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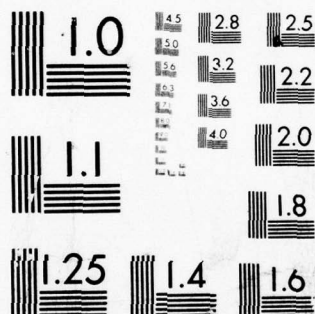
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model. Results suggest the effects from compressional damping have a relatively narrow frequency bandwidth centered at the compressional (delamination) frequency, ω_c , of the composite. Compressional damping is shown to have a minimal effect on the transverse damping response of thin three-layer damped beams for frequencies significantly less than ω_c , where a shear damping model provides a better description of dynamic response.

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TABLE OF CONTENTS

	Page
LIST OF FIGURES.	iii
NOTATION	v
LIST OF ABBREVIATIONS.	vii
ABSTRACT	1
ADMINISTRATIVE INFORMATION	1
INTRODUCTION	1
ANALYTICAL FORMULATION	3
TRANSVERSE COMPRESSIONAL DAMPING MODEL.	4
DISTRIBUTED MASS-VISCOELASTIC SPRING-MASS SYSTEM.	10
SHEAR DAMPING MODEL	12
EXPERIMENTAL EVALUATION.	17
DISCUSSION	22
CONCLUSIONS.	23
REFERENCES	25

LIST OF FIGURES

1 - Geometry and Coordinate System for the Fully Constrained Elastic-Viscoelastic-Elastic Beam. . . .	3
2 - Transverse Driving Point Mechanical Impedance and Phase Angle Spectrum for a Three-Layer Damped Beam, Fully Constrained Compressional Damping Model.	9
3 - The Mechanical Impedance Magnitude Response of the Distributed Mass-Viscoelastic Spring-Mass System	12
4 - Transverse Driving Point Mechanical Impedance Spectrum of an Elastic Beam, An Evaluation of Experimental Boundary Conditions and Instrumentation.	19

	Page
5 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 1.	20
6 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 2.	21
7 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 3.	21

NOTATION

$w(x)$	Transverse displacement
$u(x)$	Longitudinal displacement
i	$\sqrt{-1}$
m	Mass per unit length
l	Beam length
t_v	Thickness of viscoelastic core
t_i	Thickness of i^{th} layer
b	Beam width
E_i	Elastic (Young's) modulus of i^{th} layer
ρ_i	Mass density of i^{th} layer
$E^*(\omega)$	$E_v (1 + i\delta) =$ complex dynamic elastic modulus of viscoelastic core
E_v	Elastic storage modulus of viscoelastic core
δ	Elastic loss tangent of viscoelastic core
$G^*(\omega)$	$G_v (1 + i\beta) =$ complex dynamic shear modulus of viscoelastic core
G_v	Shear storage modulus of viscoelastic core
β	Shear loss tangent of viscoelastic core
ω	Radial frequency
I_i	Moment of inertia of the i^{th} layer
k^*	Compressional spring constant
ω_c	Compressional composite frequency
$Z(x, \omega)$	Mechanical impedance of beam
P_0	Applied force

ϵ, μ

Complex flexural wave number for transverse
compressional damping model

$\delta_1, \delta_2, \delta_3$

Complex flexural wave number for shear damping
model

LIST OF ABBREVIATIONS

dB	decibel
DTNSRDC	David W. Taylor Naval Ship Research and Development Center
Hz	Hertz
in.	Inch
in/sec	Inch per second
i.e.	That is
kg	Kilograms
kg/m ³	Kilograms per cubic meter
lb	Pound
m	Meter
m/sec	Meters per second
N	Newtons
NAVSEA	Naval Sea Systems Command
N/m ²	Newtons per square meter
psi	Pounds per square inch
sec	Second

ABSTRACT

The effects of transverse compressional damping on the vibratory response of three-layer elastic-viscoelastic-elastic beams are considered both analytically and experimentally in a mechanical impedance format. The relative importance of this type of damping is assessed by comparison to shear damping mechanisms inherent in the composite using the Mead and Markus model. Results suggest the effects from compressional damping have a relatively narrow frequency bandwidth centered at the compressional (delamination) frequency, ω_c , of the composite. Compressional damping is shown to have a minimal effect on the transverse dynamic response of thin three-layer damped beams for frequencies significantly less than ω_c , where a shear damping model provides a better description of dynamic response.

ADMINISTRATIVE INFORMATION

This report represents work performed under the Exploratory Development Acoustical Program, Silencing for Auxiliary Machinery Systems Program Element/Task Area 62543N, SF 434-52-702, Task 18182, Work Unit 1-2740-111.

The cognizant NAVSEA program manager is Mr. S. G. Wieczorek, NAVSEA (SEA 037T); the DTNSRDC program manager is Dr. Y. F. Wang (Code 2740).

INTRODUCTION

The transverse vibratory response of elastic-viscoelastic-elastic laminated beams has received considerable attention since Plass^{1*} and Kerwin² examined the potential of this composite in vibration control. Many investigators¹⁻⁸ have studied the dynamic response of the three-layer damped sandwich beam, concentrating predominantly on the broad-band damping inherent in the composite associated with shear damping. In a classic

*A complete listing of references is given on page 25.

paper² on this subject, Kerwin analyzed the shear damping in an infinitely long, simply supported beam with a soft viscoelastic core and a thin, stiff constraining layer, deriving an expression for the complex flexural stiffness of the beam section. DiTaranto³ extended Kerwin's work deriving a sixth-order differential equation of motion in terms of dynamic longitudinal beam displacement, $u(x)$. In a later paper, Mead and Markus⁴ derived a sixth-order differential equation in terms of the transverse motion of the beam, which is an important factor in experimentally validating the model. In the same paper, Mead and Markus also examined the form of the boundary constraints on the composite for many widely used end conditions and showed that the eigenvalues for such a system are generally complex for boundary conditions other than simply supported. Lu and Douglas⁵ evaluated the Mead and Markus model in several experiments and showed that it adequately predicted the damped resonance frequencies and damping inherent in the low-order modes of two relatively thin three-layer laminates.

