24 T(II,13)=XNHOP*SHIFTF*((FC/REFREQF)**EXSHTF)*P(I,1)            C 0021025914
      T(II,14)=SQRT(XNHOP)*SPRDF*((FC/REFREQF)**EXSPRF)*P(I,1)       C 0021025F14
   25 CONTINUE                                                       C 0021026612
C          END COMPUTATION OF SHIFTS AND SPREADS                     C 0021026612
   40 CONTINUE                                                       C 0021026612
   50 CONTINUE                                                       C 0021026813
   60 CONTINUE                                                       C 0021026A14
C          DO 70 DETERMINES MINIMUM (NONZERO) ATTENUATION            C 0021026C15
      ATTMIN=1.E+05                                                  C 0021026C15
      ATTMAX=0.                                                      C 0021026E13
      DO70I=1,48                                                     C 0021026F11
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      IF(I(I,12))70,70,61                                                  0021027610
   61 ATTMIN=AMIN1(I(I,12),ATTMIN)                                         0021027711
      ATTMAX=AMAX1(I(I,12),ATTMAX)                                         0021027810
   70 CONTINUE                                                             0021027915
      IF(ATTMAX)71,71,72                                                   0021027A10
   71 WRITE(6,950)                                                         0021027B11
      NPATH=0                                                              0021028112
      GOTO120                                                              0021028210
   72 WRITE(6,938)                                                         0021028313
      WRITE(6,939)                                                         0021028812
      WRITE(6,940)                                                         0021028A12
      IF(IPRINT.LT.PRET)GOTO101                                            0021028B12
C          DO 100 PRINTS THE COMPLETE ARRAY T(I,J)                        0021028F14
      DO100I=1,48                                                          0021028F14
      IN=I(I,1)+1.E-05                                                     0021029110
      WRITE(6,941)IN,T(I,2),T(I,3),T(I,4),T(I,5),T(I,6),T(I,7),T(I,8),T(   0021029415
     1I,9),T(I,10),T(I,11),T(I,12),T(I,13),T(I,14)                         0021029A11
  100 CONTINUE                                                             0021029B12
C          DO 110 DETERMINES NPATH AND MODIFIES ARRAY T(I,J) SO THAT       0021029B13
C          IT CONTAINS ONLY THE DESIRED RETURNS                            0021029B13
  101 ATTLIM=ATTMIN+ATTDIF                                                 0021029V13
      NPATH=0                                                              0021029E15
      DO110I=1,48                                                          0021029F13
      IF(I(I,2))110,110,102                                                0021029G10
  102 IF(I(I,12)-ATTLIM)103,103,110                                        0021029G11
  103 NPATH=NPATH+1                                                        0021029G13
      DO105J=1,14                                                          0021029G15
      T(NPATH,J)=T(I,J)                                                    0021029G10
  105 CONTINUE                                                             0021029G14
  110 CONTINUE                                                             0021029G15
      IF(IPRINT.LT.PRET)GOTO115                                            0021029U10
      GOTO115                                                              0021029Z12
C          DO 115 PRINTS THE MODIFIED ARRAY T(I,J)                        0021029Z15
  114 DO115I=1,NPATH                                                       0021029Z15
      IN=I(I,1)+1.E-05                                                     0021029A10
      WRITE(6,951)IN,T(I,2),T(I,3),T(I,5),T(I,6),T(I,7),T(I,8),T(I,9),T(   0021029B15
     1I,10),T(I,11),T(I,12),T(I,13),T(I,14)                                0021029E12
  115 CONTINUE                                                             0021029F12
      WRITE(6,900)                                                         0021029F13
      WRITE(6,952)ATTDIF                                                   0021030D12
  120 CONTINUE                                                             0021030V12
C          PUNCH DATA CARDS FOR USE IN SIGNAL ANALYSIS PART OF SIMULATOR   0021030V12
C          IF NPUNCH=1 (PUNCHES MODIFIED ARRAY T(I,J))                     0021030V12
      IF(NPUNCH-1)140,121,140                                              0021030V12
  121 PUNCH/,NCARD,N,NPATH                                                 0021030A13
                                                                                FIB IS 0008 LONG
      PUNCH 99,TIME                                                        0021031612
      PUNCH/,DELAY,GATTEN                                                  0021031D12
      DO130I=1,NPATH                                                       0021032512
      PUNCH 935,(T(I,J),J=1,7)                                             0021032610
      PUNCH 936,(T(I,J),J=8,11)                                            0021033312
      PUNCH 937,(T(I,J),J=12,14)                                           0021034012
  130 CONTINUE                                                             0021034012
  140 CONTINUE                                                             0021034F13
C          END PUNCH DATA CARDS                                            0021034F13
  900 FORMAT(1X)                                                           0021034F13
  901 FORMAT(1X,/)                                                         0021034F13
  902 FORMAT(1X,///)                                                       0021034F13
  903 FORMAT(1X,////)                                                      0021034F13
  904 FORMAT(1X,/////)                                                     0021034F13
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909  FORMAT(a6)                                                                  C  0021034F13
910  FORMAT(*6X,'SUMMARY OF TRANSMISSION PARAMETERS'///)                         C  0021034F13
911  FORMAT(1X,'SIGNAL PARAMETERS'/)                                             C  0021034F13
912  FORMAT(4X,'CARRIER FREQ. =',F7.3,' MHZ.     BANDWIDTH =',F7.2,' KHZ      C  0021034F13
     1.'/)                                                                       C  0021034F13
913  FORMAT(4X,'TRANSMITTER/RECEIVER PARAMETERS'/)                               C  0021034F13
915  FORMAT(4X,'LOCATION OF TRANSMITTER: LONGITUDE = ',F7.2,' DEGREES (          C  0021034F13
     1+ FOR WEST, - FOR EAST)')                                                  C  0021034F13
916  FORMAT(34X''LATITUDE  = '',F7.2,'' DEGREES (+ FOR NORTH, - FOR SOUTH        C  0021034F13
     1)')                                                                        C  0021034F13
917  FORMAT(4X,'DISTANCE BETWEEN TRANSMITTER AND RECEIVER =',F8.2,' KM.          C  0021034F13
     1')                                                                         C  0021034F13
918  FORMAT(4X,'DIRECTION FROM TRANSMITTER TO RECEIVER =',F6.2,' DEGREE          C  0021034F13
     1S (MEASURED FROM TRUE NORTH)')                                             C  0021034F13
919  FORMAT(4X,'TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)')            C  0021034F13
920  FORMAT(4X,'RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)')               C  0021034F13
921  FORMAT(4X,'SEA STATE =',I2/)                                                C  0021034F13
922  FORMAT(1X,'IONOSPHERE PARAMETERS'/)                                         C  0021034F13
923  FORMAT(4X,'SUNSPOT NUMBER = ',F7.2)                                         C  0021034F13
924  FORMAT(4X,'DAYTIME ELECTRON DENSITY PROFILE')                               C  0021034F13
925  FORMAT(4X,'NIGHTTIME ELECTRON DENSITY PROFILE')                             C  0021034F13
926  FORMAT(15X,'E REGION: HEIGHT OF MAX. ELECTRON DENSITY =',F7.2,' KM          C  0021034F13
     1.')                                                                        C  0021034F13
927  FORMAT(25X,'SEMITHICKNESS OF PARABOLA =',F7.2,' KM.')                       C  0021034F13
928  FORMAT(25X,'PENETRATION FREQUENCY, ORDINARY WAVE =',F7.3,' MHZ.')           C  0021034F13
929  FORMAT(15X,'F REGION: HEIGHT OF MAX. ELECTRON DENSITY =',F7.2,' KM          C  0021034F13
     1.')                                                                        C  0021034F13
930  FORMAT(25X,'PENETRATION FREQUENCY, EXTRAORDINARY WAVE =',F7.3,' MH          C  0021034F13
     1Z.')                                                                       C  0021034F13
932  FORMAT(4X,'SOLAR ZENITH ANGLE =',F7.2,' DEGREES')                           C  0021034F13
933  FORMAT(4X,'SOLAR ZENITH ANGLE = NOT APPLICABLE TO NIGHTTIME TRANSM          C  0021034F13
     1ISSION')                                                                   C  0021034F13
934  FORMAT(1X,'COMPUTED CHANNEL PARAMETERS'/)                                   C  0021034F13
935  FORMAT(1X,'GROUNDWAVE  DELAY = ',1PE10.3,' SEC.    ATTENUATION =            C  0021034F13
     1',E10.3,' DB.')                                                            C  0021034F13
936  FORMAT(1X,'IONOSPHERE RETURNS'/)                                            C  0021034F13
938  FORMAT( 3X,'MODE',3X,'SOLUTION',3X,'RAY ANGLE',3X,'PATH',4X,'CARRI          C  0021034F13
     1ER',3X,'CARRIER',3X,'  SIGNAL  ',1X,'AMPLITUDE',4X,'PHASE',5X,'ATT         C  0021034F13
     2ENUATION',2X,'DOPPLER',4X,'DOPPLER')                                       C  0021034F13
939  FORMAT(10X,'INDICATOR',2X,'(DEGREES)',2X,'LENGTH',4X,'DELAY',6X,'P          C  0021034F13
     1HASE',6X,'DELAY',3X,'DISTORTION',1X,' DISTORTION',5X,'(DB)',7X,'S          C  0021034F13
     2HIFT',5X,'SPREAD')                                                         C  0021034F13
940  FORMAT( 9X,'(UPPER SOLN)',13X,'(KM)',5X,'(SEC)',4X,'(CYCLES)',5X,'(         C  0021034F13
     1SEC)',4X,'(SEC/MHZ)',1X,'(SEC/MHZ**2)',15X,'(HZ)',7X,'(HZ)'/)             C  0021034F13
941  FORMAT(1X,I1,A2,1X,A4,4X,F3.1,7X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,          C  0021034F13
     11X,E10.3,1X,F10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)                     C  0021034F13
945  FORMAT(4X,'DOPPLER REFERENCE VALUES')                                       C  0021034F13
946  FORMAT(15X,'E REGION: SHIFT =',F8.4,' HZ,',1X,'SHIFT EXP. =',F6.3,          C  0021034F13
     1',  SPREAD =',F8.4,' HZ,',1X,'SPREAD EXP. =',F6.3,',  REF. FREQ. =',     C  0021034F13
     2F7.3,' MHZ.')                                                              C  0021034F13
947  FORMAT(15X,'F REGION: SHIFT =',F8.4,' HZ,',1X,'SHIFT EXP. =',F6.3,          C  0021034F13
     1',  SPREAD =',F8.4,' HZ,',1X,'SPREAD EXP. =',F6.3,',  REF. FREQ. =',     C  0021034F13
     2F7.3,' MHZ.')                                                              C  0021034F13
950  FORMAT(22X,'***** THERE ARE NO RETURNS FROM THE IONOSPHERE FOR THI          C  0021034F13
     1S FREQUENCY AND RANGE *****')                                              C  0021034F13
951  FORMAT(1X,I1,A2,1X,A4,4X,'NA',8X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,          C  0021034F13
     11X,E10.3,1X,F10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)                     C  0021034F13
952  FORMAT(4X,'ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THA          C  0021034F13
     1N ',F7.2,' DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED')                 C  0021034F13
953  FORMAT(4X,'NONDEVIATIVE ABSORPTION CONSTANTS')                              C  0021034F13
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954   FORMAT(15X,'MULTIPLICATIVE CONSTANT =',F8.2,', SUNSPOT NUMBER MULT          L  002,034F,13
     1IPLIER =',F9.6,', SOLAR ZENITH ANGLE EXPONENT =',F6.3)                      L  0021034F13
955   FORMAT(11,A2,A4,4E14.9)                                                     L  0021034F13
956   FORMAT(4E16.9)                                                              L  0021034F13
957   FORMAT(3E16.9)                                                              L  0021034F13
      STOP                                                                        L  0021034F13
      END                                                                         L  0021035U12

0021035213  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021000AD
0021035315  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021009C
0021035411  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002100A4
0021035613  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210084
0021035715  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210074
0021035911  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210068
0021035A13  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021005A
0021035B15  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021004D
0021035D11  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021003D
0021035E13  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210035
0021035F15  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021002D
0021036111  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210025
0021036213  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021001D
0021036315  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210013
0021036511  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210009

