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TRADEOFF OF HARDWARE AND SIGNAL-TO-NOISE RATIO IN MOVING TARGET--ETC(U)
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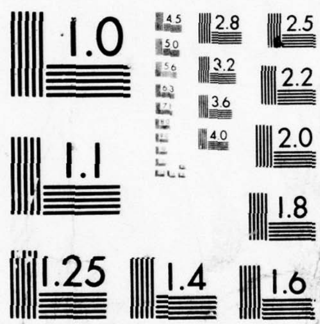

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TRADEOFF OF HARDWARE AND SIGNAL-TO-NOISE RATIO IN MOVING TARGET INDICATORS

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1 March 1978

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In many radar systems the signal processor contains a moving target indicator which is a high pass filter used for clutter rejection. It can be implemented by either an infinite or finite impulse response filter. Due to transient and hardware considerations, the finite impulse response is the configuration most often used. There are two major alternatives for realizing the finite impulse response filter. In radar terminology these have become known by names

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ABSTRACT (Concluded)

such as the moving window (or sliding window) and the fixed window (or block window) methods. The implementation methods allow a tradeoff between the amount of hardware required and the signal-to-noise improvement obtained.

In this report, the hardware requirement for various implementations of moving and fixed window moving target indicator is discussed and compared. The signal-to-noise improvement obtained by integration of moving target indicator outputs is calculated by analytical and simulation techniques for both configurations.

It is demonstrated that the moving window moving target indicator produces correlation of noise samples where it is not present in fixed-window moving target indicator systems. It is shown that a square-law detector or a linear detector does not alter the relative performance of the moving and fixed window moving target indicator systems. A tabular result that compares hardware requirements (i.e., memories, adders, multiplexers, multipliers, multiplication speed) for the moving and fixed window moving target indicator is given.

One significance of this effort is that it can be used to calculate the signal-to-noise improvement for moving and fixed window systems and relate it to the probability of detection. It also allows a decision to be made as to whether increased probability of detection is justified by the increased cost of the moving window implementation over the fixed window system.

CONTENTS

I. INTRODUCTION

Signal processors used in radar systems can efficiently solve the problems introduced by the following:

1) Time of arrival of the received signal due to range dependency.
2) Large amplitude clutter signals.
3) Incoherent phase of the returned radar signals.
4) Low signal-to-noise ratios (SNR) of returned signals.

One system configuration that has been successful in these regards is the quadrature channel digital signal processor (Figure 1). Practical implementations are possible due to reducing the RF signal to the Doppler frequency baseband range, i.e., the video signal. The range information is preserved by sampling during the interpulse period at a rate which is determined by the transmitted pulse duration, i.e., the use of range bins. However, the target velocity which is proportional to the Doppler frequency is not preserved by this type of processor. Other processors such as Doppler filter banks or fast Fourier transform (FFT) systems can achieve Doppler information at the cost of more hardware. Signal-to-clutter considerations dictate the use of moving-target-indicators (MTI), but this adds to the hardware complexity. Since the processor extracts amplitude information by a square-law or linear detector while ignoring phase information, the coherent phase of the return signal is not needed. SNR improvement is necessary for improved target detectability and is obtained by the noncoherent integration of the detector outputs.

The MTI in the quadrature channel processor must perform the clutter canceling (i.e., high pass filtering) for each range bin. Two or three pulse cancelers are used in most radars because they are simple to build; however, they are not suitable for all applications. Other design methods have been devised to meet various types of specifications, e.g., the original methods were identified as delay-line cancelers (see Chapter 4 of Reference 1). There are methods available today that will maximize "improvement factor" [2] or target passband while meeting minimum improvement factor specifications [3]. The finite impulse response (FIR) configuration is most often used because of the desirable transient response characteristics, i.e., an N coefficient filter will reach steady-state output conditions after processing N sample values. Infinite impulse response (IIR) filters are not often used because of their undesirable transient response properties. This condition may change as a result of a technique to minimize the IIR transients produced by ground clutter environment through the use of an initialization theory [4]. Only FIR filters are considered here because of the hardware savings that can be realized.

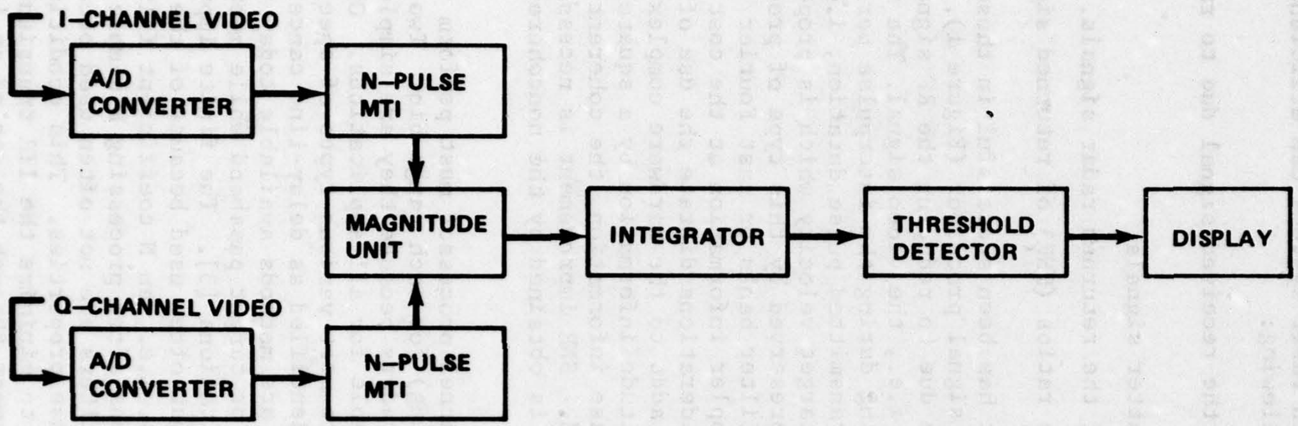


Figure 1. Block diagram of quadrature processor.

There are two major alternatives available for realizing the FIR filters used in the MTI. In radar system terminology these have become known by names such as the moving window (or sliding window) and the fixed window (or block window) methods. An N-pulse moving window MTI will produce a valid output after receiving the first N pulses and subsequently one output for each additional input. The fixed window approach provides an output after receiving the first N pulses, another output after receiving the second set of N pulses, etc. The fixed window and moving window concepts are depicted for a three coefficient filter in Figure 2. The number of outputs from the magnitude unit (i.e., residues) are determined by the total number of pulses available, N_P, and the number of pulses, N, used in the filter. The moving window filter produces K_{MW} residues

$$K_{MW} = (N_P - N) + 1 \quad , \tag{1}$$

while the number of fixed window residues are

$$K_{FW} = \text{Integer } [N_P/N] \quad . \tag{2}$$

Assuming that N_P is an integer multiple of N and that the total number of pulses remains constant between systems, it follows that

$$K_{MW} = N(K_{FW} - 1) + 1 \quad . \tag{3}$$

Two main considerations are involved in the decision to use moving window or fixed window, viz., (1) hardware complexity and (2) SNR improvement. The following sections show that a tradeoff is available between these quantities.