An important feature of the above-mentioned work was the assumption that transverse displacements, $w(x)$, of all points on a cross section are equal. For thin composites where the product of the viscoelastic layer thickness and the constraining layer thickness is small, shear damping appears to be the major factor controlling the resonance response of these beams in the audiofrequency spectrum. However, as the thickness of soft ($E_v < 10^3 \text{ N/m}^2$)* viscoelastic cores and constraining layer increase, compressional damping can be expected to play an increasingly important role in the dynamic response of such structures. This report examines, both analytically and experimentally, the contribution of transverse compressional damping on the transverse dynamic response of the three-layer damped

*Definitions of abbreviations used are given on page vii.

beam and compares the importance of this form of damping with shear damping by using the model developed by Mead and Markus.

ANALYTICAL FORMULATION

The three-layer damped beam consists of two elastic-face layers separated by a thin, relatively soft viscoelastic damping core. Figure 1 depicts the geometry and coordinate system utilized in this report.

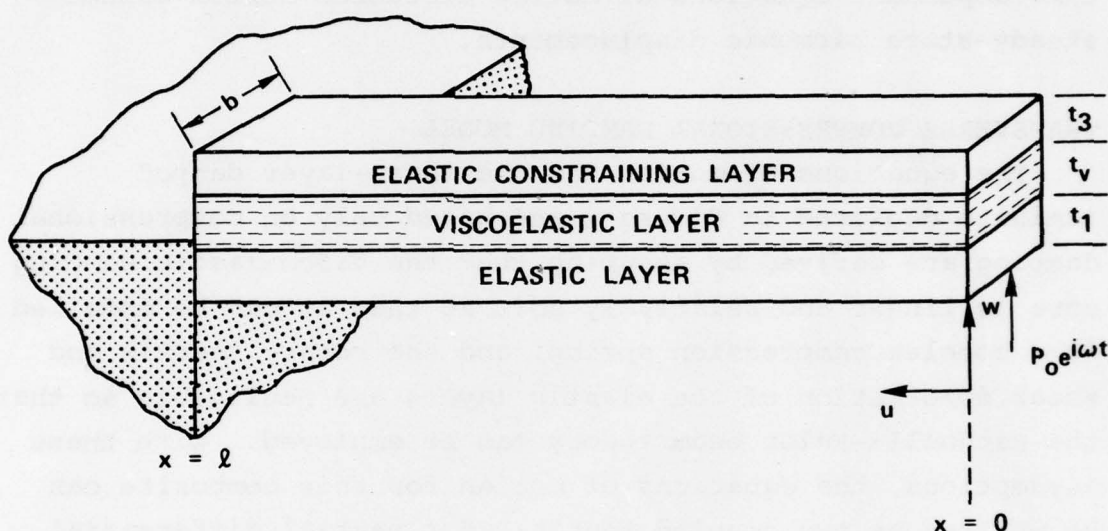


Figure 1 - Geometry and Coordinate System for the Fully Constrained Elastic-Viscoelastic-Elastic Beam

The case of fixed-free (cantilever) boundary constraints is considered with a concentrated sinusoidal load, $P_0 e^{i\omega t}$, applied at the free end to facilitate comparison between analytical and experimentally derived spectra. The dynamic response of this composite is examined in a mechanical impedance format with m_i the mass per unit length of the i^{th} layer; E_i the elastic (Young's) modulus of the i^{th} layer; $E_V^* = E_V (1 + i\delta)$ the dynamic complex modulus of the viscoelastic core, where E_V is the elastic storage modulus and δ the elastic loss tangent; and $G_V^* = G_V (1 + i\beta)$ the dynamic complex shear

modulus of the viscoelastic core where, in a similar manner, G_v is the shear storage modulus and β the shear loss tangent. The complex elastic and shear moduli are assumed to be both temperature- and frequency-dependent, and the loss tangents of the elastic layers are assumed negligible. No restrictions are placed on the densities and moduli of the layers, except that the viscoelastic layer is considered soft compared to the elastic layers, i.e., $E_v \ll E_i$, and its mass is negligible. The elastic layers need not be identical. The time-dependent equations of motion discussed herein assume steady-state harmonic displacements.

TRANSVERSE COMPRESSIONAL DAMPING MODEL

The equations of motion for the three-layer damped laminate depicted in Figure 1 and based only on compressional damping are derived by assuming that the viscoelastic damping core is linear and relatively soft so that it can be modelled as a complex compression spring, and the rotary inertia and shear deformation of the elastic layers are negligible so that the Bernoulli-Euler beam theory can be employed. With these assumptions, the equations of motion for this composite can be written as two coupled fourth-order partial differential equations:

$$\begin{aligned}
 - E_1 I_1 \frac{\partial^4 w_1}{\partial x^4} &= k^* (w_1 - w_3) + m_1 \frac{\partial^2 w_1}{\partial t^2} \\
 - E_3 I_3 \frac{\partial^4 w_3}{\partial x^4} &= k^* (w_3 - w_1) + m_3 \frac{\partial^2 w_3}{\partial t^2}
 \end{aligned} \tag{1}$$

where $w_i(x)$ is the transverse displacement of the i^{th} layer, I_i the moment of inertia of the i^{th} layer, and k^* is the viscoelastic spring constant, $(E_v^* b)/(t_v)$. Assuming harmonic

time dependence, these equations can be combined into a single eighth-order differential equation with complex coefficients for the basic cantilever beam (i.e., layer 1):

$$\begin{aligned} \frac{d^8 w_1(x)}{dx^8} + \left[\frac{k^* - m_1 \omega^2}{E_1 I_1} + \frac{k^* - m_3 \omega^2}{E_3 I_3} \right] \frac{d^4 w_1(x)}{dx^4} \\ + \left[\frac{m_3 m_1 \omega^4 - k^* \omega^2 (m_1 + m_3)}{E_1 I_1 E_3 I_3} \right] w_1(x) = 0 \end{aligned} \quad (2)$$

The response of the constraining layer can be written in terms of the response of the cantilever beam as:

$$w_3(x) = \frac{k^* - m_1 \omega^2}{k^*} w_1(x) + E_1 I_1 \frac{d^4 w_1}{dx^4} \quad (3)$$