                                                          SEGMENT 002 IS 03CF LONG
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                                                                    START OF SEGMENT 00B
      SUBROUTINE BNGMAG(PHIT,XLAMT,R,ANGLEG,ANGLEM)                 C  00B:000010
                                                                    C  00B:000010
C         SUBROUTINE COMPUTES BEARING OF THE RECEIVER FROM THE      C  00B:000010
C         TRANSMITTER IN MAGNETIC COORDINATES                       C  00B:000010
C             PHIT  = GEOGRAPHIC LATITUDE OF TRANSMITTER (DEGREES)  C  00B:000010
C             XLAMT = GEOGRAPHIC LONGITUDE OF TRANSMITTER (DEGREES) C  00B:000010
C             R     = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM)C  00B:000010
C             ANGLEG= BEARING OF RECEIVER IN GEOGRAPHIC COORD. (DEGREES) C  00B:000010
C             ANGLEM= BEARING OF RECEIVER IN MAGNETIC COORD. (DEGREES)   C  00B:000010
                                                                    C  00B:000010
      PI=3.14159265359                                              C  00B:000010
      CC=PI/180.                                                    C  00B:000510
      GG=ANGLEG/180.                                                C  00B:000610
      IF(GG)6,5,6                                                   C  00B:000712
5     PHIR=PHIT+(R/(CC*6370.))                                      C  00B:000811
      GOTO8                                                         C  00B:000A14
6     IF(GG-1.)10,7,10                                              C  00B:000B11
7     PHIR=PHIT-(R/(CC*6370.))                                      C  00B:000C12
8     XLAMR=XLAMT                                                   C  00B:000E15
      GOTO20                                                        C  00B:000F14
10    A=CC*(90.-PHIT)                                               C  00B:001011
      B=R/6370.                                                     C  00B:001210
      G=(CC*ANGLEG)/2.                                              C  00B:001313
      D1=(A-B)/2.                                                   C  00B:001512
      S1=(A+B)/2.                                                   C  00B:001711
      W1=SIN(D1)/(TAN(G)*SIN(S1))                                   C  00B:001910
      W2=COS(D1)/(TAN(G)*COS(S1))                                   C  00B:001C15
      Z1=ATAN(W1)                                                   C  00B:002014
      Z2=ATAN(W2)                                                   C  00B:002211
      W3=(TAN(S1)*COS(Z2))/COS(Z1)                                  C  00B:002414
      C=2.*ATAN(W3)                                                 C  00B:002713
      PHIR=90.-(C/CC)                                               C  00B:002913
      XLAMR=XLAMT+((Z2-Z1)/CC)                                      C  00B:002B12
20    CALL GEOMA(PHIT,XLAMT,PHITM,XLAMTM)                           C  00B:002D14
      CALL GEOMA(PHIR,XLAMR,PHIRM,XLAMRM)                           C  00B:003012
      D2=(CC/2.)*(PHIRM-PHITM)                                      C  00B:003310
      S2=(CC/2.)*(PHIRM+PHITM)                                      C  00B:003512
      D=(CC/2.)*(XLAMTM-XLAMRM)                                     C  00B:003714
      IF(D)55,50,55                                                 C  00B:003A10
50    IF(PHIRM-PHITM)51,52,52                                       C  00B:003A15
51    ANGLEM=180.                                                   C  00B:003C11
      GOTO100                                                       C  00B:003D10
52    ANGLEM=0.                                                     C  00B:003D13
      GOTO100                                                       C  00B:003E11
55    W4=SIN(D2)/(TAN(D)*COS(S2))                                   C  00B:003E14
      W5=COS(D2)/(TAN(D)*SIN(S2))                                   C  00B:004213
      Z4=ATAN(W4)                                                   C  00B:004612
      Z5=ATAN(W5)                                                   C  00B:004715
      ANGLEM=(Z5-Z4)/CC                                             C  00B:004912
100   RETURN                                                        C  00B:004A15
      END                                                           C  00B:004B12
                                                                  SEGMENT 00B IS 005C LONG
```

```
                                                                    START OF SEGMENT OUF
      SUBROUTINE GEOMA(PHIG,XLAMG,PHIM,XLAMM)                       L  OUF:0000:0
                                                                    L  OUF:0000:0
C          SUBROUTINE CONVERTS GEOGRAPHIC COORD. TO MAGNETIC COORD. L  OUF:0000:0
C              PHIG  = GEOGRAPHIC LATITUDE (DEGREES)               L  OUF:0000:0
C              XLAMG = GEOGRAPHIC LONGITUDE (DEGREES)              L  OUF:0000:0
C              PHIM  = MAGNETIC LATITUDE (DEGREES)                 L  OUF:0000:0
C              XLAMM = MAGNETIC LONGITUDE (DEGREES)                L  OUF:0000:0
                                                                    L  OUF:0000:0
      PI=3.14159265359                                              L  OUF:0000:0
      C=PI/180.                                                     L  OUF:0000:0
      A1=C*PHIG                                                     L  OUF:0000:0
      A2=C*XLAMG                                                    L  OUF:0000:0
      A3=C*78.3                                                     L  OUF:0000:2
      A4=C*69.                                                      L  OUF:0000:4
      B1=SIN(A1)*SIN(A3)+COS(A1)*COS(A3)*COS(A2-A4)                 L  OUF:0000:0
      PHIM=ARSIN(B1)                                                L  OUF:0001:4
      B2=(COS(A1)*SIN(A2-A4))/COS(PHIM)                             L  OUF:0001:1
      PHIM=PHIM/C                                                   L  OUF:0001:3
      XLAMM=ARSIN(B2)/C                                             L  OUF:0001:5
      RETURN                                                        L  OUF:0001:5
      END                                                           L  OUF:0001:2
                                                                 SEGMENT OUF IS 0028 LONG
```

```
                                                                                START OF SEGMENT 011
      SUBROUTINEATTEN(XLATM,TH,RAM,SPT,ZEN,TIME,DIST,NHOP,FC,XMAG,A,ATT)        C  011:000010
      DIMENSION A(3)                                                            C  011:000010
                                                                                C  011:000010
C           SUBROUTINE COMPUTES TOTAL ATTENUATION OF IONOSPHERE RETURNS          C  011:000010
C               XLATM = MAGNETIC LATITUDE (DEGREES)                             C  011:000010
C               TH    = RAY ANGLE MEASURED FROM VERTICAL (DEGREES)              C  011:000010
C               RAM   = BEARING OF RECEIVER IN MAGNETIC COORD. (DEGREES)        C  011:000010
C               SPT   = SUNSPOT NUMBER                                          C  011:000010
C               ZEN   = SOLAR ZENITH ANGLE (DEGREES)                            C  011:000010
C               TIME  = TIME OF DAY ('DAY' OR 'NITE')                           C  011:000010
C               NHOP  = NUMBER OF HOPS                                          C  011:000010
C               DIST  = TOTAL SIGNAL PATH LENGTH (KM.)                          C  011:000010
C               FC    = CARRIER FREQUENCY (MHZ.)                                C  011:000010
C               XMAG  = INDICATOR (+1 FOR O-WAVE, -1 FOR X-WAVE)                C  011:000010
C               A(1)  = MULTIPLICATIVE CONST. FOR NONDEVIATIVE ABSORPTION       C  011:000010
C               A(2)  = SUNSPOT NO. MULTIPLIER FOR NONDEVIATVE ABSORPTION       C  011:000010
C               A(3)  = SOLAR ZEN. ANG. EXPNT. FOR NONDEVIATVE ABSORPTION       C  011:000010
C               ATT   = TOTAL SIGNAL ATTENUATION                                C  011:000010
                                                                                C  011:000010
      PI=3.14159265359                                                          C  011:000010
      CC=PI/180.                                                                C  011:000510
      TM='NITE'                                                                 C  011:000610
      XTH=CC*TH                                                                 C  011:000813
C         COMPUTE ABSORPTION IN D REGION                                        C  011:000915
      IF(TIME.IS.TM)GOTO9                                                       C  011:000915
      ANGIND=ATAN(-2.*TAN(CC*XLATM))                                            C  011:000B11
      XMAG=CC*RAM                                                               C  011:000E13
      XA=SIN(ATH)*COS(XRAM)*COS(ANGIND)                                         C  011:000F15
      XH=COS(XTH)*SIN(ANGIND)                                                   C  011:001314
      XI=ABS(XA*XH)                                                             C  011:001612
      XMU=ABS(XA-XH)                                                            C  011:001810
      FH=.87*SQRT(1.+3.*((SIN(CC*XLATM))**2))*((6370./6440.)**3)                C  011:001914
      B=(A(1)*(1.+A(2)*SPT)*((COS(CC*ZEN))**A(3)))/COS(XTH)                     C  011:002110
      DRI=B/((FC+XMAG*FH*XI)**2)                                                C  011:002411
      DBR=B/((FC+XMAG*FH*XU)**2)                                                C  011:002610
      DBAB=FLOAT(NHOP)*(DRI+DBR)                                                C  011:002F11
      GOTO10                                                                    C  011:003110
    9 DBAB=0.                                                                   C  011:003113
   10 CONTINUE                                                                  C  011:003211
C         END COMPUTATION OF ABSORPTION                                         C  011:003211
C         COMPUTE 1/R**2 LOSS                                                   C  011:003211
      DBR2=20.*ALOG10((4.*PI*DIST*FC)/.3)                                       C  011:003211
C         END COMPUTATION OF 1/R**2 LOSS                                        C  011:003710
C         COMPUTE ANTENNA PATTERN LOSS                                          C  011:003710
      RTH=SQRT(3.72)                                                            C  011:003710
      DBANT=-40.*ALOG10(RTH*SIN(XTH))                                           C  011:003910
C         END COMPUTATION OF ANTENNA LOSS                                       C  011:003C12
      ATT=DBAB+DBR2+DBANT                                                       C  011:003C12
      RETURN                                                                    C  011:003E11
      END                                                                       C  011:003E14
                                                                        SEGMENT 011 IS 0052 LONG
```

```
                                                                    START OF SEGMENT 013
      SUBROUTINE PATH(R,FPE,FPF,F,P)                                C 0131000010
      COMMON/ML,MEM,MP,MM/TIME                                      C 0131000010
      DIMENSION P(4,8)                                              C 0131000010
                                                                    C 0131000010
C         EVALUATES ANGLE, PATH DISTANCE, DELAY AND DELAY DERIVATIVES C 0131000010
C         FOR E REGION AND F REGION                                 C 0131000010
C             AME = THICKNESS OF E REGION (KM.)                     C 0131000010
C             MEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.) C 0131000010
C             FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)        C 0131000010
C             AMF = THICKNESS OF F REGION (KM.)                     C 0131000010
C             MFM = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.) C 0131000010
C             FPF = PENETRATION FREQUENCY OF F REGION (MHZ.)        C 0131000010
C             TIME= TIME OF DAY ('DAY' OR 'NITE')                   C 0131000010
C             R   = LENGTH OF GROUND PATH (KM.)                     C 0131000010
C             F   = FREQUENCY (MHZ.)                                C 0131000010
C             X   = COS(THETA)  THETA=RAY ANGLE MEASURED FROM VERTICAL C 0131000010
C         SUBROUTINE RETURNS P(I,J)                                 C 0131000010
C             I = 1: LOW  RAY, F REGION                             C 0131000010
C               = 2: HIGH RAY, F REGION                             C 0131000010
C               = 3: LOW  RAY, F REGION                             C 0131000010
C               = 4: HIGH RAY, F REGION                             C 0131000010
C             J = 1: SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION) C 0131000010
C               = 2: R=LENGTH OF GROUND PATH (KM.)                  C 0131000010
C               = 3: X=COS(THETA)                                   C 0131000010
C               = 4: D=LENGTH OF SIGNAL PATH (KM.)                  C 0131000010
C               = 5: T=PATH DELAY (SEC.)                            C 0131000010
C               = 6: D1T=DERIVATIVE OF DELAY (SEC./MHZ.)            C 0131000010
C               = 7: D2T=SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)  C 0131000010
C               = 8: D3T=THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)   C 0131000010
                                                                    C 0131000010
      FNE=F/FPE                                                     C 0131000010
      FNF=F/FPF                                                     C 0131000112
      IF(FNE-1.)10,10,20                                            C 0131000214
   10 XMIN=1.                                                       C 0131000315
      DO15J=1,8                                                     C 0131000413
      P(2,J)=0.                                                     C 0131000610
      P(3,J)=0.                                                     C 0131000814
      P(4,J)=0.                                                     C 0131000912
   15 CONTINUE                                                      C 0131000015
      GOTO40                                                        C 0131001010
                                                                    C 0131001013
C         DO 30 DETERMINES MINIMUM VALUE OF AE(X)                   C 0131001013
                                                                    C 0131001013
   20 XLAST=1.E-10                                                  C 0131001013
      CALL AE(FPE,F,XLAST,ALAST)                                    C 0131001213
      DO30L=1,9                                                     C 0131001511
      DIFF=1./(10.**L)                                              C 0131001610
   26 XMIN=XLAST+DIFF                                               C 0131001813
      IF(FNE*XMIN-1.)27,29,29                                       C 0131001915
   27 CALL AE(FPE,F,XMIN,AMIN)                                      C 0131001813
      IF(AMIN-ALAST)28,29,29                                        C 0131001E11
   28 XLAST=XMIN                                                    C 0131002113
      ALAST=AMIN                                                    C 0131002212
      GOTO26                                                        C 0131002311
   29 IF(L-1)36,36,37                                               C 0131002414
   36 XLAST=XMIN-DIFF                                               C 0131002515
      GOTO38                                                        C 0131002611
   37 XLAST=XMIN-2.*DIFF                                            C 0131002414
   38 CALL AE(FPE,F,XLAST,ALAST)                                    C 0131002613
```

```
 30   CONTINUE                                                          C 013:002611
                                                                        C 013:002812
C          IF MIN. AE(X) .GT. P, THERE ARE NO RETURNS FROM E REGION    C 013:002812
                                                                        C 013:002812
      IF(AMIN-R)40,40,31                                                C 013:002812
 31   DO35J=1,8                                                         C 013:002914
      P(1,J)=0.                                                         C 013:002A10
      P(2,J)=0.                                                         C 013:003014
 35   CONTINUE                                                          C 013:003112
      GOTO80                                                            C 013:003513
                                                                        C 013:003810
C          DO 50 DETERMINES X = COS(THETA) FOR E REGION LOW RAY         C 013:003810
                                                                        C 013:003810
 40   XLAST=1.E-10                                                      C 013:003810
      DO50L=1,8                                                         C 013:003813
      DIFF=1./(10.**L)                                                  C 013:003A10
 46   X=XLAST+DIFF                                                      C 013:003C13
      IF(X-XMIN)47,49,49                                                C 013:003D15
 47   CALL AE(FPE,F,X,A)                                                C 013:003E15
      IF(A-R)49,49,48                                                   C 013:004113
 48   XLAST=X                                                           C 013:004215
      GOTO46                                                            C 013:004314
 49   XLAST=X-DIFF                                                      C 013:004411
 50   CONTINUE                                                          C 013:004513
      P(1,1)=1.                                                         C 013:004714
      P(1,2)=R                                                          C 013:004815
      X=AMIN1(X,1.)                                                     C 013:004A14
      P(1,3)=X                                                          C 013:004B12
      P(1,4)=H/SQRT(1.-X**2)                                            C 013:004F11
      CALL DELAYE(R,FPE,F,X,T,D1T,D2T,D3T)                              C 013:005215
      P(1,5)=T                                                          C 013:005713
      P(1,6)=D1T                                                        C 013:005912
      P(1,7)=D2T                                                        C 013:005B11
      P(1,8)=D3T                                                        C 013:005D10
      IF(FNE-1.)200,200,60                                              C 013:005E15
                                                                        C 013:006013
C          DO 70 DETERMINES X = COS(THETA) FOR E REGION HIGH RAY        C 013:006013
                                                                        C 013:006013
 60   XLAST=(1./FNE)-1.E-10                                             C 013:006013
      DO70L=1,9                                                         C 013:006314
      DIFF=1./(10.**L)                                                  C 013:006510
 66   X=XLAST-DIFF                                                      C 013:006713
      IF(X-XMIN)69,69,67                                                C 013:006815
 67   CALL AE(FPE,F,X,A)                                                C 013:006915
      IF(A-R)69,69,68                                                   C 013:006A13
 68   XLAST=X                                                           C 013:006B15
      GOTO66                                                            C 013:006E14
 69   XLAST=X+DIFF                                                      C 013:006F11
 70   CONTINUE                                                          C 013:007013
      P(2,1)=1.                                                         C 013:007214
      P(2,2)=H                                                          C 013:007411
      X=AMIN1(X,1.)                                                     C 013:007610
      P(2,3)=X                                                          C 013:007814
      P(2,4)=H/SQRT(1.-X**2)                                            C 013:007A13
      CALL DELAYE(H,FPE,F,X,T,D1T,D2T,D3T)                              C 013:007E11
      P(2,5)=T                                                          C 013:008215
      P(2,6)=D1T                                                        C 013:008414
      P(2,7)=D2T                                                        C 013:008613
      P(2,8)=D3T                                                        C 013:008812
 80   IF(FNE-1.)90,90,100                                               C 013:008A11
```