II. HARDWARE CONSIDERATIONS

One of the standard methods of realizing an N coefficient moving window FIR filter for one range bin is shown in Figure 3. This system requires (N - 1) delay units, N multipliers, and a summing unit with N input ports. Alternatively, a cascade of (N - 1) summing units, each with two input ports, could be used. The multipliers and adders must be capable of operation at the pulse repetition frequency (PRF), i.e., each time a new sample is provided for the range bin. Typically this rate is slow (≃5000 Hz) as compared to the digital hardware capability (≃10 MHz). Consequently, it is possible to reduce the number of multipliers and adders by time multiplexing single units. A multiplexed moving window configuration for one range bin is shown in Figure 4. This system requires (N - 1) delay units for the input

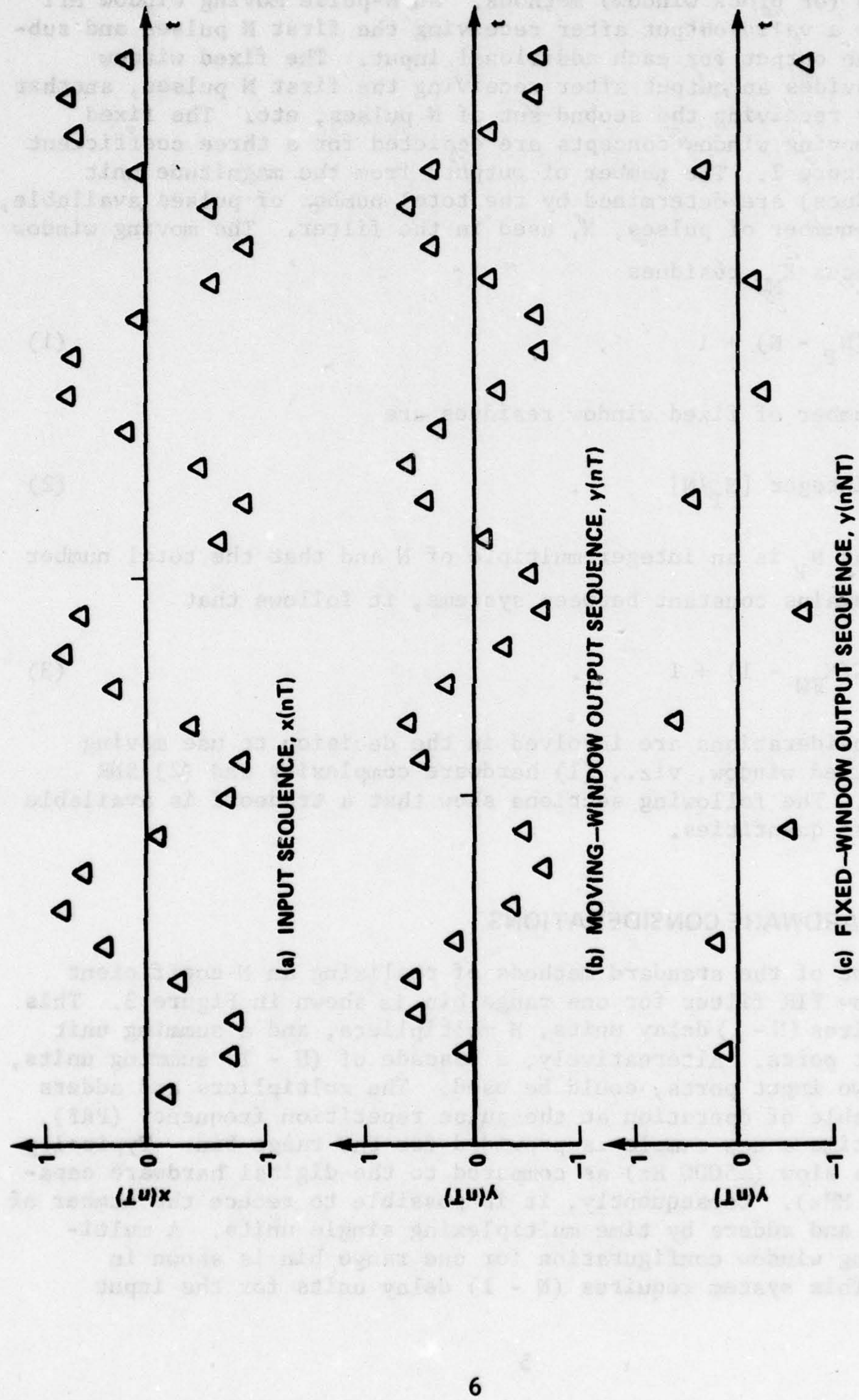


Figure 2. Three-pulse canceler input and output sequences.

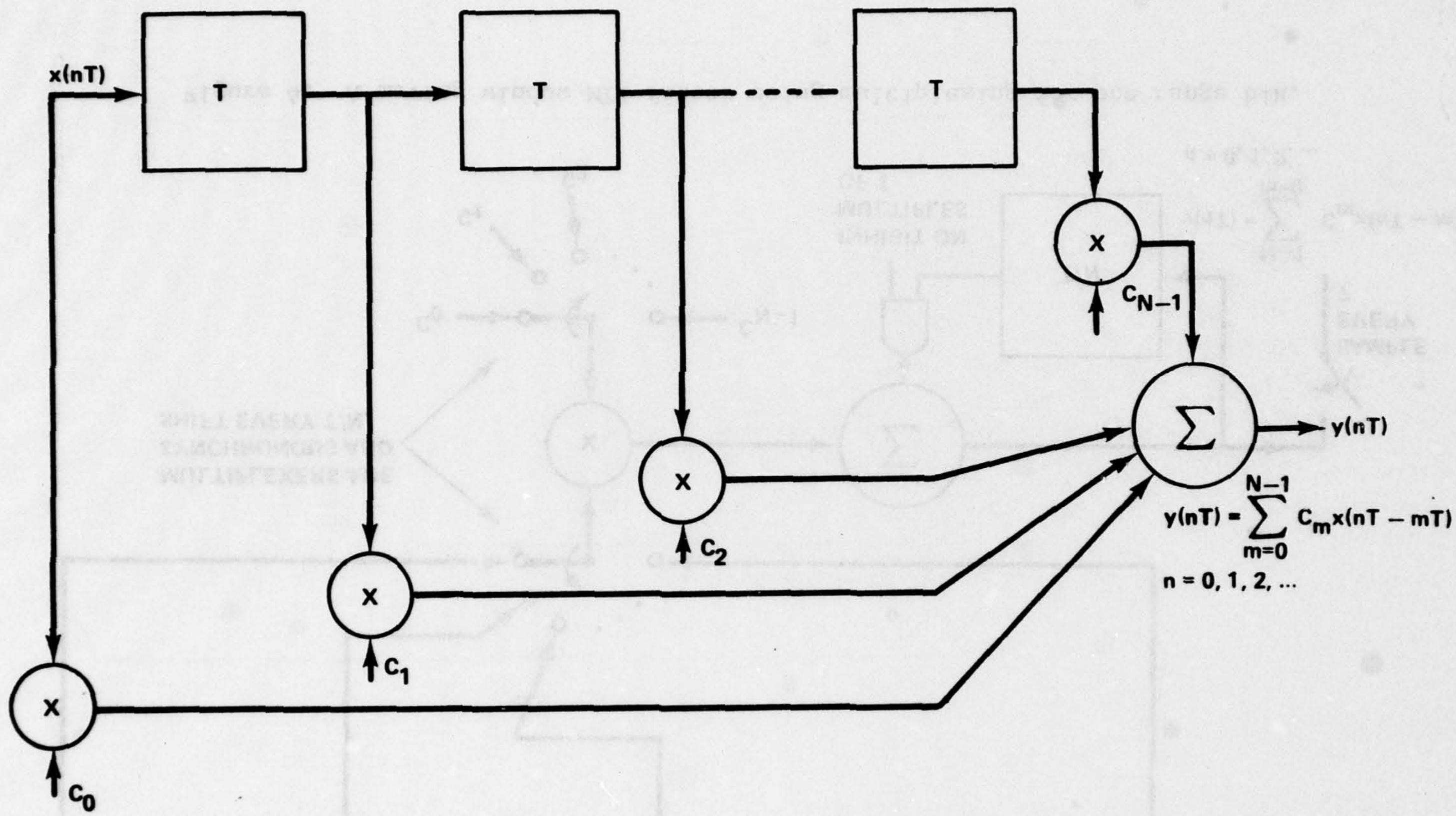


Figure 3. A moving window MTI filter for one range bin.