Using a progressive wave approach, the solution for equation (2) can be written in terms of the complex wave numbers ϵ and μ as:

$$\begin{aligned} w_1(x) = A_1 e^{\epsilon x} + A_2 e^{-\epsilon x} + A_3 e^{i\epsilon x} + A_4 e^{-i\epsilon x} \\ + A_5 e^{\mu x} + A_6 e^{-\mu x} + A_7 e^{i\mu x} + A_8 e^{-i\mu x} \end{aligned}$$

and

$$\begin{aligned} w_3(x) = M \left[A_1 e^{\epsilon x} + A_2 e^{-\epsilon x} + A_3 e^{i\epsilon x} + A_4 e^{-i\epsilon x} \right] \\ + N \left[A_5 e^{\mu x} + A_6 e^{-\mu x} + A_7 e^{i\mu x} + A_8 e^{-i\mu x} \right] \end{aligned} \quad (4)$$

where

$$\epsilon = \left[-\frac{\alpha}{2} - \left(\frac{\alpha^2}{4} - \kappa \right)^{1/2} \right]^{1/4}$$

$$\mu = \left[-\frac{\alpha}{2} + \left(\frac{\alpha^2}{4} - \kappa \right)^{1/2} \right]^{1/4}$$

$$\alpha = \frac{k^* - m_1 \omega^2}{E_1 I_1} + \frac{k^* - m_3 \omega^2}{E_3 I_3}$$

$$\kappa = \frac{m_1 m_3 \omega^4 - k^* \omega^2 (m_1 + m_3)}{E_1 I_1 E_3 I_3}$$

$$M = \left[\frac{k^* - m_1 \omega^2}{k^*} + E_1 I_1 \epsilon^4 \right]$$

and

$$N = \left[\frac{k^* - m_1 \omega^2}{k^*} + E_1 I_1 \mu^4 \right]$$

For fixed-free boundary conditions, the four equations of constraint for layer 1 require:

$$w_1(l) = 0$$

$$\left(\frac{\partial w_1}{\partial x} \right)_{x=l} = 0$$

$$\left(\frac{\partial^2 w_1}{\partial x^2} \right)_{x=0} = 0$$

and

$$\left. \frac{\partial^3 w_3}{\partial w^3} \right)_{x=0} = + \frac{P_0 e^{i\omega t}}{E_1 I_1} \quad (5)$$

As indicated, the applied concentrated sinusoidal loading is accounted for implicitly in the shear boundary constraint at $x = 0$.

The four boundary constraints for layer 3 require:

$$\frac{\partial^2 w_3}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^3 w_3}{\partial x^3} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = l \quad (6)$$

The shear condition evolved from assuming the effective shear force, $\delta k_{\text{eff}} (w_3 - w_1)$ transmitted by the viscoelastic layer at the ends of the laminate is zero.

$$\left(\text{i.e., } \lim_{\delta A \rightarrow 0} \delta k_{\text{eff}} \rightarrow \lim_{\delta A \rightarrow 0} \left(\frac{k^* \delta A}{t_v} \right) = 0 \right)$$

These equations of constraint can be placed in a matrix representation to solve for the complex coefficients, A_n , by standard matrix inversion methods or $[M] \cdot [A] = [P]$. These coefficients can then be determined at each frequency of concern by multiplying the inverse of the constraint matrix, $[M]$, and the loading matrix, $[P]$. From this information, the mechanical impedance at any arbitrary point on the surface of the cantilever beam (layer 1) can then be written:

$$Z_1(x, \omega) = \frac{P_0}{i\omega} \left[A_1 e^{\epsilon x} + A_2 e^{-\epsilon x} + A_3 e^{i\epsilon x} + A_4 e^{-i\epsilon x} + A_5 e^{\mu x} + A_6 e^{-\mu x} + A_7 e^{i\mu x} + A_8 e^{-i\mu x} \right]^{-1}$$

and the transfer impedance to layer 3 as

$$Z_3(x, \omega) = \frac{P_0}{i\omega} \left\{ M \left[A_1 e^{\epsilon x} + A_2 e^{-\epsilon x} + A_3 e^{i\epsilon x} + A_4 e^{-i\epsilon x} \right] + N \left[A_5 e^{\mu x} + A_6 e^{-\mu x} + A_7 e^{i\mu x} + A_8 e^{-i\mu x} \right] \right\}^{-1} \quad (7)$$

ϵ^3	$-\epsilon^3$	$-i\epsilon^3$	$i\epsilon^3$	μ^3	$-\mu^3$	$-i\mu^3$	$i\mu^3$	A_1	$P_0/E_1 I_1$
ϵ^2	ϵ^2	$-\epsilon^2$	$-\epsilon^2$	μ^2	μ^2	$-\mu^2$	$-\mu^2$	A_2	0
$e^{\epsilon t}$	$e^{-\epsilon t}$	$e^{i\epsilon t}$	$e^{-i\epsilon t}$	$e^{\mu t}$	$e^{-\mu t}$	$e^{i\mu t}$	$e^{-i\mu t}$	A_3	0
$\epsilon e^{\epsilon t}$	$-\epsilon e^{-\epsilon t}$	$i\epsilon e^{i\epsilon t}$	$-i\epsilon e^{-i\epsilon t}$	$\mu e^{\mu t}$	$-\mu e^{-\mu t}$	$i\mu e^{i\mu t}$	$-i\mu e^{-i\mu t}$	A_4	0
$M\epsilon^2$	$M\epsilon^2$	$-M\epsilon^2$	$-M\epsilon^2$	$N\mu^2$	$N\mu^2$	$-N\mu^2$	$-N\mu^2$	A_5	0
$M\epsilon^3$	$-M\epsilon^3$	$-iM\epsilon^3$	$iM\epsilon^3$	$N\mu^3$	$-N\mu^3$	$-iN\mu^3$	$iN\mu^3$	A_6	0
$M\epsilon^2 e^{\epsilon t}$	$M\epsilon^2 e^{-\epsilon t}$	$-M\epsilon^2 e^{i\epsilon t}$	$-M\epsilon^2 e^{-i\epsilon t}$	$N\mu^2 e^{\mu t}$	$N\mu^2 e^{-\mu t}$	$-N\mu^2 e^{i\mu t}$	$-N\mu^2 e^{-i\mu t}$	A_7	0
$M\epsilon^3 e^{\epsilon t}$	$-M\epsilon^3 e^{-\epsilon t}$	$-iM\epsilon^3 e^{i\epsilon t}$	$iM\epsilon^3 e^{-i\epsilon t}$	$N\mu^3 e^{\mu t}$	$-N\mu^3 e^{-\mu t}$	$-iN\mu^3 e^{i\mu t}$	$iN\mu^3 e^{-i\mu t}$	A_8	0