```
 90   XMIN=1.                                                    C 0131008B12
      DO95J=1,8                                                  C 0131008C10
      P(4,J)=0.                                                  C 0131008D10
 95   CONTINUE                                                   C 0131008F13
      GOTO120                                                    C 0131009114
                                                                 C 0131009211
C          DO 110 DETERMINES MINIMUM VALUE OF AF(X)              C 0131009411
                                                                 C 0131009511
100   XLAST=(1./FNF)+1.E-11                                      C 0131009611
      CALL AF(FPE,FPF,F,XLAST,ALAST)                             C 0131009814
      DO110L=1,10                                                C 0131009915
      DIFF=1./(10.**L)                                           C 0131009A10
106   XMIN=XLAST+DIFF                                            C 0131009B13
      IF(FNF*XMIN-1.)107,109,109                                 C 0131009C15
107   CALL AF(FPE,FPF,F,XMIN,AMIN)                               C 0131009E13
      IF(AMIN-ALAST)108,109,109                                  C 0131009F14
108   XLAST=XMIN                                                 C 0131009G10
      ALAST=AMIN                                                 C 0131009H15
      GOTO106                                                    C 0131009I14
109   IF(L-1)116,116,117                                         C 0131009J11
116   XLAST=XMIN-DIFF                                            C 0131009K12
      GOTO118                                                    C 0131009L14
117   XLAST=XMIN-2.*DIFF                                         C 0131009M11
118   CALL AF(FPE,FPF,F,XLAST,ALAST)                             C 0131009N10
110   CONTINUE                                                   C 0131009O11
                                                                 C 0131009P12
C          IF MIN. AF(X) .GT. R, THERE ARE NO RETURNS FROM F REGION C 0131009Q12
                                                                 C 0131009R12
      IF(AMIN-R)120,120,111                                      C 0131009S12
111   DO115J=1,8                                                 C 0131009T14
      P(3,J)=0.                                                  C 0131009U10
      P(4,J)=0.                                                  C 0131009V14
115   CONTINUE                                                   C 0131009W11
      GOTO900                                                    C 0131009X12
                                                                 C 0131009Y15
C          DO 130 DETERMINES X = COS(THETA) FOR F REGION LOW RAY  C 0131009Z15
                                                                 C 0131009A15
120   XLAST=(1./FNF)+1.E-11                                      C 0131009B15
      DO130L=1,9                                                 C 0131009C14
      DIFF=1./(10.**L)                                           C 0131009D10
126   X=XLAST+DIFF                                               C 0131009E13
      IF(X-XMIN)127,129,129                                      C 0131009F15
127   CALL AF(FPE,FPF,F,X,A)                                     C 0131009G15
      IF(A-R)129,129,128                                         C 0131009H10
128   XLAST=X                                                    C 0131009I12
      GOTO126                                                    C 0131009J11
129   XLAST=X-DIFF                                               C 0131009K14
130   CONTINUE                                                   C 0131009LA10
      P(3,1)=1.                                                  C 0131009MC11
      P(3,2)=R                                                   C 0131009N15
      X=AMIN1(X,1.)                                              C 0131009O14
      P(3,3)=X                                                   C 0131009P12
      P(3,4)=R/SQRT(1.-X**2)                                     C 0131009Q11
      CALL DELAYF(R,FPE,FPF,F,Y,T,D1T,D2T,D3T)                   C 0131009R15
      P(3,5)=1                                                   C 0131009S10
      P(3,6)=D1T                                                 C 0131009TE15
      P(3,7)=D2T                                                 C 0131009U14
      P(3,8)=D3T                                                 C 0131009VE13
      IF(FNF-1.)200,200,140                                      C 0131009WE12
                                                                 C 0131009XE10
```

```
C          DO 150 DETERMINES X = COS(THETA) FOR F REGION HIGH RAY          L  013100E610
                                                                           L  013100E610
  140  XLAST=(1./FNF)-1.E-11                                               L  013100E610
       NOISOL=1.TO                                                         L  013100E614
       DIFF=1./(10.**I.)                                                   L  013100EA10
  146  X=XLAST-DIFF                                                        L  013100EC13
       IF(X-XMIN)149,149,147                                               L  013100ED15
  147  CALL AF(FPE,FPF,F,X,A)                                              L  013100EE15
       IF(A-R)149,149,148                                                  L  013100F210
  148  XLAST=X                                                             L  013100F312
       GOTO146                                                             L  013100F411
  149  XLAST=X+DIFF                                                        L  013100F414
  150  CONTINUE                                                            L  013100F610
       P(4,1)=1.                                                           L  013100F811
       P(4,2)=M                                                            L  013100FA15
       X=AMIN1(X,1.)                                                       L  013100FB14
       P(4,3)=X                                                            L  013100FE12
       P(4,4)=R/SQRT(1.-X**2)                                              L  013101000111
       CALL DELAYF(R,FPE,FPF,F,Y,T,D1T,D2T,D3T)                            L  013101003A15
       P(4,5)=T                                                            L  013101004810
       P(4,6)=D1T                                                          L  013101005A15
       P(4,7)=D2T                                                          L  013101006E14
       P(4,8)=D3T                                                          L  013101008E13
  900  RETURN                                                              L  013101011012
       END                                                                 L  013101011A15
                                                                  SEGMENT 013 IS 011F LONG
```

```
                                                                        START OF SEGMENT 014
      SUBROUTINE AF(FPE,F,X,A)                                      C 01410000010
      COMMON/ML/HEM,HE,HFM,TIME                                     C 01410000020
                                                                    C 01410000030
C         EVALUATES A(X)=A FOR THE E REGION                         C 01410000040
C              HE = THICKNESS OF E REGION (KM.)                     C 01410000050
C              HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.) C 01410000060
C              FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)       C 01410000070
C              X   = COS(THETA)  THETA=RAY ANGLE MEASURED FROM VERTICAL C 01410000080
C         WARNING! F.X MUST BE LESS THAN FPE                        C 01410000090
                                                                    C 01410000100
      FFE=F.X/FPE                                                   C 01410000110
      A=2.*SQRT((1./X**2)-1.)*(HFM**2.*HE+HF.FFE*ALOG((1.+FFE)/(1.-FFE))) C 01410000125
      RETURN                                                        C 01410000131
      END                                                           C 01410000140
                                                                        SEGMENT 014 IS 0014 LONG
```

```
                                                                        START OF SEGMENT 016
      SUBROUTINE AF(FPE,FHF,F,X,A)                                   L  0161000010
      COMMON/HE/HEM,HF,HFM,TIME                                      L  0161000010
C                                                                    L  0161000010
C          EVALUATES A(X)=A FOR THE F REGION                         L  0161000010
C               HE = THICKNESS OF E REGION (KM.)                     L  0161000010
C               HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)  L  0161000010
C               FPE = PENETRATION FREQUENCY OF F REGION (MHZ.)       L  0161000010
C               HF = THICKNESS OF F REGION (KM.)                     L  0161000010
C               HFM = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)  L  0161000010
C               FPF = PENETRATION FREQUENCY OF F REGION (MHZ.)       L  0161000010
C               TIME= TIME OF DAY ('DAY' OR 'NITE')                  L  0161000010
C               F   = FREQUENCY (MHZ.)                               L  0161000010
C               X   = COS(THETA) THETA=RAY ANGLE MEASURED FROM VERTICAL  L  0161000010
C          WARNING: F.X MUST BE GREATER THAN FPE AND LESS THAN FPF   L  0161000010
C                                                                    L  0161000010
      TN='NITE'                                                      L  0161000010
      FFE=F.X/FPE                                                    L  0161000213
      FFF=F.X/FPF                                                    L  0161000412
      FFF2=FFF**2                                                    L  0161000611
      FPR=(FPE**2)/(FPF**2)                                          L  0161000710
      IF(TIME.IS.TN)GOTO10                                           L  0161000910
      A=HEM-2.*HE+HE*FFE*ALOG((FFE+1.)/(FFE-1.))                     L  0161000A14
      A=A+(HFM-HEM-2.*HF*SQRT(1.-FPR))/SQRT(1.-(1./FFE**2))          L  0161001012
      A=A+2.*HF*FFF*ALOG((SQRT(1.-FPR)+SQRT(FFF2-FPR))/SQRT(1.-FFF2))  L  0161001212
      GOTO20                                                         L  0161001F10
   10 A=HEM-2.*HE**2.*HE*FFE*ALOG((FFE+1.)/(FFE-1.))                 L  0161001F13
      A=A+HFM-2.*HF*HEM-2.*HE+HF*FFF*ALOG((1.+FFF)/(1.-FFF))         L  0161002314
   20 A=A+2.*SQRT((1./X**2)-1.)                                      L  0161002615
      RETURN                                                         L  0161003112
      END                                                            L  0161003115
                                                                        SEGMENT 016 IS 003D LONG
```

```
                                                                    START OF SEGMENT 017
      SUBROUTINE DELAYE(R,FPE,F,X,T,D1T,D2T,D3T)                    C  0171000010
      COMMON/HE/HEM,HL,HF4,TIME                                     C  0171000010
                                                                    C  0171000010
C         EVALUATES DELAY AND DELAY DERIVATIVES FOR E REGION        C  0171000010
C              HE = THICKNESS OF E REGION (KM.)                     C  0171000010
C              HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)  C  0171000010
C              FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)       C  0171000010
C              R   = LENGTH OF GROUND PATH (KM.)                    C  0171000010
C              F   = FREQUENCY (MHZ.)                               C  0171000010
C              X   = COS(THETA)  THETA=RAY ANGLE MEASURED FROM VERTICAL  C  0171000010
C              T   = SIGNAL PATH DELAY (SEC.)                       C  0171000010
C              D1T = DERIVATIVE OF DELAY (SEC./MHZ.)                C  0171000010
C              D2T = SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)      C  0171000010
C              D3T = THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)       C  0171000010
                                                                    C  0171000010
      C=3.E+05                                                      C  0171000010
      FNF=F/FPE                                                     C  0171000213
      FNF2=FNF**2                                                   C  0171000315
      FNF4=FNF**4                                                   C  0171000414
      FFF=FNF*X                                                     C  0171000511
      FFF2=FFF**2                                                   C  0171000713
      X2=X**2                                                       C  0171000812
      X4=X**4                                                       C  0171000913
      V1=X2/(1.-X2)                                                 C  0171000B10
      V2=V1**.5                                                     C  0171000C14
      V3=V1**1.5                                                    C  0171000F13
      V4=1./((1.-X2)**1.5)                                          C  0171001213
      V5=1./((1.-X2)**2.5)                                          C  0171001614
      V6=X2*V5                                                      C  0171001A14
      W1=1./(1.-FFE2)                                               C  0171001C10
      W2=W1**2                                                      C  0171001D13
      W3=W1**3                                                      C  0171001E12
      Z1=X/F                                                        C  0171001F15
      T=R/(C*SQRT(1.-X2))                                           C  0171002111
      T2=T**2                                                       C  0171002410
      U1=X*T/(1.-X2)                                                C  0171002415
      U2=(1.+2.*X2)/(X2*T)                                          C  0171002710
      DEN=R*V3+2.*HEM**4.*HE*W1                                     C  0171002914
      DXDF=Z1*(R*V2*2.*HEM*4.*HF*W1)/DEN                            C  0171002D15
      DXDF2=DXDF**2                                                 C  0171003110
      D1T=U1*DXDF                                                   C  0171003315
      Z2=Z1+DXDF                                                    C  0171003511
      Z3=Z2**2                                                      C  0171003613
      Z4=1.-3.*X*F*DXDF/(1.-X2)                                     C  0171003712
      D2XDF=(Z1*(R*V4*Z4+DXDF*3.*HE*F*FNE2*W2*Z3)/DEN)              C  0171003A15
      D2XDF=D2XDF+(DXDF2/X)*(DXDF/F)                                C  0171004112
      D2T=U2*(D1T**2)+U1*D2XDF                                      C  0171004315
      Z5=D2XDF-(DXDF2/X)+(DXDF/F)                                   C  0171004613
      Z6=3.*X*DXDF2+(1.-X2)*D2XDF                                   C  0171004912
      Z7=DXDF*((1.+X2)/(1.-X2))+Z1*DXDF+X*D2XDF                     C  0171004C13
      Z8=Z1+5.*DXDF+2.*F*D2XDF                                      C  0171005110
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)-Z5   C  0171005412
     1+((Z1+DXDF)/X)-(45*R*DXDF*V6+8.*HE*X*FNE2*Z2*W2)/DEN)         C  0171005A12
      D3XDF=D3XDF+(Z1/DEN)*(R*V5*Z4+Z6-3.*R*F*DXDF*V5*Z7+32.*HE*X*F*FNE4   C  0171006213
     1*W3*(Z2**3)+8.*HL*FNE2*Z2*Z8*W2)                              C  0171006A11
      D3T=-(3.*(1.+X2+Z.*X4)*(U1T**3)/(X4*T2))+3.*U2*U1*D2T+U1*D3XDF C  0171007113
      RETURN                                                        C  0171007613
      END                                                           C  0171007910
                                                                    SEGMENT 017 IS 008E LONG
```