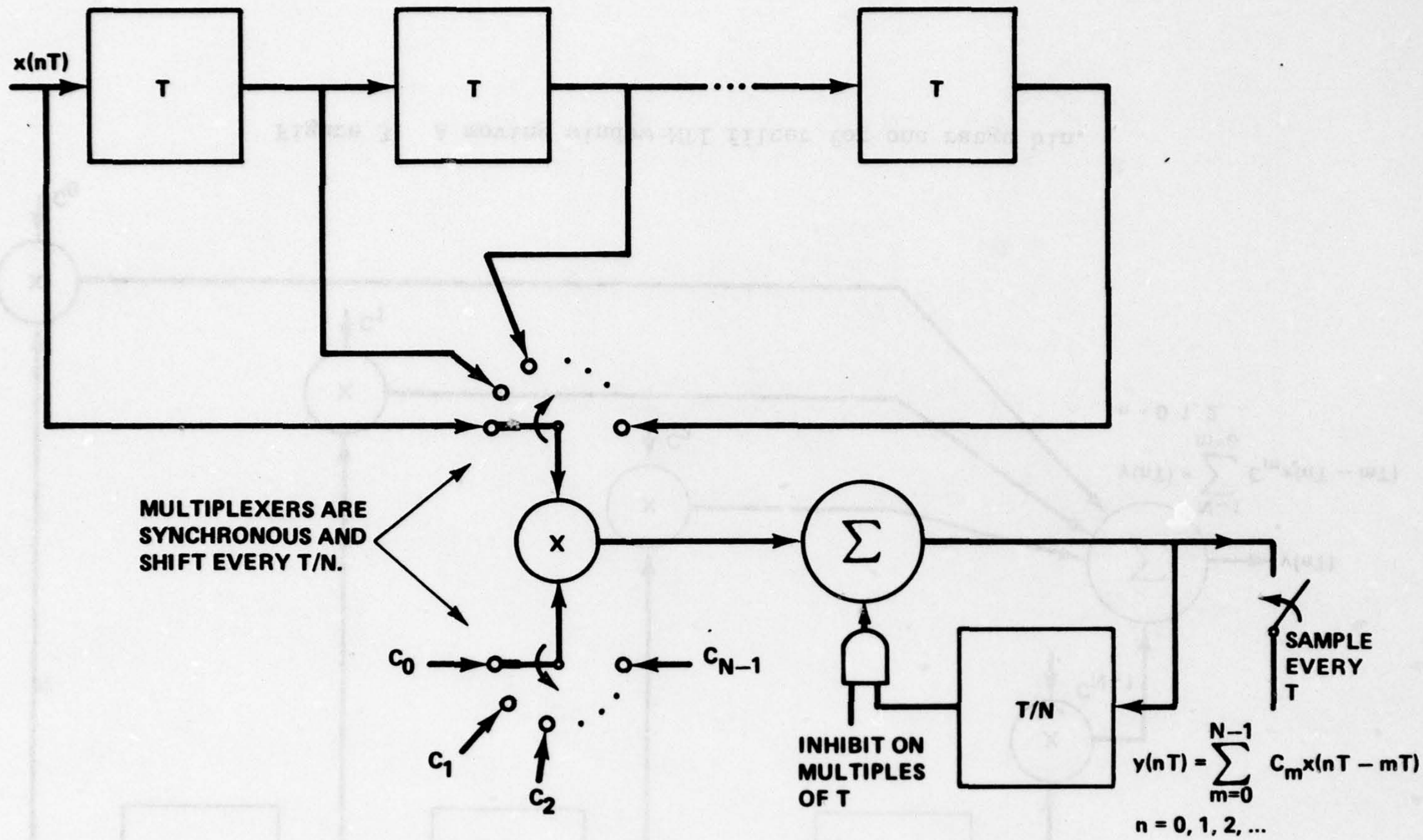


Figure 4. A moving window MTI filter using multiplexing for one range bin.

samples and one delay unit for accumulating the output during the multiplexing operations. The rate of operation for the multiplier and adder has now increased to N times the PRF. This is still a very low rate, e.g., a nine-coefficient filter and a 5000-Hz PRF would require 45,000 multiplications and sums per second. Consequently, the multiplexing can be extended to multiple range bins as shown in Figure 5. Additional control circuitry is required for routing the appropriate range bin samples into the corresponding string of (N - 1) delay units. In addition, the entire MTI requires K range bin accumulator units, K · (N - 1) sample delay units, a multiplier and one, and a two-port adder. The rate of operation is now K · N · PRF. When 1000 range bins are used with a PRF of 5000 Hz and a nine-coefficient filter, the multiplication and summing rate is 45×10^6 operations per second. This exceeds the easily obtainable rate of 10×10^6; however, special hardware and designs are capable of obtaining these rates.

It is possible to simultaneously reduce the number of delay units and the required rate of operation of the multiplier/adder by converting to the fixed window approach. Figure 6 shows a single range bin system; a multiple range bin configuration is shown in Figure 7. Note that the input samples are only used once; this eliminates the (N - 1) delay units required for each range bin in the moving window realization. The range bin accumulators are still required. The rate of operation is K · PRF, e.g., a 1000-range bin, 5000-Hz system will require 5×10^6 operations per second. This is well within the state-of-the-art hardware capability.

Table 1 presents a summary of the hardware requirements for the various configurations discussed in this section. The appreciable savings that can be obtained in hardware by using the fixed window configuration is paid for by a reduction in the system performance as discussed in the following section.

III. SNR CONSIDERATIONS

The purpose of the integrator shown in Figure 1 is to increase the SNR prior to the threshold comparison. In this analysis, clutter signals are considered to be sufficiently suppressed by the MTI filter such that they are negligible in comparison to the wide band Gaussian noise at the output of the MTI filter. The theoretical analysis is performed for the square-law detector $(I^2 + Q^2)$ but has not been performed for a linear detector $(\sqrt{I^2 + Q^2})$; however, results obtained by simulation are included for both detectors. It can be anticipated that the two detectors will have similar probability of detection performance characteristics (cf., Figure 42 of Marcum [5], or as repeated on