(8)

From this analytical model, the dynamic response of a damped three-layer laminated beam containing only compressional damping can be studied by examining the mechanical impedance spectrum of several beam geometries and material properties. Figure 2 shows the mechanical impedance spectra generated from this model of a selected case: a steel laminate with geometry outlined in the figure, and a viscoelastic damping core with an elastic storage modulus of $6.89 \cdot 10^6 \text{ N/m}^2$ (1000 lb-in^{-2}) and several values of elastic loss tangent. As is evident from this figure, the damping in this composite is negligible except for the spectral region between 250 and 2000 Hz centered at 500 Hz. In this region, the damping is strongly dependent

on the elastic loss tangent. This result can be anticipated from consideration of a model which treats the face layers as lumped masses and the viscoelastic layer as a complex distributed spring. This model gives rise to a compressional frequency inherent in composite working to delaminate the face layers:

$$\omega_c^* = \left[\left(\frac{E_v^*}{t_v} \right) \left(\frac{1}{\rho_1 t_1} + \frac{1}{\rho_3 t_3} \right) \right]^{1/2} = \omega_c' (1 + i\delta)^{1/2}$$

Since, according to this model, the viscoelastic layer receives the greatest dynamic compressional strains in this spectral region (which, in turn, is the primary mechanism converting vibratory energy to heat), it is to be expected that vibrational modes of the composite occurring near this frequency would exhibit a high degree of damping for high-loss viscoelastic materials.

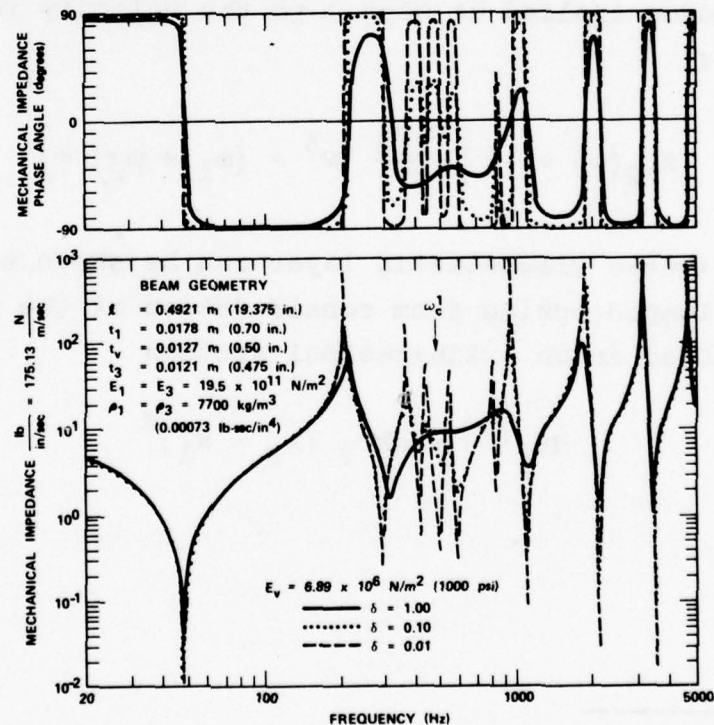


Figure 2 - Transverse Driving Point Mechanical Impedance and Phase Angle Spectrum for a Three-Layer Damped Beam, Fully Constrained Compressional Damping Model

DISTRIBUTED MASS-VISCOELASTIC SPRING-MASS SYSTEM

Since the dynamic response of the one-dimensional mass-complex viscoelastic spring-mass system can provide insight to both understand and interpret the results of the transverse compressionally damped three-layer beam models described above, as well as to evaluate the dynamic complex elastic (Young's) modulus of linear viscoelastic materials, a brief analytical development is presented below. Assuming a massless complex viscoelastic spring, the driving point mechanical impedance of mass 1 can be written as:

$$Z_{11}(\omega) = i \left(\omega m_1 + \frac{1}{\frac{1}{\omega m_2} - \frac{\omega}{k^*}} \right) \quad (9)$$

and the transfer mechanical impedance from a sinusoidal driving force applied at mass 1 to the velocity response of mass 2 as:

$$Z_{12}(\omega) = -i \left[\frac{m_1 m_2}{k^*} \omega^3 - (m_1 + m_2) \omega \right] \quad (10)$$

The distributed viscoelastic layer can be shown to behave as a simple lumped spring from consideration of the potential energy stored in an infinitesimal section

$$dV_i(v) = dk^*_i (x_2 - x_1)^2 \quad (11)$$

*The kinetic energy of the viscoelastic layer can be shown to be: $T(v) = (1/6)(\rho_v/g)t_v(\dot{x}_2 - \dot{x}_1)^2$ and then factored into the system Lagrangian to obtain modified equations of motion, if desired.

where

$$dk^* = \frac{E_v^* b dy_i}{t_v} = \frac{E_v (1 + i\delta) b dy_i}{t_v}$$

The total potential energy stored in the viscoelastic layer can be obtained from integrating along the length of the lumped system

$$V^V = \int_0^l dv_i = \frac{E_v (1 + i\delta) b l}{t_v} \quad (12)$$

so that the lumped elements of this system can be identified as:

$$k^* = \frac{E_v (1 + i\delta) b}{t_v}$$

$$m_1 = \rho_1 t_1 b \quad (13)$$

$$m_2 = \rho_2 t_2 b$$

From examination of equations (9) and (10), it is seen that the dynamic response of mass 1 exhibits an antiresonance at $\omega_A^* = \sqrt{k^*/m_2}$ and a resonance at:

$$\omega_c^* = \sqrt{\frac{k^*}{m_1} + \frac{k^*}{m_2}} \quad (14)$$

while mass 2 exhibits a resonance at the same frequency. Figure 3 graphically shows the impedance response characteristics of this system. As is easily seen, placing the lumped elements of equation (13) into the expression for the resonance compressional frequency yields

$$\omega_c^* = \left[\frac{E_v (1 + i\delta)}{t_v} \left(\frac{1}{\rho_1 t_1} + \frac{1}{\rho_2 t_2} \right) \right]^{1/2}$$

This equation identifies the spectral band where transverse compressional damping can be expected to dominate the dynamic response of distributed structures incorporating constrained viscoelastic layers.