```
                                                                    START OF SEGMENT 018
      SUBROUTINE DELAYF(R,FPE,FPF,F,X,T,D1T,D2T,D3T)                C 0181000010
      COMMON/HE/HEM,HE,HFM,TIME                                     C 0181000010
C                                                                   C 0181000010
C         EVALUATES DELAY AND DELAY DERIVATIVES FOR F REGION        C 0181000010
C             HE = THICKNESS OF E REGION (KM.)                      C 0181000010
C             HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.) C 0181000010
C             FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)        C 0181000010
C             HF = THICKNESS OF F REGION (KM.)                      C 0181000010
C             HFM = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.) C 0181000010
C             FPF = PENETRATION FREQUENCY OF F REGION (MHZ.)        C 0181000010
C             R   = LENGTH OF GROUND PATH (KM.)                     C 0181000010
C             F   = FREQUENCY (MHZ.)                                C 0181000010
C             T   = SIGNAL PATH DELAY (SEC.)                        C 0181000010
C             D1T = DERIVATIVE OF DELAY (SEC./MHZ.)                 C 0181000010
C             D2T = SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)       C 0181000010
C             D3T = THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)        C 0181000010
                                                                    C 0181000010
      TN='NITE'                                                     C 0181000010
      C=3.E+05                                                      C 0181000413
      FNF=F/FPE                                                     C 0181000413
      FNF2=FNE**2                                                   C 0181000515
      FFF=FNE*X                                                     C 0181000614
      FFF2=FFE**2                                                   C 0181000810
      FNF=F/FPF                                                     C 0181000815
      FNF2=FNF**2                                                   C 0181000A11
      FFF=FNF*X                                                     C 0181000B10
      FFF2=FFF**2                                                   C 0181000C12
      FFF3=FFF**3                                                   C 0181000D11
      FFF4=FFF**4                                                   C 0181000E14
      FFF5=FFF4*FNF                                                 C 0181001011
      FFF6=FFF2*FNF                                                 C 0181001111
      FPR=FPE/FPF                                                   C 0181001213
      FPR2=FPR**2                                                   C 0181001315
      RFPR=SQRT(1.-FPR2)                                            C 0181001414
      X2=X**2                                                       C 0181001613
      X4=X**4                                                       C 0181001714
      V1=1./SQRT(1.-X2)                                             C 0181001911
      V2=X*V1                                                       C 0181001B12
      V3=X*X2*(V1**3)                                               C 0181001C14
      V4=V1**3                                                      C 0181001F11
      W1=1./(FFF2-1.)                                               C 0181002014
      W2=W1**2                                                      C 0181002211
      W3=W1**3                                                      C 0181002310
      W4=(FFF2/(FFF2-1.))**1.5                                      C 0181002413
      W5=W1**2.5                                                    C 0181002613
      W6=W1*W5                                                      C 0181002813
      W7=1./(1.-FFF2)                                               C 0181002C15
      W8=W7**2                                                      C 0181002E12
      W9=W7**3                                                      C 0181002F11
      W10=FFF2-FPR2                                                 C 0181003014
      W11=SQRT(W10)                                                 C 0181003210
      T=R*V1/C                                                      C 0181003413
      T2=T**2                                                       C 0181003512
      Z1=X/F                                                        C 0181003611
      IF(TIME.IS.TN)GOTO10                                          C 0181003713
      C1=2.*(HFM-HFM**2.*HF*RFPR)                                   C 0181003911
      C2=RFPR*W11+W10                                               C 0181003B12
      B=2.*HFM+4.*HE*W1+C1*W4-(4.*HF*FFF3/C2)-4.*HF*FFF3*W7         C 0181003F11
      DXDF=Z1*(R*V2-B)/(R*V3+B)                                     C 0181004010
```

```
      GOTO20                                                                    C  0181004B15
10    8=2.*HFN+8.*HE*N1-8.*HF*N2                                                C  0181004L12
      DXDF=Z1*(R*V2**B)/(R*V3*B)                                                C  0181005113
20    DIF*X*T+DXDF/(1.-X2)                                                      C  0181005512
      Z2=Z1+DXDF                                                                C  0181005810
      DEN=R*V3*B                                                                C  0181005V12
      IF(TIME.IS.TN)GOTO30                                                      C  0181005B11
      ZU1=2.*(RFPR/W11)                                                         C  0181005L15
      C3=L2**2                                                                  C  0181005L14
      FFE6=FFE2*FNF                                                             C  0181005P15
      D2DF=Z2*(8.*HE*X*FNE2*W2-3.*C1*FFE6*W5-(12.*HF*FFF6/C2)+(4.*HF*FFF        C  0181006111
     15*ZU1/C3)-12.*HF*FFF6*W7-8.*HF*FFF5*W8)                                   C  0181006V13
      D2XDF=(DXDF**2/X)-(DXDF/F)+((R*Z1*DXDF*V8*(1.-(3.*X*F*DXDF/(1.-X2        C  0181007U13
     1)))-Z2*U8DF)/DEN)                                                         C  0181007614
      GOTO40                                                                    C  0181007V15
30    D2XDF=(DXDF**2/X)-(DXDF/F)+Z1*((R*DXDF*V8*(1.-(3.*X*F*DXDF/(1.-X2        C  0181007A12
     1)))+16.*HE*F*FNE2*W2*(Z2**2)+8.*HF*F*FNF2*W8*(Z2**2))/DEN)                C  0181008U12
40    D2T=((D*T**2)*(1.+2.*X2)/(X2*T))+(D2XDF*X*T/(1.-X2))                      C  0181008V15
      IF(TIME.IS.TN)GOTO50                                                      C  0181008F14
      C4=C2**3                                                                  C  0181009112
      ZU2=ZU1**2                                                                C  0181009215
      FFF7=FFF3*FFF2*FNF2                                                       C  0181009410
      FFF8=FFF*FNF2                                                             C  0181009515
      FFF9=FFF3*FNF2                                                            C  0181009711
      W12=1./(W11**3)                                                           C  0181009813
      ZU3=2.*Z1+3.*DXDF                                                         C  0181009A12
      ZU4=3.*Z1+2.*DXDF                                                         C  0181009C14
      ZU5=2.*Z1+DXDF                                                            C  0181009F10
      ZU6=5.*Z1+4.*DXDF                                                         C  0181009G15
      ZU7=5.-FFF2                                                               C  0181009J11
      ZU8=3.-FFF2                                                               C  0181009K13
      ZU9=D2XDF                                                                 C  0181009S15
      ZU10=3.*ZU4-FFF2*ZU6                                                      C  0181009U14
      D2HDF=((DXDF-Z1+F*D2XDF)/(F*Z2))*U8DF+Z2*(-8.*HE*FNE2*((FFE2*ZU3+Z        C  0181009V10
     1U5)*W3+FFF8*3.*C1*(ZU4+FFE2*ZU3)*W6+(12.*HF*FFF8*(-ZU4+FFF2*Z2+ZU        C  0181009Y11
     21)/C2)+(4.*HF*FFF9*ZU6*ZU1/C3)-(4.*HF*FFF7*RFPR*Z2*W12/C3)-(8.*HF        C  0181009B15
     3+FFF7*Z2*ZU2/C4)-16.*HF*FFF9*Z2*ZU8*W9-4.*HF*FFF8*ZU10*W8)               C  0181009H15
      DXDF2=DXDF**2                                                             C  0181009V15
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)+(         C  0181009C10
     11./DEN)*(-3.*R*X*DXDF*(V1**5)*(((1.+4.*X2)/(1.-X2)))*DXDF2+Z1*DXDF        C  0181009D10
     2+3.*X*D2XDF)+(R*V4/5)*(((1.+2.*X2)/(1.-X2)))*DXDF2-Z1*DXDF*X*ZU9)-        C  0181009E13
     3(1./F)*(2.*DXDF*Z1+2.*F*D2XDF-(DXDF2/Z1))*D2HDF-Z2*D2HDF)                 C  0181009E14
      GOTO60                                                                    C  0181009E11
50    DXDF2=DXDF**2                                                             C  0181009E12
      ZN1=D2XDF-(DXDF2/X)+(DXDF/F)                                              C  0181009E11
      ZN2=Z1+3.*DXDF+2.*F*D2XDF                                                 C  0181009E13
      ZN3=V1**5                                                                 C  0181009F12
      ZN4=Z2**3                                                                 C  0181009F15
      ZZ=Z1-DXDF                                                                C  0181009F11
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)-(         C  0181009F11
     1ZZ*ZN1/X)-(ZN1/DEN)*(R*3.*X2*DXDF*ZN3-16.*HF*X*HNE2*Z2*W2-8.*HF*X        C  0181009FA11
     2*FNF2*ZZ*W8)+(Z1/DEN)*(R*ZN3*((1.-(3.*X*F*DXDF/(1.-X2)))*(3.*X*DXD       C  0181010J11
     3F2+(1.-X2)*D2XDF)-3.*R*F*DXDF*ZN3*(((1.+X2)/(1.-X2))*DXDF2+Z1*DXD        C  0181010B12
     4F+X*D2XDF)-8.*HE*X*F*(FNE2**2)*ZN4*W3+32.*HF*X*F*(FNF2**2)*ZN4*W         C  0181011C10
     59+16.*HE*FNE2*Z2*ZN2*W2+A.*HF*FNF2*Z2*ZN2*W8)                             C  0181011B13
60    D3T=(3./(X4+T2))*(1.+X2+2.*X4)*(D[F**3)+(3./(X2+T))*(1.+2.*X2)*D          C  0181012J13
     111*D2T+(X*T/(1.-X2))*D3XDF                                                C  0181012A14
      RETURN                                                                    C  0181012L14
      END                                                                       C  0181012P11
                                                                  SEGMENT 018 IS 014B LONG
```

APPENDIX C

COMPUTER GENERATION OF RANDOM GAINS

APPENDIX C
COMPUTER GENERATION OF RANDOM GAINS

For each ionosphere return, it is desired to generate a stationary, zero-mean Gaussian sequence $\tilde{G}_j(nT_o)$, $n = 0,1,\cdots$, having the correlation function

$$\begin{aligned} C_{\tilde{G}_j}(nT_o) &= E\,\tilde{G}_j(nT_o + mT_o)\,\tilde{G}_j^*(mT_o) \\ &= A_j^2\left[1 + (2\pi\rho\sigma_j T_o|n|) + \frac{1}{3}(2\pi\rho\sigma_j T_o|n|)^2\right] \\ &\quad \cdot\exp\left[-2\pi\rho\sigma_j T_o|n| + 2\pi i\nu_j T_o n\right] \qquad (C.1) \end{aligned}$$

$$n = 0, 1,\cdots$$
$$j = 1,\cdots,N\ .$$

We will generate this sequence by passing a white noise process through an appropriately chosen discrete-time linear filter [26]. Denoting the filter input by $\{\xi_j(n)\}$ and the impulse response by $\{a_j(n)\}$ we have

$$\tilde{G}_j(nT_o) = \exp[2\pi i\nu_j nT_o]\sum_{m=0}^{\infty}\xi_j(n-m)\,a_j(m) \qquad (C.2)$$

$$j=1,\cdots N$$

where

$$a_j(m) = \begin{cases} A_j\left(\frac{8}{3}\pi\rho\sigma_j T_o\right)^{1/2}\left(2\pi\rho\sigma_j mT_o\right)^2\exp\left[-2\pi\rho\sigma_j mT_o\right], & m\geq 0 \\ 0\ , \qquad m<0\ , & j=1,\cdots N. \end{cases} \qquad (C.3)$$

The random variables $\{\xi_j(n)\}$ are independent, identically-distributed, complex Gaussian variables. They may be expressed as

$$\xi_j(n) = \xi_j^{(1)}(n) + i\xi_j^{(2)}(n) \quad , \quad n=0,1,\cdots \tag{C.4}$$

where $\xi_j^{(1)}(n)$ and $\xi_j^{(2)}(n)$ are, respectively, the real and imaginary parts of $\xi_j(n)$. For each n the variables $\xi_j^{(1)}(n)$ and $\xi_j^{(2)}(n)$ are independent, zero-mean, unit variance, real Gaussian variables, and both components of $\xi_j(n)$ are independent of each other for all values of n.

To generate $\tilde{G}_j(nT_o)$ directly using the convolution (C.2), we would have to approximate the infinite summation with a sufficiently accurate truncated summation. Because $\sigma_j T_o$ is very small, the length of this truncated summation would be large and we would expect error accumulation and an attendant loss of accuracy. However, our particular choice of an approximating correlation function (C.1) allows us to generate the random sequence $G_j(nT_o)$ iteratively. This was, of course, the primary consideration in selecting this particular approximation. The iterative scheme is given below

$$\begin{aligned}
\tilde{G}_j(nT_o + 3T_o) = {} & K_j \exp[-2\pi\rho\sigma_j T_o + 2\pi i\nu_j(n+3)T_o]\ \xi_j(n+2) \\
& + K_j \exp[-4\pi\rho\sigma_j T_o + 2\pi i\nu_j(n+3)T_o]\ \xi_j(n+1) \\
& + \exp[-6\pi\rho\sigma_j T_o + 6\pi i\nu_j T_o]\ \tilde{G}_j(nT_o) \\
& - 3\exp[-4\pi\rho\sigma_j T_o + 4\pi i\nu_j T_o]\ \tilde{G}_j(nT_o + T_o) \\
& + 3\exp[-2\pi\rho\sigma_j T_o + 2\pi i\nu_j T_o]\ \tilde{G}_j(nT_o + 2T_o)
\end{aligned} \tag{C.5}$$

$$\begin{matrix} n \geq 0 \\ j=1,\cdots N \end{matrix}$$

where

$$K_j = \frac{2}{\sqrt{3}} A_j (2\pi\rho\sigma_j T_o)^{5/2} \quad , \quad j=1,\cdots N \tag{C.6}$$

and, as discussed earlier, we will choose $\rho = 2.1$. To start this iteration we need $\tilde{G}_j(0)$, $\tilde{G}_j(T_o)$ and $\tilde{G}_j(2T_o)$ given by

$$\tilde{G}_j(0) = A_j W_j(1) \tag{C.7}$$

$$\tilde{G}_j(T_o) = \exp[2\pi i \nu_j T_o] \cdot \left[\gamma_j(1) W_j(2) + \lambda_j(1) \tilde{G}_j(0) \right] \tag{C.8}$$

$$\tilde{G}_j(2T_o) = \exp[4\pi i \nu_j T_o] \cdot \left| \gamma_j(2) W_j(3) + K_j \exp[-2\pi\rho\sigma_j T_o] \xi_j(1) \right. \tag{C.9}$$

$$\left. + \lambda_j(1)[1-\lambda_j(2)] \tilde{G}_j(T_o) + \lambda_j(2) \tilde{G}_j(0) \right|$$

$$j=1,\cdots N.$$

Here $W_j(1)$, $W_j(2)$, $W_j(3)$, and $\xi_j(1)$ are independent, complex, Gaussian variables whose real and imaginary parts are independent and have zero mean and unit variance. The other terms appearing in (C.7)-(C.9) are

$$\lambda_j(1) = [1+(4\pi\rho\sigma_j T_o) + \frac{1}{3}(4\pi\rho\sigma_j T_o)^2] \cdot \exp[-4\pi\rho\sigma_j T_o] \tag{C.10}$$

$$\lambda_j(2) = [1+(8\pi\rho\sigma_j T_o) + \frac{1}{3}(8\pi\rho\sigma_j T_o)^2] \cdot \exp[-8\pi\rho\sigma_j T_o] \tag{C.11}$$

$$\gamma_j(1) = A_j [1-\lambda^2{}_j(1)]^{1/2} \tag{C.12}$$

$$\gamma_j(2) = A_j \left[\left(1-\lambda^2{}_j(1)\right) \left(1-\lambda^2{}_j(2)\right) - \frac{4}{3} \left(2\pi\rho\sigma_j T_o\right)^5 \exp[-4\pi\rho\sigma_j T_o] \right]^{1/2} \tag{C.13}$$

The coefficients A_j are determined by the attenuation of each return via Eqn. (43).

It remains to generate the sequence of independent, Gaussian variables $\{\xi_j(n)\}$. We begin by using the congruence method to generate a sequence of uniform random variables. This method generates a sequence of random integers via the iterative scheme [24]

$$Y_{n+1} = a \cdot Y_n \ [\text{mod } M] \ . \tag{C.14}$$

For the quantities a and M, we have found $a = 7^5$ and $M = 2^{31}$ to be effective. Multiplication is done using double precision arithmetic but the final result is expressed as a single precision variable. Upon division of $\{Y_n\}$ by 2^{31} a sequence of independent, uniform [0,1] variables is produced. This sequence is then transformed into a sequence of independent, complex, Gaussian variables. If u_1 and u_2 are real, independent, uniform variables, the real and imaginary parts of the variable ξ defined by

$$\xi = (-2 \ln u_1)^{\frac{1}{2}} \exp[2\pi i u_2] \tag{C.15}$$

are independent and Gaussian with zero mean and unit variance [24]. Thus, we obtain the desired sequence of complex Gaussian variables for use as the filter input in Eqn. (C.5).

APPENDIX D
SIGNAL ANALYSIS PROGRAM