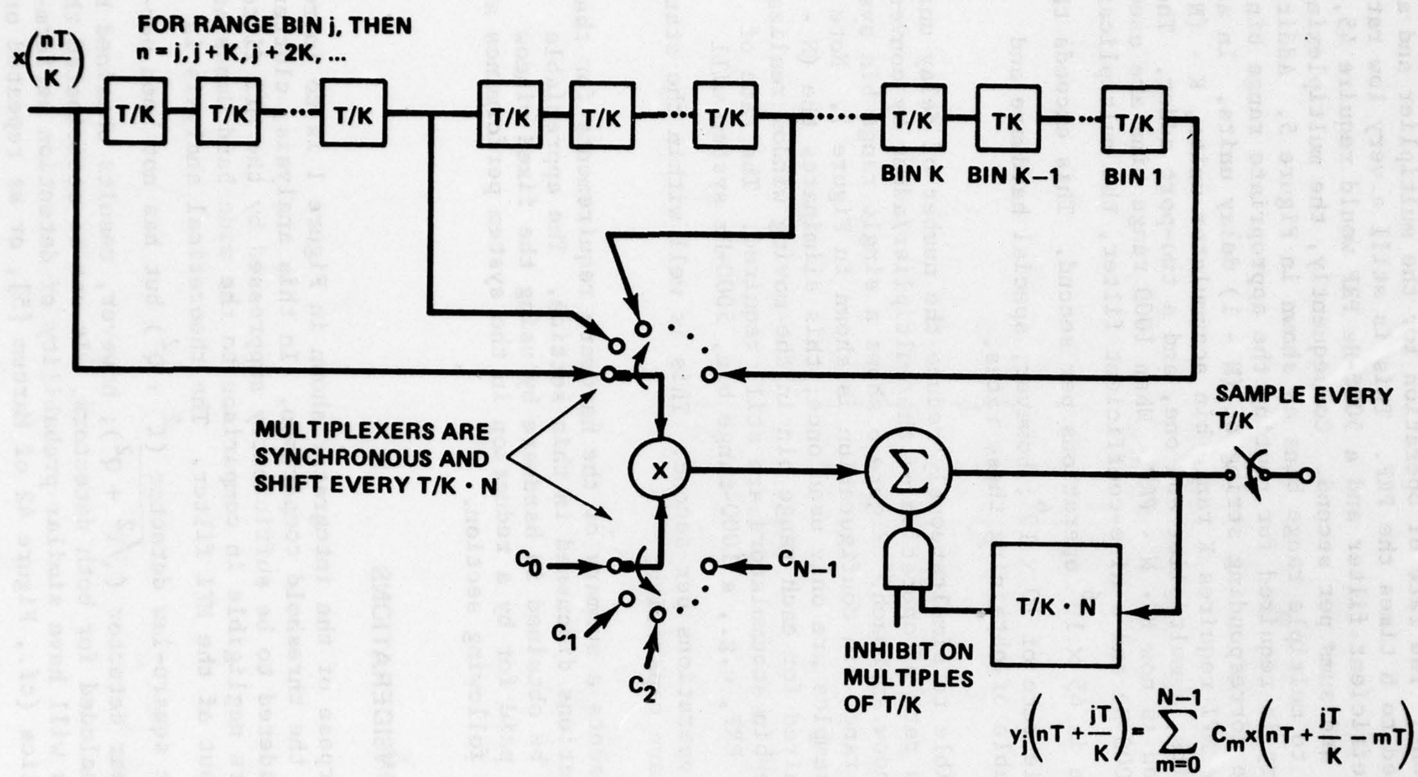


Figure 5. A moving window MTI filter using multiplexing for K range bins.

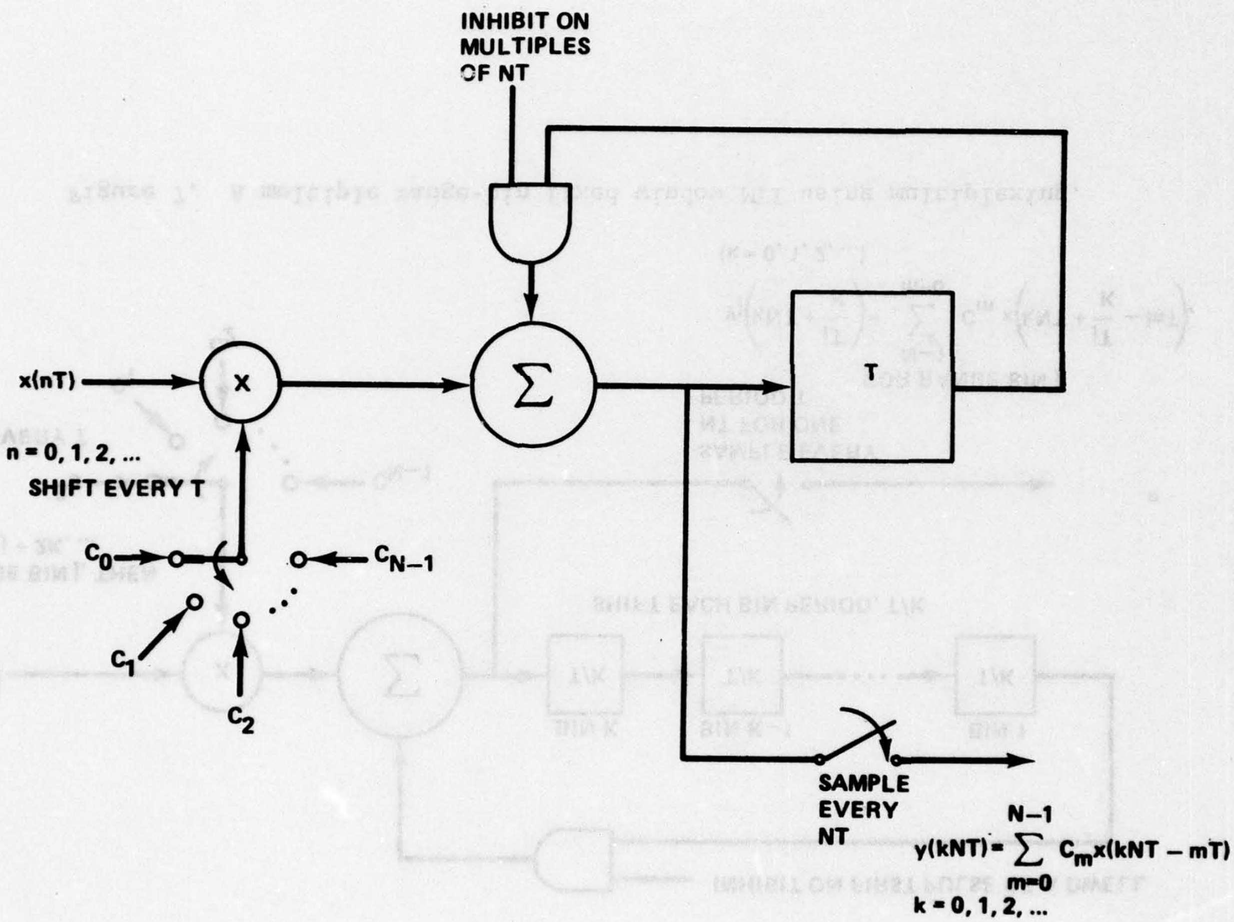
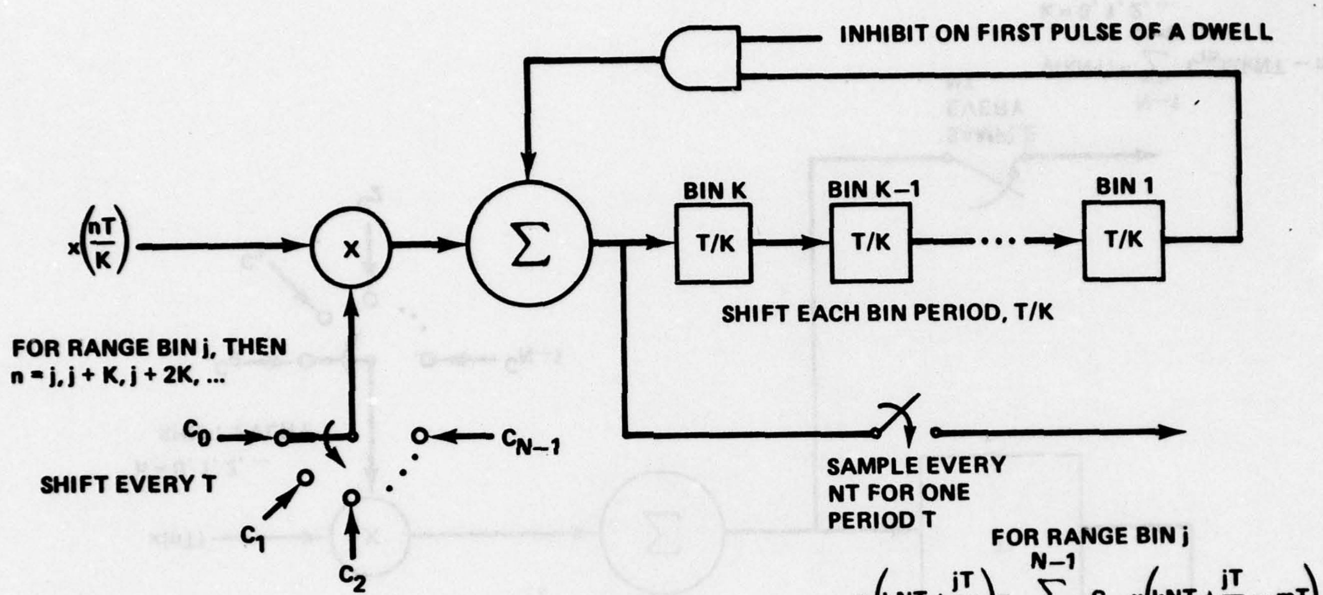


Figure 6. A fixed window MTI filter using multiplexing for one range bin.



$$y_j\left(kNT + \frac{jT}{K}\right) = \sum_{m=0}^{N-1} C_m \, x\left(kNT + \frac{jT}{K} - mT\right),$$

Figure 7. A multiple range-bin fixed window MTI using multiplexing.