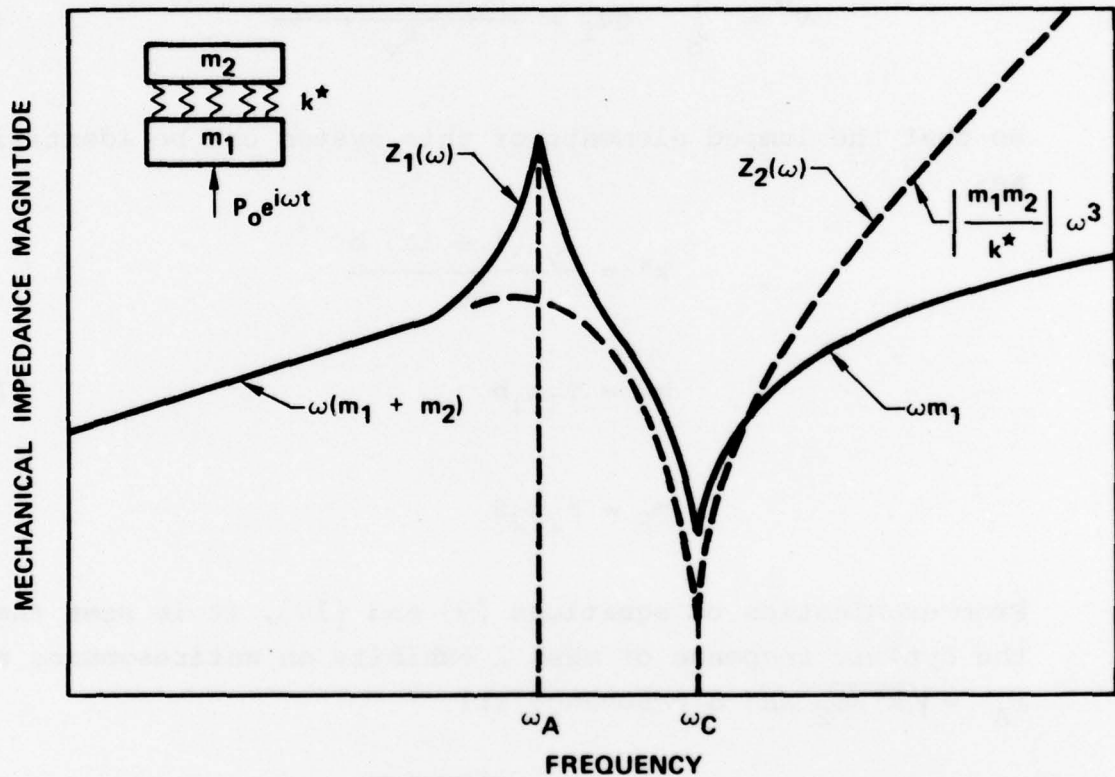


Figure 3 - The Mechanical Impedance Magnitude Response of the Distributed Mass-Viscoelastic/Spring-Mass System

SHEAR DAMPING MODEL

The equation for transverse motion for the three-layer damped laminate, based only on shear damping, was derived by Mead and Markus⁴ assuming:

1. The shear strain is constant across the depth of the damping core which is linearly viscoelastic.

2. Shear strains in the face plates and longitudinal stresses in the core are negligible.

3. Transverse direct strains in the core and face plates are negligible so that transverse displacements of all points on a beam cross section are equal.

4. The shear stress in the core acts uniformly between the midplanes of the face plates.

From these assumptions, a sixth-order partial differential equation for the damped laminate subjected to a concentrated sinusoidal loading was derived⁴ in terms of the transverse displacement variable, w :

$$\frac{\partial^6 w}{\partial w^6} - g(1 + \gamma) \frac{\partial^4 w}{\partial x^4} + \frac{m}{D_t} \frac{\partial^4 w}{\partial x^2 \partial t^2} = \frac{mg}{D_t} \frac{\partial^2 w}{\partial t^2} \quad (15)$$

where

$$g = \frac{G_v^*}{t_v} \left(\frac{1}{E_1 t_1} + \frac{1}{E_3 t_3} \right)$$

$$\gamma = \frac{d^2}{D_t} \left(\frac{E_1 t_1 E_3 t_3}{E_1 t_1 + E_3 t_3} \right)$$

$$d = t_v + 1/2 (t_1 + t_3)$$

$$D_t = \frac{(E_1 t_1^3 + E_3 t_3^3) b}{12}$$

Again, a progressive-wave solution for this equation can be written in terms of the complex wave numbers δ_1 , δ_2 , and δ_3 .

$$w(x) = c_1 e^{\delta_1 x} + c_2 e^{-\delta_1 x} + c_3 e^{\delta_2 x} + c_4 e^{-\delta_2 x} + c_5 e^{\delta_3 x} + c_6 e^{-\delta_3 x} \quad (16)$$

where

$$\delta_1 = \left[\gamma_1 + \gamma_2 - \frac{g}{3} (1 + \gamma) \right]^{1/2}$$

$$\delta_2 = \left[- \left(\frac{\gamma_1 + \gamma_2}{2} \right) + i \left(\frac{\gamma_1 - \gamma_2}{2} \right) \sqrt{3} - \frac{g}{3} (1 + \gamma) \right]^{1/2}$$

$$\delta_3 = \left[- \left(\frac{\gamma_1 + \gamma_2}{2} \right) - i \left(\frac{\gamma_1 - \gamma_2}{2} \right) \sqrt{3} - \frac{g}{3} (1 + \gamma) \right]^{1/2}$$

$$\gamma_1 = \left[- \frac{\xi_2}{2} + \left(\frac{\xi_2^2}{4} + \frac{\xi_1^3}{27} \right)^{1/2} \right]^{1/3}$$

$$\gamma_2 = \left[- \frac{\xi_1}{3\gamma_1} \right]$$

$$\xi_1 = - \frac{m\omega^2}{D_t} - \frac{1}{3} g^2 (1 + \gamma)^2$$

$$\xi_2 = \frac{m\omega^2}{D_t} g - \frac{1}{3} g (1 + \gamma) \frac{m\omega^2}{D_t} - \frac{2}{27} g^3 (1 + \gamma)$$