```
B6700/B7700  F O R T R A N  C O M P I L A T I O N  M A R K  2.8.V6V   MONDAY, 09/26/77   02:22 PM

FILE  2(KIND=TAPE,TITLE="HFOUTPUT.")                                  ; 00000001
FILE  10(KIND=TAPE,TITLE="HFINPUT.")                                  ; 00000002
                                                                        START OF SEGMENT 002
                                                                      FORMAT SEGMENT IS OVCV LONG
                                                                      FORMAT SEGMENT IS OVEA LONG
      DIMENSION T(10,14),D2(10),D3(10),N1(10)                         ; 002:000010
      COMPLEX X(10,4),G(90),Y(10),R(10),D(10,801),B(10),SIGIN(5000),AI ; 002:000010
      COMPLEX PRTOUT(10000),GNDOUT,SIGOUT                             ; 002:000010
      DOUBLE PRECISION C(10,7)                                        ; 002:000010
                                                                      ; 002:000010
C         THIS PROGRAM IS THE SIGNAL ANALYSIS PART OF THE SIMULATOR   ; 002:000010
                                                                      ; 002:000010
C             FC    = CARRIER FREQUENCY (MHZ.)                      ; 002:000010
C             BW    = BANDWIDTH (KHZ.)                              ; 002:000010
C             RANGE = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM.) ; 002:000010
C             TIME  = TIME OF DAY ('DAY' OR 'NITE')                 ; 002:000010
C             NPATH = NUMBER OF RETAINED IONOSPHERE RETURNS         ; 002:000010
C             NUMIN = NUMBER OF INPUT SAMPLES                       ; 002:000010
C             NTAPE = TAPE WRITE SELECTOR (WRITES TAPE IF NTAPE=1).  ; 002:000010
C             NGAIN = RANDOM GAIN SELECTOR (MULTIPLIES BY RANDOM GAINS ; 002:000010
C                     ONLY IF NGAIN=1)                              ; 002:000010
C             NPRINT= NUMBER OF OUTPUT VALUES TO BE PRINTED         ; 002:000010
C             NCONVL= ONE-SIDED LENGTH OF TRUNCATED IMPULSE RESPONSE ; 002:000010
C             GDELAY= GROUNDWAVE DELAY (SEC.)                       ; 002:000010
C             GATTEN= GROUNDWAVE ATTENUATION (DB)                   ; 002:000010
C             ALPHA = PARAMETER USED IN APPROXIMATE SPECTRUM        ; 002:000010
C             NGND  = GROUNDWAVE DELAY SHIFT PARAMETER              ; 002:000010
C             ISEED = SEED VALUE USED IN RANDOM NUMBER GENERATOR. IT ; 002:000010
C                     IS SET = 7**5 FOR FIRST CALL AND IT IS REPLACED ; 002:000010
C                     BY A NEW VALUE TO BE USED IN SUBSEQUENT CALLS   ; 002:000010
C             NIMIN = MINIMUM DELAY SHIFT PARAMETER                 ; 002:000010
C             NIMAX = MAXIMUM DELAY SHIFT PARAMETER                 ; 002:000010
C             GNDOUT= DELAYED, ATTENUATED GROUNDWAVE                ; 002:000010
C             SIGOUT= OUTPUT SIGNAL (GROUNDWAVE+IONOSPHERE RETURNS) ; 002:000010
                                                                      ; 002:000010
C         THE ARRAY D(I,N) CONTAINS TERMS IN IMPULSE RESPONSE. IT IS  ; 002:000010
C         PRODUCED BY THE SUBROUTINE 'IMPULS'.                        ; 002:000010
                                                                      ; 002:000010
C         THE ARRAY D2(I) CONTAINS AMPLITUDE DISTORTION COEFFICIENTS  ; 002:000010
C         THE ARRAY D3(I) CONTAINS PHASE DISTORTION COEFFICIENTS      ; 002:000010
C         THE ARRAY N1(I) CONTAINS SIGNAL DELAY SHIFT PARAMETERS      ; 002:000010
C         THE ARRAY SIGIN(N) CONTAINS INPUT SIGNAL SEQUENCE           ; 002:000010
                                                                      ; 002:000010
C         THE ARRAY G(I) CONTAINS THE RANDOM GAINS. IT IS PRODUCED    ; 002:000010
C         BY THE SUBROUTINE 'GAIN'.                                   ; 002:000010
                                                                      ; 002:000010
C         THE ARRAYS C(I,J), X(I,J) AND Y(I) CONTAIN VARIABLES USED IN ; 002:000010
C         SUBROUTINE 'GAIN', THEY ARE SET = 0 FOR FIRST CALL.         ; 002:000010
                                                                      ; 002:000010
C         THE ARRAY T(I,J) CONTAINS THE IONOSPHERE CHANNEL PARAMETERS. ; 002:000010
C         ITS ENTRIES ARE:                                            ; 002:000010
C             I =    INDEX NUMBER FOR PATH                          ; 002:000010
C             J = 1: NUMBER OF HOPS                                 ; 002:000010
C               = 2: REGION AND MAGNETOIONIC INDICATOR (E0 = E REGION - ; 002:000010
C                    ORDINARY WAVE; SIMILARLY FOR EX, F0, AND FX)     ; 002:000010
C               = 3: 'HIGH' (FOR HIGH RAY) OR 'LOW' (FOR LOW RAY)     ; 002:000010
C               = 4: SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION)   ; 002:000010
C               = 5: THETA = RAY ANGLE MEASURED FROM VERTICAL (DEGREES) ; 002:000010
```

```
C                 = 6: PATH LENGTH (KM.)                                    C 0021000010
C                 = 7: DELAY AT CARRIER FREQUENCY (SEC.)                    C 0021000010
C                 = 8: RECEIVED CARRIER PHASE (CYCLES)                      C 0021000010
C                 = 9: SIGNAL DELAY (SEC.)                                  C 0021000010
C                =10: AMPLITUDE DISTORTION (SEC/MHZ)                        C 0021000010
C                =11: PHASE DISTORTION (SEC/MHZ**2)                         C 0021000010
C                =12: ATTENUATION (DB.)                                     C 0021000010
C                =13: DOPPLER SHIFT (HZ.)                                   C 0021000010
C                =14: DOPPLER SPREAD (HZ.)                                  C 0021000010
C          WARNING: NPATH MUST NOT EXCEED 10                                C 0021000010
C          WARNING: NCONVL MUST NOT EXCEED 400 (HOWEVER SEE WARNING IN      C 0021000010
C                   SUBROUTINE 'IMPULS').                                   C 0021000010
C          WARNING: NPRINT MUST NOT EXCEED 10,000                           C 0021000010
                                                                            C 0021000010
      AI=(0.,1.)                                                            C 0021000010
      PI=3.14159265359                                                      C 0021000111
      ALPHA=2.1                                                             C 0021000610
      TNTIF='WITE'                                                          C 0021000813
C          INITIALIZE VARIABLES AND ARRAYS FOR SUBROUTINE 'GAIN'            C 0021000A13
      ISEED=7**5                                                            C 0021000A13
      NCALL=0                                                               C 0021000C13
      DO101=1,10                                                            C 0021000D11
      Y(I)=(0.,0.)                                                          C 0021000E10
      DO6J=1,7                                                              C 0021001010
      C(I,J)=CMPLX(0.)                                                      C 0021001110
    4 CONTINUE                                                              C 0021001315
      DO6J=1,4                                                              C 0021001610
      X(I,J)=(0.,0.)                                                        C 0021001710
    6 CONTINUE                                                              C 0021001A11
   10 CONTINUE                                                              C 0021001C12
C          END INITIALIZATION FOR SUBROUTINE 'GAIN'                         C 0021001E13
C          READ INPUT DATA                                                  C 0021001E13
      READ(,)FL,BW,RANGE,NPATH                                              C 0021001E13
                                                                              FIB IS 0006 LONG
      READ(5,900)TIME                                                       C 0021002610
      READ(,)GDELAY,GATTEN                                                  C 0021003312
      READ(,)NCONVL                                                         C 0021003D10
      READ(,)NUMIN                                                          C 0021004310
      READ(,)NGAIN                                                          C 0021004B10
      READ(,)NPRINT                                                         C 0021005310
      READ(,)NTAPE                                                          C 0021005B10
      DO201=1,NPATH                                                         C 0021006310
      READ(5,955)(T(I,J),J=1,7)                                             C 0021006810
      READ(5,956)(T(I,J),J=8,11)                                            C 0021007412
      READ(5,957)(T(I,J),J=12,14)                                           C 0021008212
   20 CONTINUE                                                              C 0021009012
C          END READ INPUT DATA                                              C 0021009213
C          DETERMINE IF LIMITS FOR NPATH, NCONVL OR NPRINT ARE EXCEEDED     C 0021009213
      IF(NPATH-10)22,22,21                                                  C 0021009213
   21 WRITE(6,901)                                                          C 0021009315
                                                                              FIB IS 0006 LONG
      GOTO999                                                               C 0021009812
   22 IF(NCONVL-400)26,26,24                                                C 0021009815
   24 WRITE(6,902)                                                          C 0021009A12
   26 IF(NPRINT-10000)28,28,27                                              C 0021009E12
   27 WRITE(6,905)                                                          C 0021009F15
      GOTO999                                                               C 002100A412
C          END DETERMINATION OF EXCEEDED LIMITS                             C 002100A415
C          THE SAMPLE INTERVAL TO IS HALF THE NYQUIST SAMPLE INTERVAL       C 002100A415
   28 TO=1./(2W*2000.)                                                      C 002100A415
```