TABLE 1. MOVING AND FIXED WINDOW MTI HARDWARE REQUIREMENTS FOR A K-RANGE BIN RADAR

Hardware	Moving Window			Fixed Window		
	Multiplexing			Multiplexing		
	None	for 1-Range Bin	for K-Range Bins	None	for 1-Range Bin	for K-Range Bins
Delays	K(N - 1)	KN	K(N - 1) + 1	-	K	K
Two-port adders or	K(N - 1)	K	1	-	K	1
N-port adders	K	0	0	-	0	0
Multiplexers	0	2K	2	-	K	1
Multipliers	KN	K	1	-	K	1
Multiplication time	T	T/N	T/(KN)	-	T	T/K

pp. 68 and 69 of Meyer and Mayer [6]). Results obtained for the square-law detector are in agreement with those stated by Hall and Ward [7] and Kretschmer [8] for the RF counterpart of this problem without quadrature channels.

A detailed derivation is presented in the Appendix and a summary of the more important steps are presented in this section. The main objective of the analysis is to relate the SNR at the integrator output for the fixed window and moving window structures. Only the SNR is considered, i.e., probability of detection is not specifically analyzed.

There are several forms of the SNR that could be used for this analysis. For example, Hall and Ward [7] used a noise-only analysis and defined a ratio of normalized noise variances to obtain the effective number of independent samples integrated. An equivalent procedure is to use the ratio of signal-only output to noise-only output. The signal-only output of the integrator for a sinusoid input with peak value P is

$$A = \sum_{k=1}^{K} R_k = KP^2 \quad . \tag{4}$$

For SNR purposes it follows that

$$\bar{A}^2 = K^2 P^4 \quad . \tag{5}$$

The noise-only output variance can be expressed as given in Appendix A by Equation (A.10), i.e.,

$$\sigma_A^2 = \sigma_R^2 \sum_{k=1}^{K} \sum_{j=1}^{K} \rho_{kj}$$

$$= \frac{K^2}{K_e} \sigma_R^2 \quad . \tag{6}$$

The SNR at the residue unit output is

$$SNR_R = \frac{P^4}{\sigma_R^2} \tag{7}$$

while the SNR at the integrator output is

$$SNR_A = \frac{K^2 P^4}{\sigma_A^2} = K_e \frac{P^4}{\sigma_R^2}$$

$$SNR_A = K_e \; SNR_R \quad . \tag{8}$$

For totally independent residues such as from a fixed window MTI it is shown (Appendix A) that the number of residues integrated is equal to the SNR improvement, i.e.,

$$K_e = K_{FW} \quad . \tag{9}$$

The residues are correlated for a moving window MTI and K_e depends on the total number of residues integrated, K_{MW}, and the coefficients of the MTI [cf., Equation (A.27)]. If the residues were totally correlated, the K_e would equal 1 and it follows that

$$1 \leq K_e \leq K_{MW} \quad . \tag{10}$$

There may be major limitations to using the *results* of this SNR analysis and extending this to performance considerations such as the probability of false alarm and probability of detection. The outputs of the integrator for a moving window and fixed window MTI are described by different probability density functions. Marcum [5] has analyzed the case when pulses of statistically-independent noise plus signal are noncoherently integrated. This is equivalent to analyzing the fixed window MTI.

An analysis has not been performed for the case of correlated noise. An approximation of the performance can be obtained by making a number of assumptions and then using Marcum's results. If the number of residues integrated are sufficiently large, the central limit theorem indicates that the integrator output will become Gaussian. The mean value and variance of the output are then sufficient to calculate detection or false alarm statistics. This suggests that the Marcum results for N-pulse noncoherent integration can be used for the moving window MTI if K_e is used instead of the total number of residues.

Appendix A shows the derivation of K_e when a square-law detector is used; i.e., $R = I^2 + Q^2$. A simulation program was written to verify these results. It was possible to include the simulation of a linear detector, i.e., $R = \sqrt{I^2 + Q^2}$. Comparisons of the results for a three-pulse canceler is shown in Figure 8. It is noted that this moving window MTI produces approximately three times as many residues as a fixed window MTI [cf., Equation (3)], but correlation reduces this to an effective increase of approximately 1.5 times as many residues. It is also noted that there is little difference in the linear and square-law detectors.

The effect of the coefficient design method is illustrated for the binomial technique [9] and the Houts and Burlage technique [3] in Figures 9, 10, and 11. The relative results for the design methods depend on the number of coefficients used and can be very significant. Table 2 presents the mathematical relationship between K_e and K_{FW} based on a least-squared error fit to calculated points.

IV. CONCLUSIONS

The implementation of either a moving window or a fixed window MTI in a quadrature processor determines the amount of hardware required and the SNR improvement obtained.

The difference in hardware used for either structure is inconsequential if only one range bin is implemented but, typically, radars contain a large number of range bins, i.e., greater than 250. Due to the range bin requirement, the amount of hardware for either structure proves prohibitive unless multiplexing is used. If multiplexing is used, the additional amount of hardware necessary for moving window over fixed window is not significant (except for memory); however, as previously stated, the timing restraints for the multiplexed implementation of a moving window processor is beyond the present state-of-the-art.

In the future, as component speed increases and size decreases, the implementation of a moving window MTI will become less hardware and speed critical; however, there will still be the problem of correlated noise in the MTI outputs. As previously presented, the effective SNR improvement is reduced by the correlated noise and is a function of the particular MTI used, i.e., binominal, Houts-Burlage, etc.

The utilization of either a linear or square-law detector affects the SNR improvement. Even though the linear detector is slightly better, the difference is not considered significant. The linear detector will have a dynamic range advantage; Hall [10] has stated a collapsing loss disadvantage.