For cantilever end conditions, the equations of constraint require (at the free end) the moment, χ , is zero or

$$\chi = \frac{D_t}{g} \left(-\frac{\partial^4 w}{\partial x^4} + g(1 + \gamma) \frac{\partial^2 w}{\partial x^2} + \frac{m\omega^2}{D_t} w \right) = 0$$

the shear force, $S = \partial\chi/\partial x$ equal the applied concentrated force or

$$S = \frac{D_t}{g} \left(-\frac{\partial^5 w}{\partial x^5} + g(1 + \gamma) \frac{\partial^3 w}{\partial x^3} + \frac{m\omega^2}{D_t} \frac{\partial w}{\partial x} \right) = P_0 e^{i\omega t}$$

and the longitudinal face plate displacement, $u(x)$, are unrestrained. These conditions reduce to the simple form at $x = 0$:

$$\frac{\partial^2 w}{\partial x^2} = 0; \quad \frac{\partial^3 w}{\partial x^3} = \frac{P_0 e^{i\omega t}}{D_t}; \quad \text{and} \quad \frac{\partial^4 w}{\partial x^4} - \frac{m\omega^2}{D_t} w = 0$$

At the fixed end, the equations of constraint require:

$$w(l) = 0; \quad \left. \frac{\partial w}{\partial x} \right|_{x=l} = 0; \quad \text{and} \quad u(l) = 0$$

The longitudinal displacement can be described in terms of the transverse displacement for a concentrated dynamic load by the expression:

$$u(x) = \left(\frac{-D_t}{E_1 t_1 d} \right) \left[\frac{1}{g^2} \frac{\partial^5 w}{\partial x^5} - \frac{\gamma}{g} \frac{\partial^3 w}{\partial x^3} - \left(\frac{m\omega^2}{D_t g^2} + \gamma \right) \frac{\partial w}{\partial x} \right] \quad (17)$$

These equations can be placed in a matrix representation and solved for the complex coefficients, C_n , in a manner similar to that described in the compressional damping model:

$$\begin{bmatrix}
 \delta_1^3 & -\delta_1^3 & \delta_2^3 & -\delta_2^3 & \delta_3^3 & -\delta_3^3 \\
 \delta_1^2 & \delta_1^2 & \delta_2^2 & \delta_2^2 & \delta_3^2 & \delta_3^2 \\
 R_1 & R_1 & R_2 & R_2 & R_3 & R_3 \\
 e^{\delta_1 t} & e^{-\delta_1 t} & e^{\delta_2 t} & e^{-\delta_2 t} & e^{\delta_3 t} & e^{-\delta_3 t} \\
 \delta_1 e^{\delta_1 t} & -\delta_1 e^{-\delta_1 t} & \delta_2 e^{\delta_2 t} & -\delta_2 e^{-\delta_2 t} & \delta_3 e^{\delta_3 t} & -\delta_3 e^{-\delta_3 t} \\
 s_1 e^{\delta_1 t} & -s_1 e^{-\delta_1 t} & s_2 e^{\delta_2 t} & -s_2 e^{-\delta_2 t} & s_3 e^{\delta_3 t} & -s_3 e^{-\delta_3 t}
 \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} P_0/D_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

where

$$R_1 = \delta_1^4 - \frac{m\omega^2}{D_t}$$

$$R_2 = \delta_2^4 - \frac{m\omega^2}{D_t}$$

$$R_3 = \delta_3^4 - \frac{m\omega^2}{D_t}$$

$$s_1 = \frac{\delta_1^5}{g^2} - \frac{Y}{g} \delta_1^3 - \left(\frac{m\omega^2}{D_t g^2} + Y \right) \delta_1$$

$$s_2 = \frac{\delta_2^5}{g^2} - \frac{Y}{g} \delta_2^3 - \left(\frac{m\omega^2}{D_t g^2} + Y \right) \delta_2$$

and

$$s_3 = \frac{\delta_3^5}{g^2} - \frac{Y}{g} \delta_3^3 - \left(\frac{m\omega^2}{D_t g^2} + Y \right) \delta_3$$

Upon determining the C_n coefficients by matrix inversion, the mechanical impedance at an arbitrary point on the laminate can be calculated from the expression:

$$Z(x, \omega) = \frac{P_0}{i\omega} \left(c_1 e^{\delta_1 x} + c_2 e^{-\delta_1 x} + c_3 e^{\delta_2 x} + c_4 e^{-\delta_2 x} + c_5 e^{\delta_3 x} + c_6 e^{-\delta_3 x} \right)^{-1} \quad (19)$$

Previous papers⁴⁻⁶ on this subject have shown that shear damping is a broad-band phenomenon strongly dependent on the shear loss tangent of the viscoelastic layer.

EXPERIMENTAL EVALUATION

Three damped sandwich beams were constructed to serve to evaluate the relative importance of compressional and shear damping in elastic-viscoelastic-elastic beams. Two beams were designed with the compressional frequency, ω_c , located in 20- to 5000-Hz spectrum, and a third beam was designed with ω_c above 5000 Hz. The elastic face layers of all beams were constructed from steel. Specimens 1 and 2 incorporated an acrylic base viscoelastic material with a complex dynamic shear modulus in the 20- to 5000-Hz spectral region that can be

approximated (assuming thermorheologically simple material behavior) by the expression:

$$G_v = (1.42 \times 10^5) e^{0.494 \ln(\omega/2\pi)} \text{ (N/m}^2\text{)} \quad (20)$$

$$\beta = 1.46$$

Specimen 1 contained a viscoelastic layer thickness of 0.00686 m which placed its composite compressional frequency near 900 Hz, and specimen 2 contained a viscoelastic layer thickness of 0.000102 m which placed its composite compressional frequency outside the spectral range of the experiments reported herein. The complex dynamic elastic modulus used to generate the analytical compressional damping spectrum for specimen 1 was obtained from assuming incompressibility and a real Poisson ratio (i.e., $E_v = 3G_v$ and $\delta = \beta$).