```
C          INITIALIZE ARRAYS FOR CONVOLUTION                    C 0021000A014
      DO40I=1,NPATH                                             C 0021000A014
      D2(I)=T(I,10)**2.*PI*(BW**2)                              C 0021000A810
      D3(I)=T(I,11)**2.*PI*(BW**3)*(1.E-03)                     C 0021000AC12
      NN1=ABS(T(I,9))/TO                                        C 0021000B212
      XN1=TO-IO*FLOAT(NN1)                                      C 0021000B510
      IF(I(I,V))32,34,34                                        C 0021000B614
   32 NN2=-1                                                    C 0021000B814
      GOTO36                                                    C 0021000BB13
   34 NN2=1                                                     C 0021000BA10
   36 IF(XN1-.5)38,38,39                                        C 0021000BA14
   38 N1(1)=NN1*NN2                                             C 0021000BD14
      GOTO40                                                    C 0021000C010
   39 N1(I)=(NN1+1)*NN2                                         C 0021000C013
   40 CONTINUE                                                  C 0021000C311
C          END INITIALIZATION OF ARRAYS FOR CONVOLUTION         C 0021000C512
C          DETERMINE NGND                                       C 0021000C512
      NG1=GDELAY/TO                                             C 0021000C512
      XG1=TO-IO*FLOAT(NG1)                                      C 0021000C815
      IF(XG1-.5)42,42,44                                        C 0021000C813
   42 NGND=NG1                                                  C 0021000CA14
      GOTO45                                                    C 0021000C813
   44 NGND=NG1+1                                                C 0021000C110
C          END DETERMINATION OF NGND                            C 0021000CD12
C          DETERMINE MAX. AND MIN. DELAY SHIFT PARAMETERS       C 0021000CD12
   45 LLIMIT=2*NCONVL+1                                         C 0021000CD12
      N1MAX=NGND                                                C 0021000CF11
      N1MIN=NGND                                                C 0021000D010
      DO48I=1,NPATH                                             C 0021000D015
      N1MAX=MAXO(N1MAX,N1(I))                                   C 0021000D210
      N1MIN=MINO(N1MIN,N1(I))                                   C 0021000D514
   48 CONTINUE                                                  C 0021000D912
C          END DETERMINATION OF MAX. AND MIN. SHIFT PARAMETERS  C 0021000DB13
C          READ INITIAL INPUT SEQUENCE                          C 0021000DB13
      NREAD=N1MAX-N1MIN+LLIMIT                                  C 0021000DB13
      NREAD1=NREAD-1                                            C 0021000DD13
      IF(NREAD-5000)50,50,55                                    C 0021000DE13
   55 WRITE(6,903)                                              C 0021000E010
      GOTO999                                                   C 0021000E412
   50 DO60L=1,NREAD                                             C 0021000E415
      XXLL=FLOAT(L-1)                                           C 0021000E610
      READ(10,904)SIGIN(L)                                      C 0021000E711
   60 CONTINUE                                                  C 0021000F212
C          END READ INITIAL INPUT SEQUENCE                      C 0021000F413
      NSHIFT=0                                                  C 0021000F413
      CALL IMPULS(NCONVL,NPATH,D2,D3,D)                         C 0021000F511
                                                                C 0021000F815
C          DO 200 IS THE FUNDAMENTAL LOOP FOR DETERMINING THE OUTPUT C 0021000F815
C          FOR EACH INDEX N (N CORRESPONDS TO AN ARGUMENT OF (N-1)*TO). C 0021000F815
                                                                C 0021000F815
      DO200N=1,NUMIN                                            C 0021000F815
C          DETERMINE DISTORTED IONOSPHERE RETURNS               C 0021000FA10
      DO70I=1,NPATH                                             C 0021000FA10
      B(I)=(0.,0.)                                              C 0021000FB10
   70 CONTINUE                                                  C 0021000FD10
      DO100L=1,LLIMIT                                           C 0021000FF11
      LU=2*NCONVL-1+2                                           C 0021010010
      LL=L-NCONVL+N-NSHIFT-1                                    C 0021010213
      DO80I=1,NPATH                                             C 0021010512
      LS=LL-N1(I)                                               C 0021010610
```

```
      IF(LS)80,80,75                                                    0021010
   75 B(I)=B(I)+D(I,LD)*SIGIN(LS)                                       0021010
   80 CONTINUE                                                          0021011
  100 CONTINUE                                                          0021011
      DO110I=1,NPATH                                                    0021011
      B(I)=H(I)*CEXP(-AI*2.*PI*T(I,8))                                  0021011
  110 CONTINUE                                                          0021012
C          END DETERMINATION OF DISTORTED IONOSPHERE RETURNS            0021012
C          MULTIPLY BY RANDOM GAINS IF NGAIN=1                          0021012
      IF(NGAIN-1)120,115,120                                            0021012
  115 CALL GAIN(ISEED,NCALL,BW,ALPHA,NPATH,I,G,X,Y,Q)                   0021012
      DO120I=1,NPATH                                                    0021012
      B(I)=B(I)*G(I)                                                    0021012
  120 CONTINUE                                                          0021013
C          END MULTIPLY BY RANDOM GAINS                                 0021013
C          DETERMINE DELAYED, ATTENUATED GROUNDWAVE                     0021013
      GNDOUT=(0.,0.)                                                    0021013
      LG=N-NGND-NSHIFT                                                  0021013
      IF(LG)125,125,122                                                 0021013
  122 GNDOUT=SIGIN(LG)*(10.**(-GATTEN/20.))                             0021013
  125 CONTINUE                                                          0021013
C          END DETERMINATION OF DELAYED, ATTENUATED GROUNDWAVE          0021013
C          ADD GROUNDWAVE AND IONOSPHERE RETURNS                        0021013
      SIGOUT=GNDOUT                                                     0021013
      DO130I=1,NPATH                                                    0021013
      SIGOUT=SIGOUT+B(I)                                                0021014
  130 CONTINUE                                                          0021014
C          END ADDITION OF GROUNDWAVE AND IONOSPHERE RETURNS            0021014
C          WRITE TAPE (FILE 2) IF NTAPE = 1                             0021014
      IF(NTAPE-1)134,132,134                                            0021014
  132 WRITE(2,904)SIGOUT                                                0021014
  134 CONTINUE                                                          0021015
C          END WRITE TAPE                                               0021015
      IF(NPRINT-N)136,135,135                                           0021015
  135 PRTOUT(N)=SIGOUT                                                  0021015
  136 CONTINUE                                                          0021015
      IF(N-N1MAX-NCONVL)200,200,140                                     0021015
  140 NSHIFT=NSHIFT+1                                                   0021015
      DO150L=1,NREAD1                                                   0021015
      LL=L+1                                                            0021015
      SIGIN(L)=SIGIN(LL)                                                0021015
  150 CONTINUE                                                          0021015
      IF(NREAD+NSHIFT-NUMIN)154,154,156                                 0021015
  154 READ(10,904)SIGIN(NREAD)                                          0021015
      GOTO200                                                           0021016
  156 SIGIN(NREAD)=(0.,0.)                                              0021016
  200 CONTINUE                                                          0021016
C          PRINT OUTPUT DATA SUMMARY IF NPRINT .GE. 3                   0021016
      IF(NPRINT-2)999,999,201                                           0021016
  201 WRITE(6,910)                                                      0021017
      WRITE(6,911)                                                      0021017
      IF(TIME.IS.INITE) GOTO205                                         0021017
      WRITE(6,912)                                                      0021017
      GOTO205                                                           0021017
  202 WRITE(6,913)                                                      0021017
  205 WRITE(6,914)RANGE                                                 0021018
      WRITE(6,915)FC,BW                                                 0021018
      WRITE(6,930)                                                      0021019
      WRITE(6,917)                                                      0021019
      WRITE(6,918)TO                                                    0021019
```

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      WRITE(6,920)NUMIN                                                 002101AU12
      IF(NGAI.)203,203,204                                              002101A712
  203 WRITE(6,921)                                                      002101A813
  204 WRITE(6,922)LLIMIT                                                002101AL12
      WRITE(6,923)NIMIN,NIMAX                                           002101B512
      WRITE(6,934)                                                      002101B912
      WRITE(6,935)DDELAY,RATTEN                                         002101BF12
      WRITE(6,936)                                                      0021010712
      WRITE(6,938)                                                      0021010B12
      WRITE(6,939)                                                      0021010F12
      WRITE(6,940)                                                      0021011312
      DO206I=1,NPATH                                                    0021011712
      IN=T(I,1)+1.E-05                                                  0021011B10
      WRITE(6,951)IN,T(I,2),T(I,3),T(I,5),T(I,6),T(I,7),T(I,8),T(I,9),T(  0021011B15
     1I,10),T(I,11),T(I,12),T(I,13),T(I,14)                             0021011F15
  206 CONTINUE                                                          0021012312
          WRITE(6,930)                                                  0021020U13
      WRITE(6,924)                                                      0021020412
      WRITE(6,930)                                                      0021020812
C         PRINT OUTPUT ARRAY = PRTOUT(N)                                0021020C12
      WRITE(6,906)                                                      0021020G12
      NLPR=FLOAT(NPRINT)/3.                                             0021021U12
      DO210L=1,NLPR                                                     0021021115
      LA=3*(L-1)+1                                                      0021021510
      LB=LA+1                                                           0021021911
      LC=LA+2                                                           0021021D11
      WRITE(6,907)LA,PRTOUT(LA),LB,PRTOUT(LB),LC,PRTOUT(LC)             0021021114
  210 CONTINUE                                                          0021022C12
C         END PRINTING OF OUTPUT DATA SUMMARY AND OUTPUT ARRAY          0021023U13
  900 FORMAT(A6)                                                        0021023U13
  901 FORMAT(1X,'NPATH IS GREATER THAN 10; THIS EXCEEDS THE ALLOCATED AR  0021023U13
     1RAY STORAGE. PROGRAM IS TERMINATED')                              0021023U13
  902 FORMAT(1X,'NCUNVL IS GREATER THAN 400; THIS EXCEEDS THE ALLOCATED  0021023U13
     1ARRAY STORAGE. PROGRAM IS TERMINATED')                            0021023U13
  903 FORMAT(1X,'THE ARRAY STORAGE REQUIREMENTS FOR SIGIN(L) HAVE BEEN   0021023U13
     1EXCEEDED. PROGRAM IS TERMINATED.')                                0021023U13
  904 FORMAT(4E13.6)                                                    0021023U13
  905 FORMAT(1X,'THE ARRAY STORAGE REQUIREMENTS FOR PRTOUT(N) HAVE BEEN  0021023U13
     1EXCEEDED. PROGRAM IS TERMINATED.')                                0021023U13
  906 FORMAT(5X,'INDEX',10X,'OUTPUT',17X,'INDEX',10X,'OUTPUT',17X,'INDEX  0021023U13
     1',10X,'OUTPUT'//)                                                 0021023U13
  907 FORMAT( 2X,I6, 2X,1P2E13.6,4X,I6, 2X,2E13.6, 4X,I6, 2X,2E13.6)     0021023U13
  910 FORMAT(35X,'OUTPUT DATA SUMMARY'///)                              0021023U13
  911 FORMAT(1X,'IONOSPHERE-TRANSMITTER-RECEIVER PARAMETERS'/)          0021023U13
  912 FORMAT(10X,'DAYTIME ELECTRON DENSITY PROFILE')                    0021023U13
  913 FORMAT(10X,'NIGHTTIME ELECTRON DENSITY PROFILE')                  0021023U13
  914 FORMAT(10X,'DISTANCE BETWEEN TRANSMITTER AND RECEIVER = ',F8.2,' K  0021023U13
     1M.')                                                              0021023U13
  915 FORMAT(10X,'CARRIER FREQ. = ',F7.3,' MHZ.   BANDWIDTH = ',F7.2,' K  0021023U13
     1HZ.')                                                             0021023U13
  917 FORMAT(1X,'SIGNAL ANALYSIS PARAMETERS'/)                          0021023U13
  918 FORMAT(10X,'THE SAMPLING INTERVAL,TO, EQUALS',1PF10.3,' SEC. (THIS  0021023U13
     1 IS HALF THE NYQUIST SAMPLING INTERVAL)')                         0021023U13
  920 FORMAT(10X,'THE LENGTH OF THE INPUT SEQUENCE IS ',I4)             0021023U13
  921 FORMAT(10X,'THE GAINS FOR THE IONOSPHERE RETURNS ARE SET = 1 (THEY  0021023U13
     1 ARE NONRANDOM)')                                                 0021023U13
  922 FORMAT(10X,'THE IONOSPHERE IMPULSE RESPONSE IS TRUNCATED TO ',I3,'  0021023U13
     1 SAMPLES')                                                        0021023U13
  923 FORMAT(10X,'MINIMUM DELAY SHIFT = ',I4,3X,'MAXIMUM DELAY SHIFT = '  0021023U13
     1,I4/)                                                             0021023U13
```

```
924  FORMAT(1X,'THE FOLLOWING DATA REPRESENTS THE RECEIVED, COMPLEX BAS      C  0021023013
     1EBAND SIGNAL. IT INCLUDES THE GROUNDWAVE AND ALL IONOSPHERE RETURN     C  0021023013
     2S'//)                                                                  C  0021023013
930  FORMAT(1X)                                                              C  0021023013
934  FORMAT(1X,'COMPUTER CHANNEL PARAMETERS'/)                               C  0021023013
935  FORMAT(1X,'GROUNDWAVE: DELAY = ',1PF10.3,' SEC.    ATTENUATION =       C  0021023013
     1',F10.3,' DB.')                                                        C  0021023013
936  FORMAT(1X,'IONOSPHERE RETURNS:'/)                                       C  0021023013
938  FORMAT( 3X,'MODE',3X,'SOLUTION',3X,'RAY ANGLE',3X,'PATH',4X,'CARRI     C  0021023013
     1ER',4X,'CARRIER',3X,' SIGNAL ',1X,'AMPLITUDE',4X,'PHASE',5X,'ATT      C  0021023013
     2ENUATION',2X,'DOPPLER',4X,'DOPPLER')                                   C  0021023013
940  FORMAT(10X,'INDICATOR',2X,'(DEGREES)',2X,'LENGTH',4X,'DELAY',6X,'P      C  0021023013
     1HASE',6X,'DELAY',3X,'DISTORTION',1X,' DISTORTION',5X,'(DB)', 7X,'S     C  0021023013
     2HIFT',5X,'SPREAD')                                                     C  0021023013
940  FORMAT( 9X,'(UNNO SOLN)',13X,'(KM)',5X,'(SEC)',4X,'(CYCLES)',5X,'(     C  0021023013
     1SEC)',4X,'(SEC/MHZ)',1X,'(SEC/MHZ**2)',15X,'(HZ)', 7X,'(HZ)'/)        C  0021023013
951  FORMAT(1X,I1,A2,1X,A4,4X,'NA',8X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,      C  0021023013
     1I4,E10.3,1X,E10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)                 C  0021023013
955  FORMAT(11,A2,A4,4E16.9)                                                 C  0021023013
956  FORMAT(4E16.9)                                                          C  0021023013
957  FORMAT(3E16.9)                                                          C  0021023013
999  STOP                                                                    C  0021023013
     END                                                                     C  0021023112
     0021023313  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021005D
     0021023415  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210055
     0021023611  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021004D
     0021023713  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210045
     0021023815  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021003D
     0021023A11  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210033
     0021023B13  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021001E
                                                  SEGMENT 002 IS 028B LONG
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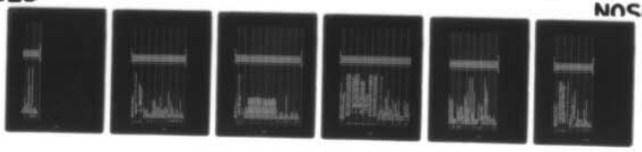
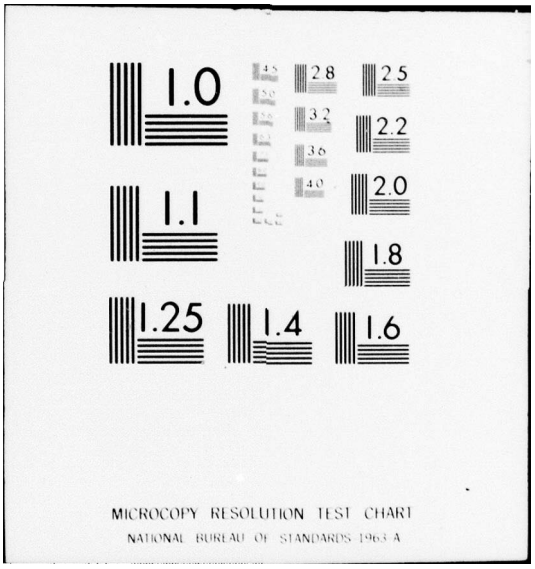
```
                                                                                    START OF SEGMENT 009
      SUBROUTINE IMPULS(NCONVL,NPATH,D2,D3,D)                                   C 009!000010
      DIMENSION D2(10),D3(10)                                                   C 009!000010
      COMPLEX D(10,801),XI(9),XIO,AI                                            C 009!000010
                                                                                C 009!000010
C          SUBROUTINE COMPUTES DELAY DISTORTION IMPULSE RESPONSES FOR            C 009!000010
C          EACH IONOSPHERE RETURN                                               C 009!000010
                                                                                C 009!000010
C               NCONVL= LENGTH OF TRUNCATED IMPULSE RESPONSE (.LE. 400)         C 009!000010
C               NPATH = NUMBER OF RETAINED IONOSPHERE RETURNS                   C 009!000010
C               D2(I) = ARRAY OF AMPLITUDE DISTORTION COEFFICIENTS              C 009!000010
C               D3(I) = ARRAY OF PHASE DISTORTION COEFFICIENTS                  C 009!000010
                                                                                C 009!000010
C          SUBROUTINE RETURNS ARRAY D(I,N). I DESIGNATES THE PARTICULAR         C 009!000010
C          IONOSPHERE RETURN. N DESIGNATES THE ARGUMENT OF THE IMPULSE          C 009!000010
C          RESPONSE. N=1 CORRESPONDS TO AN ARGUMENT OF -NCONVL*TO,              C 009!000010
C          N=2*NCONVL+1 CORRESPONDS TO AN ARGUMENT OF +NCONVL*TO.               C 009!000010
                                                                                C 009!000010
C          WARNING: THE MAXIMUM ERROR IN THE FRESNEL INTEGRAL ( FROM            C 009!000010
C                   SUBROUTINE 'FRSNEL') IS 1.0 E-09. AS A CONSEQUENCE          C 009!000010
C                   THE ITERATIVE METHOD USED TO EVALUATE D(I,N) BREAKS         C 009!000010
C                   DOWN WHEN N EXCEEDS APPROXIMATELY 25. FOR THIS              C 009!000010
C                   REASON IT IS NOT ADVISED TO CHOOSE NCONVL LARGER            C 009!000010
C                   THAN 25.                                                    C 009!000010
                                                                                C 009!000010
      PI=3.14159265359                                                          C 009!000020
      AI=(0.,1.)                                                                C 009!000030
      NLIMIT=2*NCONVL+1                                                         C 009!000811
      DO1001I=1,NPATH                                                           C 009!000810
      DD2=D2(I)                                                                 C 009!000910
      DD3=D3(I)                                                                 C 009!000a10
      DO50N=1,NLIMIT                                                            C 009!000b10
      ARG=FLOAT(N-1-NCONVL)*PI                                                  C 009!000L10
      IF(DD2)30,5,30                                                            C 009!001011
5     IF(ARG)10,6,10                                                            C 009!001112
6     XIO=(2.,0.)                                                               C 009!001213
      XI(1)=(0.,0.)                                                             C 009!001315
      XI(2)=(2./3.)*(1.,0.)                                                     C 009!001512
      XI(3)=(0.,0.)                                                             C 009!001712
      XI(4)=(.4,0.)                                                             C 009!001810
      XI(5)=(0.,0.)                                                             C 009!001914
      XI(6)=(2./7.)*(1.,0.)                                                     C 009!001f12
      XI(7)=(0.,0.)                                                             C 009!002013
      XI(8)=(2./9.)*(1.,0.)                                                     C 009!002111
      XI(9)=(0.,0.)                                                             C 009!002912
      GOTO45                                                                    C 009!002b10
10    XIO=(2.,0.)*(SIN(ARG)/ARG)                                                C 009!002b13
      XI(1)=(-2.*AI*((COS(ARG)+(SIN(ARG)/ARG)))/ARG                             C 009!002f14
      DO20K=2,9                                                                 C 009!003012
      KI=K-1                                                                    C 009!003810
      XI(K)=(-AI/ARG)*((CEXP(AI*ARG)-((-1.)**FLOAT(K))*CEXP(-AI*ARG))*FL        C 009!003412
     1OAT(K)*XI(KI))                                                            C 009!004013
20    CONTINUE                                                                  C 009!005011
      GOTO45                                                                    C 009!005212
30    AARG=ARG                                                                  C 009!005315
      DDD2=DD2                                                                  C 009!005614
      CALL FRSNEL(DDD2,AARG,XIO)                                                C 009!005713
      XI(1)=(-SIN(ARG)*CEXP(-AI*DD2)+.5*ARG*XIO)/DD2                            C 009!005e15
      XI(2)=(AI*COS(ARG)*CEXP(-AI*DD2)+.5*ARG*XI(1)-AI*.5*XIO)/DD2              C 009!006011
      DO40K=3,9                                                                 C 009!006111
```

AD-A054 726 SYSTEMS EXPLORATION INC SAN DIEGO CA F/G 17/2
HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS. A MATHEMATICAL MODEL--ETC(U)
MAR 78 R LUGANNANI, H C BOOKER, L E HOFF N00123-76-C-1090
UNCLASSIFIED NOSC-TR-208 NL

2 OF 2
AD
A054726



END
DATE FILMED
7-78
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