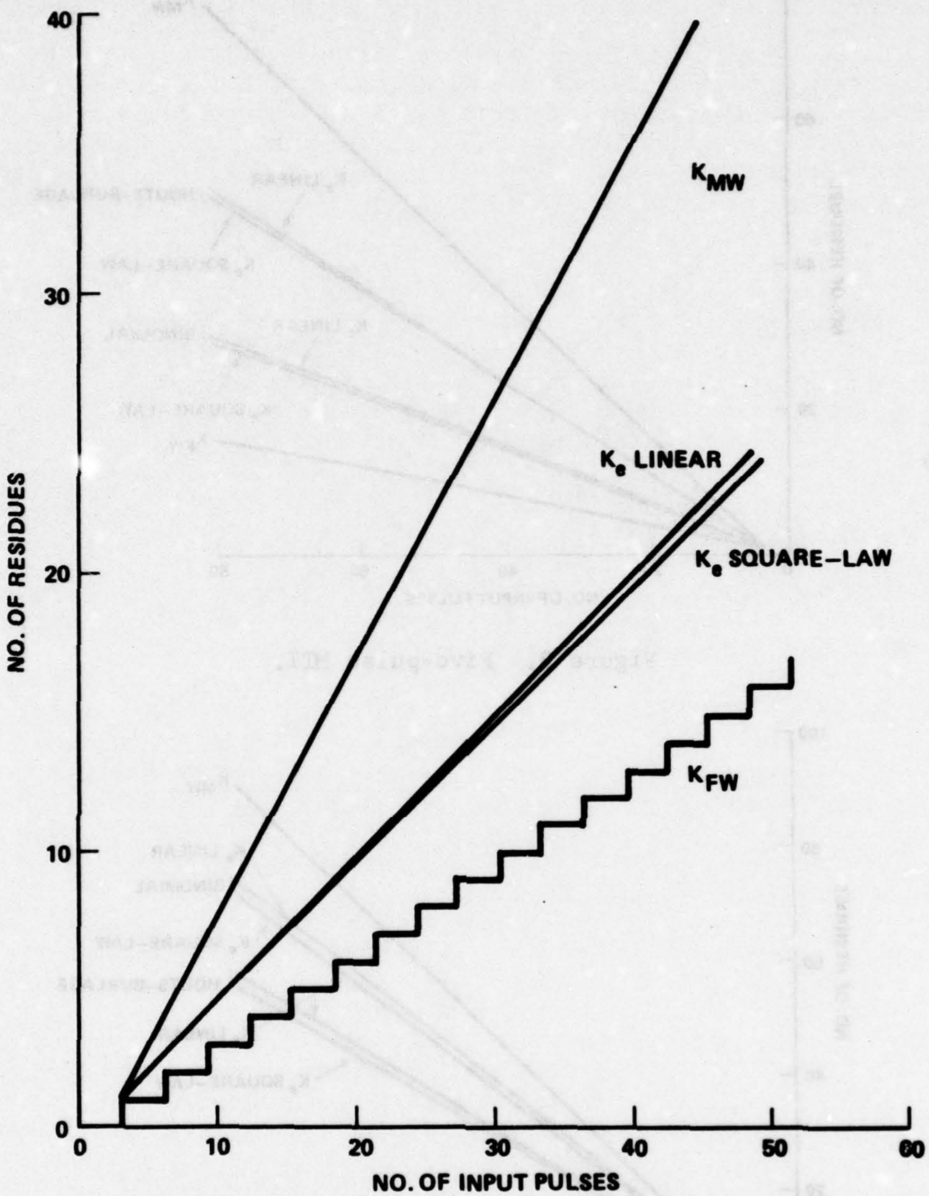


Figure 8. Three-pulse canceler.

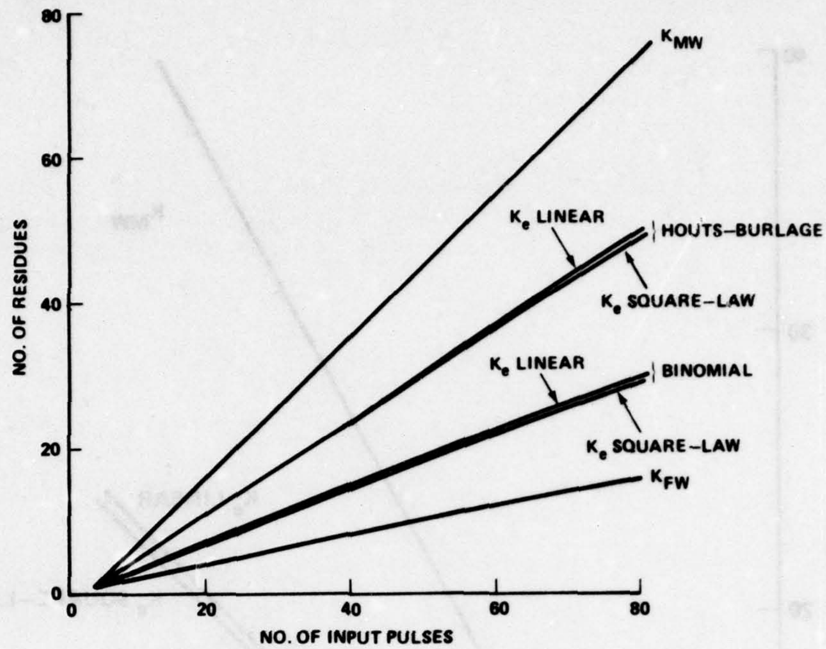


Figure 9. Five-pulse MTI.

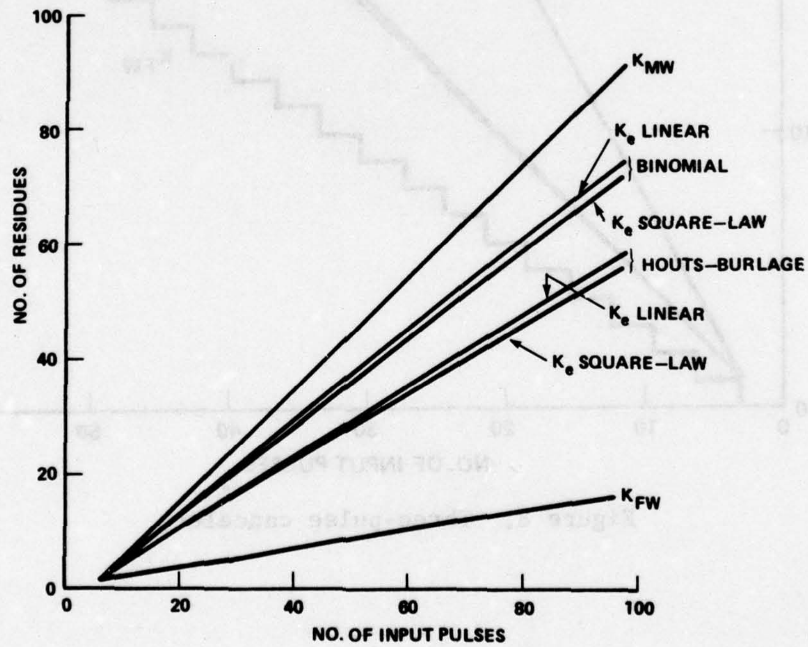


Figure 10. Six-pulse MTI.

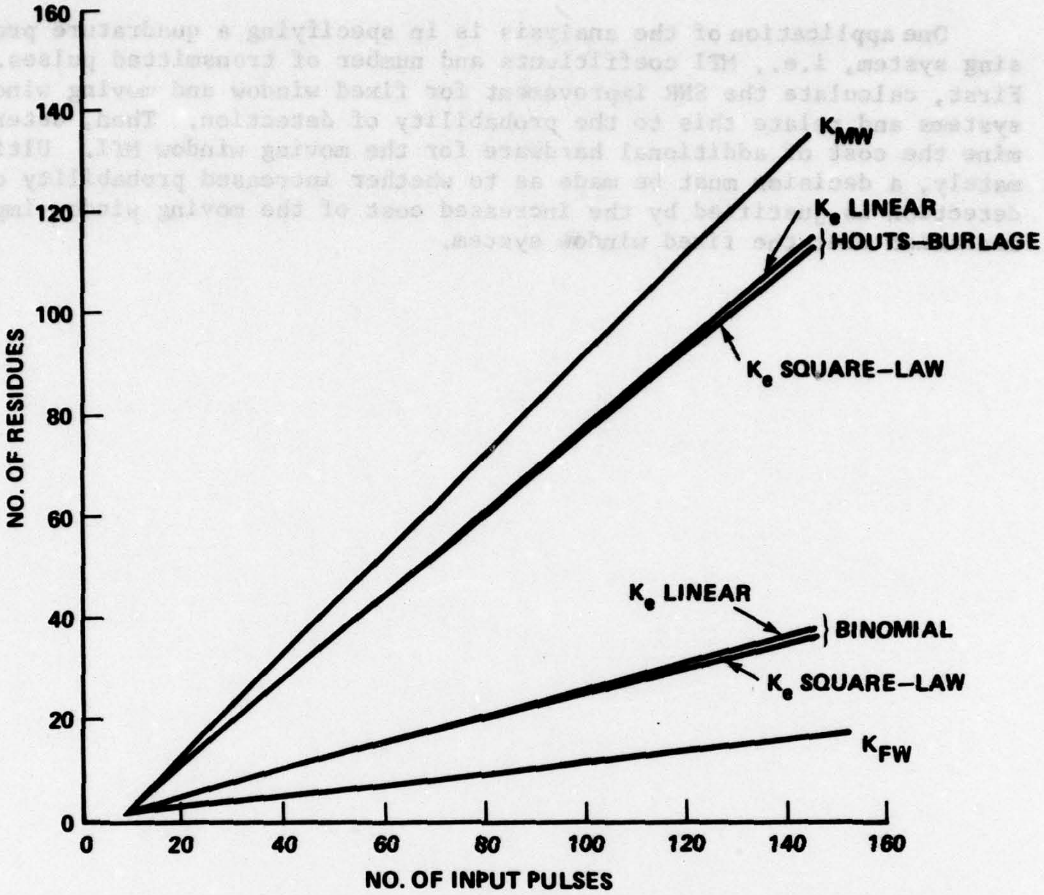


Figure 11. Nine-pulse MTI.