Specimen 3 incorporated a medium-density, closed-cell, neoprene foam layer with a complex dynamic elastic modulus that can be approximated by the expressions:

$$E_v = 1.078 \cdot 10^5 e^{0.4041 \ln(\omega/2\pi)} \text{ (N/m}^2\text{)}$$

$$\delta = \begin{cases} 0.8 & 20 \text{ Hz} < \omega/2\pi < 60 \text{ Hz} \\ 10.47 e^{-0.628 \ln(\omega/2\pi)} & 60 \text{ Hz} < \omega/2\pi < 150 \text{ Hz} \\ 0.45 & 150 \text{ Hz} < \omega/2\pi < 500 \text{ Hz} \end{cases} \quad (21)$$

The complex dynamic elastic modulus of the neoprene foam (equation (21)) was obtained from a series of resonance mass-spring experiments. The measurements to determine the expressions in equation (20) were obtained from a commercial

apparatus which utilizes dynamic stress-strain and related phase-angle measurements. The thickness of the foam layer was 0.0127 m. The compressional frequency of this composite is located near 200 Hz.

The cantilever test fixture used to mount these beams was evaluated by comparing the measured mechanical impedance of an undamped simple beam with Bernoulli-Euler theory. Figure 4 shows the 20 to 5000 Hz mechanical impedance spectrum of a steel beam 0.0178 m thick, 0.0508 m wide, and 0.4921 m long.

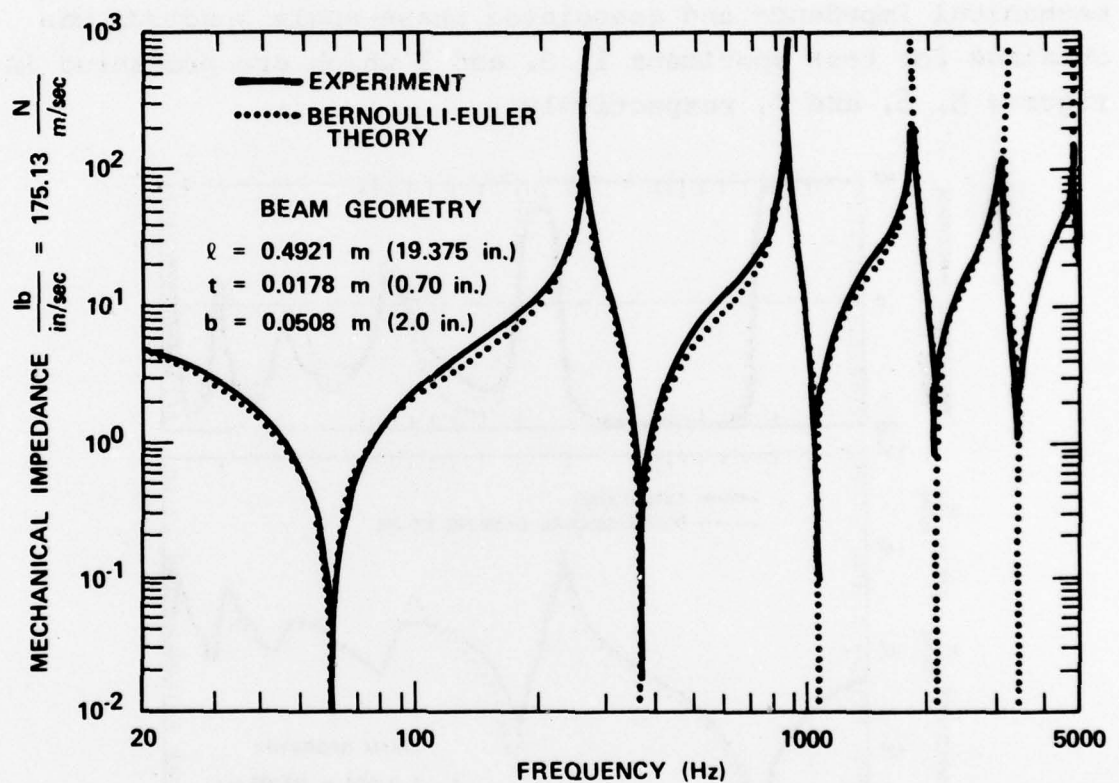


Figure 4 - Transverse Driving Point Mechanical Impedance Spectrum of an Elastic Beam, An Evaluation of Experimental Boundary Conditions and Instrumentation

With this test fixture, excellent agreement was observed between measurement and theory as to values for both resonance and antiresonance frequencies. The dynamic range between associated resonance-antiresonance pairs for the measured steel beam spectrum exceed 40 dB throughout the 20- to 5000-Hz spectrum increasing to in excess of 100 dB for the low-order vibrational modes. All measured spectra discussed in this report were obtained from transducers which had a negligible mass loading (<0.004 kg) effect on the structure.

By using the methods described above, a 20- to 5000-Hz mechanical impedance and associated phase-angle spectrum was obtained for test specimens 1, 2, and 3 which are presented in Figures 5, 6, and 7, respectively.