```
      K1=K-1                                                            C  009100B210
      K2=K+2                                                            C  009100B312
      KI=(FLOAT(K)/2.)+1.E-06                                           C  009100B415
      XIND=K-2*KI                                                       C  009100B715
      XINDC=1.-XIND                                                     C  009100B914
      XI(K)=(1.7(2.*DD2))*(2.*AI*CEXP(-AI*DD2)*(XINDC*COS(ARG)+AI*XIND*S C  009100BA15
     1IN(ARG))+ARG*XI(K1)-AI*FLOAT(K1)*XI(K2))                          C  009100C912
   40 CONTINUE                                                          C  009100CA10
   45 D(1,N)=.5*(XI0-AI*DD3*XI(3)-.5*(DD3**2)*XI(6)+AI*(1./6.)*(DD3**3)* C  009100C413
     1XI(7))                                                            C  009100C615
   50 CONTINUE                                                          C  009100CE10
  100 CONTINUE                                                          C  009100D011
      RETURN                                                            C  009100D014
      END                                                               C  009100D014
                                                                  SEGMENT 009 IS 0000 LONG
```

```
                                                                    START OF SEGMENT 00C
      SUBROUTINE FRSNEL(D2,ARG,XIO)                                 L  00C:0000010
      COMPLEX AI,C2,C3,C4,C5,HD,HS,XIO                              L  00C:0000020
                                                                    L  00C:0000030
C         SUBROUTINE COMPUTES COMPLEX FRESNEL INTEGRAL USING RATIONAL L  00C:0000040
C         APPROXIMATION.                                            L  00C:0000050
                                                                    L  00C:0000060
C              D2    = AMPLITUDE DISTURTION COEFFICIENT            L  00C:0000070
C              ARG   = ARGUMENT OF FRESNEL INTEGRAL                L  00C:0000080
C              XIO   = COMPLEX FRESNEL INTEGRAL                    L  00C:0000090
                                                                    L  00C:0000100
      PI=3.14159265359                                              L  00C:0000110
      AI=(0.,1.)                                                    L  00C:0000120
      ARG=ABS(ARG)                                                  L  00C:0000130
      IF(D2)1,35,2                                                  L  00C:0000140
1     SGN=-1.                                                       L  00C:0000150
      GOTO3                                                         L  00C:0000160
2     SGN=1.                                                        L  00C:0000170
3     D2=ABS(D2)                                                    L  00C:0000180
      SUM=ABS(1.+(ARG/(2.*D2)))                                     L  00C:0000190
      DIFF=ABS(1.-(ARG/(2.*D2)))                                    L  00C:0000200
      C1=SQRT(PI/(2.*D2))                                           L  00C:0001150
      C2=CEXP(-AI*D2)                                               L  00C:0001160
      C3=CEXP(AI*ARG)                                               L  00C:0001170
      C4=CEXP(-AI*ARG)                                              L  00C:0001180
      C5=CEXP((AI*(ARG**2))/(4.*D2))                                L  00C:0002000
      ASUM=SUM/C1                                                   L  00C:0002010
      ADIFF=DIFF/C1                                                 L  00C:0002020
      W=ARG-2.*D2                                                   L  00C:0002030
      CALL H(ADIFF,HD)                                              L  00C:0002040
      CALL H(ASUM,HS)                                               L  00C:0002050
      IF(W)5,10,10                                                  L  00C:0002060
5     XIO=(1.-AI)*C1*C5-C1*C2*(HD*C3+HS*C4)                         L  00C:0002070
      GOTO20                                                        L  00C:0002080
10    XIO=C1*C2*(HD*C3-HS*C4)                                       L  00C:0002090
20    IF(SGN)25,25,30                                               L  00C:0002100
25    XIO=CONJG(XIO)                                                L  00C:0002110
30    CONTINUE                                                      L  00C:0002120
      GOTO40                                                        L  00C:0002130
35    IF(ARG)37,36,37                                               L  00C:0002150
36    XIO=(2.,0.)                                                   L  00C:0002160
      GOTO40                                                        L  00C:0002170
37    XIO=((2.,0.)*SIN(ARG))/ARG                                    L  00C:0002180
40    CONTINUE                                                      L  00C:0002190
      RETURN                                                        L  00C:0002200
      END                                                           L  00C:0002210
                                                                  SEGMENT 00C IS 008A LONG
```

```
                                                                    START OF SEGMENT OOE
      SUBROUTINE H(W,HH)                                          C  OOE:0000:10
      COMPLEX A(12),C(12),AI,HH,S                                 C  OOE:0000:10
                                                                  C  OOE:0000:10
C           SUBROUTINE COMPUTES FUNCTION H(W) USED IN RATIONAL APPROX.  C  OOE:0000:10
C           OF COMPLEX FRESNEL INTEGRAL                           C  OOE:0000:10
                                                                  C  OOE:0000:10
C           H(W) = HH                                             C  OOE:0000:10
                                                                  C  OOE:0000:10
      AI=(0.,1.)                                                  C  OOE:0000:10
      PI=3.14159265359                                            C  OOE:0001:11
      E=(PI*(W**2))/2.                                            C  OOE:0006:10
      A(1)=(1.595769140,-.00000A033)                              C  OOE:0007:15
      A(2)=(-.000C01702,4.2553A75?4)                              C  OOE:000C:14
      A(3)=(-6.808508654,-.000A9281)                              C  OOE:0011:13
      A(4)=(-.0005763615,-7.780n?04)                              C  OOE:0016:14
      A(5)=(6.920?019620,-.0095?0A95)                             C  OOE:001B:14
      A(6)=(-.016896657,5.0751A1?98)                              C  OOE:0021:14
      A(7)=(-4.05C485660,-.13B341947)                             C  OOE:0025:13
      A(8)=(-.075752419,-1.3637?9124)                             C  OOE:002A:14
      A(9)=(.850663781,-.A033402?6)                               C  OOE:002F:14
      A(10)=(-.025A3?0491,.7022?7016)                             C  OOE:0034:14
      A(11)=(-.15023096,-.2161?5929)                              C  OOE:0039:13
      A(12)=(.03*40*779,-.01954703?)                              C  OOE:003E:14
      C(1)=(0.,.19947114)                                         C  OOE:0043:13
      C(2)=(-.02*933975,.00000A023)                               C  OOE:0046:13
      C(3)=(.00006936,-.009351341)                                C  OOE:004B:13
      C(4)=(.005770956,.0000?3n08)                                C  OOE:0050:14
      C(5)=(-.00068984?,-.0048?1A64)                              C  OOE:0055:13
      C(6)=(-.0094971?6,.001903?18)                               C  OOE:005A:13
      C(7)=(.0119A8809,-.01712?914)                               C  OOE:005F:13
      C(8)=(-.00?67*8873,-.*290?4?67)                             C  OOE:0064:14
      C(9)=(.00024642,-.0079280?5)                                C  OOE:0069:13
      C(10)=(-.00210?967,.0184973?8)                              C  OOE:006E:14
      C(11)=(-.00121793,-.0055?8515)                              C  OOE:0073:13
      C(12)=(-.000?33939,.00083?3?8)                              C  OOE:0078:14
      IF(E-4.)10,10,30                                            C  OOE:007D:13
10    Y=E/4.                                                      C  OOE:007E:15
      P=1.                                                        C  OOE:0080:11
      S=(0.,0.)                                                   C  OOE:0086:15
      DO?ON=1,12                                                  C  OOE:008?:10
      S=S+A(N)*P                                                  C  OOE:0083:10
      P=P*Y                                                       C  OOE:0088:14
20    CONTINUE                                                    C  OOE:008A:10
      HH=.5*(1.-AI)*CEXP(AI*E)-S*SQRT(Y)                          C  OOE:008E:11
      GOTO100                                                     C  OOE:0096:13
30    Y=4./E                                                      C  OOE:0098:10
      P=1.                                                        C  OOE:009C:12
      S=(0.,0.)                                                   C  OOE:009E:10
      DO?ON=1,12                                                  C  OOE:009F:11
      S=S+C(N)*P                                                  C  OOA0:10
      P=P*Y                                                       C  OOA5:14
40    CONTINUE                                                    C  OOA7:10
      HH=-S*SQRT(Y)                                               C  OOA9:11
100   RETURN                                                      C  OOAD:14
      END                                                         C  OOAE:11
                                                                SEGMENT OOE IS OOBC LONG
```