TABLE 2. LINEAR RELATIONSHIP OF K_e AND K_{FW} USING LEAST SQUARES METHOD

	K_e	
	Houts-Burlage	Binomial
Three-pulse MTI	$1.537\ K_{FW} - 0.689$	(Same as Houts-Burlage)
Five-pulse MTI	$3.222\ K_{FW} - 2.130$	$1.895\ K_{FW} - 1.128$
Six-pulse MTI	$3.667\ K_{FW} - 2.611$	$4.721\ K_{FW} - 3.583$
Nine-pulse MTI	$7.456\ K_{FW} - 6.322$	$2.469\ K_{FW} - 1.776$

One application of the analysis is in specifying a quadrature processing system, i.e., MTI coefficients and number of transmitted pulses. First, calculate the SNR improvement for fixed window and moving window systems and relate this to the probability of detection. Then, determine the cost of additional hardware for the moving window MTI. Ultimately, a decision must be made as to whether increased probability of detection is justified by the increased cost of the moving window implementation over the fixed window system.

Appendix A. COMPARISON OF SNR AT OUTPUTS OF MOVING AND FIXED WINDOW MOVING TARGET INDICATORS

The system configuration used in this analysis is discussed in Section I and shown in Figure 1. This is a baseband system with quadrature channels I and Q. The kth residue R_k is assumed to be calculated as

$$R_k = I_k^2 + Q_k^2 \quad . \tag{A.1}$$

The integrator forms an input for the decision element as

$$A = \sum_{k=1}^{K} R_k \quad . \tag{A.2}$$

An analysis by Hall and Ward [7] has considered the RF counterpart of this problem but does not use quadrature channels. The main point of their analysis is to show the equivalent number of statistically independent residues that are obtained in a moving window MTI.

1. SNR ANALYSIS FOR FIXED WINDOW MTI

The I or Q channel outputs for the kth residue determination for a fixed window MTI are given by

$$y_\alpha(kNT) = \sum_{n=o}^{N-1} C_n x_\alpha(kNT - nT), \{\alpha = I,Q\},\ k = 1,2,\ ...,\ K_{FW} \quad , \tag{A.3}$$

where the filter has N coefficients $C_0, C_1, ... C_{N-1}$, and $x_\alpha(\ )$ represents the input to the channel. The signal component of the input and the noise component of the input are analyzed separately. Since the output is taken at multiples of N input samples, it follows that the output will have reached a steady-state value for the sinusoidal signal component. The same statement is true for the moving window MTI. Also, the signal component inputs to the I and Q channel are in quadrature and the residue calculation will yield an answer that is proportional to the magnitude-squared value of this component. Consequently, only the noise component is considered in the details of the analysis.

Assuming white Gaussian noise with variance σ^2 and zero mean value as the input signal in Equation (A.3) and then determining the variance of the I or Q channel output, σ_α^2, gives

$$\sigma_\alpha^2 = E\left\{y_\alpha^2(kNT)\right\}$$

$$\sigma_\alpha^2 = E\left\{\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} C_n C_m x(kNT - nT)x(kNT - mT)\right\}$$

$$= \sigma^2 \sum_{n=0}^{N-1} C_n^2 \quad , \qquad \text{(A.4)}$$

where the statistical independence is used for the x( ) terms when $n \neq m$.

The residue term and the decision term are calculated per Equations (A.1) and (A.2), and the variance of A can be related to the variance of R_k as follows

$$\sigma_A^2 = E\{(A - \bar{A})^2\} = E\{A^2\} - \bar{A}^2$$

$$= E\left\{\sum_{k=1}^{K_{FW}}\sum_{j=1}^{K_{FW}} R_k R_j\right\} - E^2\left\{\sum_{k=1}^{K_{FW}} R_k\right\}$$

$$= K_{FW}\left[E\left\{R_k^2\right\} - E^2\{R_k\}\right] = K_{FW}\sigma_R^2 \quad . \qquad \text{(A.5)}$$

This result uses the statistical independence of the residues in the fixed window MTI. Since the variance of A has been increased by the factor of K_{FW}, it follows that the SNR has increased by a corresponding amount since the signal component will increase by K_{FW}, i.e.,

$$SNR_A = K_{FW} \cdot SNR_R \quad . \qquad \text{(A.6)}$$

2. SNR ANALYSIS FOR MOVING WINDOW MTI

The analysis for the moving window MTI follows the same pattern presented in the preceding section. Two differences are encountered. First, the number of residues calculated is not the same as for the fixed window MTI. If the same number of input samples are processed in an N-coefficient filter, the number of residues for the moving window MTI are

$$K_{MW} = N(K_{FW} - 1) + 1 \quad , \qquad \text{(A.7)}$$

where K_{FW} is the number of residues calculated by the fixed window MTI. The second difference in the analysis is that the residues are not necessarily statistically independent. It is desirable, and possible, to perform an analysis similar to Equation (A.5) and obtain an equivalent number K_e of statistically independent residues, i.e.,

$$SNR_A = K_e \cdot SNR_R \quad . \qquad \text{(A.8)}$$

This form results by expressing the variance of A as in the first part of Equation (A.5), i.e.,

$$\sigma_A^2 = E\left\{\sum_{k=1}^{K_{MW}} \sum_{j=1}^{K_{MW}} R_k R_j\right\} - E^2\left\{\sum_{k=1}^{K_{MW}} R_k\right\}$$

$$= \sum_k \sum_j \left[\overline{R_k R_j} - \bar{R}^2\right] \quad . \qquad \text{(A.9)}$$

Manipulations are performed to obtain

$$\sigma_A^2 = \sigma_R^2 \left[\sum_k \sum_j \frac{\overline{R_k R_j} - \bar{R}^2}{\sigma_R^2}\right] = \sigma_R^2 \sum_k \sum_j \rho_{jk} \quad , \qquad \text{(A.10)}$$

where ρ_{kj} is the normalized covariance of the k and j residues. Comparing to Equations (A.5) and (A.8) yields

$$K_e = \frac{K_{MW}^2}{\sum_k \sum_j \rho_{kj}} \cdot \qquad \text{(A.11)}$$

The analysis can be completed by a process that evaluates ρ_{kj} in terms of the filter coefficients.

The I or Q channel output for the kth residue determination in a moving window MTI is

$$y_\alpha(hT) = \sum_{n=0}^{N-1} C_n x_\alpha(hT - nT), \; \{\alpha = I,Q\} \quad , \qquad \text{(A.12)}$$

where the requirements on h are

$$h = N, N + 1, \ldots, K_{MW} \quad . \qquad \text{(A.13)}$$

The kth residue occurs at $h = N + (K - 1)$ and the relationship between K_{MW} and K_{FW} is given by Equation (A.7).