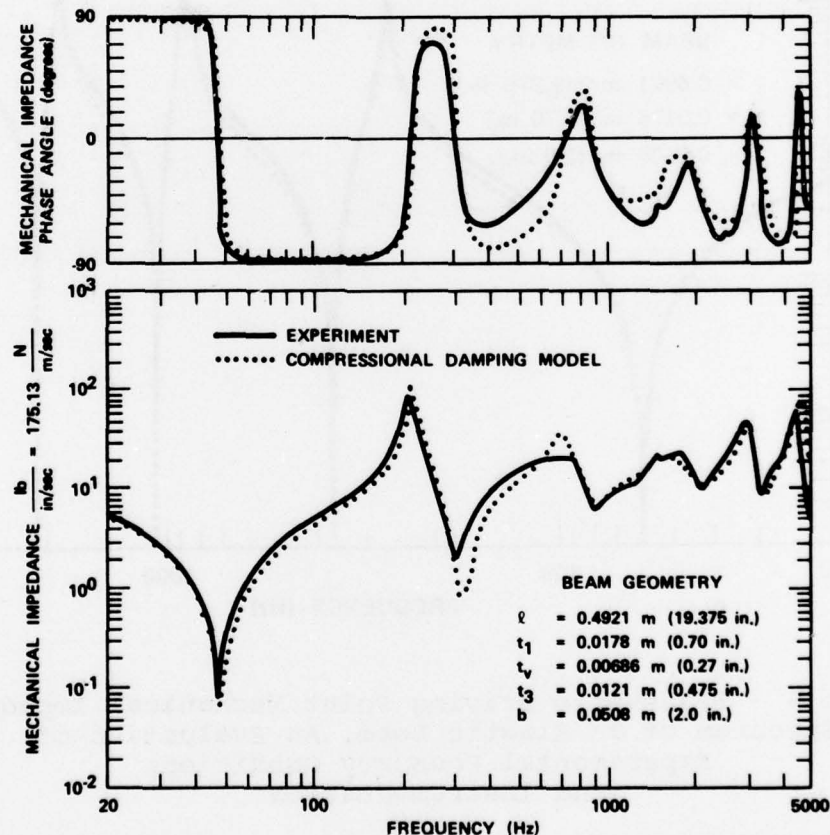


Figure 5 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 1

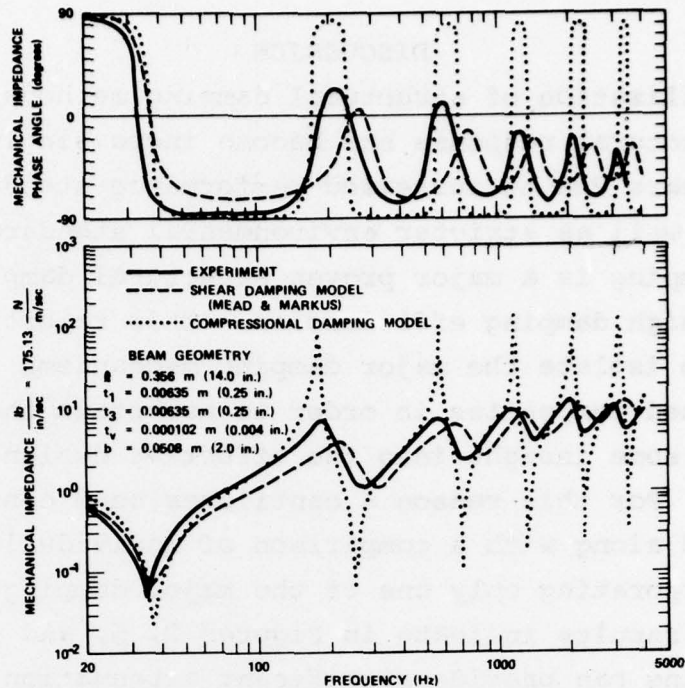


Figure 6 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 2

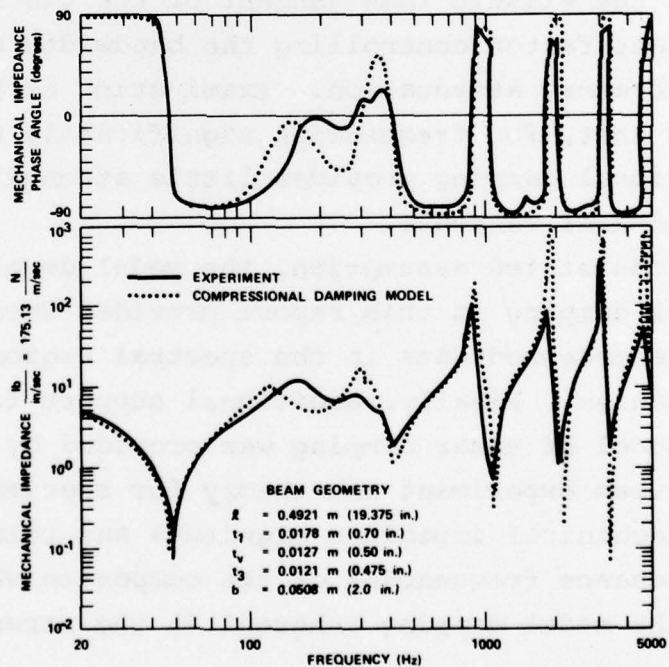


Figure 7 - Transverse Driving Point Mechanical Impedance and Phase-Angle Spectrum for the Fully Constrained Specimen 3

DISCUSSION

The utilization of structural damping methods to control dynamic structural response has become increasingly widespread in recent years due to increased performance standards for vehicles as well as stricter environmental standards. Constrained damping is a major proven structural damping technique with high damping efficiencies. This report has attempted to isolate the major damping mechanisms inherent in constrained composites in order to ascertain their potential and provide some insight into the effective design of such structures. For this reason a cantilever beam configuration was selected along with a comparison of individual analytical models incorporating only one of the major damping mechanisms.

As the results indicate in Figures 2, 5, and 7, compressional damping can provide significant attenuation in the vibrational energy of resonant structures in a narrow frequency band centered at the compressional frequency of the composite. In addition, the elastic loss tangent of the viscoelastic layer is an important factor controlling the bandwidth and amount of effective vibratory attenuation. Examination of Figure 6 demonstrates that, for frequencies significantly removed from ω_c , compressional damping provides little attenuation to the dynamic structural response.

Within the stated assumption, the model developed for compressional damping in this report provided excellent agreement with the measured data in the spectral regions governed by this mechanism. Finally, additional support for the Mead and Markus model of shear damping was provided by the agreement observed between experiment and theory for specimen 2, especially the mechanical impedance magnitude and relative bandwidth near the resonance frequencies of the composite which is a measure of the modal damping inherent in the structure.

CONCLUSIONS

It is concluded that shear damping is a broad-band mechanism which adequately, for most engineering purposes, describes the damping inherent in the transverse dynamic response of elastic-viscoelastic-elastic beams, outside the spectral influence of compressional effects. Inside this spectral band the relative displacement between the elastic layers of the composite must be considered in dynamic calculations.

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