```
                                                                        START OF SEGMENT 001
      SUBROUTINE GAIN(ISEED,NCALL,BW,ALPHA,NPATH,T,C,X,Y,G)             C  001 1000010
      DIMENSION T(10,14)                                                C  001 1000010
      COMPLEX X(10,4),G(10),Y(10),R(10),AI                              C  001 1000010
      DOUBLE PRECISION C(10,7),D1,B1,D2,B2,DD2                          C  001 1000010
                                                                        C  001 1000010
C          SUBROUTINE COMPUTES RANDOM GAINS FOR IONOSPHERE RETURNS. ON   C  001 1000010
C          EACH CALL THE ARRAY G(I) IS RETURNED. THE NUMBER OF ENTRIES   C  001 1000010
C          IN G(I) = NPATH. THE ARRAY G(I) IS ASSOCIATED WITH THE SAMPLE C  001 1000010
C          TIME N*TO (N=0 FOR FIRST CALL, N=1 FOR SECOND CALL, ETC.).    C  001 1000010
                                                                        C  001 1000010
C              ISEED = SEED VALUE USED IN RANDOM NUMBER GENERATOR. IT    C  001 1000010
C                      SHOULD BE SET = 7**9 FOR FIRST CALL. IT IS        C  001 1000010
C                      REPLACED BY A NEW VALUE USED IN SUBSEQUENT CALLS. C  001 1000010
C              NCALL = NUMBER OF TIMES THE SUBROUTINE HAS BEEN CALLED.   C  001 1000010
C                      NCALL SHOULD BE SET = 0 BEFORE FIRST CALL. IT IS  C  001 1000010
C                      REPLACED BY A NEW VALUE USED IN SUBSEQUENT CALLS. C  001 1000010
C                BW    = BANDWIDTH (KHZ.)                              C  001 1000010
C                ALPHA = PARAMETER USED IN APPROXIMATE SPECTRUM         C  001 1000010
C                NPATH = NUMBER OF RETAINED IONOSPHERE RETURNS          C  001 1000010
C          THE ENTRIES OF T(I,J) USED IN THIS SUBROUTINE ARE:           C  001 1000010
C                I =  N: INDEX NUMBER FOR PATH (I .LE. NPATH)           C  001 1000010
C                J = 12: ATTENUATION (DB)                               C  001 1000010
C                  = 13: DOPPLER SHIFT (HZ.)                            C  001 1000010
C                  = 14: DOPPLER SPREAD (HZ.)                           C  001 1000010
C          THE ARRAYS C(I,J), X(I,J) AND Y(I) CONTAIN VARIABLES USED IN  C  001 1000010
C          THE ALGORITHM. THEY SHOULD BE SET = 0 FOR FIRST CALL. THEY    C  001 1000010
C          ARE REPLACED BY NEW ARRAYS TO BE USED IN SUBSEQUENT CALLS.    C  001 1000010
                                                                        C  001 1000010
C          DOUBLE PRECISION ARITHMETIC IS USED FOR SOME CALCULATIONS TO   C  001 1000010
C          OBTAIN THE DESIRED ACCURACY IN THE RANDOM GAINS. EARLIER       C  001 1000010
C          CALCULATIONS INDICATE THAT THE DOUBLE PRECISION IS ESSENTIAL.  C  001 1000010
                                                                        C  001 1000010
      AI=(0.,1.)                                                        C  001 1000010
      PI=3.14159265359                                                  C  001 1000111
C          THE SAMPLE INTERVAL TO IS HALF THE NYQUIST SAMPLE INTERVAL     C  001 1000610
      TO=1./(BW*2000.)                                                  C  001 1000610
      IF(NCALL)10,10,21                                                 C  001 1000715
   10 DO151=1,NPATH                                                     C  001 1000910
      C(I,1)=DBLE(10.**(-T(I,12)/10.))                                  C  001 1000A10
      C(I,2)=DBLE(PI*ALPHA*T(I,14)*TO)                                  C  001 1000F14
      C(I,3)=DSQRT(((4.+C(I,1)**2/3.)*C(I,2)**2.5)                      C  001 1001510
      C(I,4)=DEXP(-C(I,2))                                              C  001 1001E13
      C(I,5)=C(I,4)**2                                                  C  001 1002310
      C(I,6)=C(I,4)**3                                                  C  001 1002811
      C(I,7)=DBLE(2.*PI*T(I,13)*TO)                                     C  001 1002D12
   15 CONTINUE                                                          C  001 1003214
C          COMPUTE G(I) FOR N=0                                           C  001 1003415
      CALL RANDOM(ISEED,NPATH,R)                                        C  001 1003415
      DO301=1,NPATH                                                     C  001 1003711
      X(I,1)=SQRT(SNGL(C(I,3)))*R(I)                                    C  001 1003810
      G(I)=X(I,1)                                                       C  001 1003F13
   20 CONTINUE                                                          C  001 1004215
      NCALL=1                                                           C  001 1004510
      GOTO100                                                           C  001 1004514
C          END COMPUTATION FOR N=0                                        C  001 1004611
   21 IF(NCALL-1)25,25,31                                               C  001 1004611
C          COMPUTE G(I) FOR N=1                                           C  001 1004715
   25 CALL RANDOM(ISEED,NPATH,R)                                        C  001 1004715
      DO301=1,NPATH                                                     C  001 1004A11
```

```
      D1=(1.+G(I,2)+(1./3.)*(C(I,2)**2))*C(I,4)                          G  00F10048 10
      R1=DSQRT(C(I,1))*DSQRT(1.-(D1**2))                                 G  00F10054 13
      X(I,2)=SNGL(R1)*R(I)+SNGL(D1)*X(I,1)                               G  00F10059 11
      G(I)=CEXP(A1*SNGL(C(I,7)))*X(I,2)                                  G  00F10064 11
   30 CONTINUE                                                           G  00F10069 11
      NCALL=2                                                            G  00F10070 12
      GOTO100                                                            G  00F10071 11
C          END COMPUTATION FOR N=1                                        G  00F10071 14
   31 IF(NCALL-2)35,35,41                                                G  00F10071 14
C          COMPUTE G(I) FOR N=2                                           G  00F10073 13
   35 CALL RANDUM(ISEED,NPATH,R)                                         G  00F10073 13
      CALL RANDOM(ISEED,NPATH,V)                                         G  00F10075 15
      DO401=1,NPATH                                                      G  00F10078 12
      D1=(1.+G(I,2)+(1./3.)*(C(I,2)**2))*G(I,4)                          G  00F10078 10
      B1=DSQRT(C(I,1))*DSQRT(1.-(D1**2))                                 G  00F10081 13
      DD2=(1.+2.*C(I,2)+(4./3.)*(C(I,2)**2))*G(I,5)                      G  00F10087 11
      D2=(DD2-D1**2)/(1.-D1**2)                                          G  00F10090 12
      B2=DSQRT(C(I,1))*DSQRT( (1.+D2)*(1.-DD2)-(4./3.)*(C(I,2)**5)*C(I,5
     1))                                                                 G  00F10095 11
      X(I,3)=SNGL(B2)*R(I)+SNGL(C(I,3)*C(I,4))*V(I)+SNGL(D1*(1.-D2))*X(I
     1,2)+SNGL(D2)*X(I,1)                                                G  00F10099 13
      G(I)=CEXP(A1*SNGL(G(I,7)))*FLOAT(NCALL))*X(I,3)                    G  00F10106 12
   40 CONTINUE                                                           G  00F10111 15
      NCALL=3                                                            G  00F10115 13
      GOTO100                                                            G  00F10116 14
C          END COMPUTATION FOR N=2                                        G  00F10116 13
C          COMPUTE G(I) FOR N=3                                           G  00F10120 10
   41 CALL RANDOM(ISEED,NPATH,R)                                         G  00F10120 10
      DO501=1,NPATH                                                      G  00F10120 10
      X(I,4)=SNGL(C(I,3)*C(I,4))*(R(I)+SNGL(C(I,4)))*Y(I))+SNGL(C(I,6))*X  G  00F10126 12
     1(I,1)-3.*SNGL(C(I,5))*X(I,2)+3.*SNGL(C(I,4))*X(I,3)                G  00F10130 10
      G(I)=CEXP(A1*SNGL(C(I,7)))*FLOAT(NCALL))*X(I,4)                    G  00F10135 15
      X(I,1)=X(I,2)                                                      G  00F10140 13
      X(I,2)=X(I,3)                                                      G  00F10141 11
      X(I,3)=X(I,4)                                                      G  00F10144 14
      Y(I)=R(I)                                                          G  00F10153 12
   50 CONTINUE                                                           G  00F10160 10
      NCALL=NCALL+1                                                      G  00F10165 15
  100 RETURN                                                             G  00F10170 10
      END                                                                G  00F10171 12
                                                                         G  00F10176 15
                                SEGMENT 00F IS 01IF LONG
```

```
                                                                    START OF SEGMENT 013
      SUBROUTINE RANDOM(ISEED,NPATH,R)                          G  01310000010
      COMPLEX R(10),AI                                          G  01310000010
      DOUBLE PRECISION D2P31M,D2PN31,Z                          G  01310000010
                                                                G  01310000010
C           THIS SUBROUTINE RETURNS THE ARRAY R(I), EACH ENTRY OF R(I)  G  01310000010
C           IS A COMPLEX GAUSSIAN VARIABLE, THE REAL AND IMAGINARY PARTS  G  01310000010
C           ARE INDEPENDENT, IDENTICALLY-DISTRIBUTED, ZERO-MEAN, UNIT-  G  01310000010
C           VARIANCE GAUSSIAN VARIABLES. ALL ENTRIES OF R(I) ARE  G  01310000010
C           INDEPENDENT AND ARE INDEPENDENT OF THE ARRAY PRODUCED  G  01310000010
C           ON PREVIOUS CALLS                                     G  01310000010
                                                                G  01310000010
C           ISEED IS REPLACED BY A NEW ISEED TO BE USED IN SUBSEQ. CALLS  G  01310000010
                                                                G  01310000010
      AI=(0.,1.)                                                G  01310000010
      TPI=6.2831853071B                                         G  01310000111
C           D2P31M = (2**31)-1; D2PN31 = 1/(2**31)              G  01310000010
      DATA        D2P31M/2147483647.D0/,                        G  01310000010
     *            D2PN31/Z'3E00000000000000'/                   G  01310000010
      Z=ISEED                                                   G  01310000010
      DO 50 L=1,NPATH                                           G  01310000110
    1 Z=DMOD(16807.D0*Z,D2P31M)                                 G  01310000010
      U1=Z*D2PN31                                               G  01310000111
      IF(U1).3,.2                                               G  01310000112
    3 Z=.57706925                                               G  01310000111
      GOTO1                                                     G  01310000114
    2 Z=DMOD(16807.D0*Z,D2P31M)                                 G  01310000111
      U2=Z*D2PN31                                               G  01310001112
      R(L)=SQRT(-2.*ALOG(U1))*CEXP(AI*TPI*U2)                   G  01310001213
   50 CONTINUE                                                  G  01310001314
      ISEED=Z                                                   G  01310001415
      RETURN                                                    G  01310002015
      END                                                       G  01310002112
                                                                    SEGMENT 013 IS 0029 LONG
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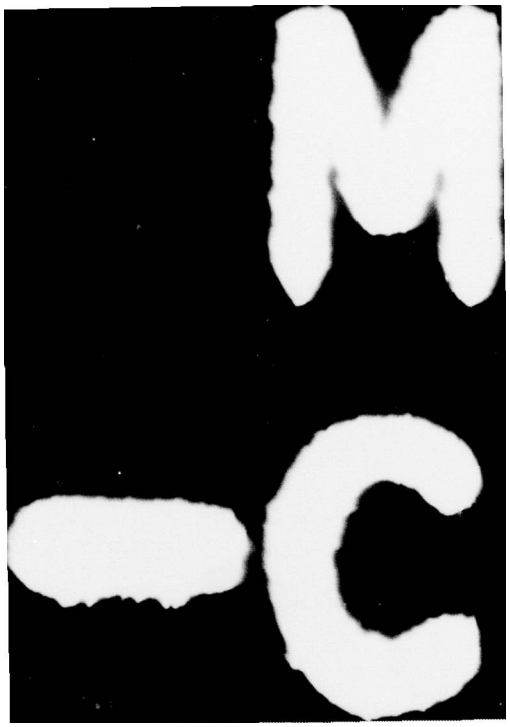
M
C