The determination of the normalized covariance begins by considering

$$\rho_{kj} = \frac{\overline{R_k R_j} - \bar{R}^2}{\sigma_R^2} \quad , \qquad \text{(A.14)}$$

and evaluating the various terms. First,

$$\sigma_R^2 = \bar{R^2} - \bar{R}^2 \quad . \qquad \text{(A.15)}$$

Since I and Q are assumed identically distributed, zero mean, and statistically independent, then

$$\bar{R}^2 = \left(\overline{I^2 + Q^2}\right)^2 = 4\bar{I^2}^2$$

$$\bar{R}^2 = 4\sigma_I^4 \quad , \qquad \text{(A.16)}$$

and

$$\overline{R^2} = (\overline{I^4 + 2I^2Q^2 + Q^4}) = 2\overline{I^4} + 2\overline{I^2}^2 = 2\overline{I^4} + 2\sigma_I^4 \quad ,$$

For the white Gaussian noise as the input x( ), it follows that I and Q are also Gaussian and that the fourth moment is related to the mean squared value as,

$$\overline{I^4} = 3\left(\overline{I^2}\right)^2 = 3\sigma_I^4 \quad . \tag{A.17}$$

Thus, from Equation (A.16)

$$\overline{R^2} = 2(3\sigma_I^4) + 2\sigma_I^4 = 8\sigma_I^4 \quad . \tag{A.18}$$

Using Equations (A.18) and (A.16) in Equation (A.15) gives

$$\sigma_R^2 = 8\sigma_I^4 - 4\sigma_I^4 = 4\sigma_I^4 \quad . \tag{A.19}$$

and

$$\rho_{kj} = \frac{\overline{R_k R_j}}{4\sigma_I^4} - 1 \tag{A.20}$$

The second part of the evaluation takes

$$\overline{R_k R_j} = \overline{\left(I_k^2 + Q_K^2\right)\left(I_j^2 + Q_j^2\right)} = 2\overline{I_k^2 I_j^2} + 2\sigma_I^4 \tag{A.21}$$

and substitutes in the preceding expression to obtain

$$\rho_{kj} = \frac{\overline{I_k^2 I_j^2}}{2\sigma_I^4} - \frac{1}{2} = \frac{3}{2}\left[\frac{\overline{I_k^2 I_j^2}}{\overline{I^4}} - \frac{1}{3}\right] \quad . \tag{A.22}$$

Using the procedure for the expected value of the product of the squares of Gaussian variables as suggested on pages 205 and 206 of Wozencraft and Jacobs [11] allows a simplification to occur, viz.,

$$\overline{I_k^2 I_j^2} = \overline{I_k^2}\,\overline{I_j^2} + \overline{I_k I_j}\;\overline{I_k I_j} + \overline{I_k I_j}\;\overline{I_k I_j} = \left(\overline{I^2}\right)^2 + 2\left(\overline{I_k I_j}\right)^2$$

$$= \frac{1}{3}\,\overline{I^4} + 2\left(\overline{I_k I_j}\right)^2 \quad . \qquad \text{(A.23)}$$

Substituting into Equation (A.22) yields

$$\rho_{kj} = \frac{3}{2}\left[\frac{1}{3} + \frac{2\left(\overline{I_k I_j}\right)^2}{\overline{I^4}} - \frac{1}{3}\right] = \left(\frac{\overline{I_k I_j}}{\sigma_I^2}\right)^2 . \qquad \text{(A.24)}$$

From Equation (A.11) it is seen that ρ_{kj} is summed over all k and j. The evaluation of ρ_{kj} will depend only on the magnitude of the difference between k and j. By constructing a matrix containing ρ_{kj} as elements and then recognizing that $\rho_{kj} = \rho_{jk}$ and that $\rho_{kk} = 1$, it can be shown that

$$\sum_{k=1}^{K_{MW}} \sum_{j=1}^{K_{MW}} \rho_{kj} = K_{MW} + 2 \sum_{i=1}^{K_{MW}-1} (K_{MW} - 1) \left(\frac{\overline{I_k I_{k+i}}}{\sigma_I^2}\right)^2 . \qquad \text{(A.25)}$$

The evaluation cintinues by forming

$$\overline{I_k I_{k+i}} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} C_n C_m \overline{x(kT - nT)x(kT - mT + iT)}$$

$$= \sum_{m=0}^{N-1} C_{m-i} C_m \sigma^2 \quad , \qquad \text{(A.26)}$$

where $C_{m-i} = 0$ for $(m - i) < 0$. The σ_I^2 term in Equation (A.25) can be evaluated as in Equation (A.4). This allows the equivalent number of residues to be formulated from Equation (A.11) as

$$K_e = \frac{K_{MW}^2}{K_{MW} + 2 \sum_{i=1}^{K_{MW}-1} (K_{MW} - i) \left[\frac{\sum_{m=0}^{N-1} C_{m-i} C_m}{\sum_{n=0}^{N-1} C_n^2} \right]} \quad . \qquad (A.27)$$

This can be expressed in terms of the number of residues calculated by the fixed window MTI by using Equation (A.7) and can be related to the output SNR by using Equation (A.8).

A computer program was written to compute the results of Equation (A.27). A listing of this program is included in Appendix B.

Appendix B. PROGRAM LISTING

```
      PROGRAM KEON(INPUT,OUTPUT,TAPE5,TAPE6,TAPE1)
C
C     THIS PROGRAM CALCULATES THE SNR GAIN POSSIBLE WHEN
C     CHANGING FROM FIXED TO MOVING WINDOW MTI.  THE ANALYSIS
C     ASSUMES THAT RESIDUE = I**2 + Q**2.  THE PARAMETERS AR
C          KFW  = NUMBER OF RESIDUES FOR FIXED WINDOW
C          NC = NUMBER OF FILTER COEFFICIENTS
C          KMW = NUMBER OF RESIDUES FOR MOVING WINDOW
C          KE  = EQUIVALENT NUMBER OF RESIDUES FOR
C                MOVING WINDOW
C          DB = KE/KFW   EXPRESSED IN DB
C          C( ) = COEFFICIENT ARRAY
C          CK( ) = SHIFTED VERSION OF COEF. ARRAY
C          CTC( ) = PRODUCT OF C() AND CK()
      DIMENSION C(15),CK(15),CTC(15)
      DIMENSION X(15),Y(15)
      REAL KE
      WRITE(6,104)
  104 FORMAT(1H1)
      READ(5,99) NC
   99 FORMAT(I2)
      READ(5,100) (C(I),I=1,NC)
  100 FORMAT(4F20.10)
      WRITE(6,101)
  101 FORMAT(//2X,21HTHE COEFFICIENTS ARE /)
      WRITE(6,102) (C(I),I=1,NC)
  102 FORMAT(1X,4(F20.10,2X))
      WRITE(6,111)
  111 FORMAT(//8X,2HNC,6X,4H KFW,8X,3HKMW,6X,3H KE,8X,2HDB/)
C
C         SUM CN**2
C
      SUMCSQ=0.
      DO  9 J=1,NC
      SUMCSQ=SUMCSQ+C(J)*C(J)
    9 CONTINUE
      NCM1=NC-1
      DO 12  KFW=2,16
C
C     SUMMING COVARIANCE TERMS
C
      I=KFW-1
      KMWM=NC*15+1
      KMWM=KMWM+5
      FKM=FLOAT(KMWM)/5.
      FKM=5.0*AINT(FKM)
      KMW=NC*(KFW -1)+1
      SUMDEN=KMW
      DO 10 K=1,NCM1
C
C          CORRELATION PRODUCT CM-I*CM
C
      SUMCTC=0.
      DO 11 MP1=1,NC
      CTC(MP1)=0.0
      IF((MP1-K).GT.0) CTC(MP1)=C(MP1-K)*C(MP1)
      SUMCTC=SUMCTC+CTC(MP1)
```

```
 11   CONTINUE
      RHO=SUMCTC/SUMCSO
      RHOSQ=RHO**2
      KDEL=KMW-K
      SUMDEN=SUMDEN+2.0*FLOAT(KDEL)*RHOSQ
 10   CONTINUE
      KE=FLOAT(KMW)**2/SUMDEN
      FREW=FLOAT(KFW)
      DB=10.*ALOG10(KE/FREW)
      WRITE(6,103) NC,KFW,KMW,KE,DB
103   FORMAT(1X,I9,2I10,2F10.2)
      X(I)=FLOAT(KMW)
      Y(I)=KE
 12   CONTINUE
      STOP
      END
```

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-78